Satisfying Train Timetables: Rolling Stock Rostering

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joint work with

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Overview

Part I:

- Problem Definition
- Flow Based Model:
  - Train length problem
  - Other Solvable Cases

Part II:

- Hard Cases
- Open Problems

Problem Definition

Goal

Given: Periodic train schedule (times, stations)

Produce: Minimum cost train assignment

Example: Minimize number of cars

Constraints and Variants

Maintenance: Cleaning, supplies, safety inspections, repairs...

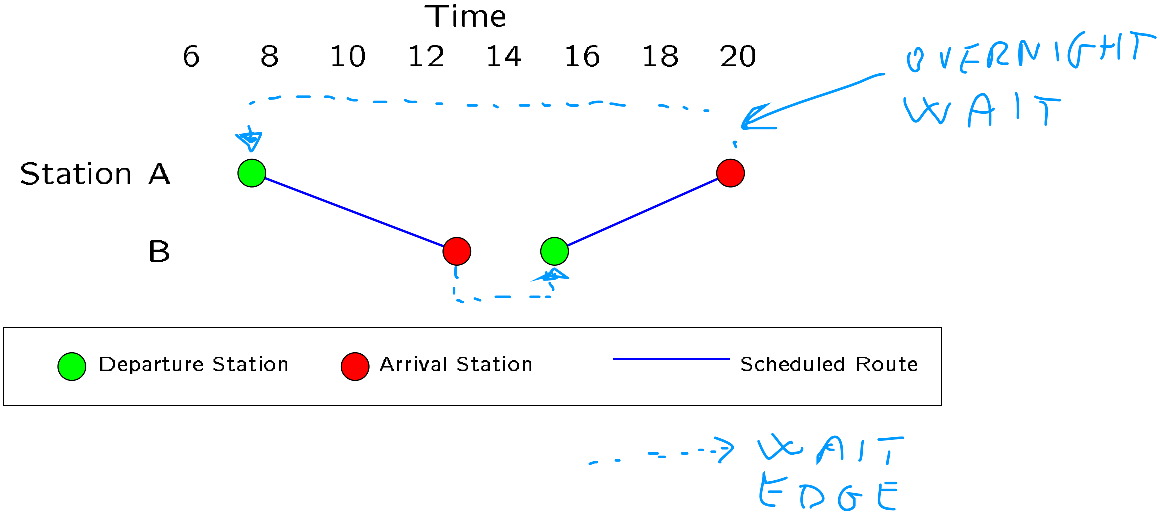
Routes: Empty movements allowed? Variable train sizes? Identical cars?...

Station Limits: Capacity, track topologies and shunting, train arrival safety time buffers...

CISALPINO Example

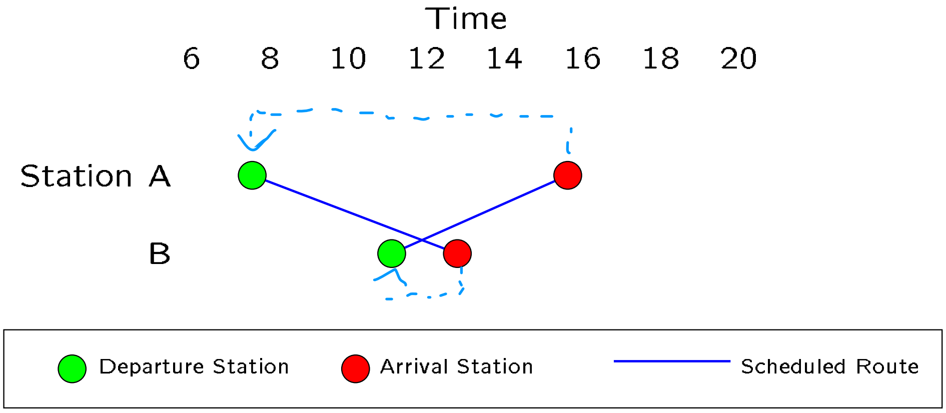
Basel 6:17	to Milano 10:50
Firenze 16:00	to Zürich 22:53
Geneve 6:05	to Milano 9:50
Geneve 9:05	to Venezia 15:57
Milano 7:15	to Stuttgart 14:00
Milano 11:10	to Stuttgart 17:00
Milano 17:10	to Basel 21:41
Milano 17:55	to Zürich 21:31
Milano 18:10	to Geneve 21:55
Stuttgart 10:02	to Milano 16:45
Stuttgart 16:02	to Milano 22:45
Venezia 17:00	to Geneve 23:55
Zürich 6:33	to Milano 10:15
Zürich 7:04	to Firenze 14:06

Departure		Arrival	
station	time	station	time
A	7:45	B	12:45
B	15:00	A	20:00



1 CYCLE OF LENGTH 1 DAY $\Rightarrow$ 1 TRAIN

Departure		Arrival	
station	time	station	time
A	7:45	B	12:45
B	10:30	A	15:30

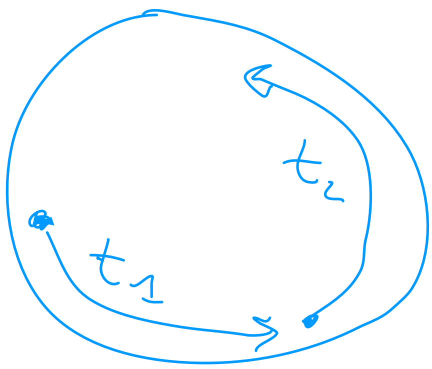


1 CYCLE OF LENGTH 2 DAYS $\Longrightarrow$ 2 TRAINS

How Many Trains?

- Solution: A set of **cycles**
- Cycle: Represents what a **set of trains** do
- Cycle Length $\Rightarrow$ Number of Trains

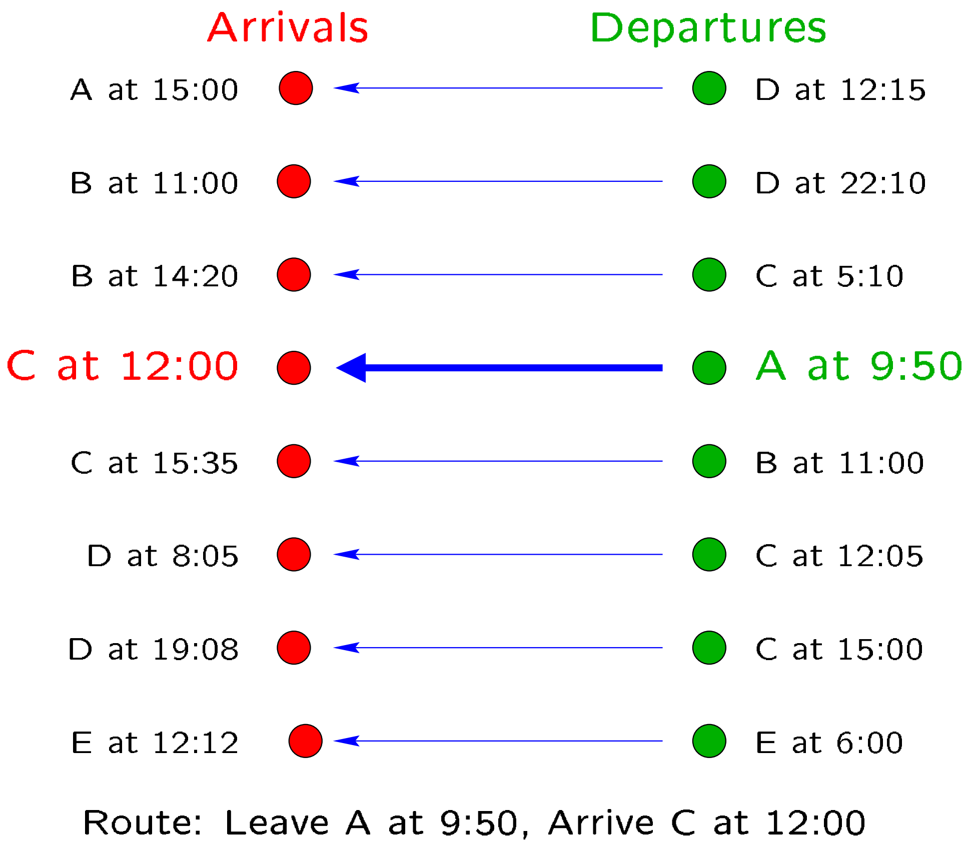
EX:



CYCLE OF LENGTH 3 DAYS

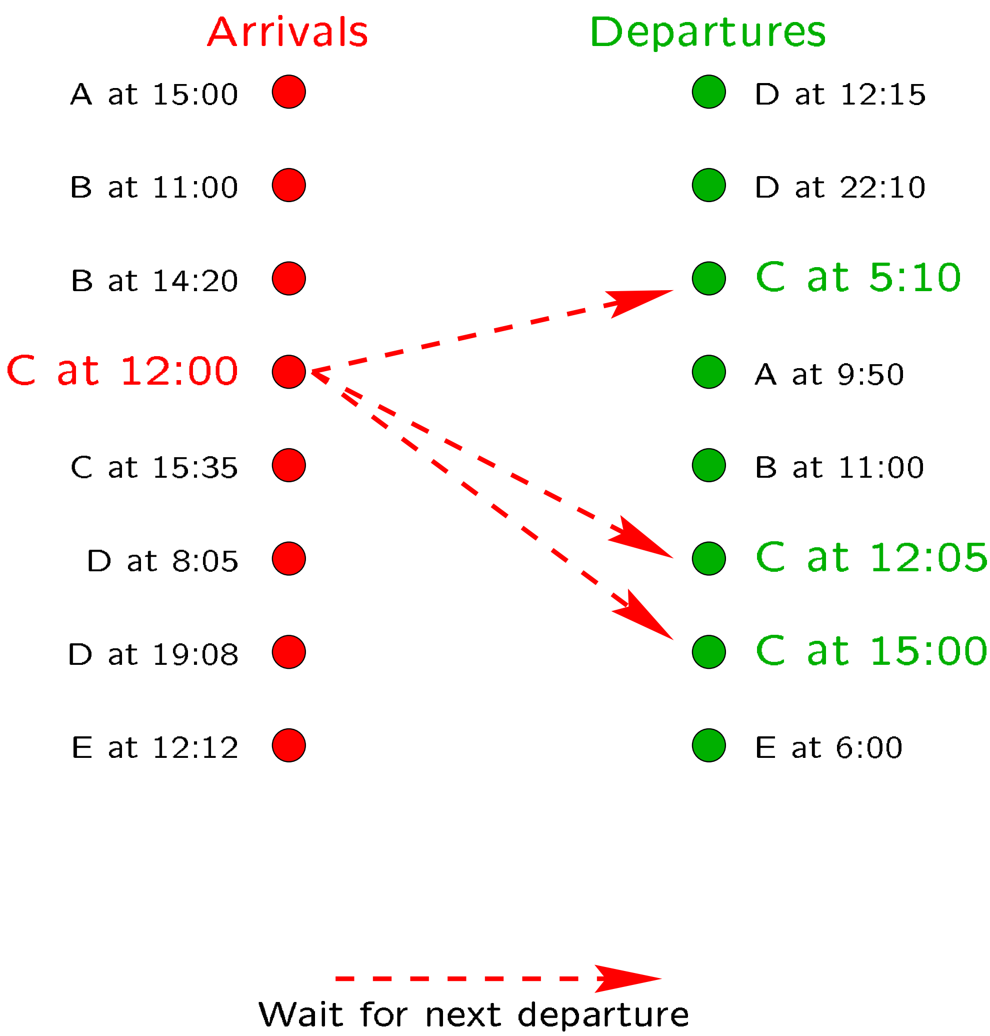
2 TRAINS ARE NOT ENOUGH TO COVER ALL ROUTES IN ONE DAY

Transform to Graph



Map routes to arrival and departure nodes.

Bipartite Matching



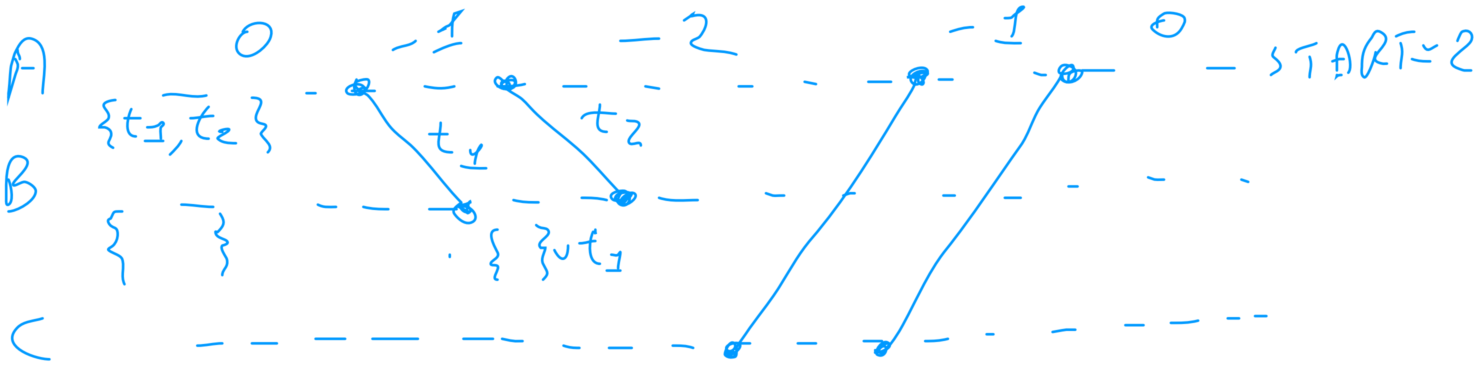
Edge weights = Train waiting times

Faster Basic Rostering

Compute:

1. Number of trains to start with

2. Assign trains to routes

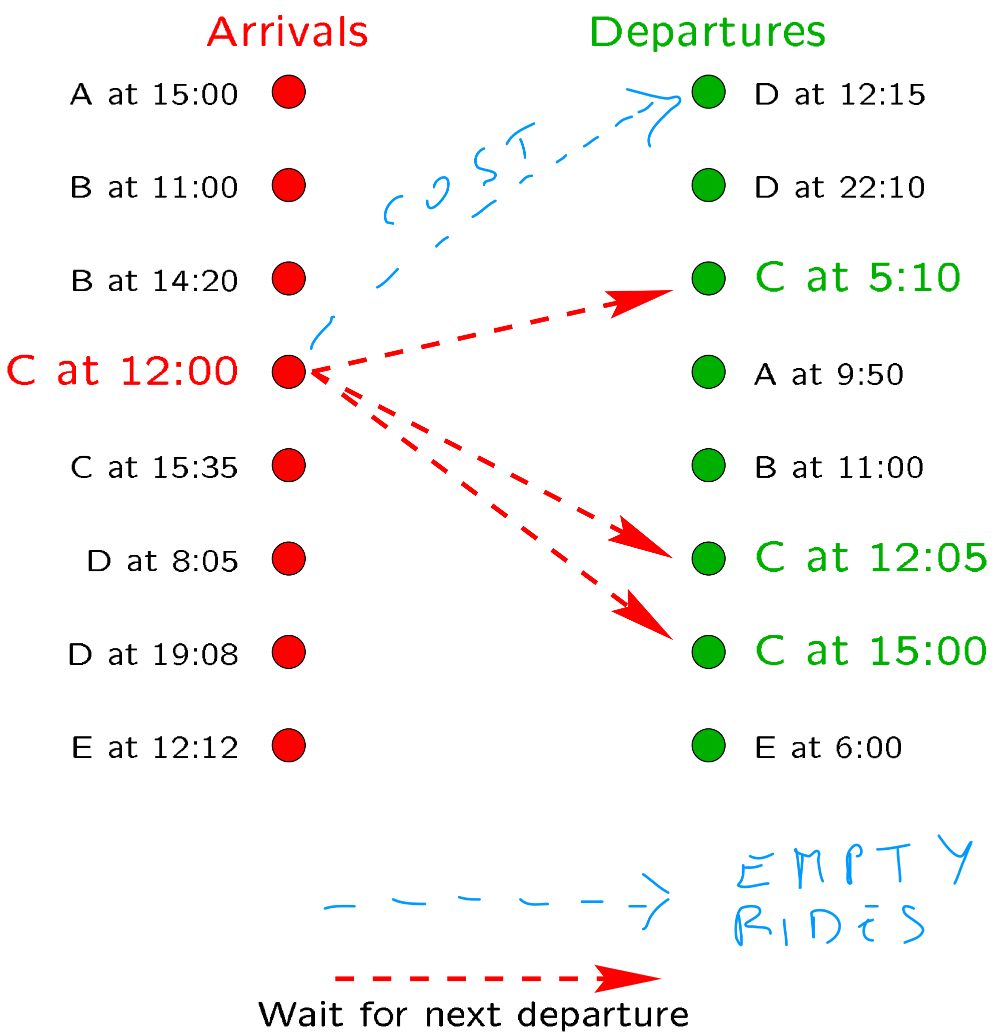


Complexity: $O(n \log n)$-time

What did we solve?

- Cost: Number of trains:
  - Each train is an identical unit.
  - Each route needs one train.
- There are no empty train movements.
- No trains need maintenance. Ever.

Bipartite Matching



Edge weights = Train waiting times

The Train Length Problem

- Cost: Number of **cars** (train units)
  - For each route a **minimum number of units** is needed
  - Each train unit is **self powered** (i.e. no locomotives)

Min Cost Flow Circulation

For every route:

1. Min Train Length: l_{ij}

2. Linear Cost Function: $c_{ij} \cdot x_{ij}$

<u>Min Cost Flow Circulation:</u>

Minimize

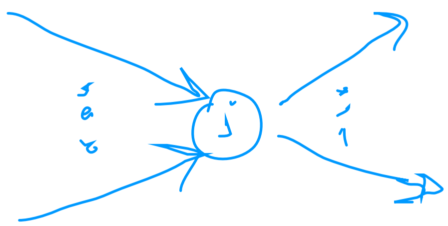
$$\sum_{(i,j)\in E} c_{ij} \cdot x_{ij}$$

$$\bigcirc \xrightarrow{c_{ij}} \bigcirc$$

s.t.

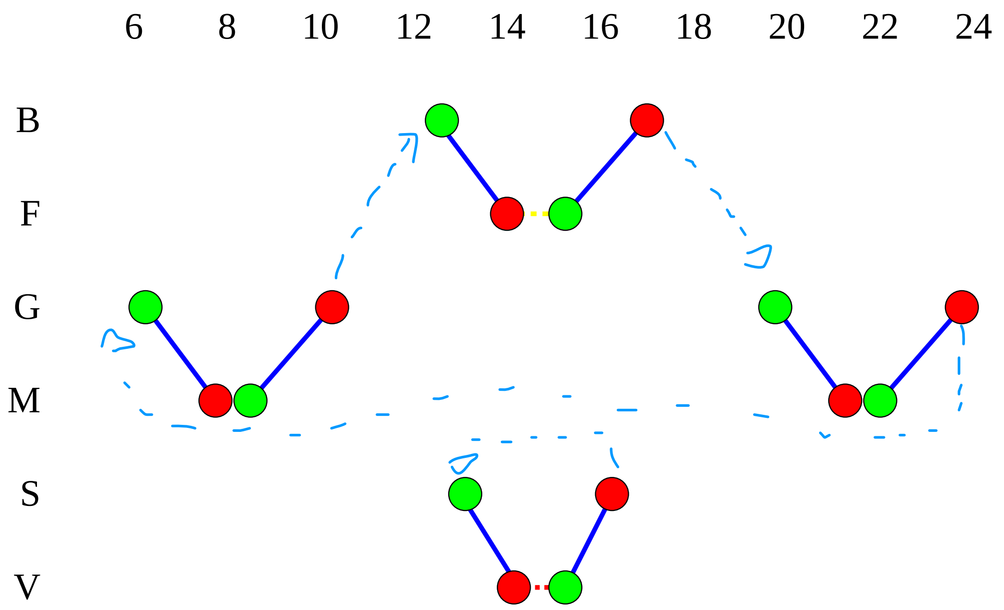
$$\sum_{i:(i,j)\in E} x_{ij} - \sum_{k:(j,k)\in E} x_{jk} = 0 \quad \forall k \in V$$

$$l_{ij} \le x_{ij} \le u_{ij} \quad \forall (i,j) \in E$$



Periodic Timetable ⇒ Periodic Solution?

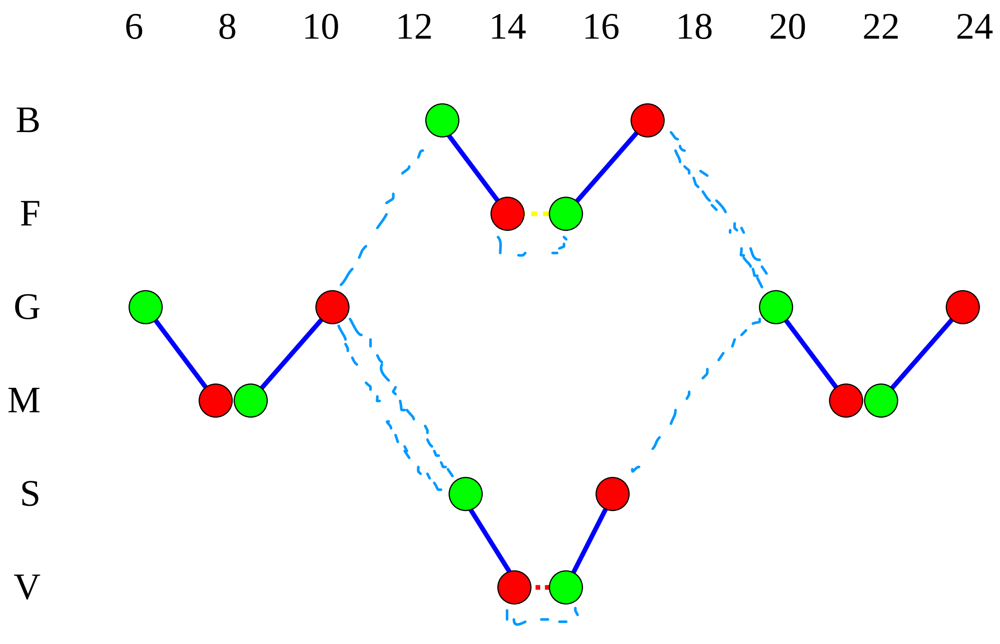
For a daily schedule, does a station start each day with the same number of trains?



THE UPPER CYCLE
IS LONGER (AS # Km)
THAN THE LOWER ONE

Periodic Timetable ⇒ Periodic Solution?

For a daily schedule, does a station start each day with the same number of trains?



A SOLUTION WITH PERIOD 2 DAYS:

= DAY 1

= DAY 2

STILL 2 TRAINS AND EQUALIZES MILEAGE

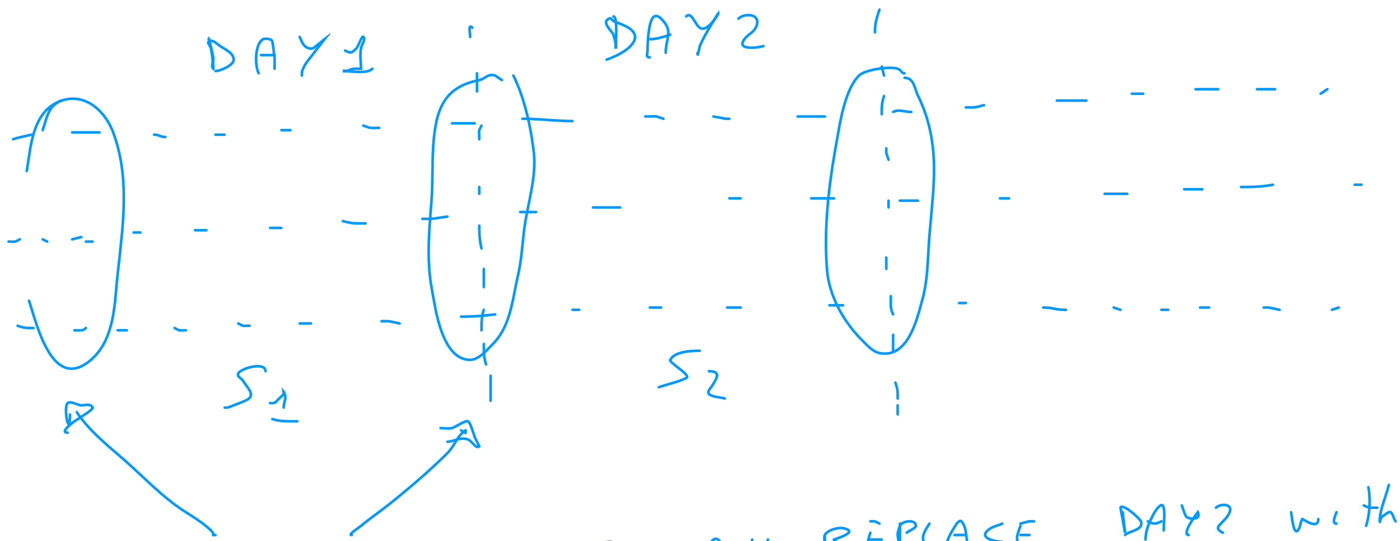
Periodic Solutions

- easy to follow
- used by railway companies

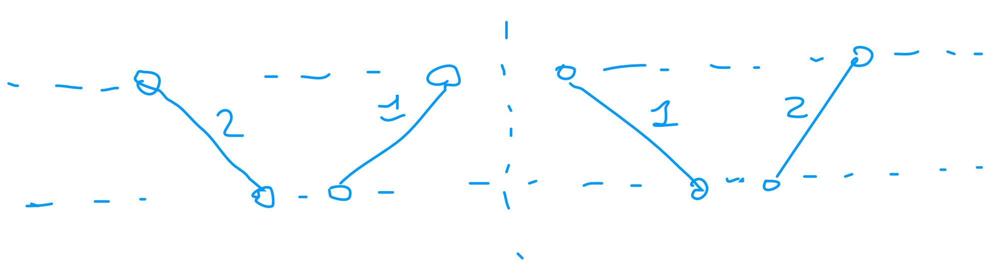
Are they optimal (w.r.t. more general ones)?

	NO EMPTY	EMPTY
NO MAINTEN.	YES	YES
MAINTEN.	YES	NO

EASY CASE: BASIC RSR



SAME $\Rightarrow$ WE CAN REPLACE DAY 2 with DAY 1 (FOR INSTANCE)



$x_{ij} \Longrightarrow x_{ij}^{d} \Leftarrow$ DAY

1) INFINITE $x^1 x^2 \ldots \Rightarrow$

FINITE SUBSEQUENCE

$x^{\alpha} x^{\alpha+1} \ldots x^{\alpha+c}$

2) $x^{\alpha} \ldots x^{\alpha+c} \Rightarrow x'$

1

$x^1, \ldots, x^{\alpha} \ldots x^{\alpha+c}$

SAME

$x^{\alpha} \ldots x^{\alpha+c-1}\, x^{\alpha} \ldots x^{\alpha+c-1} \ldots$

2

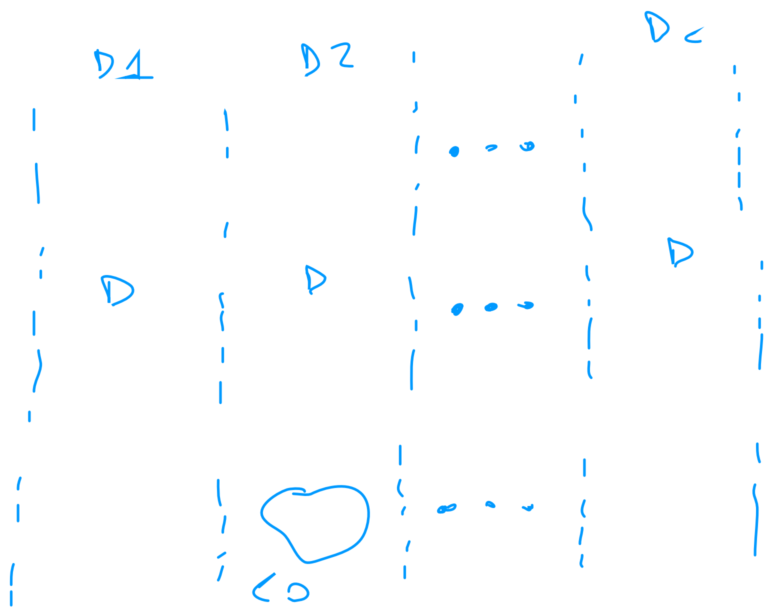
C-PERIODIC

BEST 1-DAY

$\Downarrow$

$[1\text{-DAY}]^c$

IF DIFF < 0



$\Uparrow$

WE CAN IMPROVE THE 1-DAY SOLUTION

Further Desiderata

- Equalize Mileage

- Overnight wait vs Station security