Rail Car Allocation Problems

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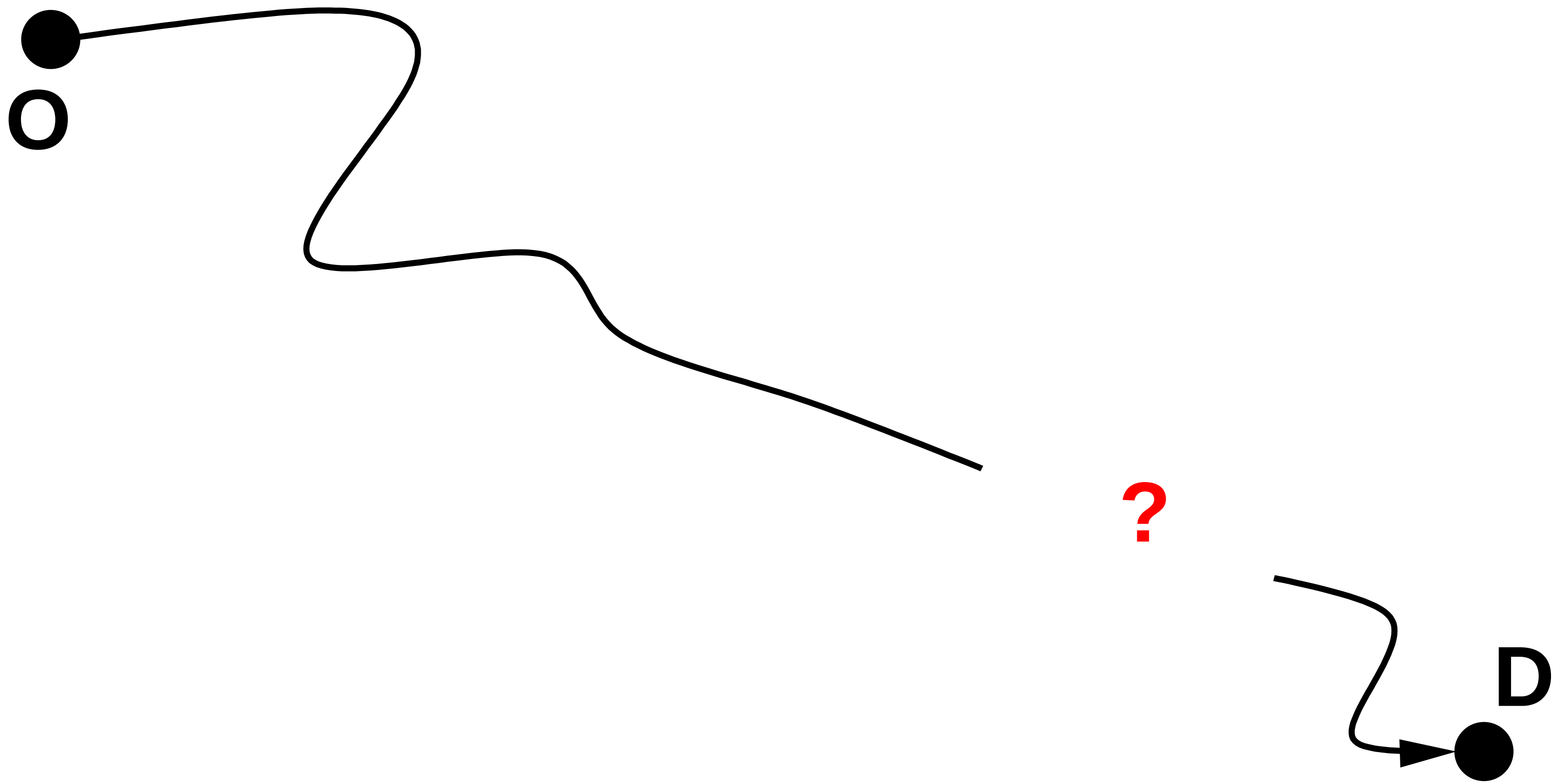
Mathematical Optimization · Braunschweig · Germany

Freight Cars . . .

Freight Cars . . .

. . . sit and wait

Freight Cars . . .

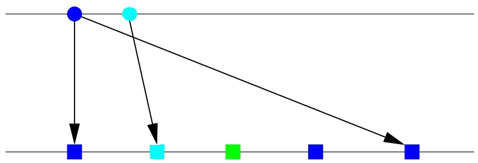


Empty Car Distribution

Cars are repositioned due to imbalances in demands

$$
\begin{array}{ccc}
\text{Delayed empty cars} & \longrightarrow & \text{Car shortage} \\
\uparrow & & \downarrow \\
\text{Inaccurate data} & \longleftarrow & \text{Safety inventories} \\
 & & \downarrow \\
 & & \text{Low utilization} \\
 & & \downarrow \\
 & & \text{Large fleet sizes}
\end{array}
$$

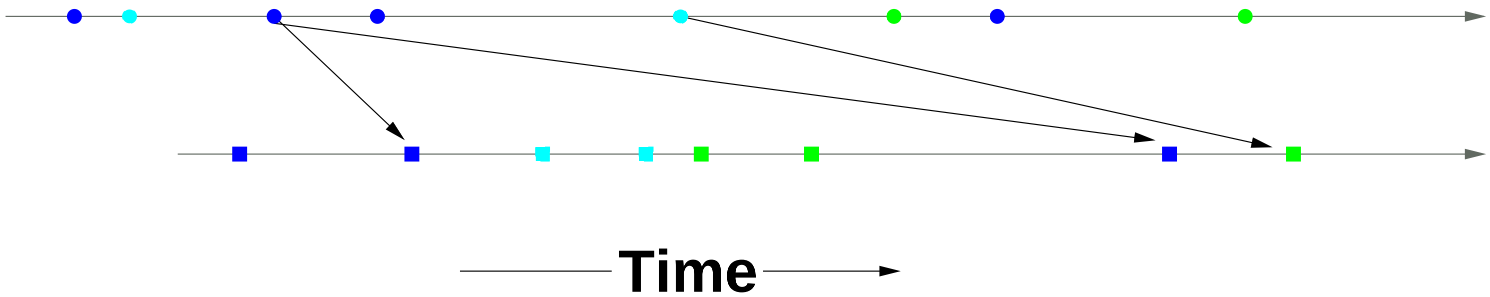
Empty Car Distribution



Supply

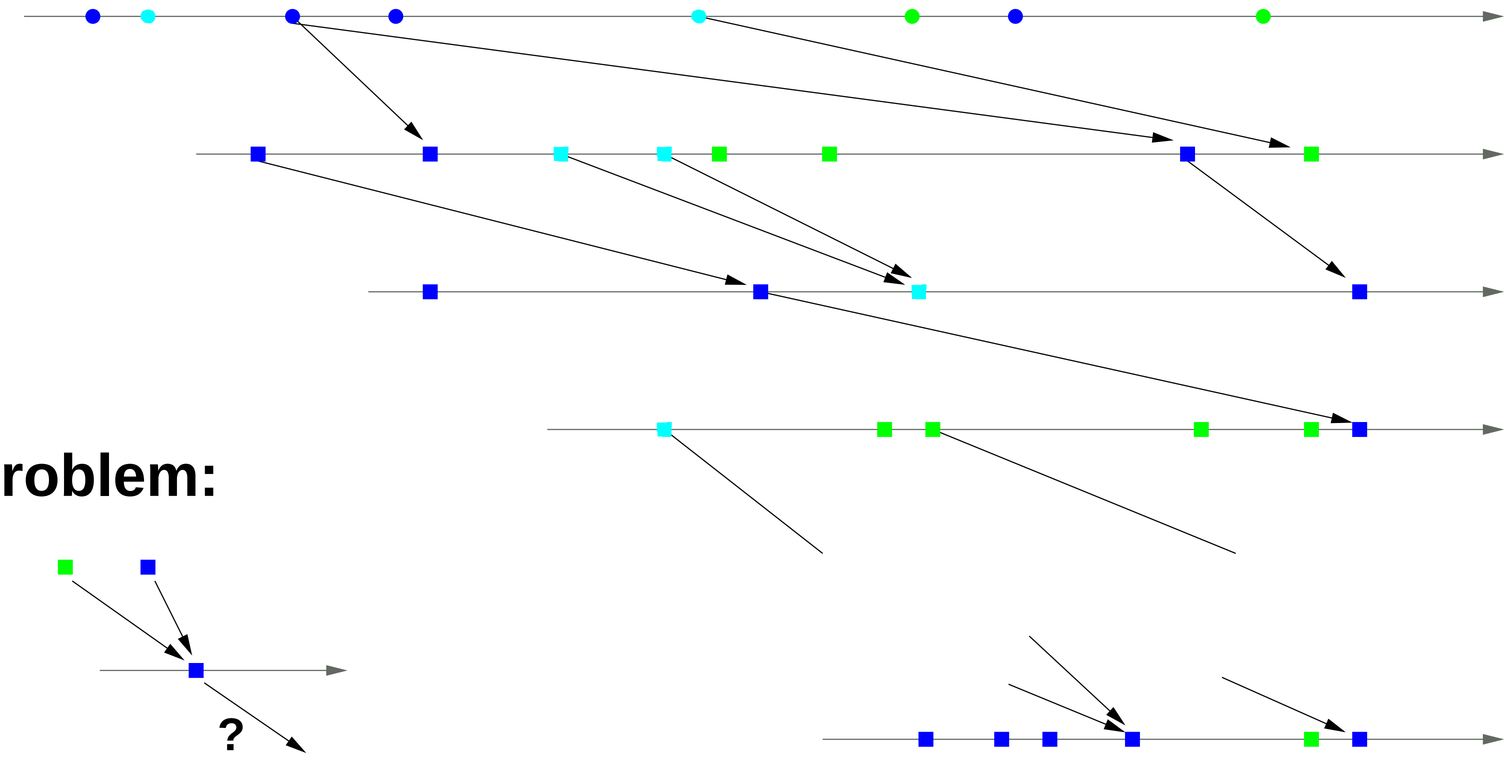
Demand

Empty Car Distribution



Empty Car Distribution

Problem:



Time expanded integer multi commodity flow problem

Empty Car Distribution

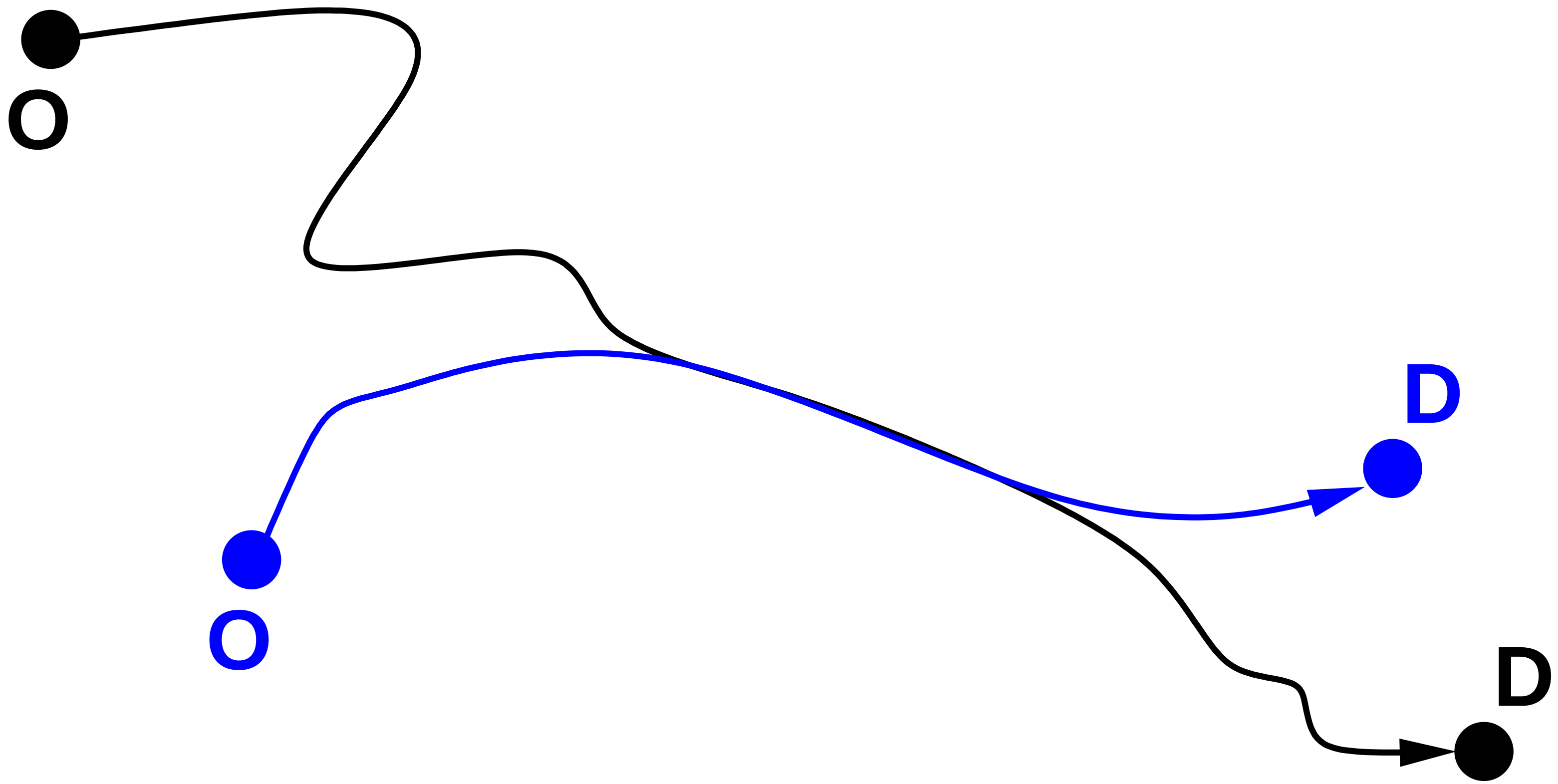
- Glickman, Sherali (1985): Pooling of cars
- Holmberg, Joborn, and Lundgren (1996, 1998): Explicit train schedules, train capacities

Many more papers
5,000 cars, 18 (aggregated) types, 50 terminals

- Future:
  Simultaneously plan empty and loaded cars

Railroad Blocking

A *block* is a group of cars with common OD pair



Ideally: Only *direct* blocks

Source of delay and unreliable service

Railroad Blocking

Assign each car a sequence of blocks, observing

- the maximal tractable volume of cars per station

minimizing the total

- number of reclassifications *or*
- delay *or*
- mileage

Considerable body of literature

Barnhart, Jin, and Vance (1997): 1080 stations, 12,000 shipments

Yard Operations

- Shunting with capacity constraints
- Dahlhaus *et al.* (2000): Regrouping of cars
- older papers: e.g., queueing models

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Also from passenger transportation:

- Routing trough stations (Kroon, Zwaneveld 1996)
- Shunting trams (Winter, Zimmermann 2000)
- This afternoon session
- . . .

Further Issues

- Assign blocks to trains
- What is a good fleet size?
- Where and when to clean, maintain, and repair?
- . . .

Integrated Planning

- Why not propose *one* big model for *all* stages?

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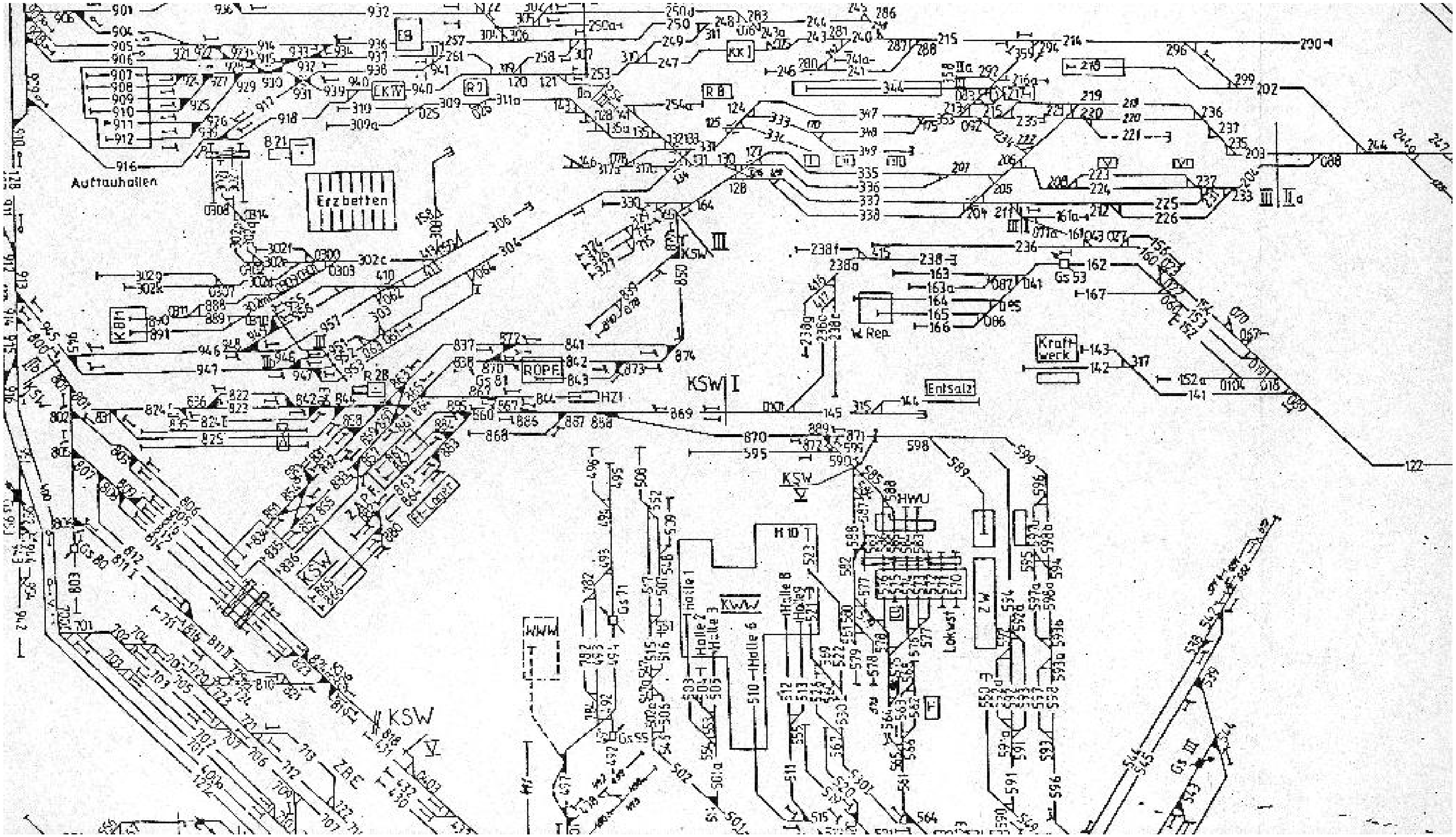
- But . . .

In-Plant Railroads



In-Plant Railroads

Customer oriented!

Rail Car Management

① Transportation request specified by

- ➡ Terminal/track
- ➡ Quantity
- ➡ Goods type/car type
- ➡ Deadline
- ➡ Substitution car types

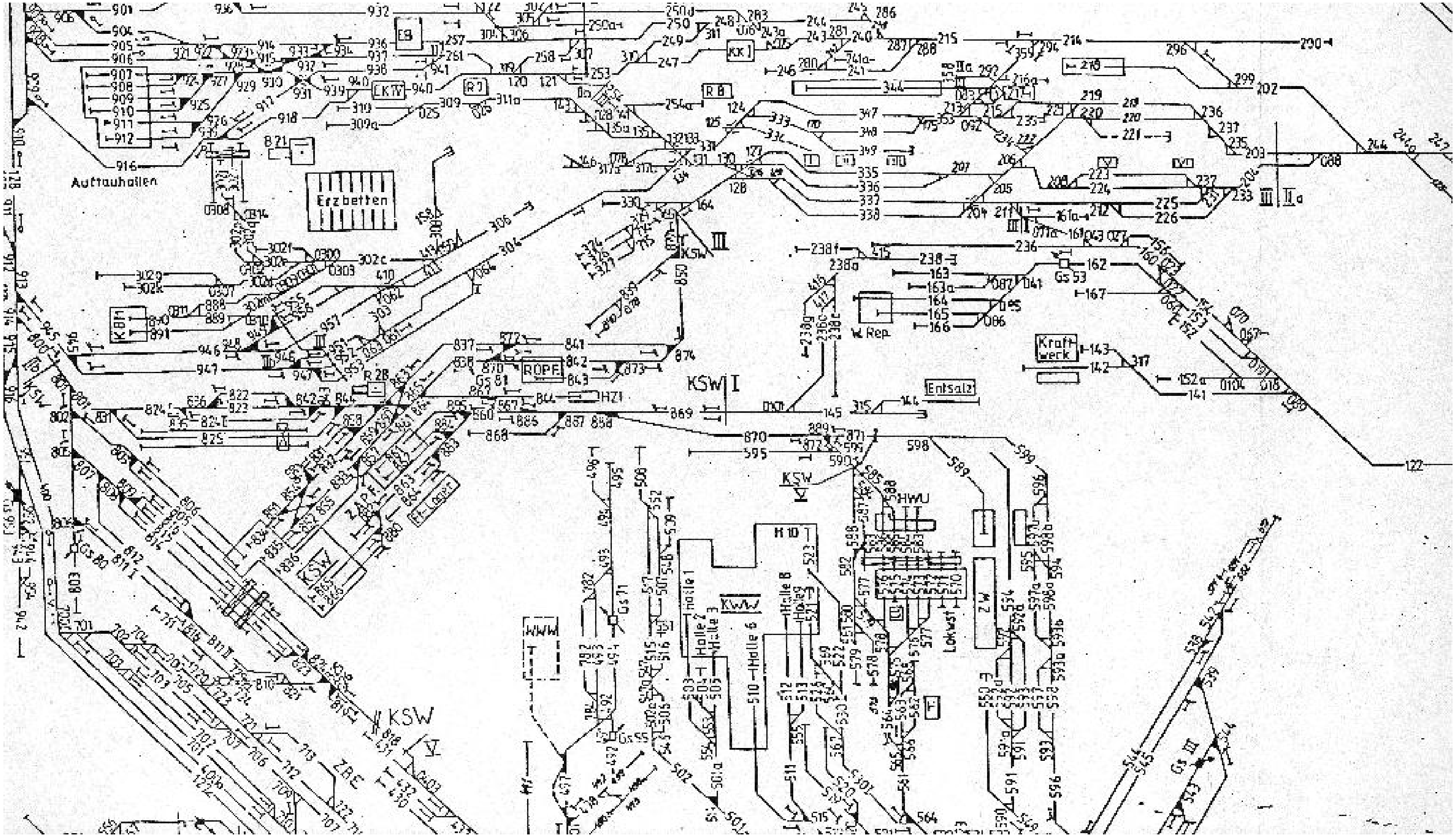
② As long as available, assign cars

- ➡ dedicated, pooled, incoming, in repair . . .

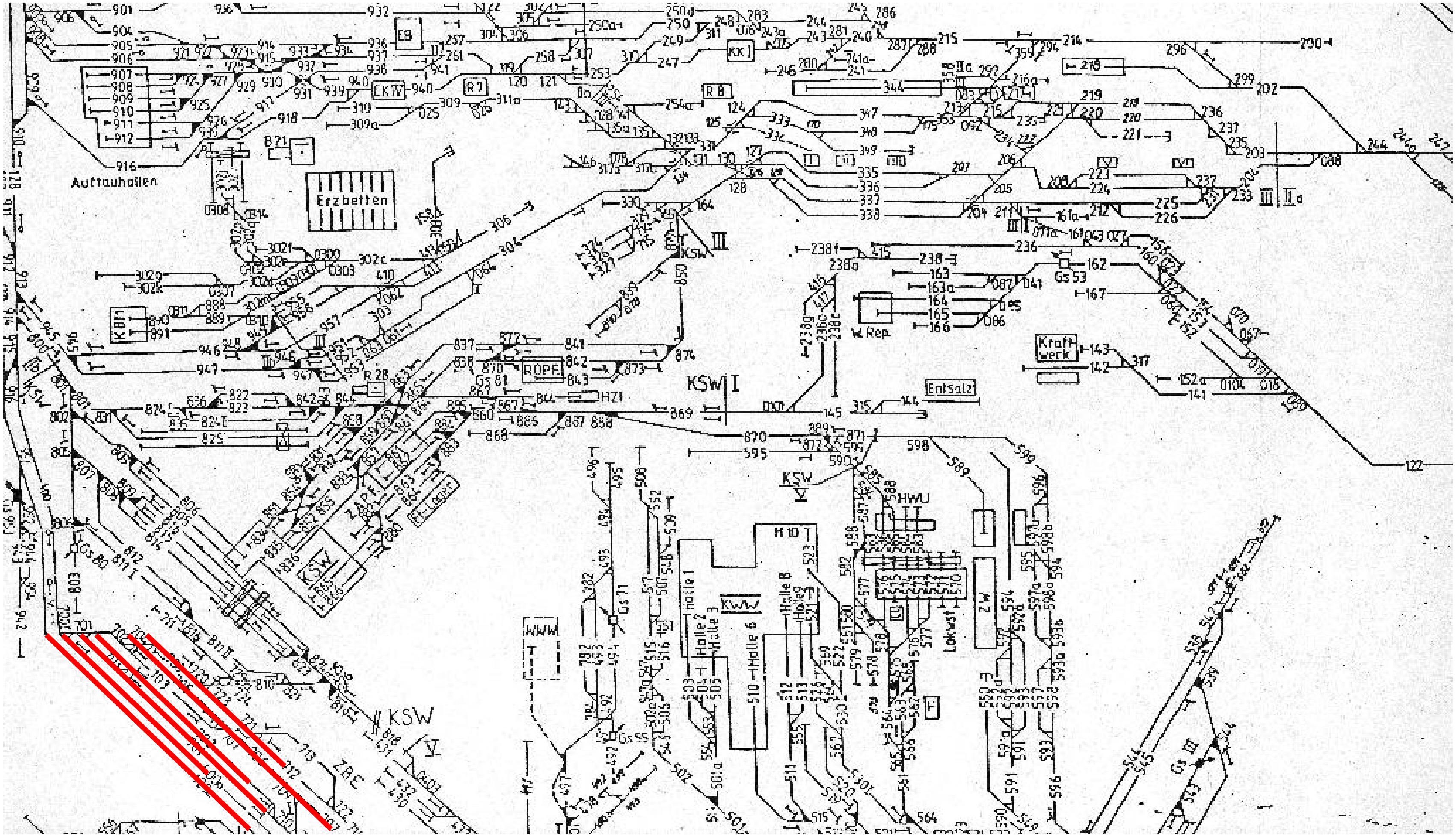
and build blocks

③ Otherwise: Rent additional rail cars

Blocks



Blocks



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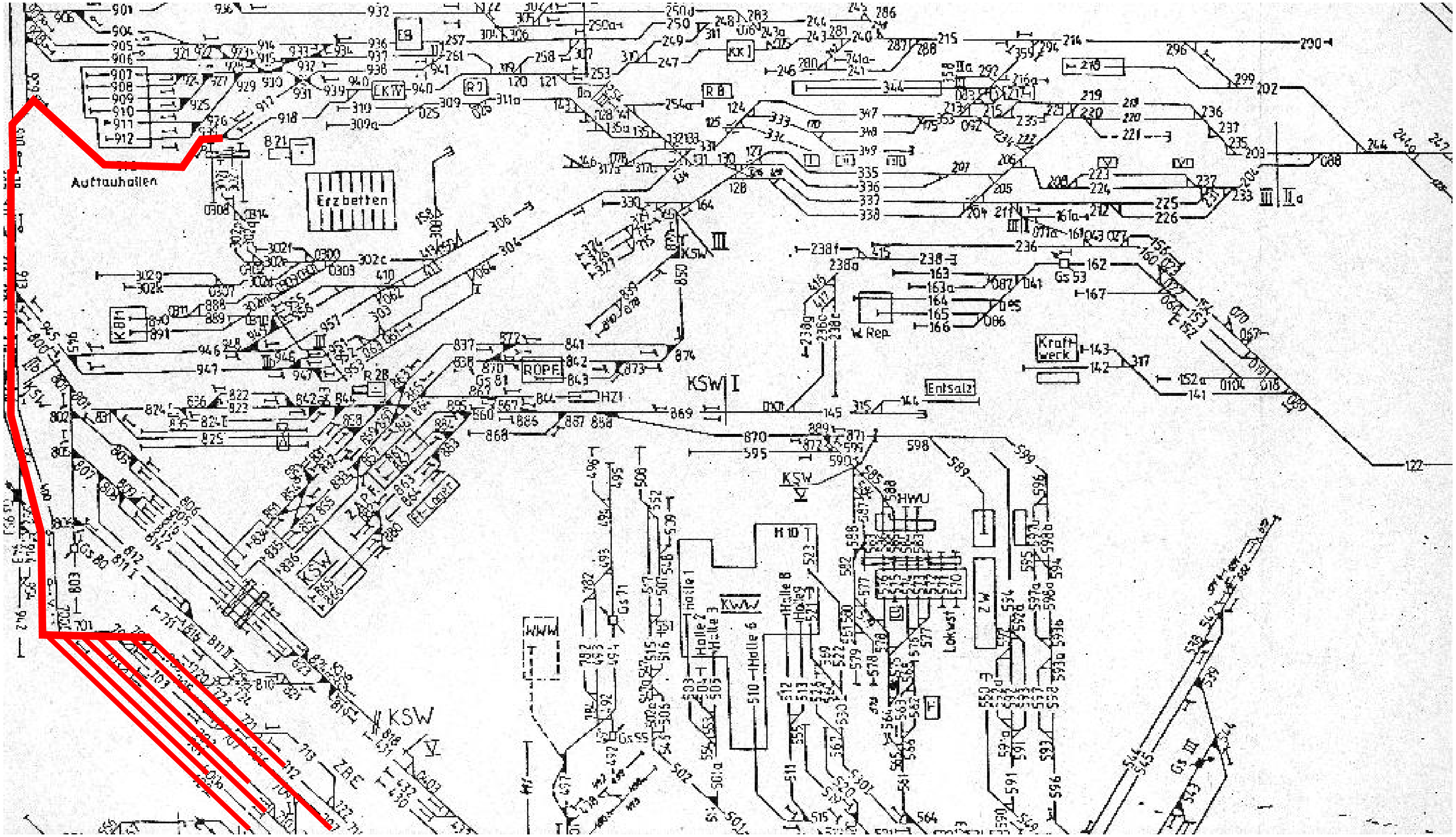

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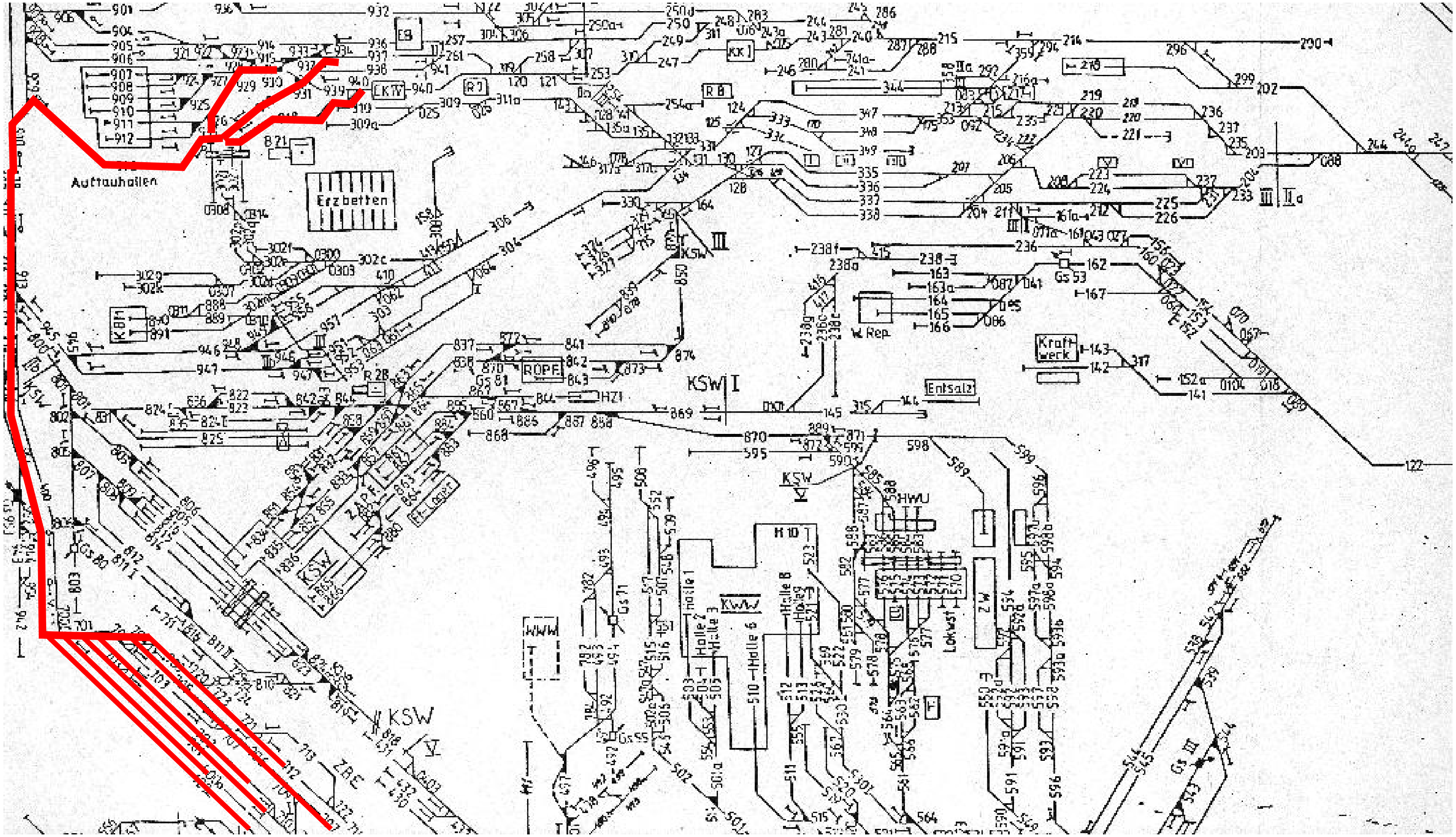
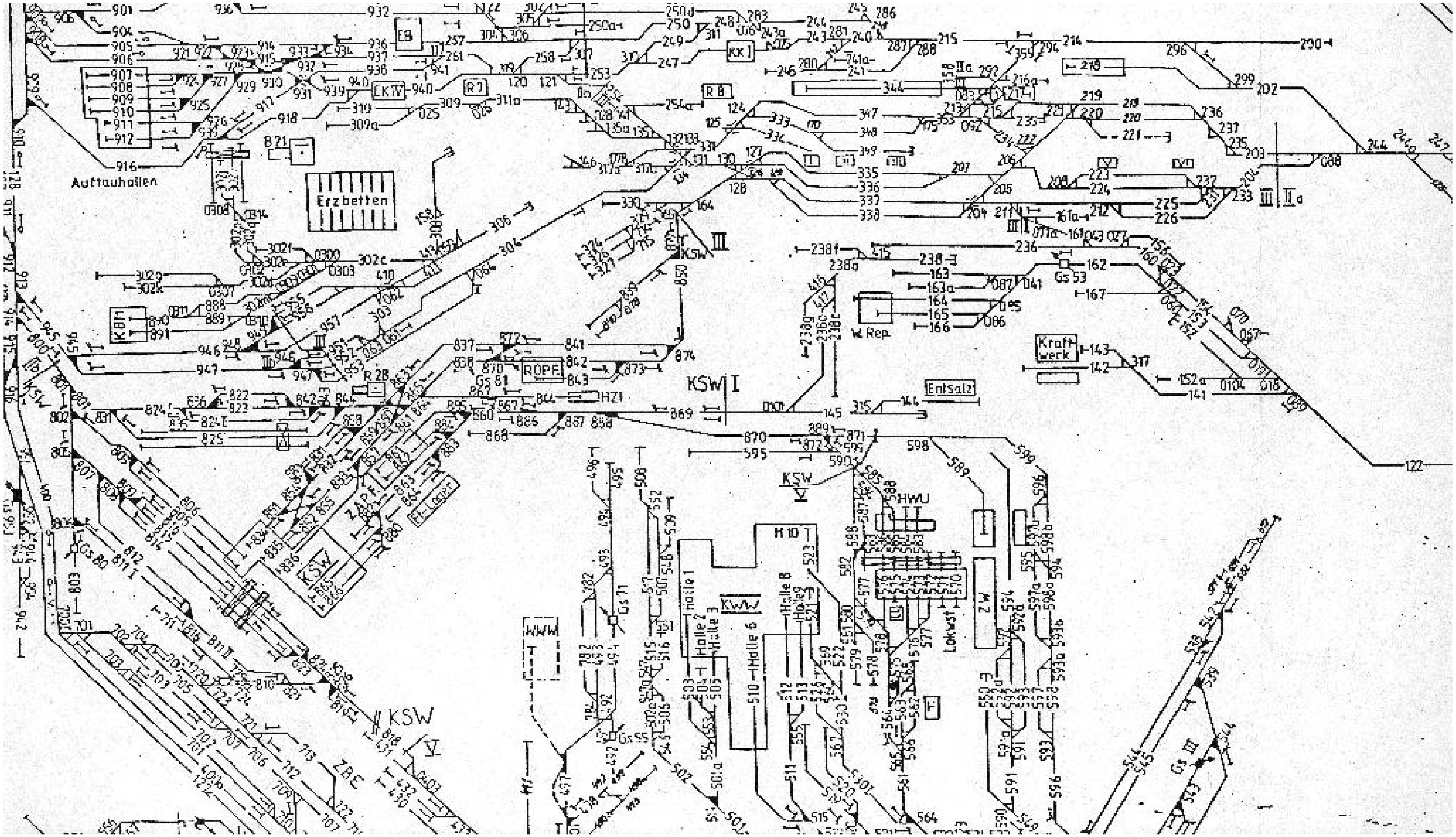


Split into Regions



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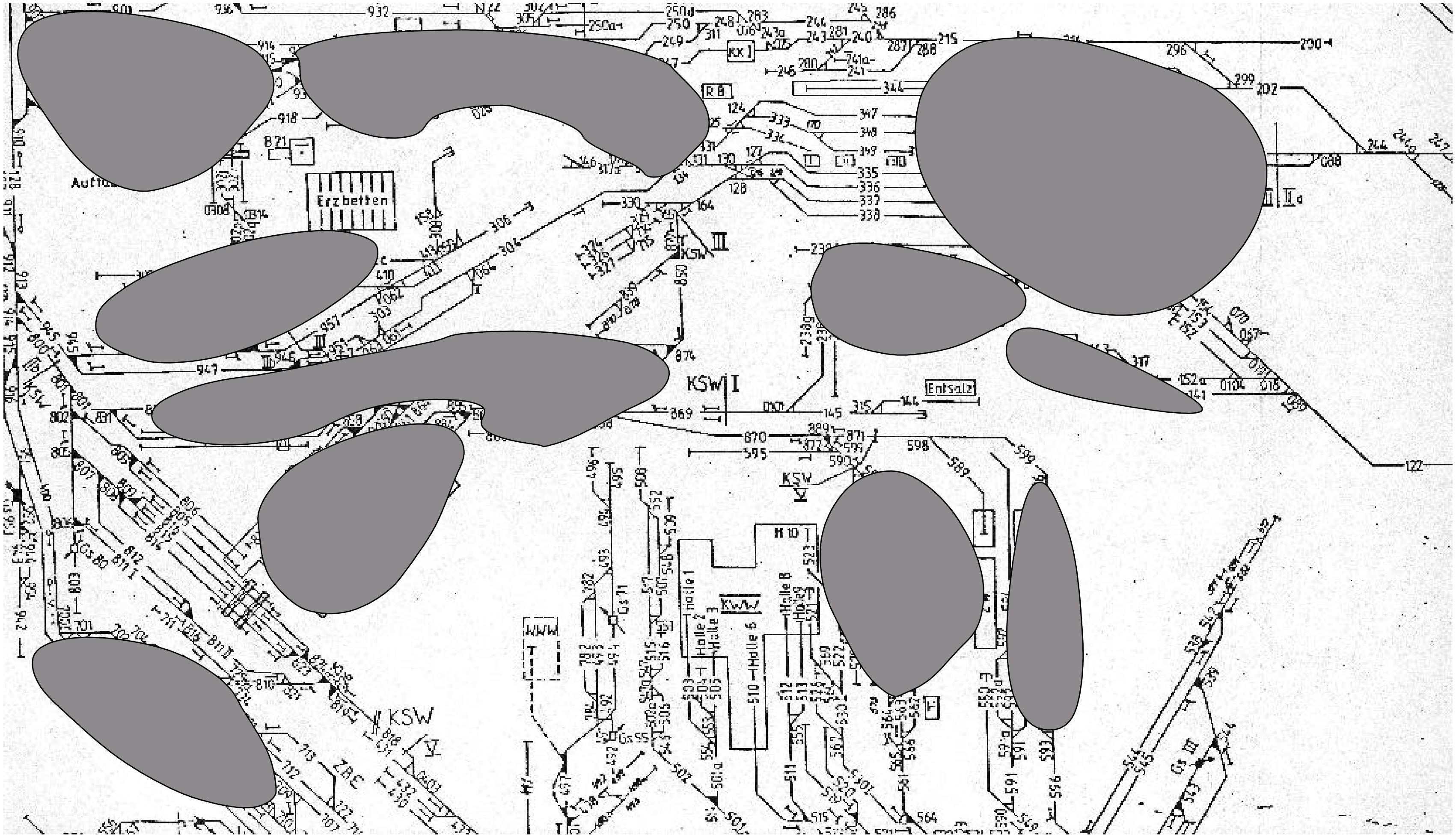

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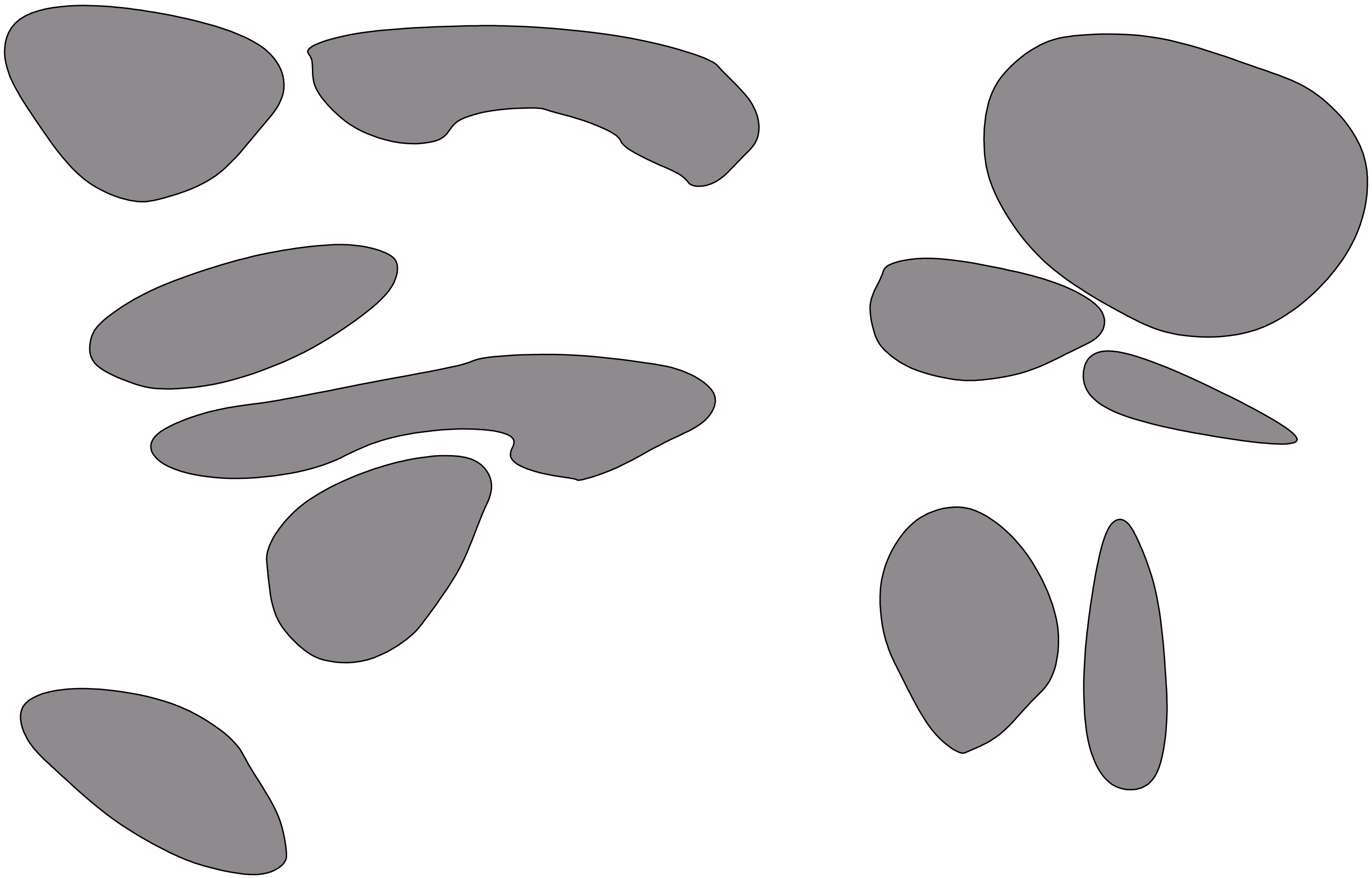

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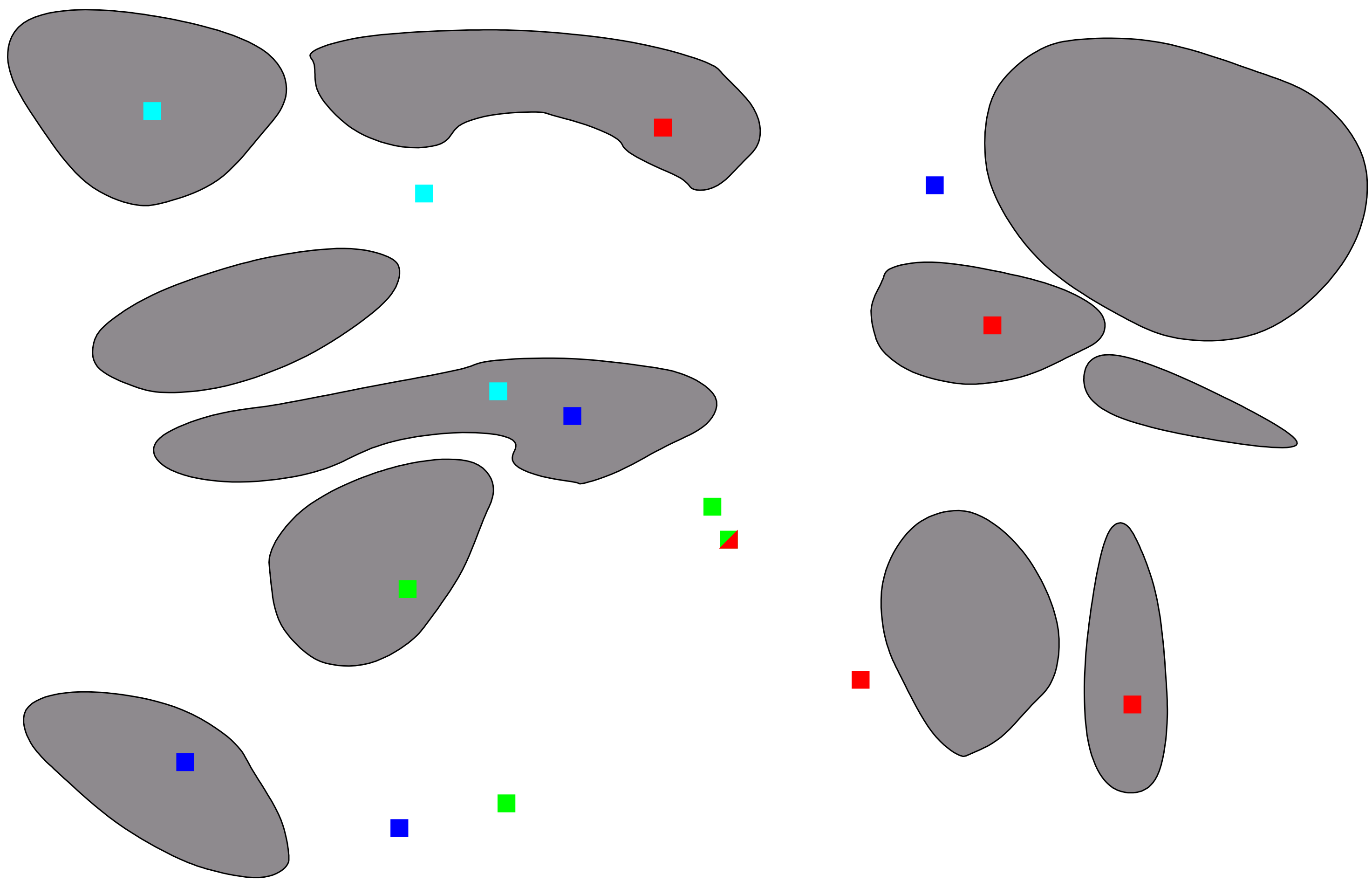

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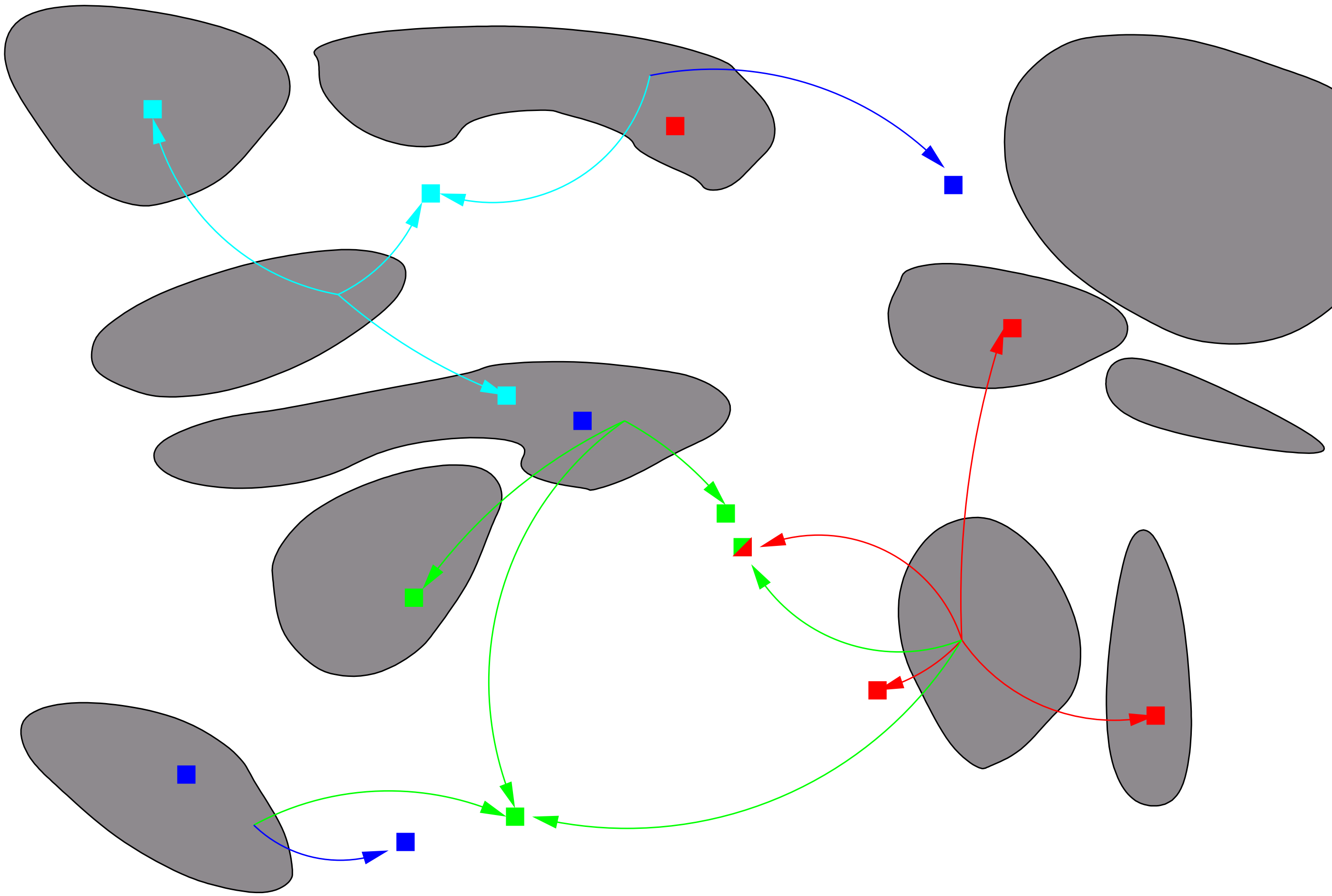


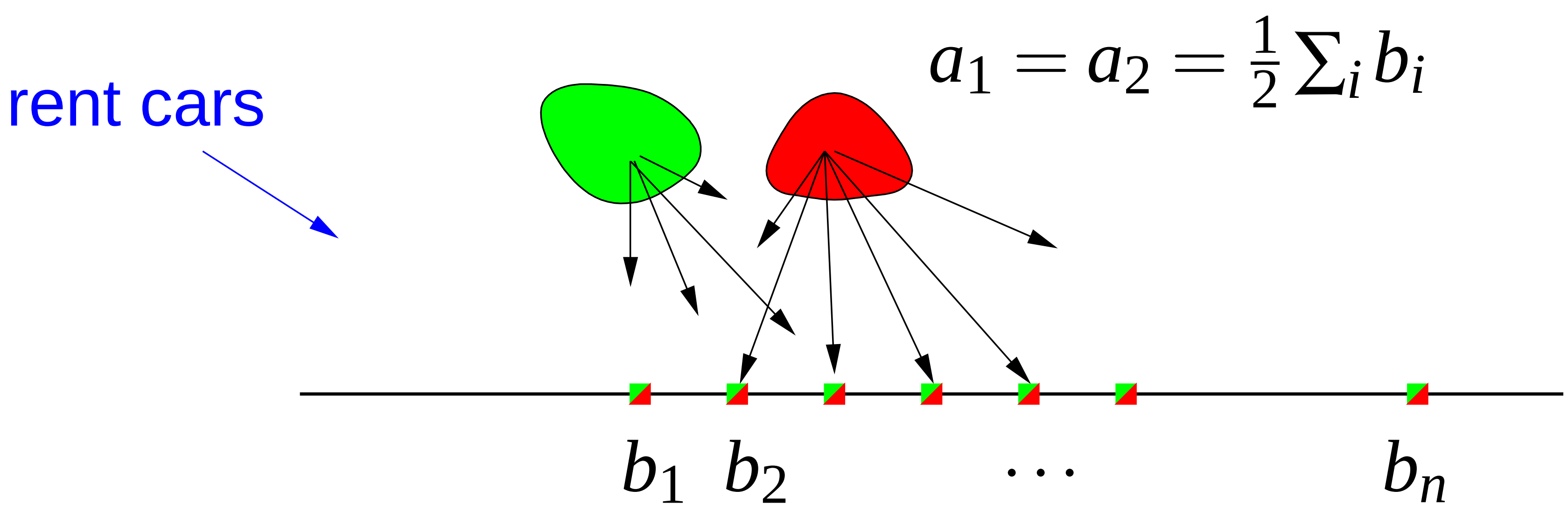
Transportation Problem

x_{ij}^{τ} : Cars of type τ from region i for request j

$$
\begin{array}{llcll}
\text{min} & \displaystyle\sum_{i,r,\tau\in\mathcal{T}_r} c_{i,r}\cdot x_{i,r}^{\tau} + \sum_{r,\tau\in\mathcal{T}_r} M_{\tau}\cdot x_{S_\tau,r}^{\tau} & & & \\
\text{s.t.} & \displaystyle\sum_{r:\tau\in\mathcal{T}_r} x_{i,r}^{\tau} & \leq & a_i^{\tau} & \forall i,\tau \\
 & \displaystyle\sum_{i,\tau\in\mathcal{T}_r} x_{i,r}^{\tau} + \sum_{\tau\in\mathcal{T}_r} x_{S_\tau,r}^{\tau} & \geq & b_r & \forall r \\
 & x_{i,r}^{\tau} & \geq & 0 & \forall i,r,\tau\in\mathcal{T}_r
\end{array}
$$

Unsplit Supply

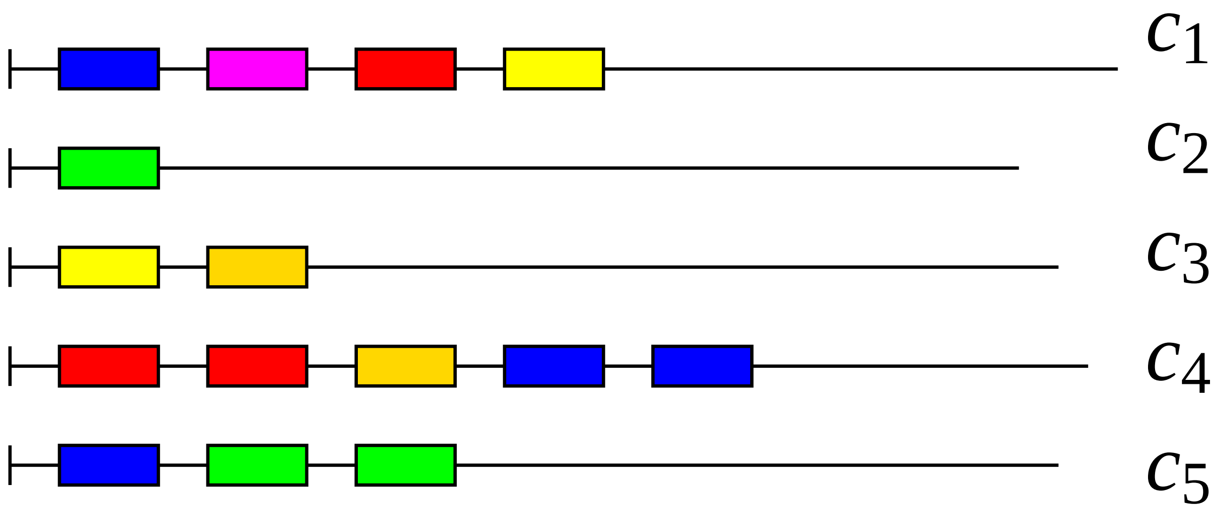
- Fulfill a demand from a single origin?



$\exists$ Solution without car rental $\iff$
$\exists$ Partition of $\{b_1, b_2, \ldots, b_n\}$

Shunting Minimization

Demand: $D = (1, 2, 2)$



Each moved car on track i costs $c_i \in \mathbb{Q}_+$,

No space limitations, no ordering/sequence

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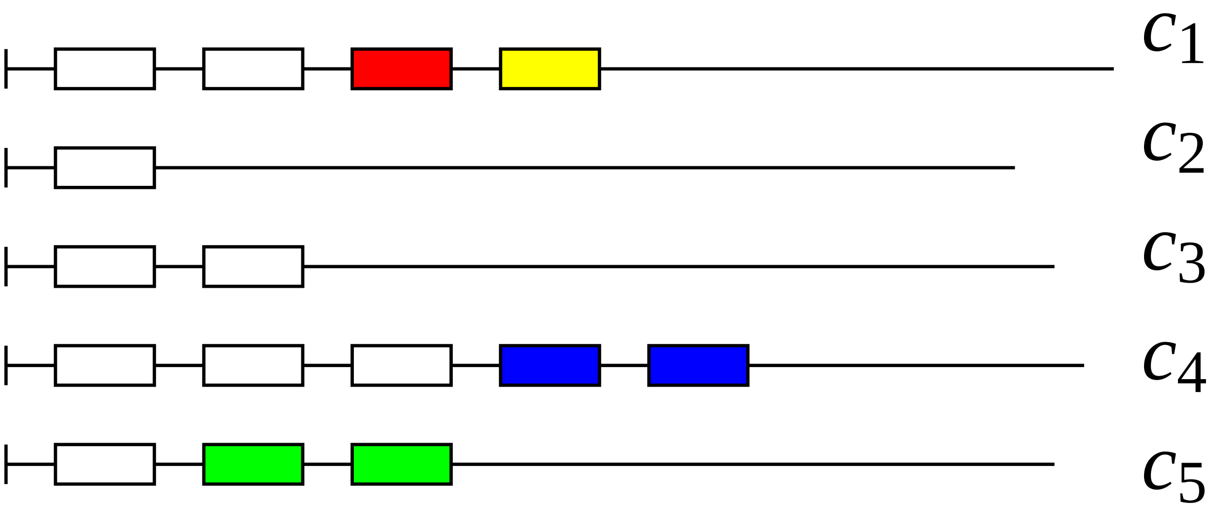


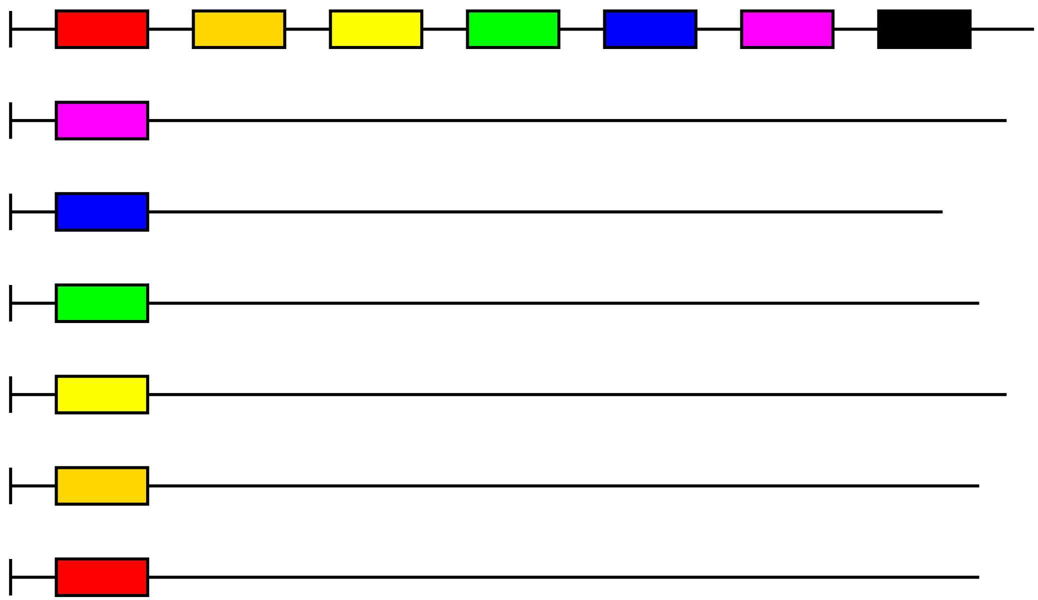
Each moved car on track i costs $c_i \in \mathbb{Q}_+$,
e.g. $(1+1) \cdot c_1 + 2 \cdot c_4 + 2 \cdot c_5$

No space limitations, no ordering/sequence

Greedy

Take cheapest car(s) for each color, respectively

Demand: $D = (1, 1, 1, 1, 1, 1)$, $|D| = n$



1

$2-\varepsilon$

$3-\varepsilon$

$4-\varepsilon$

$5-\varepsilon$

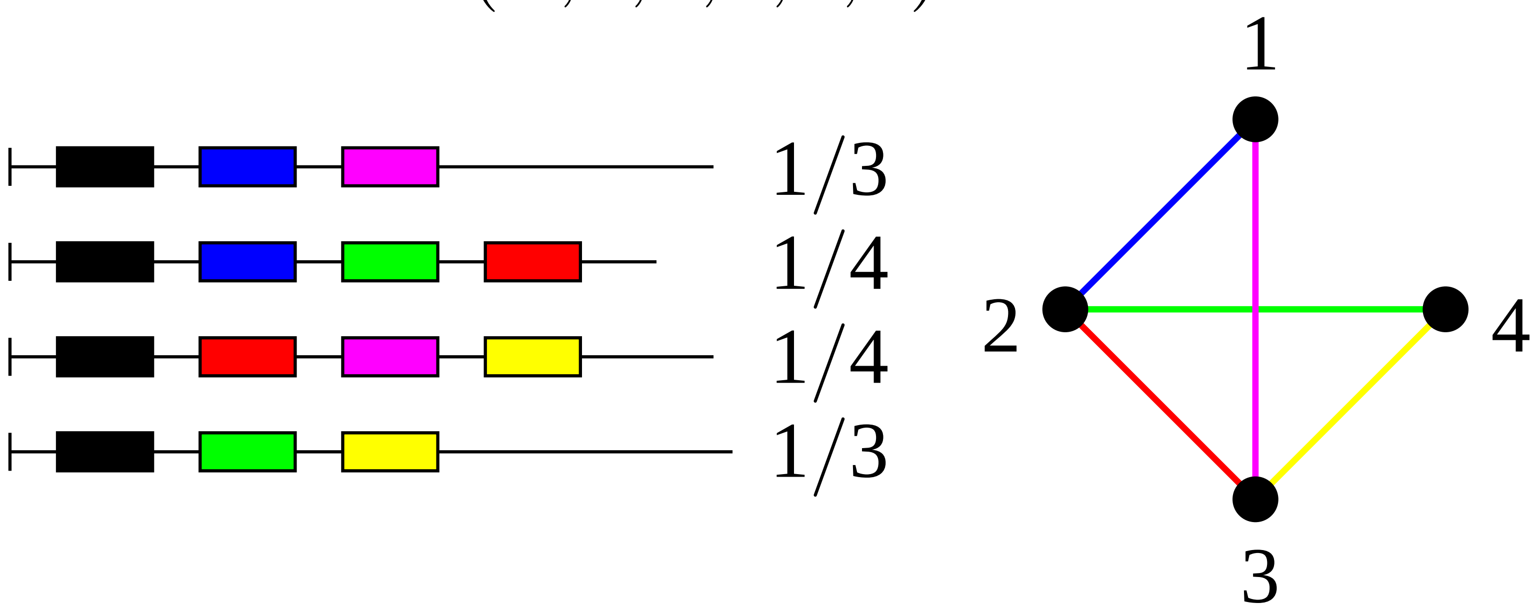
$\vdots$

$n-\varepsilon$

Cost: Greedy $O(n^2)$, optimal $O(n)$

Complexity

Demand: $D = (K, 1, 1, 1, 1, 1)$



$\exists$ feasible shunting plan at cost K $\iff$

$\exists$ vertex cover of cardinality K

Integer Program

$z_{t,g}$: Access group g on track t?

$y_{t,g}$: Number of chosen cars from group g on track t, at most $Q_{t,g}$

$$\min \quad \sum_{t,g} c_t \cdot [(Q_{t,g} - y_{t,g}) \cdot z_{t,g+1} + y_{t,g}]$$

s.t.

$$\begin{aligned}
z_{t,g} &\leq z_{t,g-1} && \forall t, 1 < g \\
y_{t,g} &\leq Q_{t,g} \cdot z_{t,g} && \forall t, g \\
\sum_{t,g:color(t,g)=\tau} y_{t,g} &\geq D_\tau && \forall \text{ types } \tau \\
y_{t,g} &\geq 0 && \forall t, g \\
z_{t,g} &\in \{0,1\} && \forall t, g
\end{aligned}$$

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$$\min \quad \sum_{t,g} c_t \cdot Q_{t,g} \cdot z_{t,g}$$

$$\begin{aligned}
\text{s.t.} \quad z_{t,g} &\leq z_{t,g-1} && \forall t, 1 < b \\
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$$

Computational Results

- $\sim$ 683 tracks, 168 terminals, 42 regions;
- $\sim$ 1575 cars, 123 car types; 18 requests

Conclusion

- Almost generic approach yields relevant results
- Experiments feed back into practice
- Test runs planned for September 2002
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