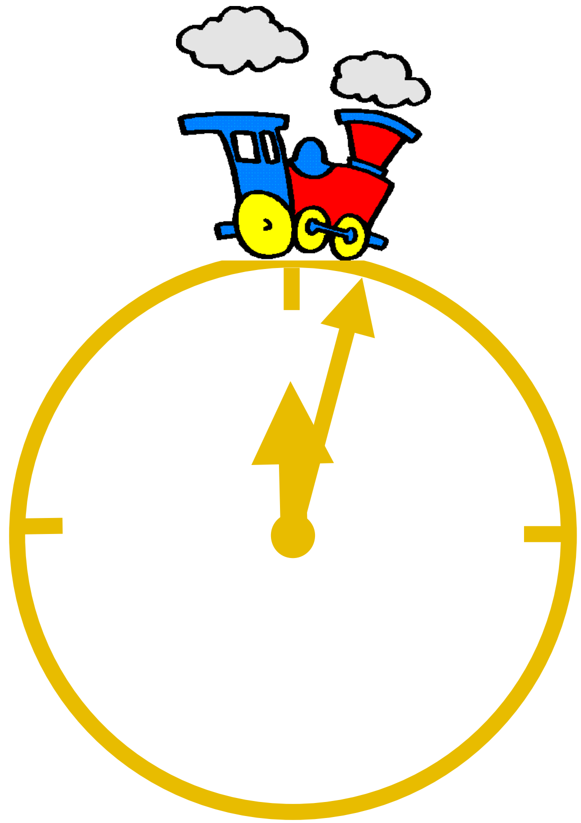
Cyclic Railway Timetabling & Cycle Bases of Graphs



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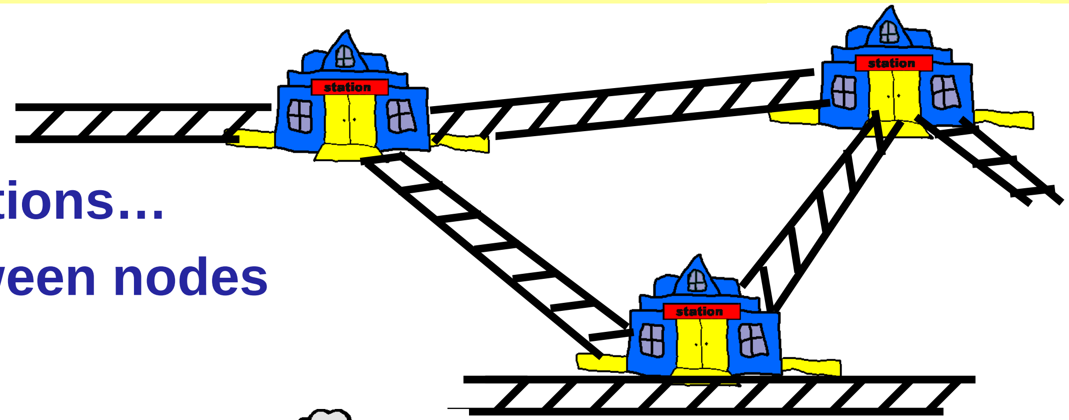
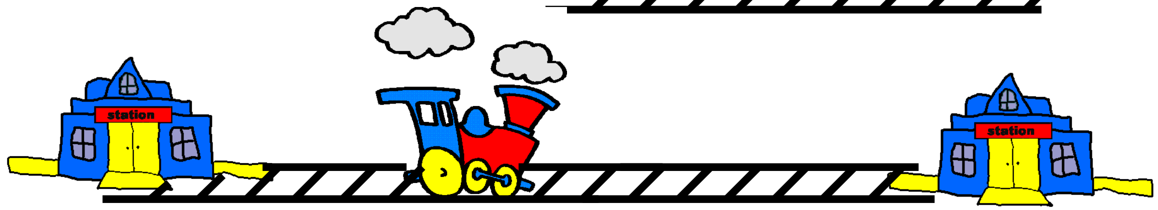
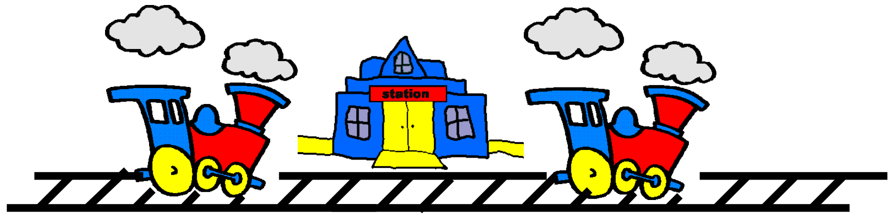
Assumptions



- Infrastructure
  - nodes: stations, junctions…
  - eventless tracks between nodes
- Train lines
  - route & type
  - frequency
  - fixed trip times



- Various requirements
  - safety system
  - commercial wishes
  - logistic restrictions

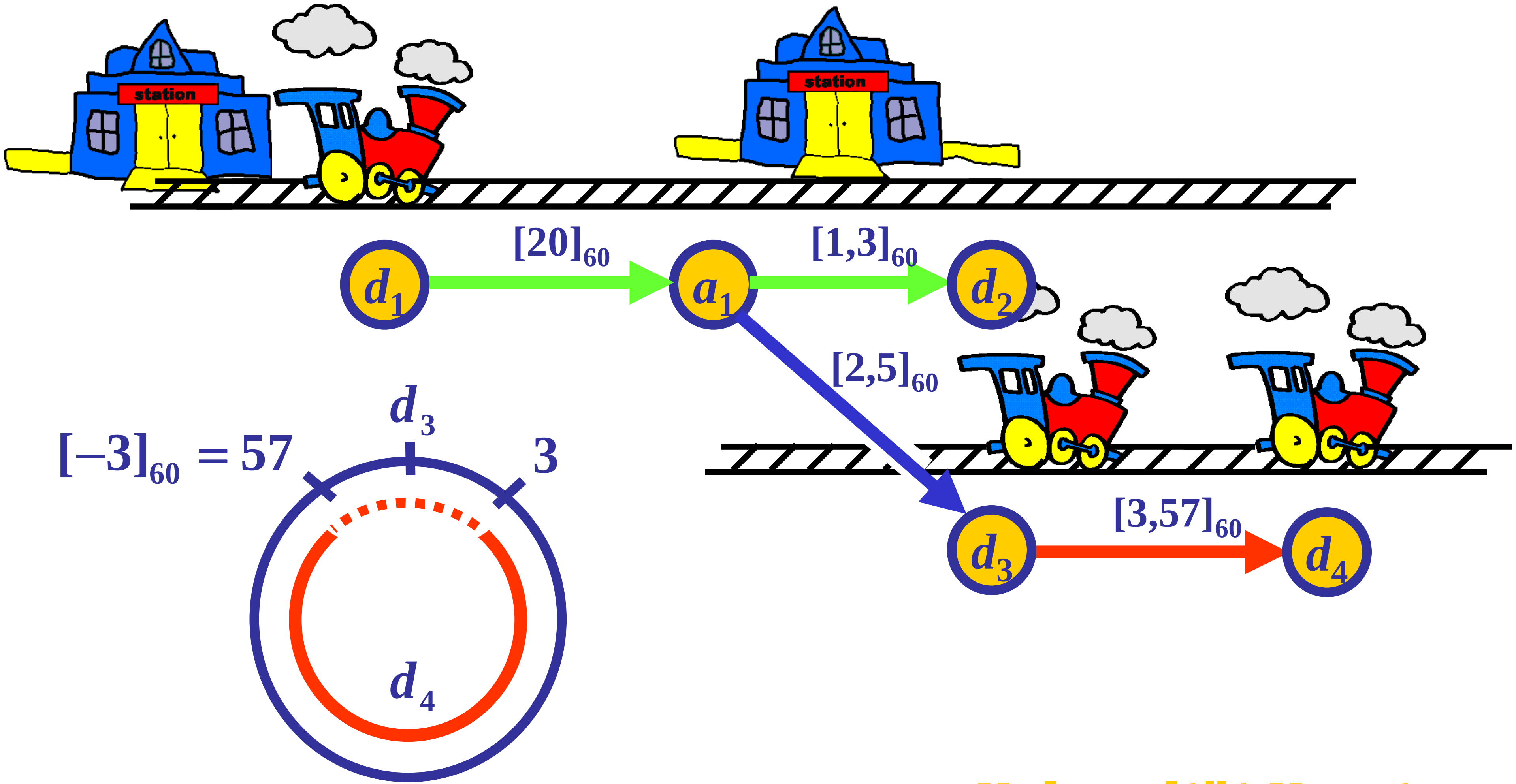


Problem Definition

- Construct a timetable:
  - for each train, and
  - for every visited station,
  - compute arrival and departure times
- Cyclic timetable with cycle time T (60 or 120 minutes)
- Example: IC Rotterdam – Leiden – Schiphol Airport

•	06 : 02	07 : 02	08 : 01	22 : 01	23 : 01
05 : 31	06 : 31	07 : 31	08 : 31	22 : 31	23 : 31
•	•	07 : 42	08 : 42	22 : 42	•

Example Constraints



Periodic Potentials and Tensions

- Periodic constraints with a potential v :

$$l_{ij} \le v_j - v_i + Tp_{ij} \le u_{ij}$$

- Periodic tension x :

$$x_{ij} = v_j - v_i + Tp_{ij}$$

- Theorem: x is a periodic tension if and only if

$$\sum_{a \in C^+} x_a - \sum_{a \in C^-} x_a = Tq_C$$

for all cycles $C \in G$

Cycle Formulation

- **Schrijver, Nachtigall (1999)**
- **Find a solution** (x, q)

satisfying

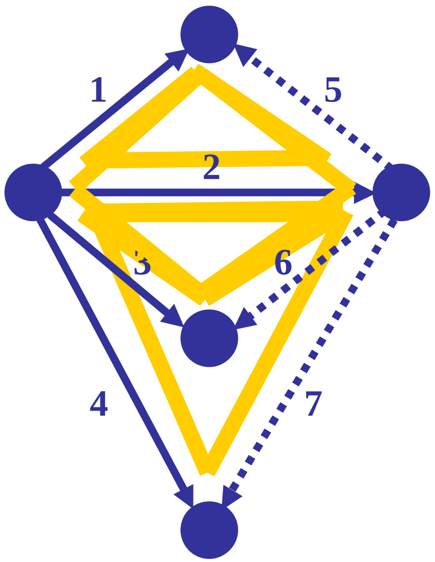
$$\sum_{a \in C^+} x_a - \sum_{a \in C^-} x_a = T q_C \qquad \forall C \in G$$

$$l_a \leq x_a \leq u_a \qquad \forall a \in A$$

$$x \in \mathbf{R}^m$$

$$q \in \mathbf{Z}^c$$

Cycle Bases of Graphs



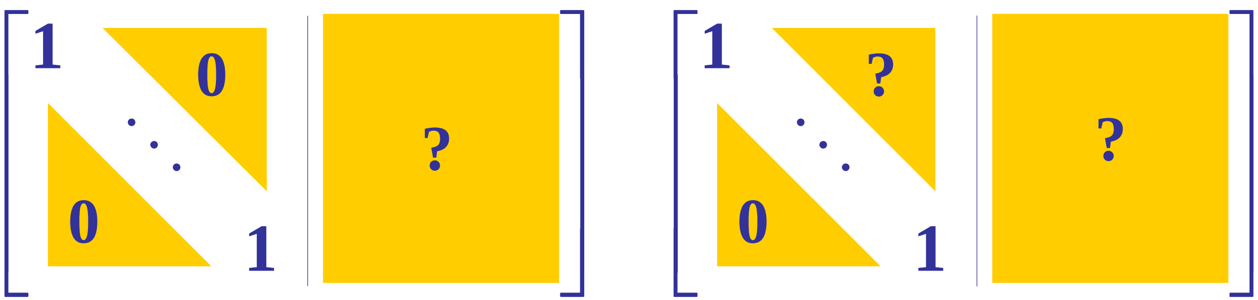
$$\begin{array}{c} \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \left[\begin{array}{cccc|ccc} -1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \end{array}$$

$$\gamma_4 \begin{bmatrix} 1 & 0 & -1 & 0 & -1 & 1 & 0 \end{bmatrix}$$

$$\gamma_4 = -\gamma_1 + \gamma_2$$

Cycle Bases

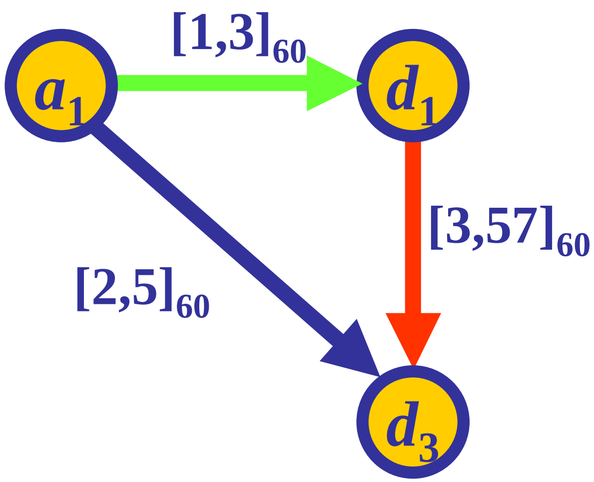
- Cycle basis of G has dimension $k = m - n + 1$
- We need integer cycle bases
- Strictly fundamental cycle basis corresponds to a spanning tree
- For a fundamental cycle basis, there exists some ordering such that: $C_{i+1} \setminus (C_i \cup \cdots \cup C_1) \neq \emptyset \quad \forall i$



Odijk's Cycle Bounds (1998)

- Bounds on the cycle integer variables:

$$a_C = \left\lceil \frac{1}{T} \sum_{a \in C^+} l_a - \sum_{a \in C^-} u_a \right\rceil \le q_C \le \left\lfloor \frac{1}{T} \sum_{a \in C^+} u_a - \sum_{a \in C^-} l_a \right\rfloor = b_C$$



$$a_C = \left\lceil \frac{1+3-5}{60} \right\rceil = \left\lceil \frac{-1}{60} \right\rceil = 0$$

$$b_C = \left\lfloor \frac{3+57-2}{60} \right\rfloor = \left\lfloor \frac{58}{60} \right\rfloor = 0$$

Constructing a Good Cycle Basis

- Ideally, cycle basis minimizing

$$W = \prod_{C \in B} (b_C - a_C + 1) \Rightarrow \sum_{C \in B} \log(b_C - a_C + 1)$$

- Heuristic: forget about rounding and minimize

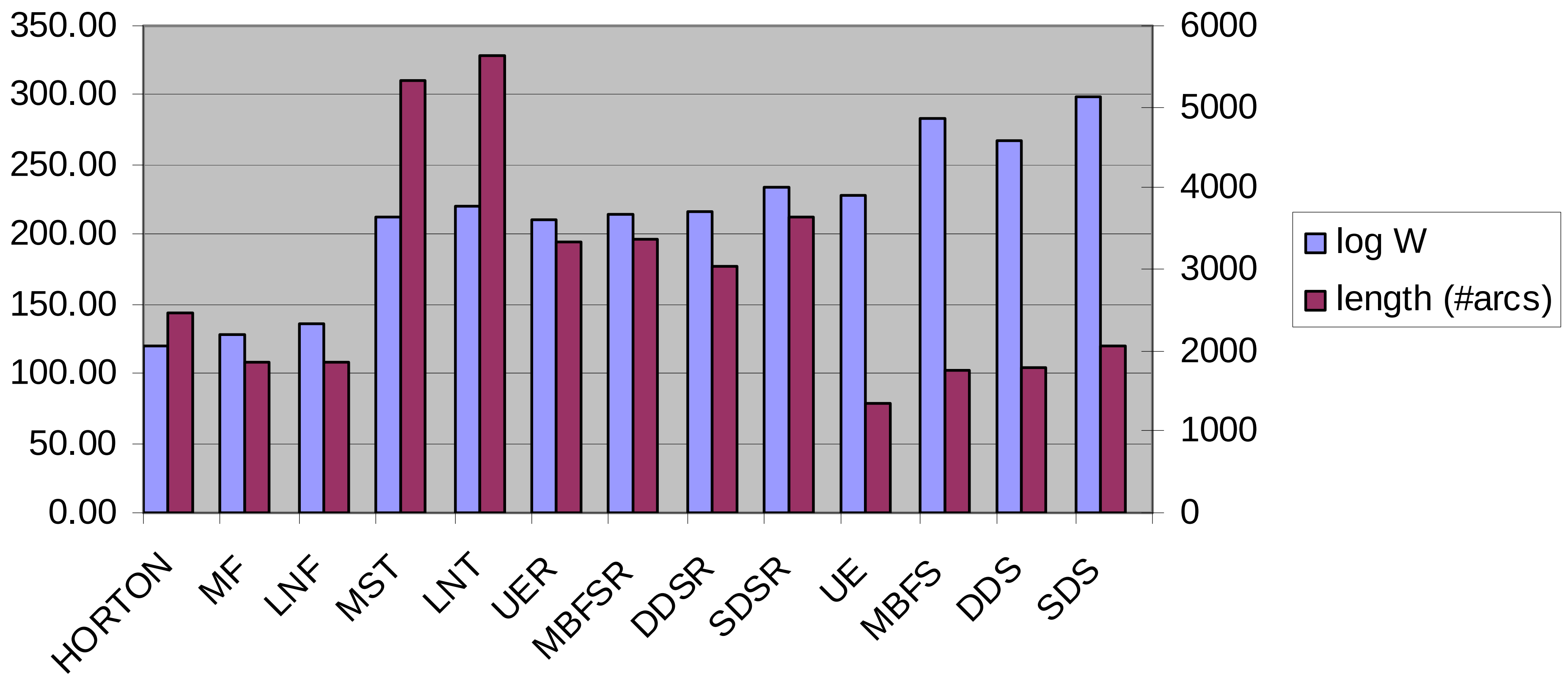
$$\sum_{C \in B} \sum_{a \in C} (u_a - l_a)$$

- Polynomial algorithm by Horton (1987)
- Minimum strictly fundamental cycle basis is NP-complete (Deo et al, 1982)

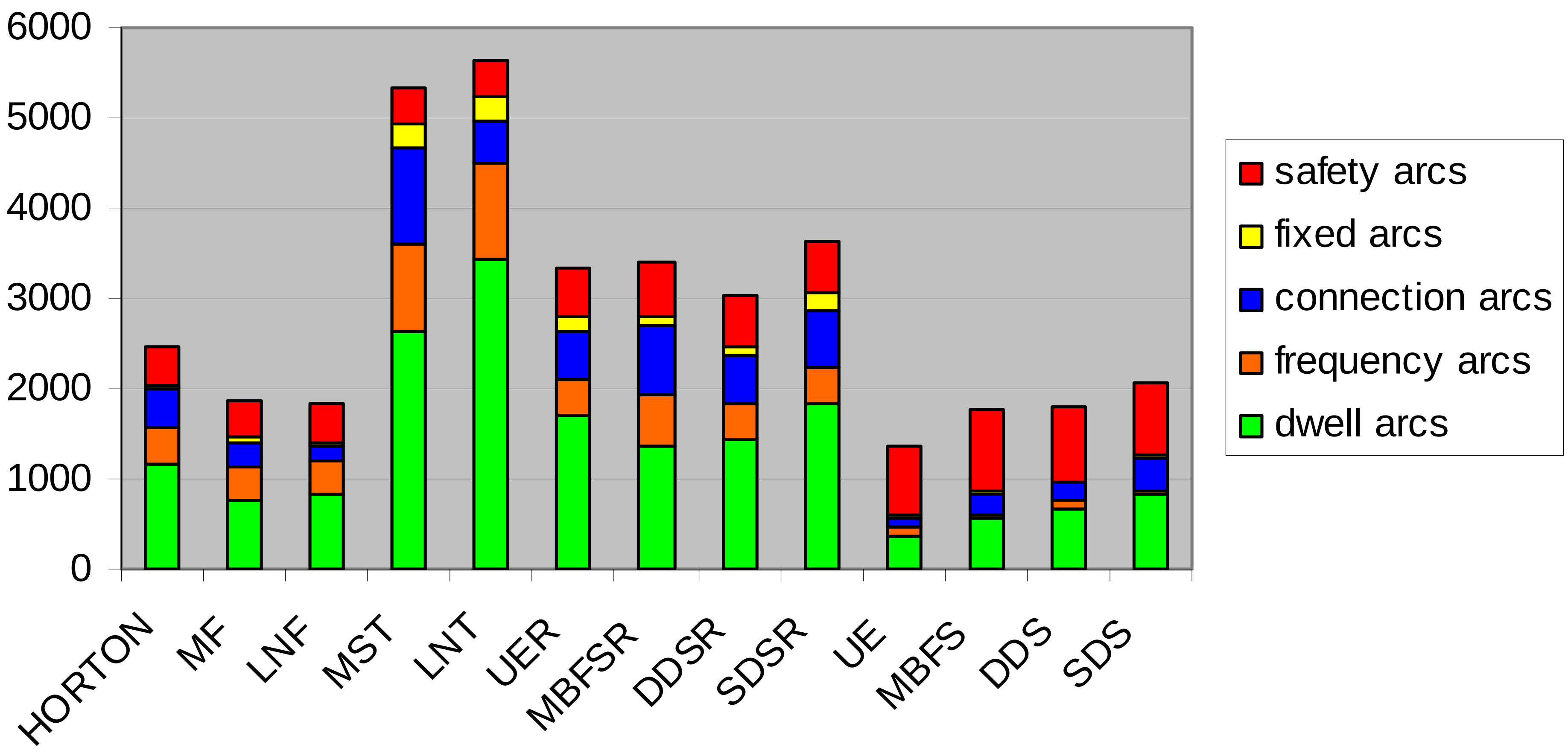
Cycle Basis Algorithms

- Horton (1987)
- Deo et al (1982): DDS, SDS, UE, MBFS
- MST with respect to edge weights $(u_a - l_a)$
- MST with respect to adjusted edge weights
- Fundamental improvements to MST (Berger)

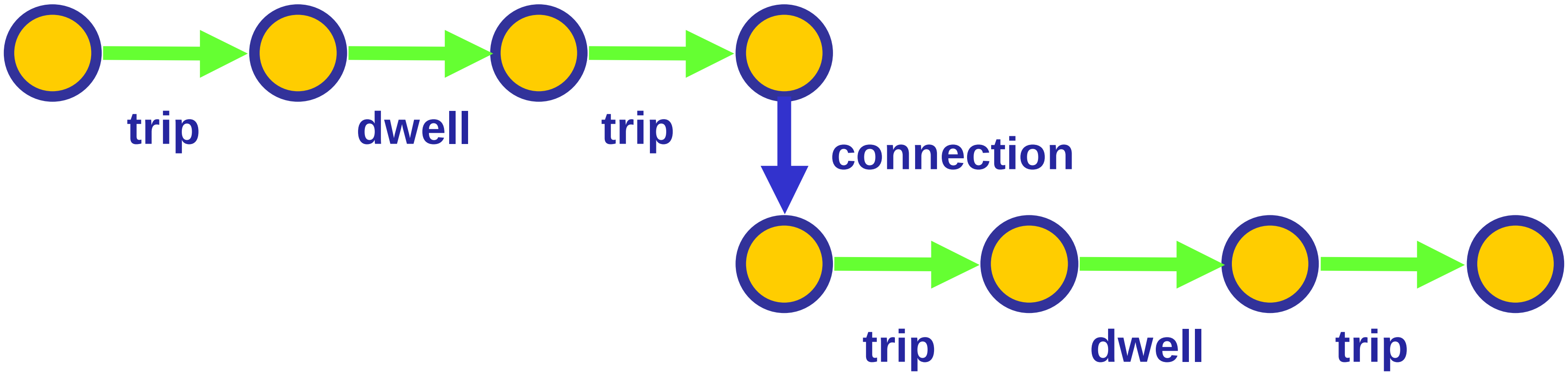
IC97: Cycle Basis Width & Length



IC97: Edge composition

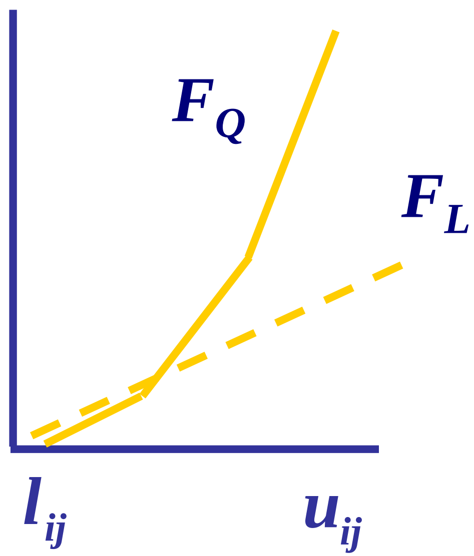


Minimizing Passenger Travel Time

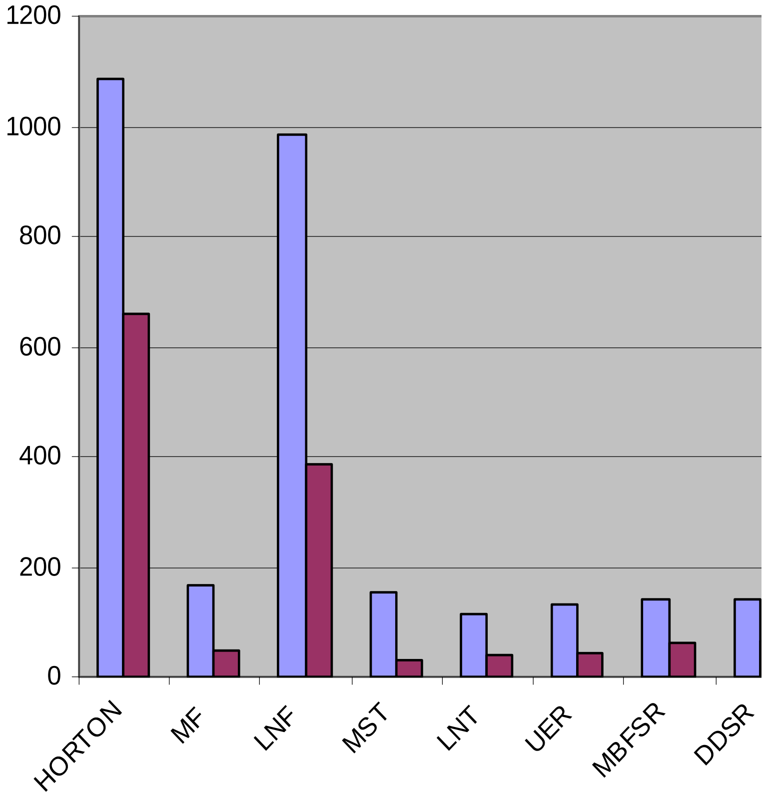


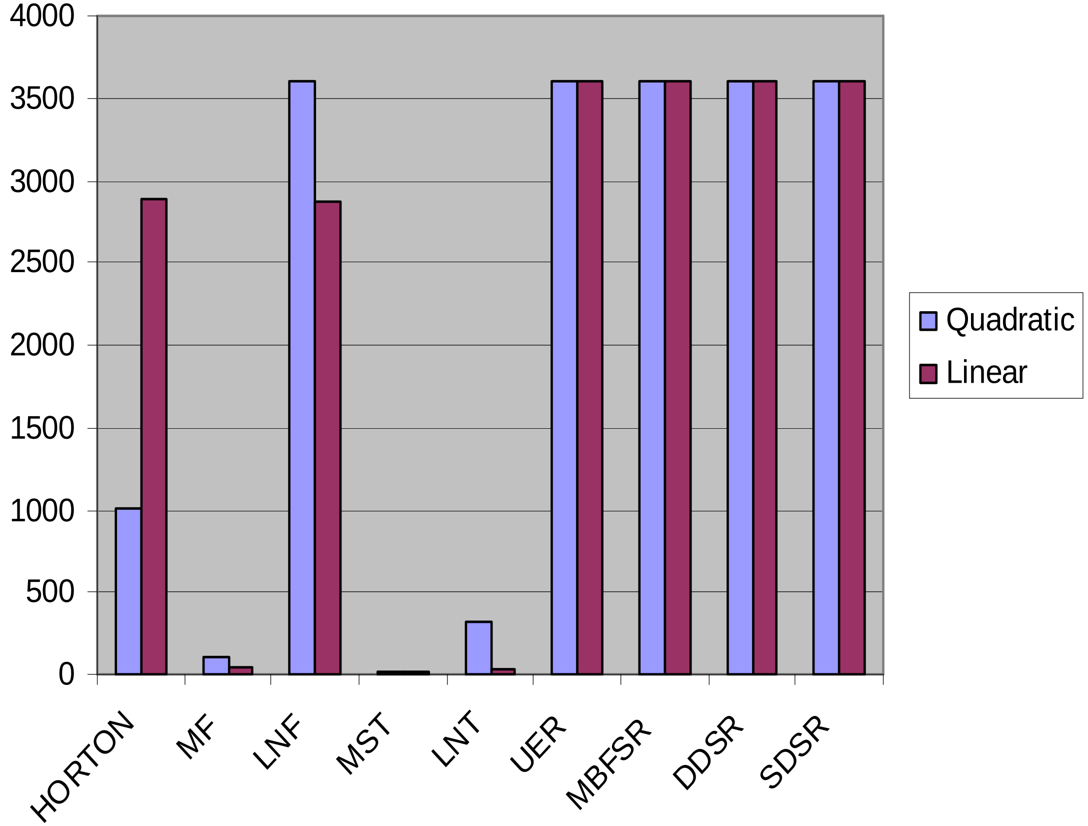
Minimize $F_L = \sum_{(i,j)\in A^*} \left(x_{ij} - l_{ij}\right)$

Minimize $F_Q = \sum_{(i,j)\in A^*} \left(x_{ij} - l_{ij}\right)^2$



IC97 & NH97: Solving the MIP





- **Computation times in seconds using CPLEX 7.5**
- **AMD Athlon 1300**