Preventing Critical Edges when Designing Transmission Networks

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ABSTRACT

When designing an electric transmission grid, it is important to ensure that the resulting grid is reliable. In particular, it should remain operable if one piece of equipment fails ($N-1$ criterion). In this work we focus on the failure of single transmission lines. We consider a criticality measure by Witthaut et al. [21], which captures the dynamic behavior of failing lines. The criterion itself is based on maximum graph-theoretic flows in suitably defined residual networks. In a first step, we compare it to the $N-1$ criterion and find that networks without critical edges tend to satisfy the $N-1$ criterion. We then formulate the criticality measure as set of linear constraints, which may form a building block in transmission network design problems.

In particular, we introduce these constraints into a basic Transmission Network Expansion Planning (TNEP) formulation, obtaining models for the two problems Criticality-Constrained Transmission Network Expansion Planning (CC-TNEP) and Criticality Minimal Expansion (CME). We study the effects of adding these constraints on the time needed for solving the models. Moreover, we provide a simple heuristic for CME, which often finds optimal solutions but in less time.

CCS CONCEPTS

• **Mathematics of computing** → *Network flows*; • **Theory of computation** → *Algorithm design techniques*; *Mixed discrete-continuous optimization.*

KEYWORDS

Transmission Network Expansion Planning, Critical Edges, Greedy Algorithm

ACM Reference Format:
Dorothea Wagner and Matthias Wolf. 2020. Preventing Critical Edges when Designing Transmission Networks. In *The Eleventh ACM International Conference on Future Energy Systems (e-Energy'20), June 22–26, 2020, Virtual Event, Australia.* ACM, New York, NY, USA, 15 pages. https://doi.org/10.1145/3396851.3397735

e-Energy'20, June 22–26, 2020, Virtual Event, Australia

ACM ISBN 978-1-4503-8009-6/20/06…$15.00
https://doi.org/10.1145/3396851.3397735

1 INTRODUCTION

When designing a new electric transmission grid or re-designing an existing one, one strives to make the grid robust against potential equipment failures. Otherwise, a single failure may cause a collapse of large parts of the grid leading to a widespread black out. To prevent such widespread failures, one therefore has to take these equipment failures into account both during the design and the operation phase. In this work we focus on the design phase.

A widely used reliability criterion is the $N-1$ *criterion* [22]. It roughly states that the grid must be operable even if a single piece of equipment (e.g., a transmission line or a generator) fails. It is widely used both while designing [6, 13, 18] and operating transmission and distribution grids [16]. This criterion may also be extended to include the failure of up to k pieces of equipment. This is then called the $N-k$ *criterion* [17].

Typically, these criteria consider the static behavior of the grid after the failure. They do not include the dynamic behavior of, e.g., a line failure. In contrast, Witthaut et al. [21] consider the dynamic behavior of the voltage angles in transmission grids. They classify lines as either *critical* if their failure causes a widespread outage or *non-critical* if the failure still leaves the grid operable. They then develop two simple measures that try to predict for each line whether they are critical or not. The first measure bases the decision on the maximum graph-theoretic flow between the endpoints of the line in the residual network after the line is removed; see Section 3 for a formal definition. If the fraction of the power flow on the line and the value of the maximum is larger than some threshold h, then they predict the line to be critical. In their evaluation they obtain an optimal value of $h = 0.614$. The second measure is based on the linear response on small perturbations in the existing power flow. They claim that their measures more accurately predict critical edges than standard load flow analyses. In this work, we employ the first measure and present how to incorporate it into transmission network design problems, e.g., Transmission Network Expansion Planning.

1.1 Related Work

There is a large body of research on Transmission Network Expansion Planning (TNEP). An overview over the various aspects and solution methods for TNEP is given by Mahdavi et al. [15]. For a literature review focusing on the solution methods see [20]. Here, we focus on the works that incorporate reliability criteria such as the $N-1$ criterion [6, 18].

Domínguez et al. [7] present a mixed-integer linear program (MILP) formulation for TNEP including both AC and DC links considering the $N-1$ criterion. Moreover, they present a method to reduce the search space, which significantly decreases the time to solve the model. Choi et al. [6] formulate an integer programming formulation for TNEP taking the $N - k$ criterion and variations thereof into account. This means, they consider contingencies with up to k components failures. However, to reduce the complexity of the model, they ignore contingencies with probabilities below a given threshold (10^{-9} in their study). Moreira et al. [17] address the computational complexity of explicitly modeling the $N-k$ criterion differently. They formulate this problem as a trilevel mixed-integer program and solve it using Benders decomposition.

Instead of employing the deterministic $N - 1$ criterion, Shortle et al. [19] consider the stochastic nature of blackouts. They present a variant of the transmission expansion planning problem with the goal of minimizing the probability of blackouts.

Reliability criteria are not only important for designing but also for operating transmission grids. A relaxed version of the $N - 1$ criterion is proposed by Zima and Andersson [22]. They find that using this relaxed criterion during dispatch reduces the expected blackout size. Heylen et al. [11] compare six reliability criteria including the $N - 1$ criterion and various probabilistic criteria. Their case study shows that probabilistic criteria allow for lower expected total cost of reliability management.

1.2 Contribution and Outline

We present a linear program formulation of the criticality criterion by Witthaut et al. [21]. This formulation is general in the sense that it can be in included in various transmission network design problems. As an example application, we include it in a basic version of the Transmission Network Expansion Planning (TNEP) problem. This yields the two problems Criticality-Constrained Transmission Network Expansion Planning (CC-TNEP) and Criticality Minimal Expansion (CME). In the former problem, the total criticality of all edges is bounded and the expansion costs are minimized. In the latter problem, the goal is to minimize the total criticality in the expansion subject to budget constraints. We study the effect of the criticality constraints in these problems by simulations on example networks. We also compare the criticality constraints with the more standard $N - 1$ constraints both theoretically and by simulations on the example networks. For CME we present a greedy heuristic, which is both fast and gives very good results compared to solving the mixed-integer linear program formulation with Gurobi.

The paper is structured as follows. In the following section we define basic terms and models that we use throughout this work. In Section 3 we reformulate the criticality criterion by Witthaut et al. [21] and relate it to the $N - 1$ criterion in a theoretical analysis. We formulate the criticality criterion as linear constraints, which are then included in a model for TNEP resulting in models for CC-TNEP and CME. A theoretical comparison to the $N - 1$ criterion is presented in Section 3.2. We develop a greedy heuristic for CME in Section 4. In Section 5 we evaluate and compare the presented models and algorithms on example networks. We further compare the criticality criterion and the $N - 1$ criterion. We finally conclude with a summary of the results and give an outlook on potential future work on this topic in Section 6.

2 PRELIMINARIES

As input we are given a graph $G = (V, E)$, where the edge set E is partitioned into *existing edges* E_0 and *candidate edges* E_1. Vertices of the graph correspond to buses in a power grid. The existing edges are lines in the power grid, and the candidate edges correspond to lines that may be added to the power grid. For notational convenience we assume that all edges are directed but the choice of the direction is arbitrary. It has no physical significance; we only use it to define the direction of a flow on the edge. Each edge $e \in E$ further has a *capacity* $\operatorname{cap}(e) \in \mathbb{R}_{\geq 0}$ and a susceptance $b(e) \in \mathbb{R}$. The cost for building a candidate edge $e \in E_1$ is given by $\operatorname{cost}(e)$.

Additionally, we are given a set of timestamps T. For each timestamp $t \in T$ and vertex $v \in V$, the generation $g_t(v)$ and load $l_t(v)$ at v is fixed. We call the input tuple $(G, \operatorname{cap}, b, \operatorname{cost}, T, g, l)$ containing the graph G, the edge properties cap, b, cost, the timestamps T, and generation and load functions (g and l), an *instance*.

A graph $H = (V_H, E_H)$ is an *expansion* of G if $V = V_H$ and $E_0 \subseteq E_H \subseteq E$. That is, an expansion of G contains all existing edges of G and possibly some candidate edges. We call the edges in $E_H \cap E_1$ *selected*. The *cost of an expansion* H is given by the sum of the costs of the candidate edges selected by H; i.e., $\operatorname{cost}(H) := \sum_{e \in E_H \cap E_1} \operatorname{cost}(e)$. Note that the graph (V, E_0) with only the existing edges is an expansion without any selected edges. Therefore, any notion we define for expansions applies to the existing graph as well.

2.1 Power Flows

As it is often done in the context of Transmission Network Expansion Planning problems, we use a linearized version of the AC power flow, which is called the *DC power flow* [14]; see [9] for the underlying assumptions and a derivation of the power flow model.

We represent the power flow on an edge $e = (u, v) \in E$ at time $t \in T$ by $f_t(e)$. We interpret positive flow on e as flow from u to v and negative flow on e as flow from v to u. Here, the arbitrary direction of the edges comes into play.

Definition 2.1. Given an expansion $H = (V, E_H)$, a *flow* in H at time $t \in T$ is a function $f_t : E_H \to \mathbb{R}$ such that

(1) the flow is conserved at all vertices (Kirchhoff's Current Law),

$$\sum_{(x,v) \in E_H} f_t(x, v) - \sum_{(v,x) \in E_H} f_t(v, x) = l_t(v) - g_t(v) \quad \forall v \in V,$$

(2) the edge capacities are not exceeded,

$$|f_t(e)| \leq \operatorname{cap}(e) \qquad \forall e \in E_H.$$

A flow f_t is a *power flow* if additionally

(3) there is a function $\theta_t : V \to \mathbb{R}$ such that Kirchhoff's Voltage Law is satisfied, i.e.,

$$f_t(u, v) = b(u, v) \cdot (\theta_t(v) - \theta_t(u)) \qquad \forall (u, v) \in E_H.$$

The value $\theta_t(v)$ in the last condition is called the *power angle* at the vertex v. We say an expansion H *admits a power flow* at time $t \in T$ if there is a power flow in H at time t. It is known that a power flow is unique if it exists, and if the capacities are large enough there always is a power flow [4].

2.2 Transmission Network Expansion Planning

In terms of expansions a basic version of the Transmission Network Expansion Planning problem can be formulated as follows.

Definition 2.2 (Transmission Network Expansion Planning (TNEP)). Given an instance $\mathcal{N}$, find an expansion H admitting a power flow at all times $t \in T$ and that has minimum cost cost(H) among all such expansions.

In order to formulate this problem as a mixed-integer linear program, we introduce the binary variables $z(e)$ for all $e \in E_1$. We interpret $z(e) = 1$ as "the candidate e is selected in the expansion", and $z(e) = 0$ as "it is not selected in the expansion". Further, we have the continuous variables $f_t(e)$ for all $t \in T$ and all $e \in E$ representing the power flows and $\theta_t(v)$ for all $t \in T$ and all $v \in V$ representing the power angles. The conservation of flow is described by the linear equations for all $v \in V$ and $t \in T$

$$\sum_{(x,v)\in E_H} f_t(x,v) - \sum_{(v,x)\in E_H} f_t(v,x) = l_t(v) - g_t(v). \quad (1)$$

For the existing edges the capacity constraints and Kirchhoff's Voltage Law are described as in Definition 2.1. That is, we have for all $(u,v) \in E_0$ and all $t \in T$

$$|f_t(u,v)| \le \text{cap}(u,v), \quad (2)$$

$$f_t(u,v) = b(u,v)\cdot(\theta_t(v) - \theta_t(u)). \quad (3)$$

The flow on a candidate edge (u,v) has to obey the conditions of Definition 2.1 if the edge is selected in the expansion ($z(u,v) = 1$), and it must be 0 if (u,v) is not selected. We therefore have for all $(u,v) \in E_1$ and all $t \in T$

$$|f_t(u,v)| \le \text{cap}(u,v)\cdot z(u,v), \quad (4)$$

$$f_t(u,v) = z(u,v)\cdot b(u,v)\cdot(\theta_t(v) - \theta_t(u)). \quad (5)$$

Equation (4) ensures that the flow on the edges does not exceed the capacity. Equation (5) requires Kichhoff's Voltage Law to be satisfied if $z(u,v) = 1$, and places no restriction on the power angles if $z(u,v) = 0$. The latter equation is non-linear, but it can be linearized using big-M-constraints.

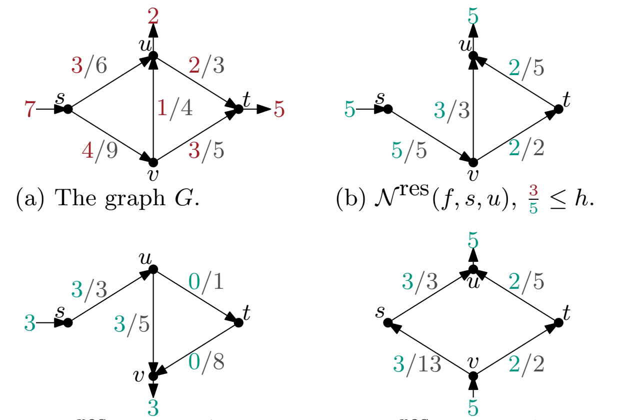
$$|f_t(u,v) - b(u,v)\cdot(\theta_t(v) - \theta_t(u))| \le M_{(u,v)}\cdot(1 - z(u,v)). \quad (6)$$

The minimal values for $M_{(u,v)}$ can be determined by computing shortest paths in G equipped with a suitable metric [3].

Now, Equations (1) to (4) and (6) represent the basic constraints for the Transmission Network Expansion Planning problem. The objective is to minimize

$$\sum_{(u,v)\in E_1} z(u,v)\cdot\text{cost}(u,v).$$

This basic version can be extended in various ways, e.g., by additionally minimizing the operation costs (e.g., [1, 2, 6]), or by considering that lines may be added over a longer time horizon [1, 2, 12]. Mahdavi et al. [15] present an overview over the various modeling possibilities for TNEP. However, such extensions are out of scope in this paper. We focus on the effect of including a criticality measure into expansion planning problems.



(a) The graph G. (b) $\mathcal{N}^{\text{res}}(f,s,u)$, $\frac{3}{5} \le h$. (c) $\mathcal{N}^{\text{res}}(f,s,v)$, $\frac{4}{3} > h$. (d) $\mathcal{N}^{\text{res}}(f,v,u)$, $\frac{1}{5} \le h$.

Figure 1: An example graph and three residual networks. The edges are marked by $f(e)/\text{cap}(e)$. The susceptance of all edges is 1. Vertices acting as generators/consumers are marked by arrows into/out of the vertices. In the residual networks only the direction of the edges carrying flow is shown, and we assume $h = 1$. (a) The graph with its power flow. (b) The residual graph $N^{\text{res}}(f,s,u)$ admits a maximum flow of $F^{\text{res}}(f,s,u) = 5 \ge 3 = h\cdot f(s,u)$. Thus, (s,u) is not critical. (c) $F^{\text{res}}(f,s,v) = 3 < 4 = h\cdot f(s,v)$. The criticality of (s,v) is 1. (d) $F^{\text{res}}(f,v,u) = 5 \ge 1$. The edge (v,u) is not critical.

3 CRITICALITY

Witthaut et al. [21] propose and evaluate simple classifiers for determining whether the failure of a line in a power grid causes the grid to desynchronize. In this work we consider a classifier based on maximal (graph-theoretic) flows in the residual networks. In the following sections we reformulate this classifier in graph theoretic terms (Section 3.1) and compare it to the $N-1$ criterion (Section 3.2). We formulate it as a set of linear constraints (Section 3.3), which we then include in a mixed-integer linear program formulation for extensions of TNEP (Section 3.4).

3.1 Formulation of the Criticality Criterion

Given an expansion $H = (V, E_H)$ let f_t be the power flow in H at time $t \in T$ and let $(u,v) \in E_H$ be an edge. The *residual network* $N^{\text{res}}(f_t,u,v)$ consists of a directed graph with vertex set $V^{\text{res}} := V$ and edge set E^{res}. For each edge $(x,y) \in E_H \setminus \{(u,v)\}$ the residual network contains the edges (x,y) and (y,x). The *residual capacity* $\text{cap}^{\text{res}}(x,y)$ of an edge $(x,y) \in E^{\text{res}}$ is $\text{cap}(x,y) - f_t(x,y)$ if $(x,y) \in E_H$ and $\text{cap}(y,x) + f_t(y,x)$ if $(y,x) \in E_H$. We further require that the flow on each directed residual edge is at least 0. That is, flow on the residual edges is only allowed in the direction of the edges. We denote the value of the maximum (graph-theoretic) flow in $N^{\text{res}}(f_t,u,v)$ from u to v by $F_t^{\text{res}}(u,v)$. Similarly, $F_t^{\text{res}}(v,u)$ is the maximum flow in $N^{\text{res}}(f_t,u,v)$ from v to u.

Figure 1 shows an example power flow and three residual networks. In Figure 1 (a) the power flow in the graph is shown. The generation and load at the vertices is indicated by arrows pointing into and coming from the vertices, respectively. Figures 1 (b), (c), and (d) show the residual networks for the edges (s, u), (s, v), and (v, u), respectively. Note that even though only one direction of each edge is shown, the residual networks actually contain edges in both directions; showing both directions and their capacities would clutter the illustration.

Definition 3.1. The *criticality* of an edge $(u, v) \in E$ at time $t \in T$ is defined by

$$\text{crit}_t(u,v) = \begin{cases} \max\{0, f_t(u,v) - h \cdot F_t^{\text{res}}(u,v)\}, & f_t(u,v) \geq 0, \\ \max\{0, -f_t(u,v) - h \cdot F_t^{\text{res}}(v,u)\}, & f_t(u,v) < 0, \end{cases}$$

where f_t is the power flow in H at time t and $h \in \mathbb{R}_{\geq 0}$ is a parameter.

The edge (u, v) is *critical* if $\text{crit}_t(u, v) > 0$. If $f_t(u, v)$ is non-negative, being critical is equivalent to $f_t(u, v)/F_t^{\text{res}}(u, v) > h$. In that sense, the parameter h represents a threshold when we classify an edge as critical.

In the example in Figure 1, we assume $h = 1$ for simplicity. For the edge (s, u) (Figure 1 (b)) we see that $f(s, u) = 3 \leq 5 = h \cdot F^{\text{res}}(f, s, u)$. Hence, (s, u) is not critical. In contrast, the edge (s, v) is critical (Figure 1 (c)). The maximum residual flow $F^{\text{res}}(f, s, v)$ is only 3, which is not sufficient as $f(s, v) = 4$. Hence, its criticality is $\text{crit}(s, v) = \max\{0, 4 - 1 \cdot 3\} = 1$.

Note that the criticality of an edge does not directly depend on the capacity of the edge but rather on the residual capacities of the other edges. Hence, criticality is different to the line congestion level, which relates the flow on an edge to the capacity of the edge.

The criticality is measured in MW and can be interpreted as the amount by which the flow on (x, y) shall be reduced until the edge is not critical anymore. From a different point of view, $\text{crit}_t(x, y)/h$ can be interpreted as by how much the maximum flow in the residual network needs to be increased until (x, y) is not critical anymore. In the example of Figure 1 (c) this means that we would need to increase the maximum flow in the residual network $N^{\text{res}}(f, s, v)$ by at least $\text{crit}(s, v)/h = 1$ in order to make (s, v) non-critical.

The *total criticality* of the expansion $H = (V, E_H)$ is defined as the sum of the criticalities over all edges and all time stamps,

$$\text{crit}(H) := \sum_{t \in T} \sum_{(u,v) \in E_H} \text{crit}_t(u, v).$$

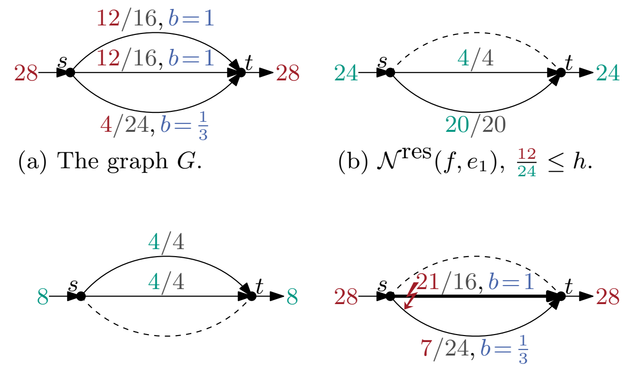
The equation above directly gives a way to compute the total criticality of an expansion H. Note that for each timestamp the power flow in H only needs to be computed once.

Lemma 3.2. *The total criticality of an expansion* (V, E_H) *can be computed in* $O(|T| \cdot (T_{PF} + |E_H| \cdot T_{MF}))$ *time, where* T_{PF} *and* T_{MF} *are the times for computing one power flow and one maximum flow, respectively.*

We observe that the value of the maximum flow in the residual graph is at least 0. If $f_t(u, v) < 0$, we therefore have $\max\{0, f_t(u, v) - h \cdot F_t^{\text{res}}(u, v)\} = 0$. Similarly, we obtain $\max\{0, -f_t(u, v) - h \cdot F_t^{\text{res}}(v, u)\} = 0$ if $f_t(u, v) \geq 0$. Hence, we can equivalently compute the criticality of an edge (u, v) as follows

Lemma 3.3.

$$\text{crit}_t(u,v) = \max \left\{ \begin{array}{l} 0, \\ f_t(u,v) - h \cdot F_t^{res}(u,v), \\ -f_t(u,v) - h \cdot F_t^{res}(v,u) \end{array} \right\}.$$



(a) The graph G. (b) $\mathcal{N}^{\text{res}}(f, e_1)$, $\frac{12}{24} \leq h$.

(c) $\mathcal{N}^{\text{res}}(f, e_2)$, $\frac{4}{8} \leq h$. (d) No power flow in $G - e_1$.

Figure 2: An example network without critical edges for $h \geq 0.5$, but removing any edge prevents a power flow. (a) The power flow f in the grid. The vertex s acts as a generator with $g(s) = 28$, and t acts as a load with $l(t) = 28$. The edges are marked by $f(e)/\text{cap}(e), b(e)$. (b) The maximum residual flow in $N^{\text{res}}(f, e_1)$ has value $24 \geq f(e_1)/h = 12/h$. (c) The maximum residual flow in $N^{\text{res}}(f, e_1)$ has value $8 \geq f(e_3)/h = 4/h$. (d) If e_1 fails there is no power flow in the resulting network because the edge e_2 is overloaded.

This equation lends itself more easily to be formulated as a linear program than the original formulation. We therefore base our linear constraints on this formulation; see Section 3.3.

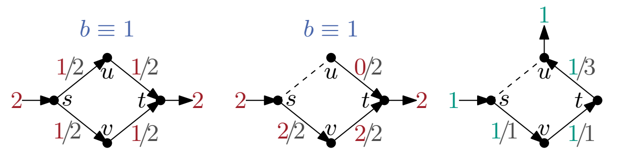
3.2 Relation to the N-1 Criterion

An expansion H satisfies the $N - 1$ criterion under edge failures if and only if removing one edge still yields a network that admits a power flow [22]. As the criticality criterion considers edge failures (and not vertex or other equipment failures), we restrict ourselves to the $N - 1$ criterion under edge failures. In the remainder of this work, we simply call this the $N - 1$ criterion without explicitly mentioning edge failures.

It is similar in to the criticality criterion by Witthaut et al. [21] in the sense that both consider the failure of one edge. Both criteria aim to establish whether such an edge failure causes the network to fail. However, in the $N - 1$ criterion only the static behavior is considered. The network after one edge failure must still admit a (static) power flow. The dynamics of the failing edge are ignored. In contrast, the criticality criterion tries to capture whether one failing edge causes the network to desynchronize.

In general, if a network satisfies one of the two criteria, it does not need to satisfy the other. Figures 2 and 3 show networks that satisfy one criterion but not the other. However, we shall see in Section 5.4 that the two criteria are related empirically.

The network in Figure 2 has no critical edge for $h \geq 0.5$. If either of the edges is removed, the maximum residual flow is twice the flow on the removed edge. The maximum residual flows in $N^{\text{res}}(f, e_1)$ and $N^{\text{res}}(f, e_3)$ are shown in Figures 2 (b) and (c); the case of removing e_2 is symmetric to the case of removing e_1. But if any of the edges fails, the remaining network does not admit a power flow. For



(a) The graph G. (b) $G - (s, u)$. (c) $\mathcal{N}^{\text{res}}(f, u, v)$.

Figure 3: An example grid, in which all edges are critical but which still admits a power flow even if one edge is removed. (a) The power flow f in the grid. The vertex s acts as a generator with $g(s) = 2$, and t acts as a load with $l(t) = 2$. The edges are marked by $f(e)/\text{cap}(e)$. All edges have susceptance 1. (b) The power flow after the edge (s, u) fails. (c) The maximum flow in the residual network $N^{\text{res}}(f, s, u)$ has value 1. For $h < 1$, the edge (s, u) is critical.

example, if e_1 fails (see Figure 2 (d)), any flow satisfying Kirchhoff's Voltage Law violates the capacity constraint of the edge e_2.

The network in Figure 3 satisfies the $N - 1$ criterion because if either of the four edges fails, the other path is sufficient to transport the required 2 units of power from s to t; see Figure 3 b. However, for $h < 1$ all edges are critical. For example, if the edge (s, u) is removed, the maximum residual flow from s to u is 1, but the required residual flow is $f(s, u)/h = 1/h > 1$.

However, in the case that we choose $h \geq 1$, then any network that satisfies the $N - 1$ criterion also satisfies the criticality criterion. To see this consider the power flows f and f' before and after an edge (u, v) is removed. As the network satisfies the $N - 1$ criterion both flows exist. We may assume without loss of generality that $f(u, v) \geq 0$. Then, the difference $f'' := f' - f$ is a flow in the residual network $N^{\text{res}}(f, u, v)$ with value $f(u, v)$. Hence, $f(u, v) \leq F^{\text{res}}(u, v) \leq h \cdot F^{\text{res}}(u, v)$ and thus (u, v) is not critical.

Note, however, that Witthaut et al. [21] empirically determined a value of $h = 0.614$ in their case study. While the optimal choice of h may depend on the graph topology, their value is far less than 1. Hence, we expect that one should choose $h < 1$ in realistic grids. This means that we are in the regime where neither the criticality criterion implies the $N - 1$ criterion nor vice versa. Hence, it may make sense to consider both criteria together.

3.3 Criticality as Linear Program

In this section we present how the criticality can be incorporated as part of a (mixed-integer) linear program. In Section 3.4 we describe how to actually incorporate these constraints in Transmission Network Expansion Planning models. The formulation of the criticality constraints is not limited to expansion planning problems. It may also be incorporated in other transmission network design problems, for example Optimal Transmission Switching [8].

The criticality of an edge (u, v) at time $t \in T$ depends on the maximum flow in the residual flow $N^{\text{res}}(f_t, u, v)$, where f_t is the power flow in the original network at time t. In $N_{\text{res}}(f_t, u, v)$ the vertices u and v act as unbounded generators and consumers. That is, the net flow at u and v is unrestricted. At all other vertices the amount of flow entering the vertex equals the flow leaving it. Note that in our model we do not explicitly set one of the vertices u and v as a consumer and the other as a load. Their roles are implicitly determined when optimizing the model. This is different to the original formulation of the criticality condition. In this formulation u acts as a generator if there is positive flow from u to v (i.e., $f_t(u, v) > 0$) and v acts as a generator if there is positive flow from v to u (i.e., $f_t(u, v) < 0$).

Suppose $f_t \colon E \to \mathbb{R}$ is a power flow in the original network. We model the criticality of an edge $e \in E$ as follows. We have one continuous variable $f^{\text{res}}_{e,t}(e')$ for each edge $e' = (u, v) \in E$, which models the residual flow on e'. As for the power flow, we interpret positive values of $f^{\text{res}}_{e,t}(u, v)$ as flow from u to v and negative values as $-f^{\text{res}}_{e,t}(u, v)$ units flowing from v to u. There further is one continuous variable $c_t(e)$ representing the criticality of the edge e.[1]

For the ease of presentation, we consider the edge e and the time t as fixed, and drop the subscripts e and t. That is, we write $f^{\text{res}}(e')$ and $c(e)$ instead of $f^{\text{res}}_{e,t}(e')$ and $c_t(e)$. In our models, all constraints below are repeated for all edges e and all time stamps t. We first model the capacity constraints for the residual flow.

$$f^{\text{res}}(e) = 0, \tag{7}$$

$$f^{\text{res}}(e') \leq \text{cap}(e') - f(e') \qquad \forall e' \in E_0, \tag{8}$$

$$f^{\text{res}}(e') \geq -\text{cap}(e') - f(e') \qquad \forall e' \in E_0, \tag{9}$$

$$f^{\text{res}}(e') \leq (\text{cap}(e') - f(e')) \cdot z(e') \qquad \forall e' \in E_1, \tag{10}$$

$$f^{\text{res}}(e') \geq (-\text{cap}(e') - f(e')) \cdot z(e') \qquad \forall e' \in E_1. \tag{11}$$

Equation (7) ensures that there is no residual flow on e since we consider the residual flow network where e is removed. Equations (8) and (9) restrict the flow on existing edges to the residual capacities. The residual capacity constraints for the candidate edges are modeled by Equations (10) and (11). This includes requiring a flow of 0 on candidate edges that are not included in the resulting expansion ($z(e') = 0$). The last two equations are non-linear, but they can be linearized in the following way.

$$f^{\text{res}}(e') \leq 2\,\text{cap}(e') \cdot z(e') \qquad \forall e' \in E_1, \tag{12}$$

$$f^{\text{res}}(e') \geq -2\,\text{cap}(e') \cdot z(e') \qquad \forall e' \in E_1, \tag{13}$$

$$f^{\text{res}}(e') \leq \text{cap}(e') - f(e') \qquad \forall e' \in E_1, \tag{14}$$

$$f^{\text{res}}(e') \geq -2\,\text{cap}(e') - f(e') \qquad \forall e' \in E_1. \tag{15}$$

The first two inequalities ensure that if the edge $e' \in E_1$ is not in the final solution ($z(e') = 0$), then $f^{\text{res}}(e') = 0$. If $z(e') = 1$, the last two inequalities ensure that the residual capacity of e' is not exceeded. Note that in this case the first two equations do not restrict $f^{\text{res}}(e')$ any further since $|f(e')| \leq \text{cap}(e')$.

As stated above we ensure that the flow is conserved at all vertices except at the endpoints u and v of e. For those two vertices we impose no restriction on their net flow.

$$\sum_{(x,w) \in E} f^{\text{res}}(x, w) - \sum_{(w,x) \in E} f^{\text{res}}(w, x) = 0 \quad \forall w \in V \setminus \{u, v\}. \tag{16}$$

Equations (8), (9) and (12) to (16) model a flow from u to v (or vice versa) in the residual network $N_{\text{res}}(f, u, v)$. So far, however, we

[1] Actually, the constraints only ensure that $c_t(e)$ is an upper bound for the criticality of e. As we minimize over $c_t(e)$, we may think of it as representing the criticality of e.

do not require any minimum flow between u and v. In particular, setting all variables $f^{\text{res}}(e')$ to 0 satisfies all constraints.

We base the constraints for the criticality of the edge e, represented by $c(e)$, on the criticality formulation in Lemma 3.3.

$$c(e) \geq 0, \tag{17}$$

$$c(e) \geq f(e) - h \cdot \left(\sum_{(x,v)\in E} f^{\text{res}}(x,v) - \sum_{(v,x)\in E} f^{\text{res}}(v,x) \right), \tag{18}$$

$$c(e) \geq -f(e) - h \cdot \left(\sum_{(x,u)\in E} f^{\text{res}}(x,u) - \sum_{(u,x)\in E} f^{\text{res}}(u,x) \right). \tag{19}$$

Here, h is the threshold used to classify an edge as critical (see Definition 3.1). These constraints ensure that $c(e)$ is at least the criticality of the edge e in the resulting expansion.

3.4 Criticality in Transmission Network Expansion Planning

Having seen how to formulate the criticality criterion as a set of linear constraints, we show how to include them in a TNEP model. There are two direct ways to modify the TNEP problem. The first way is to require that the resulting expansion does not exceed a given maximum total criticality $\text{Crit}_{\max}$.

Definition 3.4 (Criticality Constrained Transmission Network Expansion Planning (CC-TNEP)). Given an instance $\mathcal{N}$ and a maximum total criticality $\text{Crit}_{\max} \in \mathbb{R}_{\geq 0}$, find an expansion H with $\text{crit}(H) \leq \text{Crit}_{\max}$ that admits a feasible power flow and has minimum costs.

Alternatively, we can directly consider the total criticality of the expansion as our objective.

Definition 3.5 (Criticality Minimial Expansion (CME)). Given an instance $\mathcal{N}$ and a budget $\text{Cost}_{\max} \in \mathbb{R}_{\geq 0}$, find an expansion H with $\text{cost}(H) \leq \text{Cost}_{\max}$ that admits a feasible power flow and has minimum total criticality.

Adding Equations (7) to (19) to the basic TNEP problem and requiring that the resulting total criticality is at most $\text{Crit}_{\max} \in \mathbb{R}_{\geq 0}$, i.e.,

$$\sum_{t\in T} \sum_{e\in E} c_t(e) \leq \text{Crit}_{\max}, \tag{20}$$

we obtain an MILP-formulation of the Criticality Constrained Transmission Network Expansion Planning (CC-TNEP) problem; see Appendix A for a presentation of all constraints together. Recall that in any solutions the values of the variables $c_t(e)$ are just an upper bound for $\text{crit}_t(e)$. But with Equation (20) this implies that the total criticality of the resulting expansion is at most $\text{Crit}_{\max}$.

Similarly, we can model Criticality Minimal Expansion as an MILP, taking the basic TNEP-constraints as well as the criticality constraints (Equations (7) to (19)). Different to CC-TNEP, we require the total cost of the expansion to be bounded by $\text{Cost}_{\max}$, i.e.,

$$\sum_{e\in E_1} z(e) \cdot \text{cost}(e) \leq \text{Cost}_{\max}, \tag{21}$$

and we minimize the total criticality, which is

$$\sum_{t\in T} \sum_{e\in E} c_t(e). \tag{22}$$

A full presentation of the model is given in Appendix B. Note that as we minimize over the sum of the variables $c_t(e)$, we have $c_t(e) = \text{crit}_t(e)$ in any optimal solution. Hence, the objective value of an optimal solution can directly be interpreted as the criticality of the resulting expansion.

4 A GREEDY HEURISTIC FOR CRITICALITY MINIMAL EXPANSION

In addition to the MILP formulation for Criticality Minimal Expansion we develop a simple greedy heuristic. We start with the network $N_0 = (V, E_0)$, which contains all existing edges but no candidate edges. The initial remaining budget r_0 is $\text{Cost}_{\max}$. At each step i, we have an expansion $N_i = (V, E_i')$ and budget r_i. We then determine which candidate reduces the total criticality the most if it is added to N_i. To this end, we consider each candidate edge $e \in E_1$ that has not been selected in the expansion N_i, i.e., $e \notin E_i'$. If $\text{cost}(e) > r_i$, the candidate edge is too expensive, and we ignore it. Otherwise, we compute the total criticality of the expansion $(V, E_i \cup \{e\})$.

Afterwards, if there is a candidate that reduces the total criticality, we choose the edge e for which the total criticality $\text{crit}(V, E_i \cup \{e\})$ is minimal. We then set $N_{i+1} := (V, E_i' \cup \{e\})$ and $r_{i+1} := r_i - \text{cost}(e)$. If all candidates are either too expensive or do not reduce the total criticality, we stop and return N_i as the resulting expansion. In particular, if we have reached an expansion without critical edges, we stop since no expansion further reduces the total criticality.

Note that it may happen that an expansion does not admit a power flow because the power flow would violate some capacity constraints. If this happens during the check whether added an edge is worthwhile, we simply ignore this expansion. However, if the initial network N_0 already does not admit a feasible power flow, we proceed differently. We define the *total capacity violation* $\text{viol}(V, E')$ of an expansion (V, E') by

$$\text{viol}(V, E') := \sum_{t\in T} \sum_{e\in E'} \max\{0, |f_t'(e)| - \text{cap}(e)\}, \tag{23}$$

where f_t' is a power flow in N' at time t except that it may violate some edge capacity constraints. We now proceed similar to the main part of the algorithm. But instead of minimizing the total criticality, we greedily minimize the total capacity violation. If we reach a point where the resulting expansion admits a feasible power flow, we switch to greedily minimizing the total criticality.

Lemma 4.1. *The greedy algorithm runs in $O(|E_1|^2 \cdot T_{crit})$ time, where T_{crit} is the time needed for computing the total criticality of an expansion.*

Proof. As in each iteration one candidate edge is added, there are at most $|E_1|$ iterations. In each iteration we compute the total criticalities of at most $|E_1|$ expansions, which each takes time T_{crit}. ◻

Table 1: The properties of the networks in the evaluation. The candidate edges are parallel to the existing edges.

Country	$\|V\|$	$\|E_0\|$	Country	$\|V\|$	$\|E_0\|$
AT	23	29	IE	8	12
BE	27	32	NL	33	40
BG	12	17	NO	41	65
CH	15	23	PT	16	22
CZ	21	35	RO	17	27
DK	11	11	SE	46	72
HR	6	6	SI	4	4
HU	13	21	SK	9	13

5 EVALUATION

Based on the theoretical analysis of the models we formulate hypotheses that guide our evaluation. These hypotheses are then verified or falsified empirically on 16 sample networks that are extracted from the data available in PyPSA [5]. Each network is a clustered version of the transmission grid of one European country. The networks have between 4 and 46 vertices, and between 4 and 72 edges; see Table 1 for details. There is a candidate edge parallel to each existing edge, i.e., the total number of edges in the graph is twice the number of existing edges. The data for the maximum generation and load at each vertex are available in hourly resolution over the course of one year. We restricted our evaluation to four days. To alleviate the impact of seasonal and weekly variation, we chose one Tuesday in winter (22 January 2013) and the following Sunday (27 January) and the same in summer (16 July, 21 July).

In the data, however, only the maximum generation is available and not the actual generation. But different generation distributions induce different power flows and thus different criticality values. To exclude the impact of different generation distributions, we fixed the generations based on a merit order principle. We assigned the required power to the cheapest generators. While an optimal power flow [9] may be more desirable from an optimization point of view, it has the disadvantage that it depends on the network topology. Since the topology changes when adding edges, one would have to re-calculate the optimal power flow. This makes the optimization more complex. Moreover, using the merit order is realistic in the sense that it is used to decide which generators are active, e.g., in the European Union [10].

The models and the heuristic presented above are able to deal with multiple timestamps. We were interested how the number of timestamps considered together affects the solution. Therefore, we split the 96 timestamps in total in groups of k timestamps each, with $k \in \{1, 2, 3, 4, 6, 12, 24\}$. That is, for each network we had $96/k$ groups. For the criticality threshold parameter h, we use $h = 0.614$, which is the value Witthaut et al. [21] determined as optimal in their case study.

We implemented our algorithms in C++17, compiled with GCC 8.2.1, and used Gurobi 9.0.0 to solve the mixed-integer linear programs. The algorithms were executed on a server with 64-bit architecture, four 12-core AMD CPUs running at 2.1 GHz, 256 GB of RAM under openSUSE Leap 15.1. We ran tests on 40 instances in parallel, but each algorithm was only allowed to use a single thread.

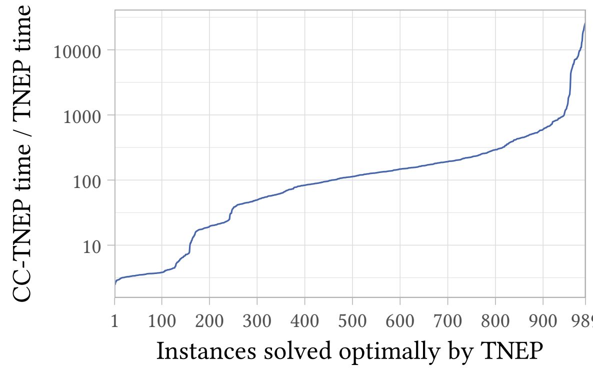


Figure 4: The ratio of the solution times for CC-TNEP to the solution times for TNEP on the 989 instances with one timestamp where both TNEP and CC-TNEP found solutions. The instances are sorted by increasing ratio.

The latter was done to ensure a fair comparison between Gurobi and our heuristic, which is unable to utilize multiple threads.

We gave the Gurobi one hour time to solve the models. In cases where this was not sufficient to prove optimality or that the instance is infeasible, we report the best solution found within this time frame.

5.1 TNEP vs. CC-TNEP

In a first step we assess the effect of including the criticality constraints in the TNEP model. To this end we compare solving the plain TNEP model to solving the CC-TNEP model. For CC-TNEP, we choose $\mathrm{Crit}_{\max} = 0$, which means that we require the resulting expansions to have not critical edges. We expect the solver to obtain the optimal results faster for the TNEP model than for the CC-TNEP model as the CC-TNEP model in more complex.

Hypothesis 1. *Optimal solutions for TNEP can be obtained faster than optimal solutions for CC-TNEP.*

The number of constraints grows quadratically in the number of edges for CC-TNEP but only linearly for TNEP. Hence, we expect the ratio between the solution times for CC-TNEP and TNEP to grow with increasing graph sizes.

Hypothesis 2. *The ratio between the solution times for CC-TNEP and TNEP increases with increasing number of edges.*

To verify or falsify these hypotheses, we compare solving TNEP and CC-TNEP on the instances with one timestamp. There are 96 timestamps for each of the 16 countries. Hence, we have 1536 instances in total. Out of these, there are 336 instances where no expansion admits a power flow. That is, TNEP (and consequently CC-TNEP) has no feasible solution. For one other instance Gurobi was not able to find any solution in one hour. Ignoring these instances, we have 1199 instances left. For all these instances, Gurobi was able to find optimal solutions for TNEP. Out of these 1199 instances, 209 instances do not admit an expansion without any critical edges. Additionally, there is one instance for which Gurobi was unable to find any feasible solution of CC-TNEP in one hour.

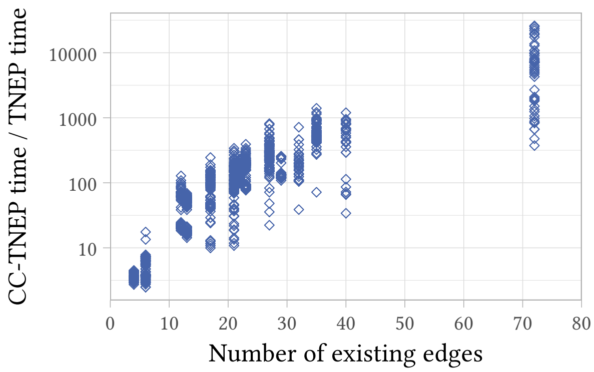


Figure 5: The ratio of the solution times for CC-TNEP to TNEP plottet against the number of existing edges. Each mark corresponds to one instance. Only instances with one timestamp where both TNEP and CC-TNEP found solutions are shown.

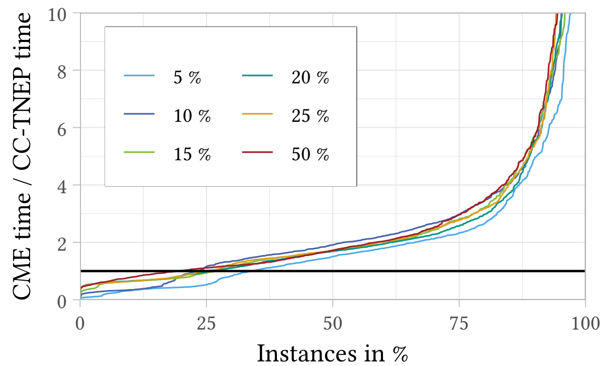


Figure 6: The ratio of the solution times for CME vs. CC-TNEP on instances with one timestamp. For each budget, the instances are sorted by increasing ratio.

This leaves us with 989 instances for which Gurobi found optimal solutions of both TNEP and CC-TNEP.

In the following analysis, we focus on how long it takes until the optimal solution is found by Gurobi and not until Gurobi can actually prove optimality. We therefore use the *solution time*, which we define as the time when Gurobi found the optimal solution.

Figure 4 shows the ratio of solution times for CC-TNEP and for TNEP. The instances are sorted by increasing ratio. The minimum solution time ratio is about 2.5, which means that Gurobi takes more than twice as long on all instances. The median and maximum ratios are 111.5, and 25940. Plots for more than one timestamp have a similar shape; see Appendix C. These findings confirm Hypothesis 1.

In absolute terms, however, both models could be solved quite fast. The solution times for TNEP are all below one second (with a maximum of 670 ms). For CC-TNEP slightly more than half of the instances (528, 53.4 %) could be solved within one second, and only 49 instances (5.0 %) needed more than one minute.

When we plot the solution time ratio in comparison to the number of edges (Figure 5), we can clearly see a trend that this ratio becomes larger the more edges there are. This observation confirms Hypothesis 2.

5.2 CC-TNEP vs. CME

Criticality Minimal Expansion has a budget as an additional parameter. For our tests, we set the budget to certain fractions (5 %, 10 %, 15 %, 20 %, 25 %, 50 %) of the total costs of all candidate edges.

The mixed-integer linear programs for CC-TNEP and CME are similar as the objective for one problem has a hard bound in the other problem and vice versa. We therefore expect the running times to be similar.

Hypothesis 3. *On the same instances the running times for CME and CC-TNEP are the same.*

To evaluate this hypothesis, we analyze those instances for which Gurobi found a feasible solution within one hour for both problems. These are between 958 instances for a 5 %-budget and 984 instaces for both 10 %- and 15 %-budgets. We plot the ratio of the solution times of Gurobi for CME vs. CC-TNEP in Figure 6. Each line shows the solution time ratio for one the budgets; the instances are sorted by these ratios individually for each line. We can see that except for the 5 % budget, CME can be solved faster than CC-TNEP on slightly less than 25 % of the instances. But on most instances solving CC-TNEP is faster. On 40 % of the instances the ratio exceeds 2, compared to only 8.0 % of the instances with a ratio below 0.5. We therefore conclude that solving CME is on average slower than solving CC-TNEP and therefore reject Hypothesis 3

5.3 Evalution of the Greedy Heuristic for CME

We now compare solving the CME model with Gurobi to running the greedy heuristic presented in Section 4. When Gurobi is given enough time, we are guaranteed to find the optimal solution if the instance is feasible. The greedy algorithm, however, does not have this guarantee. It may even fail to find any feasible solution if the existing graph does not admit a power flow. But as the greedy heuristic tries to deal with this case, we nevertheless expect it to usually find feasible solutions.

Hypothesis 4. *There are instances where Gurobi finds a solution but the greedy algorithm does not.*

Moreover, we expect the greedy algorithm to find good solutions in much less time than Gurobi.

Hypothesis 5. *The greedy algorithm finds optimal solutions on the majority of instances.*

Hypothesis 6. *The greedy algorithm is faster than Gurobi.*

The running time of the greedy algorithm only linearly depends on the number of timestamps. We therefore expect the time advantage of the greedy heuristic compared to Gurobi to increase if the number of timestamps increases.

Hypothesis 7. *The time advantage of the greedy algorithm compared to Gurobi increases with the number of timestamps.*

Similarly, the running time of the greedy algorithm only indirectly depends on the budget. If it finds a solution with total

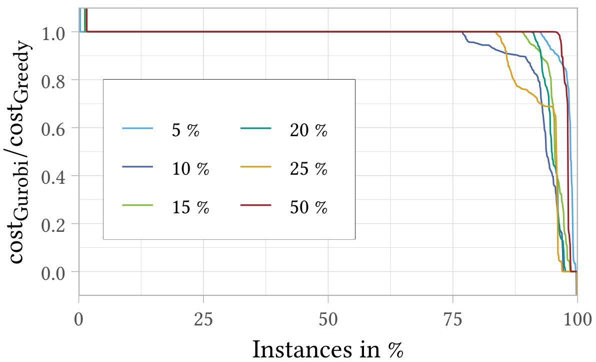


Figure 7: The ratio of the total criticalities of the expansions computed by Gurobi for CME vs. the greedy algorithm on the instances with one timestamp. Instances where neither method found a solution are omitted. For each budget, the instances are sorted by decreasing ratio.

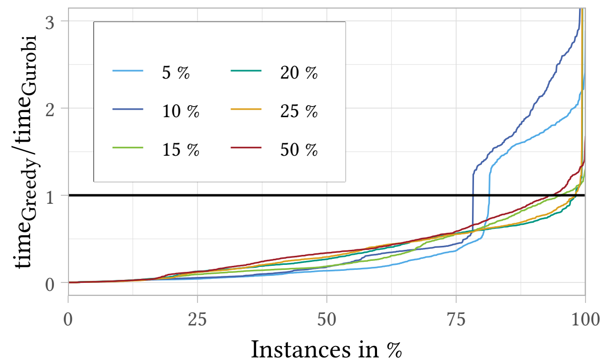


Figure 8: The ratio of solution times for the greedy algorithm and Gurobi solving CME on instances with one timestamp. Only the instances where both solution methods found the same result are plotted. For each budget, the instances are sorted by increasing ratio.

criticality 0 early, it stops independent on the budget available. However, if more budget is available the space of feasible solutions becomes larger. This may impede Gurobi in finding good solutions.

Hypothesis 8. *The time advantage of the greedy algorithm compared to Gurobi increases with the budget.*

Among the 9216 instances with one timestamp (over all six budget choices) there are 2221 infeasible instances and 9 instances where neither Gurobi nor the greedy algorithm found any solution. We use the remaining 6986 instances in the following analysis. Gurobi solved 6914 instances (99.0 %) optimally within one hour. There are 7 instances (0.1 %) where Gurobi found a solution (the optimal solution in all cases) but the greedy algorithm was unable to find any feasible solution. Conversely, there are 43 instances (0.6 %), where the greedy algorithm found a feasible solution but Gurobi did not. On most instances both Gurobi and the greedy algorithm found a solution. In fact, on 86.6 % of the instances they computed expansions with the same criticality. This observation is also visible in Figure 7, where the ratio of the total criticalities for the expansions computed by the greedy algorithm and Gurobi are plotted. Each line corresponds to one budget. For each budget choice, the instances are sorted by decreasing ratios. Values below 1 mean that the expansion computed by Gurobi has a smaller total criticality than the one by the greedy algorithm. It is clearly visible that for most instances the ratio is 1. This observation confirms Hypotheses 4 and 5. That is, the greedy algorithm is competitive to the MILP solver Gurobi in terms of resulting total criticality.

To assess the running time, we plot the ratio of the solution times of the greedy algorithm to the solution times of Gurobi. As being fast but providing much worse results is not useful, we consider only those instances where Gurobi and the greedy algorithm provide expansions with the same total criticality; see Figure 8. As before, each line in the plot corresponds to one budget choice. We observe that the greedy algorithm is faster on 91.0 % of the instances over all budget choices, but for small budgets this portion is notably smaller: 81.4 % for a 5 %-budget and 78.2 % for a 10 %-budget. On slightly more than half of the instances (51.0 %) the greedy algorithm is faster by a factor of at least 4; and on 28.4 % of the instances it is faster by a factor of at least 10. These results confirm Hypothesis 6. Regarding the dependence on the budget, however, our results only support Hypothesis 8 insofar as there are more instances for smaller budgets where the greedy algorithm is slower. For well over 75 % of the instances, the budget does not make any significant difference.

For a larger number of timestamps, the time advantage of the greedy algorithm increases. For three timestamps, it is faster by a factor of 10 on already more than half of the instances (59.4 %). This portion raises to 66.7 % for instances with six timestamps; see also Appendix D. Hence, we confirm Hypothesis 7.

5.4 N-1 Criterion and Criticality

In Section 3.2 we observe that in general the $N-1$ criterion and the criticality criterion do not imply each other. However, both criteria require redundancy in the resulting expansion. Hence, we expect expansions without critical edges to often satisfy the $N-1$ criterion as well. In an expansion H, we call an edge e *vital* if $H - e$ does not admit a power flow. Hence, an expansion satisfies the $N - 1$ criterion if and only if it has no vital edges. The *fraction of vital edges* of an expansion H is the number of vital edges divided by the total number of edges in H.

Hypothesis 9. *An optimal solution to CC-TNEP with* $\mathrm{Crit}_{max} = 0$ *tends to satisfy the* $N - 1$ *criterion, i.e., its fraction of vital edges is close to* 0.

Compared to the cost-minimal expansions (i.e., optimal solutions of TNEP), we expect cost-minimal expansions with total criticality 0 to have fewer vital edges.

Hypothesis 10. *If* H_{TNEP} *and* $H_{CC\text{-}TNEP}$ *are optimal solutions for TNEP and CC-TNEP, then the fraction of vital edges in* $H_{CC\text{-}TNEP}$ *is at most the fraction of vital edges in* H_{TNEP}.

To verify or falsify these hypotheses, we consider the expansions that result from CC-TNEP and compare them to cost-minimal expansions, which result from solving TNEP. Expansions for instances

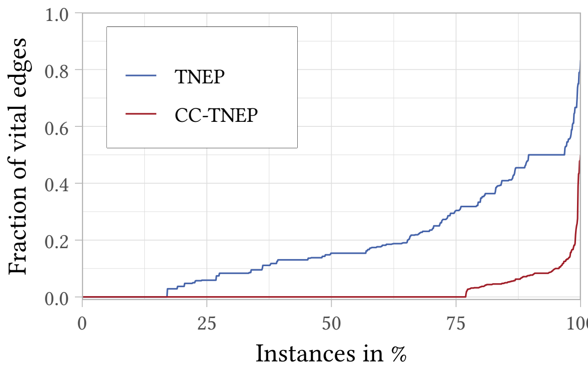


Figure 9: The fraction of vital edges in optimal expansions as computed by solving TNEP and CC-TNEP. Only instances with one timestamp are shown.

with multiple timestamps at once tend to include more candidate edges than the expansions for the individual timestamps. Hence, they should be more likely to have few vital edges. In this sense, the instances with only one timestamp are the hardest instances. We therefore focus on those. As in Section 5.1, we only consider the 989 instances solved by Gurobi for both TNEP and CC-TNEP.

We find that there is no instance for which the fraction of vital edges is larger in the expansion computed by CC-TNEP than in the cost-minimal expansion. Hence, we maintain Hypothesis 10. In Figure 9 each line corresponds to the expansions computed by solving either TNEP or CC-TNEP. For each line the instances are sorted by their fraction of vital edges. Note, however, that the values shown at the same x-coordinate do not necessarily correspond to the same instance as the instances are sorted per curve. We observe that for 76.9 % of the instances, the expansion computed by CC-TNEP has no vital edge. That is, the expansion satisfies the $N-1$ criterion. For TNEP, this holds only for 17.1 % of the instances. Hence, we conclude that requiring no critical edges tends to result in expansions that satisfy the $N-1$ criterion. We thus confirm Hypothesis 9.

6 CONCLUSION

In this paper we present how to extend any (mixed-integer) linear program formulation of any transmission network design problem by the criticality criterion introduced by Witthaut et al. [21]. To this end we formulate the criticality criterion as a set of linear constraints. These may then be used as a building block when formulating transmission network optimization problems. To introduce those constraints only variables (or constants) representing the power flow on each edge and at each timestamp are needed.

As an example we analyzed the effects of adding the criticality criterion to a basic version of the Transmission Network Expansion Planning (TNEP) problem. We formulated two problems: Criticality-Constrained Transmission Network Expansion Planning (CC-TNEP), where the total criticality is bounded by a hard constraint, and Criticality Minimal Expansion (CME), where the criticality is minimized. We further present a greedy heuristic for CME, which is both fast and produces optimal solutions in more than 75 % of the instances. We further observed that minimizing the criticality subject to a budget constraint subject to a budget constraint (CME) seems to be harder than minimizing the cost subject to a maximum criticality (CC-TNEP).

While models including the criticality criterion take longer to solve, the resulting networks are much more reliable. In particular, they tend to satisfy the $N-1$ criterion as evidenced by our simulations. Hence, one may also consider using the criticality criterion instead of (or in addition to) the $N-1$ criterion in transmission network design problems.

It would be interesting to include the criticality constraints in more comprehensive variants of TNEP; for example, by including the operation costs in the optimization criterion or by considering expansions over a longer time scale. Moreover, the criticality constraints can be incorporated in other transmission network optimization problems, e.g., Optimal Power Flow [9] or Optimal Transmission Switching [8]. One may analyze the applicability of the criticality criterion in online settings such as transmission grid operation. One could also investigate larger networks or analyze the relation to other reliability criteria such as the $N-k$ criterion [17].

ACKNOWLEDGEMENTS

This work was funded by the Helmholtz Association Program Storage and Cross-linked Infrastructures, Topic 6 Superconductivity, Networks and System Integration and by the German Research Foundation (DFG) as part of the Research Training Group GRK 2153: Energy Status Data – Informatics Methods for its Collection, Analysis and Exploitation.

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A MILP-FORMULATION OF CC-TNEP

Minimize

$$\sum_{e \in E_1} \operatorname{cost}(e) \cdot z(e)$$

such that

$$\begin{aligned}
l_t(v) - g_t(v) &= \sum_{(x,v)\in E} f_t(x,v) - \sum_{(v,x)\in E} f_t(v,x) && \forall v \in V, t \in T,\\
f_t(u,v) &= b(u,v) \cdot (\theta_t(v) - \theta_t(u)) && \forall (u,v) \in E_0, t \in T,\\
|f_t(e)| &\leq \operatorname{cap}(e) && \forall e \in E_0, t \in T,\\
|f_t(u,v) - b(u,v) \cdot (\theta_t(v) - \theta_t(u))| &\leq M_{(u,v)} \cdot (1 - z(u,v)) && \forall (u,v) \in E_1, t \in T,\\
|f_t(e)| &\leq \operatorname{cap}(e) \cdot z(e) && \forall e \in E_1, t \in T,\\
\sum_{t\in T}\sum_{e\in E} c_t(e) &\leq \mathrm{Crit}_{\max}\\
f^{\mathrm{res}}_{e,t}(e) &= 0 && \forall e \in E, t \in T,\\
f^{\mathrm{res}}_{e,t}(e') &\leq \operatorname{cap}(e') - f_t(e') && \forall e \in E, e' \in E_0, t \in T,\\
f^{\mathrm{res}}_{e,t}(e') &\geq -\operatorname{cap}(e') - f_t(e') && \forall e \in E, e' \in E_0, t \in T,\\
f^{\mathrm{res}}_{e,t}(e') &\leq 2\operatorname{cap}(e') \cdot z(e') && \forall e \in E, e' \in E_1, t \in T,\\
f^{\mathrm{res}}_{e,t}(e') &\geq -2\operatorname{cap}(e') \cdot z(e') && \forall e \in E, e' \in E_1, t \in T,\\
f^{\mathrm{res}}_{e,t}(e') &\leq \operatorname{cap}(e') - f_t(e') && \forall e \in E, e' \in E_1, t \in T,\\
f^{\mathrm{res}}_{e,t}(e') &\geq -2\operatorname{cap}(e') - f_t(e') && \forall e \in E, e' \in E_1, t \in T,\\
0 &= \sum_{(x,w)\in E} f^{\mathrm{res}}_{e,t}(x,w) - \sum_{(w,x)\in E} f^{\mathrm{res}}_{e,t}(w,x) && \forall e = (u,v) \in E, w \in V \setminus \{u,v\}, t \in T,\\
c_t(e) &\geq 0, && \forall e \in E, t \in T,\\
c_t(e) &\geq f_t(e) - h \cdot \left(\sum_{(x,v)\in E} f^{\mathrm{res}}_{e,t}(x,v) - \sum_{(v,x)\in E} f^{\mathrm{res}}_{e,t}(v,x) \right), && \forall e = (u,v) \in E, t \in T,\\
c_t(e) &\geq -f_t(e) - h \cdot \left(\sum_{(x,u)\in E} f^{\mathrm{res}}_{e,t}(x,u) - \sum_{(u,x)\in E} f^{\mathrm{res}}_{e,t}(u,x) \right), && \forall e = (u,v) \in E, t \in T,\\
f_t(e) &\in \mathbb{R} && \forall e \in E, t \in T,\\
\theta_t(v) &\in \mathbb{R} && \forall v \in V, t \in T,\\
c_t(e) &\in [0, \infty) && \forall e \in E, t \in T,\\
f^{\mathrm{res}}_{e,t}(e') &\in \mathbb{R} && \forall e \in E, e' \in E, t \in T,\\
z_t(e) &\in \{0, 1\} && \forall e \in E_1, t \in T.
\end{aligned}$$

B MILP-FORMULATION OF CME

Minimize

$$\sum_{t\in T}\sum_{e\in E} c_t(e)$$

such that

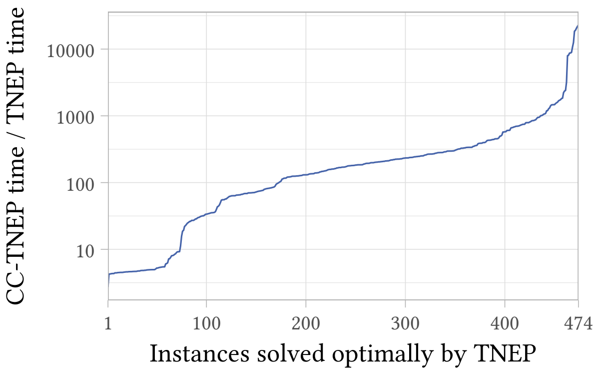
$$\begin{aligned}
l_t(v) - g_t(v) &= \sum_{(x,v)\in E} f_t(x,v) - \sum_{(v,x)\in E} f_t(v,x) && \forall v \in V, t \in T,\\
f_t(u,v) &= b(u,v)\cdot(\theta_t(v)-\theta_t(u)) && \forall (u,v) \in E_0, t \in T,\\
|f_t(e)| &\le \mathrm{cap}(e) && \forall e \in E_0, t \in T,\\
|f_t(u,v) - b(u,v)\cdot(\theta_t(v)-\theta_t(u))| &\le M_{(u,v)}\cdot(1-z(u,v)) && \forall (u,v) \in E_1, t \in T,\\
|f_t(e)| &\le \mathrm{cap}(e)\cdot z(e) && \forall e \in E_1, t \in T,\\
\sum_{e\in E_1} \mathrm{cost}(e)\cdot z(e) &\le \mathrm{Cost}_{\max}
\end{aligned}$$

$$\begin{aligned}
f^{\mathrm{res}}_{e,t}(e) &= 0 && \forall e \in E, t \in T,\\
f^{\mathrm{res}}_{e,t}(e') &\le \mathrm{cap}(e') - f_t(e') && \forall e \in E, e' \in E_0, t \in T,\\
f^{\mathrm{res}}_{e,t}(e') &\ge -\mathrm{cap}(e') - f_t(e') && \forall e \in E, e' \in E_0, t \in T,\\
f^{\mathrm{res}}_{e,t}(e') &\le 2\,\mathrm{cap}(e')\cdot z(e') && \forall e \in E, e' \in E_1, t \in T,\\
f^{\mathrm{res}}_{e,t}(e') &\ge -2\,\mathrm{cap}(e')\cdot z(e') && \forall e \in E, e' \in E_1, t \in T,\\
f^{\mathrm{res}}_{e,t}(e') &\le \mathrm{cap}(e') - f_t(e') && \forall e \in E, e' \in E_1, t \in T,\\
f^{\mathrm{res}}_{e,t}(e') &\ge -2\,\mathrm{cap}(e') - f_t(e') && \forall e \in E, e' \in E_1, t \in T,\\
0 &= \sum_{(x,w)\in E} f^{\mathrm{res}}_{e,t}(x,w) - \sum_{(w,x)\in E} f^{\mathrm{res}}_{e,t}(w,x) && \forall e=(u,v) \in E, w \in V\setminus\{u,v\}, t \in T,
\end{aligned}$$

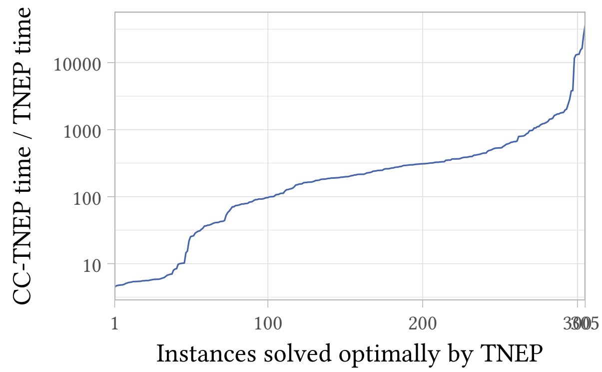
$$\begin{aligned}
c_t(e) &\ge 0, && \forall e \in E, t \in T,\\
c_t(e) &\ge f_t(e) - h\cdot\left(\sum_{(x,v)\in E} f^{\mathrm{res}}_{e,t}(x,v) - \sum_{(v,x)\in E} f^{\mathrm{res}}_{e,t}(v,x)\right), && \forall e=(u,v) \in E, t \in T,\\
c_t(e) &\ge -f_t(e) - h\cdot\left(\sum_{(x,u)\in E} f^{\mathrm{res}}_{e,t}(x,u) - \sum_{(u,x)\in E} f^{\mathrm{res}}_{e,t}(u,x)\right), && \forall e=(u,v) \in E, t \in T,
\end{aligned}$$

$$\begin{aligned}
f_t(e) &\in \mathbb{R} && \forall e \in E, t \in T,\\
\theta_t(v) &\in \mathbb{R} && \forall v \in V, t \in T,\\
c_t(e) &\in [0,\infty) && \forall e \in E, t \in T,\\
f^{\mathrm{res}}_{e,t}(e') &\in \mathbb{R} && \forall e \in E, e' \in E, t \in T,\\
z_t(e) &\in \{0,1\} && \forall e \in E_1, t \in T.
\end{aligned}$$

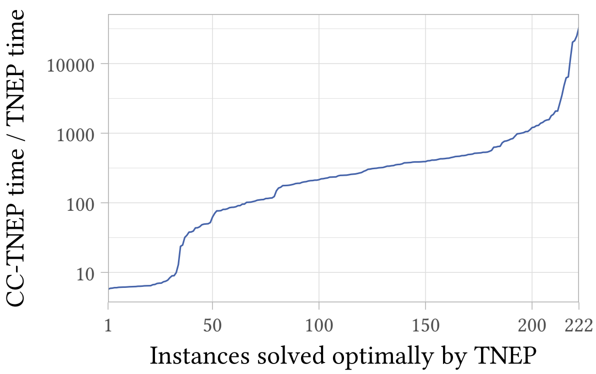
C PLOTS FOR CC-TNEP VS. TNEP



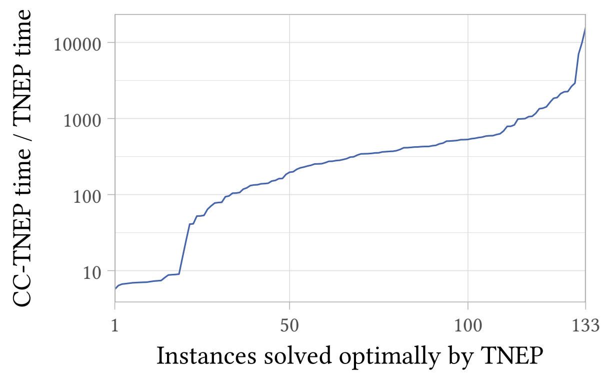
(a) Two timestamps.



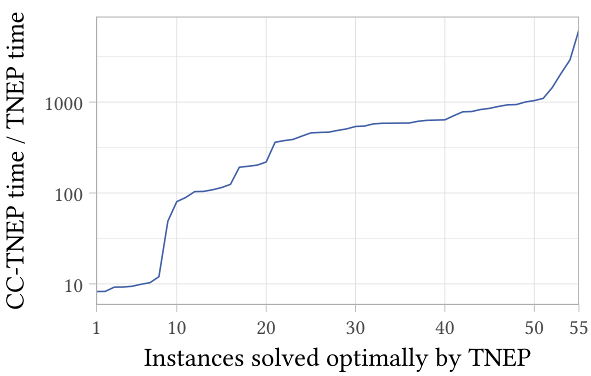
(b) Three timestamps.



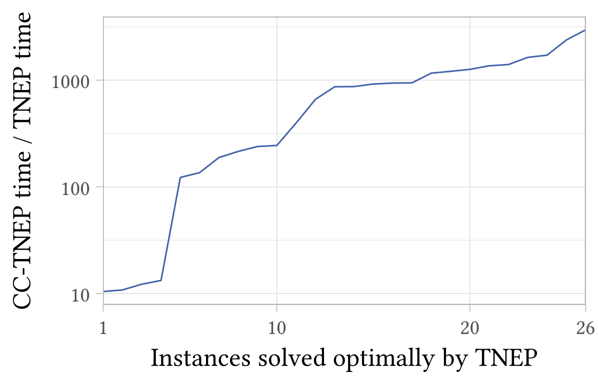
(c) Four timestamps.



(d) Six timestamps.



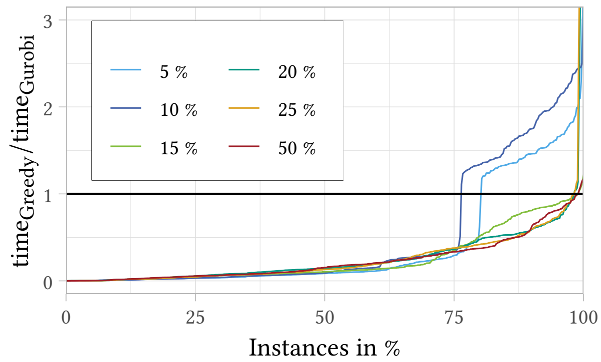
(e) Twelve timestamps.



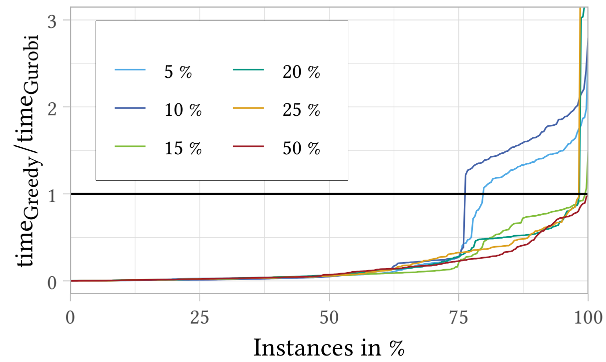
(f) 24 timestamps.

Figure 10: The ratio of the solution times for CC-TNEP to TNEP on instances where both TNEP and CC-TNEP found solutions. Each plot shows the results for instances with a fixed number of timestamps. The instances are sorted by increasing ratio. See also Figure 4.

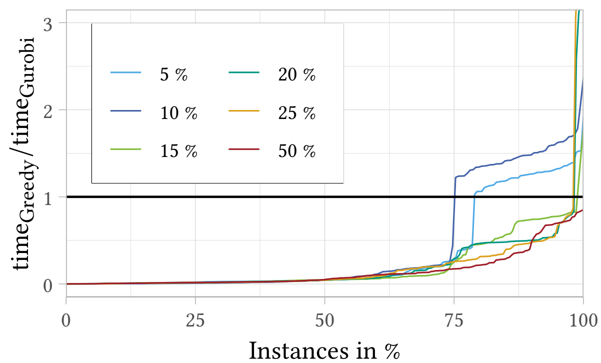
D PLOTS FOR GREEDY VS. GUROBI FOR CME



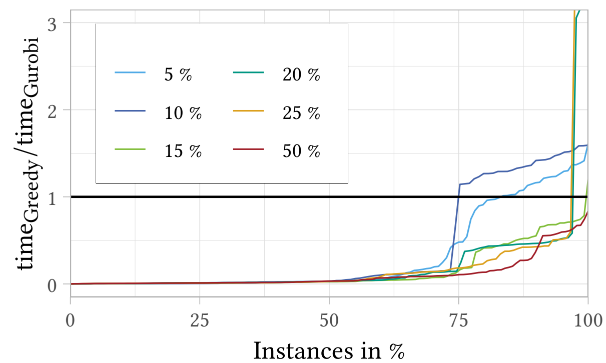
(a) Two timestamps.



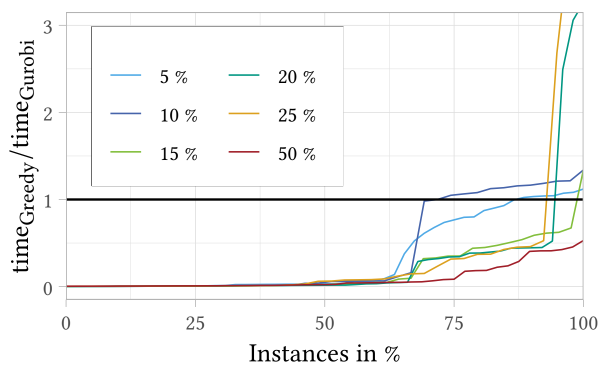
(b) Three timestamps.



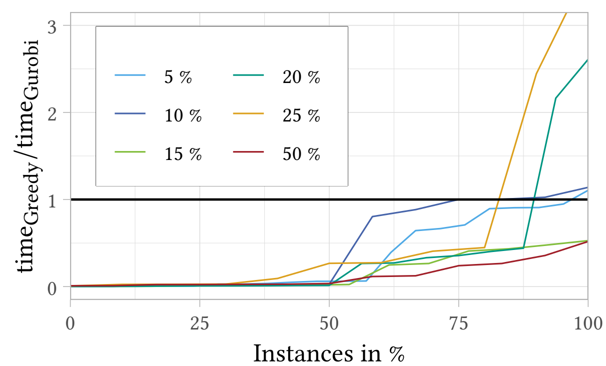
(c) Four timestamps.



(d) Six timestamps.



(e) Twelve timestamps.



(f) 24 timestamps.

Figure 11: The ratio of solution times for the greedy algorithm and Gurobi solving CME. Only the instances where both solution methods found the same result are plotted. Each plot shows instances with the same number of timestamps. For each budget, the instances are sorted by increasing ratio. See also Figure 8.