Exercise Sheet 1

Discussion: 22. October 2019

Exercise 1: Tree Layouts ⋆⋆

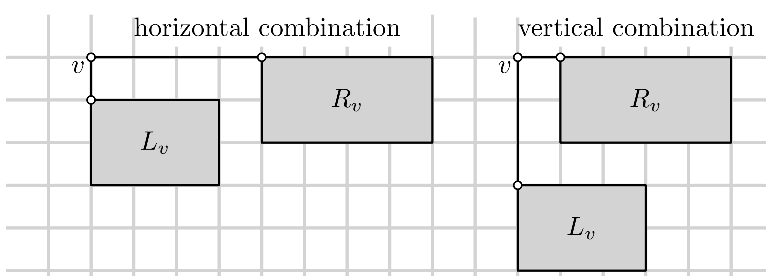
Let $T = (V, E)$ be a rooted binary tree. For a vertex $v \in V$, we denote its x-coordinate by $x(v)$ and its y-coordinate by $y(v)$.

(a) We draw the tree T as follows. For each vertex v of T, we set $x(v)$ equal to the rank of v in a post-order traversal of T, and $y(v)$ equal to its depth in T.
   (i) Show that the resulting straight-line drawing is planar.
   (ii) What is the area of the drawing?
   (iii) What happens if instead of a post-order traversal we use a pre-order traversal?
   (iv) Can the algorithm be extended to rooted ordered trees?

(b) We draw the tree T as follows. For each vertex v of T, we set $x(v)$ equal to the rank of v in a pre-order traversal of T, and $y(v)$ equal to the rank of v in a post-order traversal of T.
   (i) Show that the resulting straight-line drawing is planar and *strictly downward* (for each edge (u, v), with $depth(u) < depth(v)$, it holds that $y(u) > y(v)$).
   (ii) Show that a vertex v is in the subtree rooted at vertex u if and only if $x(v) > x(u)$ and $y(v) < y(u)$.
   (iii) Do isomorphic subtrees have congruent drawings?

Exercise 2: HV-Layouts ⋆⋆

An *HV-Layout* of a tree $T = (V, E)$ is a drawing on a grid where each vertex is on a grid point and each edge is either horizontal (H) or vertical (V). Moreover, for a vertex v, drawings of the two subtrees L_v and R_v are position according to one of the following two construction rules.



Give an algorithm that for a given n-vertex binary tree constructs an HV-layout with minimum area in $O(n^2)$ time. Consider both ordered an non-ordered trees.

Exercise 3: Layouts of General Trees ⋆

Let $T = (V, E)$ be an arbitrary rooted tree (i.e., not necessarily binary). Prove that a planar straight-line drawing Γ of T such that siblings (vertices with the same parent) have the same y-coordinate, parent-vertices are centered with respect to their children, and the area of Γ is in $O(n^2)$, can be computed in $O(n)$ time.

Exercise 4: Minimal-Width Layout ⋆

Let $T = (V, E)$ be a rooted binary tree with a BFS-ordering and let $\text{depth}(v)$ be the respective BFS-level of a vertex $v \in V$. Formulate a linear program (LP) that computes a planar straight-line drawing Γ of T with minimal width such that Γ respects the BFS-ordering, parent nodes are centered with respect to its children, and each vertex v has $-\text{depth}(v)$ as y-coordinate. Is the running time of the resulting algorithm polynomial in the size of T?