«Magic: The Gathering» is Turing Complete

Alex Churchill, Stella Biderman, Austin Herrick

Presentation by Thomas Bläsius
What does that even mean?

**Theorem**
Determining the outcome of a game of *Magic: The Gathering* in which all remaining moves are forced is undecidable.
What does that even mean?

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Determining the outcome of a game of *Magic: The Gathering* in which all remaining moves are forced is undecidable.

**General plan**
- create game state where every move is forced by the game rules
- remaining moves simulate a universal TM with arbitrary input
- first player (Alice) wins if and only if TM halts
What does that even mean?

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- first player (Alice) wins if and only if TM halts

⇒ deciding whether Alice wins is as hard has the halting problem
Rogozhin's Universal Turing machine

UTM(2, 18) [Rogozhin, 1996]
- has two states: 1 and 2
- alphabet of size 18:
- interprets input on the tape as TM and simulates it

--- C C C D A S A | J R H C C C ---

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Questions to be answered

Core components
- How to represent the tape?
- How to implement the transition function?
  - How does the head read and write symbols on the tape?
  - How does the head move left or right?
  - How can we represent different states?

Interplay between components
- How can players trigger the components?
- How to force players to behave as intended?
- How to win the game (a.k.a. the TM halts)?
- What to do, if we are out of tape?

Setup
- How can one player set up the initial game state?
- Can I do that at a tournament?
Whenever Rotlung Reanimator or another Cleric is put into a graveyard from play, put a 2/2 black Zombie creature token into play.
Tape

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Rotlung Reanimator

Whenever Rotlung Reanimator or another Cleric is put into a graveyard from play, put a 2/2 black Zombie creature token into play.
There are other creature types:

- Aetherborn
- Basilisk
- Cephalid
- Demon
- Elf
- Faerie
- Giant
- Harpy
- Illusion
- Juggernaut
- Kavu
- Leviathan
- Myr
- Noggle
- Orc
- Pegasus
- Rhino
- Sliver

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Transition function: reading and writing

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**Transition function: reading and writing**

**Read symbol**
- A: S
- B: E
- C: S
- D: A
- E: D
- F: H
- G: J

**Write symbol**
- S
- E
- A
- D

**Head movement**
- ←
- →
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Giant 6/6 Sliver 5/5 Rhino 4/4 Faerie 3/3 Elf 2/2 Demon 3/3 Giant 4/4 Orc 5/5 Elf 6/6

Illus. Thomas M. Baxter

Whenever Rotlung Reanimator or another Elf is put into a graveyard from play, put a 2/2 black Demon creature token into play.

Change the text of target spell or permanent by replacing all instances of one creature type with another. The new creature type can’t be Legend or Wall. (This effect doesn’t end at end of turn.)
Transition function: reading and writing

Whenever Rotlung Reanimator or another Elf is put into a graveyard from play, put a 2/2 black Demon creature token into play.

Artificial Evolution

Change the text of target spell or permanent by replacing all instances of one creature type with another. The new creature type can’t be Legend or Wall. (This effect ends at end of turn.)

Glamerdye

Change the text of target spell or permanent by replacing all instances of one color word with another. Retrace (You may play this card from your graveyard by discarding a land card in addition to paying its other costs.)

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Transition function: reading and writing

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Transition function: reading and writing

read symbol
write symbol
head movement

A S ←
B E →
C S ←
D A →
E D ←
F H →
G J →

Giant  Sliver  Rhino  Faerie
6/6  5/5  4/4  3/3

Demon  Giant  Orc  Elf
3/3  4/4  5/5  6/6

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Transition function: reading and writing

Read symbol: A S ↔
Write symbol: B E →
Head movement: C S ←
D A →
E D ←
F H →
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Rotlung Reanimator
Creature — Zombie Cleric
Whenever Rotlung Reanimator or another Elf is put into a graveyard from play, put a 2/2 white Demon creature token into play.

Artificial Evolution
Instant
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Transition function: moving the head (left)
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When Soul Snuffers enters the battlefield, put a -1/-1 counter on each creature.

They once sought to unite with the flame of transcendence. Now they seek only to extinguish the lights of others.
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Transition function: moving the head (left)

Soul Snuffers

Creature — Elemental Shaman

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Cleansing Beam

Instant

Radiance — Cleansing Beam deals 2 damage to target creature and each other creature that shares a color with it.

“Justice is toothless without punishment. Righteousness cannot succeed without the suffering of the guilty.” — Razia, Boros archangel

Vigor

Creature — Elemental Incarnation

Trample
If damage would be dealt to another creature you control, prevent that damage. Put a +1/+1 counter on that creature for each 1 damage prevented this way.

When Vigor is put into a graveyard from anywhere, shuffle it into its owner’s library.

Elf

Giant

Sliver

Rhino

Faerie

Demon

Demon

Giant

Orc

Elf

5/5

4/4

3/3

2/2

1/1

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Giant
5/5

Orc
6/6

Elf
7/7

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Transition function: changing state

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Whenever an Elf dies, create a 2/2 white Demon.

Whenever a Faerie dies, create a 2/2 green Harpy.

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2

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Whenever a Faerie dies, create a 2/2 green Harpy.
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Cloak of Invisibility

Enchanted creature gains phasing and cannot be blocked except by Walls.

“Useful... if you can find it.”

—Pashad ibn Asim, Suq’Ata trader

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What the players do

Two players: Alice and Bob

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- Alice repeatedly takes four turns
- she sometimes skips turn 3 to change state (remember: phasing)
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All creatures get -2/-2 until end of turn.

“It is from Phage that this evil springs, and to her it shall return.”
—Akroma, angelic avenger
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*Infest*

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**Turn 2**

*Cleansing Beam*

Instant

Radiance — Cleansing Beam deals 2 damage to target creature and each other creature that shares a color with it.

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**Turn 2**

- *Cleansing Beam* (Instant)
  
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  —Razia, Dorvi’s archangel

**Turn 3**

- *Coalition Victory* (Sorcery)
  
  You win the game if you control a land of each basic land type and a creature of each color.
  
  “You can build a perfect machine out of imperfect parts.”
  
  —Urza
What the players do

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—Uzra

Turn 3

**Soul Snuffers**

*Creature — Elemental Shaman*

When Soul Snuffers enters the battlefield, put a -1/-1 counter on each creature.

They once sought to unite with the flame of transcendence. Now they seek only to extinguish the lights of others.

Turn 4

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Forcing Alice to cast spells

(controlled by Bob)
Forcing Alice to cast spells

Wild Evocation

At the beginning of each player’s upkeep, that player reveals a card at random from his or her hand. If it’s a land card, the player puts it onto the battlefield. Otherwise, the player casts it without paying its mana cost if able.

Wheel of Sun and Moon

Enchant player
If a card would be put into enchanted player’s graveyard from anywhere, instead that card is revealed and put on the bottom of that player’s library.

Every life ends, but life itself never does.

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Forcing Alice to cast spells

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Dread of Night
Enchantment
black
All white creatures get -1/-1.

“These moonless, foreign skies keep me in thrall. Dark whispers echo in the night, and I cannot resist.”
— Selenia, dark angel
Changing the state by skipping turn 3

**Rotlung Reanimator**
Creature — Zombie Cleric

Whenever Rotlung Reanimator or another Cleric is put into a graveyard from play, put a 2/2 black Zombie creature token into play.

**Xathrid Necromancer**
Creature — Human Wizard

Whenever Xathrid Necromancer or another Human creature you control dies, put a 2/2 black Zombie creature token onto the battlefield tapped.

“My commands shall echo forever in their dusty skulls.”

**Mesmeric Orb**
Artifact

Whenever a permanent becomes untapped, that permanent’s controller puts the top card of his or her library into his or her graveyard.

A step in one direction is two steps away from another.

(controlled by Alice)
Forcing Alice to pick the right target

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Forcing Alice to pick the right target

protecting Bob’s creatures

protecting Alice’s creatures

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Sliver 5/5
Rhino 4/4
Faerie 3/3
Elf 2/2
Demon 3/3
Giant 4/4
Orc 5/5
Elf 6/6

Thomas Bläsius
Forcing Alice to pick the right target

- Protecting Bob’s creatures
- Protecting Alice’s creatures
- Only legal target: newest creature

**Sliver** Bob 5/5
**Rhino** Bob 4/4
**Faerie** Bob 3/3
**Elf** Alice 2/2
**Demon** Bob 3/3
**Giant** Bob 4/4
**Orc** Bob 5/5
**Elf** Bob 6/6

Magic: The Gathering is Turing Complete

Thomas Bläsius
Keeping your engine alive

Vigor

Creature — Elemental Incarnation

Trample
If damage would be dealt to another creature you control, prevent that damage. Put a +1/+1 counter on that creature for each 1 damage prevented this way. When Vigor is put into a graveyard from anywhere, shuffle it into its owner’s library.

6/6

Prismatic Lace

Instant
Target permanent becomes the color(s) of your choice. Costs to tap, maintain, or use an ability of that permanent remain unchanged.

Fungus Sliver

Creature — Fungus Sliver

All Slivers have “Whenever this creature is dealt damage, put a +1/+1 counter on it.” (The damage is dealt before the counter is put on.)

“When a sliver of this breed enters the hive, the others claw each other in frenzied fits, thereby ensuring their rapid growth.”

—Rukarumel, field journal

2/2

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Locking everything else down

(controlled by Bob)  (controlled by Alice)  (controlled by both)
Halting

<table>
<thead>
<tr>
<th>current state</th>
<th>read symbol</th>
<th>write symbol</th>
<th>head movement</th>
<th>new state</th>
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<td>halt</td>
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goal: TM halts $\iff$ Alice wins with:
Halting

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**goal**: TM halts $\iff$ Alice wins with:

*Whenever a Rhino dies, create a 2/2 blue Assassin.*

*Prismatic Omen*

*Enchantment*

Lands you control are every basic land type in addition to their other types.

_In times of portent, the land sculpts itself in accordance with the signs burned on the sky._

*Coalition Victory*

*Sorcery*

You win the game if you control a land of each basic land type and a creature of each color.

*“You can build a perfect machine out of imperfect parts.”* —Urza

*Rotlung Reanimator*

*Creature — Zombie Cleric*

Whenever a Rhino dies, create a 2/2 blue Assassin.

*Prismatic Lace*

*Instant*

Target permanent becomes the color(s) of your choice. Costs to tap, maintain, or use an ability of that permanent remain unchanged. (white, black, red, green)
Out of tape exception
Out of tape exception

Giant 5/5  
Faerie 4/4  
Orc 3/3  
Rat 2/2
Out of tape exception

Whenever a Rat dies, create a 2/2 white Rat.
(controlled by Bob)

Whenever a Rat dies, create a 2/2 black Cephalid.
(controlled by Alice)
Out of tape exception

Whenever a Rat dies, create a 2/2 white Rat.

Whenever a Rat dies, create a 2/2 black Cephalid.

(controlled by Bob) (controlled by Alice)
Out of tape exception

Whenever a Rat dies, create a 2/2 white Rat.

Whenever a Rat dies, create a 2/2 black Cephalid.

(controlled by Bob) (controlled by Alice)
Out of tape exception

Whenever a Rat dies, create a 2/2 white Rat.

(controlled by Bob) (controlled by Alice)

Magic: The Gathering is Turing Complete

Thomas Bläsius
Out of tape exception

Giant  Faerie  Orc  Cephalid  Rat
5/5  4/4  3/3  2/2  3/3

Whenever a Rat dies, create a 2/2 white Rat.
Whenever a Rat dies, create a 2/2 black Cephalid.

Rat (controlled by Bob) (controlled by Alice)

Magic: The Gathering is Turing Complete
Out of tape exception

Whenever a Rat dies, create a 2/2 white Rat.
Whenever a Rat dies, create a 2/2 black Cephalid.

(called by Bob) (called by Alice)

Magic: The Gathering is Turing Complete

Thomas Bläsius
Out of tape exception

Whenever a Rat dies, create a 2/2 white Rat.

Whenever a Rat dies, create a 2/2 black Cephalid.

As Shared Triumph comes into play, choose a creature type. Creatures of the chosen type get +1/+1. (Rat)

"Win together, die alone."

—Selenia, dark angel

Magic: The Gathering is Turing Complete

Thomas Bläsius
Questions to be answered

Core components

- How to represent the tape?
- How to implement the transition function?
  - How does the head read and write symbols on the tape?
  - How does the head move left or right?
  - How can we represent different states?

Interplay between components

- How can players trigger the components?
- How to force players to behave as intended?
- How to win the game (a.k.a. the TM halts)?
- What to do, if we are out of tape?

Setup

- How can one player set up the initial game state?
- Can I do that at a tournament?
Werbepause

**Vertragliches**
- April bis Juli (4 Monate)
- 40 h/Monat
- 12 €/h

**Bewerben**
- bis spätestens 8.1.23
- gerne früher
- portal.wiwi.kit.edu/forms/form/640
Questions to be answered

Core components
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Magic: The Gathering is Turing Complete

Thomas Bläsius
Setup: getting started

- Magic: The Gathering is Turing Complete

<table>
<thead>
<tr>
<th>MANA</th>
</tr>
</thead>
<tbody>
<tr>
<td>colorless white blue black red green</td>
</tr>
</tbody>
</table>

**Ancient Tomb**
- Add two colorless mana to your mana pool. Ancient Tomb deals 2 damage to you.
- There is no glory to be gained in the kingdom of the dead.

—Vẹc tomb inscription

**Grim Manslaughter**
- Artificer
- Add one colorless mana to your mana pool. You may use one mana of any color to cast this creature.

**Lotus Petal**
- Artificer
- # Sacrifice Lotus Petal: Add one mana of any color to your mana pool. "Hard to imagine."

**Staff of Domination**
- Artificer
- # Unattach Staff of Domination. You gain 1 life.

**Magic: The Gathering is Turing Complete**
- Thomas Bläsius
Magic: The Gathering is Turing Complete

Setup: getting started

BATTLEFIELD

HAND

COLORLESS

WHITE

BLUE

BLACK

RED

GREEN

2

Grim Moonlithe

- Does not untap during your untap phase.
- Add one colorless mana to your mana pool. Play this ability as a mana source.

Lotos Petal

- Sacrifice Lotus Petal: Add one mana of any color to your mana pool.
- "Hard to imagine," wailed Flavas, watching the petals, "such a lovely flower inspiring such pride."

Lotos Petal

- Sacrifice Lotus Petal: Add one mana of any color to your mana pool.
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Enchanted artifact

Enchanted artifact's activated abilities cost 1 less to activate. This effect can't reduce the amount of mana an ability costs to activate to less than one mana.

Ancient Tomb

- Add two colorless mana to your mana pool. Ancient Tomb deals 2 damage to you.
- "There is no glory to be gained in the kingdom of the dead." — Véc tomb inscription

Illus. Colin MacNeil

Magic: The Gathering is Turing Complete

Thomas Bläsius
Setup: getting started

- Magic: The Gathering does not untap during your untap phase.
- Add three colorless mana to your mana pool. Play this ability as a mana source.
- Untap Grim Monolith.
- Part prison, part home.

**Grim Monolith**

<table>
<thead>
<tr>
<th>Mana</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>colorless</td>
</tr>
</tbody>
</table>

**Artifact**

- Grish Monolith does not untap during your untap phase.
- Add three colorless mana to your mana pool. Play this ability as a mana source.
- Untap Grim Monolith.
- Part prison, part home.

**Grim Monolith**

- Grish Monolith does not untap during your untap phase.
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**Lotus Petal**

- **Artifact**

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Setup: getting started

**Magic: The Gathering** is Turing Complete

Thomas Bläsius
Setup: getting started

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Thomas Bläsius
Setup: getting started

Lotus Petal

Sacrifice Lotus Petal: Add one mana of any color to your mana pool.

"Hard to imagine," mused Hanna, stroking the petal, "such a lovely flower inspiring such greed."

<table>
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Setup: getting started

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Thomas Bläsius
Setup: getting started

**BATTLEFIELD**

- **Grim Vanquish**
  - Grey, white, black, and blue.
  - Enchantment
  - Enchant artifact
  - Active abilities cost 2 less to activate. This effect can't reduce the amount of mana an ability costs to activate to less than one mana.

**HAND**

- **Lightwood Axe**
  - Blue, white, and black.
  - Enchantment
  - Enchant artifact
  - Active abilities cost 1 less to activate. This effect can't reduce the amount of mana an ability costs to activate to less than one mana.

- **Staff of Domination**
  - Black, blue, and green.
  - Artifact
  - Untap Staff of Domination.
  - Gain 1 life.
  - Untap target creature.
  - Tap target creature.
  - Draw a card.

** Mana**

- Colorless
- White
- Blue
- Black
- Red
- Green

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Setup: getting started.

Magic: The Gathering is Turing Complete

Thomas Bläsius
Setup: getting started

Grim Monolith

- Colorless
- White
- Blue

- Black
- Red
- Green

Grim Monolith does not untap during your untap phase.
- Add three colorless mana to your mana pool. Play this ability as a mana source.
- Untap Grim Monolith.

Part prison, part home.

Magic: The Gathering is Turing Complete

Thomas Bläsius
Setup: getting started
Setup: getting started

*Mana*

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*Battlefield*

*Grim Monolith*

- Artfact
- Part prison, part home.
- Grim Monolith does not untap during your untap phase.
- Add three colorless mana to your mana pool. Play this ability as a mana source.
- Untap Grim Monolith.

*Hand*

*Staff of Domination*

- Untap Staff of Domination.
- Gain 1 life.
- Untap target creature.
- Tap target creature.
- Draw a card.

Magic: The Gathering is Turing Complete

Thomas Bläsius
Setup: getting started

**Mana**
- Colorless
- White
- Blue
- Black
- Red
- Green

**Mana Pool**
- 4

**Grim Monolith**
- Artifact
- Grim Monolith does not untap during your untap phase.
- 0: Add three colorless mana to your mana pool. Play this ability as a mana source.
- 4: Untap Grim Monolith.
- Part prison, part home.

**Staff of Domination**
- Artifact
- Untap Staff of Domination.
- : You gain 1 life.
- : Untap target creature.
- : Tap target creature.
- : Draw a card.
Magic: The Gathering is Turing Complete

Setup: getting started

Mana
- colorless
- white
- blue
- black
- red
- green

Mana
- 2

Grim Monolith
- Artifact
- Colorless
- 2

"Grim Monolith does not untap during your untap phase.

1: Add three colorless mana to your mana pool. Play this ability as a mana source.
2: Untap Grim Monolith.
3: Part prison, part home.

Staff of Domination
- Artifact
- Colorless
- 2

1: Untap Staff of Domination.
2: You gain 1 life.
3: Untap target creature.
4: Tap target creature.
5: Draw a card.

Hand

Battlefield

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Setup: getting started

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Setup: getting started

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Setup: getting started
Setup: getting started

Magic: The Gathering is Turing Complete Thomas Bläsius

- **Mana**:
  - Colorless
  - White
  - Blue
  - Black
  - Red
  - Green

- **Staff of Domination**
  - **Artifact**
  - 1: Untap Staff of Domination.
  - 2: You gain 1 life.
  - 3: Untap target creature.
  - 4: Tap target creature.
  - 5: Draw a card.

- **Grim Vanquisher**
  - Artifact
  - Whenever another creature enters the battlefield, that creature gains haste.

- **Blazing Archon**
  - Creature — Archon
  - Whenever a player casts a spell, that player discards a card.

- **Reaching, Reawaken**
  - Creature — Elemental
  - Enchant creature
  - Target permanent color(s) of your choice. Untap and tap that permanent.

- **Cleansing Beams**
  - Instant
  - Target permanent color(s) of your choice. Untap and tap that permanent.

- **Ancient Zephyr**
  - Creature — Elemental
  - Enchant creature
  - Target permanent color(s) of your choice. Untap and tap that permanent.

- **Recycle**
  - Artifact
  - When a creature and an artifact enter the battlefield simultaneously, untap them.

- **Firstborn Lotus**
  - Enchantment
  - Whenever a creature enters the battlefield, untap it.

- **Recall**
  - Artifact
  - Return to hand.

- **Enchantment**
  - Enchantment
  - Untap any number of untappable artifacts.

- **Islands**
  - Colorless
  - Add colorless

- **Death**
  - Instant
  - Target creature becomes colorless.
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Setup: getting startet

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Thomas Bläsius
Setup: getting started

**Magic: The Gathering is Turing Complete**

Thomas Bläsius

---

**Mana**

- colorless: n
- white: n
- blue: n
- black: n
- red: n
- green: n

---

**Artifact**

1. Put a charge counter on Gemstone Array.
2. Remove a charge counter from Gemstone Array: Add one mana of any color to your mana pool.

*Outside, it reflects the sun's light. Inside, it harvests the sun's power.*

---

**Gemstone Array**

- rarity: uncommon
- set: 4th Edition
- type: Artifact
- set emblem: none

---

**Grim Vanilith**

- rarity: common
- set: Urza's Legacy
- type: Treasure
- set emblem: none

---

**Staff of Domination**

- rarity: rare
- set: 8th Edition
- type: Artifact
- set emblem: none

---

**Retrolung Reanimator**

- rarity: rare
- set: Urza's Legacy
- type: Creature — Zombie Cleric
- set emblem: none

---

**Innoka**

*“One day we shall walk where once man roamed.”* — Eldanor, Lord of Lorien

---

**Magic: The Gathering is Turing Complete**

Thomas Bläsius
Setup: reusing cards

Stolen Identity

Sorcery

Put a token onto the battlefield that's a copy of target artifact or creature.

Cipher (Then you may exile this spell card encoded on a creature you control. Whenever that creature deals combat damage to a player, its controller may cast a copy of the encoded card without paying its mana cost.)

Memnarch

Legendary Artifact Creature — Wizard

1, 4: Target permanent becomes an artifact in addition to its other types.

3, 4: Gain control of target artifact.

In the blur between metal and flesh, Memnarch found madness.

Reito Lantern

Artifact

3: Put target card from a graveyard on the bottom of its owner's library.

Lanterns carved from the mystic stones of the Reito Mines were said to light the way of lost souls.

Capsize

Instant

Buyback 3 (You may pay an additional 3 as you cast this spell. If you do, put this card into your hand as it resolves.)

Return target permanent to its owner's hand.

Clint Cearley

Carl Critchlow

Greg Staples

Tom Wiberg
Setup: creating the tape

As Riptide Replicator comes into play, choose a color and a creature type. Riptide Replicator comes into play with X charge counters on it.

\[\text{4,}\ \textcolor{red}{\text{\langle}}\text{\rangle:}\ \text{Put an X/X creature token of the chosen color and type into play, where X is the number of charge counters on Riptide Replicator.}\]

Illus. Doug Chaffee
Setup: cleaning up
Setup: cleaning up
Conclusion

Theorem
Determining the outcome of a game of Magic: The Gathering in which all remaining moves are forced is undecidable.

Note
- a player can set up the corresponding game state in the first turn
- with a tournament legal deck (in the Legacy format)
Conclusion

Theorem
Determining the outcome of a game of Magic: The Gathering in which all remaining moves are forced is undecidable.

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Related fun facts
- deciding whether a game action (blocking) is legal is coNP-complete
  [Chatterjee and Ibsen-Jensen, 2016]
Conclusion

**Theorem**
Determining the outcome of a game of *Magic: The Gathering* in which all remaining moves are forced is undecidable.

**Note**
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**Related fun facts**
- deciding whether a game action (blocking) is legal is coNP-complete [Chatterjee and Ibsen-Jensen, 2016]
- the mate-in-\(n\) problem is \(\Delta^0_n\) hard [Biderman, 2020]