



«Magic: The Gathering» is Turing Complete

Alex Churchill, Stella Biderman, Austin Herrick

Presentation by Thomas Bläsius

What does that even mean?

Theorem

Determining the outcome of a game of **Magic: The Gathering** in which all remaining moves are forced is undecidable.

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Determining the outcome of a game of **Magic: The Gathering** in which all remaining moves are forced is undecidable.

General plan

- create game state where every move is forced by the game rules
- remaining moves simulate a universal TM with arbitrary input
- first player (Alice) wins if and only if TM halts

What does that even mean?

Theorem

Determining the outcome of a game of **Magic: The Gathering** in which all remaining moves are forced is undecidable.

General plan

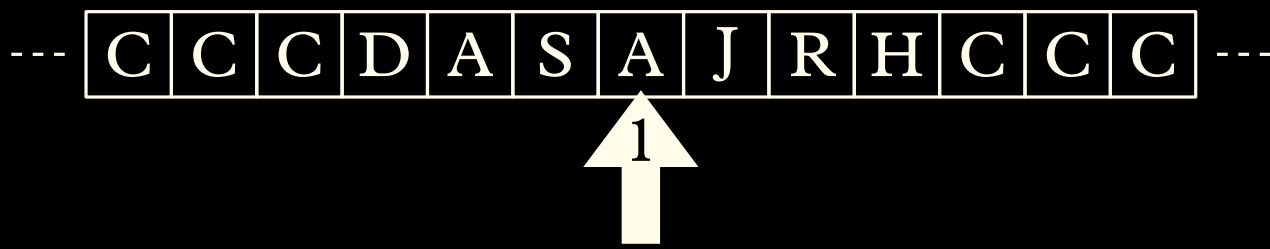
- create game state where every move is forced by the game rules
 - remaining moves simulate a universal TM with arbitrary input
 - first player (Alice) wins if and only if TM halts
- ⇒ deciding whether Alice wins is as hard as the halting problem

Rogozhin's Universal Turing machine

UTM(2, 18)

[Rogozhin, 1996]

- has two states: 1 and 2
- alphabet of size 18:
 {A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, R, S}
- interprets input on the tape as TM and simulates it



	current state	read symbol	write symbol	head movement	new state
1	A	S	←		1
1	B	E	→		1
1	C	S	←		1
1	D	A	→		1
1	E	D	←		1
1	F	H	→		1
1	G	J	→		1
1	H	F	←		1
1	I	F	→		1
1	J	I	←		1
1	K	L	←		2
1	L	I	←		2
1	M	B	←		2
1	N	O	→		1
1	O	P	←		1
1	P	R	→		2
1	R		halt		
1	S	C	→		1
2	A	...			

Questions to be answered

Core components

- How to represent the tape?
- How to implement the transition function?
 - How does the head read and write symbols on the tape?
 - How does the head move left or right?
 - How can we represent different states?

Interplay between components

- How can players trigger the components?
- How to force players to behave as intended?
- How to win the game (a.k.a. the TM halts)?
- What to do, if we are out of tape?

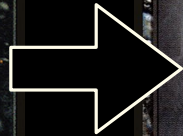
Setup

- How can one player set up the initial game state?
- Can I do that at a tournament?

Tape



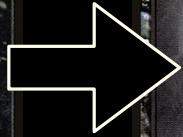
Tape



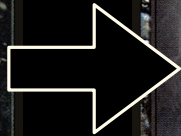
Tape



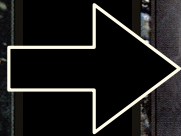
Tape



Tape



Tape

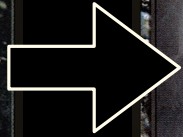


There are other creature types

- Aetherborn
- Basilisk
- Cephalid
- Demon
- Elf
- Faerie
- Giant
- Harpy
- Illusion
- Juggernaut
- Kavu
- Leviathan
- Myr
- Noggle
- Orc
- Pegasus
- Rhino
- Sliver

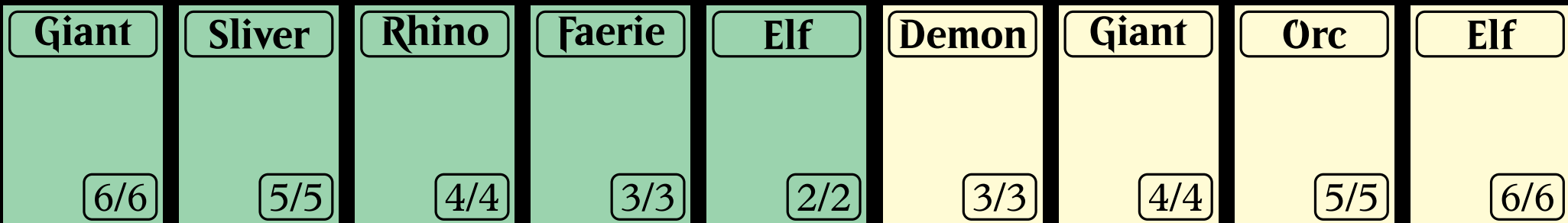


Tape



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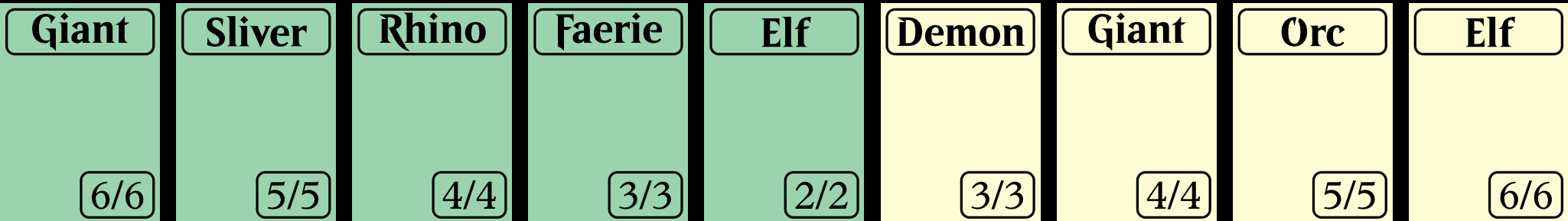
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- Rhino
- Sliver



Transition function: reading and writing



	read symbol	write symbol	head movement
A	S	←	
B	E	→	
C	S	←	
D	A	→	
E	D	←	
F	H	→	
G	J	→	



Transition function: reading and writing



read symbol
write symbol
head movement

A	S	←
B	E	→
C	S	←
D	A	→
E	D	←
F	H	→
G	J	→

Giant	Sliver	Rhino	Faerie	Elf	Demon	Giant	Orc	Elf
6/6	5/5	4/4	3/3	2/2	3/3	4/4	5/5	6/6

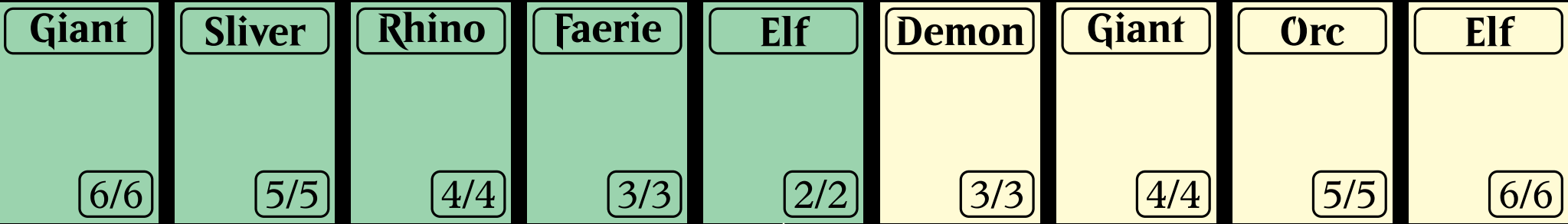


Transition function: reading and writing



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write symbol
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B	E	→
C	S	←
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E	D	←
F	H	→
G	J	→

Giant	Sliver	Rhino	Faerie	Elf	Demon	Giant	Orc	Elf
6/6	5/5	4/4	3/3	2/2	3/3	4/4	5/5	6/6



Transition function: reading and writing



read symbol
write symbol
head movement

A	S	←	■
B	E	→	■
C	S	←	■
D	A	→	■
E	D	←	■
F	H	→	■
G	J	→	■

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Transition function: reading and writing



read symbol
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A	S	←	■
B	E	→	■
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D	A	→	■
E	D	←	■
F	H	→	■
G	J	→	■

Giant	Sliver	Rhino	Faerie
6/6	5/5	4/4	3/3

Demon	Giant	Orc	Elf
3/3	4/4	5/5	6/6



Transition function: reading and writing

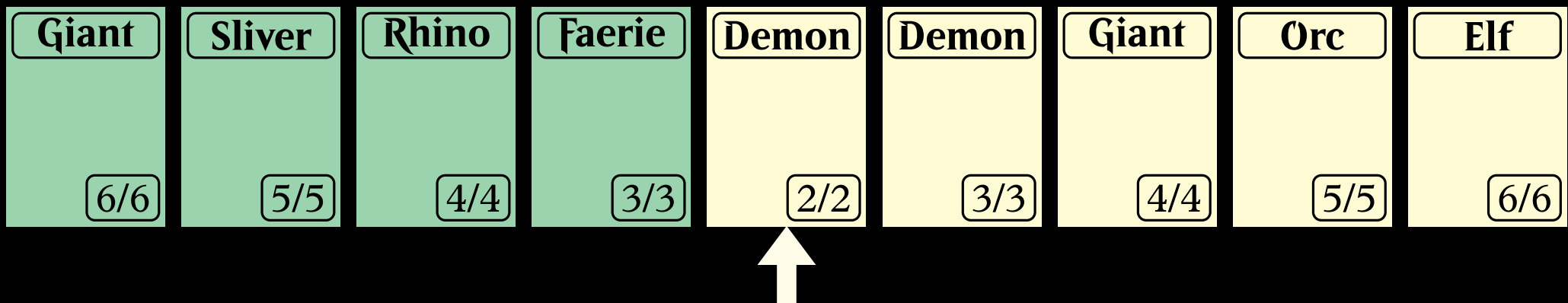


	read symbol	write symbol	head movement
A	S	←	■
B	E	→	■
C	S	←	■
D	A	→	■
E	D	←	■
F	H	→	■
G	J	→	■

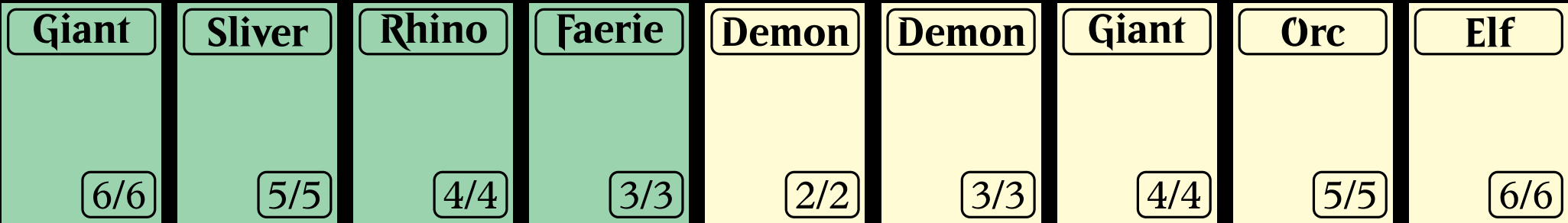
Giant	Sliver	Rhino	Faerie	Demon	Demon	Giant	Orc	Elf
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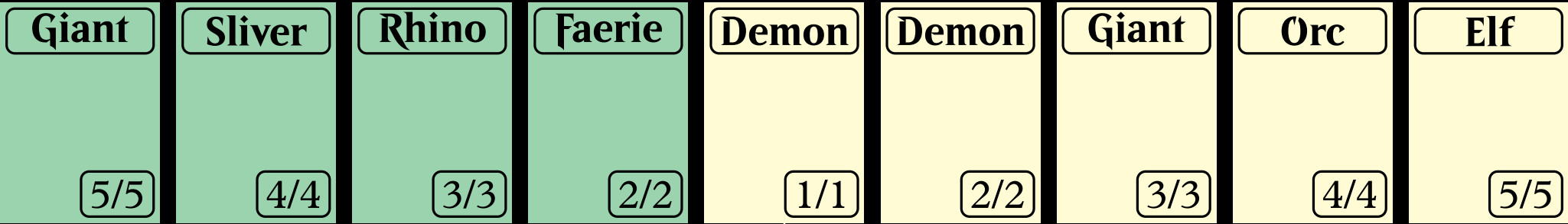
Transition function: moving the head (left)



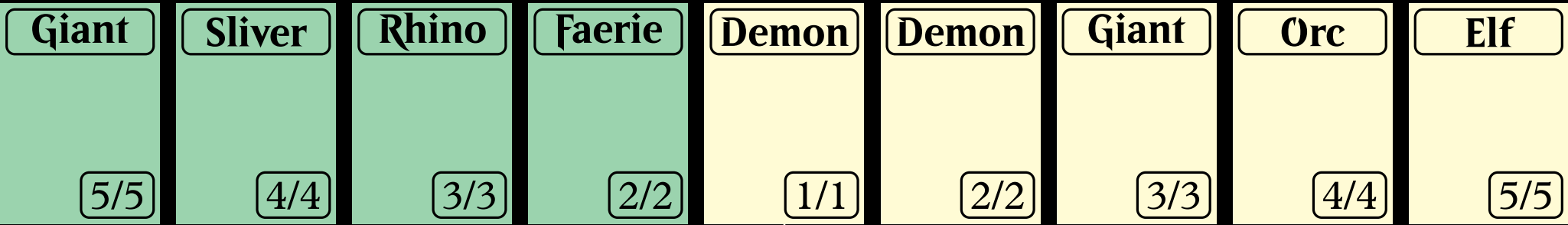
Transition function: moving the head (left)



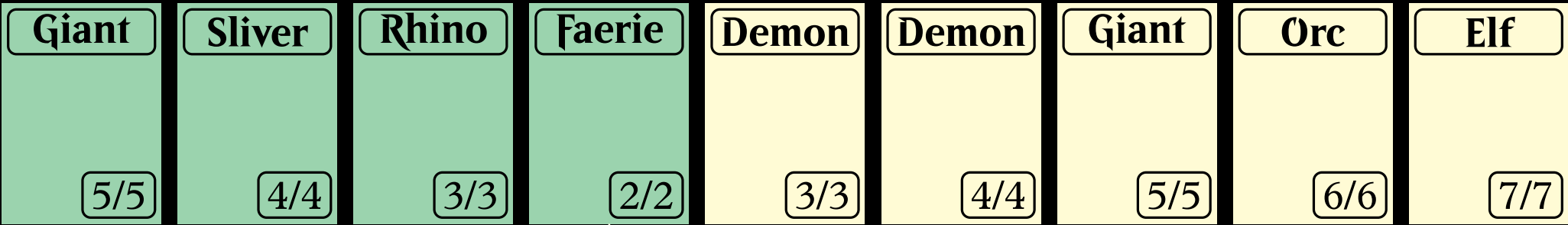
Transition function: moving the head (left)



Transition function: moving the head (left)



Transition function: moving the head (left)



Transition function: changing state

	current state	read symbol	write symbol	head movement	new state
	⋮	⋮	⋮	⋮	⋮
	1	E	D	←	1
	1	F	H	→	1
	⋮	⋮	⋮	⋮	⋮
	2	E	A	←	2
	2	F	K	→	1
	⋮	⋮	⋮	⋮	⋮

Transition function: changing state



1

current state	read symbol	write symbol	head movement	new state
:	:	:	:	:
1	E	D	←	1
1	F	H	→	1
:	:	:	:	:
2	E	A	←	2
2	F	K	→	1
:	:	:	:	:

Transition function: changing state



1

current state	read symbol	write symbol	head movement	new state
:	:	:	:	:
1	E	D	←	1
1	F	H	→	1
:	:	:	:	:
2	E	A	←	2
2	F	K	→	1
:	:	:	:	:



2

Transition function: changing state



1

current state	read symbol	write symbol	head movement	new state
:	:	:	:	:
1	E	D	←	1
1	F	H	→	1
:	:	:	:	:
2	E	A	←	2
2	F	K	→	1



2



Transition function: changing state



1

current state	read symbol	write symbol	head movement	new state
:	:	:	:	:
1	E	D	←	1
1	F	H	→	1
:	:	:	:	:
2	E	A	←	2
2	F	K	→	1



2



Transition function: changing state



1

current state	read symbol	write symbol	head movement	new state
⋮	⋮	⋮	⋮	⋮
1	E	D	←	1
1	F	H	→	1
⋮	⋮	⋮	⋮	⋮
2	E	A	←	2
2	F	K	→	1



2



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Setup

- How can one player set up the initial game state?
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What the players do

Two players: Alice and Bob

- Bob can't do anything
- Alice repeatedly takes four turns
- she sometimes skips turn 3 to change state (remember: phasing)

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Turn 1



What the players do

Two players: Alice and Bob

- Bob can't do anything
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- she sometimes skips turn 3 to change state (remember: phasing)

Turn 1



Turn 2



What the players do

Two players: Alice and Bob

- Bob can't do anything
- Alice repeatedly takes four turns
- she sometimes skips turn 3 to change state (remember: phasing)

Turn 1



Turn 2



Turn 3



What the players do

Two players: Alice and Bob

- Bob can't do anything
- Alice repeatedly takes four turns
- she sometimes skips turn 3 to change state (remember: phasing)

Turn 1



Turn 2



Turn 3



Turn 4



Forcing Alice to cast spells



(controlled by Bob)

Forcing Alice to cast spells



(controlled by Bob)

Forcing Alice to cast spells



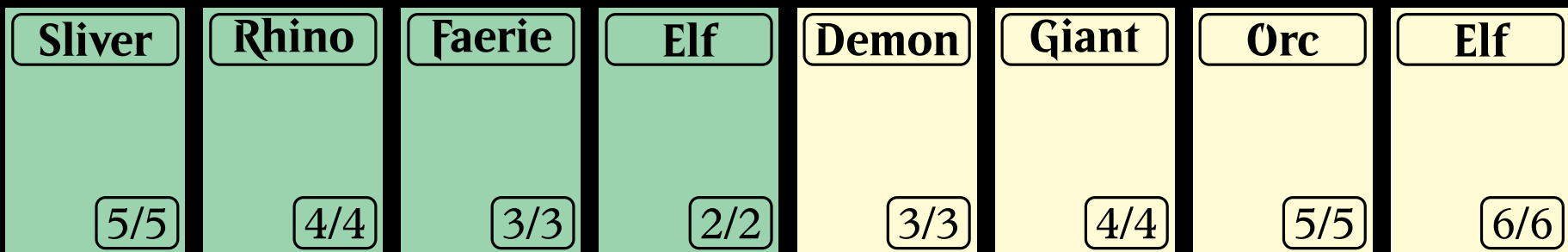
(controlled by Bob)

Changing the state by skipping turn 3



(controlled by Alice)

Forcing Alice to pick the right target

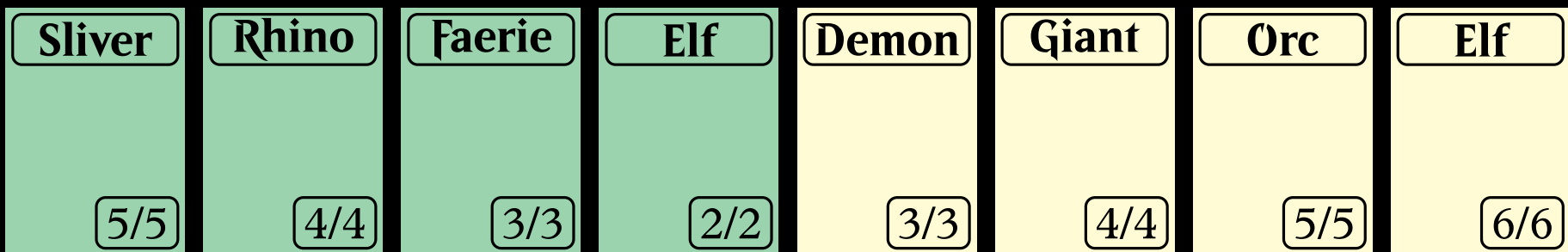


Forcing Alice to pick the right target

protecting Bob's creatures



protecting Alice's creatures



Forcing Alice to pick the right target

protecting Bob's creatures



protecting Alice's creatures



only legal target:
newest creature



controller:

Sliver	Rhino	Faerie	Elf	Demon	Giant	Orc	Elf
Bob	Bob	Bob	Alice	Bob	Bob	Bob	Bob
5/5	4/4	3/3	2/2	3/3	4/4	5/5	6/6



Keeping your engine alive

Vigor 3



Creature — Elemental Incarnation

Trample
If damage would be dealt to another creature you control, prevent that damage. Put a +1/+1 counter on that creature for each 1 damage prevented this way.
When Vigor is put into a graveyard from anywhere, shuffle it into its owner's library.

6/6

215 / 254 R
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Prismatic Lace



Instant

Target permanent becomes the color(s) of your choice. Costs to tap, maintain, or use an ability of that permanent remain unchanged.

Illus. David O'Connor
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Fungus Sliver 3



Creature — Fungus Sliver

All Slivers have “Whenever this creature is dealt damage, put a +1/+1 counter on it.” (The damage is dealt before the counter is put on.)
“When a sliver of this breed enters the hive, the others claw each other in frenzied fits, thereby ensuring their rapid growth.”
—Rukarumel, field journal

2/2

Daniel Gelon
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Locking everything else down



(controlled by Bob)



(controlled by Alice)



(controlled by both)

Halting

	current state	read symbol	write symbol	head movement	new state
:	:	:	:	:	:
1	R		halt		
:	:	:	:	:	:

goal: TM halts \Leftrightarrow Alice wins with:



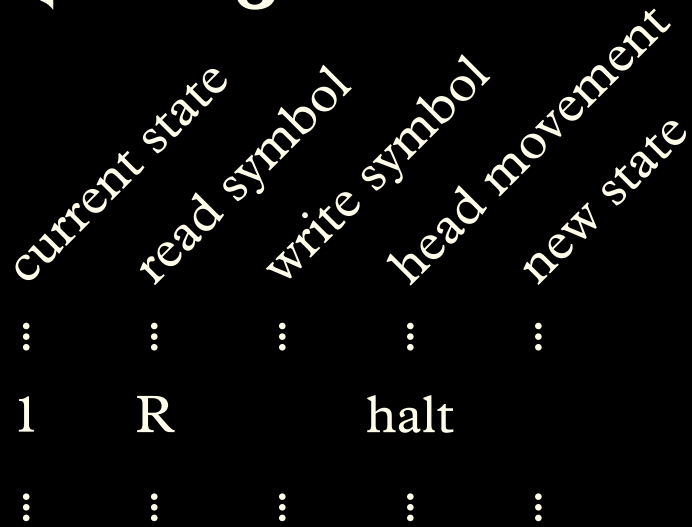
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current state	read symbol	write symbol	head movement	new state
:	:	:	:	:
1	R	halt		
:	:	:	:	:

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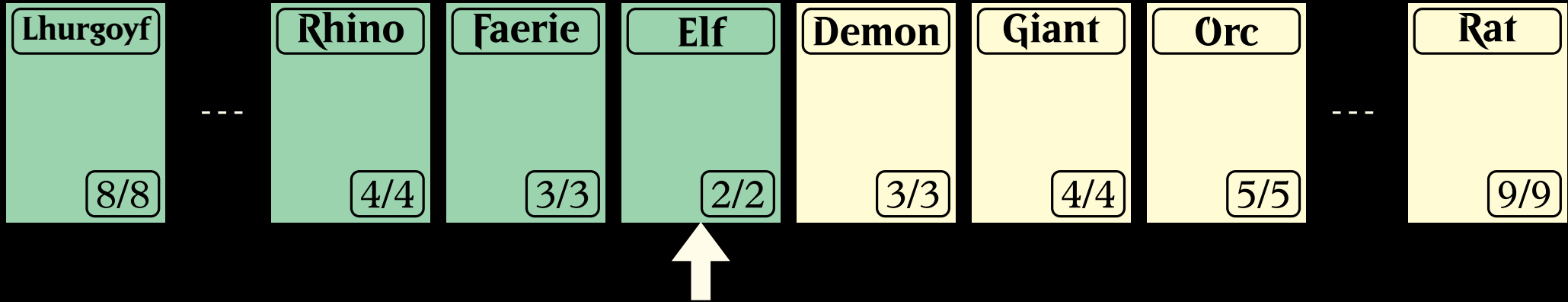
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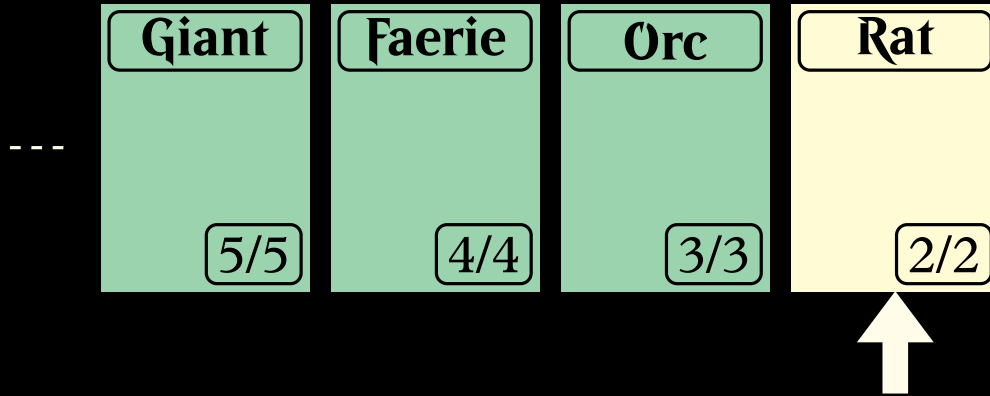
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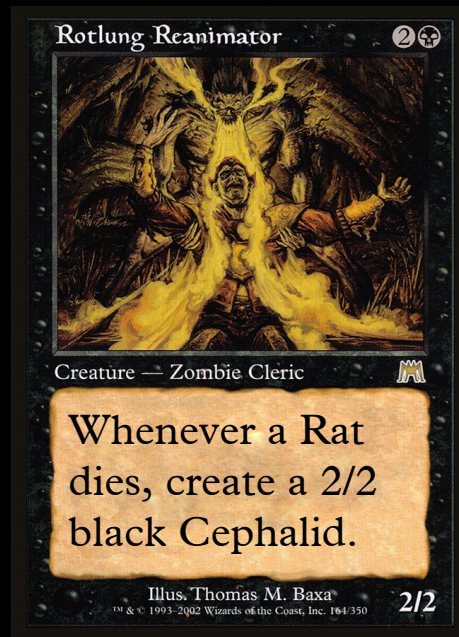
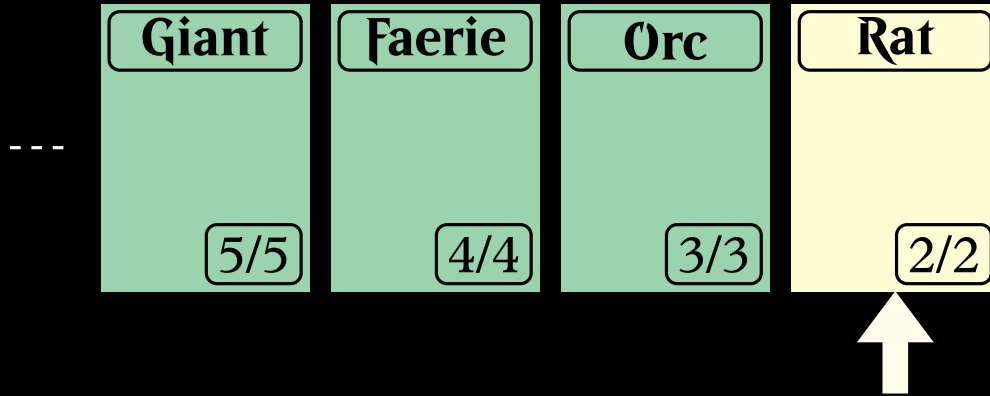
Out of tape exception



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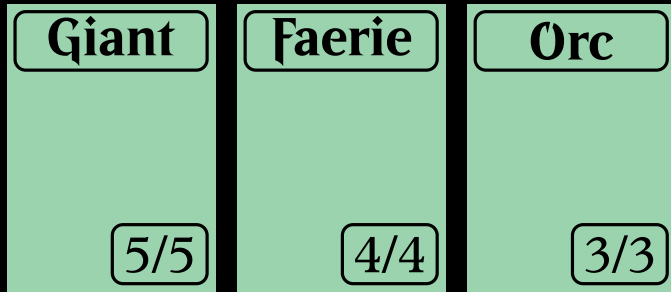


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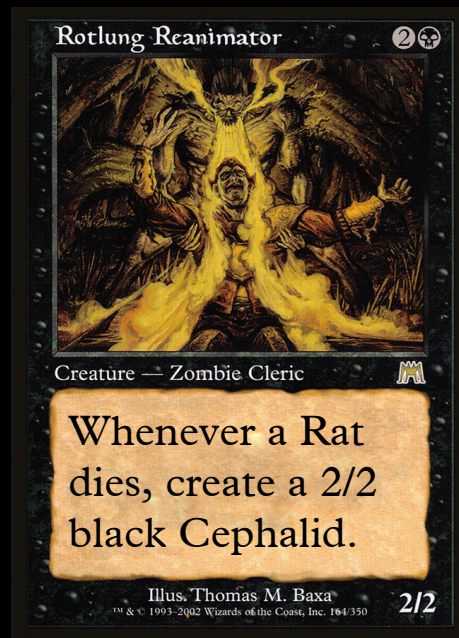
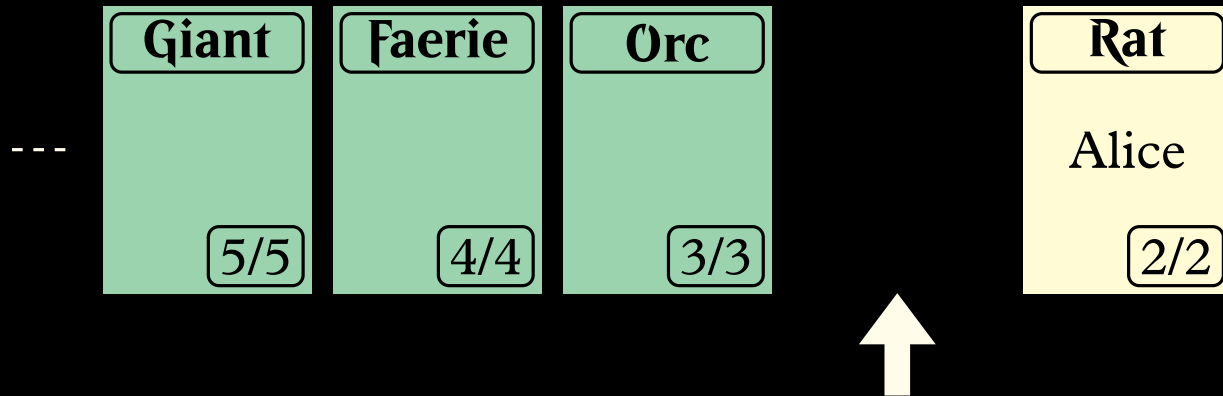
(controlled by Bob) (controlled by Alice)

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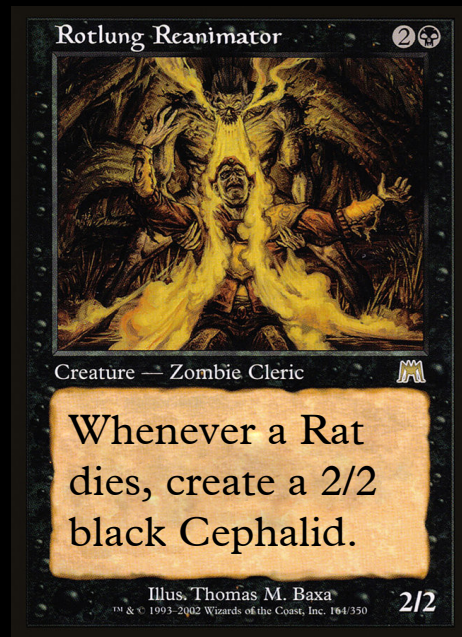
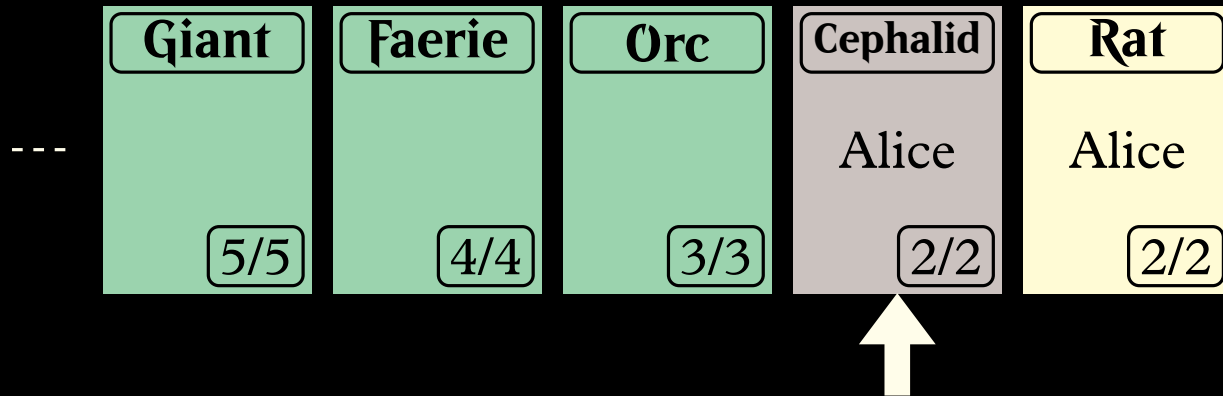
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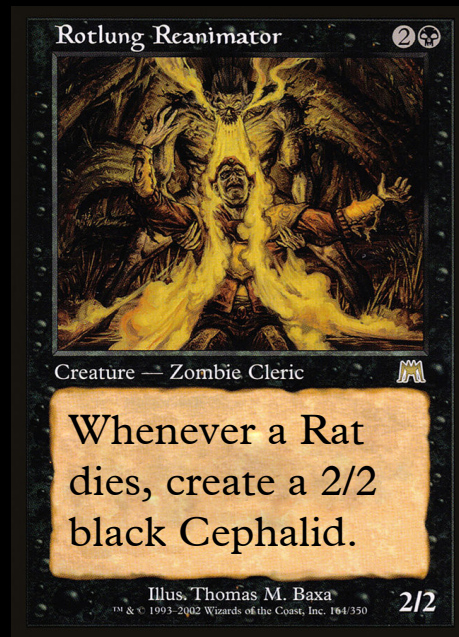
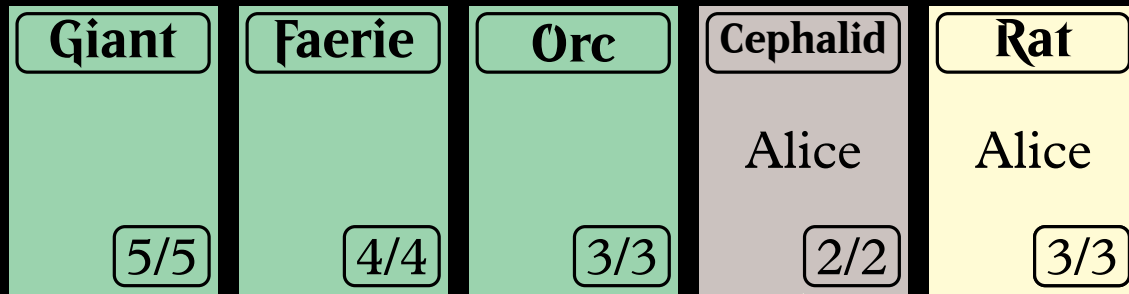
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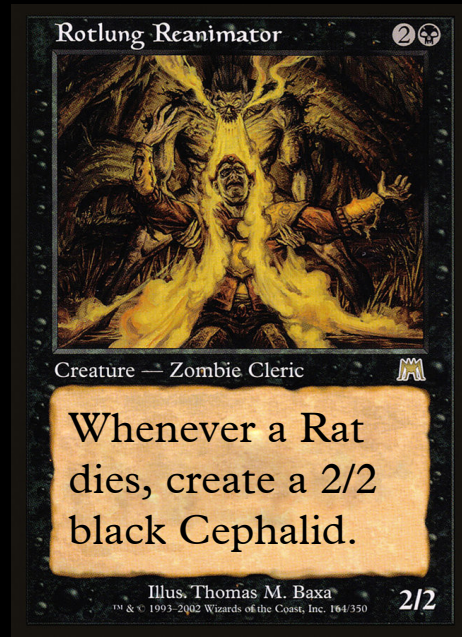
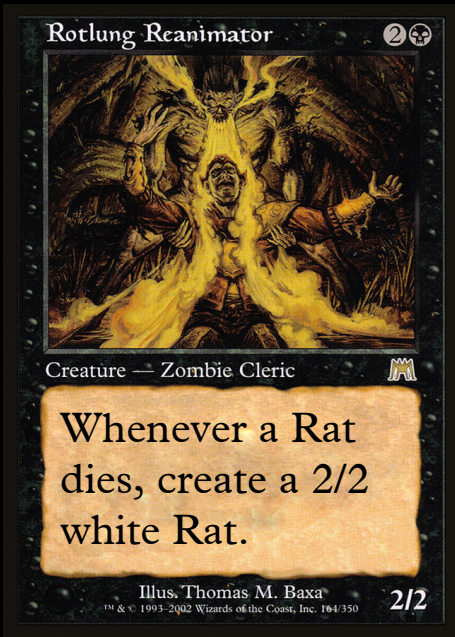
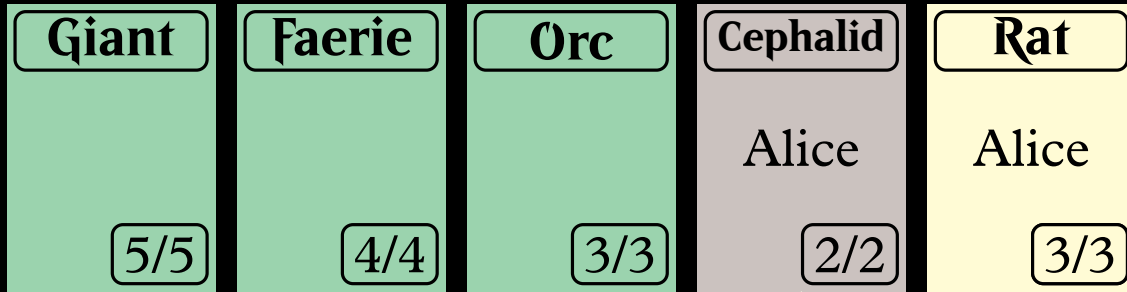
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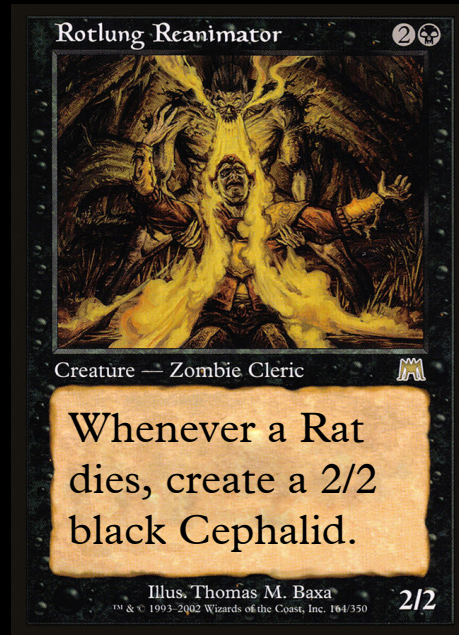
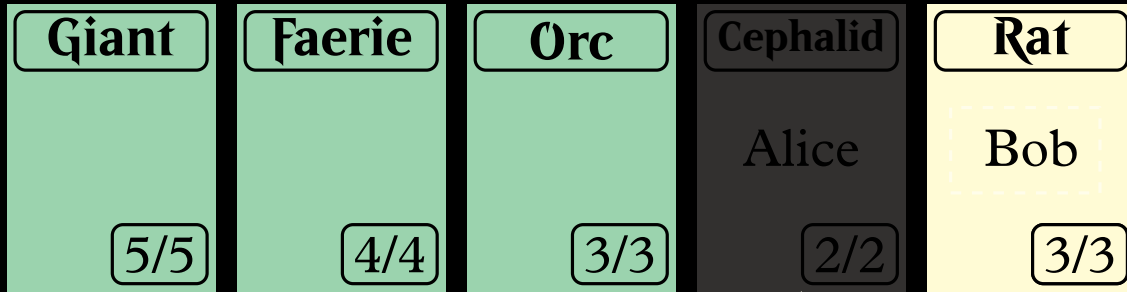
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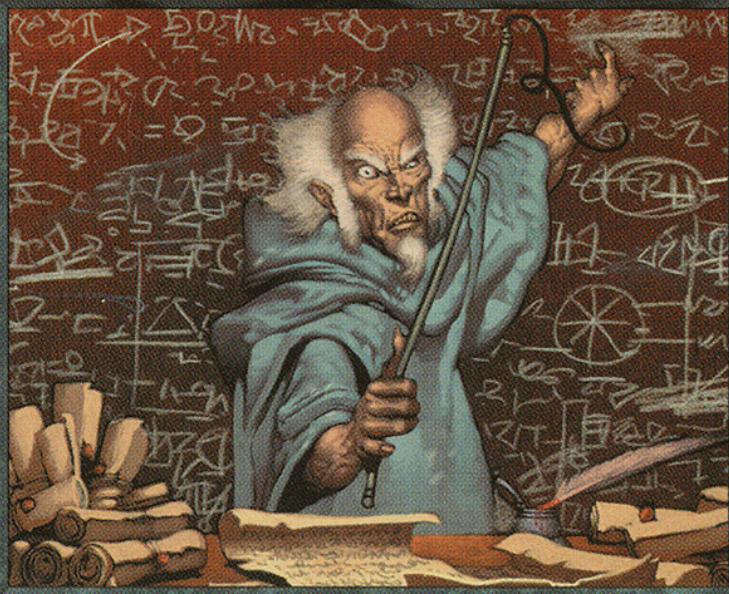
Setup

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Werbepause

Algo1-Tutor:in

12



Legendäre Kreatur – Mensch



Lege zu Beginn des Versorgungssegments 480€ auf Algo1-Tutor:in. Wenn $4 \times 480€ = 1920€$ auf Algo1-Tutor:in liegen, opfere Algo-Tutor:in.

☞: Algo1-Tutor:in hält Tutorium, korrigiert Übungsblatt und begeistert für Algorithmen.

"Meine Wochenstunden zähle ich an zwei Händen ab!"

Illus. Key Walker

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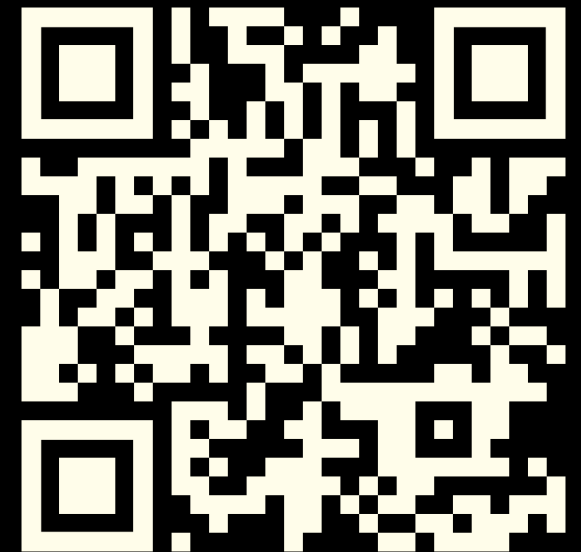


Vertragliches

- April bis Juli (4 Monate)
- 40 h/Monat
- 12 €/h

Bewerben

- bis spätestens 8.1.23
- gerne früher
- portal.wiwi.kit.edu/forms/form/640



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Setup: getting started

BATTLEFIELD

MANA	colorless	white	blue
	black	red	green

Ancient Tomb



Land

☞: Add two colorless mana to your mana pool. Ancient Tomb deals 2 damage to you.

There is no glory to be gained in the kingdom of the dead.
 —Vec tomb inscription

Illus. Colin MacNeil
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HAND

Ancient Tomb



Land

☞: Add two colorless mana to your mana pool. Ancient Tomb deals 2 damage to you.

There is no glory to be gained in the kingdom of the dead.
 —Vec tomb inscription

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Grim Monolith 2



Artifact

Grim Monolith does not untap during your untap phase.

☞: Add three colorless mana to your mana pool. Play this ability as a mana source.

⚡: Untap Grim Monolith.
Part prison, part home.

Illus. Chippy
 ©1993-1999 Wizards of the Coast, Inc. 120/143

Lotus Petal 0



Artifact

☞, Sacrifice Lotus Petal: Add one mana of any color to your mana pool.

"Hard to imagine," mused Hanna, stroking the petal, "such a lovely flower inspiring such greed."

228/269 U
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Power Artifact 0



Enchantment — Aura

Enchant artifact

Enchanted artifact's activated abilities cost 2 less to activate. This effect can't reduce the amount of mana an ability costs to activate to less than one mana.

Illus. Douglas Shuler
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Staff of Domination 3



Artifact

1: Untap Staff of Domination.
2, ☞: You gain 1 life.
3, ☞: Untap target creature.
4, ☞: Tap target creature.
5, ☞: Draw a card.

Illus. Ben Thompson
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Setup: getting started

BATTLEFIELD



MANA	colorless	white	blue
	black	red	green



HAND



Setup: getting started

BATTLEFIELD



HAND



MANA	colorless	white	blue
	2		
	black	red	green



Setup: getting started

BATTLEFIELD



MANA	colorless	white	blue
	2		
	black	red	green



HAND



Setup: getting started

BATTLEFIELD



MANA

colorless	white	blue
black	red	green



HAND



Setup: getting started

BATTLEFIELD



MANA	colorless	white	blue
	black	red	green



HAND



Setup: getting started

BATTLEFIELD



MANA	colorless	white	blue
	black	red	green



HAND



Setup: getting started

BATTLEFIELD



HAND



MANA	colorless	white	blue 2
	black	red	green



Setup: getting started

BATTLEFIELD



MANA	colorless	white	blue 2
	black	red	green



HAND



Setup: getting started

BATTLEFIELD



MANA

colorless	white	blue
black	red	green



HAND



Setup: getting started

BATTLEFIELD



HAND



MANA

colorless	white	blue
3		
black	red	green



Setup: getting started

BATTLEFIELD



HAND



MANA	colorless	white	blue
	1		
	black	red	green



Setup: getting started

BATTLEFIELD



HAND



MANA

colorless 4	white	blue
black	red	green



Setup: getting started

BATTLEFIELD



HAND



MANA	colorless	white	blue
	2		
	black	red	green



Setup: getting started

BATTLEFIELD



HAND



MANA

colorless n	white	blue
black	red	green



Setup: getting started

BATTLEFIELD



HAND



MANA	colorless <i>n</i>	white	blue
	black	red	green



Setup: getting started

BATTLEFIELD



HAND

MANA	colorless n	white	blue
	black	red	green



Setup: getting started

BATTLEFIELD



MANA	colorless <i>n</i>	white	blue
	black	red	green



HAND



Setup: getting started

BATTLEFIELD



MANA	colorless	white	blue
	n		
	black	red	green



HAND



Setup: getting started

BATTLEFIELD



MANA	colorless <i>n</i>	white <i>n</i>	blue <i>n</i>
	black <i>n</i>	red <i>n</i>	green <i>n</i>



HAND



Setup: getting started

BATTLEFIELD



MANA

colorless <i>n</i>	white <i>n</i>	blue <i>n</i>
black <i>n</i>	red <i>n</i>	green <i>n</i>

Gemstone Array 4

Artifact

2: Put a charge counter on Gemstone Array.
Remove a charge counter from Gemstone Array: Add one mana of any color to your mana pool.

Outside, it reflects the suns' light. Inside, it harvests the suns' power.

—Ittoku

HAND



Setup: reusing cards

Stolen Identity 4



Sorcery

Put a token onto the battlefield that's a copy of target artifact or creature.

Cipher (Then you may exile this spell card encoded on a creature you control. Whenever that creature deals combat damage to a player, its controller may cast a copy of the encoded card without paying its mana cost.)

Clint Cearley

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Memnarch 7



Legendary Artifact Creature — Wizard

1 : Target permanent becomes an artifact in addition to its other types.

3 : Gain control of target artifact.

In the blur between metal and flesh, Memnarch found madness.

4/5

Carl Critchlow

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Reito Lantern 2



Artifact

3 : Put target card from a graveyard on the bottom of its owner's library.

Lanterns carved from the mystic stones of the Reito Mines were said to light the way of lost souls.

Greg Hildebrandt

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Capsize 1



Instant

Buyback 3 (You may pay an additional 3 as you cast this spell. If you do, put this card into your hand as it resolves.)

Return target permanent to its owner's hand.

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Setup: creating the tape



Setup: cleaning up



Setup: cleaning up



Conclusion

Theorem

Determining the outcome of a game of **Magic: The Gathering** in which all remaining moves are forced is undecidable.

Note

- a player can set up the corresponding game state in the first turn
- with a tournament legal deck (in the Legacy format)

Conclusion

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Related fun facts

- deciding whether a game action (blocking) is legal is coNP-complete
[Chatterjee and Ibsen-Jensen, 2016]

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- deciding whether a game action (blocking) is legal is coNP-complete
[Chatterjee and Ibsen-Jensen, 2016]
- the mate-in- n problem is Δ_n^0 hard [Biderman, 2020]