ERASMUS RESEARCH INSTITUTE OF MANAGEMENT

Erasmus Research Institute of Management

Variable trip times for cyclic railway timetabling

Leon Peeters
Leo Kroon

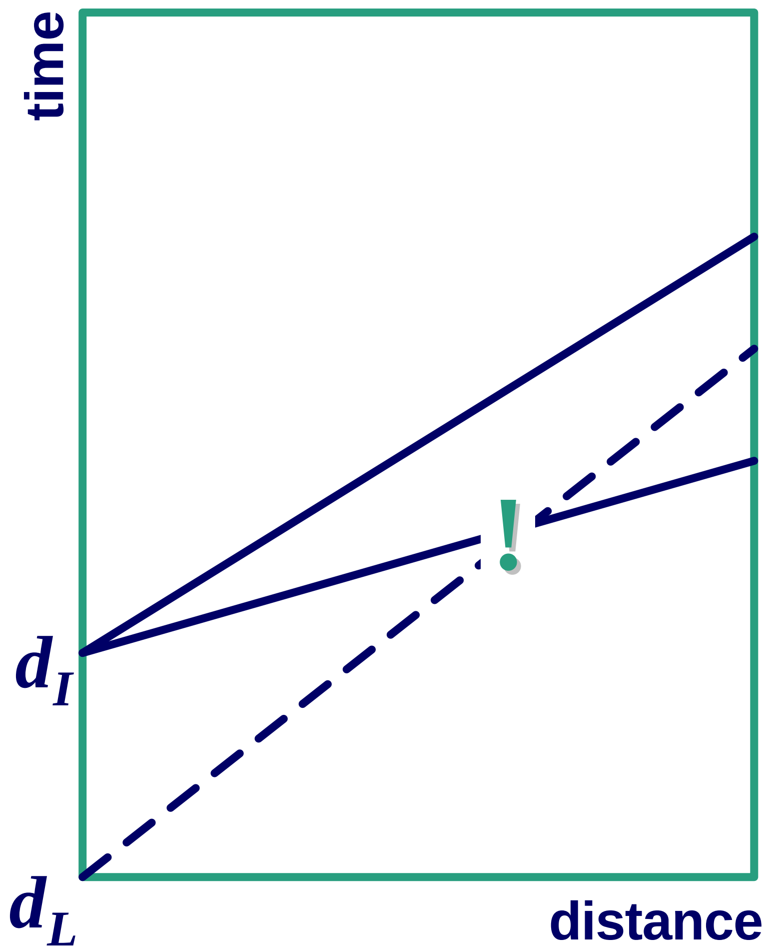
AMORE Research Seminar

Contents

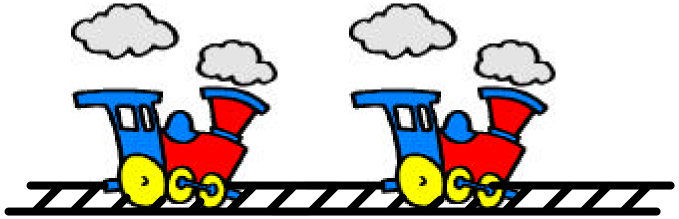
- Introduction
- Fixed trip times
- Variable trip times
  - General case
  - Disjoint time windows
- Practical experience & conclusion

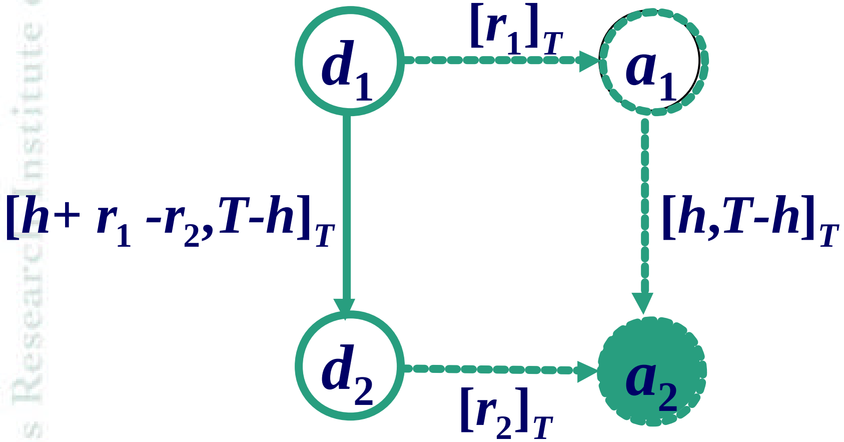
Why study variable trip times?

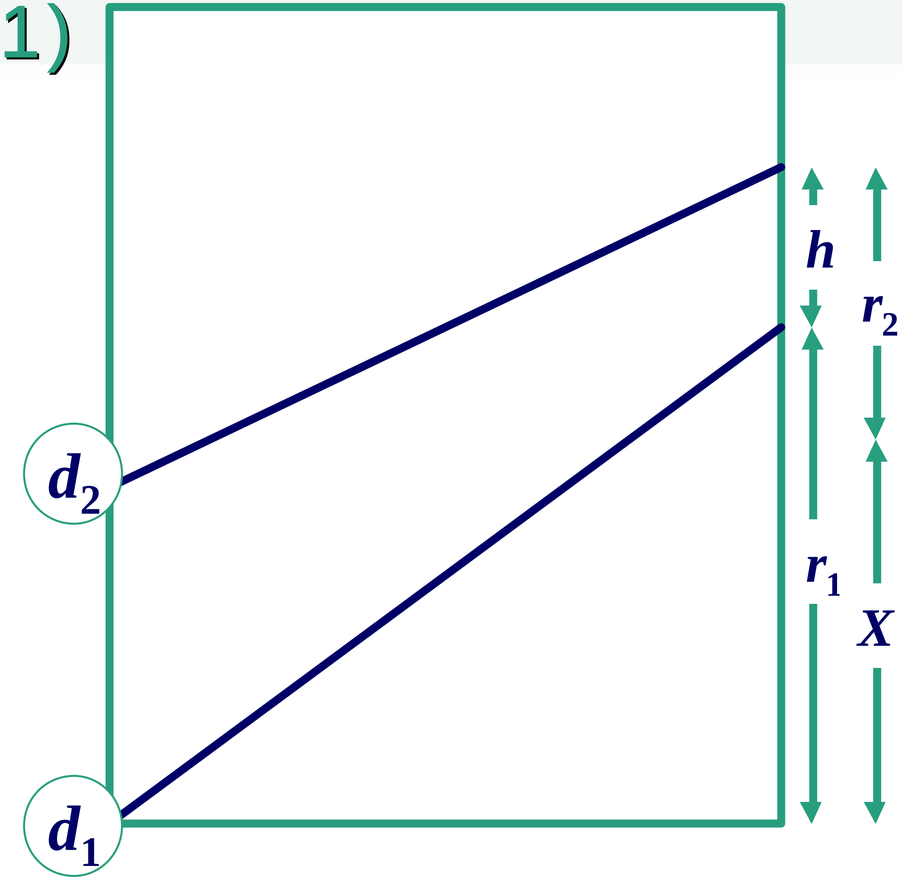
- Basic model: fixed trip times
- Larger solution space
- Possibly better solutions
- Used in practice!



Fixed trip times(1)

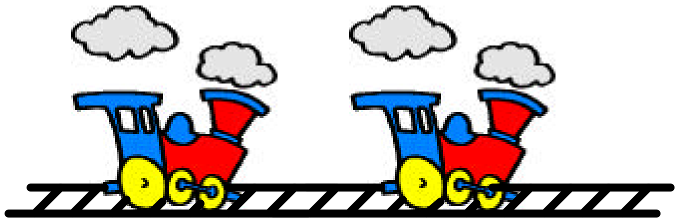


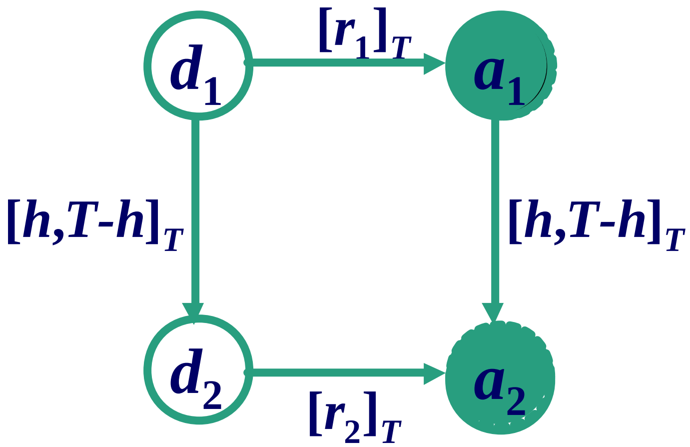




$$d_2 - d_1 \notin X_T = [0, h + r_1 - r_2)_T$$

Fixed trip times(2)





Theorem (Lindner)

Suppose

$$d_2 - d_1 + Tp_d \in [h, T - h]$$

$$a_2 - a_1 + Tp_a \in [h, T - h]$$

Trains 1 and 2 do not overtake eachother if and only if

$$p_d = p_a$$

Problems with variable trip time

- **Trip times straightforward**

$$a_i - d_i \in [r_i, R_i]$$

- **Safety constraints**

$$d_2 - d_1 \notin X_T = [-h, h + r_1 - r_2]_T$$

…but $r_1, r_2 = ?$

- **Lindners' result requires JPESP (Joined constraints) for expressing** $p_d = p_a$

Variable trip times

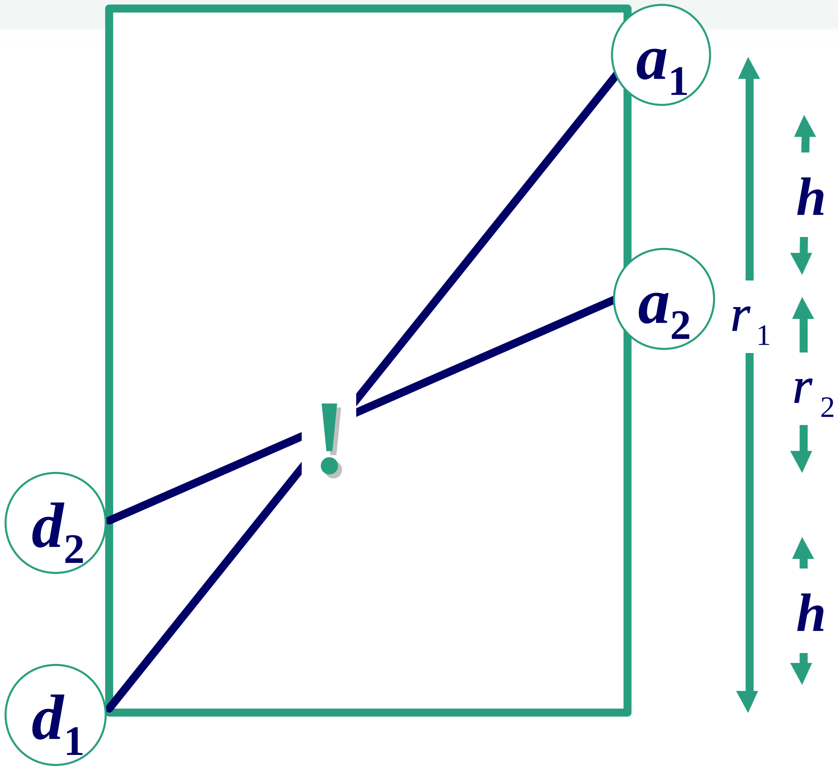
Theorem

The constraints

$$d_2 - d_1 \notin (-h, h)_T$$
$$a_2 - a_1 \notin (-h, h)_T$$

are necessary to guarantee

- **non-overtaking of 1 and 2**
- **respecting headways**



If

$$R_1 - r_2 < 2h$$
$$R_2 - r_1 < 2h$$

then also sufficient.

$$r_1 > r_2 + 2h$$
$$\Rightarrow \max\{ r_1 - r_2 \} < 2h$$
$$\Rightarrow R_1 - r_2 < 2h$$

Disjoint trip time windows

Assume $r_1 > R_2$

Theorem

$$d_2 - d_1 \notin (-h, h + r_1 - R_2)_T$$

$$a_1 - a_2 \notin (-h, h + r_1 - R_2)_T$$

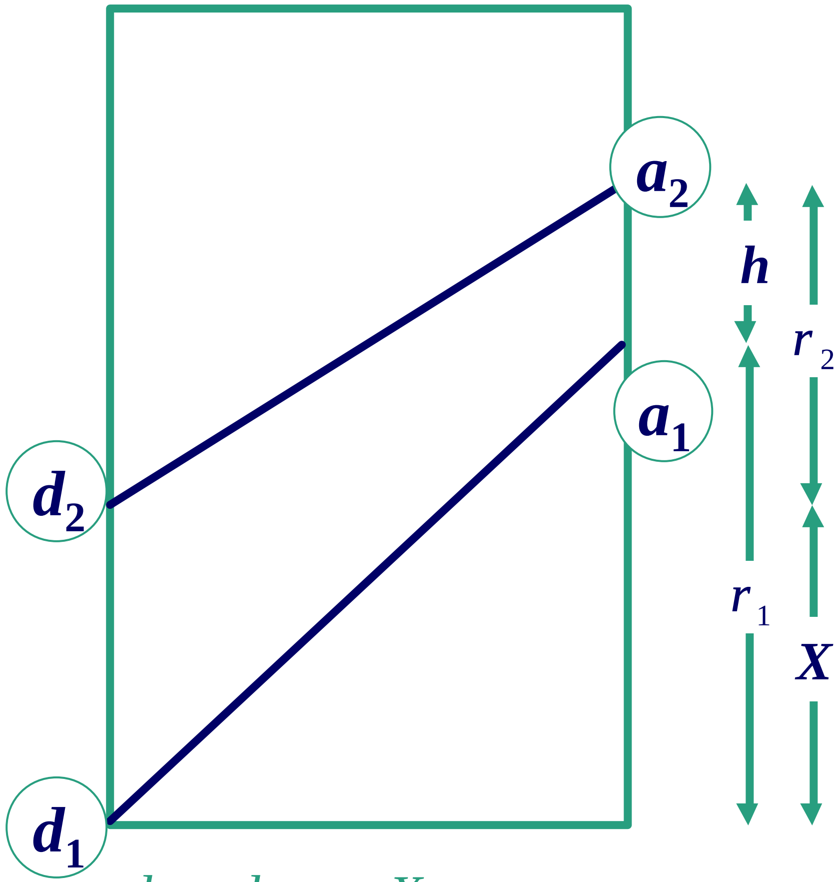
are necessary to guarantee

- **non-overtaking of 1 and 2**
- **respecting headways**

If

$$(R_1 - r_1) + (R_2 - r_2)$$
$$< 2h + (r_1 - R_2)$$

then also sufficient.



$$d_2 - d_1 \notin X_T$$
$$= [0, h + r_1 - r_2]_T$$

Conclusion & Practical experience

- Similar result for opposite trains on single track
- General case: trip window at most 5 wide
- Disjoint case: more space

- Uses PESP constraints only

- Works well in practice
  - First without variable trip times
  - If not feasible, use variable trip times

ERIM ERASMUS RESEARCH INSTITUTE OF MANAGEMENT

Erasmus Research Institute of Management

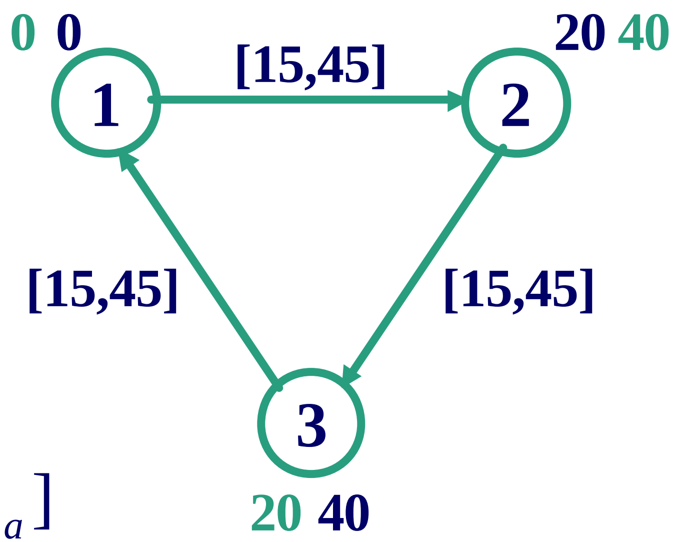
Computational experiments for the Netherlands' Intercity network

Leon Peeters
Leo Kroon

AMORE Research Seminar

Model - cycles

- **Consider a cycle:**

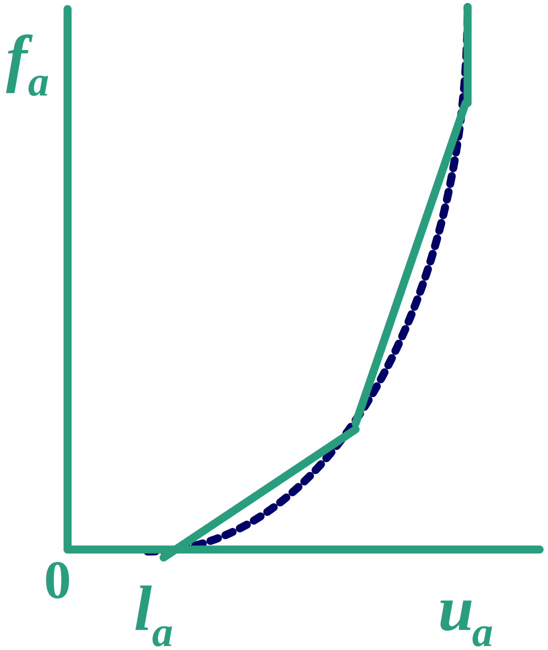


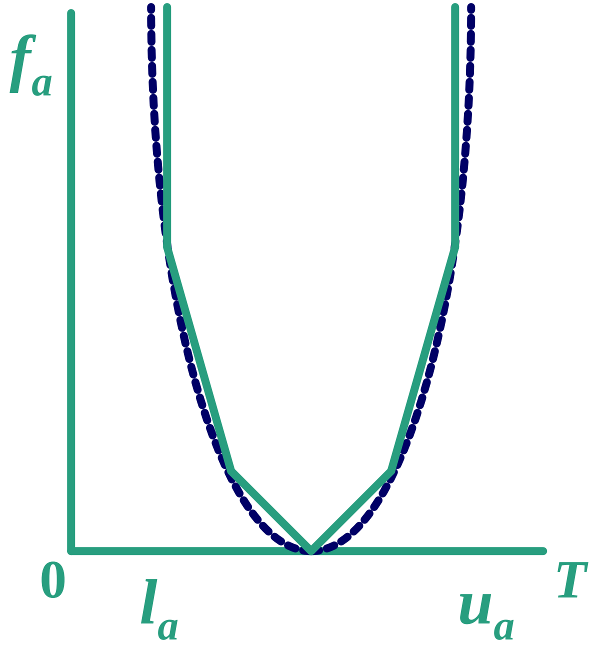
- **Cycle directions** $c = c^+ \cup c^-$
- **Decision variables** $d_a \in [l_a, u_a]$
- **Integer variable** q_c
- **For every cycle** $c \in C$ **:**

$$\sum_{a \in c^+} d_a - \sum_{a \in c^-} d_a = Tq_c$$

Objective function: d_a-variables

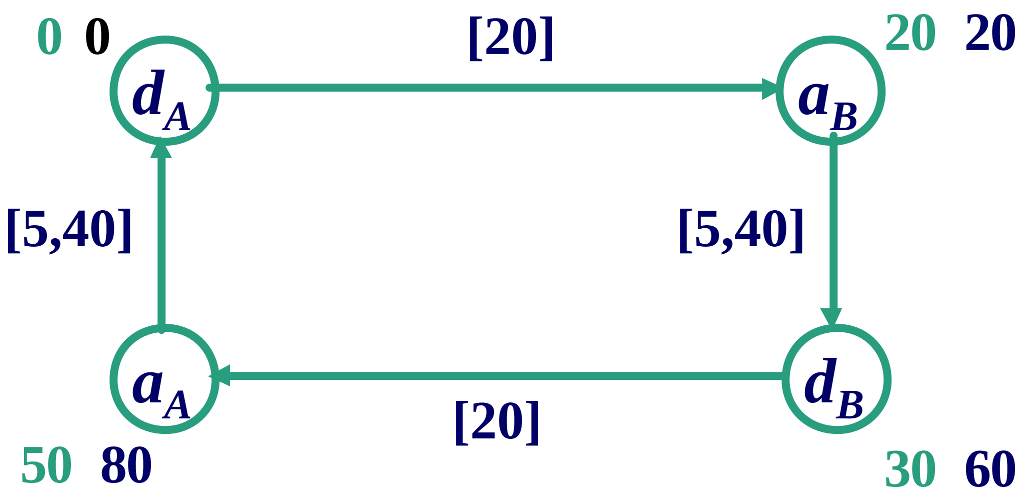
- **Minimize** $\sum_{a \in A_{obj}} f_a(d_a)$
- **Shape of functions** f_a **:**





Alternative objective function - rolling stock

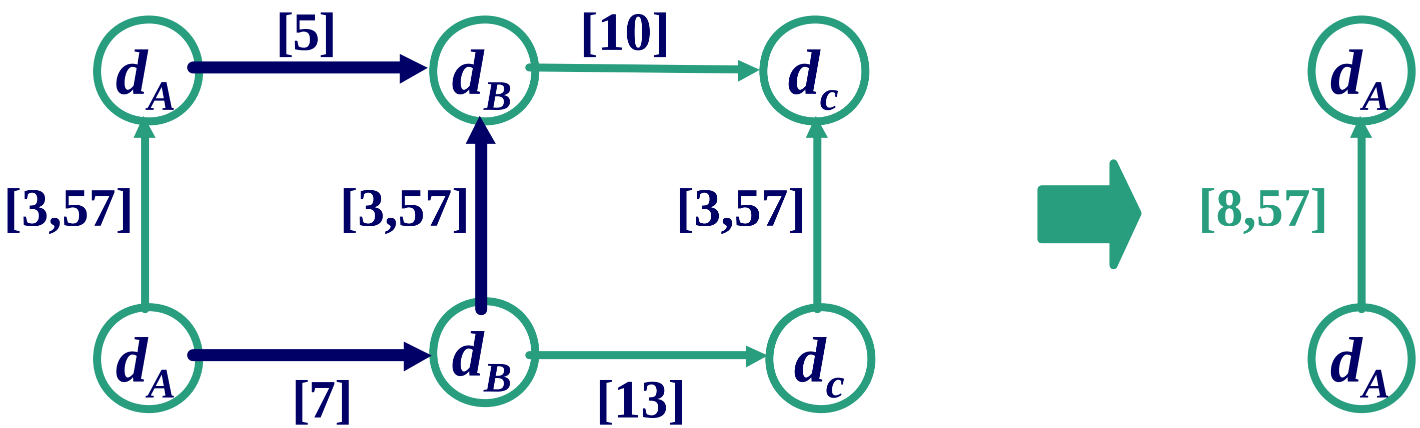
- Minimize number of rolling stock compositions
- Consider train cycle:



- Minimize $\sum_{c \in C_t} q_c$, $C_t \subseteq C$ set of train cycles

Preprocessing

- **Contract nodes with degree 1 or 2**
- **Shrink subsequent safety constraints:**



- **Substitute remaining equality constraints**

Calculating a cycle basis

- **Find a MST with respect to** $w_a = u_a - l_a$
- **Widest arcs each in only one cycle**
- **Bounds on the** q_c **-variables:**

$$\sum_{a \in c} l_a \leq \sum_{a \in c} d_a = T q_c \leq \sum_{a \in c} u_a$$

$$\Downarrow$$

$$\left\lceil \frac{1}{T} \sum_{a \in c} l_a \right\rceil \leq q_c \leq \left\lfloor \frac{1}{T} \sum_{a \in c} u_a \right\rfloor$$

Characteristics IC-97

■ **Train lines**	**25**
■ **Stations**	**50**
■ **Original size**	**n=1475, m=3342**
■ **After pre-processing**	**n= 207, m= 525**
■ **# connections**	**34**
■ **# stops**	**162**
■ **# safety**	**300**

- **MIP solver CPLEX 6.5**
- **Sun Enterprise/250, two 400MHz UltraSPARC processors, 1 Gb mem.**

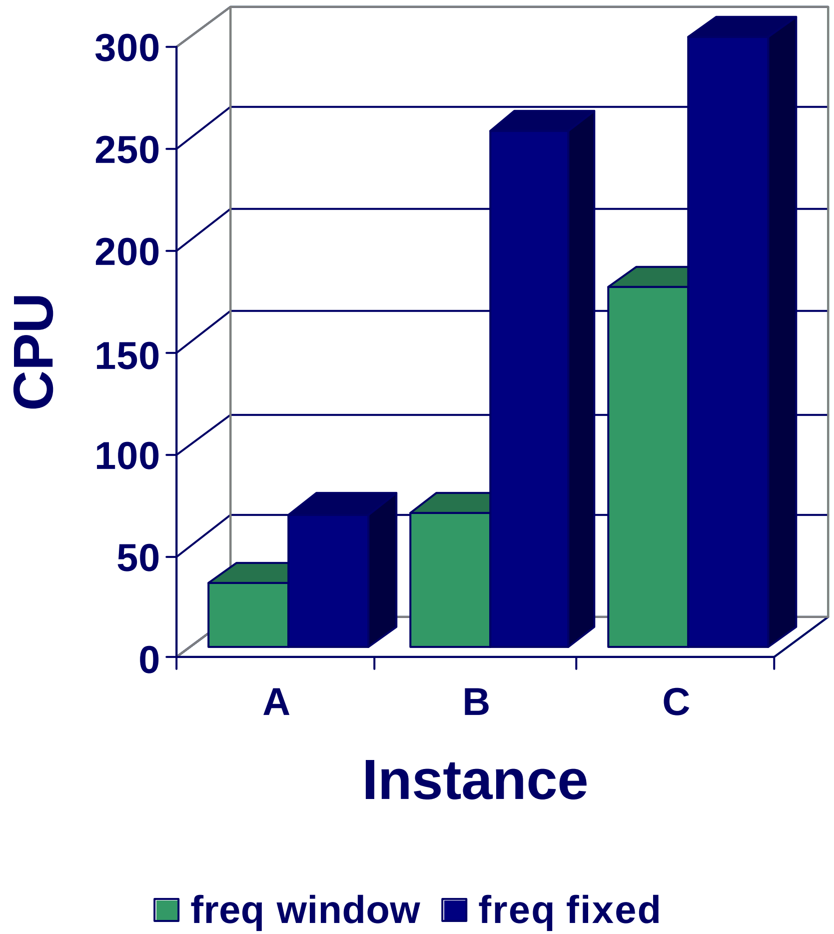
Minimizing waiting time

- Minimize $\sum_{a \in A_{conn} \cup A_{stop}} f_a(d_a)$

- Variants:

Instance	D_{conn}	D_{stop}
A	3	2
B	5	3
C	8	5

frequency	$[l_a, u_a]$	z_{opt}
window	[28,32]	114
fixed	[30]	154

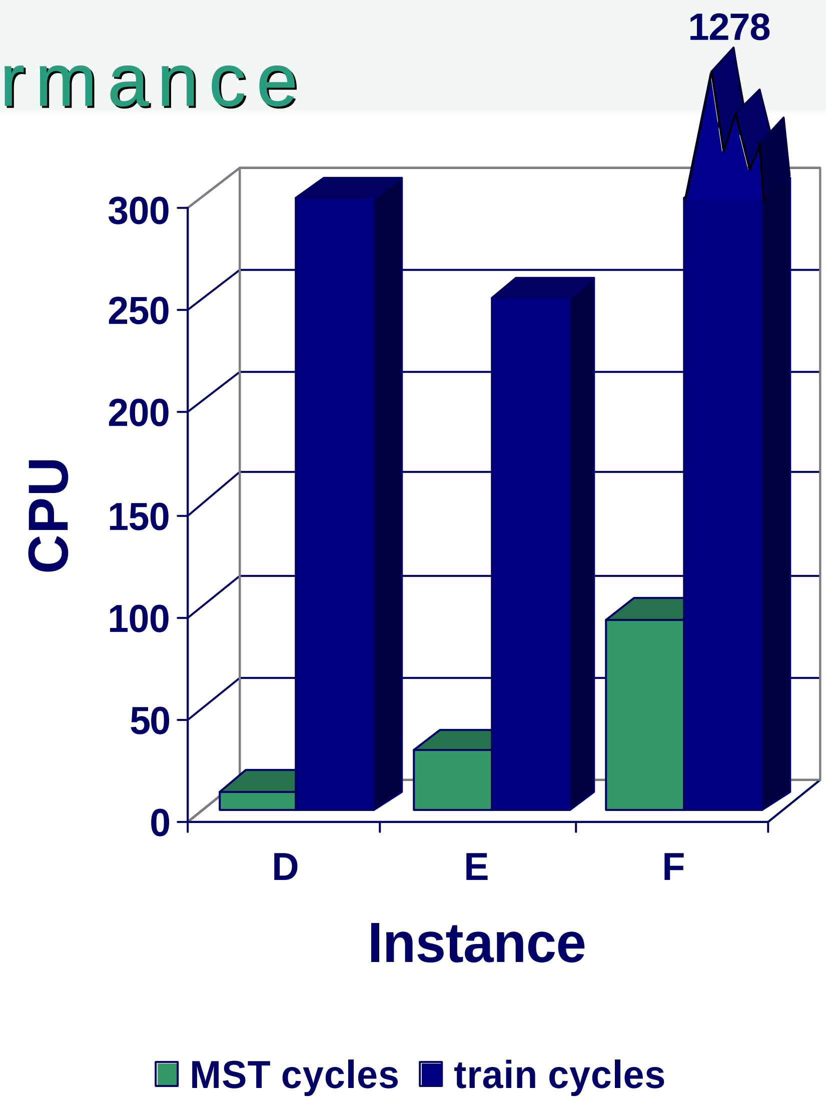


Cycle basis performance

- **Cycle bases**
  - **MST**
  - **Train cycles**

- **Variants:**

Instance	l_{turn}	u_{turn}	z_{opt}
D	10	40	764
E	5	40	247
F	5	59	154



Minimizing rolling stock & waiting time

- **Minimize** $\sum_{a \in A_{conn} \cup A_{stop}} f_a(d_a) + K \sum_{c \in C_t} q_c$
- **Train cycle basis**
- **Variants:**

Instance	z_{opt}
D	62.765
E	57.247
F	55.171