Rail Car Allocation Problems

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Mathematical Optimization · Braunschweig · Germany
Freight Cars . . .
Freight Cars . . .

. . .sit and wait
Empty Car Distribution

Cars are repositioned due to imbalances in demands

Delayed empty cars $\rightarrow$ Car shortage

$\uparrow$

Inaccurate data $\leftarrow$ Safety inventories

$\downarrow$

Low utilization

$\downarrow$

Large fleet sizes
Empty Car Distribution

Supply

Demand
Empty Car Distribution

Time

Rail Car Allocation Problems – p.5
Empty Car Distribution

Problem:

Time expanded integer multi commodity flow problem
Empty Car Distribution

- Glickman, Sherali (1985): Pooling of cars

Many more papers
5,000 cars, 18 (aggregated) types, 50 terminals

- Future:
  Simultaneously plan empty and loaded cars
A block is a group of cars with common OD pair

Ideally: Only direct blocks
Source of delay and unreliable service
Railroad Blocking

Assign each car a sequence of blocks, observing

- the maximal tractable volume of cars per station

minimizing the total

- number of reclassifications or
- delay or
- mileage

Considerable body of literature

Barnhart, Jin, and Vance (1997): 1080 stations, 12,000 shipments
Yard Operations

- Shunting with capacity constraints
- Dahlhaus et al. (2000): Regrouping of cars
- Older papers: e.g., queueing models
Yard Operations

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Also from passenger transportation:
- Routing trough stations (Kroon, Zwaneveld 1996)
- Shunting trams (Winter, Zimmermann 2000)
- This afternoon session
- ...
Further Issues

- Assign blocks to trains
- What is a good fleet size?
- Where and when to clean, maintain, and repair?
- ...

Integrated Planning

- Why not propose *one* big model for *all* stages?
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- But (today) it is illusive to solve it optimally!

- But ...
In-Plant Railroads
Customer oriented!
Rail Car Management

① Transportation request specified by
  ➞ Terminal/track ➞ Quantity
  ➞ Goods type/car type ➞ Deadline
  ➞ Substitution car types

② As long as available, assign cars
  ➞ dedicated, pooled, incoming, in repair . . .
  and build blocks

③ Otherwise: Rent additional rail cars
Blocks
Blocks
Split into Regions
Split into Regions
Split into Regions
Split into Regions
Transportation Problem

\( x_{i,j}^\tau \): Cars of type \( \tau \) from region \( i \) for request \( j \)

\[
\begin{align*}
\text{min } & \quad \sum_{i,r,\tau \in T_r} c_{i,r} \cdot x_{i,r}^\tau + \sum_{r,\tau \in T_r} M_{\tau} \cdot x_{S\tau,r}^\tau \\
\text{s.t. } & \quad \sum_{r: \tau \in T_r} x_{i,r}^\tau \leq a_i^\tau \quad \forall i, \tau \\
& \quad \sum_{i,\tau \in T_r} x_{i,r}^\tau + \sum_{\tau \in T_r} x_{S\tau,r}^\tau \geq b_r \quad \forall r \\
& \quad x_{i,r}^\tau \geq 0 \quad \forall i, r, \tau \in T_r
\end{align*}
\]
Fulfill a demand from a single origin?

\[ a_1 = a_2 = \frac{1}{2} \sum_i b_i \]

∃ Solution without car rental  \( \iff \exists \text{ Partition of } \{ b_1, b_2, \ldots, b_n \} \)
Shunting Minimization

Demand: \( D = (1, 2, 2) \)

- Each moved car on track \( i \) costs \( c_i \in \mathbb{Q}_+ \).

No space limitations, no ordering/sequence.
Shunting Minimization

Demand: $D = (1, 2, 2)$

Each moved car on track $i$ costs $c_i \in \mathbb{Q}_+$, e.g. $(1 + 1) \cdot c_1 + 2 \cdot c_4 + 2 \cdot c_5$

No space limitations, no ordering/sequence
Greedy

Take cheapest car(s) for each color, respectively
Demand: $D = (1, 1, 1, 1, 1, 1)$, $|D| = n$

Cost: Greedy $O(n^2)$, optimal $O(n)$
Demand: $D = (K, 1, 1, 1, 1, 1, 1)$

∃ feasible shunting plan at cost $K$ $\iff$

∃ vertex cover of cardinality $K$
Integer Program

$z_{t,g}$ : Access group $g$ on track $t$?

$y_{t,g}$ : Number of chosen cars from group $g$ on track $t$, at most $Q_{t,g}$

$$\min \sum_{t,g} c_t \cdot [(Q_{t,g} - y_{t,g}) \cdot z_{t,g} + 1 + y_{t,g}]$$

s.t.

$$z_{t,g} \leq z_{t,g-1} \quad \forall t, 1 < g$$

$$y_{t,g} \leq Q_{t,g} \cdot z_{t,g} \quad \forall t, g$$

$$\sum_{t,g: \text{color}(t,g) = \tau} y_{t,g} \geq D_\tau \quad \forall \text{types } \tau$$

$$y_{t,g} \geq 0 \quad \forall t, g$$

$$z_{t,g} \in \{0, 1\} \quad \forall t, g$$
Integer Program

\(z_{t,g}\) : Access group \(g\) on track \(t\)?

\(y_{t,g}\) : Number of chosen cars from group \(g\) on track \(t\), at most \(Q_{t,g}\)

\[
\min \sum_{t,g} c_t \cdot [(Q_{t,g} - y_{t,g}) \cdot z_{t,g+1} + y_{t,g}]
\]

\[
\min \sum_{t,g} c_t \cdot Q_{t,g} \cdot z_{t,g}
\]

s.t.

\[
z_{t,g} \leq z_{t,g-1} \quad \forall t, 1 < b
\]

\[
y_{t,g} \leq Q_{t,g} \cdot z_{t,g} \quad \forall t, g
\]

\[
\sum_{t,g: color(t,g) = \tau} y_{t,g} \geq D_\tau \quad \forall \text{types } \tau
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\[
y_{t,g} \geq 0 \quad \forall t, g
\]

\[
z_{t,g} \in \{0, 1\} \quad \forall t, g
\]
\[
\text{min} \quad \sum_{i,r,\tau \in \mathcal{T}_r} c_{i,r} \cdot x_{i,r}^\tau + \sum_{r,\tau \in \mathcal{T}_r} M_{\tau} \cdot x_{s_{\tau},r}^\tau \\
\text{s.t.} \quad \sum_{r: \tau \in \mathcal{T}_r} x_{i,r}^\tau \leq a_r^\tau \quad \forall i, \tau \\
\sum_{i,\tau \in \mathcal{T}_r} x_{i,r}^\tau + \sum_{\tau \in \mathcal{T}_r} x_{s_{\tau},r}^\tau \geq b_r \quad \forall r \\
x_{i,r}^\tau \in \mathbb{Z}_+ \quad \forall i, r, \tau \in \mathcal{T}_r
\]
Integrated Planning

\[
\begin{align*}
\min & \quad \sum_{i,r,\tau \in T_r} c_{i,r} \cdot x_{i,r}^\tau + \sum_{r,\tau \in T_r} M_{\tau} \cdot x_{S_{\tau},r}^\tau \\
\text{s.t.} & \quad \sum_{r: \tau \in T_r} x_{i,r}^\tau \leq \sum_{i,t,g: \text{color}(i,t,g)=\tau} y_{i,t,g} \quad \forall i, \tau \\
& \quad \sum_{i,\tau \in T_r} x_{i,r}^\tau + \sum_{\tau \in T_r} x_{S_{\tau},r}^\tau \geq b_r \quad \forall r \\
& \quad x_{i,r}^\tau \in \mathbb{Z}_+ \quad \forall i, r, \tau \in T_r
\end{align*}
\]
\[
\begin{align*}
\text{min} \quad & \sum_{i,r,\tau \in T_r} c_{i,r} \cdot x_{i,r}^\tau + \sum_{r,\tau \in T_r} M_{\tau} \cdot x_{S_{\tau},r}^\tau + \sum_{i=1}^{n} \sum_{t,g} c_{t} \cdot Q_{i,t,g} \cdot z_{i,t,g} \\
\text{s.t.} \quad & \sum_{r:\tau \in T_r} x_{i,r}^\tau \leq \sum_{i,t,g:color(i,t,g) = \tau} y_{i,t,g} \quad \forall i, \tau \\
& \sum_{i,\tau \in T_r} x_{i,r}^\tau + \sum_{\tau \in T_r} x_{S_{\tau},r}^\tau \geq b_{r} \quad \forall r \\
& z_{i,t,g} \leq z_{i,t,g} - 1 \quad \forall i, t, 1 < b \\
& y_{i,t,g} \leq Q_{i,t,g} \cdot z_{i,t,g} \quad \forall i, t, g \\
& x_{i,r}^\tau \in \mathbb{Z}_+ \quad \forall i, r, \tau \in T_r \\
& y_{i,t,g} \geq 0 \quad \forall i, t, g \\
& z_{i,t,g} \in \{0, 1\} \quad \forall i, t, g
\end{align*}
\]
Computational Results

~ 683 tracks, 168 terminals, 42 regions;
~ 1575 cars, 123 car types; 18 requests
Conclusion

➤ Almost generic approach yields relevant results
➤ Experiments feed back into practice
➤ Test runs planned for September 2002
➤ Results encourage extensions...
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