«Magic: The Gathering» is Turing Complete
Alex Churchill, Stella Biderman, Austin Herrick

Presentation by Thomas Bläsius
What does that even mean?

**Theorem**

Determining the outcome of a game of *Magic: The Gathering* in which all remaining moves are forced is undecidable.
Theorem
Determining the outcome of a game of Magic: The Gathering in which all remaining moves are forced is undecidable.

General plan
- create game state where every move is forced by the game rules
- remaining moves simulate a universal TM with arbitrary input
- first player (Alice) wins if and only if TM halts
What does that even mean?

**Theorem**
Determining the outcome of a game of *Magic: The Gathering* in which all remaining moves are forced is undecidable.

**General plan**
- create game state where every move is forced by the game rules
- remaining moves simulate a universal TM with arbitrary input
- first player (Alice) wins if and only if TM halts

$\Rightarrow$ deciding whether Alice wins is as hard has the halting problem
Rogozhin's Universal Turing machine

**UTM(2, 18)**
- has two states: 1 and 2
- interprets input on the tape as TM and simulates it

<table>
<thead>
<tr>
<th>current state</th>
<th>read symbol</th>
<th>write symbol</th>
<th>head movement</th>
<th>new state</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>S</td>
<td>←</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>B</td>
<td>E</td>
<td>→</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>C</td>
<td>S</td>
<td>←</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>D</td>
<td>A</td>
<td>→</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>E</td>
<td>D</td>
<td>←</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>F</td>
<td>H</td>
<td>→</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>G</td>
<td>J</td>
<td>→</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>H</td>
<td>F</td>
<td>←</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>I</td>
<td>I</td>
<td>←</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>J</td>
<td>I</td>
<td>←</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>K</td>
<td>L</td>
<td>←</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>L</td>
<td>I</td>
<td>←</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>M</td>
<td>B</td>
<td>←</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>N</td>
<td>O</td>
<td>→</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>O</td>
<td>P</td>
<td>←</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>P</td>
<td>R</td>
<td>→</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>R</td>
<td>C</td>
<td>→</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>...</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Magic: The Gathering is Turing Complete
Questions to be answered

Core components
- How to represent the tape?
- How to implement the transition function?
  - How does the head read and write symbols on the tape?
  - How does the head move left or right?
  - How can we represent different states?

Interplay between components
- How can players trigger the components?
- How to force players to behave as intended?
- How to win the game (a.k.a. the TM halts)?
- What to do, if we are out of tape?

Setup
- How can one player set up the initial game state?
- Can I do that at a tournament?
Whenever Rotlung Reanimator or another Cleric is put into a graveyard from play, put a 2/2 black Zombie creature token into play.
Magic: The Gathering is Turing Complete

Tape

Rotlung Reanimator
Creature — Zombie Cleric

Whenever Rotlung Reanimator or another Cleric is put into a graveyard from play, put a 2/2 black Zombie creature token into play.

Illus. Thomas M. Baxter
Tape

Rotlung Reanimator
Creature — Zombie Cleric

Whenever Rotlung Reanimator or another Cleric is put into a graveyard from play, put a 2/2 black Zombie creature token into play.

Illus. Thomas M. Baxter
Magic: The Gathering is Turing Complete

Thomas Bläsius
There are other creature types
- Aetherborn
- Basilisk
- Cephalid
- Demon
- Elf
- Faerie
- Giant
- Harpy
- Illusion
- Juggernaut
- Kavu
- Leviathan
- Myr
- Noggle
- Orc
- Pegasus
- Rhino
- Sliver

Tape

Whenever Rotlung Reanimator or another Cleric is put into a graveyard from play, put a 2/2 black Zombie creature token into play.

Illus. Thomas M. Baxter

Magic: The Gathering is Turing Complete
Thomas Bläsius
Magic: The Gathering is Turing Complete

There are other creature types

- Aetherborn
- Basilisk
- Cephalid
- Demon
- Elf
- Faerie
- Giant
- Harpy
- Illusion
- Juggernaut
- Kavu
- Leviathan
- Myr
- Noggle
- Orc
- Pegasus
- Rhino
- Sliver
Magic: The Gathering is Turing Complete

Transition function: reading and writing

<table>
<thead>
<tr>
<th>Read symbol</th>
<th>Write symbol</th>
<th>Head movement</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>S</td>
<td>←</td>
</tr>
<tr>
<td>B</td>
<td>E</td>
<td>→</td>
</tr>
<tr>
<td>C</td>
<td>S</td>
<td>←</td>
</tr>
<tr>
<td>D</td>
<td>A</td>
<td>→</td>
</tr>
<tr>
<td>E</td>
<td>D</td>
<td>←</td>
</tr>
<tr>
<td>F</td>
<td>H</td>
<td>→</td>
</tr>
<tr>
<td>G</td>
<td>J</td>
<td>→</td>
</tr>
</tbody>
</table>

Whenever Rotlung Reanimator or another Cleric is put into a graveyard from play, put a 2/2 black Zombie creature token into play.
Magic: The Gathering is Turing Complete

Transition function: reading and writing

<table>
<thead>
<tr>
<th>Read symbol</th>
<th>Write symbol</th>
<th>Head movement</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>S</td>
<td>←</td>
</tr>
<tr>
<td>B</td>
<td>E</td>
<td>→</td>
</tr>
<tr>
<td>C</td>
<td>S</td>
<td>←</td>
</tr>
<tr>
<td>D</td>
<td>A</td>
<td>→</td>
</tr>
<tr>
<td>E</td>
<td>D</td>
<td>←</td>
</tr>
<tr>
<td>F</td>
<td>H</td>
<td>→</td>
</tr>
<tr>
<td>G</td>
<td>J</td>
<td>→</td>
</tr>
</tbody>
</table>

Whenever Rotlung Reanimator or another Cleric is put into a graveyard from play, put a 2/2 black Zombie creature token into play.

Change the text of target spell or permanent by replacing all instances of one creature type with another. The new creature type can’t be Legend or Wall. (This effect doesn’t end at end of turn.)

- Giant 6/6
- Sliver 5/5
- Rhino 4/4
- Faerie 3/3
- Elf 2/2
- Demon 3/3
- Giant 4/4
- Orc 5/5
- Elf 6/6
Magic: The Gathering is Turing Complete

Transition function: reading and writing

- **Read symbol**
  - A → S
  - B → E
  - C → S
  - D → A
  - E → D

- **Write symbol**
  - F → H
  - G → J

- **Head movement**
  - t ← t
  - t → t

Creature — Zombie Cleric

Whenever Rotlung Reanimator or another Elf is put into a graveyard from play, put a 2/2 black Demon creature token into play.

Illus. Thomas M. Baxter

Artificial Evolution

Instant

Change the text of target spell or permanent by replacing all instances of one creature type with another. The new creature type can’t be Legend or Wall. (This effect doesn’t end at end of turn.)

Illus. Grey Staple

Giant  Sliver  Rhino  Faerie  Elf  Demon  Giant  Orc  Elf

6/6  5/5  4/4  3/3  2/2  3/3  4/4  5/5  6/6

Magic: The Gathering is Turing Complete

Thomas Bläsius
Magic: The Gathering is Turing Complete

Thomas Bläsius

Transition function: reading and writing

read symbol
write symbol
head movement

A S ←
B E →
C S ←
D A →
E D ←
F H →
G J →

Giant 6/6
Sliver 5/5
Rhino 4/4
Faerie 3/3
Elf 2/2
Demon 3/3
Giant 4/4
Orc 5/5
Elf 6/6

Magic: The Gathering is Turing Complete
Transition function: reading and writing

- **Read symbol**: A, B, C, D, E, F, G
- **Write symbol**: S, E, A, D
- **Head movement**: ←, →

### Magic: The Gathering is Turing Complete

**Rotlung Reanimator**
- **Creature** — Zombie Cleric
- **Instant**
- Whenever Rotlung Reanimator or another Elf is put into a graveyard from play, put a 2/2 white Demon creature token into play.

**Artificial Evolution**
- **Instant**
- Change the text of target spell or permanent by replacing all instances of one creature type with another. The new creature type can't be Legend or Wall. (This effect ends at end of turn.)

**Glamerdye**
- **Instant**
- Change the text of target spell or permanent by replacing all instances of one color word with another. Retrace (You may play this card from your graveyard by discarding a land card in addition to paying its other costs.)

<table>
<thead>
<tr>
<th>Giant</th>
<th>Sliver</th>
<th>Rhino</th>
<th>Faerie</th>
<th>Elf</th>
<th>Demon</th>
<th>Giant</th>
<th>Orc</th>
<th>Elf</th>
</tr>
</thead>
<tbody>
<tr>
<td>6/6</td>
<td>5/5</td>
<td>4/4</td>
<td>3/3</td>
<td>2/2</td>
<td>3/3</td>
<td>4/4</td>
<td>5/5</td>
<td>6/6</td>
</tr>
</tbody>
</table>

**Magic: The Gathering is Turing Complete**

Thomas Bläsius
Magic: The Gathering is Turing Complete

Thomas Bläsius

Transition function: reading and writing

[Images of Magic: The Gathering cards and symbols]

<table>
<thead>
<tr>
<th>read symbol</th>
<th>write symbol</th>
<th>head movement</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>S</td>
<td>←</td>
</tr>
<tr>
<td>B</td>
<td>E</td>
<td>→</td>
</tr>
<tr>
<td>C</td>
<td>S</td>
<td>←</td>
</tr>
<tr>
<td>D</td>
<td>A</td>
<td>→</td>
</tr>
<tr>
<td>E</td>
<td>D</td>
<td>←</td>
</tr>
<tr>
<td>F</td>
<td>H</td>
<td>→</td>
</tr>
<tr>
<td>G</td>
<td>J</td>
<td>→</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Giant</th>
<th>Sliver</th>
<th>Rhino</th>
<th>Faerie</th>
</tr>
</thead>
<tbody>
<tr>
<td>6/6</td>
<td>5/5</td>
<td>4/4</td>
<td>3/3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Demon</th>
<th>Giant</th>
<th>Orc</th>
<th>Elf</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/3</td>
<td>4/4</td>
<td>5/5</td>
<td>6/6</td>
</tr>
</tbody>
</table>

Magic: The Gathering is Turing Complete

Thomas Bläsius
**Transition function: reading and writing**

<table>
<thead>
<tr>
<th>Read symbol</th>
<th>Write symbol</th>
<th>Head movement</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>S</td>
<td>←</td>
</tr>
<tr>
<td>B</td>
<td>E</td>
<td>→</td>
</tr>
<tr>
<td>C</td>
<td>S</td>
<td>←</td>
</tr>
<tr>
<td>D</td>
<td>A</td>
<td>→</td>
</tr>
<tr>
<td>E</td>
<td>D</td>
<td>←</td>
</tr>
<tr>
<td>F</td>
<td>H</td>
<td>→</td>
</tr>
<tr>
<td>G</td>
<td>J</td>
<td>→</td>
</tr>
</tbody>
</table>

**Creature Cards**

- **Giant**: 6/6
- **Sliver**: 5/5
- **Rhino**: 4/4
- **Faerie**: 3/3
- **Demon**: 2/2
- **Giant**: 4/4
- **Orc**: 5/5
- **Elf**: 6/6

**Text**

- **Rotlung Reanimator**: Whenever Rotlung Reanimator or another Elf is put into a graveyard from play, put a 2/2 white Demon creature token into play.
- **Artificial Evolution**: Change the text of target spell or permanent by replacing all instances of one creature type with another. The new creature type can’t be Legend or Wall. (This effect ends at end of turn.)
- **Glamerdye**: Change the text of target spell or permanent by replacing all instances of one color word with another. Retrace (You may play this card from your graveyard by discarding a land card in addition to paying its other costs.)

---

**Magic: The Gathering is Turing Complete**

**Thomas Bläsius**
Magic: The Gathering is Turing Complete

Transition function: moving the head (left)
Magic: The Gathering is Turing Complete

Thomas Bläsius

Transition function: moving the head (left)

Soul Snuffers
Creature — Elemental Shaman

When Soul Snuffers enters the battlefield, put a +1/+1 counter on each creature.
They once sought to unite with the flame of transcendence. Now they seek only to extinguish the lights of others.

Giant 6/6
Sliver 5/5
Rhino 4/4
Faerie 3/3
Demon 2/2
Demon 3/3
Giant 4/4
Orc 5/5
Elf 6/6
Magic: The Gathering is Turing Complete

Thomas Bläsius
Magic: The Gathering is Turing Complete

Thomas Bläsius

Transition function: moving the head (left)

Soul Snuffers
Creature — Elemental Shaman

Cleansing Beam
Instant

Vigor
Creature — Elemental Incarnation

Giant
Sliver
Rhino
Faerie

Demon
Demon
Giant
Orc
Elf

1/1
2/2
3/3
4/4
5/5

When Soul Snuffers enters the battlefield, put a -1/-1 counter on each creature.
They once sought to unite with the flame of transcendence. Now they seek only to extinguish the lights of others.

Radiance — Cleansing Beam deals 2 damage to target creature and each other creature that shares a color with it.
"Justice is toothless without punishment. Righteousness cannot succeed without the suffering of the guilty."
— Razia, Boros archangel

Trample
If damage would be dealt to another creature you control, prevent that damage.
Put a +1/+1 counter on that creature for each 1 damage prevented this way.
When Vigor is put into a graveyard from anywhere, shuffle it into its owner’s library.
Transition function: moving the head (left)

- Soul Snuffers: 2/3
- Cleansing Beam: 3/4
- Vigor: 3/6

Creatures:
- Giant: 5/5
- Sliver: 4/4
- Rhino: 3/3
- Faerie: 2/2
- Demon: 3/3
- Demon: 4/4
- Giant: 5/5
- Orc: 6/6
- Elf: 7/7

Magic: The Gathering is Turing Complete

Thomas Bläsius
### Transition function: changing state

<table>
<thead>
<tr>
<th>current state</th>
<th>read symbol</th>
<th>write symbol</th>
<th>head movement</th>
<th>new state</th>
</tr>
</thead>
<tbody>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>1</td>
<td>E</td>
<td>D</td>
<td>←</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>F</td>
<td>H</td>
<td>→</td>
<td>1</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>2</td>
<td>E</td>
<td>A</td>
<td>←</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>F</td>
<td>K</td>
<td>→</td>
<td>1</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
</tbody>
</table>
Transition function: changing state

<table>
<thead>
<tr>
<th>current state</th>
<th>read symbol</th>
<th>write symbol</th>
<th>head movement</th>
<th>new state</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E</td>
<td>D</td>
<td>←</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>F</td>
<td>H</td>
<td>→</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>E</td>
<td>A</td>
<td>←</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>F</td>
<td>K</td>
<td>→</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Whenever an Elf dies, create a 2/2 white Demon.
Whenever a Faerie dies, create a 2/2 green Harpy.
**Transition function: changing state**

<table>
<thead>
<tr>
<th>current state</th>
<th>read symbol</th>
<th>write symbol</th>
<th>head movement</th>
<th>new state</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E</td>
<td>D</td>
<td>←</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>F</td>
<td>H</td>
<td>→</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>E</td>
<td>A</td>
<td>←</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>F</td>
<td>K</td>
<td>→</td>
<td>1</td>
</tr>
</tbody>
</table>

Whenever an Elf dies, create a 2/2 white Demon.

Whenever a Faerie dies, create a 2/2 green Harpy.

Whenever an Elf dies, create a 2/2 white Aetherborn.

Whenever a Faerie dies, create a 2/2 green Kavu.

Magic: The Gathering is Turing Complete

Thomas Bläsius
Magic: The Gathering is Turing Complete

Transition function: changing state

<table>
<thead>
<tr>
<th>Current state</th>
<th>Read symbol</th>
<th>Write symbol</th>
<th>Head movement</th>
<th>New state</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>E</td>
<td>D</td>
<td>←</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>F</td>
<td>H</td>
<td>→</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>E</td>
<td>A</td>
<td>←</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>F</td>
<td>K</td>
<td>→</td>
<td>1</td>
</tr>
</tbody>
</table>

Whenever an Elf dies, create a 2/2 white Demon.

Whenever a Faerie dies, create a 2/2 green Harpy.

Whenever an Elf dies, create a 2/2 white Aetherborn.

Whenever a Faerie dies, create a 2/2 green Kavu.

“Useful... if you can find it.”
—Pashad ibn Asim, Suq’Ata trader
Transition function: changing state

<table>
<thead>
<tr>
<th>current state</th>
<th>read symbol</th>
<th>write symbol</th>
<th>head movement</th>
<th>new state</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>E</td>
<td>D</td>
<td>←</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>F</td>
<td>H</td>
<td>→</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>E</td>
<td>A</td>
<td>←</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>F</td>
<td>K</td>
<td>→</td>
<td>1</td>
</tr>
</tbody>
</table>

Whenever an Elf dies, create a 2/2 white Demon.

Whenever a Faerie dies, create a 2/2 green Harpy.

Whenever an Elf dies, create a 2/2 white Aetherborn.

Whenever a Faerie dies, create a 2/2 green Kavu.
Transition function: changing state

<table>
<thead>
<tr>
<th>current state</th>
<th>read symbol</th>
<th>write symbol</th>
<th>head movement</th>
<th>new state</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>E</td>
<td>D</td>
<td>←</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>F</td>
<td>H</td>
<td>→</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>E</td>
<td>A</td>
<td>←</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>F</td>
<td>K</td>
<td>→</td>
<td>1</td>
</tr>
</tbody>
</table>

Whenever an Elf dies, create a 2/2 white Demon.
Whenever a Faerie dies, create a 2/2 green Harpy.
Whenever an Elf dies, create a 2/2 white Aetherborn.
Whenever a Faerie dies, create a 2/2 green Kavu.

Magic: The Gathering is Turing Complete
Thomas Bläsius
Questions to be answered

Core components
- How to represent the tape?
- How to implement the transition function?
  - How does the head read and write symbols on the tape?
  - How does the head move left or right?
  - How can we represent different states?

Interplay between components
- How can players trigger the components?
- How to force players to behave as intended?
- How to win the game (a.k.a. the TM halts)?
- What to do, if we are out of tape?

Setup
- How can one player set up the initial game state?
- Can I do that at a tournament?
What the players do

Two players: Alice and Bob

- Bob can’t do anything
- Alice repeatedly takes four turns
- she sometimes skips turn 3 to change state (remember: phasing)
What the players do

Two players: Alice and Bob

- Bob can’t do anything
- Alice repeatedly takes four turns
- she sometimes skips turn 3 to change state (remember: phasing)

Turn 1

[Image of Magic: The Gathering card: Infest]
What the players do

Two players: Alice and Bob
- Bob can’t do anything
- Alice repeatedly takes four turns
- she sometimes skips turn 3 to change state (remember: phasing)

Turn 1

- Infest
  - Sorcery
  - All creatures get -2/-2 until end of turn.
  - “It is from Phage that this evil springs, and to her it shall return.”
  - —Akroma, angelic avenger

Turn 2

- Cleansing Beam
  - Instant
  - Radiance — Cleansing Beam deals 2 damage to target creature and each other creature that shares a color with it.
  - “Justice is toothless without punishment. Righteousness cannot succeed without the suffering of the guilty.”
  - —Razia, Boros archangel
What the players do

Two players: Alice and Bob

- Bob can’t do anything
- Alice repeatedly takes four turns
- she sometimes skips turn 3 to change state (remember: phasing)

Turn 1

Infest

All creatures get -2/-2 until end of turn.

“It is from Phage that this evil springs, and to her it shall return.”
—Akroma, angelic avenger

Turn 2

Cleansing Beam

Radiance — Cleansing Beam deals 2 damage to target creature and each other creature that shares a color with it.

“Justice is toothless without punishment. Righteousness cannot succeed without the suffering of the guilty.”
—Razia, Boros archangel

Turn 3

Coalition Victory

You win the game if you control a land of each basic land type and a creature of each color.

“You can build a perfect machine out of imperfect parts.”
—Urza
What the players do

Two players: Alice and Bob

- Bob can’t do anything
- Alice repeatedly takes four turns
- she sometimes skips turn 3 to change state (remember: phasing)

Turn 1

- **Infest**
  - Sorcery
  - All creatures get -2/-2 until end of turn.
  - “It is from Phage that this evil springs, and to her it shall return.”
  - —Akroma, angelic avenger

Turn 2

- **Cleansing Beam**
  - Instant
  - Radiance — Cleansing Beam deals 2 damage to target creature and each other creature that shares a color with it.
  - “Justice is toothless without punishment. Righteousness cannot succeed without the suffering of the guilty.”
  - —Razia, Dorv's archangel

Turn 3

- **Coalition Victory**
  - Sorcery
  - You win the game if you control a land of each basic land type and a creature of each color.
  - “You can build a perfect machine out of imperfect parts.”
  - —Urza

Turn 4

- **Soul Snuffers**
  - Creature — Elemental Shaman
  - When Soul Snuffers enters the battlefield, put a -1/-1 counter on each creature.
  - They once sought to unite with the flame of transcendence. Now they seek only to extinguish the lights of others.
Forcing Alice to cast spells

At the beginning of each player's upkeep, that player reveals a card at random from his or her hand. If it's a land card, the player puts it onto the battlefield. Otherwise, the player casts it without paying its mana cost if able.

(controlled by Bob)
Forcing Alice to cast spells

Wild Evocation

At the beginning of each player's upkeep, that player reveals a card at random from his or her hand. If it's a land card, the player puts it onto the battlefield. Otherwise, the player casts it without paying its mana cost if able.

Wheel of Sun and Moon

Enchant player
If a card would be put into enchanted player's graveyard from anywhere, instead that card is revealed and put on the bottom of that player's library.

Every life ends, but life itself never does.

(controlled by Bob)
Forcing Alice to cast spells

Wild Evocation

At the beginning of each player’s upkeep, that player reveals a card at random from his or her hand. If it’s a land card, the player puts it onto the battlefield. Otherwise, the player casts it without paying its mana cost if able.

(controlled by Bob)

Wheel of Sun and Moon

Enchant player

If a card would be put into enchanted player’s graveyard from anywhere, instead that card is revealed and put on the bottom of that player’s library.

“Every life ends, but life itself never does.”

Dread of Night

black

All white creatures get -1/-1.

“All moonless, foreign skies keep me in thrall. Dark whispers echo in the night, and I cannot resist.”

—Selenia, dark angel
Changing the state by skipping turn 3

Rotlung Reanimator
Creature — Zombie Cleric
Whenever Rotlung Reanimator or another Cleric is put into a graveyard from play, put a 2/2 black Zombie creature token into play.

Xathrid Necromancer
Creature — Human Wizard
Whenever Xathrid Necromancer or another Human creature you control dies, put a 2/2 black Zombie creature token onto the battlefield tapped.
“\textit{My commands shall echo forever in their dusty skulls.}”

Mesmeric Orb
Artifact
Whenever a permanent becomes untapped, that permanent’s controller puts the top card of his or her library into his or her graveyard.
\textit{A step in one direction is two steps away from another.}

(controlled by Alice)
Forcing Alice to pick the right target

Cleansing Beam

Radiance — Cleansing Beam deals 2 damage to target creature and each other creature that shares a color with it.

“Justice is toothless without punishment. Righteousness cannot succeed without the suffering of the guilty.”

— Razia, Boros archangel

Vigor

Creature — Elemental Incarnation

Trample

If damage would be dealt to another creature you control, prevent that damage. Put a +1/+1 counter on that creature for each 1 damage prevented this way.

When Vigor is put into a graveyard from anywhere, shuffle it into its owner's library.

Sliver  Elf  Demon

5/5  2/2  3/3

Rhino  Faerie  Giant

4/4  3/3  4/4

Faerie  Elf  Orc

3/3  6/6  5/5

Magic: The Gathering is Turing Complete

Thomas Bläsius
Forcing Alice to pick the right target

- protecting Bob’s creatures
- protecting Alice’s creatures
Forcing Alice to pick the right target

protecting Bob’s creatures

protecting Alice’s creatures

only legal target: newest creature

controller:

Magic: The Gathering is Turing Complete

Thomas Bläsius
Keeping your engine alive

Vigor
Creature — Elemental Incarnation

Trample
If damage would be dealt to another creature you control, prevent that damage. Put a +1/+1 counter on that creature for each 1 damage prevented this way.
When Vigor is put into a graveyard from anywhere, shuffle it into its owner’s library.

Prismatic Lace
Instant

Target permanent becomes the color(s) of your choice. Costs to tap, maintain, or use an ability of that permanent remain unchanged.

Fungus Sliver
Creature — Fungus Sliver

All Slivers have “Whenever this creature is dealt damage, put a +1/+1 counter on it.” (The damage is dealt before the counter is put on.)

“When a sliver of this breed enters the hive, the others claw each other in frenzied fits, thereby ensuring their rapid growth.”
—Rukarumel, field journal

215/254
R
000 D
Tim Murray
2015–2016
Wizards of the Coast

Illus., David O'Connor

8/8
© 2018 Wizards of the Coast

Magic: The Gathering is Turing Complete
Thomas Bläsius
Locking everything else down

(controlled by Bob)  (controlled by Alice)  (controlled by both)
Halting

<table>
<thead>
<tr>
<th>current state</th>
<th>read symbol</th>
<th>write symbol</th>
<th>head movement</th>
<th>new state</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>l</td>
<td>R</td>
<td>halt</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

goal: TM halts ⇔ Alice wins with:
### Halting

<table>
<thead>
<tr>
<th>current state</th>
<th>read symbol</th>
<th>write symbol</th>
<th>head movement</th>
<th>new state</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>R</td>
<td></td>
<td></td>
<td>halt</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**goal:** TM halts $\iff$ Alice wins with:

*Prismatic Omen*

Lands you control are every basic land type in addition to their other types.

In times of portent, the land sculpts itself in accordance with the sigils burned on the sky.

*Coalition Victory*

You win the game if you control a land of each basic land type and a creature of each color.

“You can build a perfect machine out of imperfect parts.”

—Urza

*Prismatic Lace*

Target permanent becomes the color(s) of your choice. Costs to tap, maintain, or use an ability of that permanent remain unchanged. 

(white, black, red, green)
Halting

<table>
<thead>
<tr>
<th>current state</th>
<th>read symbol</th>
<th>write symbol</th>
<th>head movement</th>
<th>new state</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>R</td>
<td></td>
<td>halt</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

goal: TM halts $\iff$ Alice wins with:

Whenever a Rhino dies, create a 2/2 blue Assassin.

Magic: The Gathering is Turing Complete

Thomas Bläsius
Out of tape exception

<table>
<thead>
<tr>
<th>Giant</th>
<th>Faerie</th>
<th>Orc</th>
<th>Rat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bob</td>
<td>Bob</td>
<td>Bob</td>
<td>Alice</td>
</tr>
<tr>
<td>5/5</td>
<td>4/4</td>
<td>3/3</td>
<td>2/2</td>
</tr>
</tbody>
</table>
Out of tape exception

Whenever a Rat dies, create a 2/2 white Rat.

Whenever a Rat dies, create a 2/2 black Cephalid.

(controlled by Bob) (controlled by Alice)
Out of tape exception

Whenever a Rat dies, create a 2/2 white Rat.

Whenever a Rat dies, create a 2/2 black Cephalid.

(controlled by Bob) (controlled by Alice)
Out of tape exception

Whenever a Rat dies, create a 2/2 white Rat.

Whenever a Rat dies, create a 2/2 black Cephalid.

(controlled by Bob) (controlled by Alice)
Out of tape exception

Whenever a Rat dies, create a 2/2 white Rat.

Whenever a Rat dies, create a 2/2 black Cephalid.

Magic: The Gathering is Turing Complete

Thomas Bläsius
Out of tape exception

Whenever a Rat dies, create a 2/2 white Rat.

Whenever a Rat dies, create a 2/2 black Cephalid.

As Shared Triumph comes into play, choose a creature type.
Creatures of the chosen type get +1/+1.

"Win together, die alone."
Out of tape exception

Whenever a Rat dies, create a 2/2 white Rat.

Whenever a Rat dies, create a 2/2 black Cephalid.

As Shared Triumph comes into play, choose a creature type. Creatures of the chosen type get +1/+1.

“Win together, die alone.”

—Selenium, dark angel

Magic: The Gathering is Turing Complete

Thomas Bläsius
Out of tape exception

---

<table>
<thead>
<tr>
<th>Giant</th>
<th>Faerie</th>
<th>Orc</th>
<th>Cephalid</th>
<th>Rat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bob</td>
<td>Bob</td>
<td>Bob</td>
<td>Alice</td>
<td>Bob</td>
</tr>
<tr>
<td>5/5</td>
<td>4/4</td>
<td>3/3</td>
<td>2/2</td>
<td>3/3</td>
</tr>
</tbody>
</table>

Whenever a Rat dies, create a 2/2 white Rat.
Whenever a Rat dies, create a 2/2 black Cephalid.

Rat (controlled by Bob) (controlled by Alice)

Magic: The Gathering is Turing Complete

Thomas Bläsius
Questions to be answered

Core components
- How to represent the tape?
- How to implement the transition function?
  - How does the head read and write symbols on the tape?
  - How does the head move left or right?
  - How can we represent different states?

Interplay between components
- How can players trigger the components?
- How to force players to behave as intended?
- How to win the game (a.k.a. the TM halts)?
- What to do, if we are out of tape?

Setup
- How can one player set up the initial game state?
- Can I do that at a tournament?
Setup: getting started

- Add two colorless mana to your mana pool. Ancient Tomb deals 2 damage to you.
- There is no glory to be gained in the kingdom of the dead. —Ve'ec tomb inscription

Mana

<table>
<thead>
<tr>
<th>Colorless</th>
<th>White</th>
<th>Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black</td>
<td>Red</td>
<td>Green</td>
</tr>
</tbody>
</table>

Magic: The Gathering is Turing Complete

Thomas Bläsius
Setup: getting started

- Add two colorless mana to your mana pool. Ancient Tomb deals 2 damage to you.
  - There is no glory to be gained in the kingdom of the dead.
    - Véc tomb inscription

**Ancient Tomb**
- Add two colorless mana to your mana pool. Ancient Tomb deals 2 damage to you.
- There is no glory to be gained in the kingdom of the dead.
  - Véc tomb inscription

**Mana Color**
- Colorless
- White
- Blue
- Black
- Red
- Green

**Mana Cost**
- Colorless

**Hand**
- **Grim Manselth**
  - Artifice
  - Grim Manselth does not untap during your untap phase.
  - Add four colorless mana to your mana pool. Play this ability as a mana source.
  - Unmaker Gravemolder.
  - Cost: 4 mana.

- **Lotus Petal**
  - Artifice
  - Lotus Petal: Add one mana of any color to your mana pool.
  - "Hard to imagine," said Flammar, scanning the petals. "Such a lovely flower inspiring such greed."

- **Lotus Petal**
  - Artifice
  - Lotus Petal: Add one mana of any color to your mana pool.
  - "Hard to imagine," said Flammar, scanning the petals. "Such a lovely flower inspiring such greed."

- **Piers Dadwick**
  - Artifice
  - Dadwick instrument: activated ability costs 1 less to activate. This effect can't reduce the amount of mana an ability costs to activate to less than one mana.

- **Staff of Domination**
  - Artifice
  - Staff of Domination: Unmaker Staff of Domination.
  - You gain 1 life.
  - Untap target creature.
  - Tap target creature.
  - Draw a card.

---

Magic: The Gathering is Turing Complete

- Thomas Bläsius
Setup: getting started

**BATTLEFIELD**

**HAND**

**MANA**

- Add two colorless mana to your mana pool. Ancient Tomb deals 2 damage to you.

*There is no glory to be gained in the kingdom of the dead.*

― Véc tomb inscription

---

**Magic: The Gathering is Turing Complete**

Thomas Bläsius
Magic: The Gathering is Turing Complete

Thomas Bläsius

Setup: getting started

BATTLEFIELD

Mana

colorless
white
blue
black
red
green

Battlefield

Grim Monolith

Artifact

Grim Monolith does not untap during your untap phase.

• Add three colorless mana to your mana pool. Play this ability as a mana source.
• Untap Grim Monolith.
• Untap Grim Monolith.
Part prison, part home.

Hand

Lotus Petal

• Sacrifice Lotus Petal: Add one mana of any color to your mana pool.
  “Hard to imagine,” said Flamme, striking the petals. “Such a lovely flower inspiring such pain.”

Lotus Petal

• Sacrifice Lotus Petal: Add one mana of any color to your mana pool.
  “Hard to imagine,” said Flamme, striking the petals. “Such a lovely flower inspiring such pain.”

Pirate Arachne

Enchant artifact

Enchanted artifact’s activated abilities cost 1 less to activate. This effect can’t reduce the amount of mana an ability costs to activate to less than one mana.

Staff of Domination

• Untap Staff of Domination.
• • You gain 1 life.
• • Untap target creature.
• • Tap target creature.
• • Draw a card.
Magic: The Gathering is Turing Complete

Thomas Bläsius
Setup: getting started

**Lotus Petal**

- **Artifact**
- **Mana**
  - colorless
  - white
  - blue
  - black
  - red
  - green

- **Magic: The Gathering**
  - **Grim Marshwolfs**
    - **Blue Wizard**
    - **Mana**
      - colorless
      - white
      - blue
      - black
      - red
      - green
    - **Battlefield**
    - **Mana**
      - colorless
      - white
      - blue
      - black
      - red
      - green
  - **Grim Marshwolfs** is a creature with the following abilities:
    - **Sacrifice Lotus Petal**: Add one mana of any color to your mana pool.
      - "Hard to imagine," mused Hanna, "a lovely flower inspiring such greed."

- **Lost Petals**
  - **Enchant Lotus Petal**: Add one mana of any color to your mana pool.
  - **Enchant Lotus Petal**: Add one mana of any color to your mana pool.
    - "Hard to imagine," mused Hanna, "a lovely flower inspiring such greed."

- **Staff of Domination**
  - **Artifact**
    - **Mana**
      - colorless
      - white
      - blue
      - black
      - red
      - green
    - **Magic: The Gathering**
      - **Staff of Domination**
        - **Mana**
          - colorless
          - white
          - blue
          - black
          - red
          - green
        - **Ability**: Unenchant Staff of Domination.
        - **Ability**: You gain 1 life.
        - **Ability**: Unenchant target creature.
        - **Ability**: Tap target creature.
        - **Ability**: Draw a card.
Setup: getting started
Setup: getting started

Lotus Petal

\[ \text{Sacrifice Lotus Petal: Add one mana of any color to your mana pool.} \]

“Hard to imagine,” mused Hanna, stroking the petal, “such a lovely flower inspiring such greed.”
Setup: getting started

**MAGIC**
- **Mana**: colorless, white, blue
  - Black
  - Red
  - Green

**Ability**: Enchantment Aura
- Enchant artifact
- Enchanted artifact's activated abilities cost 2 less to activate. This effect can't reduce the amount of mana an ability costs to activate to less than one mana.

**Hand**
- [Artifact Card]
- [Artifact Card]

**Battlefield**
- [Artifact Card]
- [Artifact Card]

**Magic: The Gathering is Turing Complete**

Thomas Bläsius
Setup: getting started

Magic: The Gathering is Turing Complete

Thomas Bläsius
Setup: getting started

Magic: The Gathering is Turing Complete
Thomas Bläsius
Setup: getting started

- **Mana**
  - Colorless
  - White
  - Blue
  - Black
  - Red
  - Green

- **Battlefield**
  - Grim Monolith
  - Artfact
  - Does not untap during your untap phase.
  - : Add three colorless mana to your mana pool. Play this ability as a mana source.
  - 4: Untap Grim Monolith.

- **Hand**
  - Staff of Domination
  - : Untap Staff of Domination.
  - : You gain 1 life.
  - : Untap target creature.
  - : Tap target creature.
  - : Draw a card.

---

Magic: The Gathering is Turing Complete

Thomas Bläsius
Setup: getting started

Magic: The Gathering does not untap during your untap phase.  
Add three colorless mana to your mana pool. Play this ability as a mana source.  
Untap Grim Monolith.

Part prison, part home.
Setup: getting started

**Mana**
- Colorless
- White
- Blue
- Black
- Red
- Green

**Grim Monolith**
- Artifact
- Part prison, part home.
- Grim Monolith does not untap during your untap phase.
- Add three colorless mana to your mana pool. Play this ability as a mana source.
- Untap Grim Monolith.

**Staff of Domination**
- Artifact
- Untap Staff of Domination.
- Gain 1 life.
- Untap target creature.
- Tap target creature.
- Draw a card.
Setup: getting started

**Mana**
- Colorless: 2
- Black
- Red
- Blue
- Green

**Battlefield**
- Grim Monolith
  - Artifact
  - Grim Monolith does not untap during your untap phase.
  - Add three colorless mana to your mana pool. Play this ability as a mana source.
  - Untap Grim Monolith.
  - Part prison, part home.

**Hand**
- Staff of Domination
  - Artifact
  - Untap Staff of Domination.
  - You gain 1 life.
  - Untap target creature.
  - Tap target creature.
  - Draw a card.

Magic: The Gathering is Turing Complete

Thomas Bläsius
Setup: getting started

**Mana**

- colorless
- white
- blue
- black
- red
- green

**Battleground**

- Magic: The Gathering does not untap during your untap phase.

- Add three colorless mana to your mana pool. Play this ability as a mana source.


- Part prison, part home.

**Hand**

- Magic: The Gathering is Turing Complete

Thomas Bläsius
Setup: getting started

Magic: The Gathering is Turing Complete

Thomas Bläsius
Setup: getting started

**Mana**
- Colorless
- White
- Blue
- Black
- Red
- Green

**Battlefield**
- Staff of Domination
- Grim Vanquisher
- Blue Clippy

**Hand**

**Magic: The Gathering is Turing Complete**

Thomas Bläsius
Setup: getting started

Magic: The Gathering is Turing Complete
Thomas Bläsius
Setup: getting started

Magic: The Gathering is Turing Complete

Thomas Bläsius
Setup: getting started

Magic: The Gathering is Turing Complete

Thomas Bläsius
Setup: getting started

**Gemstone Array**

- **Artifact**
  - 2: Put a charge counter on Gemstone Array.
  - Remove a charge counter from Gemstone Array: Add one mana of any color to your mana pool.
  - Outside, it reflects the sun’s light. Inside, it harvests the sun’s power.

**Rottling Reanimator**

- **Creature — Zombie Cleric**
  - Whenever Rottling Reanimator or another Cleric is put into a graveyard from play, put a 2/2 black Zombie creature token into play.

**Trickery**

- **Artifact**
  - 1: Put a charge counter on Gemstone Array.
  - Remove a charge counter from Gemstone Array: Add one mana of any color to your mana pool.

**Colorless**

- **White**
- **Blue**
- **Black**
- **Red**
- **Green**
Setup: reusing cards

**Stolen Identity**

- **Sorcery**
- Put a token onto the battlefield that’s a copy of target artifact or creature.
- Cipher (When you cast a spell, you may put this spell card in your hand. If you do, copy it for yourself.)

**Memnarch**

- **Legendary Artifact Creature — Wizard**
- 1: Target permanent becomes an artifact in addition to its other types.
- 4: Gain control of target artifact.
- In the blur between metal and flesh, Memnarch found madness.

**Reito Lantern**

- **Artifact**
- 3: Put target card from a graveyard on the bottom of its owner's library.
- Lanterns carved from the mystic stones of the Reito Mines were said to light the way of lost souls.

**Capsize**

- **Instant**
- Buyback 3 (You may pay an additional 3 as you cast this spell. If you do, put this card into your hand as it resolves.)
- Return target permanent to its owner's hand.
Setup: creating the tape

As Riptide Replicator comes into play, choose a color and a creature type.
Riptide Replicator comes into play with X charge counters on it.

\[ \text{x} , \Rightarrow \] Put an X/X creature token of the chosen color and type into play, where X is the number of charge counters on Riptide Replicator.

Illus. Doug Chaffee

Setup: cleaning up

Donate

Sorcery

Target player gains control of target permanent you control.

Campus pranksters initiate new students with the old “beeble bomb” routine.

Reality Ripple

Instant

Target artifact, creature, or land phases out.

"Can you prepare for the unexpected? No, you cannot. You can prepare only to be surprised.”

—Teferi

Illus. Jeff Miracoli
(c) 1993 World of the Crescent Moon

Illus. Alan Rapoport
(c) 1993 World of the Crescent Moon
Setup: cleaning up
Conclusion

Theorem
Determining the outcome of a game of Magic: The Gathering in which all remaining moves are forced is undecidable.

Note
- a player can set up the corresponding game state in the first turn
- with a tournament legal deck (in the Legacy format)
Conclusion

Theorem
Determining the outcome of a game of *Magic: The Gathering* in which all remaining moves are forced is undecidable.

Note
- a player can set up the corresponding game state in the first turn
- with a tournament legal deck (in the Legacy format)

Related fun fact
- deciding whether a game action (blocking) is legal is coNP-complete
  
  [Chatterjee and Ibsen-Jensen, 2016]