Exercise Sheet 3

Discussion: 27. November 2018

Exercise 1: Visibility Representation of Maximal Planar Graphs ⋆

Recall the definition of *visibility representation* from the previous exercise set.

Lemma 1 *Every maximal planar graph admits a visibility representation.*

Hint: Use canonical ordering.

Exercise 2: Barycentric Representation ⋆⋆

A *Barycentric Representation* of a graph $G = (V, E)$ is an assignment of barycentric coordinates to the vertices of G, i.e., it is an *injective* function $f : V \to \mathbb{R}^3, v \mapsto (v_a, v_b, v_c)$, such that:

- $v_a + v_b + v_c = 1$ for all $v \in V$
- for each $(x, y) \in E$ and each vertex $z \in V \setminus \{x, y\}$ there is an index $k \in \{a, b, c\}$ such that $\max\{x_k, y_k\} < z_k$.

Lemma 2 *Let f be a barycentric representation of a planar graph G and let a, b, c be three non-collinear points in the plane. The straight-line drawing Γ_f of G obtained by placing every vertex v at $av_a + bv_b + cv_c$ is planar.*

Exercise 3: Linear Time Construction of a Schnyder Realizer ⋆

Lemma 3 *Let G be a maximal planar graph with n vertices. A Schnyder labeling and a Schnyder realizer of G can be constructed in $O(n)$ time.*

Hint: Find a connection between a canonical ordering and the ordering in which the edge contraction for the construction of a Schnyder labeling is executed.

Exercise 4: Induced Path in a Schnyder Realizer ⋆⋆

A path of a graph G is called *induced* if the vertices of this path are connected only by the edges of the path, i.e., path on vertices $v_1, \ldots v_k$ is *induced* if for any $1 \leq i, j \leq n$ such that $|i-j| > 1$, edge (v_i, v_j) does not belong to G.

Lemma 4 *Let G be a maximal planar graph and let T_a, T_b, T_c be a Schnyder realizer of G. Assume that the edges of T_a, T_b, T_c are colored red, blue and green, respectively. A directed monochromatic path in T_a, T_b, T_c is an induced path of G.*