Exercise 1: Visibility Representation of Maximal Planar Graphs

Recall the definition of visibility representation from the previous exercise set.

Lemma 1 Every maximal planar graph admits a visibility representation.

Hint: Use canonical ordering.

Exercise 2: Barycentric Representation

A Barycentric Representation of a graph $G = (V, E)$ is an assignment of barycentric coordinates to the vertices of $G$, i.e., it is an injective function $f: V \rightarrow \mathbb{R}^3, v \mapsto (v_a, v_b, v_c)$, such that:

- $v_a + v_b + v_c = 1$ for all $v \in V$
- for each $(x, y) \in E$ and each vertex $z \in V \setminus \{x, y\}$ there is an index $k \in \{a, b, c\}$ such that $\max\{x_k, y_k\} < z_k$.

Lemma 2 Let $f$ be a barycentric representation of a planar graph $G$ and let $a, b, c$ be three non-collinear points in the plane. The straight-line drawing $\Gamma_f$ of $G$ obtained by placing every vertex $v$ at $av_a + bv_b + cv_c$ is planar.

Exercise 3: Linear Time Construction of a Schnyder Realizer

Lemma 3 Let $G$ be a maximal planar graph with $n$ vertices. A Schnyder labeling and a Schnyder realizer of $G$ can be constructed in $O(n)$ time.

Hint: Find a connection between a canonical ordering and the ordering in which the edge contraction for the construction of a Schnyder labeling is executed.
Exercise 4: Induced Path in a Schnyder Realizer

A path of a graph $G$ is called induced if the vertices of this path are connected only by the edges of the path, i.e., path on vertices $v_1, \ldots, v_k$ is induced if for any $1 \leq i, j \leq n$ such that $|i - j| > 1$, edge $(v_i, v_j)$ does not belong to $G$.

**Lemma 4** Let $G$ be a maximal planar graph and let $T_a, T_b, T_c$ be a Schnyder realizer of $G$. Assume that the edges of $T_a, T_b, T_c$ are colored red, blue and green, respectively. A directed monochromatic path in $T_a, T_b, T_c$ is an induced path of $G$. 