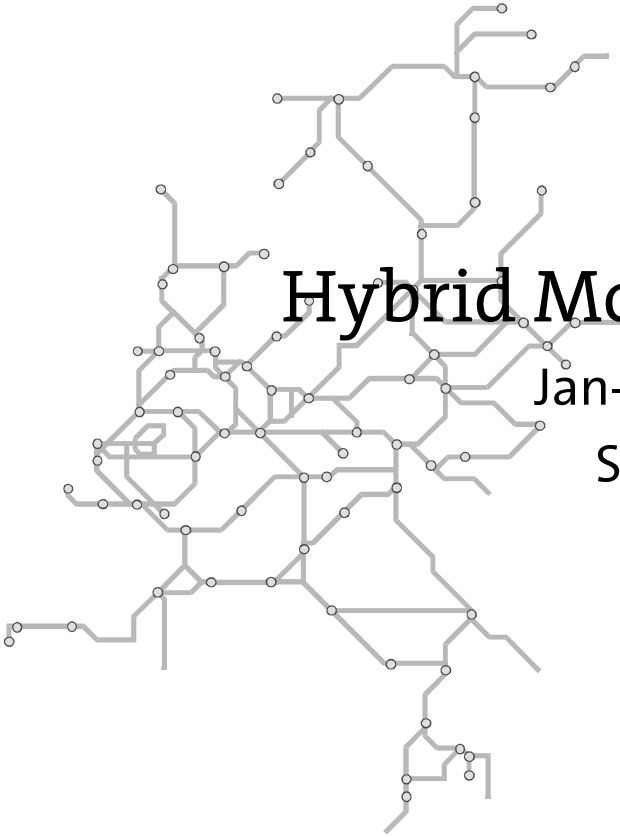
Line Planning



Hybrid Model and Extensions

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AMORE Research Seminar, Konstanz, 19/10/2000
University of Maastricht

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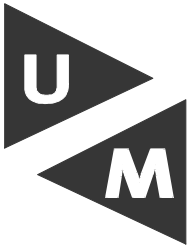
Operations Research Group

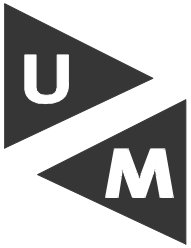
Agenda

- Passenger routing problem
- Rolling stock resource
- Hybrid model
  - Model formulation
  - Practical application
- Extension: Multiple train types
  - Model formulation
  - Practical application



Introduction

- Network graph $G=(V,E)$, where V is the set of stations and E is the set of tracks.
- Line plan: cost / resources
- Passenger routing

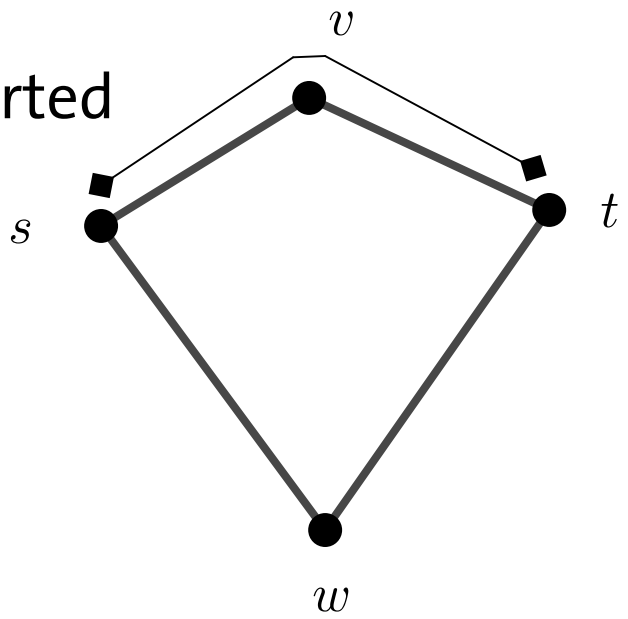


Line planning: cost / resources

- Line planning problem:

  Determine a set of lines

  s.t. all passengers can be transported
- Resource cost:
  - Per carriages
  - Per carriages per hour
  - …

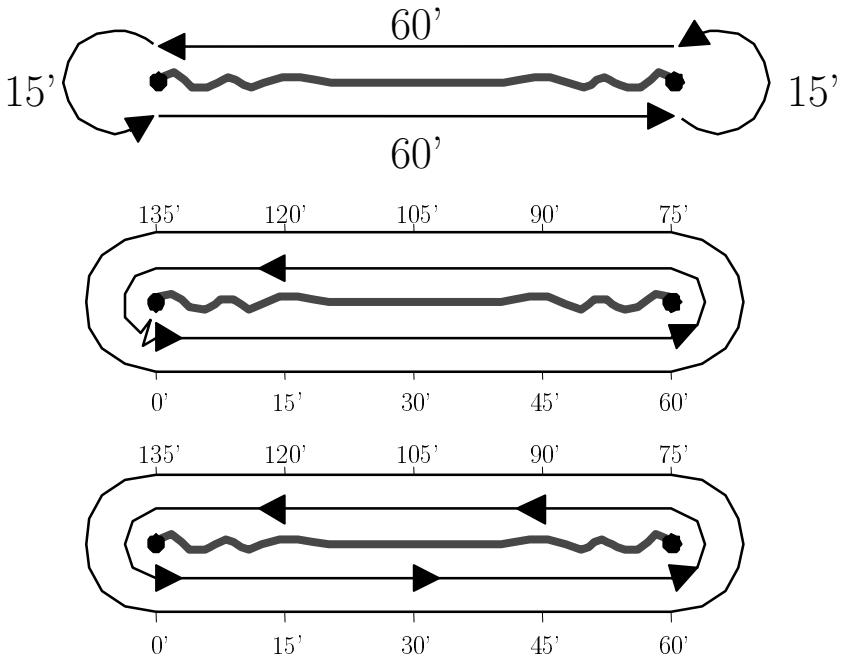


Line planning: cost / resources

Resource requirements:

$$cp_l := \frac{2 \cdot (\text{driving} + \text{turning})}{60}$$

trains = $\lceil cp_l \cdot f \rceil$



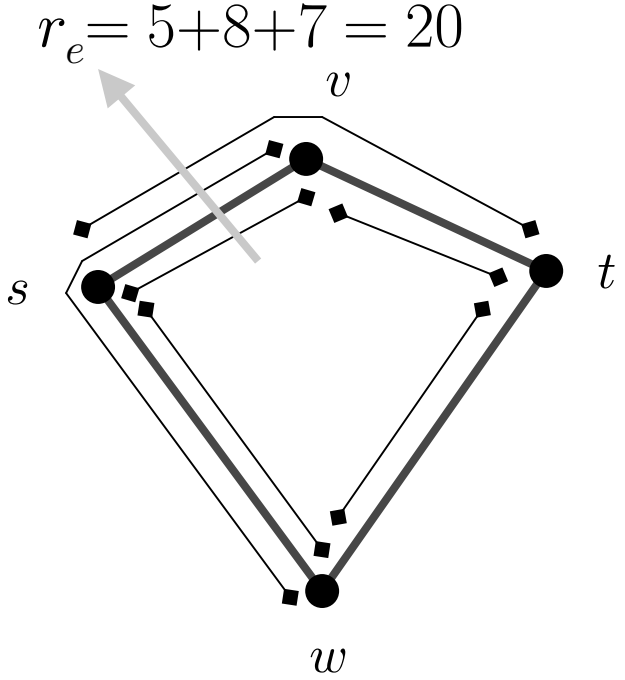
Passenger routing

Allocate passengers to tracks: fixing passenger routes

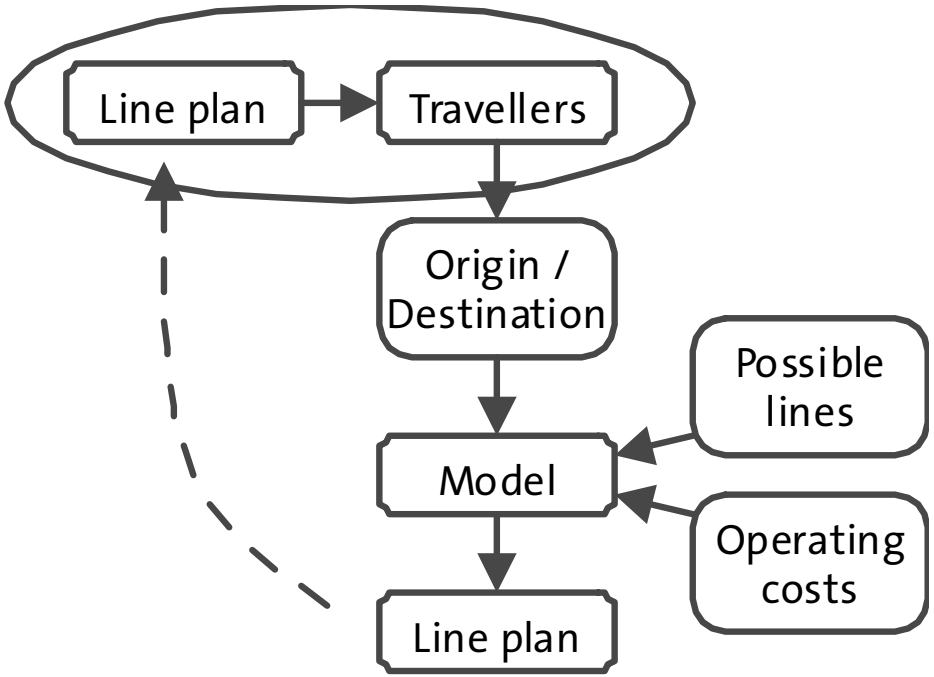
$$\forall e \in E : r_e := \sum_{s,t:e \in P_{st}} D_{st}$$

D:

	s	v	w	t
s	-	5	6	7
v		-	8	9
w			-	10
t				-

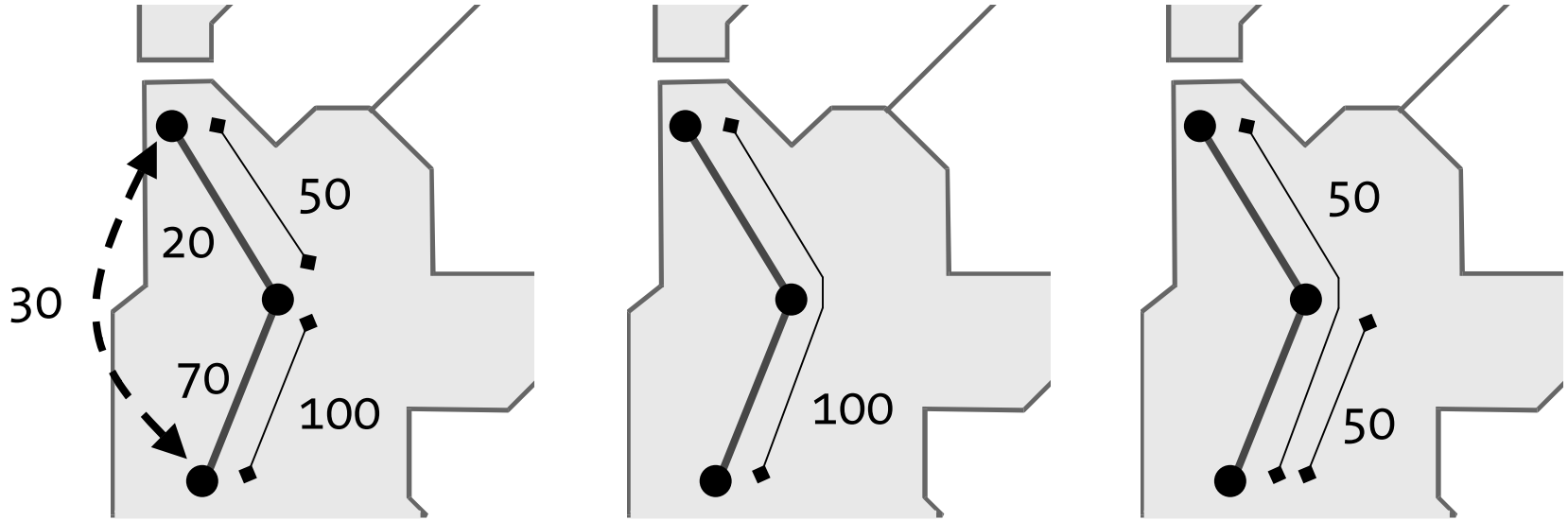


Passenger routing



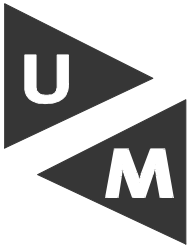
Hybrid model

- Combine cost minimization with direct traveler restrictions



Notation

$G=(V,E)$	Undirected graph
L,F,C	Set of lines, frequencies and #cars
$P_{st} \subseteq E$	Fixed route in G between $s,t \in V$
D_{st}	Number of travelers from s to t
λ	Number of travelers per car
k_{lfc}	Cost of deploying line l at f and c
r_e	Number of travelers on edge e
$l(s,t) \subseteq E$ for $s,t \in l$	Edges of line l between s and t



Model formulation

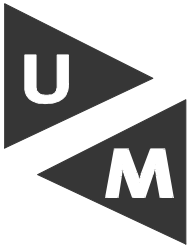
- Standard variables for cost oriented line planning:

  $x_{lfc} \in \{0,1\}$ with $l \in L$, $f \in F$, $c \in C$

- Route options: either direct, else over fixed path (e.g. shortest):

  Number of passengers using line l as direct connection for traveling from s to t :

  $p_{lst} \in \mathbb{R}_+$ with $l \in L$, $s,t \in V$



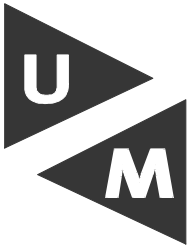
Model formulation

$$\text{minimize} \quad \sum_{(l,f,c)} k_{lfc} x_{lfc}$$

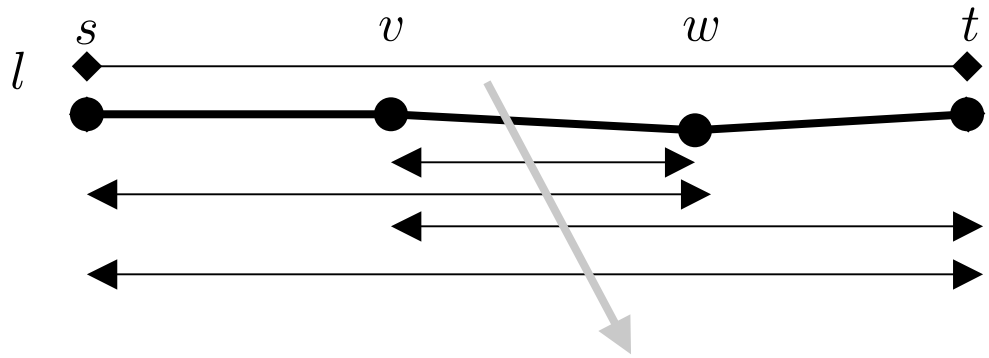
s.t.

$$\sum_{(f,c)} x_{lfc} \leq 1 \qquad \forall l \in L$$

$$\sum_{l:s \in l, t \in l} p_{lst} \leq D_{st} \qquad \forall s,t \in V$$



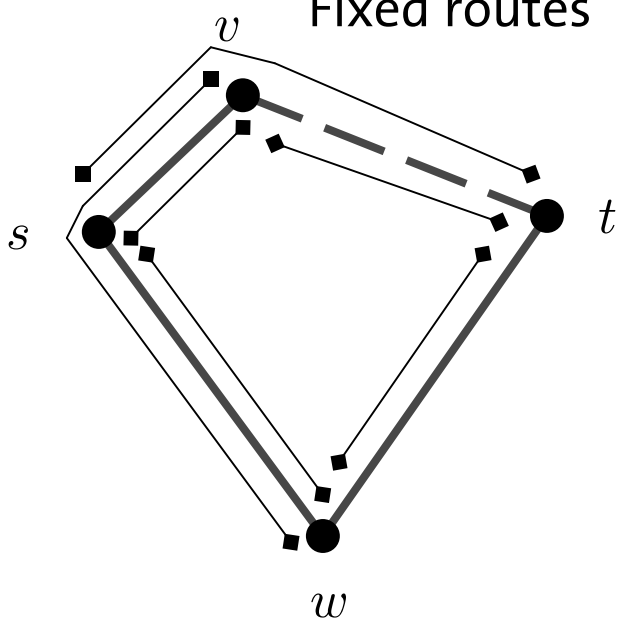
Model formulation



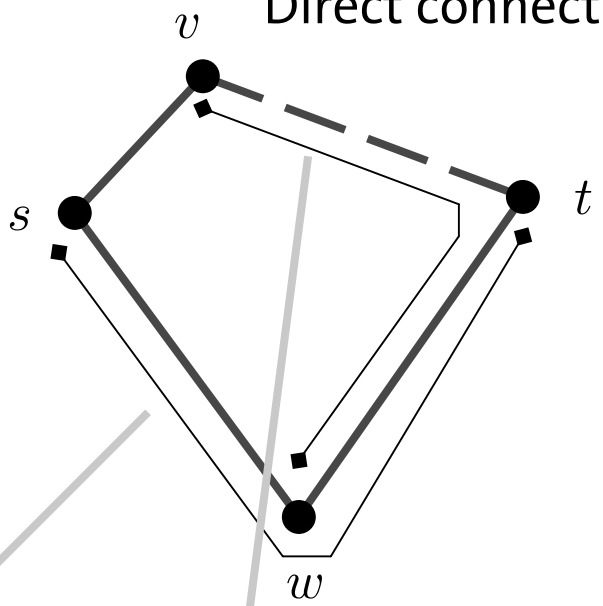
$$\lambda \sum_{(f,c)} fcx_{lfc} \geq \sum_{(s,t):e \in l(s,t)} p_{lst} \qquad \forall l \in L, e \in l$$

Model formulation

Fixed routes



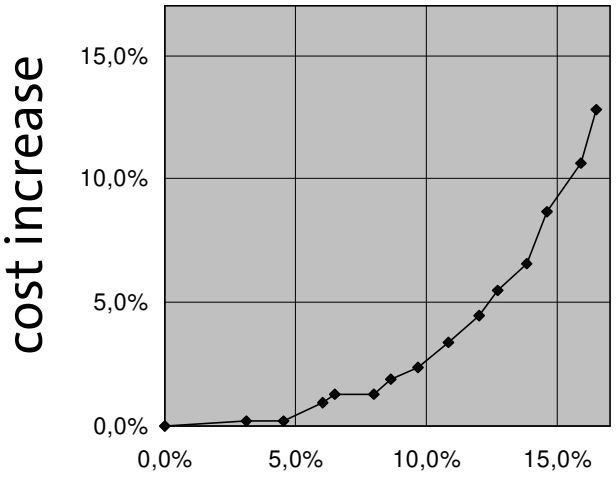
Direct connections



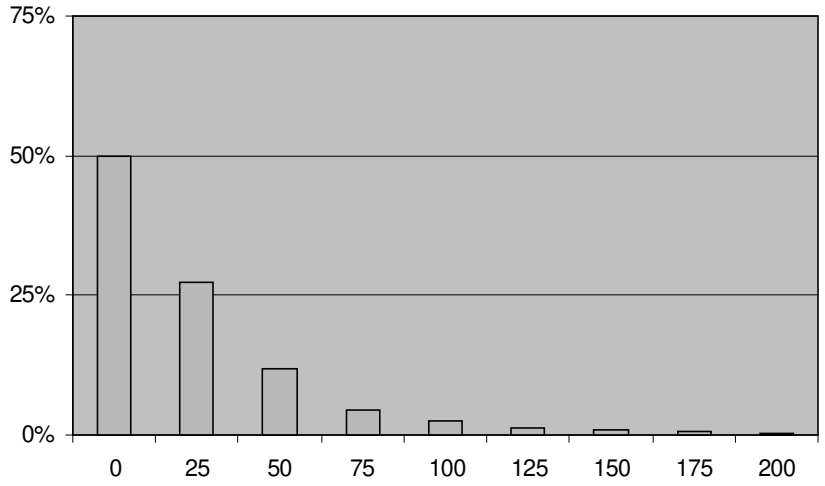
$$\lambda \sum_{(l,f,c):e\in l} fcx_{lfc} \geq r_e - \sum_{(l,s,t):e\notin l(s,t),e\in P_{st}} p_{lst} + \sum_{(l,s,t):e\in l(s,t),e\notin P_{st}} p_{lst} \qquad \forall e \in E$$

Practical applications

- Dual objective models ("price a direct traveler")
- Minimum % direct travelers per $s,t \in V$
- Minimum % direct travelers overall (relative to $\sum_{(s,t)} D_{st}$)



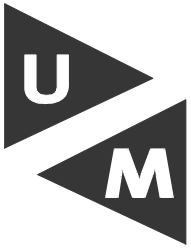
direct travelers increase



traveled distance

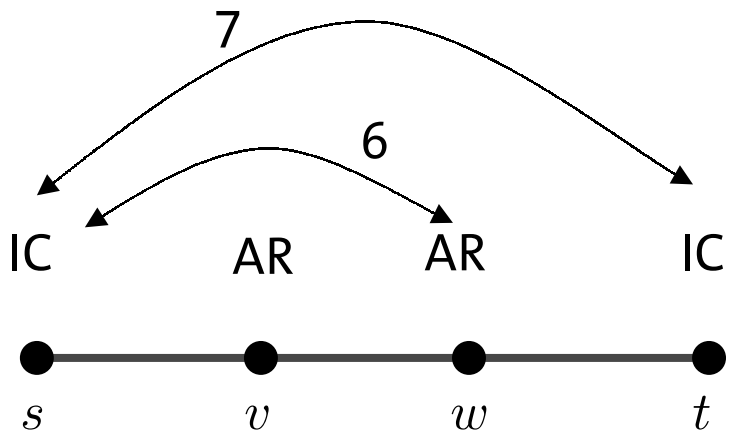
Extension: Multiple Train Types

- Avoid splitting the system into AR, IR and IC travelers (Oltrogge 1994)
- Model assigns passengers to train types



Extension: Multiple Train Types

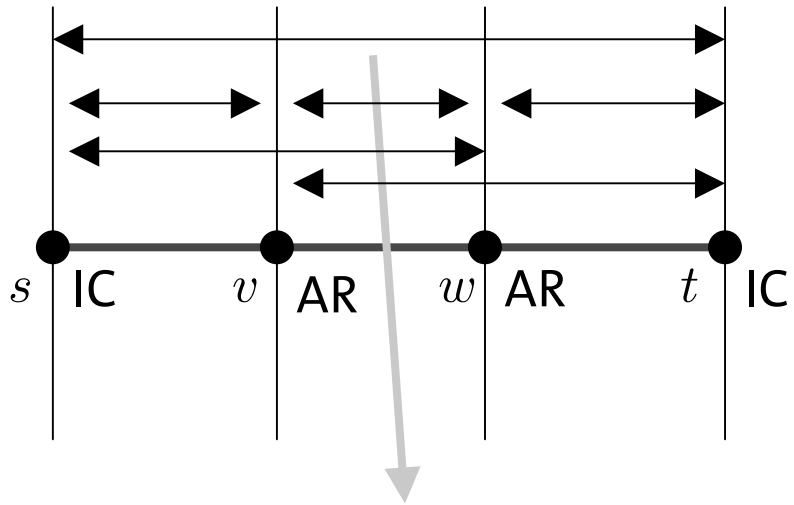
- Example



	s	v	w	t
s	-	5	6	7
v		-	8	9
w			-	10
t				-

Extension: Multiple Train Types

Passengers



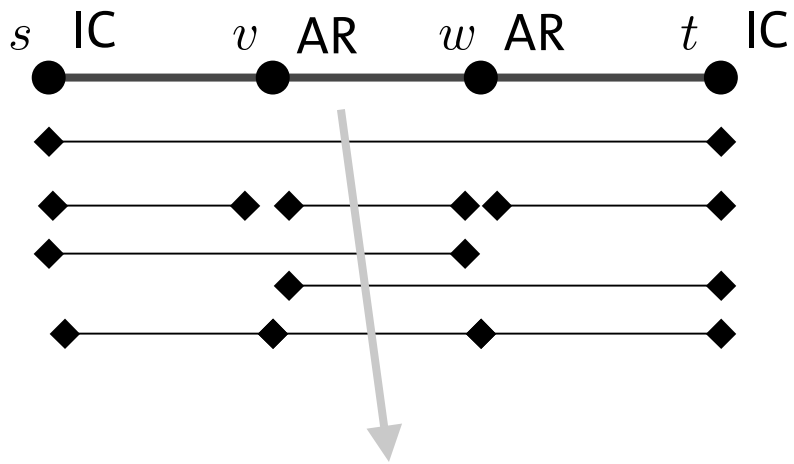
	s	v	w	t
s	-	5	6	7
v		-	8	9
w			-	10
t				-

AR $\quad D_{vw} + D_{sw} + D_{vt}$

AR + IC $\quad D_{vw} + D_{sw} + D_{vt} + D_{st}$

Extension: Multiple Train Types

Trains



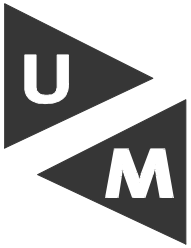
	s	v	w	t
s	-	5	6	7
v		-	8	9
w			-	10
t				-

$$\overbrace{\text{cap}(AR,v,w) + \text{cap}(AR,s,w) + \text{cap}(AR,v,t) + \text{cap}(AR,s,t)}^{r^{AR}_{(v,w)}} \geq D_{vw} + D_{sw} + D_{vt}$$

$$\ldots + \text{cap}(IC,s,t) \geq \underbrace{D_{vw} + D_{sw} + D_{vt}}_{r^{AR}_{(v,w)}} + \underbrace{D_{st}}_{r^{IC}_{(v,w)}}$$

Notation

$G=(V,E)$	Undirected graph
L,F_t,C_t	Set of lines, frequencies and #cars
T, $t(l)\rightarrow T$	Set of train types and mapping
$P_{st} \subseteq E$	Fixed route in G between $s,t \in V$
D_{st}	Number of travelers from s to t
λ	Number of travelers per car
k_{lfc}	Cost of deploying line l at f and c
r_e^t	Number of travelers on edge e that can traverse e using t, but not $t' < t$

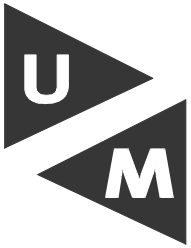


Extension: Multiple Train Types

$$\sum_{t \in \{AR\}} \lambda^t \sum_{(l,f,c):t(l)=t} fcx_{lfc} \geq r_e^{AR} \qquad \forall e \in E$$

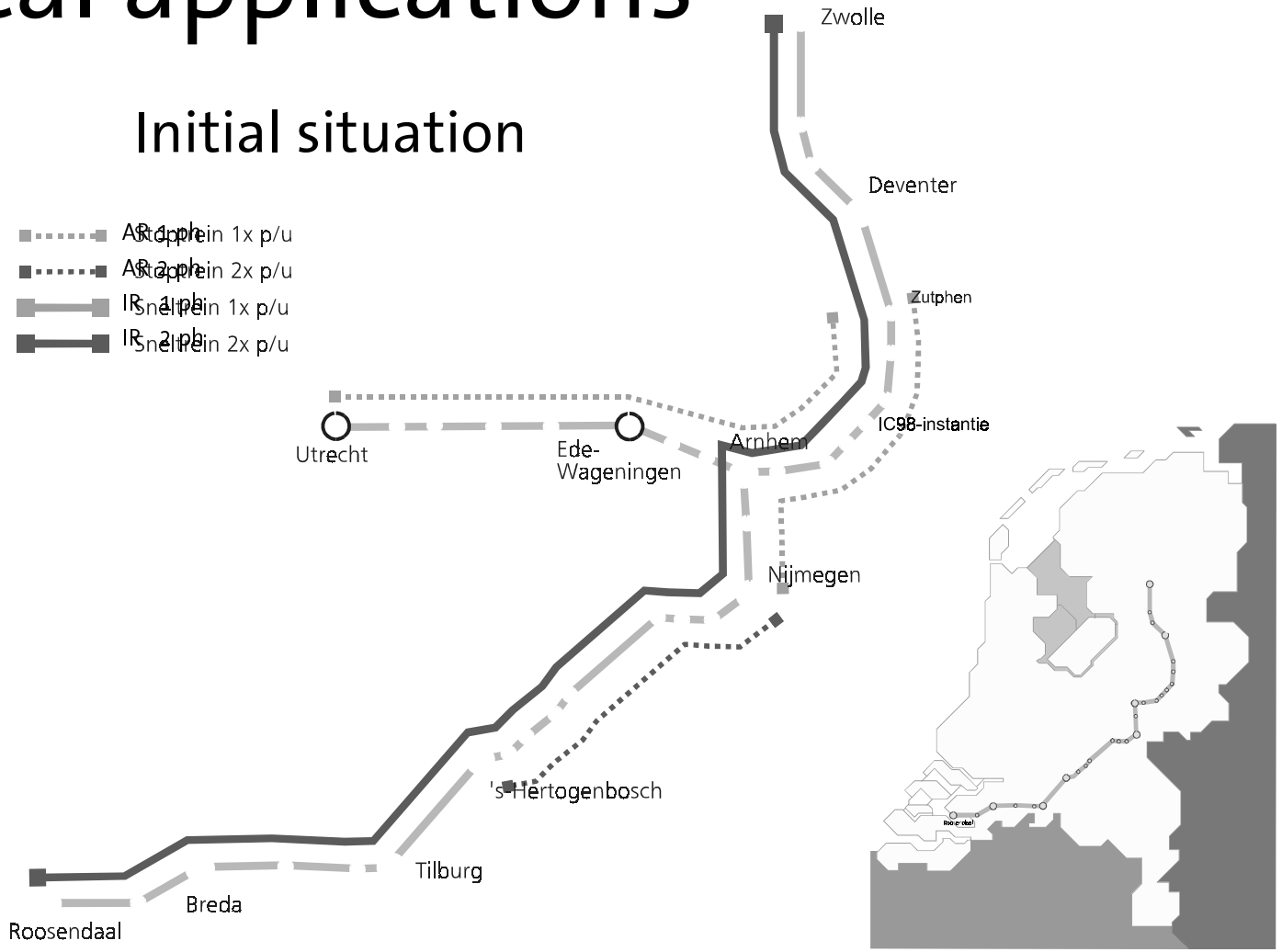
$$\sum_{t \in \{IR,AR\}} \lambda^t \sum_{(l,f,c):t(l)=t} fcx_{lfc} \geq r_e^{IR} + r_e^{AR} \qquad \forall e \in E$$

$$\sum_{t \in \{IC,IR,AR\}} \lambda^t \sum_{(l,f,c):t(l)=t} fcx_{lfc} \geq r_e^{IC} + r_e^{IR} + r_e^{AR} \qquad \forall e \in E$$



Practical applications

Initial situation



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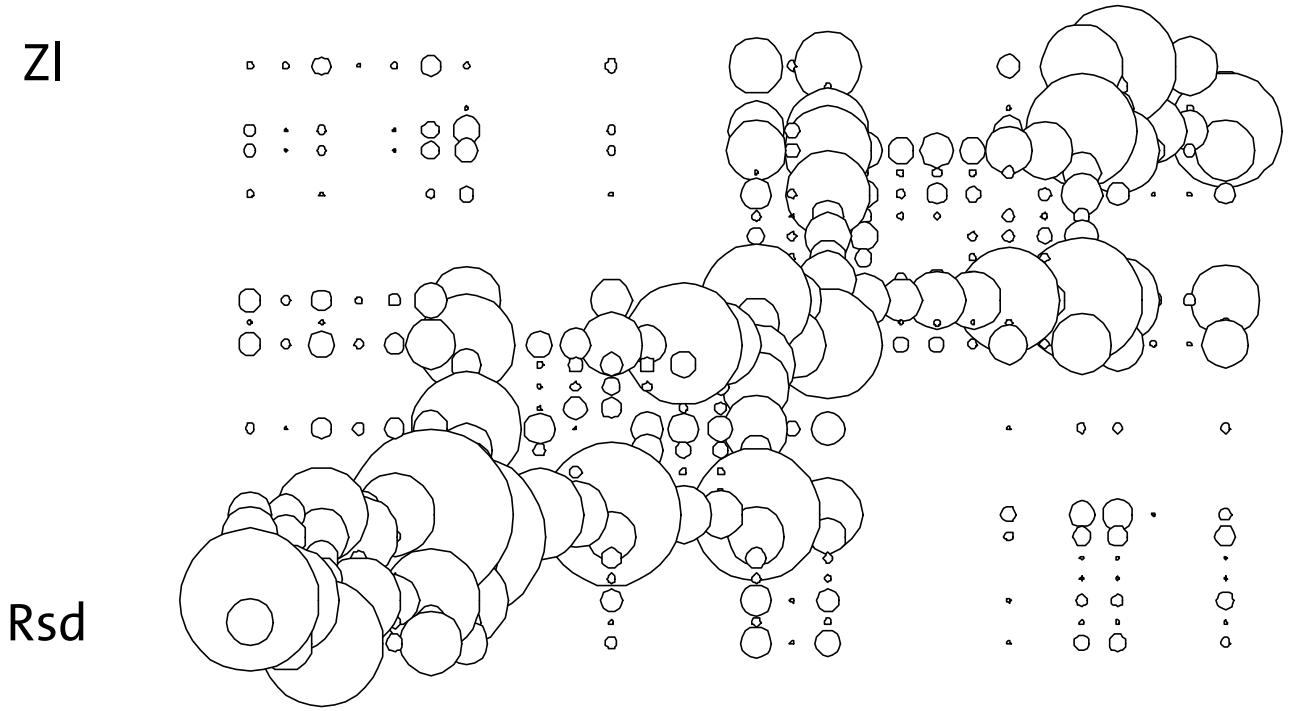
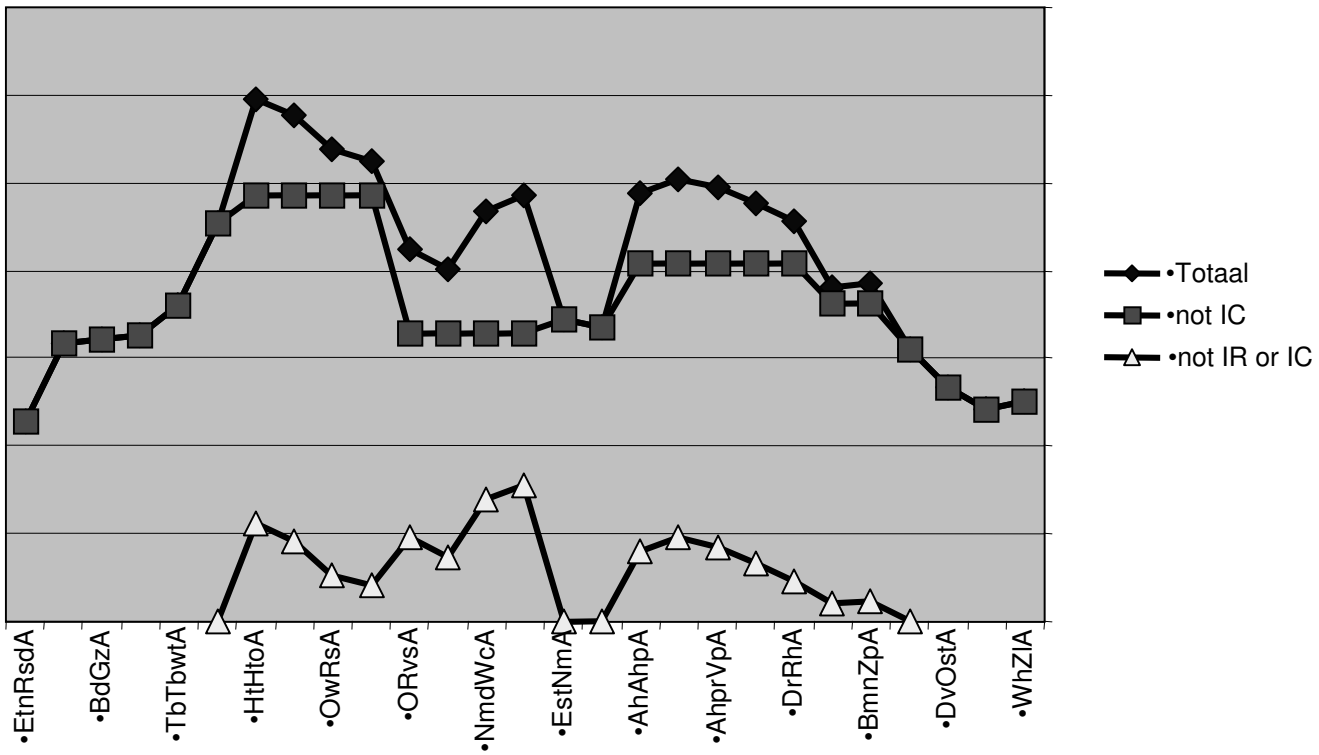
Practical applications

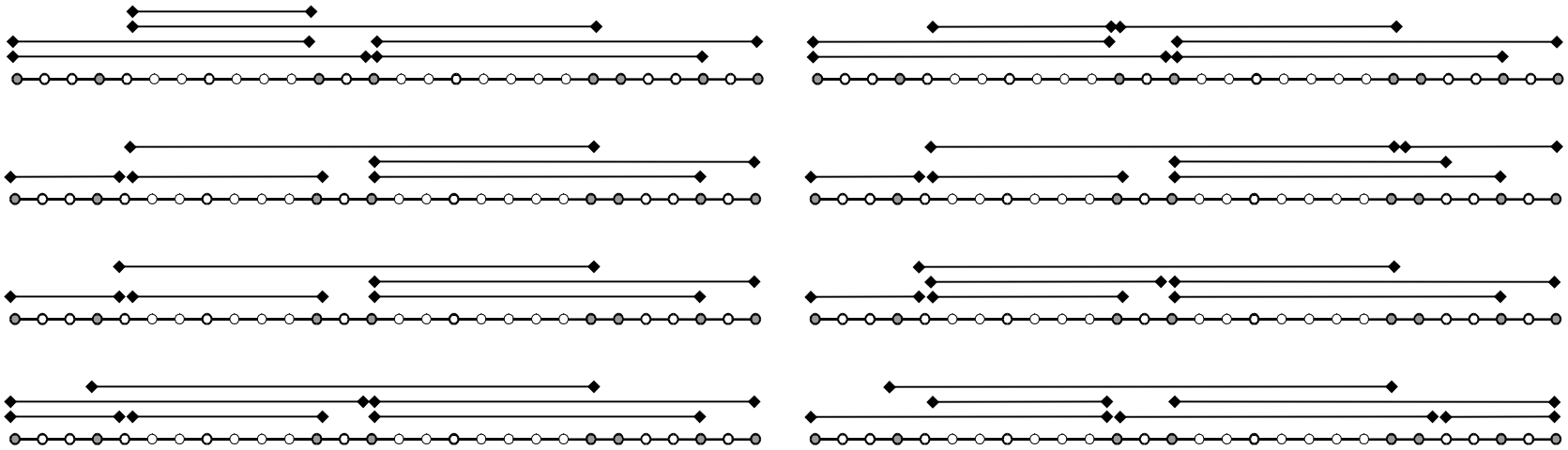


Practical applications



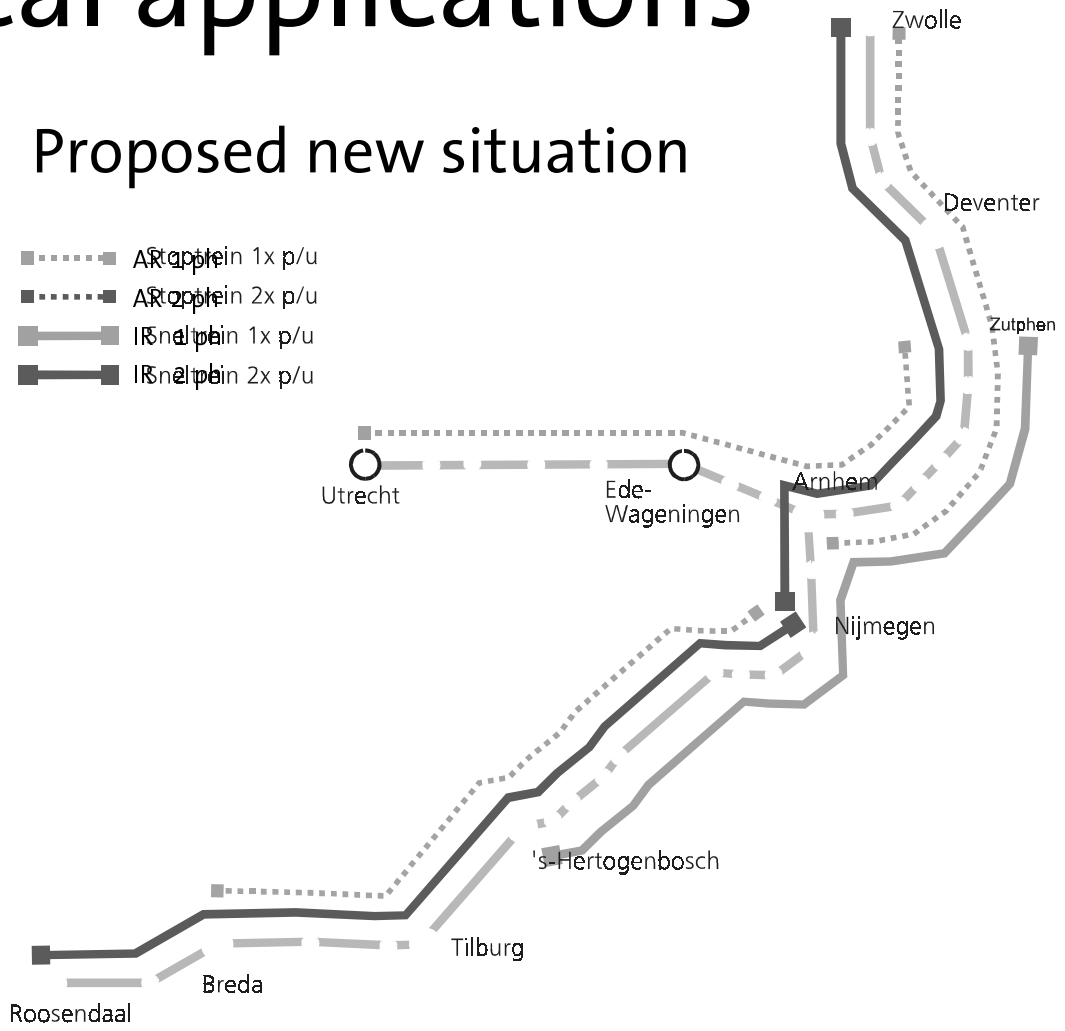
Practical applications

- Produce a range of solutions



Practical applications

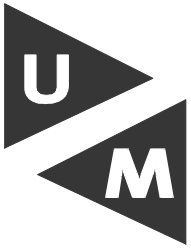
Proposed new situation



Practical applications

Serie	3600				7500	4400	7600
Average	527,9				98,5	283	106,9
StDev.	107,7				37,5	105,7	33,8
Max.	815				174	476	171

Serie	101	102	104	105	7500	103	
Average	128,9	198,4	446,8	308	105,7	141,1	
StDev.	24,2	93,9	90,4	66,6	53	48,2	
Max.	169	396	581	371	204	262	



The end

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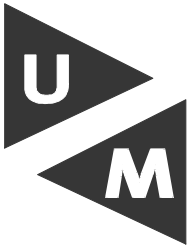
Short reference list:

M. Bussieck, *Optimal lines in public rail transportations*, PhD thesis, University of Braunschweig, 1998

M.T. Claessens et al., *Cost optimal allocation of rail passenger lines*, EJOR, 1998-110