«Magic: The Gathering» is Turing Complete

Alex Churchill, Stella Biderman, Austin Herrick

Presentation by Thomas Bläsius

What does that even mean?

Theorem
Determining the outcome of a game of **Magic: The Gathering** in which all remaining moves are forced is undecidable.

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General plan

- create game state where every move is forced by the game rules
- remaining moves simulate a universal TM with arbitrary input
- first player (Alice) wins if and only if TM halts

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Determining the outcome of a game of **Magic: The Gathering** in which all remaining moves are forced is undecidable.

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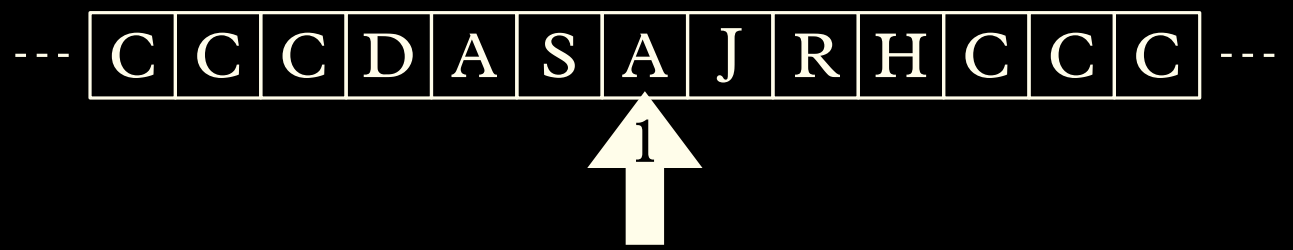
$\Rightarrow$ deciding whether Alice wins is as hard has the halting problem

Rogozhin's Universal Turing machine

UTM(2, 18) [Rogozhin, 1996]

- has two states: 1 and 2
- alphabet of size 18:
  {A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, R, S}
- interprets input on the tape as TM and simulates it



current state	read symbol	write symbol	head movement	new state
1	A	S	←	1
1	B	E	→	1
1	C	S	←	1
1	D	A	→	1
1	E	D	←	1
1	F	H	→	1
1	G	J	→	1
1	H	F	←	1
1	I	F	→	1
1	J	I	←	1
1	K	L	←	2
1	L	I	←	2
1	M	B	←	2
1	N	O	→	1
1	O	P	←	1
1	P	R	→	2
1	R		halt	
1	S	C	→	1
2	A	…		

Questions to be answered

Core components

- How to represent the tape?
- How to implement the transition function?
  - How does the head read and write symbols on the tape?
  - How does the head move left or right?
  - How can we represent different states?

Interplay between components

- How can players trigger the components?
- How to force players to behave as intended?
- How to win the game (a.k.a. the TM halts)?
- What to do, if we are out of tape?

Setup

- How can one player set up the initial game state?
- Can I do that at a tournament?

Tape

Rotlung Reanimator
Creature — Zombie Cleric
Whenever Rotlung Reanimator or another Cleric is put into a graveyard from play, put a 2/2 black Zombie creature token into play.
Illus. Thomas M. Baxa
™ & © 1993–2002 Wizards of the Coast, Inc. 164/350
2/2

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2/2
Zombie
Token Creature — Zombie
2/2
008/016 T
UMA • EN
Anna Steinbauer
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Tape

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Tape

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Tape

There are other creature types

- Aetherborn
- Basilisk
- Cephalid
- Demon
- Elf
- Faerie
- Giant
- Harpy
- Illusion
- Juggernaut
- Kavu
- Leviathan
- Myr
- Noggle
- Orc
- Pegasus
- Rhino
- Sliver

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- Pegasus
- Rhino
- Sliver

Giant	Sliver	Rhino	Faerie	Elf	Demon	Giant	Orc	Elf
6/6	5/5	4/4	3/3	2/2	3/3	4/4	5/5	6/6

Transition function: reading and writing

	read symbol	write symbol	head movement
A		S	←
B		E	→
C		S	←
D		A	→
E		**D**	**←**
F		H	→
G		J	→

Giant	Sliver	Rhino	Faerie	Elf	Demon	Giant	Orc	Elf
6/6	5/5	4/4	3/3	2/2	3/3	4/4	5/5	6/6

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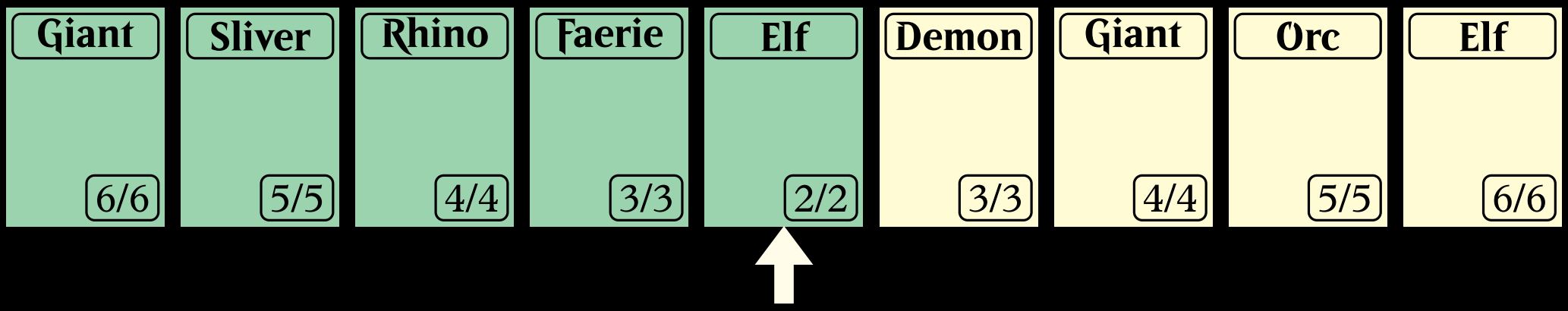
G		J	→



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B		E	→
C		S	←
D		A	→
E		D	←
F		H	→
G		J	→

Giant	Sliver	Rhino	Faerie	Elf	Demon	Giant	Orc	Elf
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B	E	→
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D	A	→
E	D	←
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Transition function: reading and writing

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E		D	←	
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G		J	→	

Giant	Sliver	Rhino	Faerie	Elf	Demon	Giant	Orc	Elf
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F		H	→	
G		J	→	

Giant	Sliver	Rhino	Faerie
6/6	5/5	4/4	3/3

Demon	Giant	Orc	Elf
3/3	4/4	5/5	6/6

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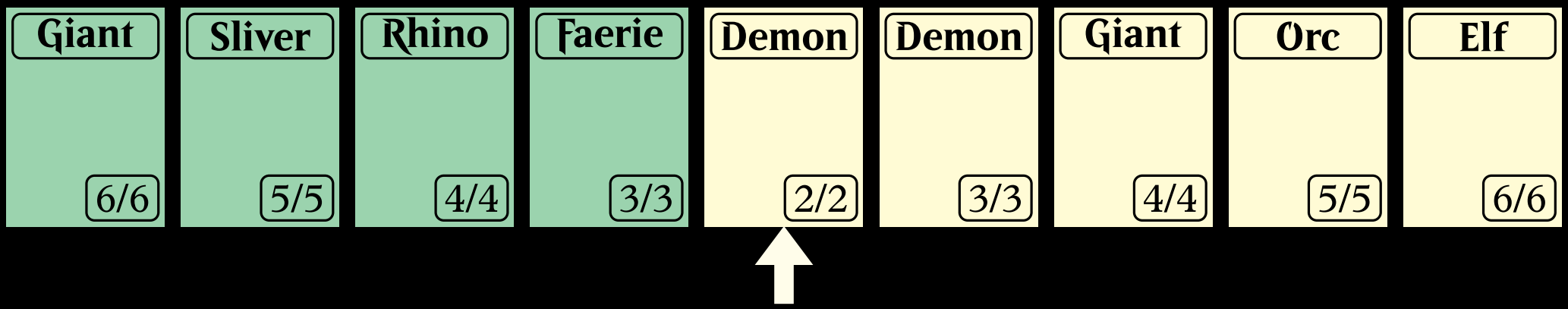
C		S	←	
D		A	→	
E		D	←	
F		H	→	
G		J	→	

Giant	Sliver	Rhino	Faerie	Demon	Demon	Giant	Orc	Elf
6/6	5/5	4/4	3/3	2/2	3/3	4/4	5/5	6/6

Transition function: moving the head (left)



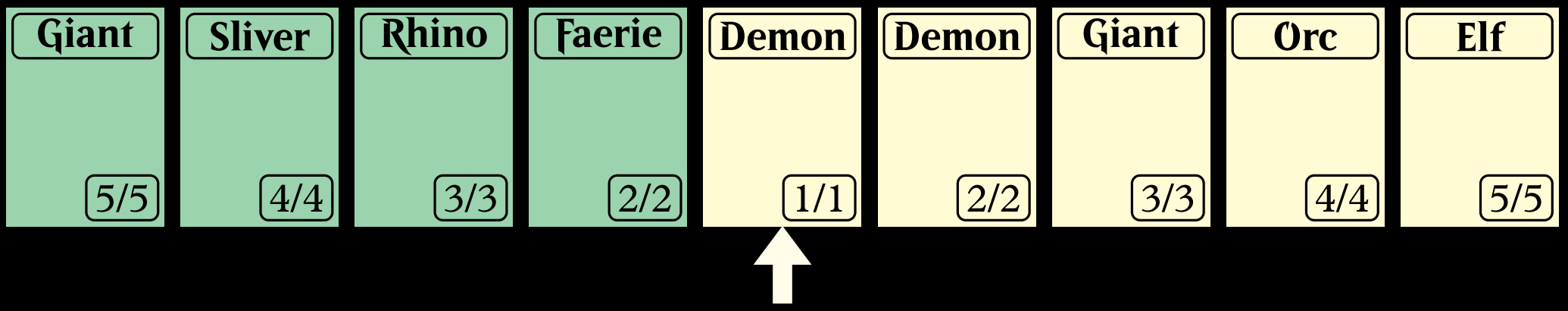
Transition function: moving the head (left)

Giant	Sliver	Rhino	Faerie	Demon	Demon	Giant	Orc	Elf
6/6	5/5	4/4	3/3	2/2	3/3	4/4	5/5	6/6

Transition function: moving the head (left)

Giant	Sliver	Rhino	Faerie	Demon	Demon	Giant	Orc	Elf
5/5	4/4	3/3	2/2	1/1	2/2	3/3	4/4	5/5

Transition function: moving the head (left)



Transition function: moving the head (left)

Giant	Sliver	Rhino	Faerie	Demon	Demon	Giant	Orc	Elf
5/5	4/4	3/3	2/2	3/3	4/4	5/5	6/6	7/7

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Transition function: changing state

current state	read symbol	write symbol	head movement	new state
⋮	⋮	⋮	⋮	⋮
1	E	D	←	1
1	F	H	→	1
⋮	⋮	⋮	⋮	⋮
2	E	A	←	2
2	F	K	→	1
⋮	⋮	⋮	⋮	⋮

Transition function: changing state

1

current state	read symbol	write symbol	head movement	new state
⋮	⋮	⋮	⋮	⋮
1	E	D	←	1
1	F	H	→	1
⋮	⋮	⋮	⋮	⋮
2	E	A	←	2
2	F	K	→	1
⋮	⋮	⋮	⋮	⋮

Transition function: changing state

1

2

current state	read symbol	write symbol	head movement	new state
⋮	⋮	⋮	⋮	⋮
1	E	D	←	1
1	F	H	→	1
⋮	⋮	⋮	⋮	⋮
2	E	A	←	2
2	F	K	→	1
⋮	⋮	⋮	⋮	⋮

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Transition function: changing state

1

2

current state	read symbol	write symbol	head movement	new state
⋮	⋮	⋮	⋮	⋮
1	E	D	←	1
1	F	H	→	1
⋮	⋮	⋮	⋮	⋮
2	E	A	←	2
2	F	K	→	1

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Transition function: changing state

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2

current state	read symbol	write symbol	head movement	new state
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Transition function: changing state

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Questions to be answered

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Interplay between components

- How can players trigger the components?
- How to force players to behave as intended?
- How to win the game (a.k.a. the TM halts)?
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Setup

- How can one player set up the initial game state?
- Can I do that at a tournament?

What the players do

Two players: Alice and Bob

- Bob can't do anything
- Alice repeatedly takes four turns
- she sometimes skips turn 3 to change state (remember: phasing)

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Turn 1

What the players do

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- she sometimes skips turn 3 to change state (remember: phasing)

Turn 1

Turn 2

What the players do

Two players: Alice and Bob

- Bob can’t do anything
- Alice repeatedly takes four turns
- she sometimes skips turn 3 to change state (remember: phasing)

Turn 1

Turn 2

Turn 3

What the players do

Two players: Alice and Bob

- Bob can’t do anything
- Alice repeatedly takes four turns
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Turn 1

Turn 2

Turn 3

Turn 4

Forcing Alice to cast spells

(controlled by Bob)

Forcing Alice to cast spells

(controlled by Bob)

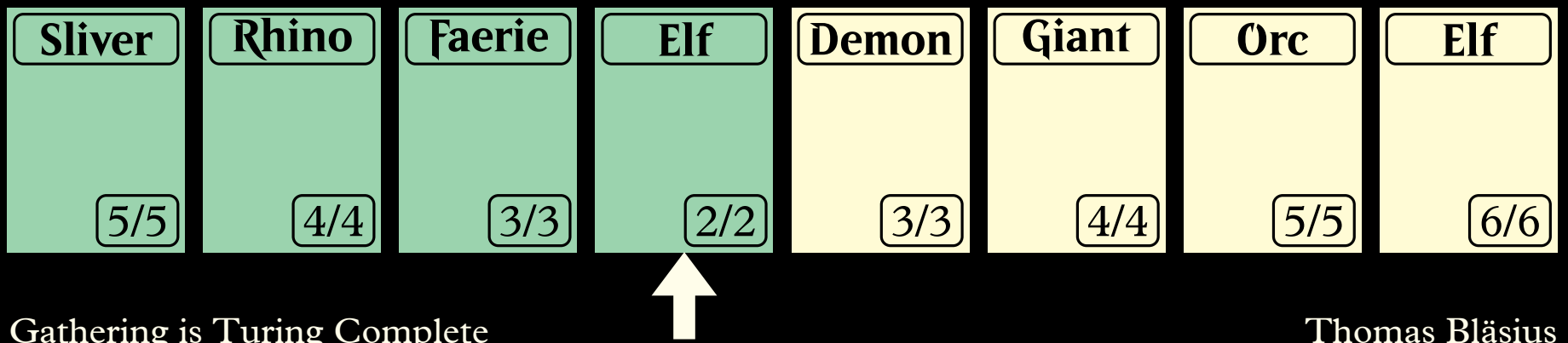
Forcing Alice to cast spells

(controlled by Bob)

Changing the state by skipping turn 3

(controlled by Alice)

Forcing Alice to pick the right target



Forcing Alice to pick the right target

protecting Bob's creatures

protecting Alice's creatures

Sliver	Rhino	Faerie	Elf	Demon	Giant	Orc	Elf
5/5	4/4	3/3	2/2	3/3	4/4	5/5	6/6

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Forcing Alice to pick the right target

protecting Bob's creatures

protecting Alice's creatures

only legal target: newest creature

	Sliver	Rhino	Faerie	Elf	Demon	Giant	Orc	Elf
controller:	Bob	Bob	Bob	Alice	Bob	Bob	Bob	Bob
	5/5	4/4	3/3	2/2	3/3	4/4	5/5	6/6

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Keeping your engine alive

Locking everything else down

(controlled by Bob)

(controlled by Alice)

(controlled by both)

Halting

current state	read symbol	write symbol	head movement	new state
⋮	⋮	⋮	⋮	⋮
1	R		halt	
⋮	⋮	⋮	⋮	⋮

goal: TM halts $\Leftrightarrow$ Alice wins with:

Halting

current state	read symbol	write symbol	head movement	new state
⋮	⋮	⋮	⋮	⋮
1	R		halt	
⋮	⋮	⋮	⋮	⋮

goal: TM halts ⇔ Alice wins with:

Halting

current state	read symbol	write symbol	head movement	new state
⋮	⋮	⋮	⋮	⋮
1	R		halt	
⋮	⋮	⋮	⋮	⋮

goal: TM halts ⇔ Alice wins with:

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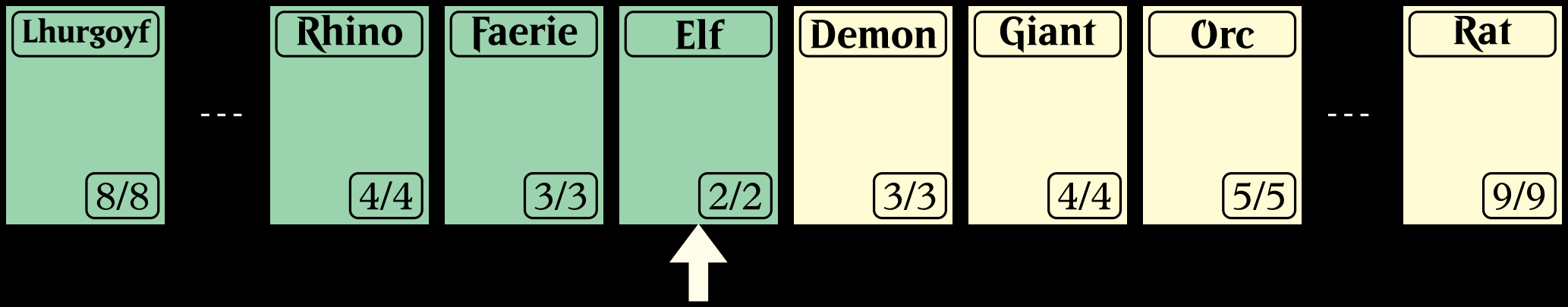
Out of tape exception



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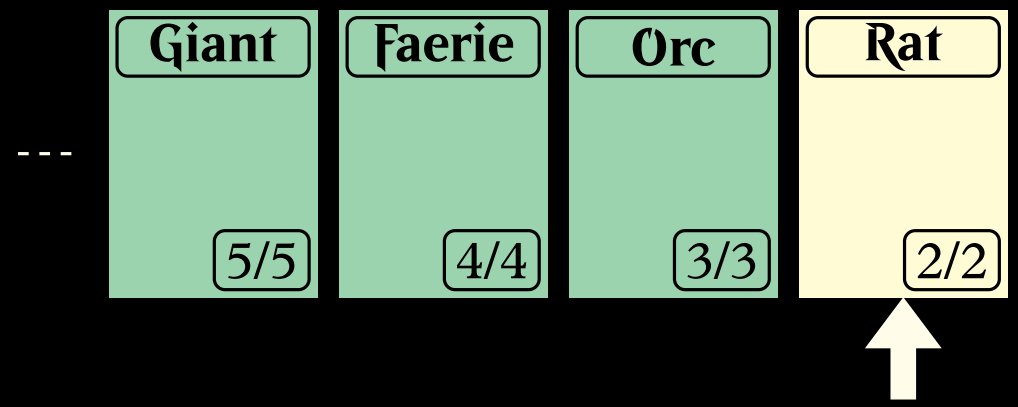

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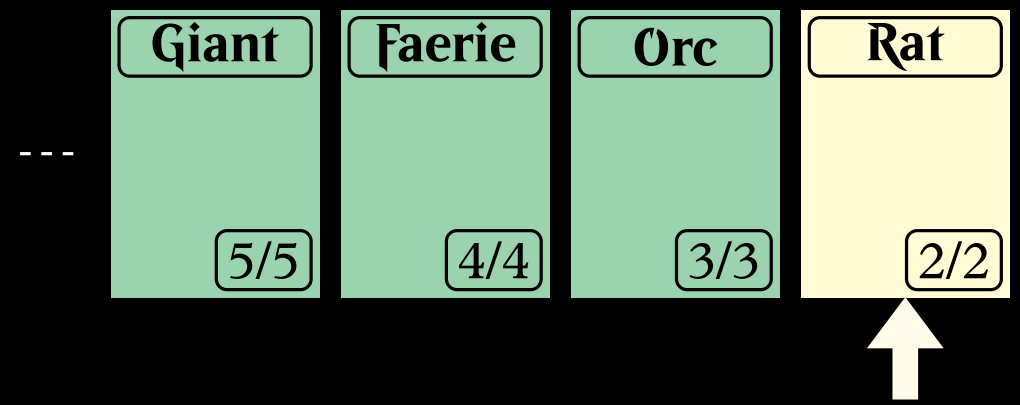
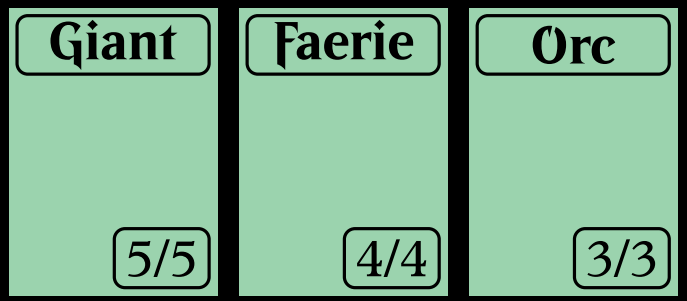


(controlled by Bob) (controlled by Alice)

Out of tape exception



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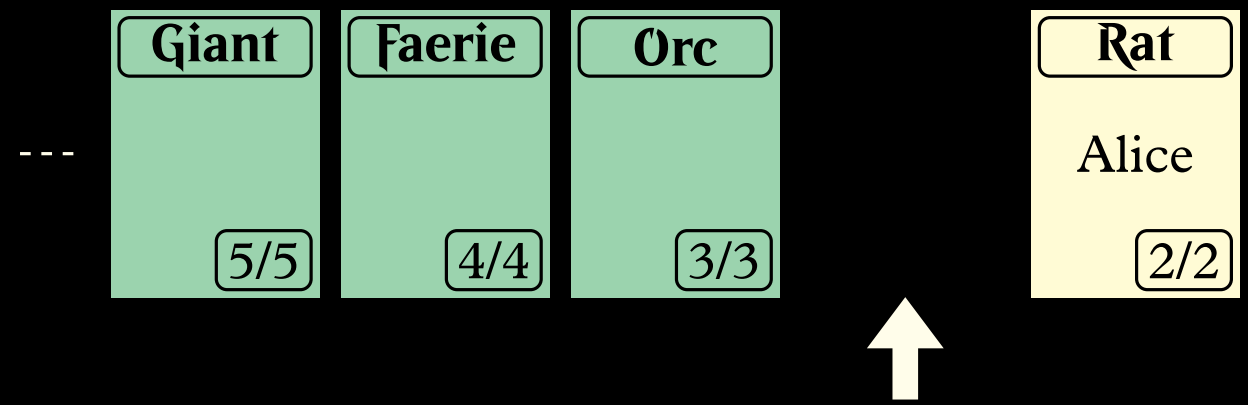

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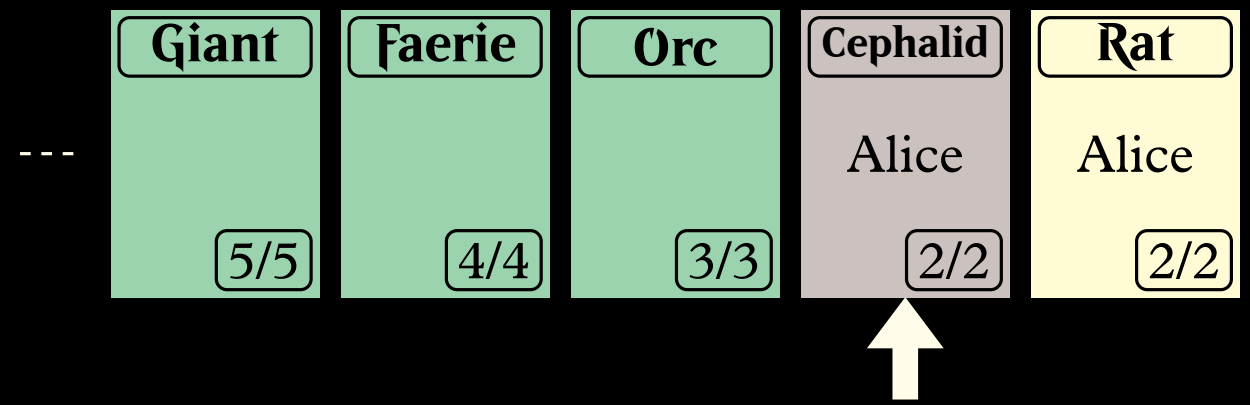

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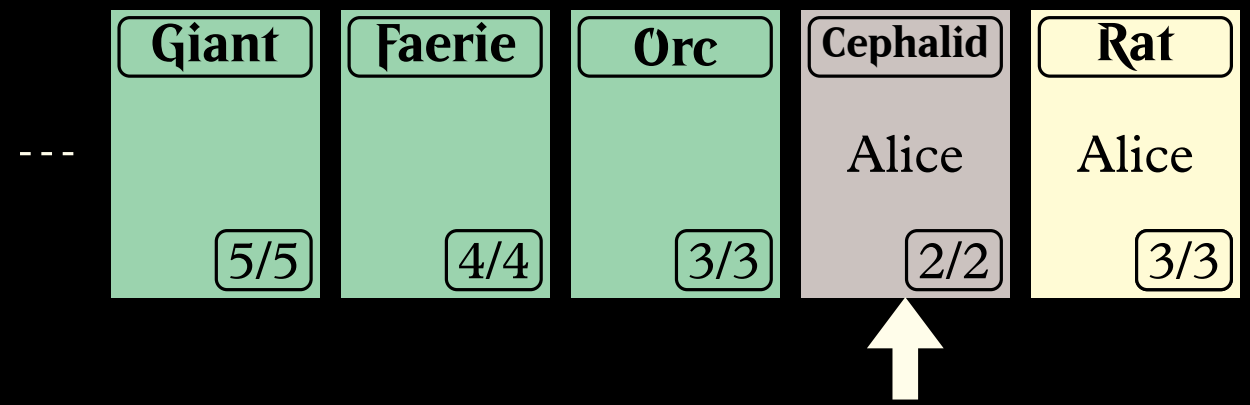

(controlled by Bob) (controlled by Alice)

Out of tape exception

Giant	Faerie	Orc	Cephalid	Rat
			Alice	Alice
5/5	4/4	3/3	2/2	3/3

(controlled by Bob) (controlled by Alice)

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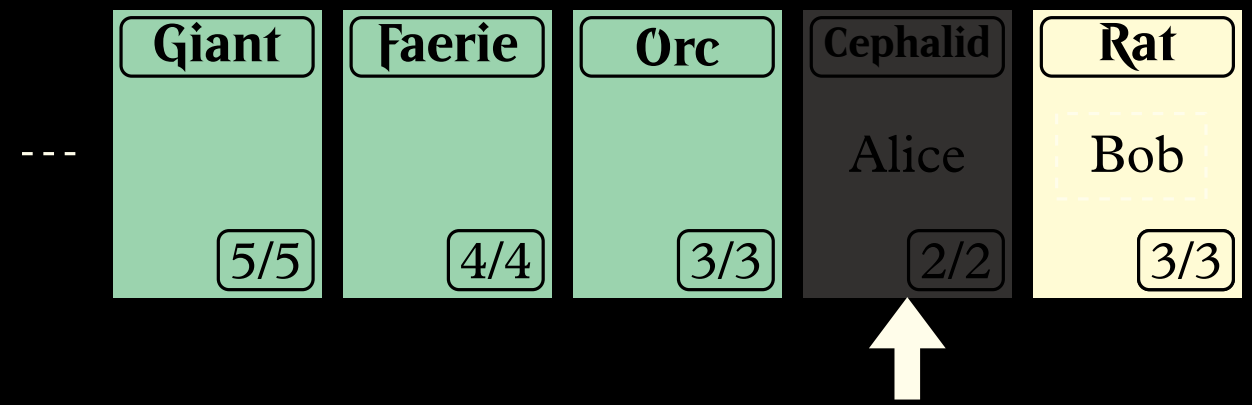
Out of tape exception



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Werbepause

Vertragliches

- April bis Juli (4 Monate)
- 40 h/Monat
- 12 €/h

Bewerben

- bis spätestens 8.1.23
- gerne früher
- portal.wiwi.kit.edu/forms/form/640

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Setup

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Setup: getting startet

MANA

colorless	white	blue
black	red	green

BATTLEFIELD

HAND

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Setup: getting startet

MANA

colorless	white	blue
black	red	green

BATTLEFIELD

HAND

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Setup: getting startet

MANA

colorless	white	blue
2		
black	red	green

BATTLEFIELD

HAND

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Setup: getting startet

MANA

colorless 2	white	blue
black	red	green

BATTLEFIELD

HAND

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Setup: getting startet

MANA

colorless	white	blue
black	red	green

BATTLEFIELD

HAND

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colorless	white	blue
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BATTLEFIELD

HAND

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Setup: getting startet

MANA

colorless	white	blue
		2
black	red	green

BATTLEFIELD

HAND

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Setup: getting startet

MANA

colorless	white	blue
		2
black	red	green

BATTLEFIELD

HAND

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Setup: getting startet

MANA

colorless	white	blue
black	red	green

BATTLEFIELD

HAND

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Setup: getting startet

MANA

colorless	white	blue
black	red	green

BATTLEFIELD

HAND

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Setup: getting startet

MANA

colorless 3	white	blue
black	red	green

BATTLEFIELD

HAND

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Setup: getting startet

MANA

colorless 1	white	blue
black	red	green

BATTLEFIELD

HAND

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Setup: getting startet

MANA

colorless 4	white	blue
black	red	green

BATTLEFIELD

HAND

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Setup: getting startet

MANA

colorless 2	white	blue
black	red	green

BATTLEFIELD

HAND

Magic: The Gathering is Turing Complete

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Setup: getting startet

MANA

colorless *n*	white	blue
black	red	green

BATTLEFIELD

HAND

Magic: The Gathering is Turing Complete

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Setup: getting startet

MANA

colorless n	white	blue
black	red	green

BATTLEFIELD

HAND

Magic: The Gathering is Turing Complete

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Setup: getting startet

MANA

colorless n	white	blue
black	red	green

BATTLEFIELD

HAND

Magic: The Gathering is Turing Complete

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Setup: getting startet

MANA

colorless *n*	white	blue
black	red	green

BATTLEFIELD

HAND

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Setup: getting startet

MANA

colorless *n*	white	blue
black	red	green

BATTLEFIELD

HAND

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Setup: getting startet

MANA			
colorless *n*	white *n*	blue *n*	
black *n*	red *n*	green *n*	

BATTLEFIELD

HAND

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Setup: getting startet

MANA

colorless	white	blue
n	*n*	*n*
black	**red**	**green**
n	*n*	*n*

BATTLEFIELD

HAND

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Setup: reusing cards

Setup: creating the tape

Setup: cleaning up

Setup: cleaning up

Conclusion

Theorem
Determining the outcome of a game of **Magic: The Gathering** in which all remaining moves are forced is undecidable.

Note

- a player can set up the corresponding game state in the first turn
- with a tournament legal deck (in the Legacy format)

Conclusion

Theorem
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- the mate-in-n problem is Δ_n^0 hard [**Biderman**, 2020]