Karlsruhe Institute of Technology

Algorithms for Graph Visualization

Flow Methods: Orthogonal Layouts – Part I

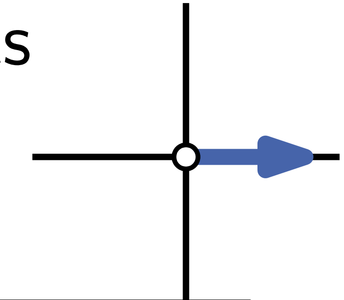
INSTITUT FÜR THEORETISCHE INFORMATIK · FAKULTÄT FÜR INFORMATIK

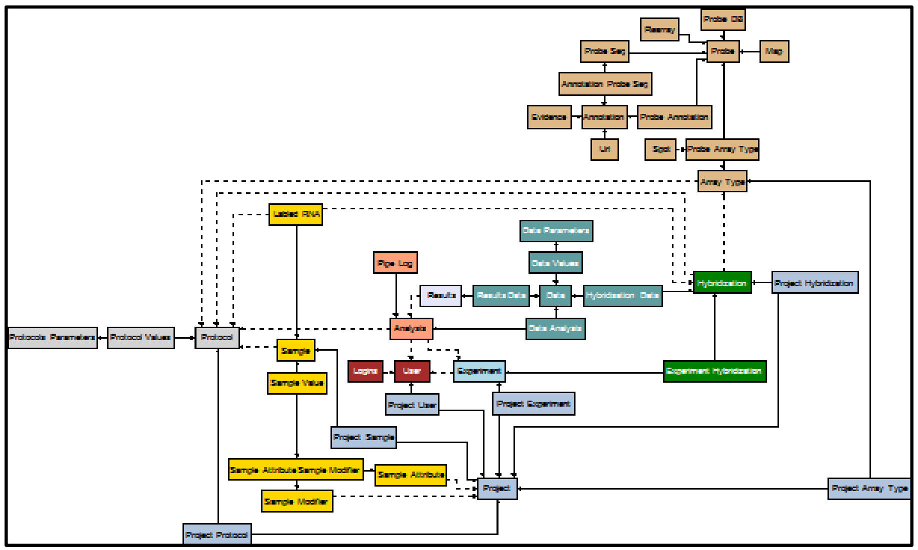
Torsten Ueckerdt

20.11.2019

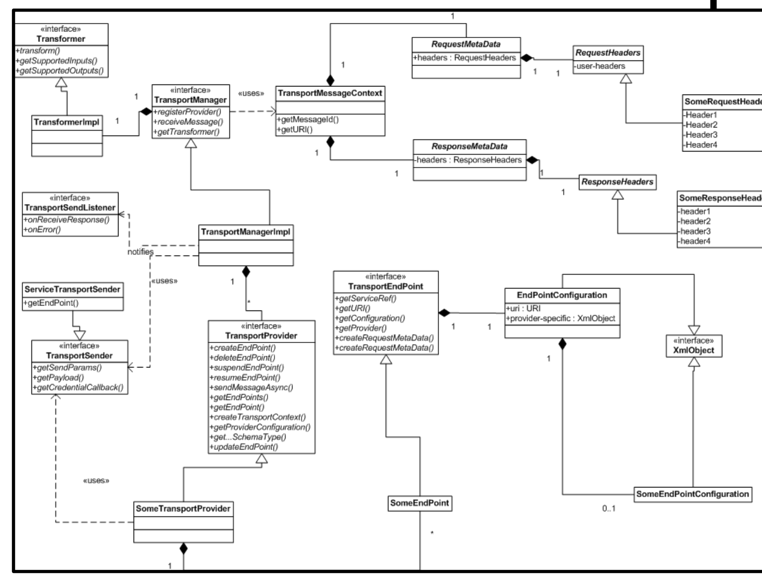
Orthogonal Layouts

- Edges consist of vertical and horizontal segments
- Applied in many areas

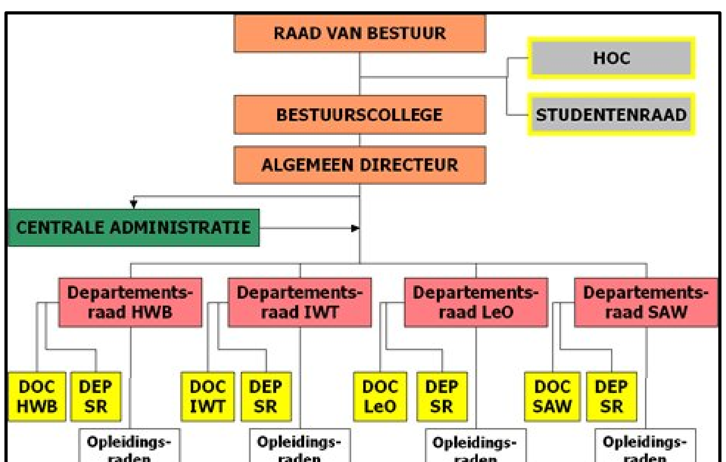




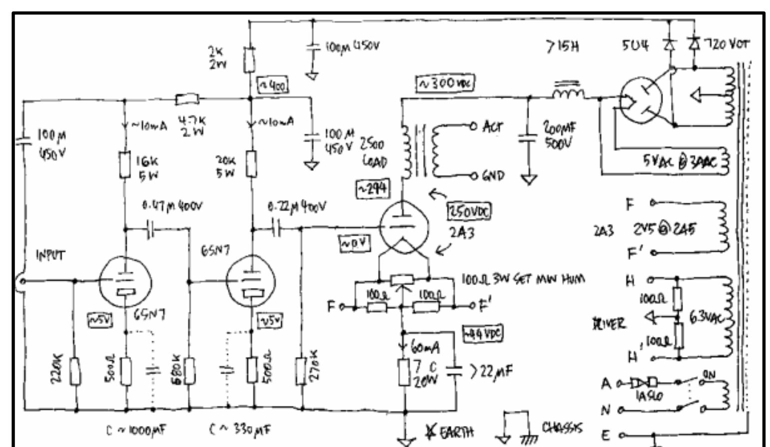
ER diagram in OGDF



UML diagram by Oracle



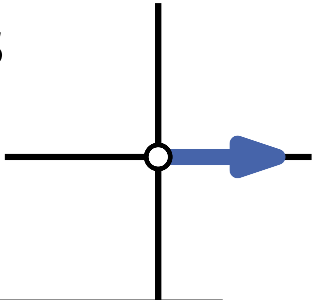
Organigram of HS Limburg

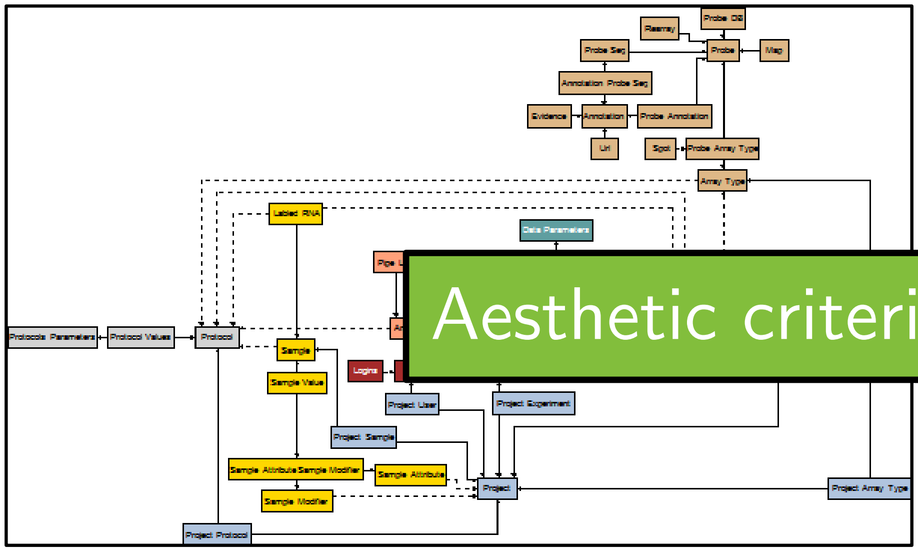


Circuit diagram by Jeff Atwood

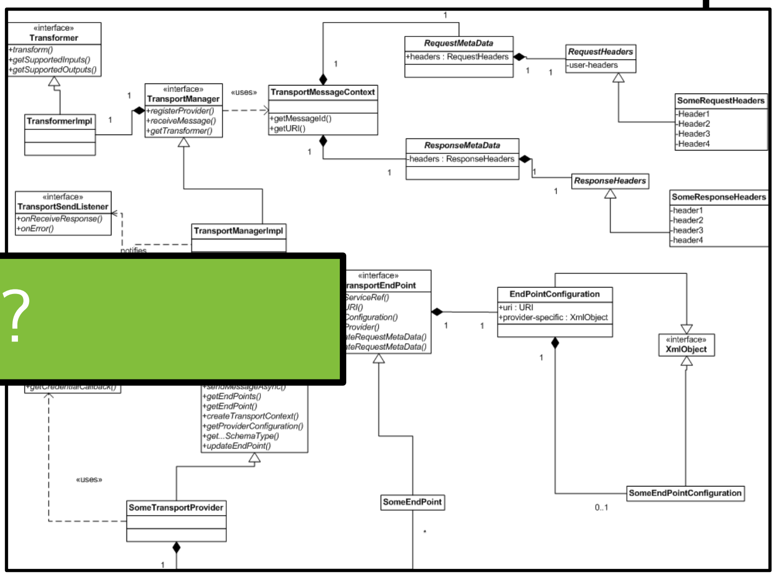
Orthogonal Layouts

- Edges consist of vertical and horizontal segments
- Applied in many areas



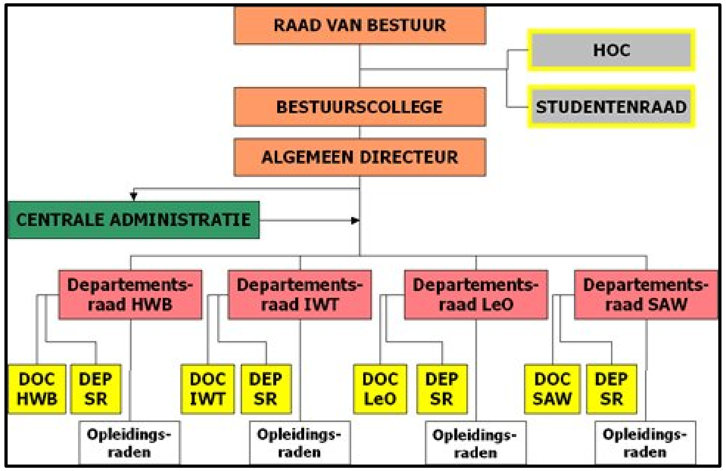


ER diagram in OGDF

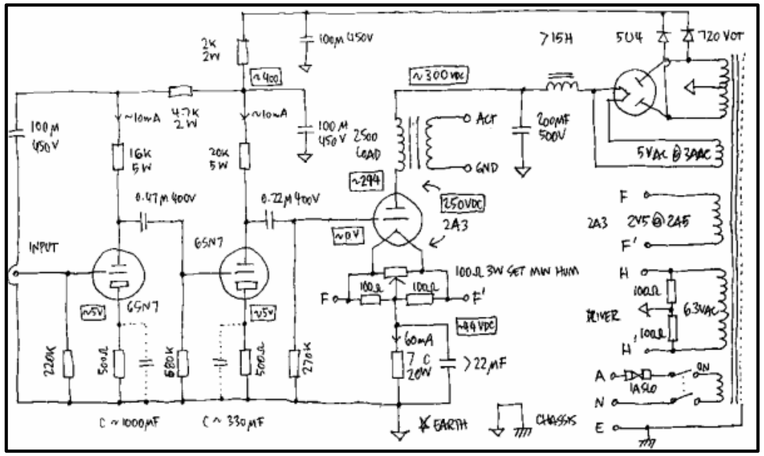


UML diagram by Oracle

Aesthetic criteria?



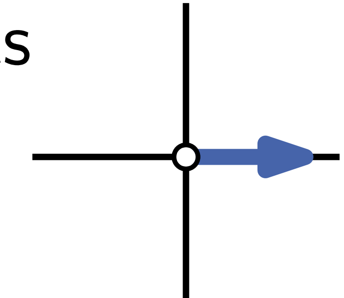
Organigram of HS Limburg



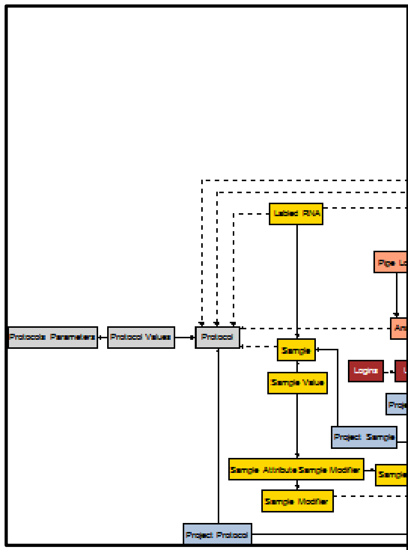
Circuit diagram by Jeff Atwood

Orthogonal Layouts

- Edges consist of vertical and horizontal segments
- Applied in many areas

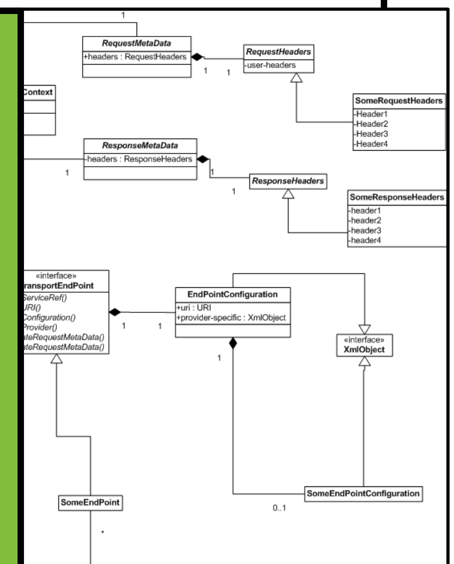


ER diagram in OGDF

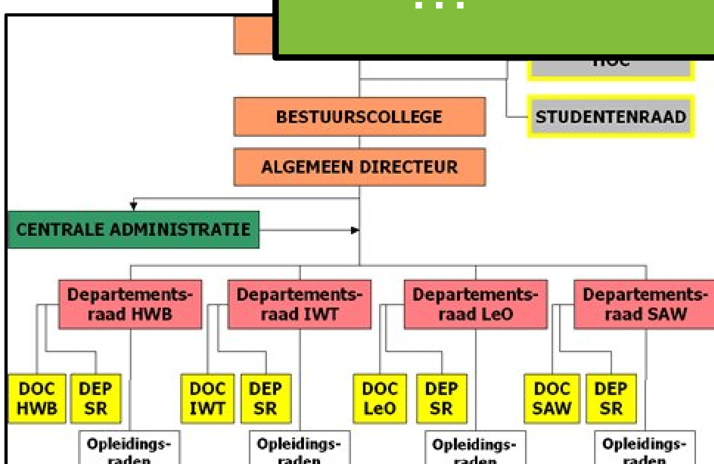


Aesthetic criteria:

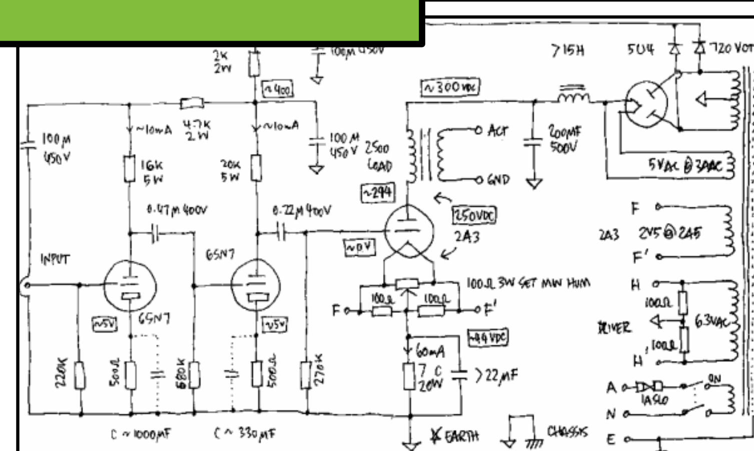
- number of bends
- length of edges
- width, height, area
- monotonicity of edges
- ...



UML diagram by Oracle



Organigram of HS Limburg



Circuit diagram by Jeff Atwood

(Planar) Orthogonal Drawings

Three-step approach: *Topology – Shape – Metrics*

[Tamassia SIAM J. Comput. 1987]

$V = \{v_1, v_2, v_3, v_4\}$
$E = \{v_1v_2, v_1v_3, v_1v_4, v_2v_3, v_2v_4\}$

(Planar) Orthogonal Drawings

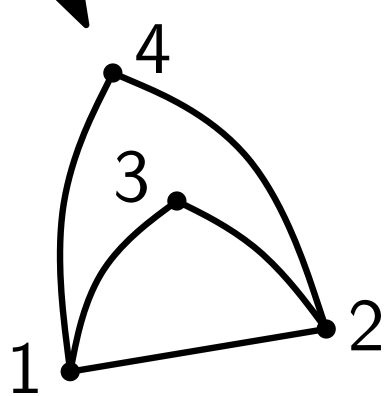
Three-step approach: *Topology – Shape – Metrics*

[Tamassia SIAM J. Comput. 1987]

$V = \{v_1, v_2, v_3, v_4\}$
$E = \{v_1v_2, v_1v_3, v_1v_4, v_2v_3, v_2v_4\}$

combinatorial embedding/ planarization

Reduce Crossings

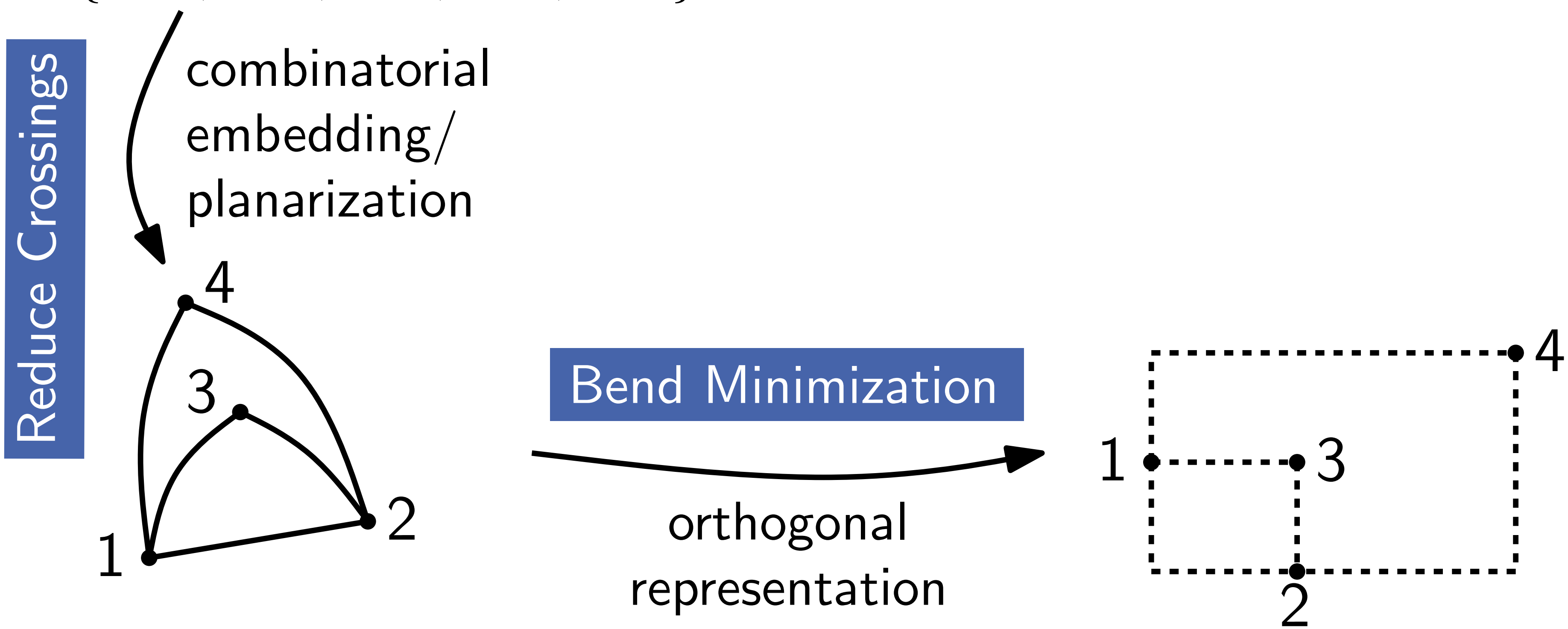


(Planar) Orthogonal Drawings

Three-step approach: *Topology – Shape – Metrics*

[Tamassia SIAM J. Comput. 1987]

$V = \{v_1, v_2, v_3, v_4\}$
$E = \{v_1v_2, v_1v_3, v_1v_4, v_2v_3, v_2v_4\}$

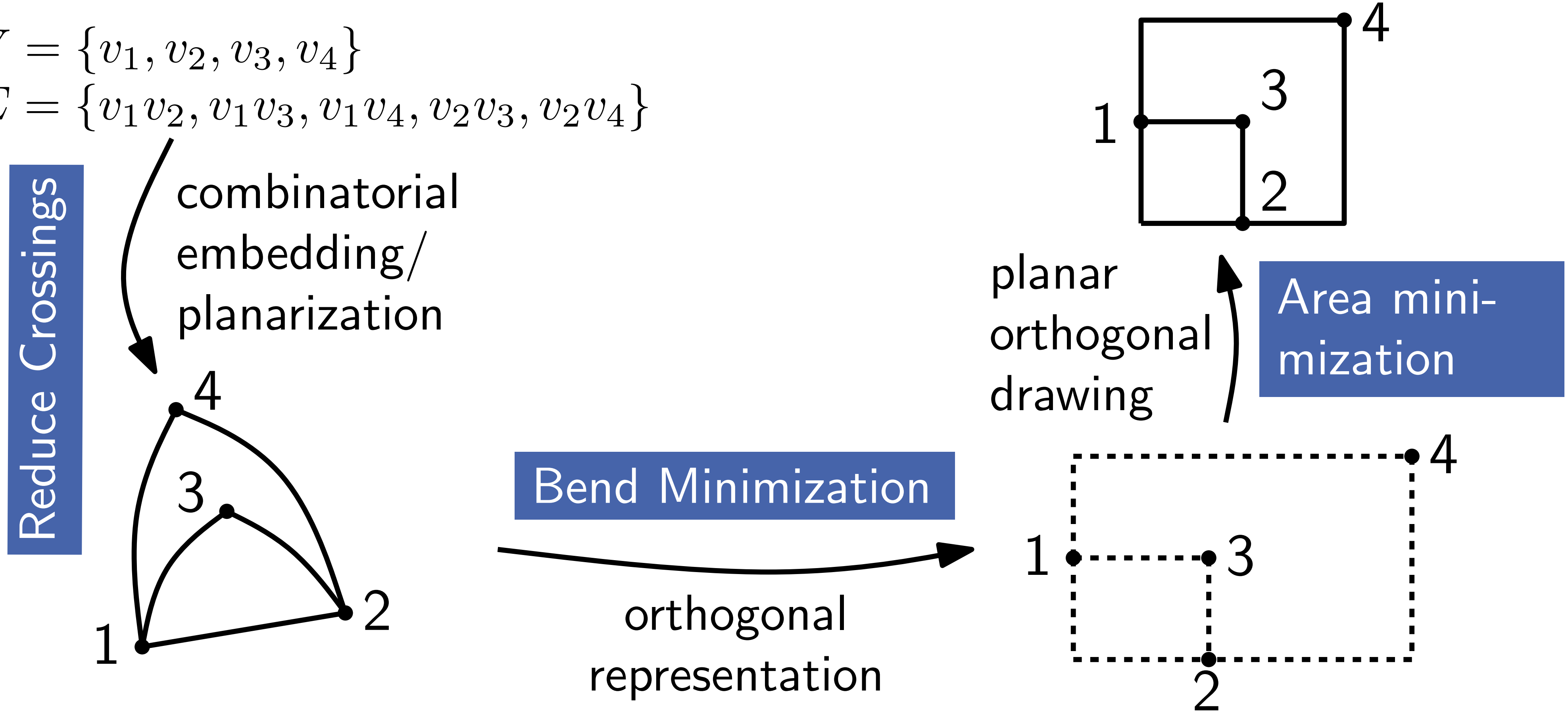


(Planar) Orthogonal Drawings

Three-step approach: *Topology – Shape – Metrics*

[Tamassia SIAM J. Comput. 1987]

$$V = \{v_1, v_2, v_3, v_4\}$$
$$E = \{v_1v_2, v_1v_3, v_1v_4, v_2v_3, v_2v_4\}$$

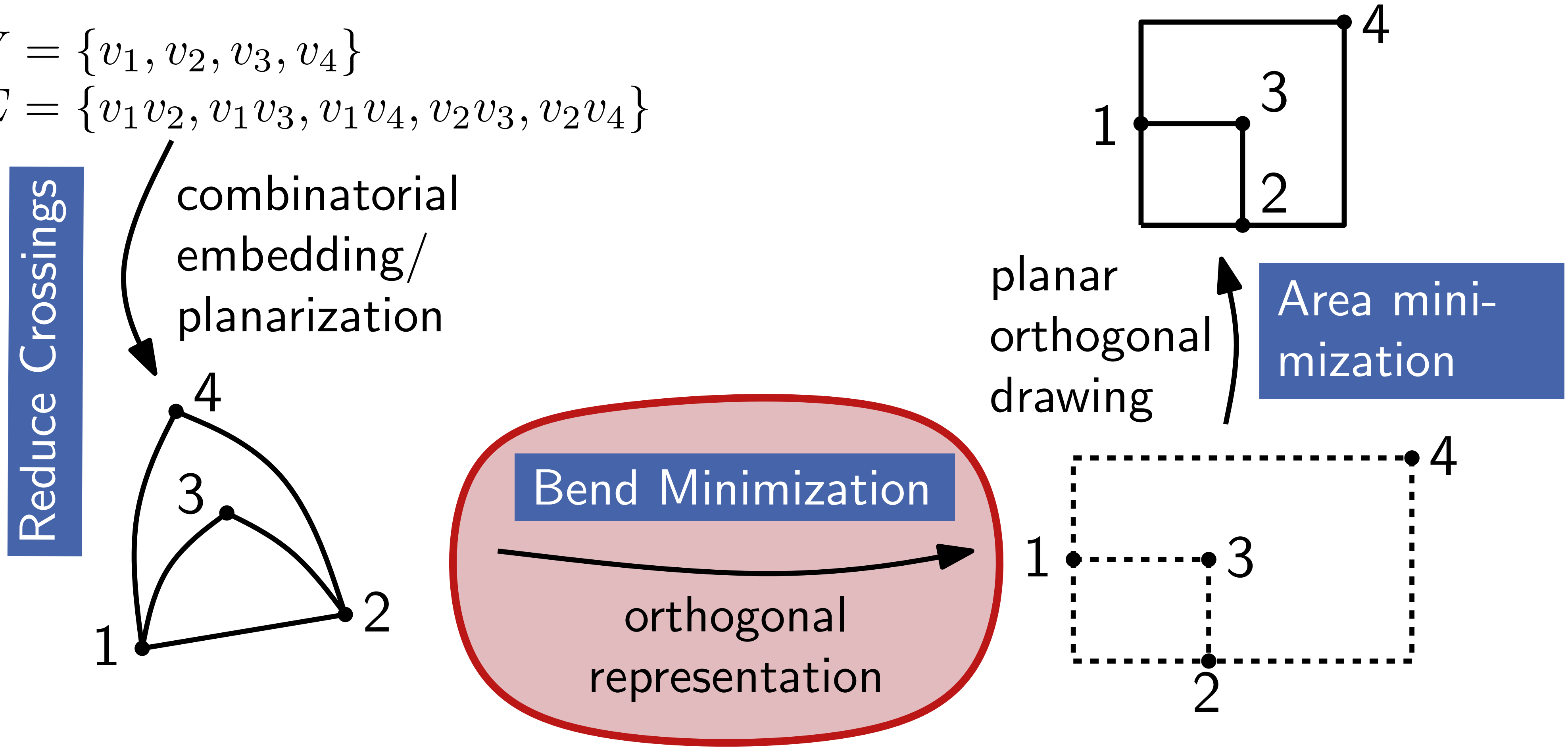


(Planar) Orthogonal Drawings

Three-step approach: *Topology – Shape – Metrics*

[Tamassia SIAM J. Comput. 1987]

$V = \{v_1, v_2, v_3, v_4\}$
$E = \{v_1v_2, v_1v_3, v_1v_4, v_2v_3, v_2v_4\}$



Orthogonal Representation

Given: planar graph $G = (V, E)$, set of faces $\mathcal{F}$,
outer face f_0

Find: orthogonal representation $H(G) = \{H(f) \mid f \in \mathcal{F}\}$

Face representation $H(f)$: of f is a clockwise ordered sequence of edge descriptions (e, δ, α) with

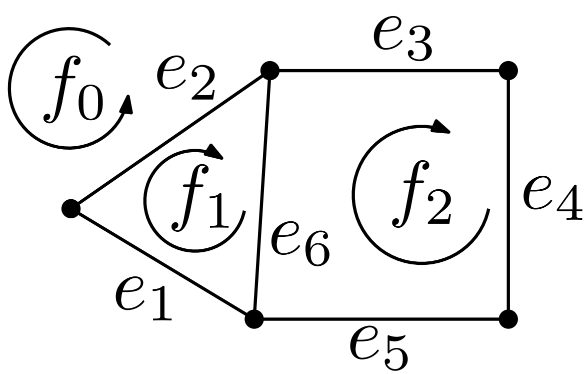
- e edge of f
- δ is sequence of $\{0, 1\}^*$ (0 = right bend, 1 = left bend)
- α is angle $\in \{\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\}$ between e and next edge e'

Orthogonal Representation: Example

$$H(f_0) = ((e_1, 11, \tfrac{\pi}{2}), (e_5, 111, \tfrac{3\pi}{2}), (e_4, \emptyset, \pi), (e_3, \emptyset, \pi), (e_2, \emptyset, \tfrac{\pi}{2}))$$

$$H(f_1) = ((e_1, 00, \tfrac{3\pi}{2}), (e_2, \emptyset, \tfrac{\pi}{2}), (e_6, 00, \pi))$$

$$H(f_2) = ((e_5, 000, \tfrac{\pi}{2}), (e_6, 11, \tfrac{\pi}{2}), (e_3, \emptyset, \pi), (e_4, \emptyset, \tfrac{\pi}{2}))$$



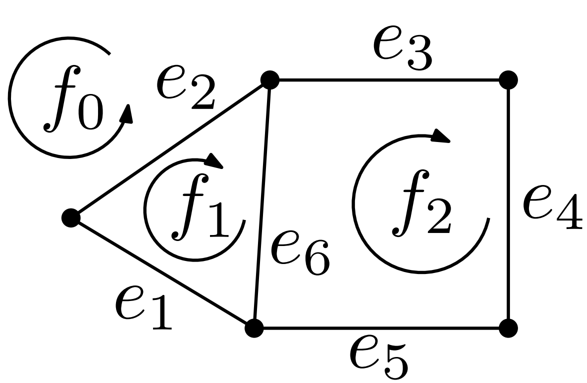
Combinatorial "drawing" of $H(G)$?

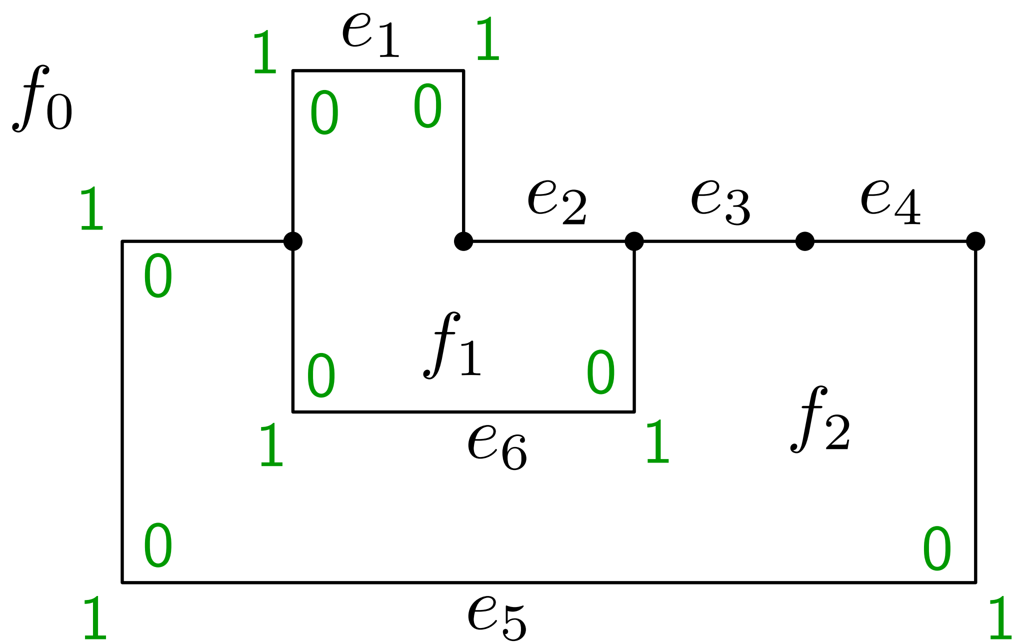
Orthogonal Representation: Example

$$H(f_0) = ((e_1, 11, \tfrac{\pi}{2}), (e_5, 111, \tfrac{3\pi}{2}), (e_4, \emptyset, \pi), (e_3, \emptyset, \pi), (e_2, \emptyset, \tfrac{\pi}{2}))$$

$$H(f_1) = ((e_1, 00, \tfrac{3\pi}{2}), (e_2, \emptyset, \tfrac{\pi}{2}), (e_6, 00, \pi))$$

$$H(f_2) = ((e_5, 000, \tfrac{\pi}{2}), (e_6, 11, \tfrac{\pi}{2}), (e_3, \emptyset, \pi), (e_4, \emptyset, \tfrac{\pi}{2}))$$





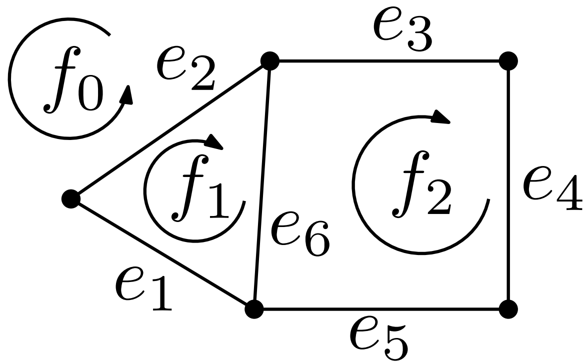
Orthogonal Representation: Example

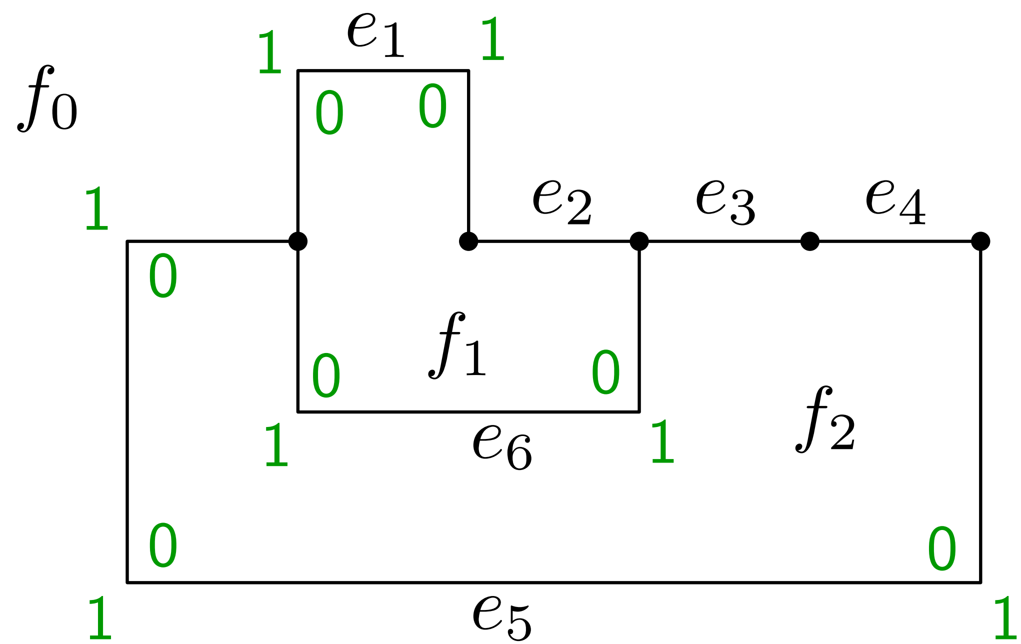
$$H(f_0) = ((e_1, 11, \tfrac{\pi}{2}), (e_5, 111, \tfrac{3\pi}{2}), (e_4, \emptyset, \pi), (e_3, \emptyset, \pi), (e_2, \emptyset, \tfrac{\pi}{2}))$$

$$H(f_1) = ((e_1, 00, \tfrac{3\pi}{2}), (e_2, \emptyset, \tfrac{\pi}{2}), (e_6, 00, \pi))$$

is f_0 listed wrongly!?

$$H(f_2) = ((e_5, 000, \tfrac{\pi}{2}), (e_6, 11, \tfrac{\pi}{2}), (e_3, \emptyset, \pi), (e_4, \emptyset, \tfrac{\pi}{2}))$$



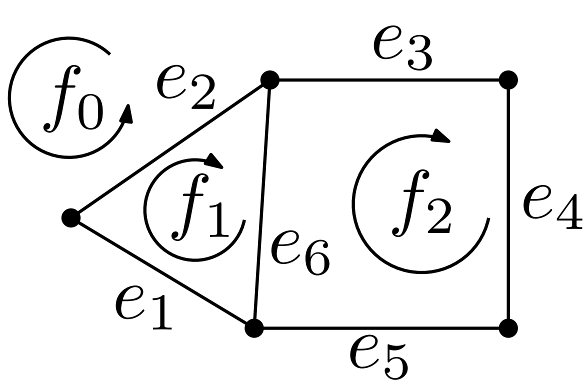


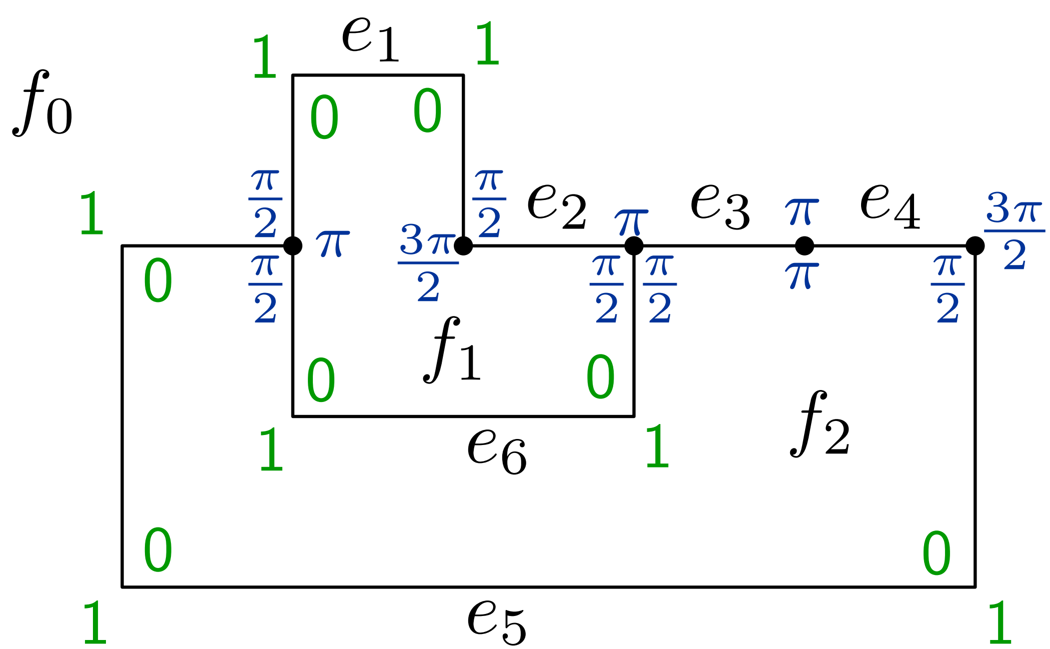
Orthogonal Representation: Example

$$H(f_0) = ((e_1, 11, \tfrac{\pi}{2}), (e_5, 111, \tfrac{3\pi}{2}), (e_4, \emptyset, \pi), (e_3, \emptyset, \pi), (e_2, \emptyset, \tfrac{\pi}{2}))$$

$$H(f_1) = ((e_1, 00, \tfrac{3\pi}{2}), (e_2, \emptyset, \tfrac{\pi}{2}), (e_6, 00, \pi))$$

$$H(f_2) = ((e_5, 000, \tfrac{\pi}{2}), (e_6, 11, \tfrac{\pi}{2}), (e_3, \emptyset, \pi), (e_4, \emptyset, \tfrac{\pi}{2}))$$



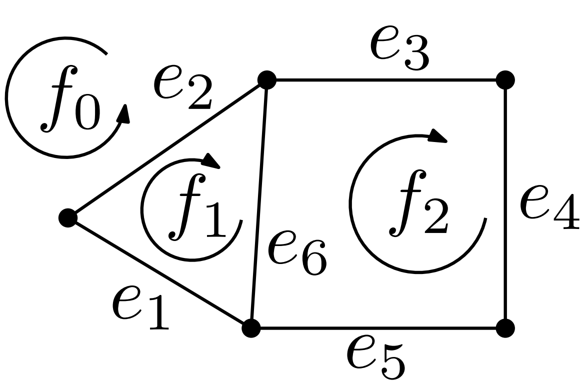


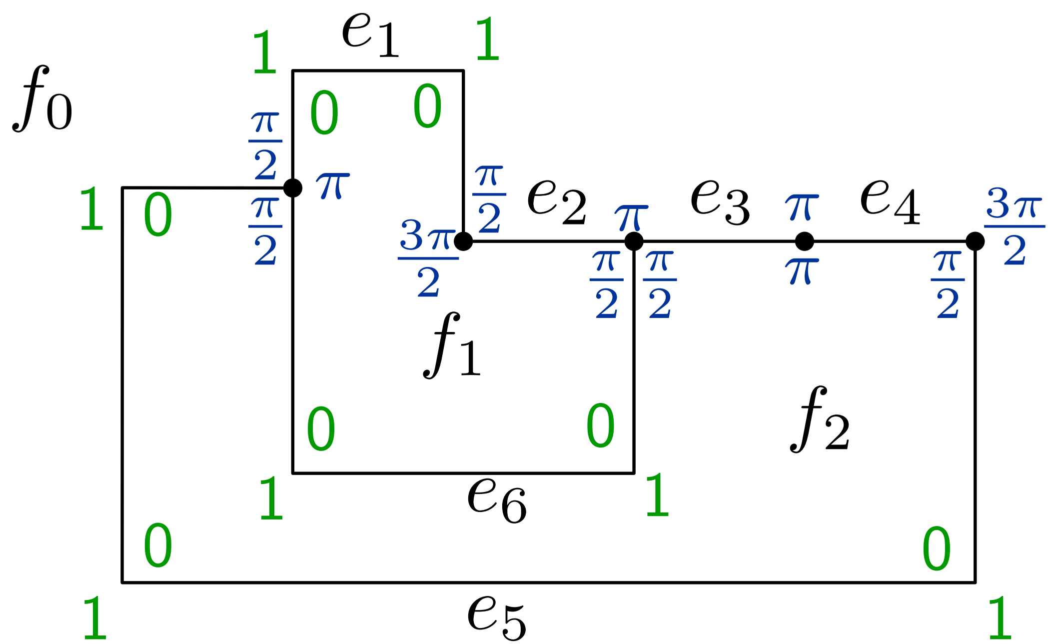
Orthogonal Representation: Example

$$H(f_0) = ((e_1, 11, \tfrac{\pi}{2}), (e_5, 111, \tfrac{3\pi}{2}), (e_4, \emptyset, \pi), (e_3, \emptyset, \pi), (e_2, \emptyset, \tfrac{\pi}{2}))$$

$$H(f_1) = ((e_1, 00, \tfrac{3\pi}{2}), (e_2, \emptyset, \tfrac{\pi}{2}), (e_6, 00, \pi))$$

$$H(f_2) = ((e_5, 000, \tfrac{\pi}{2}), (e_6, 11, \tfrac{\pi}{2}), (e_3, \emptyset, \pi), (e_4, \emptyset, \tfrac{\pi}{2}))$$



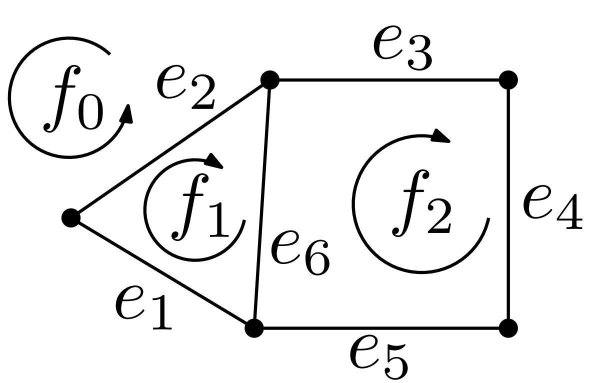


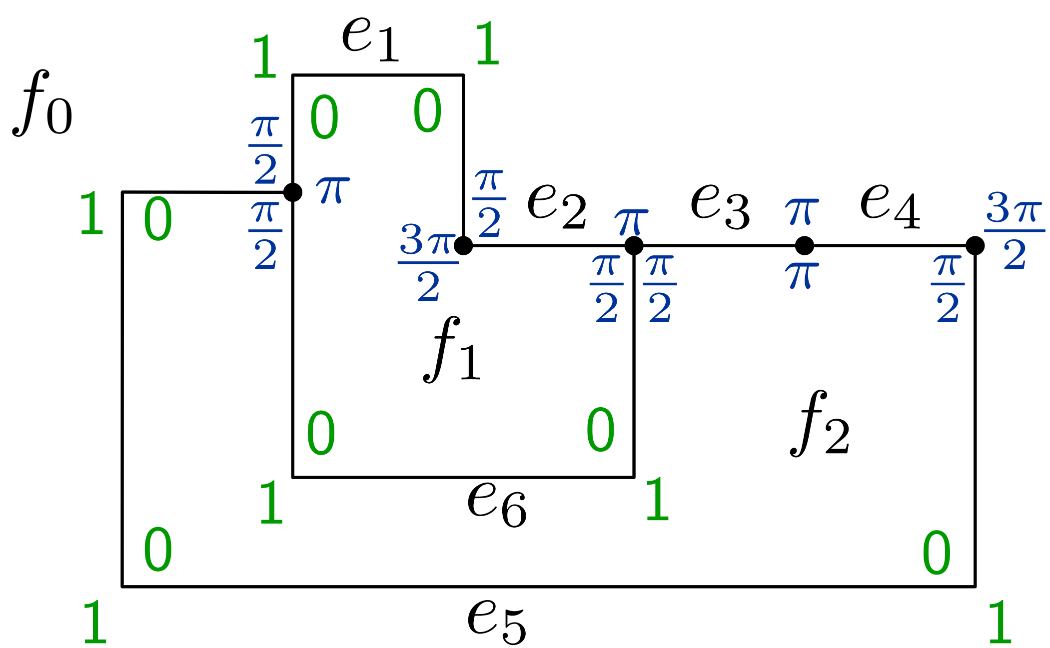
Orthogonal Representation: Example

$$H(f_0) = ((e_1, 11, \tfrac{\pi}{2}), (e_5, 111, \tfrac{3\pi}{2}), (e_4, \emptyset, \pi), (e_3, \emptyset, \pi), (e_2, \emptyset, \tfrac{\pi}{2}))$$

$$H(f_1) = ((e_1, 00, \tfrac{3\pi}{2}), (e_2, \emptyset, \tfrac{\pi}{2}), (e_6, 00, \pi))$$

$$H(f_2) = ((e_5, 000, \tfrac{\pi}{2}), (e_6, 11, \tfrac{\pi}{2}), (e_3, \emptyset, \pi), (e_4, \emptyset, \tfrac{\pi}{2}))$$





concrete coordinates are not fixed yet!

Correctness of an Orthogonal Representation

(H1) $H(G)$ corresponds to $\mathcal{F}, f_0$

Correctness of an Orthogonal Representation

(H1) $H(G)$ corresponds to $\mathcal{F}, f_0$

(H2) for an edge $\{u, v\}$ shared by faces f and g with $((u, v), \delta_1, \alpha_1) \in H(f)$ and $((v, u), \delta_2, \alpha_2) \in H(g)$ sequence δ_1 is reversed and inverted δ_2

Correctness of an Orthogonal Representation

(H1) $H(G)$ corresponds to $\mathcal{F}, f_0$

(H2) for an edge $\{u, v\}$ shared by faces f and g with $((u, v), \delta_1, \alpha_1) \in H(f)$ and $((v, u), \delta_2, \alpha_2) \in H(g)$ sequence δ_1 is reversed and inverted δ_2

(H3) Let $|\delta|_0$ (resp. $|\delta|_1$) be the number of zeros (resp. ones) in δ and $r = (e, \delta, \alpha)$. For $C(r) := |\delta|_0 - |\delta|_1 + 2 - 2\alpha/\pi$ it holds that:
$\sum_{r \in H(f)} C(r) = 4$ for $f \neq f_0$ and $\sum_{r \in H(f_0)} C(r) = -4$

Correctness of an Orthogonal Representation

(H1) $H(G)$ corresponds to $\mathcal{F}, f_0$

(H2) for an edge $\{u, v\}$ shared by faces f and g with $((u,v), \delta_1, \alpha_1) \in H(f)$ and $((v,u), \delta_2, \alpha_2) \in H(g)$ sequence δ_1 is reversed and inverted δ_2

(H3) Let $|\delta|_0$ (resp. $|\delta|_1$) be the number of zeros (resp. ones) in δ and $r = (e, \delta, \alpha)$. For $C(r) := |\delta|_0 - |\delta|_1 + 2 - 2\alpha/\pi$ it holds that:
$\sum_{r \in H(f)} C(r) = 4$ for $f \neq f_0$ and $\sum_{r \in H(f_0)} C(r) = -4$

(H4) For each node v the sum of incident angles is 2π

Correctness of an Orthogonal Representation

(H1) $H(G)$ corresponds to $\mathcal{F}, f_0$

(H2) for an edge $\{u,v\}$ shared by faces f and g with $((u,v),\delta_1,\alpha_1) \in H(f)$ and $((v,u),\delta_2,\alpha_2) \in H(g)$ sequence δ_1 is reversed and inverted δ_2

(H3) Let $|\delta|_0$ (resp. $|\delta|_1$) be the number of zeros (resp. ones) in δ and $r = (e,\delta,\alpha)$. For $C(r) := |\delta|_0 - |\delta|_1 + 2 - 2\alpha/\pi$ it holds that:

$$\sum_{r \in H(f)} C(r) = 4 \text{ for } f \neq f_0 \text{ and } \sum_{r \in H(f_0)} C(r) = -4$$

(H4) For each node v the sum of incident angles is 2π

Pair, think and share:

What does the condition (H3) mean intuitively?

5 min

Bend Minimization with Given Embedding

Karlsruhe Institute of Technology

Problem: Geometric Bend Minimization

Given:
- planar Graph $G = (V, E)$ with maximum degree 4
- combinatorial embedding $\mathcal{F}$ and outer face f_0

Find: orthogonal drawing with minimum number of bends that preserves the embedding

Bend Minimization with Given Embedding

Problem: Geometric Bend Minimization

Given:
- planar Graph $G = (V, E)$ with maximum degree 4
- combinatorial embedding $\mathcal{F}$ and outer face f_0

Find: orthogonal drawing with minimum number of bends that preserves the embedding

compare with the following variation

Problem: Combinatorial Bend Minimization

Given:
- planar Graph $G = (V, E)$ with maximum degree 4
- combinatorial embedding $\mathcal{F}$ and outer face f_0

Find: **orthogonal representation** $H(G)$ with minimum number of bends that preserves the embedding

Combinatorial Bend Minimization

Problem Combinatorial Bend Minimization

Given:
- Graph $G = (V, E)$ with maximum degree 4
- combinatorial embedding $\mathcal{F}$ and outer face f_0

Find: **orthogonal representation** $H(G)$ with minimum number of bends that preserves the embedding

Combinatorial Bend Minimization

Problem Combinatorial Bend Minimization

Given:
- Graph $G = (V, E)$ with maximum degree 4
- combinatorial embedding $\mathcal{F}$ and outer face f_0

Find: **orthogonal representation** $H(G)$ with minimum number of bends that preserves the embedding

Idea: formulate as a network flow problem
- a unit of flow represents an angle of $\pi/2$
- flow from vertices to faces represents the angles at the vertices
- flow between adjacent faces represent the bends at the edges

Reminder: s-t Flow Network

Flow network $(D = (V, A); s, t; u)$ with

- directed graph $D = (V, A)$
- edge capacity $u\colon A \to \mathbb{R}_0^+$
- source $s \in V$, sink $t \in V$

A function $X\colon A \to \mathbb{R}_0^+$ is called s-t-**flow**, if:

$$0 \leq X(i,j) \leq u(i,j) \qquad \forall (i,j) \in A \tag{1}$$

$$\sum_{(i,j)\in A} X(i,j) - \sum_{(j,i)\in A} X(j,i) = 0 \qquad \forall i \in V \setminus \{s,t\} \tag{2}$$

Reminder: General Flow Network

Flow network $(D = (V, A); \ell; u; b)$ with

- directed graph $D = (V, A)$
- edge **lower bound** $\ell\colon A \to \mathbb{R}_0^+$
- edge **capacity** $u\colon A \to \mathbb{R}_0^+$
- node **production/consumption** $b\colon V \to \mathbb{R}$ with $\sum_{i \in V} b(i) = 0$

A function $X\colon A \to \mathbb{R}_0^+$ is called **valid flow**, if:

$$\ell(i,j) \leq X(i,j) \leq u(i,j) \qquad \forall (i,j) \in A \tag{3}$$

$$\sum_{(i,j)\in A} X(i,j) - \sum_{(j,i)\in A} X(j,i) = b(i) \qquad \forall i \in V \tag{4}$$

Problems for Flow Networks

Karlsruhe Institute of Technology

(A) Valid Flow Problem:

Find a valid flow $X\colon A \to \mathbb{R}_0^+$, i.e., such that

- lower bounds and capacities $\ell(e), u(e)$ are respected (inequalities (3))
- consumption/production $b(i)$ satisfied (inequalities (4))

Problems for Flow Networks

(A) Valid Flow Problem:
Find a valid flow $X\colon A \to \mathbb{R}_0^+$, i.e., such that

- lower bounds and capacities $\ell(e), u(e)$ are respected (inequalities (3))
- consumption/production $b(i)$ satisfied (inequalities (4))

Additionally provided: **Cost function** $\mathrm{cost}\colon A \to \mathbb{R}_0^+$

Def: $\mathrm{cost}(X) := \sum_{(i,j)\in A} \mathrm{cost}(i,j) \cdot X(i,j)$

Problems for Flow Networks

(A) Valid Flow Problem:
Find a valid flow $X\colon A \to \mathbb{R}_0^+$, i.e., such that

- lower bounds and capacities $\ell(e), u(e)$ are respected (inequalities (3))
- consumption/production $b(i)$ satisfied (inequalities (4))

Additionally provided: **Cost function** $\mathrm{cost}\colon A \to \mathbb{R}_0^+$

Def: $\mathrm{cost}(X) := \sum_{(i,j)\in A} \mathrm{cost}(i,j) \cdot X(i,j)$

(B) Miminum Cost Flow Problem:
Find a valid flow $X\colon A \to \mathbb{R}_0^+$, that minimizes cost function $\mathrm{cost}(X)$ (over all valid flows)

Flow Network for Bend Minimization

Define flow network $N(G) = ((V \cup \mathcal{F}, A); \ell; u; b; \text{cost})$:

- $A = \{(v, f) \in V \times \mathcal{F} \mid v \text{ incident to} f\} \cup$
  $\{(f, g) \in \mathcal{F} \times \mathcal{F} \mid f, g \text{ adjacent through edge } e\}$

Flow Network for Bend Minimization

Define flow network $N(G) = ((V \cup \mathcal{F}, A); \ell; u; b; \text{cost})$:

- $A = \{(v, f) \in V \times \mathcal{F} \mid v \text{ incident to} f\} \cup$
  $\{(f, g) \in \mathcal{F} \times \mathcal{F} \mid f, g \text{ adjacent through edge } e\}$
- $b(v) = 4 \quad \forall v \in V$

Flow Network for Bend Minimization

Define flow network $N(G) = ((V \cup \mathcal{F}, A); \ell; u; b; \text{cost})$:

- $A = \{(v, f) \in V \times \mathcal{F} \mid v \text{ incident to} f\} \cup$
  $\{(f, g) \in \mathcal{F} \times \mathcal{F} \mid f, g \text{ adjacent through edge } e\}$
- $b(v) = 4 \quad \forall v \in V$
- $b(f) = -2(d_G(f) - 2) \quad \forall f \in \mathcal{F} \setminus \{f_0\}$
- $b(f_0) = -2(d_G(f_0) + 2)$

Flow Network for Bend Minimization

Define flow network $N(G) = ((V \cup \mathcal{F}, A); \ell; u; b; \text{cost})$:

- $A = \{(v, f) \in V \times \mathcal{F} \mid v \text{ incident to} f\} \cup$
  $\{(f, g) \in \mathcal{F} \times \mathcal{F} \mid f, g \text{ adjacent through edge } e\}$
- $b(v) = 4 \quad \forall v \in V$
- $b(f) = -2(d_G(f) - 2) \quad \forall f \in \mathcal{F} \setminus \{f_0\}$
- $b(f_0) = -2(d_G(f_0) + 2)$

$\left.\right\} \Rightarrow \sum_w b(w) \stackrel{?}{=} 0$

Flow Network for Bend Minimization

Define flow network $N(G) = ((V \cup \mathcal{F}, A); \ell; u; b; \text{cost})$:

- $A = \{(v, f) \in V \times \mathcal{F} \mid v \text{ incident to} f\} \cup$
  $\{(f, g) \in \mathcal{F} \times \mathcal{F} \mid f, g \text{ adjacent through edge } e\}$
- $b(v) = 4 \quad \forall v \in V$
- $b(f) = -2(d_G(f) - 2) \quad \forall f \in \mathcal{F} \setminus \{f_0\}$
- $b(f_0) = -2(d_G(f_0) + 2)$

$\Rightarrow \sum_i b(i) = 0$ (Euler)

Flow Network for Bend Minimization

Define flow network $N(G) = ((V \cup \mathcal{F}, A); \ell; u; b; \text{cost})$:

- $A = \{(v, f) \in V \times \mathcal{F} \mid v$ incident to $f\} \cup$
  $\{(f, g) \in \mathcal{F} \times \mathcal{F} \mid f, g$ adjacent through edge $e\}$
- $b(v) = 4 \quad \forall v \in V$
- $b(f) = -2(d_G(f) - 2) \quad \forall f \in \mathcal{F} \setminus \{f_0\}$
- $b(f_0) = -2(d_G(f_0) + 2)$

$\Rightarrow \sum_i b(i) = 0$ (Euler)

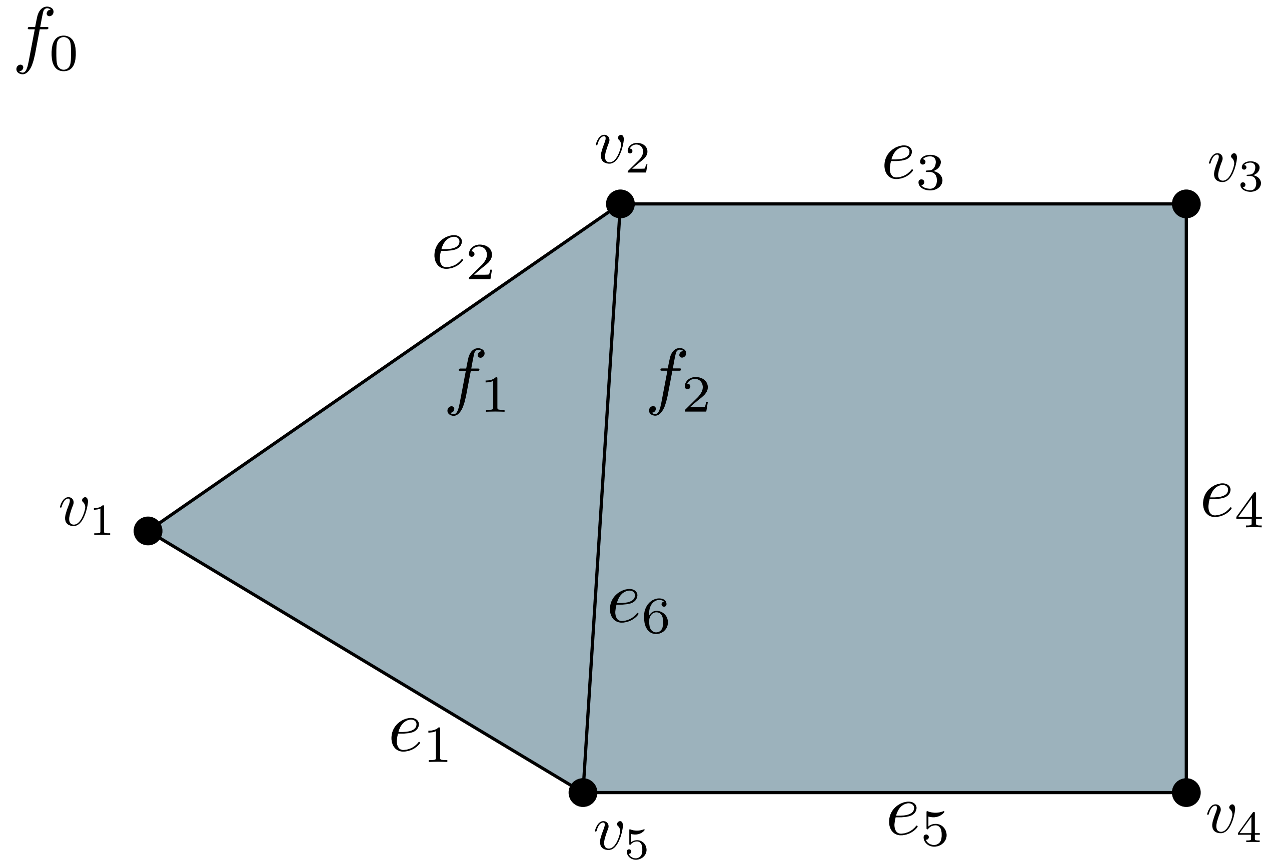
$\forall (f, g) \in A, f, g \in \mathcal{F}$ $\qquad \ell(f, g) := 0 \leq X(f, g) \leq \infty =: u(f, g$

$\text{cost}(f, g) = 1$

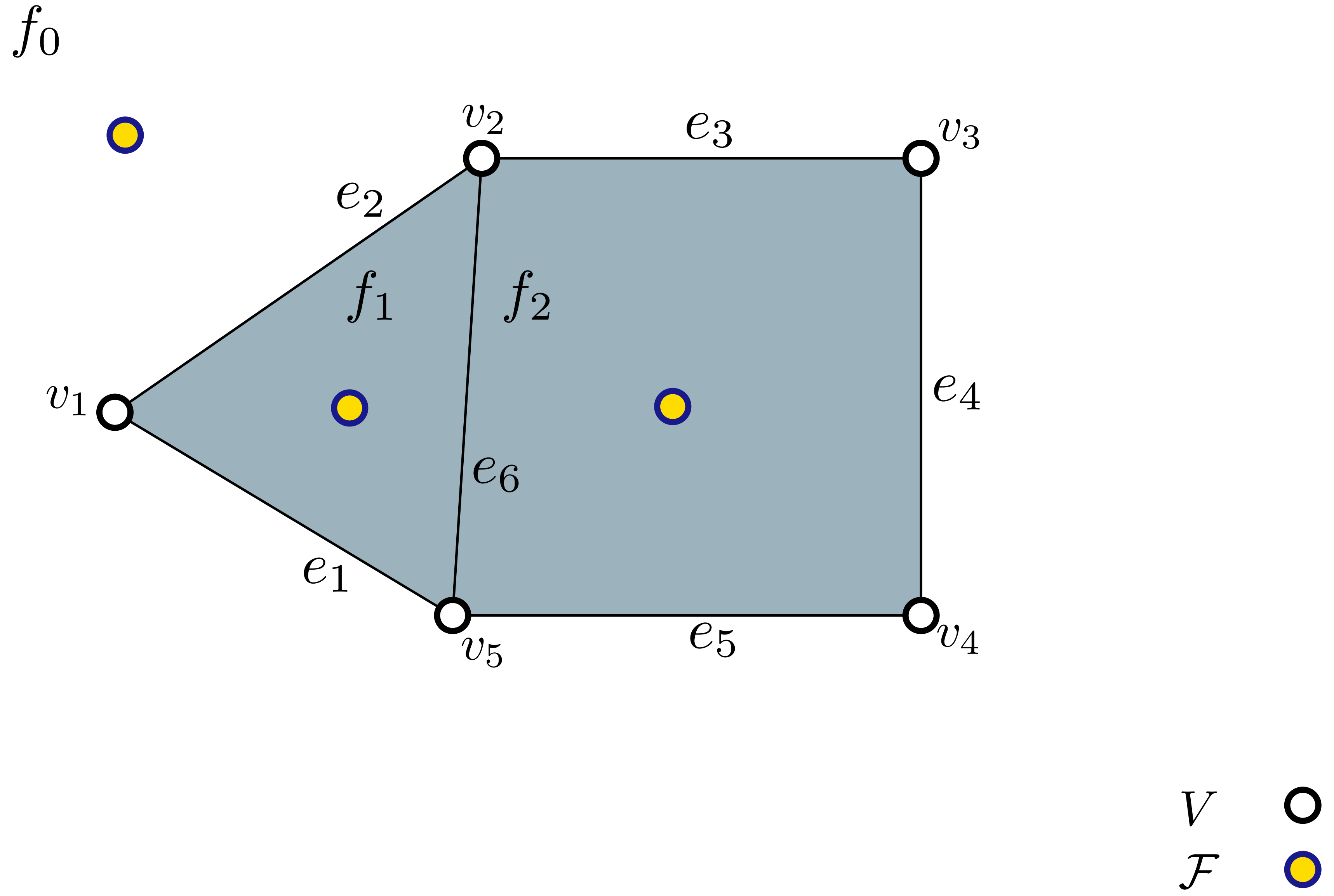
$\forall (v, f) \in A, v \in V, f \in \mathcal{F}$ $\qquad \ell(v, f) := 1 \leq X(v, f) \leq 4 =: u(v, f)$

$\text{cost}(v, f) = 0$

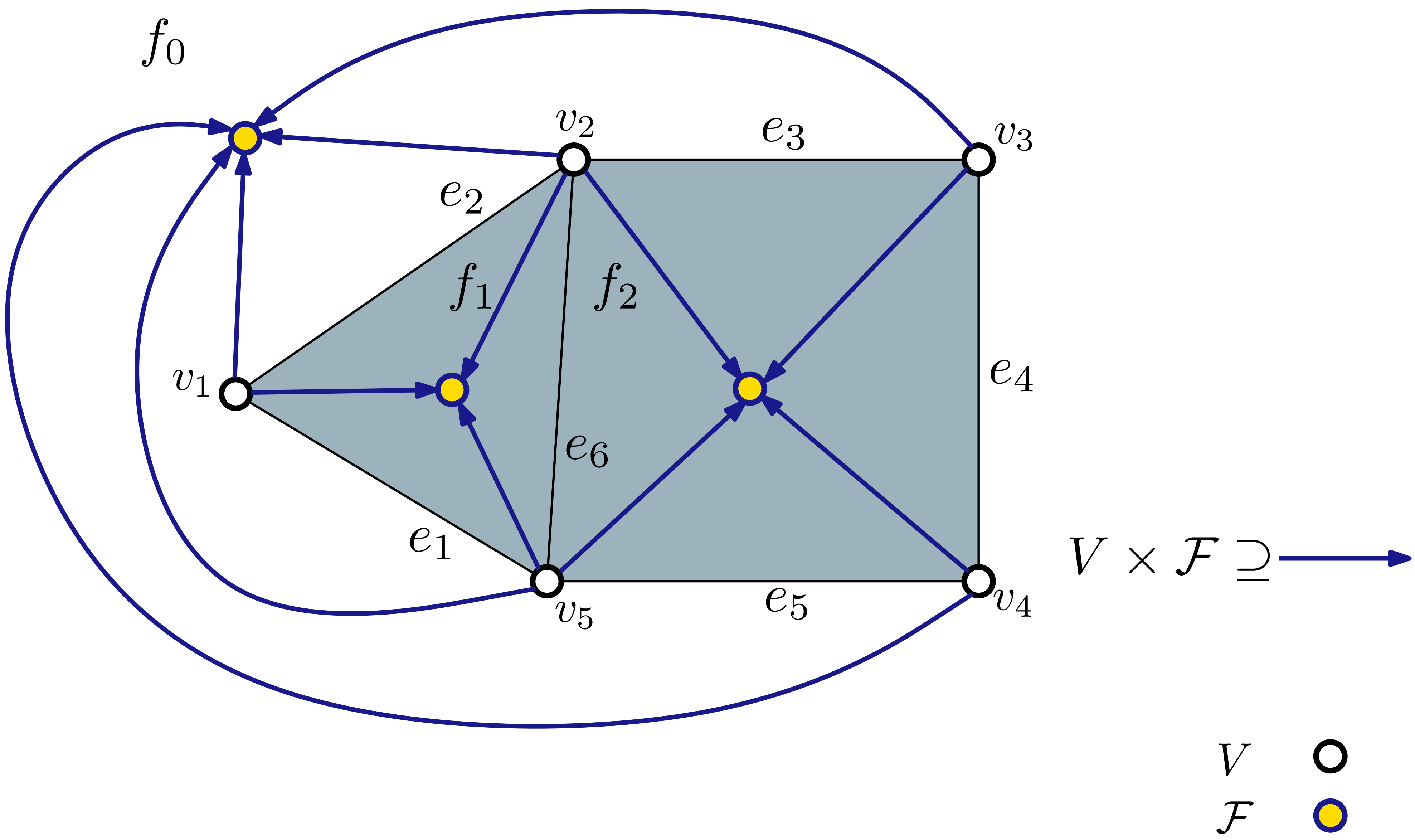
Example Flow Network



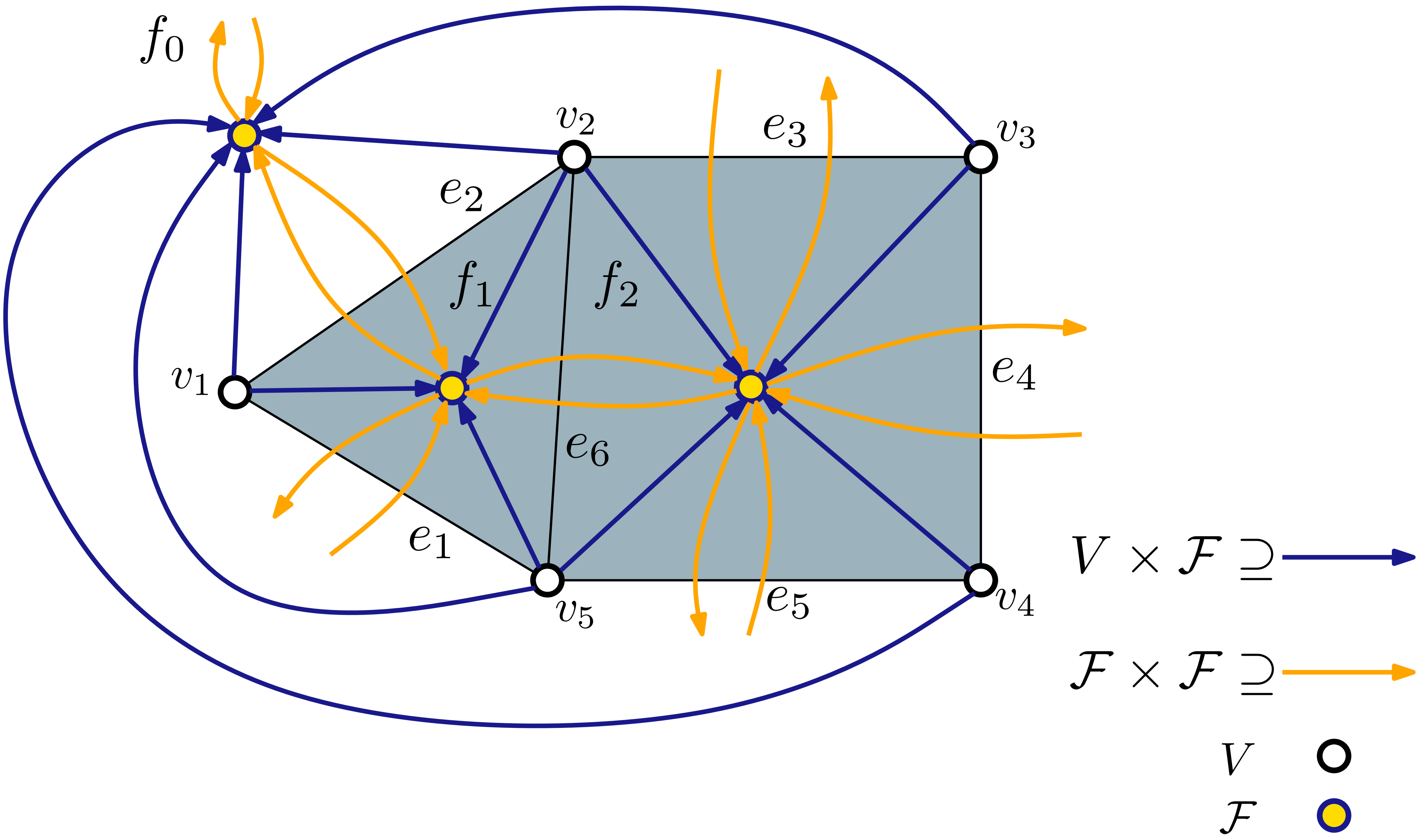
Example Flow Network



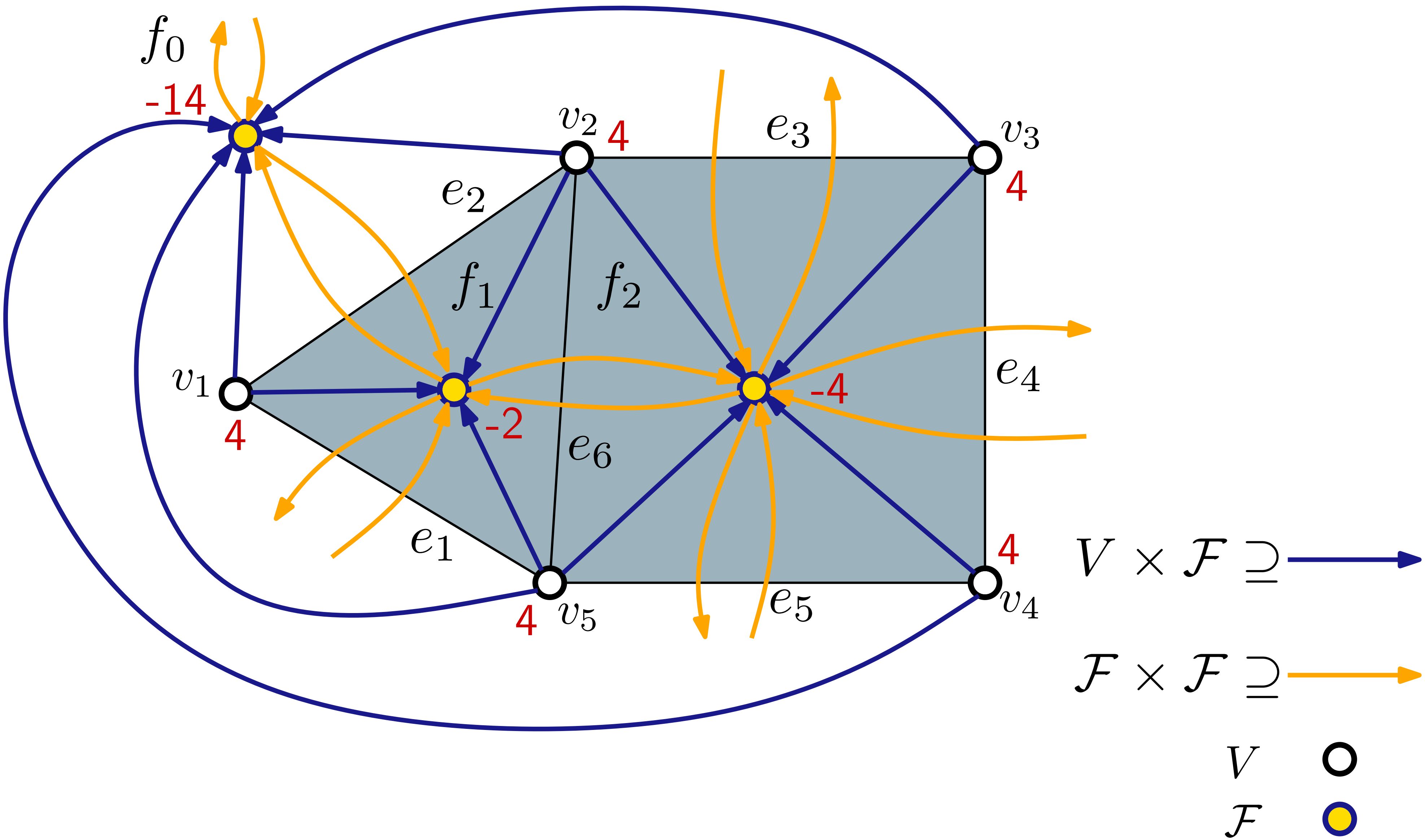
Example Flow Network



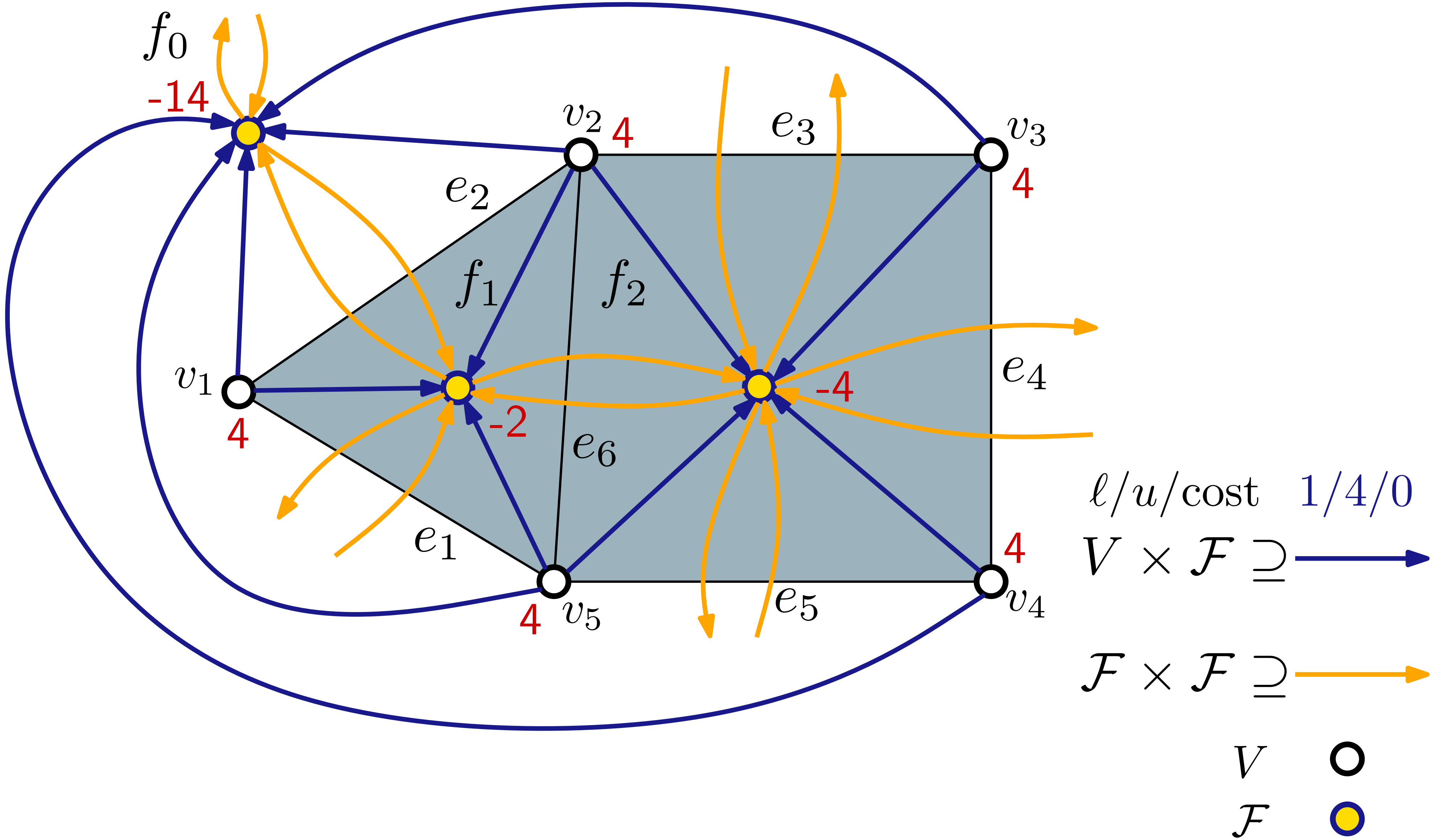
Example Flow Network



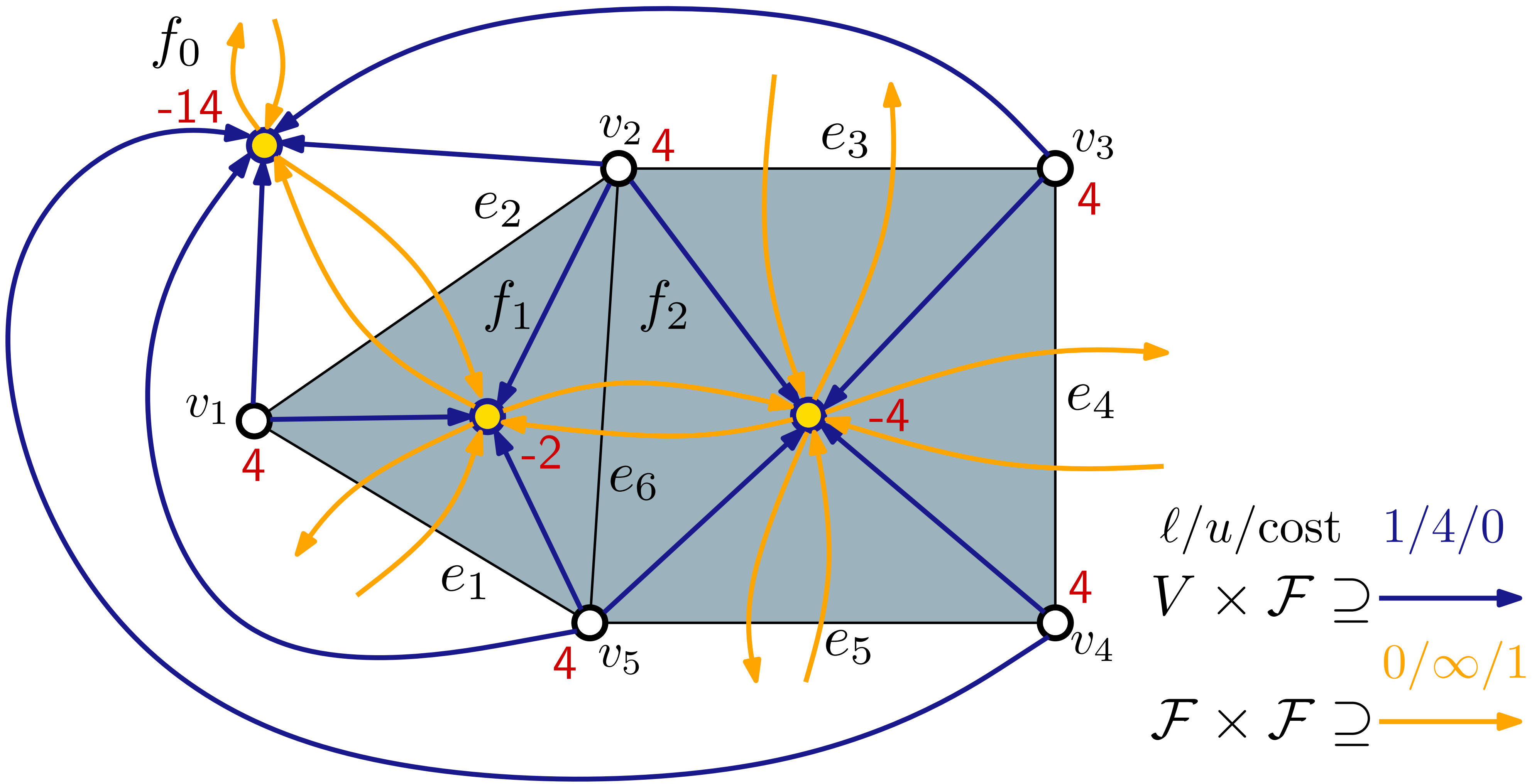
Example Flow Network



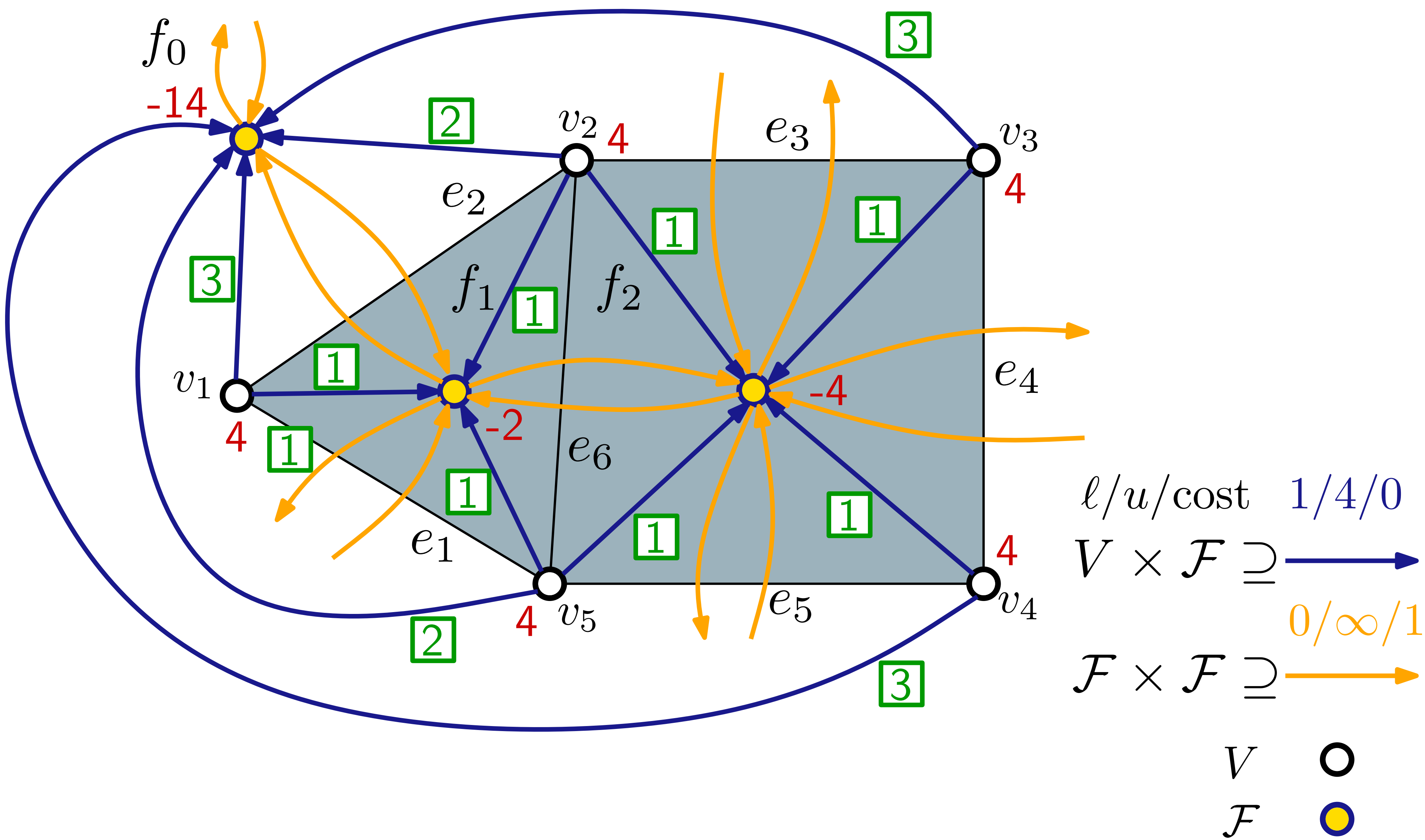
Example Flow Network



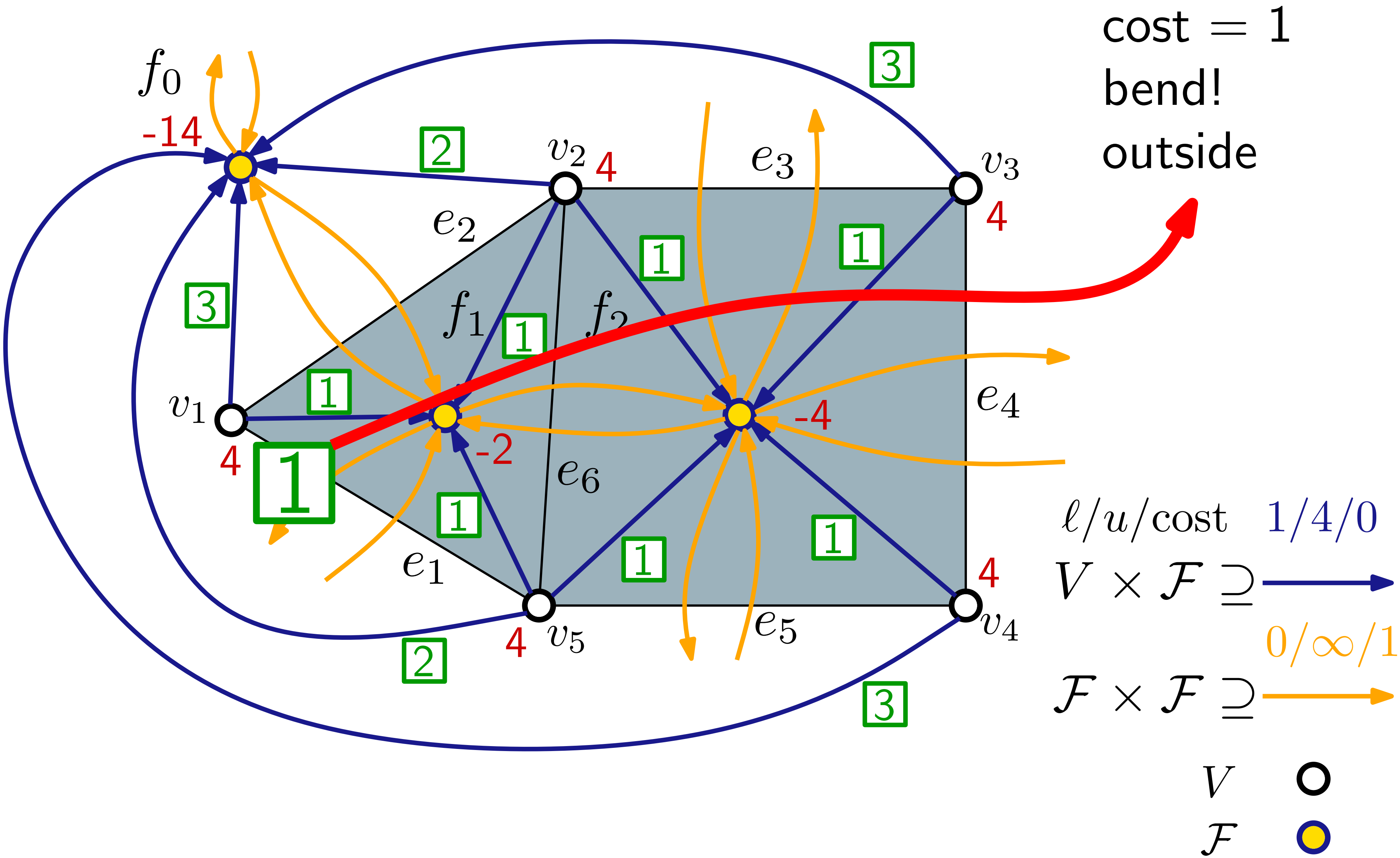
Example Flow Network



Example Flow Network



Example Flow Network



Main Statement

Karlsruhe Institute of Technology

Thm 1: A planar embedded graph $(G, \mathcal{F}, f_0)$ has a valid orthogonal representation $H(G)$ with k bends if and only if the flow network $N(G)$ has a valid flow X with cost k.

Main Statement

Thm 1: A planar embedded graph $(G, \mathcal{F}, f_0)$ has a valid orthogonal representation $H(G)$ with k bends if and only if the flow network $N(G)$ has a valid flow X with cost k.

Proof:

$\Leftarrow$ Given valid flow X in $N(G)$ with cost k
Construct orthogonal representation $H(G)$ with k bends

Main Statement

Thm 1: A planar embedded graph $(G, \mathcal{F}, f_0)$ has a valid orthogonal representation $H(G)$ with k bends if and only if the flow network $N(G)$ has a valid flow X with cost k.

Proof:

$\Leftarrow$ Given valid flow X in $N(G)$ with cost k
Construct orthogonal representation $H(G)$ with k bends

- transform from flow to orthogonal description
- show properties (H1)–(H4)

Main Statement

Thm 1: A planar embedded graph $(G, \mathcal{F}, f_0)$ has a valid orthogonal representation $H(G)$ with k bends if and only if the flow network $N(G)$ has a valid flow X with cost k.

Proof:

$\Leftarrow$ Given valid flow X in $N(G)$ with cost k
Construct orthogonal representation $H(G)$ with k bends

- transform from flow to orthogonal description
- show properties (H1)–(H4)

$\Rightarrow$ Given an orthogonal representation $H(G)$ with k bends
Construct valid flow X in $N(G)$ with cost k

Main Statement

Thm 1: A planar embedded graph $(G, \mathcal{F}, f_0)$ has a valid orthogonal representation $H(G)$ with k bends if and only if the flow network $N(G)$ has a valid flow X with cost k.

Proof:

$\Leftarrow$ Given valid flow X in $N(G)$ with cost k
Construct orthogonal representation $H(G)$ with k bends

- transform from flow to orthogonal description
- show properties (H1)–(H4)

$\Rightarrow$ Given an orthogonal representation $H(G)$ with k bends
Construct valid flow X in $N(G)$ with cost k

- define flow $X \colon A \to \mathbb{R}_0^+$
- show that X is a valid flow and has cost k

Summary of Bend Minimization

- From Theorem 1 it follows that the combinatorial orthogonal bend minimization problem for embedded planar graphs can be solved using an algorithm for the Min-Cost-Flow Problem.

Summary of Bend Minimization

- From Theorem 1 it follows that the combinatorial orthogonal bend minimization problem for embedded planar graphs can be solved using an algorithm for the Min-Cost-Flow Problem.

- This special flow problem for a planar network $N(G)$ can be solved in $O(n^{3/2})$ time. [Cornelsen, Karrenbauer GD 2011]

Summary of Bend Minimization

- From Theorem 1 it follows that the combinatorial orthogonal bend minimization problem for embedded planar graphs can be solved using an algorithm for the Min-Cost-Flow Problem.

- This special flow problem for a planar network $N(G)$ can be solved in $O(n^{3/2})$ time. [Cornelsen, Karrenbauer GD 2011]

- Bend minimization without a given combinatorial embedding is an NP-hard problem. [Garg, Tamassia SIAM J. Comput. 2001]

(Planar) Orthogonal Drawings

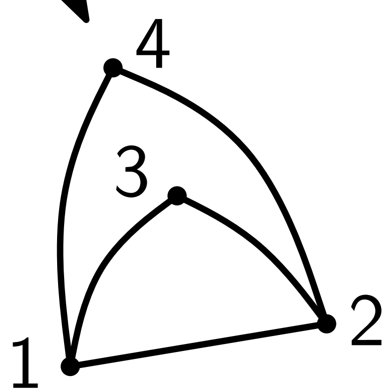
Three-step approach: *Topology – Shape – Metrics*

[Tamassia SIAM J. Comput. 1987]

$V = \{v_1, v_2, v_3, v_4\}$
$E = \{v_1v_2, v_1v_3, v_1v_4, v_2v_3, v_2v_4\}$

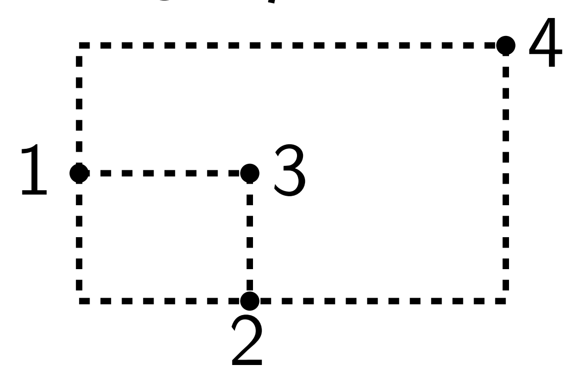
Reduce Crossings

combinatorial embedding/ planarization



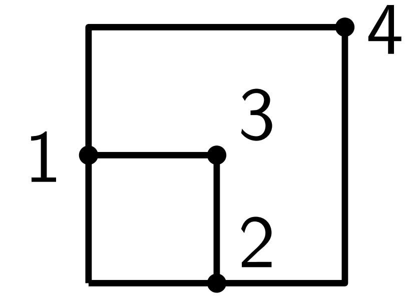
Bend Minimization

orthogonal representation



planar orthogonal drawing

Area minimization

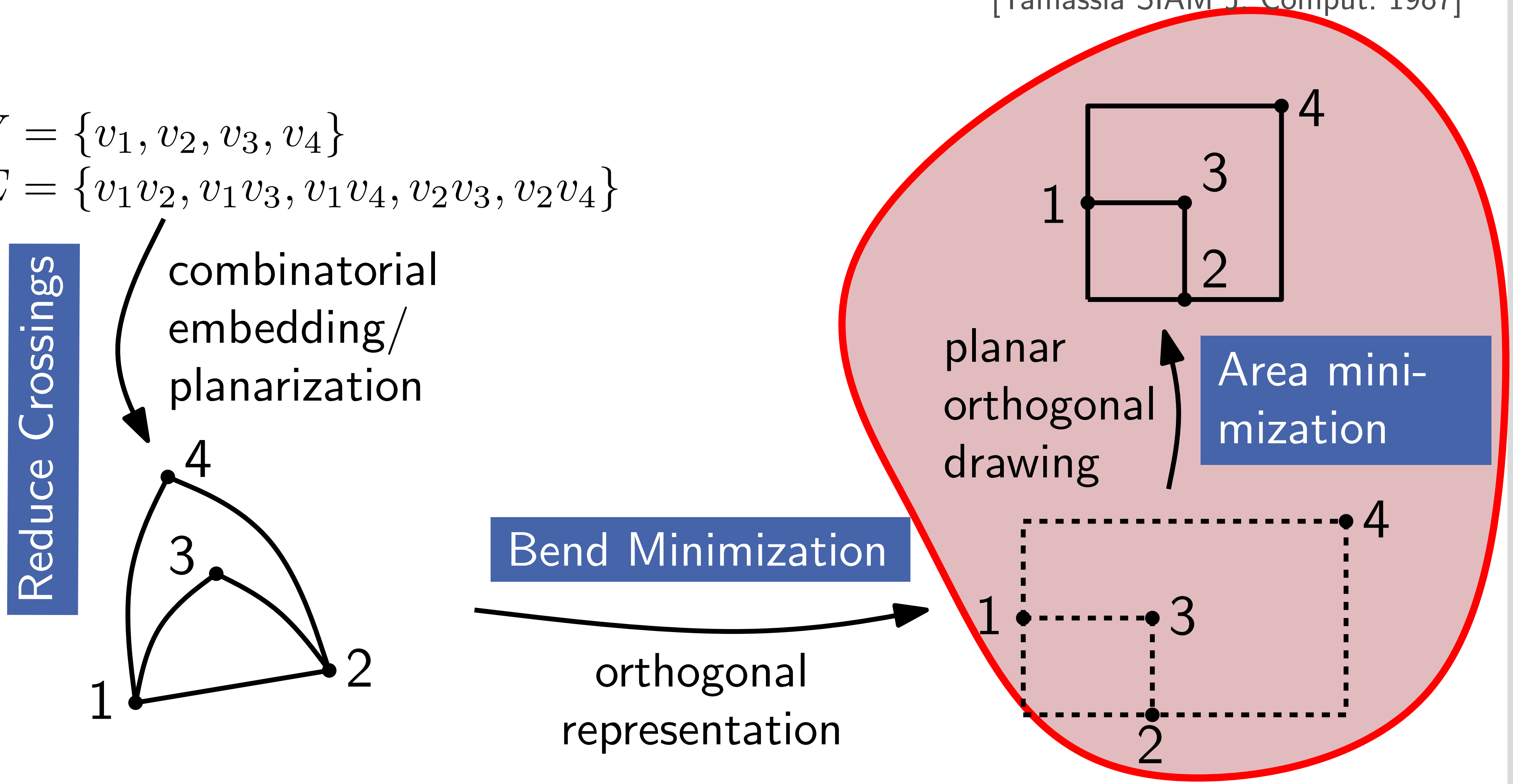


(Planar) Orthogonal Drawings

Three-step approach: *Topology – Shape – Metrics*

[Tamassia SIAM J. Comput. 1987]

$V = \{v_1, v_2, v_3, v_4\}$
$E = \{v_1v_2, v_1v_3, v_1v_4, v_2v_3, v_2v_4\}$



Compaction

Compaction Problem:

Given:
- planar graph $G = (V, E)$ with maximum degree 4
- orthogonal representation $H(G)$

Find: compact orthogonal layout of G that realizes $H(G)$

Compaction

Compaction Problem:

Given:
- planar graph $G = (V, E)$ with maximum degree 4
- orthogonal representation $H(G)$

Find: compact orthogonal layout of G that realizes $H(G)$

Special Case: all faces are rectangles

$\rightarrow$ Guarantees possible
- minimum total edge length
- minimum area

Compaction

Compaction Problem:

Given:
- planar graph $G = (V, E)$ with maximum degree 4
- orthogonal representation $H(G)$

Find: compact orthogonal layout of G that realizes $H(G)$

Special Case: all faces are rectangles

$\rightarrow$ Guarantees possible
- minimum total edge length
- minimum area

Properties:

- bends only on the outer face
- opposite sides of a face have the same length

Compaction

Compaction Problem:

Given:
- planar graph $G = (V, E)$ with maximum degree 4
- orthogonal representation $H(G)$

Find: compact orthogonal layout of G that realizes $H(G)$

Special Case: all faces are rectangles

$\rightarrow$ Guarantees possible
- minimum total edge length
- minimum area

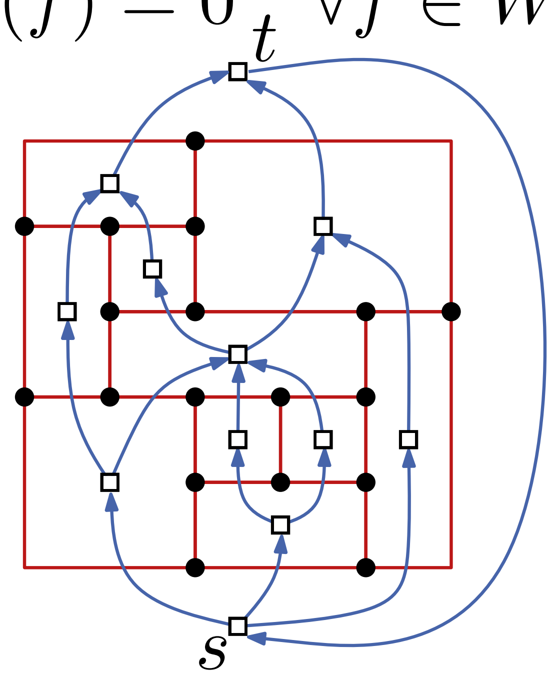
Properties:
- bends only on the outer face
- opposite sides of a face have the same length

We will formulate a flow network for (horizontal) compaction

Flow Network for Edge Length Computation

Def: Flow Network $N_{\text{hor}} = ((W_{\text{hor}}, A_{\text{hor}}); \ell; u; b; \text{cost})$

- $W_{\text{hor}} = \mathcal{F} \setminus \{f_0\} \cup \{s, t\}$
- $A_{\text{hor}} = \{(f, g) \mid f, g$ share a horizontal segment and f lies below $g\} \cup \{(t, s)\}$
- $\ell(a) = 1 \quad \forall a \in A_{\text{hor}}$
- $u(a) = \infty \quad \forall a \in A_{\text{hor}}$
- $\text{cost}(a) = 1 \quad \forall a \in A_{\text{hor}}$
- $b(f) = 0 \quad \forall f \in W_{\text{hor}}$

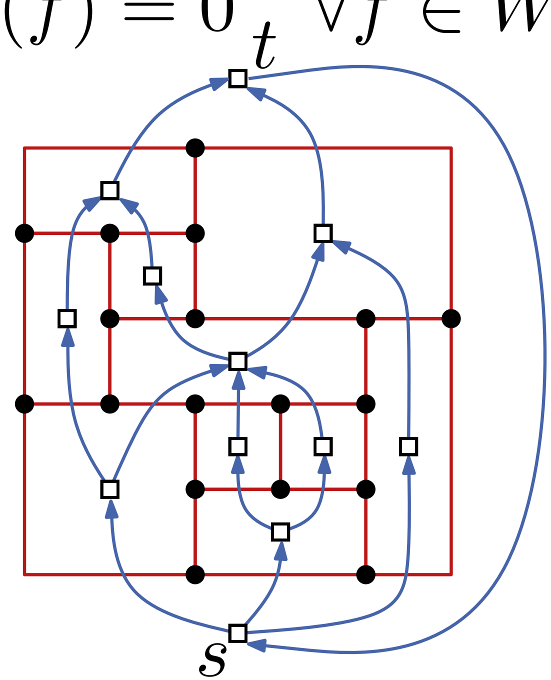


Flow Network for Edge Length Computation

Def: Flow Network $N_{\text{hor}} = ((W_{\text{hor}}, A_{\text{hor}}); \ell; u; b; \text{cost})$

- $W_{\text{hor}} = \mathcal{F} \setminus \{f_0\} \cup \{s, t\}$
- $A_{\text{hor}} = \{(f, g) \mid f, g$ share a horizontal segment and f lies below $g\} \cup \{(t, s)\}$
- $\ell(a) = 1 \quad \forall a \in A_{\text{hor}}$
- $u(a) = \infty \quad \forall a \in A_{\text{hor}}$
- $\text{cost}(a) = 1 \quad \forall a \in A_{\text{hor}}$
- $b(f) = 0 \quad \forall f \in W_{\text{hor}}$

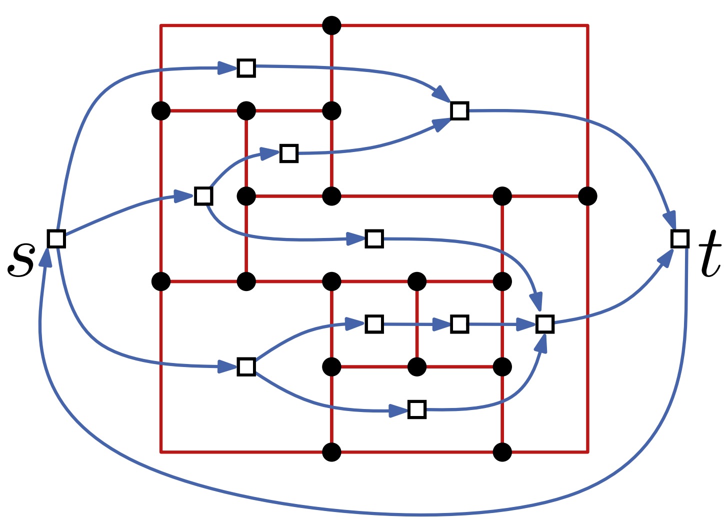
s and t represent lower and upper side of f_0



Flow Network for Edge Length Computation

Def: Flow Network $N_{\text{ver}} = ((W_{\text{ver}}, A_{\text{ver}}); \ell; u; b; \text{cost})$

- $W_{\text{ver}} = \mathcal{F} \setminus \{f_0\} \cup \{s, t\}$
- $A_{\text{ver}} = \{(f, g) \mid f, g$ share a vertical segment and f lies to the left of $g\} \cup \{(t, s)\}$
- $\ell(a) = 1 \quad \forall a \in A_{\text{ver}}$
- $u(a) = \infty \quad \forall a \in A_{\text{ver}}$
- $\text{cost}(a) = 1 \quad \forall a \in A_{\text{ver}}$
- $b(f) = 0 \quad \forall f \in W_{\text{ver}}$

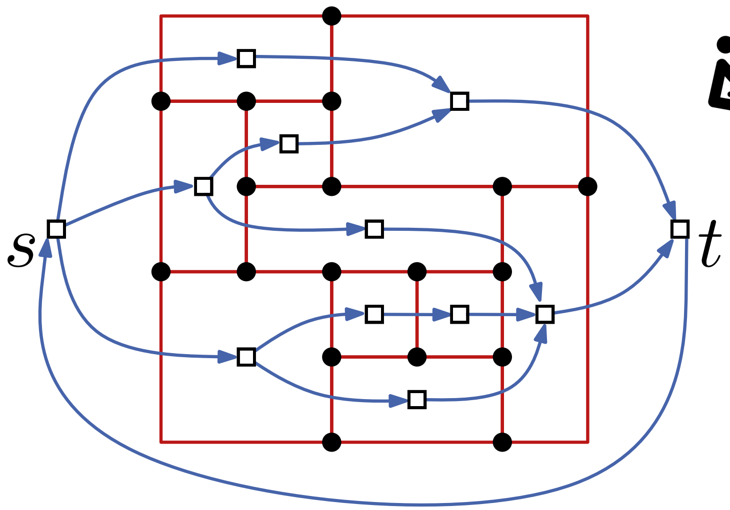


Flow Network for Edge Length Computation

Karlsruhe Institute of Technology

Def: Flow Network $N_{\text{ver}} = ((W_{\text{ver}}, A_{\text{ver}}); \ell; u; b; \text{cost})$

- $W_{\text{ver}} = \mathcal{F} \setminus \{f_0\} \cup \{s, t\}$
- $A_{\text{ver}} = \{(f, g) \mid f, g$ share a vertical segment and f lies to the left of $g\} \cup \{(t, s)\}$
- $\ell(a) = 1 \quad \forall a \in A_{\text{ver}}$
- $u(a) = \infty \quad \forall a \in A_{\text{ver}}$
- $\text{cost}(a) = 1 \quad \forall a \in A_{\text{ver}}$
- $b(f) = 0 \quad \forall f \in W_{\text{ver}}$



Pair, think, share: **3 min**

What values of the drawing represent the following?

- $|X_{\text{hor}}(t, s)|$ and $|X_{\text{ver}}(t, s)|$?
- $\sum_{a \in A_{\text{hor}}} X_{\text{hor}}(a) + \sum_{a \in A_{\text{ver}}} X_{\text{ver}}(a)$