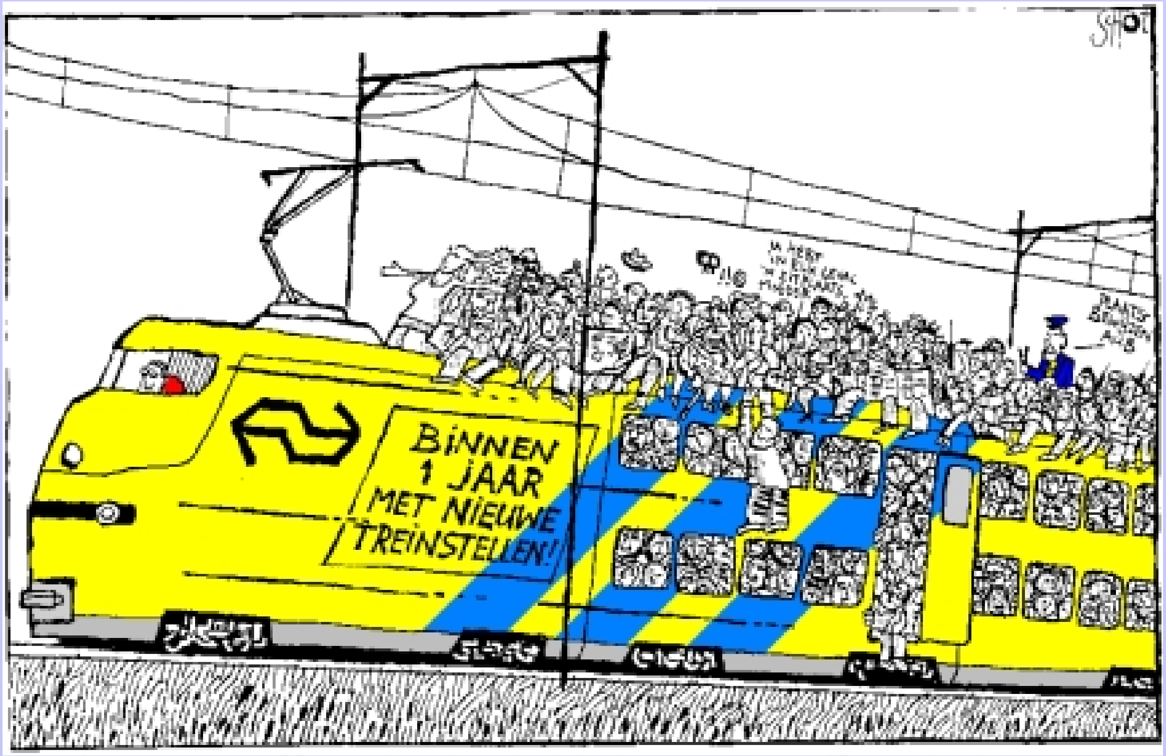
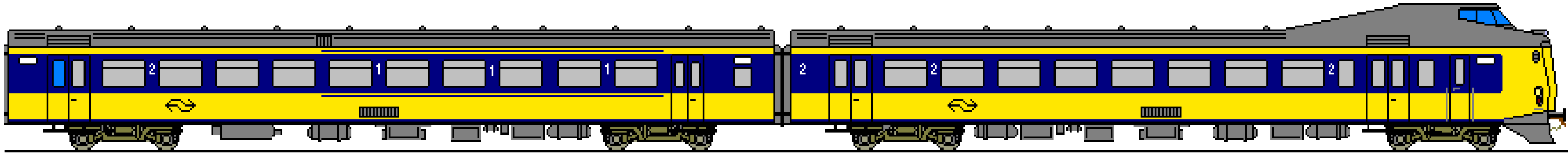
Models for rolling stock planning

Leo Kroon, AMORE meeting Patras, Oct/Nov 2001

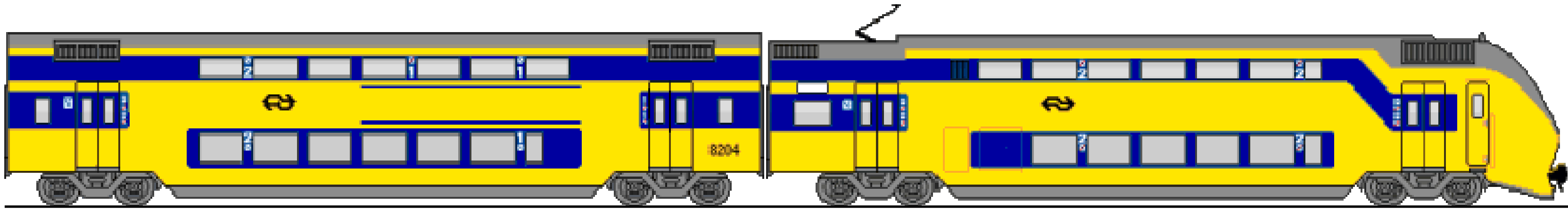


Rolling stock planning

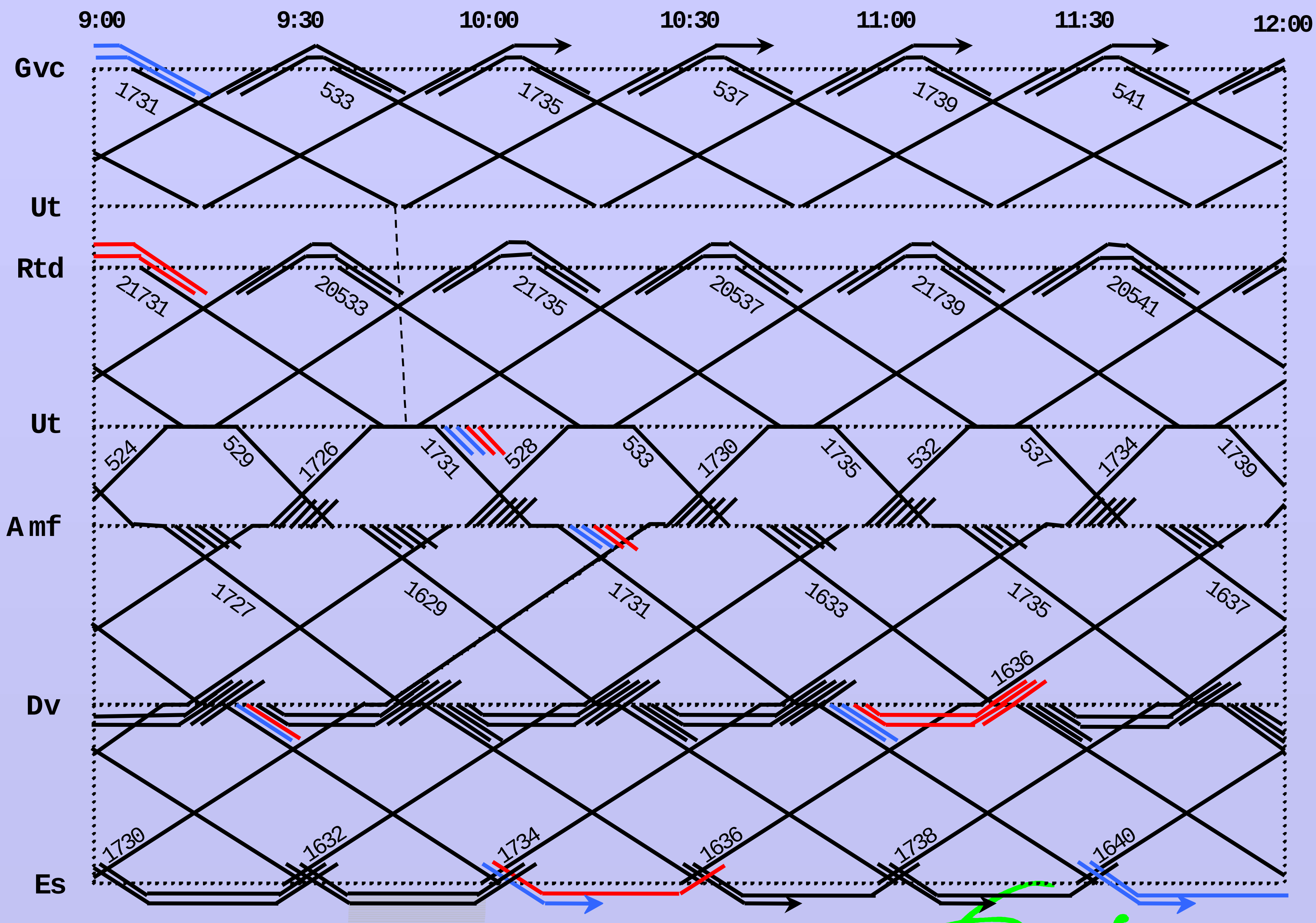
- In rush hours: Allocation of scarce capacity
- Outside rush hours: Efficiency



Koploper with 3 or 4 carriages



Double Decker with 3 or 4 carriages



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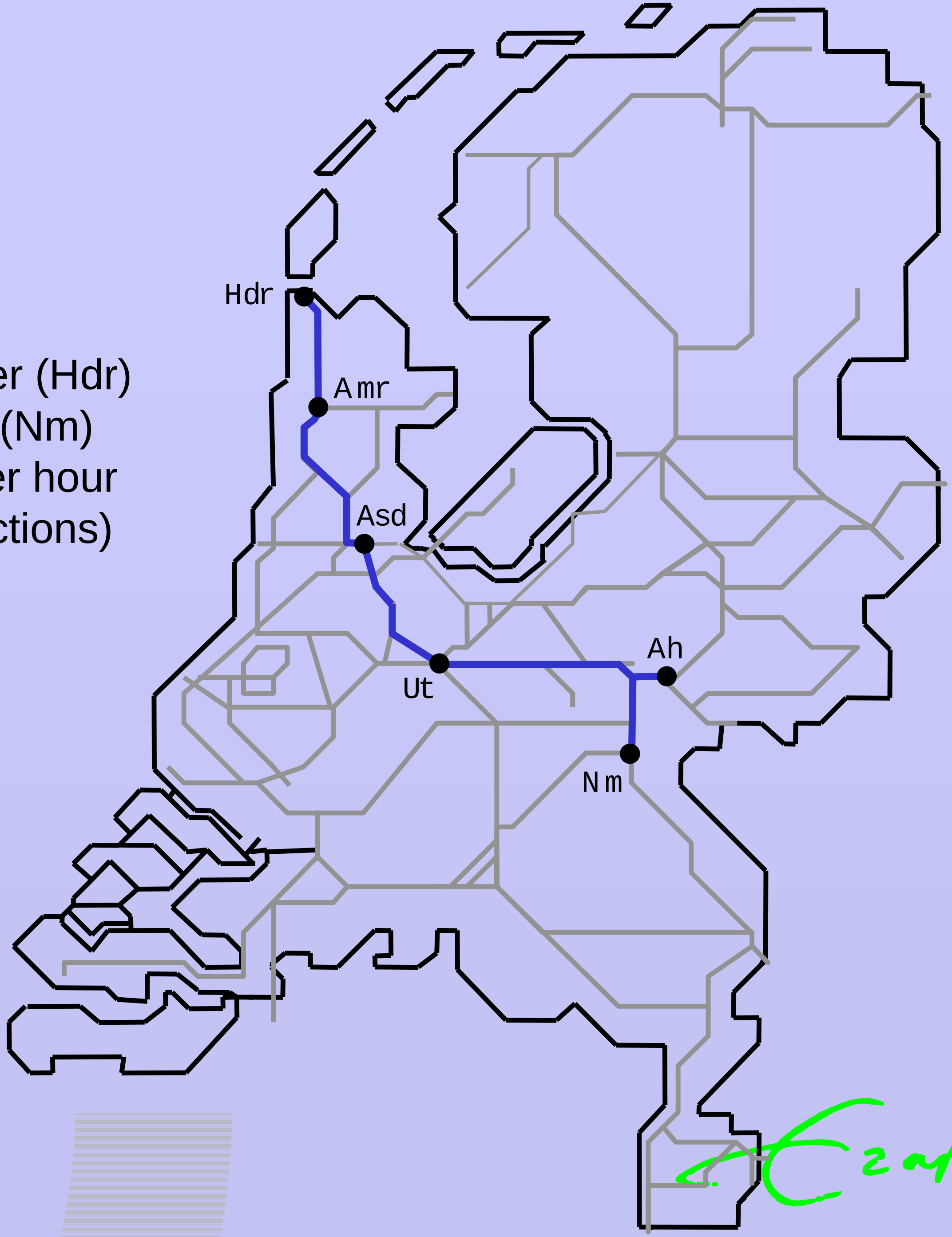
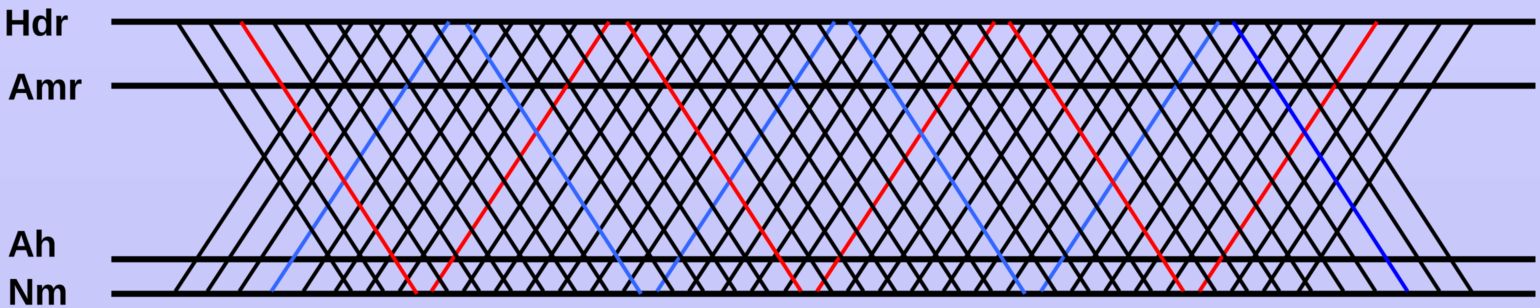
Line 3000

From: Den Helder (Hdr)
To: Nijmegen (Nm)
Freq: 2 trains per hour
(both directions)



Line 3000: 12 compositions (5 Koploper, 7 Double Decker)



Details per station

Hdr: Incoming compositions of the trains are unchanged

Amr: Going north: units may be uncoupled from the rear
Going south: units may be coupled to the front

Ah: Front and rear of the train are interchanged

Nm: Units may be coupled or uncoupled (not both) at the (incoming) front

Details per track

Hdr-Amr: Max. length of the trains is 9 carriages

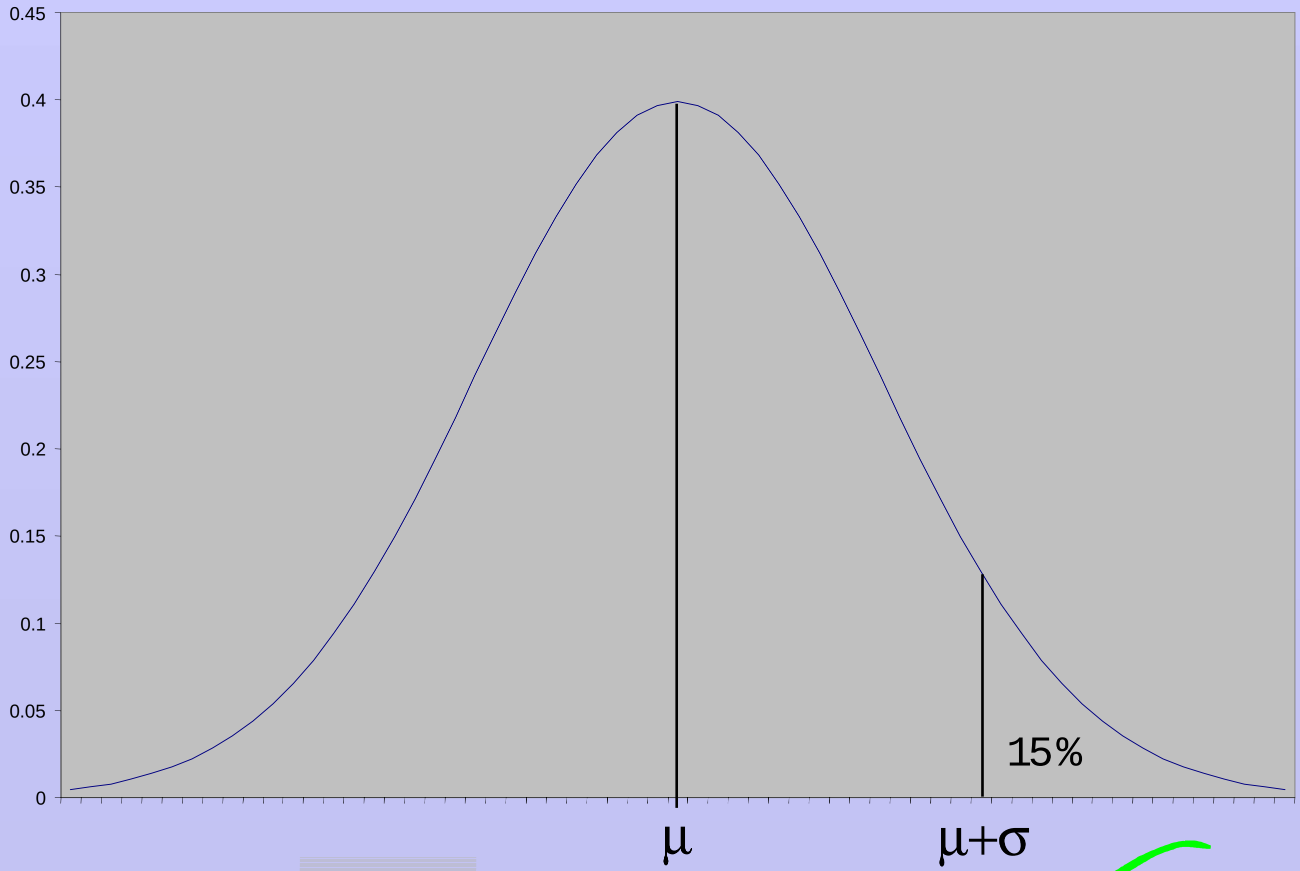
Amr-Nm: Max. length of the trains is 12 carriages

Questions:

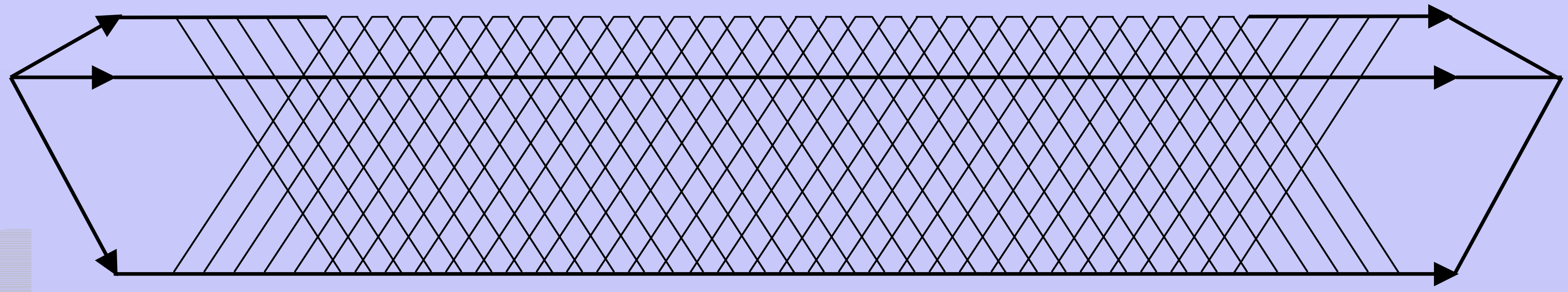
- What is the minimum capacity of the rolling stock that is necessary for providing a certain service level?
- What is the minimum number of carriage kilometers that is necessary for providing a certain service level?
- What is the maximum service level that can be provided with a given capacity of rolling stock?
- What is the maximum service level that can be provided with a given capacity of rolling stock and within a given number of carriage kilometers?
- …

Minimum required capacity per trip:

- Based on counting figures by conductors



1 type of train units: min cost flow problem
with additional constraints



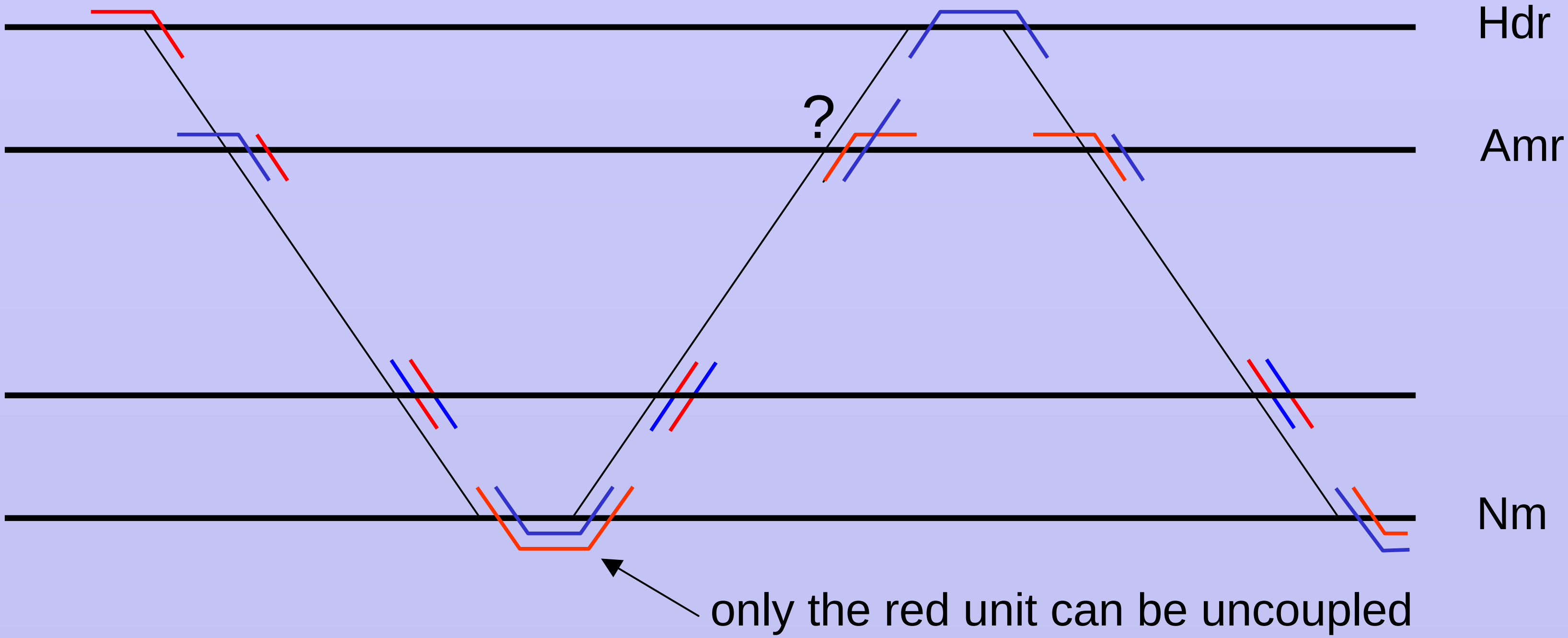
Additional constraints

- Service constraints
- Going north in Amr: units must not be be coupled
- Going south in Amr: units must not be be uncoupled
- Max. parking space at/near stations
- Circulation constraints

2 types of train units: multi-commodity flow problem

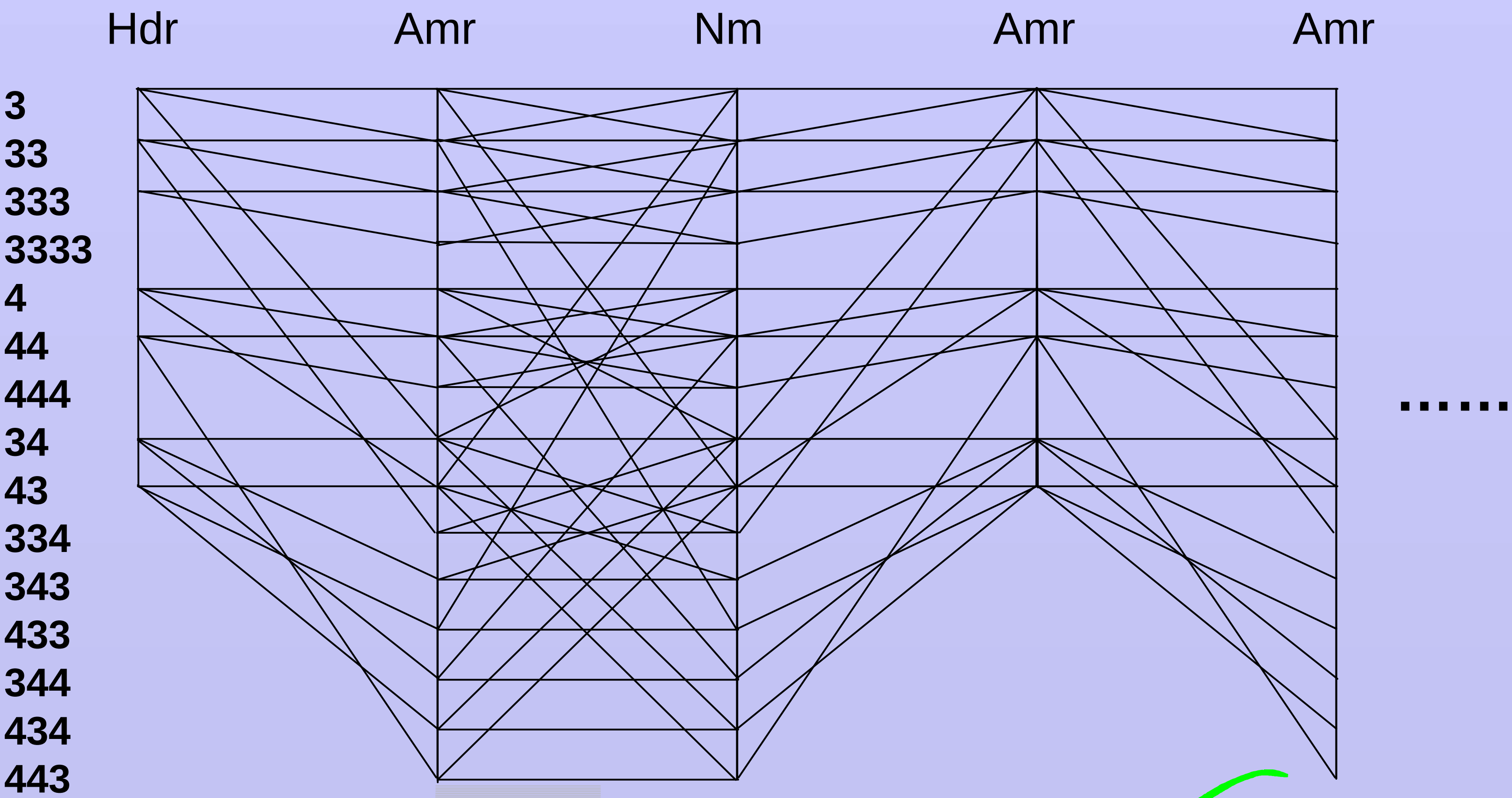
However: not only the # of train units per train, but also their order in the train is relevant, because of the shunting possibilities at the stations

Example



The approach of Schrijver & Groot:

- Composition graph per composition: look for paths in these graphs
- Link this graph to the multi-commodity flow problem



The approach of Schrijver & Groot:

Algorithm:

- Solve the integer multi-commodity flow problem without taking into account the composition graphs

- Fix the total number of units of the two types
- For each of the trips do

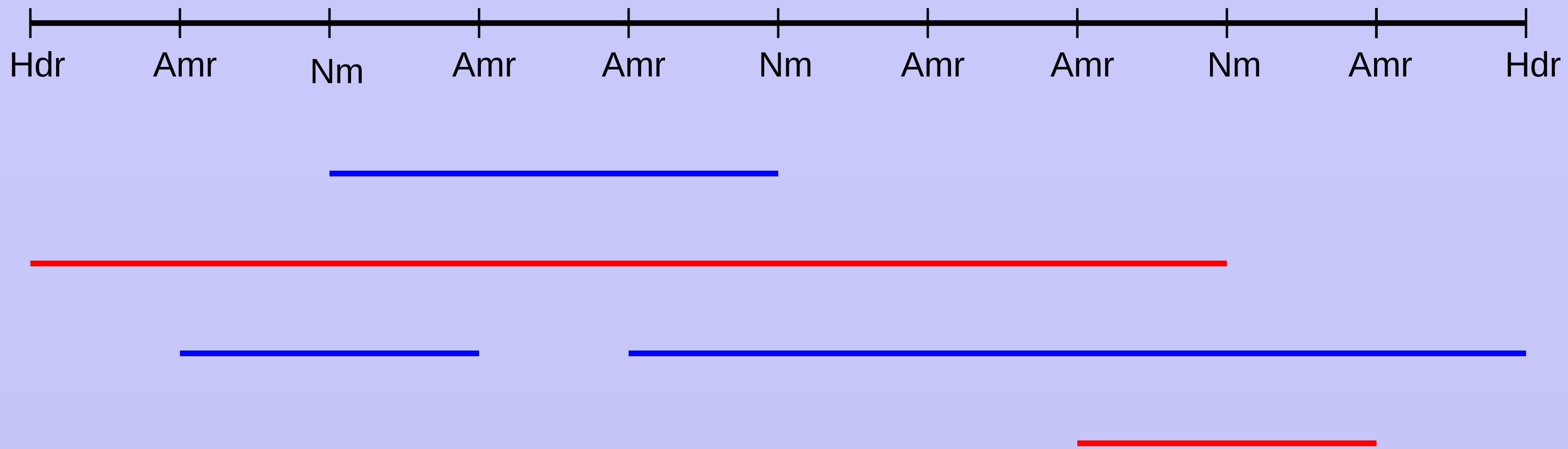
  For each of the possible capacities do

  - solve the continuous multi-commodity flow problem
  - if the capacity does not fit for the trip, then delete the corresponding nodes from the composition graph
- If one of the composition graphs becomes disconnected, then increase the total number of train units, and restart

- Otherwise solve the integer multi-commodity flow problem, thereby taking into account the reduced composition graphs

An alternative approach (Set Covering ++):

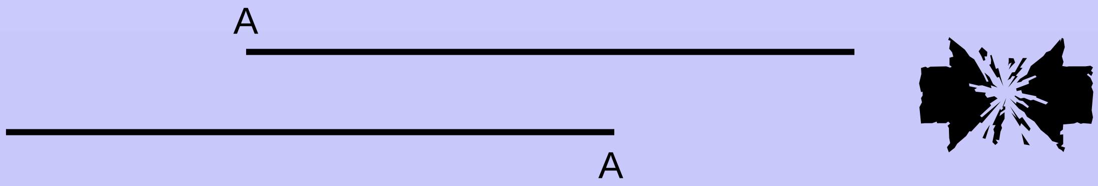
Example for 1 composition:



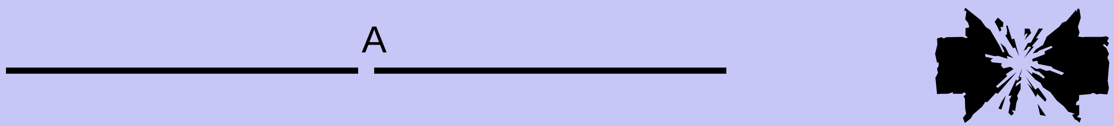
- Generate *all* potential duties
- Select an appropriate subset of the potential duties that *fit together* and are *optimal* in some sense

Relevant constraints:

- LIFO coupling and uncoupling for each side of the train



- Coupling and uncoupling cannot take place at the same time and station



- If a unit is coupled underway in X and uncoupled underway in Y, then $X=Y$

Objective:

A combination of shortages of seats (in rush hours)
and efficiency (outside rush hours)

Decision Variables:

X_d = potential duty d is/is not used

T_d = # train units of length 3 in duty d

F_d = # train units of length 4 in duty d

Decision variables for shortages on trips (1st and 2nd class)

Decision variables for stock keeping of train units in stations (Hdr, Amr, Nm)

Constraints:

$$X_d \le T_d + F_d \le M \cdot X_d$$ for all duties d

$$S_t^2 \ge P_t^2 - \sum_{d:t \in d} (c_{23} T_d + c_{24} F_d)$$ for all trips t

$$S_t^1 \ge P_t^1 - \sum_{d:t \in d} (c_{13} T_d + c_{14} F_d)$$ for all trips t

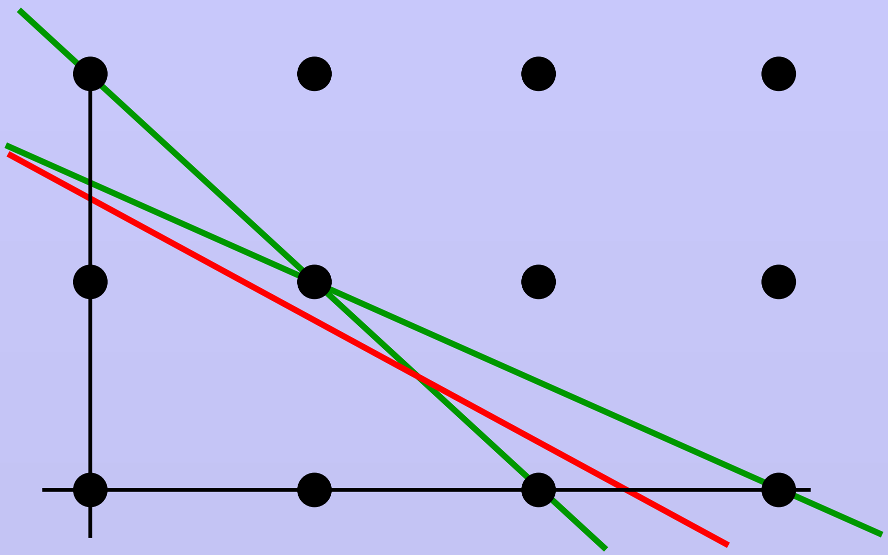
$$X_d + \frac{1}{M} \sum_{\{d' : d' \triangleleft\triangleright d\}} X_{d'} \le 1$$ for all duties d

Maximum/minimum train lengths on tracks

Stock keeping of both types of train units in stations

Valid inequalities for trips outside rush hours:

Example: suppose $2c_{23} \le P_t^2 \le c_{23} + c_{24}$



Original constraint:

$$\sum_{d:t\in d} (c_{23}T_d + c_{24}F_d) \ge P_t^2$$

Valid inequalities:

$$\sum_{d:t\in d} (T_d + F_d) \ge 2$$

$$\sum_{d:t\in d} (T_d + 2F_d) \ge 3$$

Single Deck and Double Deck combined:

$$X_d \leq T_d + F_d \leq M \cdot X_d$$ for all duties d

$$XX_d \leq TT_d + FF_d \leq M \cdot XX_d$$ for all duties d

$$X_d + XX_d \leq 1$$ for all duties d

$$\left.\begin{aligned} X_d &\leq 1 - Double_c \\ XX_d &\leq Double_c \end{aligned}\right\}$$ for all compositions c and duties d involving composition c

Extension of the valid inequalities for trips outside rush hours

Computational results

Implemented in OPL Studio (ILOG), based on 2001 timetable
Solved by CPLEX 7.0 on PC with 900 MHz, 256 M memory

Only Double Deck (3 & 4)

# variables / #constraints:	- about 1100 / 2000
computation time:	- minutes - hours
	- valid inequalities sometimes useful

Double Deck and Single Deck combined (3 & 4)

# variables / # constraints:	- about 2000 / 3200
computation time:	- highly dependent on available capacity: minutes - days
	- valid inequalities crucial

General remarks	- closing the gap takes time
	- minor improvements by allowing shortages outside rush hours

Computational results (Double Decker)

W1 = 2 W2 = 1 WS = 4 WCK = 1

No shortages outside rush hours: (9:30-15:30) + (18:30,->)

		tot	short	ckm	t(VI)	t(no VI)
inf	8361	1533	2229	15	15	
18/14		12546	2600	2146	218	524
18/12		18193	4028	2081	2651	1310
18/10		24878	5726	1974	551	1449
16/10		30322	7104	1906	250	132
15/10		33621	7943	1849	194	138
16/9		34811	8246	1827	99	99
15/9		38897	9280	1777	89	63

Computational results (Double Decker)

W1 = 2 W2 = 1 WS = 4 WCK = 1

Full optimisation without Valid Inequalities

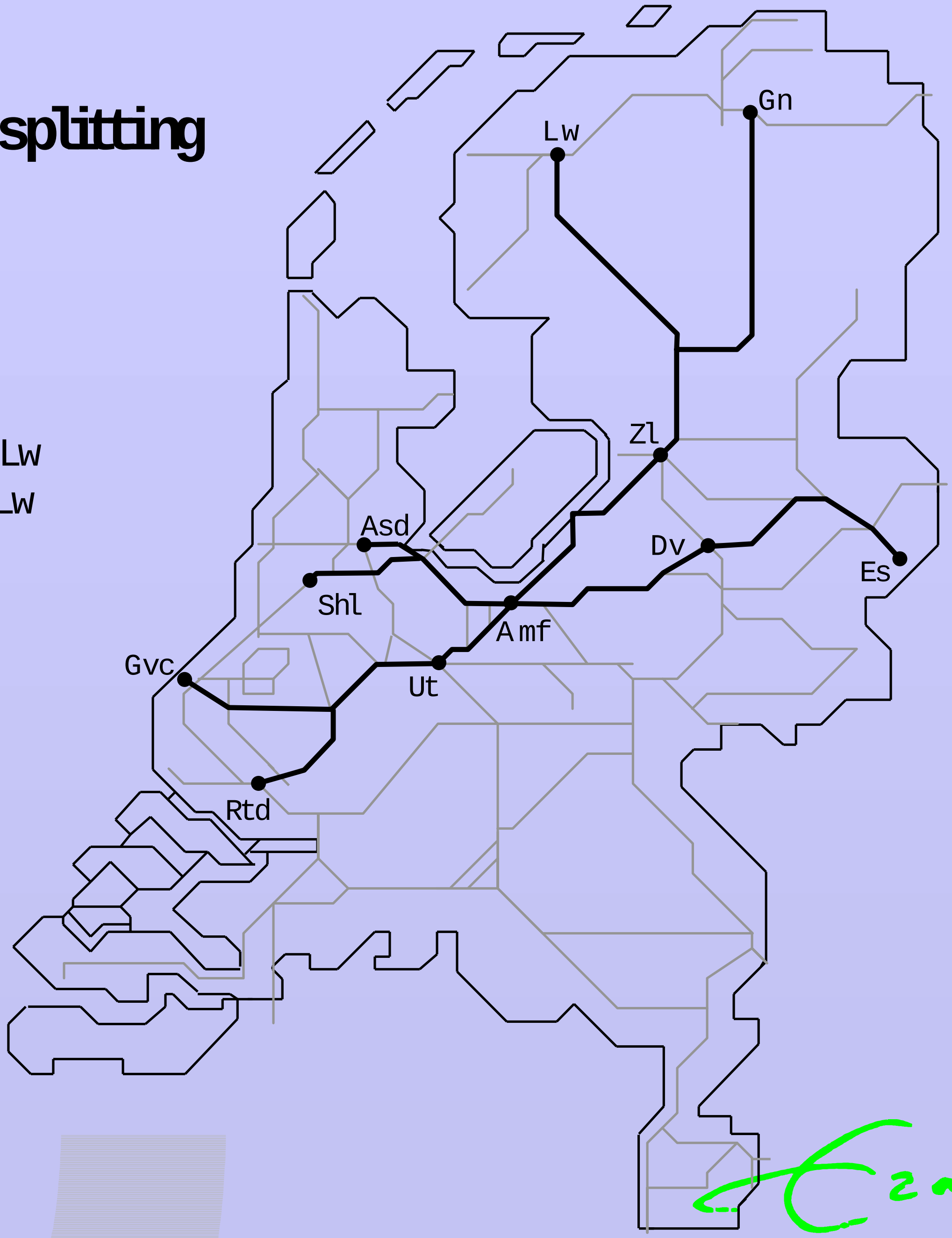
		tot	short	ckm	t	Δt	-Δtot
inf	8361	1533	2229	20	5	0	
18/14		12546	2600	2146	448	230	0
18/12		18193	4028	2081	2048	738	0
18/10	24806	5702	1998	1190	639	72	
16/10		30118	7055	1898	197	65	204
15/10		33126	7819	1850	597	359	495
16/9	34083	8059	1847	183	84	728	
15/9	37423	8912	1775	115	52	1474	

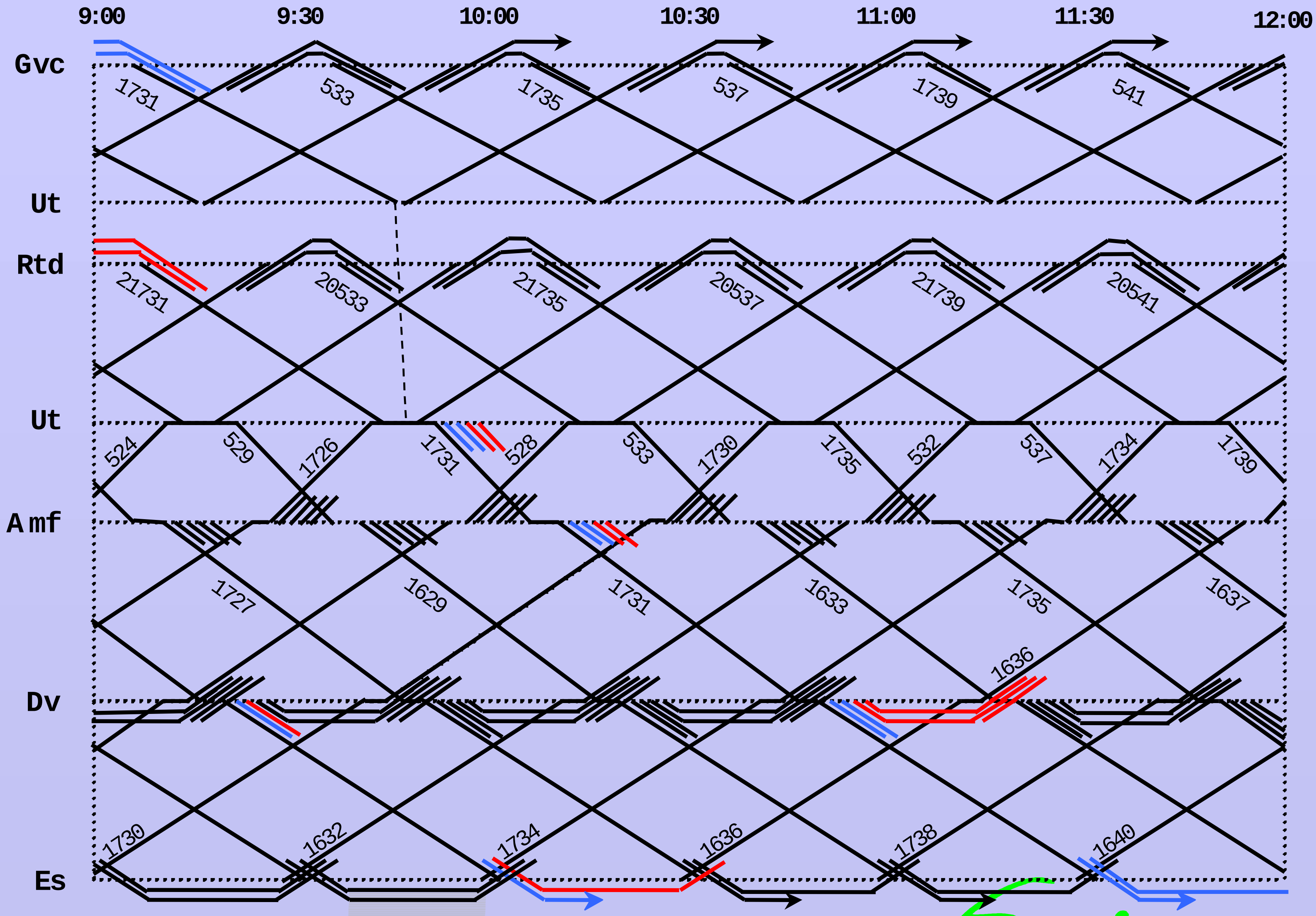
Combining and splitting

Noord-Oost
North-East

500 = Rtd/Gvc - Gn/Lw
700 = Asd/Shl - Gn/Lw
1600 = Asd/Shl - Es
1700 = Rtd/Gvc - Es

All series: 1 x per hour
All tracks: 2 x per hour




9:00
9:30
10:00
10:30
11:00
11:30
12:00
Gvc
Ut
Rtd
Ut
Amf
Dv
Es
1731
533
1735
537
1739
541
21731
20533
21735
20537
21739
20541
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Objective:

- Minimize shortages of seats during the rush hours

Decision Variables:

- $T_{from\_to\_t}$ = # units of length 3 on train t on the route from *from* to *to*

- $F_{from\_to\_t}$ = # units of length 4 on train t the route from *from* to *to*

- Decision variables for shortages on trips (1st and 2nd class)

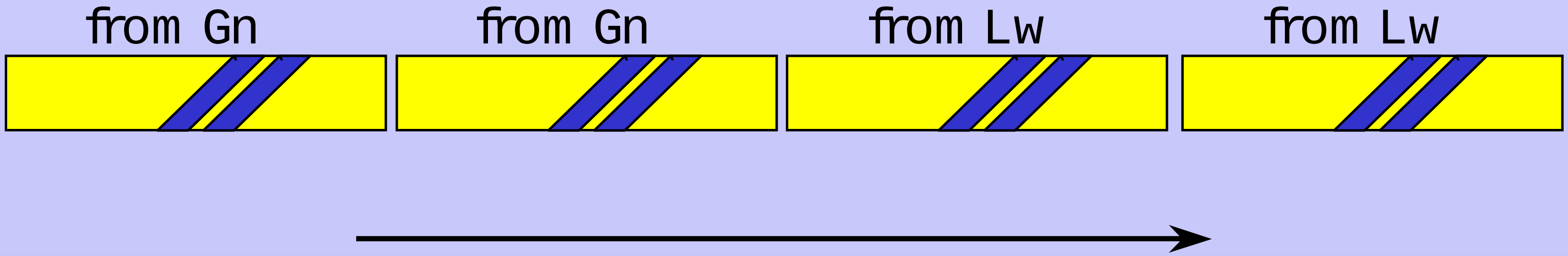
- Decision variables for stock keeping of train units in stations

Constraints:

- Link between the allocated capacities per train and the shortages per trip
- Stock keeping of both types of train units in stations
- Maximum/minimum train lengths on tracks
- Combining and splitting in Utrecht, Amersfoort, Zwolle
- Return trips on end points
- Shunting in Deventer

Complicating factor: combining in Zl and splitting in Amf

Composition between Zl and Amf:



Gn-Asd direct or **Lw-Shl** direct, but ***not both***

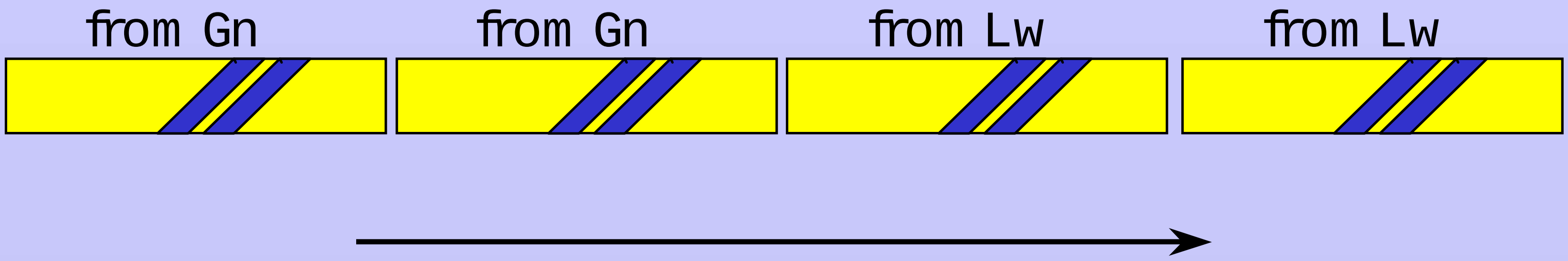
$$T_{\mathtt{Gn\_Asd\_t}} + F_{\mathtt{Gn\_Asd\_t}} \leq 5(1 - Y_{\mathtt{t}})$$

$$T_{\mathtt{Lw\_Shl\_t}} + F_{\mathtt{Lw\_Shl\_t}} \leq 5(1 - Y_{\mathtt{t}})$$

$$Y_{\mathtt{t}} \in \{0,1\}$$

Complicating factor: return in Asd

Composition upon arrival in Asd:



Capacity can be reallocated into **maximally one** direction

$$T_{Asd\_Gn\_r} \leq T_{Gn\_Asd\_t} + 5Z_t \qquad Z_t \in \{0,1\}$$

$$F_{Asd\_Gn\_r} \leq F_{Gn\_Asd\_t} + 5Z_t$$

$$T_{Asd\_Lw\_r} \leq T_{Lw\_Asd\_t} + 5(1 - Z_t)$$

$$F_{Asd\_Lw\_r} \leq F_{Lw\_Asd\_t} + 5(1 - Z_t) \qquad r = r(t)$$

Model for the Noord Oost

Implemented in OPL Studio (ILOG)

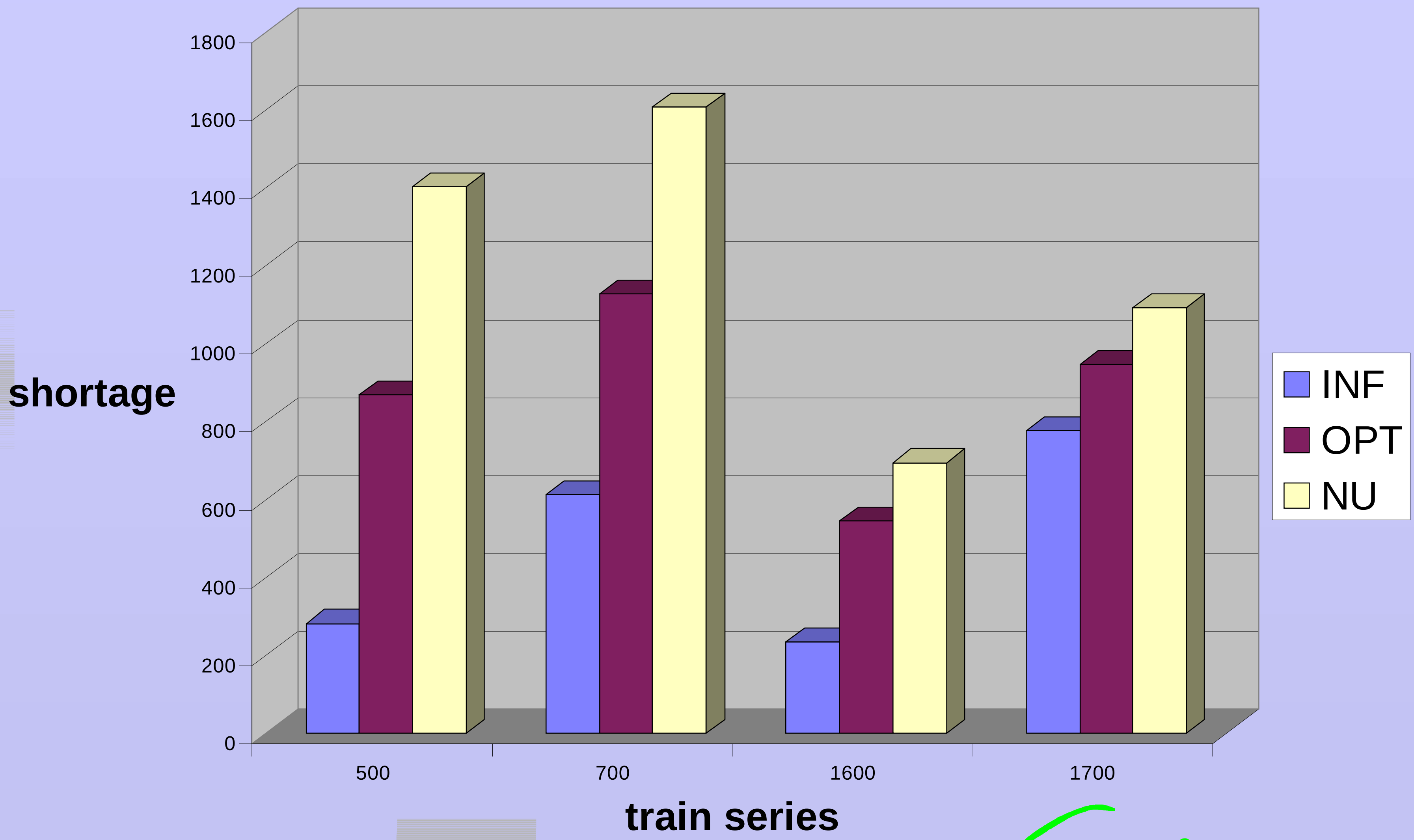
Solved by CPLEX 7.0 on PC with 900 MHz, 256 M

Tested on preliminary NSR 2001 timetable (Monday)

4 lines, 5 trips in each direction

Computation time:	< 3 minutes	
# variables:	about 1000	
# constraints:	about 1000	
Manual solution:	4528	seats short (2nd class)
	188	seats short (1st class)
Optimal solution:	3385	seats short (2nd class)
	105	seats short (1st class)
Unlimited capacity:	1874	seats short (2nd class)
	34	seats short (1st class)

Shortages in the North-East



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