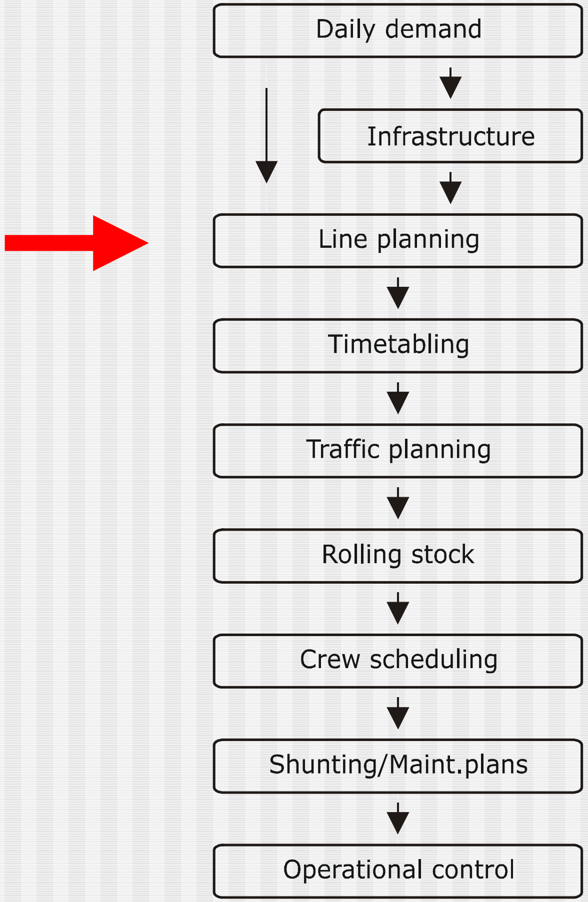
Passenger line optimization with multiple train types

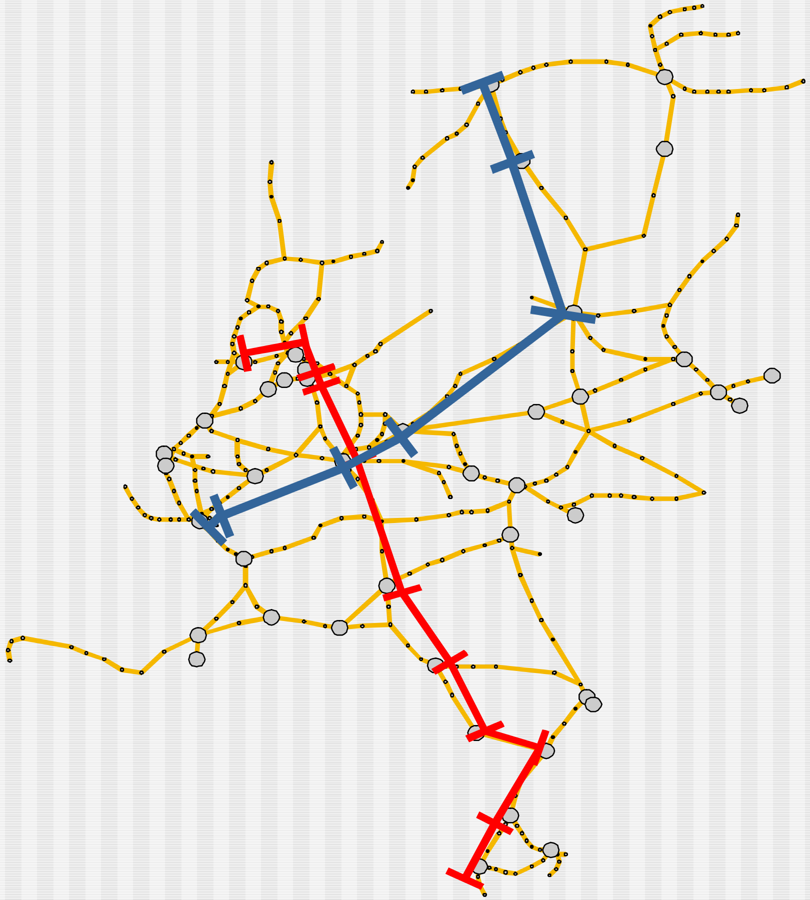
Stan van Hoesel
Leo Kroon
Jan-Willem Goossens

Outline

- Brief introduction to line planning
- Multi-type line planning
  - Three models
  - Comparing models
  - Computational experiments
- Conclusion & final remarks

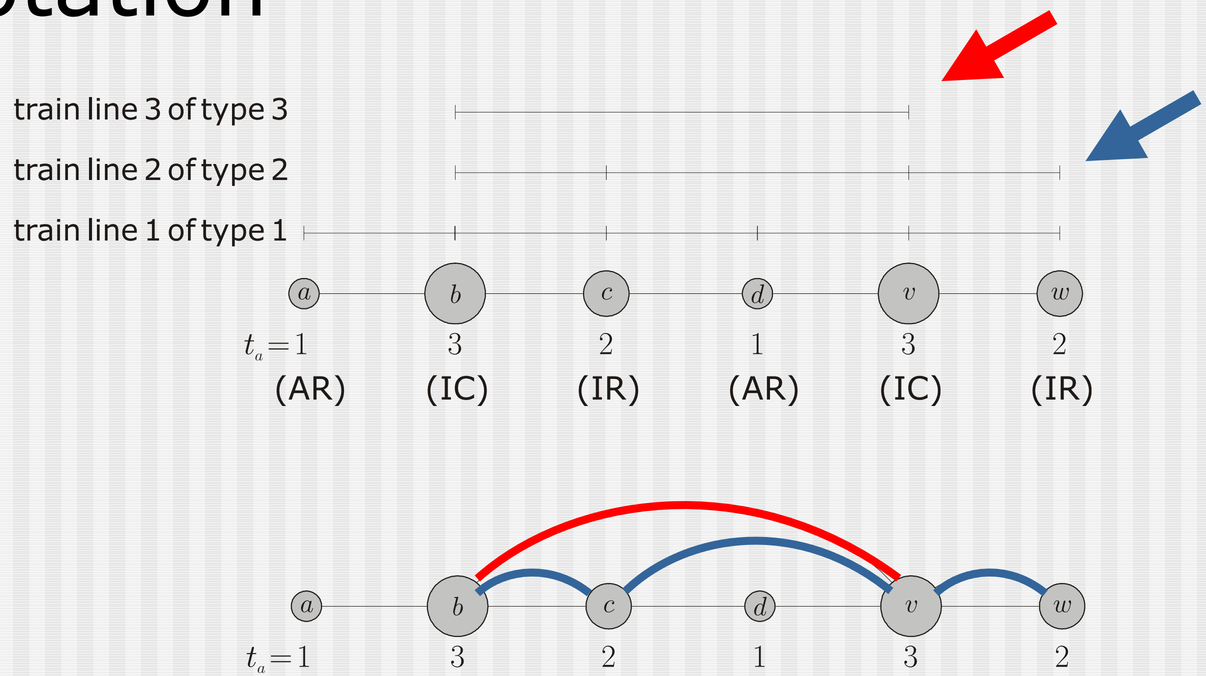
Line planning problems





Multi-type line planning

Notation



Track graph $G = (V, E)$

Type graph $G^T = (V, E^T)$

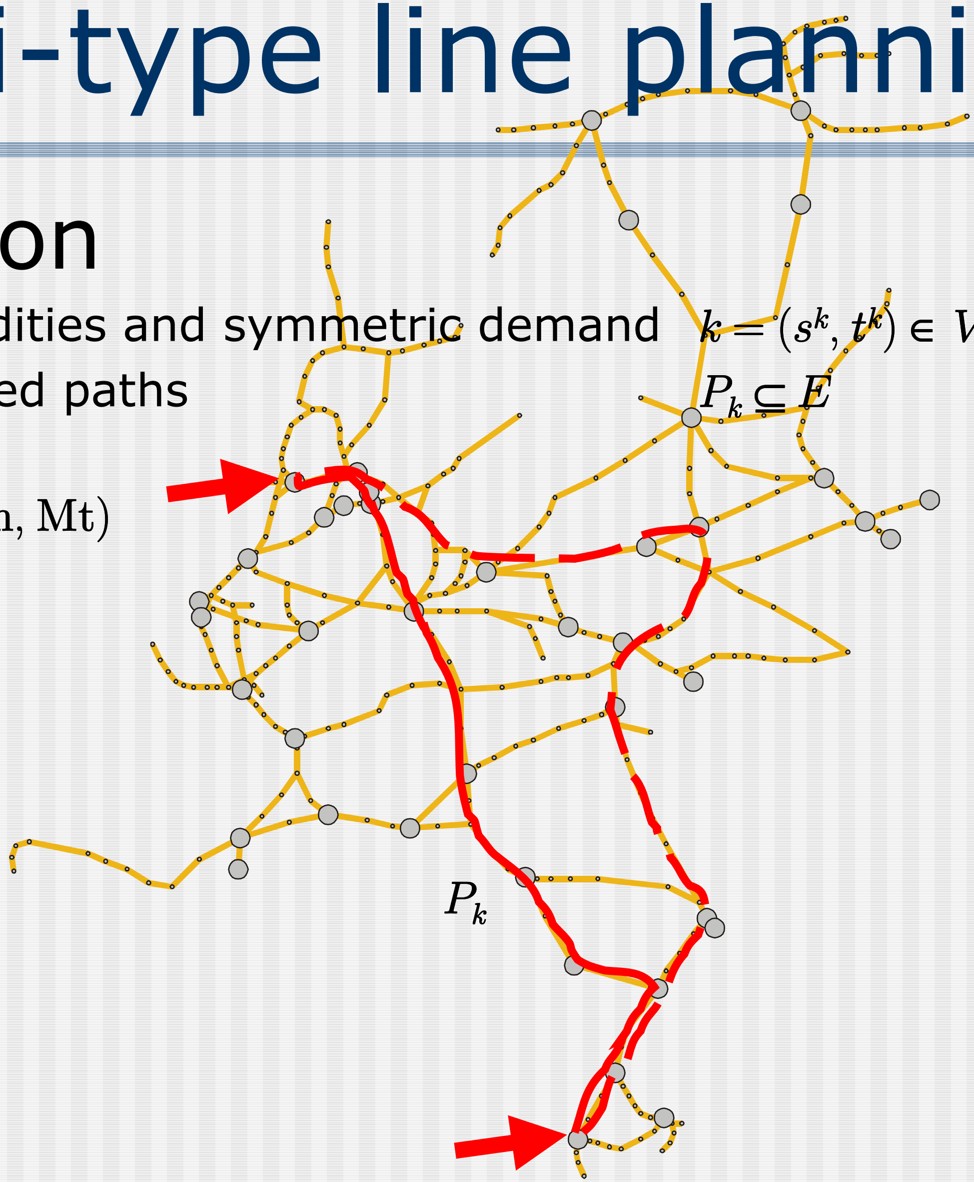
Route $R(e)$ of a type edge $e \in E^T$: $R(e) \subseteq E$ e.g. $R(\{c,v\}) = \{\{c,d\}, \{d,v\}\}$

Multi-type line planning

Notation

- Commodities and symmetric demand $k = (s^k, t^k) \in V \times V$ H^k or $H^{st} = H^{ts}$
- Prescribed paths $P_k \subseteq E$ $P_k^T \subseteq E^T$

$k = (\mathrm{Hlm}, \mathrm{Mt})$

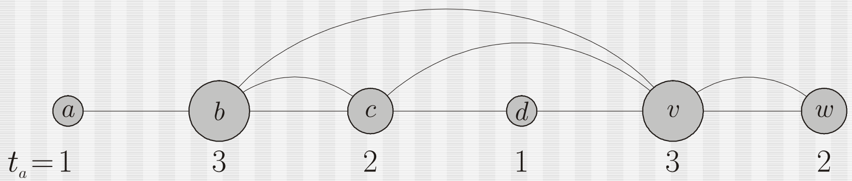


Multi-type line planning

Edge Capacity Problem (ECP)

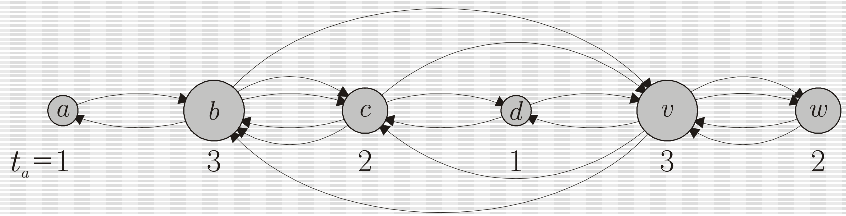
Determine minimum cost capacities for all type edges, such that all passengers can be transported through the network.

- Capacity variables for type edges $e \in E^T : x(e)$
- For every type edge, the combined line capacities must be at least equal to $x(e)$



$G^T = (V, E^T)$

Multi-commodity flow (MCF)



Directed type graph

$D^T=(V,A^T)$

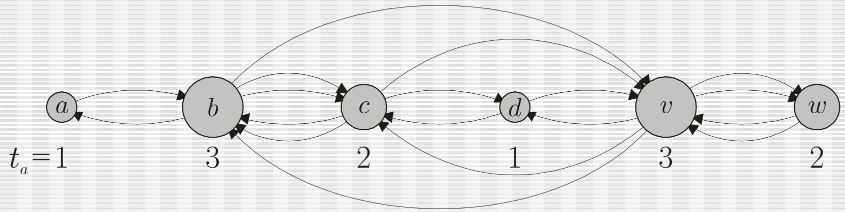
$$\min \quad \sum_{e \in E^T} f(x(e))$$

$$\text{s.t.} \quad x(e) \geq \sum_k F_{ij}^k \qquad \forall (i,j) \in A^T, e = \{i,j\} \in E^T$$

$$\sum_j F_{ij}^k - \sum_j F_{ji}^k = \begin{cases} H^k & \text{if } k \text{ starts at } i \\ 0 & \text{otherwise} \\ -H^k & \text{if } k \text{ stops at } i \end{cases} \qquad \forall i \in V, \forall k \in V \times V$$

$$x(e) \in C \qquad \forall e \in E^T$$

Multi-commodity flow (MCF)



Commodity Decomposition

$k=(a,w),\ P_k=\{(a,b),(b,c),(c,d),(d,v),(v,w)\} \Leftrightarrow P_k^T = \{(a,b),(b,v),(v,w)\}$

$k_1=(a,b),\ P_{k1}^T=\{(a,b)\}$ $k_2=(b,v),\ P_{k2}^T=\{(b,v)\}$ $k_3=(v,w),\ P_{k3}^T=\{(v,w)\}$

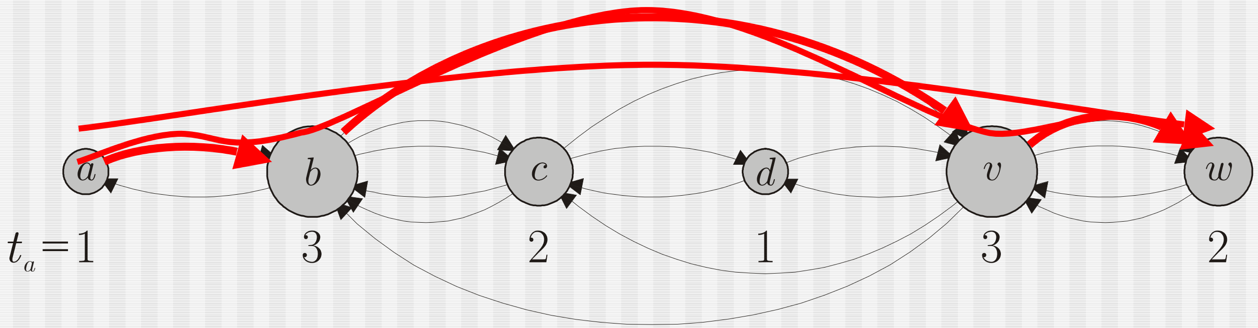
Commodity Aggregation

$k_1=(a,b),\ P_{k1}^T=\{(a,b)\}$ $k_1'=(a,b),\ P_{k'1}^T=\{(a,b)\}$ $k_1''=(a,b),\ P_{k''1}^T=\{(a,b)\}$

$k=(a,b),\ P_k^T=\{(a,b)\}$

Symmetric Flow

Multi-commodity flow (MCF)



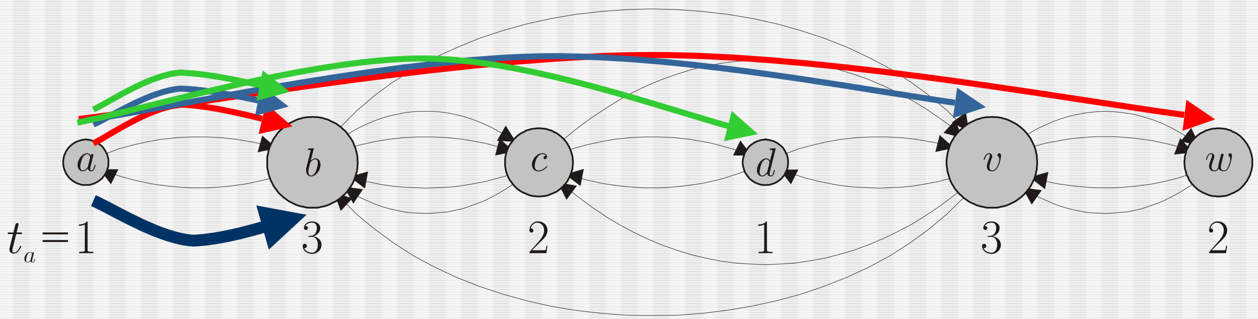
Commodity decomposition

$k=(a,w),\ P_k=\{(a,b),(b,c),(c,d),(d,v),(v,w)\} \Leftrightarrow P_k^T = \{(a,b),(b,v),(v,w)\}$

$k_1=(a,b),\ P_{k1}^T=\{(a,b)\}$ $\quad k_2=(b,v),\ P_{k2}^T=\{(b,v)\}$ $\quad k_3=(v,w),\ P_{k3}^T=\{(v,w)\}$

Every commodity is defined on exactly one type edge

Multi-commodity flow (MCF)



Commodity aggregation

$k_1=(a,b)$, $P_{k1}{}^T=\{(a,b)\}$ $k_1'=(a,b)$, $P_{k'1}{}^T=\{(a,b)\}$ $k_1''=(a,b)$, $P_{k''1}{}^T=\{(a,b)\}$

$k=(a,b)$, $P_k{}^T=\{(a,b)\}$

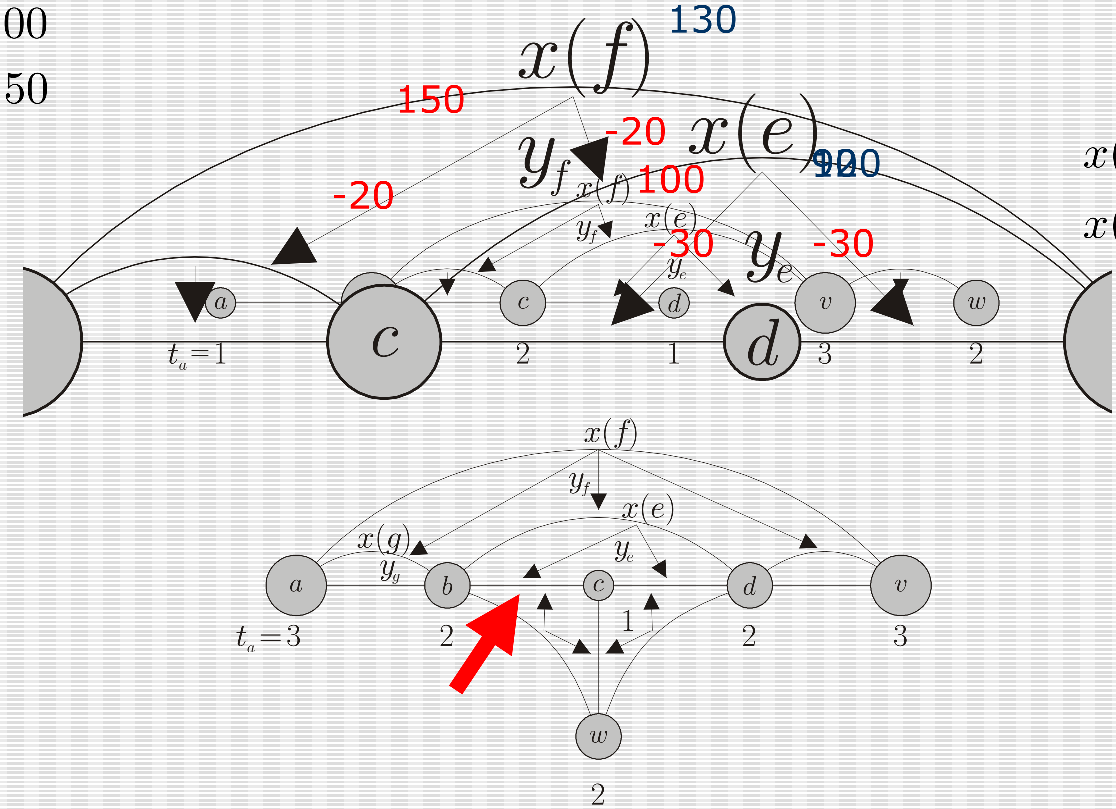
Symmetric flow

Integer programming (IP_{XY})

Assign travelers to the lower type edges

$H^e = 100$

$H^f = 150$



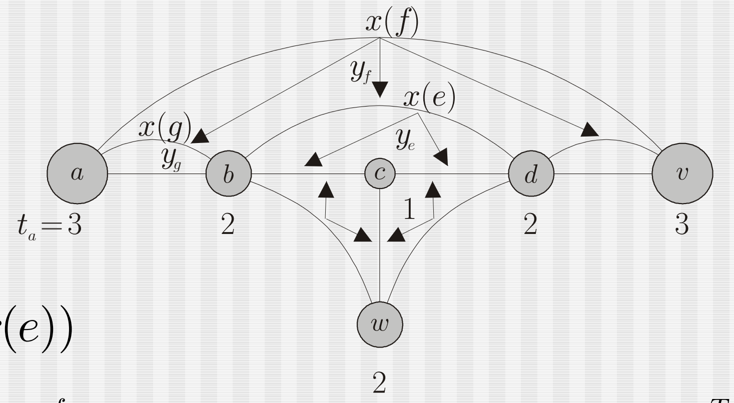
150 20

$x(f) \geq H^f - y_f$

$x(e) \geq H^e + y_f - y_e$

100 20 30

Integer programming (IP_{XY})



$$
\begin{array}{lll}
\min & \sum_{e \in E^T} f(x(e)) & \\
\text{s.t.} & x(f) \geq H^f - y_f & \forall f \in E^T_{T_{\max}} \\
& x(e) \geq H^e + \sum_{f \in E^T_{t+1} | R(f) \supseteq R(e)} y_f - y_e & \forall 1 < t < T_{\max}, e \in E^T_t \\
& x(e) \geq H^e + \sum_{f \in E^T_{t+1} | R(f) \supseteq R(e)} y_f & \forall e \in E^T_1 \\
& x(e) \in C & \forall e \in E^T \\
& y_e \in N & \forall e \in E^T
\end{array}
$$

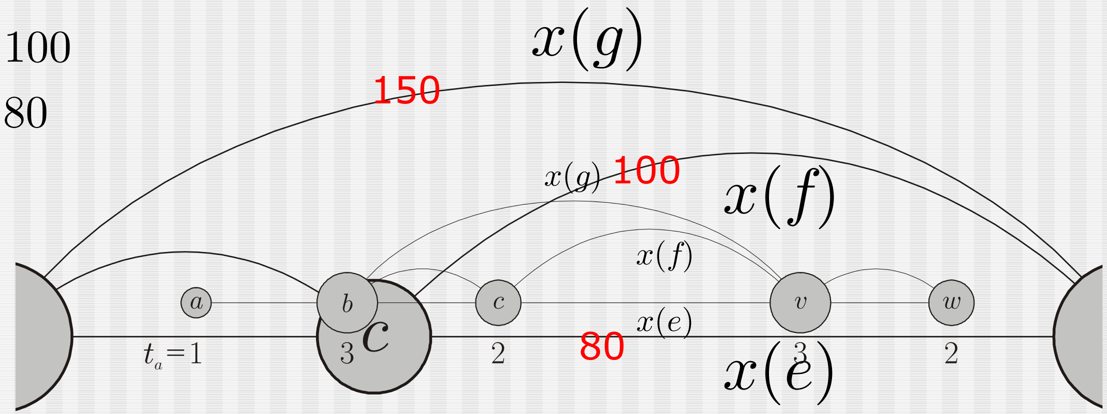
Integer programming (IP_X)

Impose additional capacity restrictions

$H^g = 150$

$H^f = 100$

$H^e = 80$



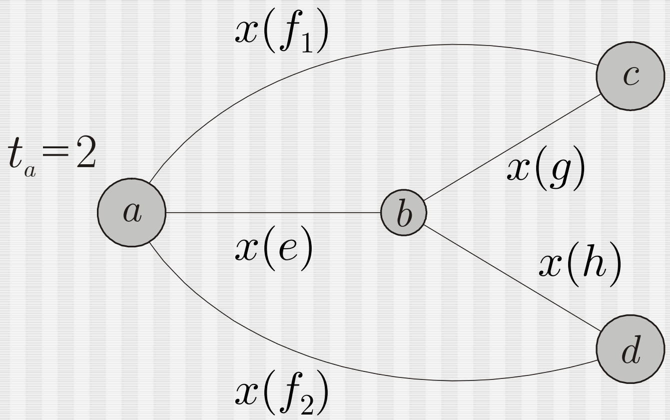
330

$x(e) + x(f) + x(g) \geq H^e + H^f + H^g$

$x(e) + x(f) \geq H^e + H^f$ 80 + 100

$x(e) \geq H^e$ 80

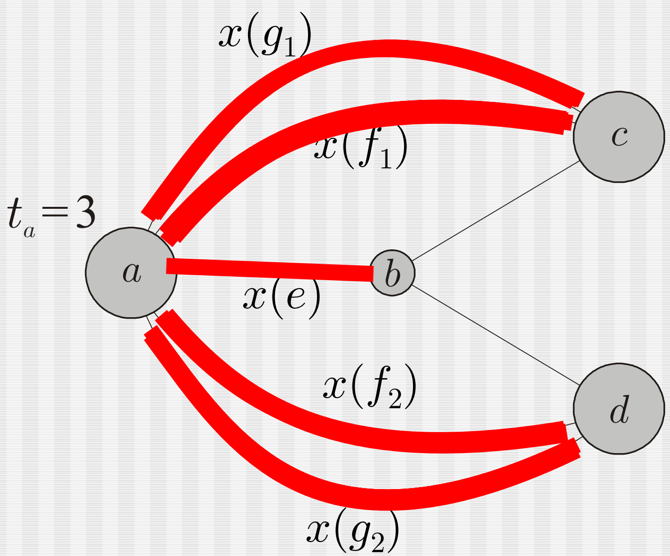
Integer programming (IP_X)



$$
\begin{aligned}
x(e) & & & \geq H^e \\
x(e) + x(f_1) & & & \geq H^e + H^{f1} \\
x(e) + x(f_1) & + x(f_2) & & \geq H^e + H^{f1} + H^{f2} \\
x(e) & + x(f_2) & & \geq H^g + H^{f2} \\
\\
x(g) & & & \geq H^g \\
x(g) + x(f_1) & & & \geq H^g + H^{f1} \\
\\
x(h) & & & \geq H^h \\
x(h) & + x(f_2) & & \geq H^h + H^{f2}
\end{aligned}
$$

Integer programming (IP_X)

Exponentially many subset restrictions



$\{e\}$
$\{e, f_1\}$
$\{e, f_1, g_1\}$
$\{e, f_2\}$
$\{e, f_2, g_2\}$
$\{e, f_1, f_2\}$
$\{e, f_1, g_1, f_2\}$
$\{e, f_1, f_2, g_2\}$
$\{e, f_1, g_1, f_2, g_2\}$
$\ldots$

not e.g.: $\{e, g_1\}$
$\{e, f_2, g_1\}$

Integer programming (IP_X)

$$\begin{aligned}
\min \quad & \sum_{e \in E^T} f(x(e)) \\
\text{s.t.} \quad & x(e) + \sum_{t>1} \sum_{f \in S_t^e} x(f) \geq H^e + \sum_{t>1} \sum_{f \in S_t^e} H^f && \forall e \in E_1^T, \forall S_2^e, \ldots, \forall S_{T_{\max}}^e \\
& x(e) \in C && \forall e \in E^T \\
& y_e \in N && \forall e \in E^T
\end{aligned}$$

$$S_t^e \subseteq \{ g \in E_t^T \mid \exists f \in S_{t-1}^e : R(f) \subseteq R(e) \}$$

$$S_1^e = \{\text{e}\}$$

Comparing the models

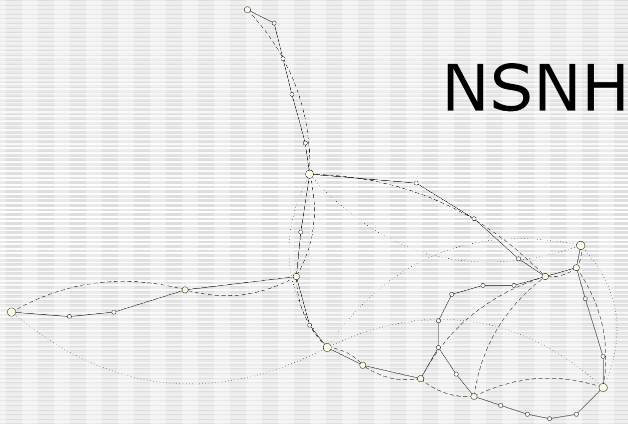
	MCF	IP_{XY}	IP_X
# Vars	$\lvert E^T\rvert + O(\lvert E^T\rvert^2)$	$\lvert E^T\rvert + \lvert E^T \setminus E\rvert$	$\lvert E^T\rvert$
# Cons	$\lvert E^T\rvert + O(\lvert E^T\rvert\lvert V\rvert)$	$\lvert E^T\rvert$	$O(2^{\lvert E^T\rvert})$

- IP_{XY} has $\lvert E^T \setminus E\rvert$ more continuous variables, compared to IP_X.
- IP_X can have *many* (exponentially) more constraints than IP_{XY}.

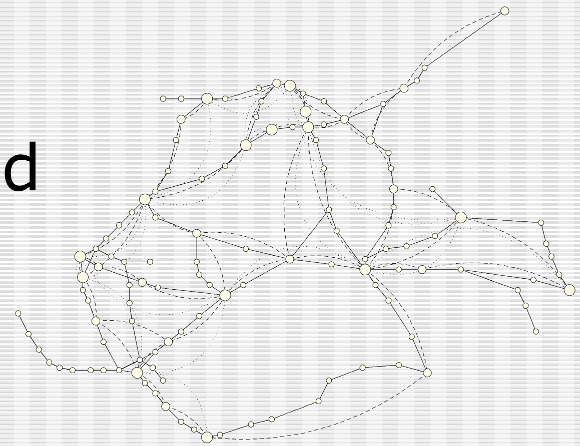
Computational experiments

NS3600

NSNH

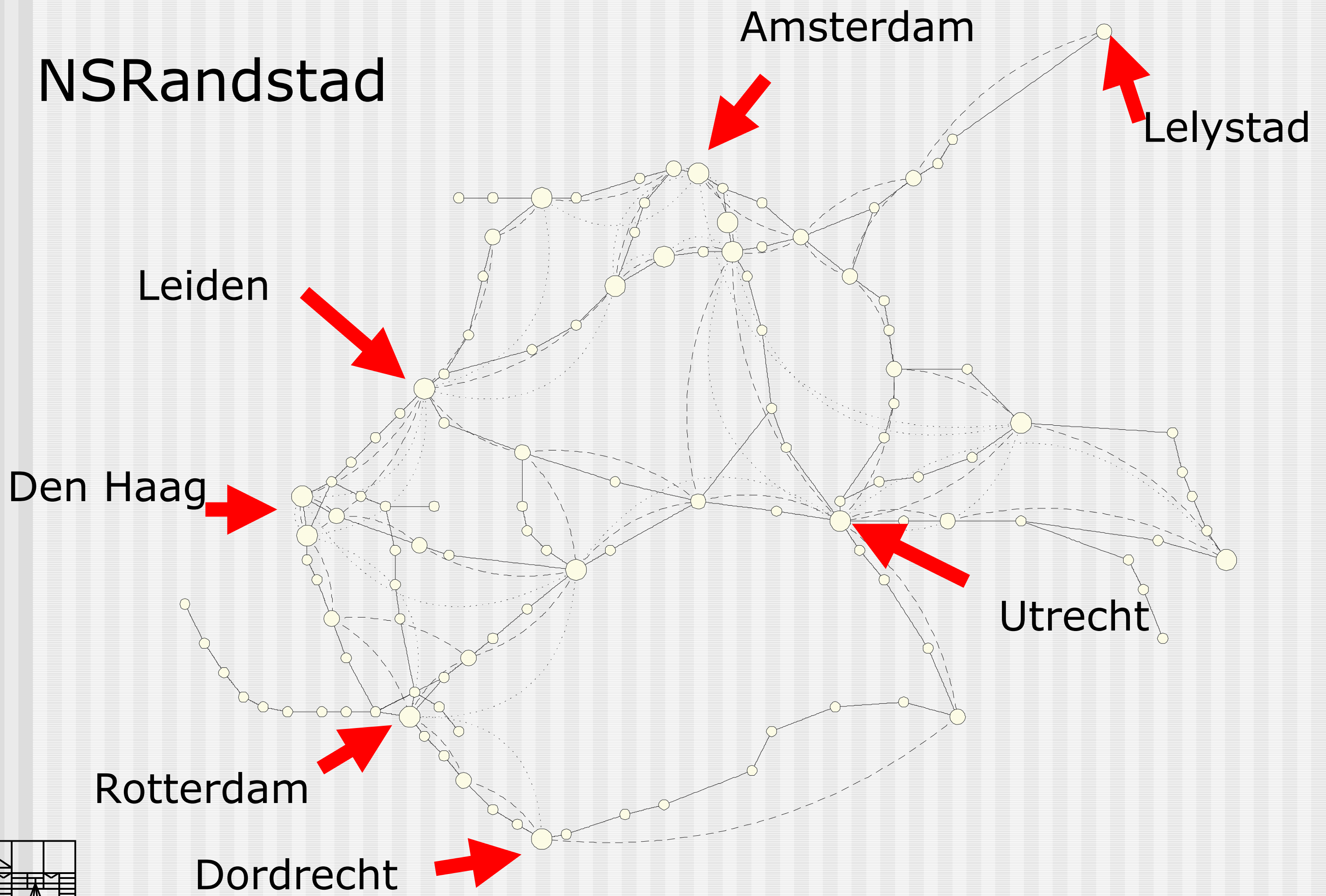


NSRandstad



Computational experiments

NSRandstad



Computational experiments

Instance	Model	# vars.	# cons.	# subsets
NS3600	IPX	1280	145	81
NS3600	IPXY	1303	114	
NSNH	IPX	1620	230	149
NSNH	IPXY	1641	139	
NSRandstad	IPX	6620	734	403
NSRandstad	IPXY	6686	535	

Instance	Model	best	root LP	best LP	Gap	# sec.	# nodes
NS3600	IPX	7520	7213	7520	0.00%	0.81	60
NS3600	IPXY	7520	6430	7173	4.61%	-	1206968
NSNH	IPX	13760	13133	13760	0.00%	6.66	407
NSNH	IPXY	13760	12501	13313	3.25%	-	747874
NSRandstad	IPX	52480	48880	50510	3.75%	-	25008
NSRandstad	IPXY	55360	46146	48733	11.97%	-	104234

CPLEX 7.5 on AMD Athlon 800Mhz, 512 MB, Linux kernel 2.4.8

Time limit of 3600 seconds

Conclusions & final remarks

- IP_X model wins
  - Number of necessary subset inequalities in real-life instances is modest
  - Higher root LP values -> smaller b&b tree
- Additional topics
  - Use e.g. branch-and-cut
  - New classes of model inequalities
    - Track/station utilization
    - Delay sensitivity

“On solving multi-type line planning problems”, J.G., S. van Hoesel, and L. Kroon. METEOR Research Memorandum RM/02/009, University of Maastricht, 2001

email: j.goossens@ke.unimaas.nl