Exercise Sheet 6

Discussion: 5. February 2020

Exercise 1: Counting Crossings ⋆⋆

Prove the following lemmas.

Let $\pi : \{1, \dots, n\} \to \{1, \dots, n\}$ be a permutation. A pair (i, j) with $1 \leq i < j \leq n$ is an *inversion*, if $\pi(i) > \pi(j)$.

Lemma 1 *The number of inversions of a permutation π can be counted in $O(n \log n)$ time.*

Hint: Use an approach similar to merge sort.

Lemma 2 *Let Γ be a straight-line drawing of a bipartite graph $G = (A \dot\cup B, E)$ where the vertices of A and B are drawn on separate layers. Then the number of crossing in Γ can be counted in $O(|E| \log |V|)$ time.*

Can all crossings be reported in the same time?

Exercise 2: Crossings in Layered Layouts ⋆

Prove the following.

Lemma 3 *The barycenter heuristic computes an optimal solution of the one-sided crossing minimization problem, if the instance admits a planar drawing.*

Exercise 3: Feedback Arc Set ⋆⋆

In the lecture we introduced the Minimum Feedback Arc Set and the Minimum Feedback Set problems. Let $D = (V, A)$ be a directed graph and A' be a subset of A. The set A' is a *feedback arc set of* D if $D_f = (V, A \setminus A')$ is acyclic. If $D_r = (V, (A \setminus A') \cup \{vu \mid uv \in A'\})$ is acyclic, then A' is a *feedback set of* D. Every feedback set is a feedback arc set, the reverse is not necessarily true. Prove the following.

Lemma 4 *A set $A' \subset A$ is a minimum feedback arc set of D if and only if A' is a minimum feedback set of D.*

Exercise 4: Contact Representation of Maximal Planar Graphs ⋆⋆

The figure below gives an example of contact representation of a planar graph with T-shapes. Prove the following Lemma.

Lemma 5 *Every maximal planar graph admits a contact representation with T-shapes.*

Hint: Use canonical ordering in the way similar to the construction of a visibility representation.