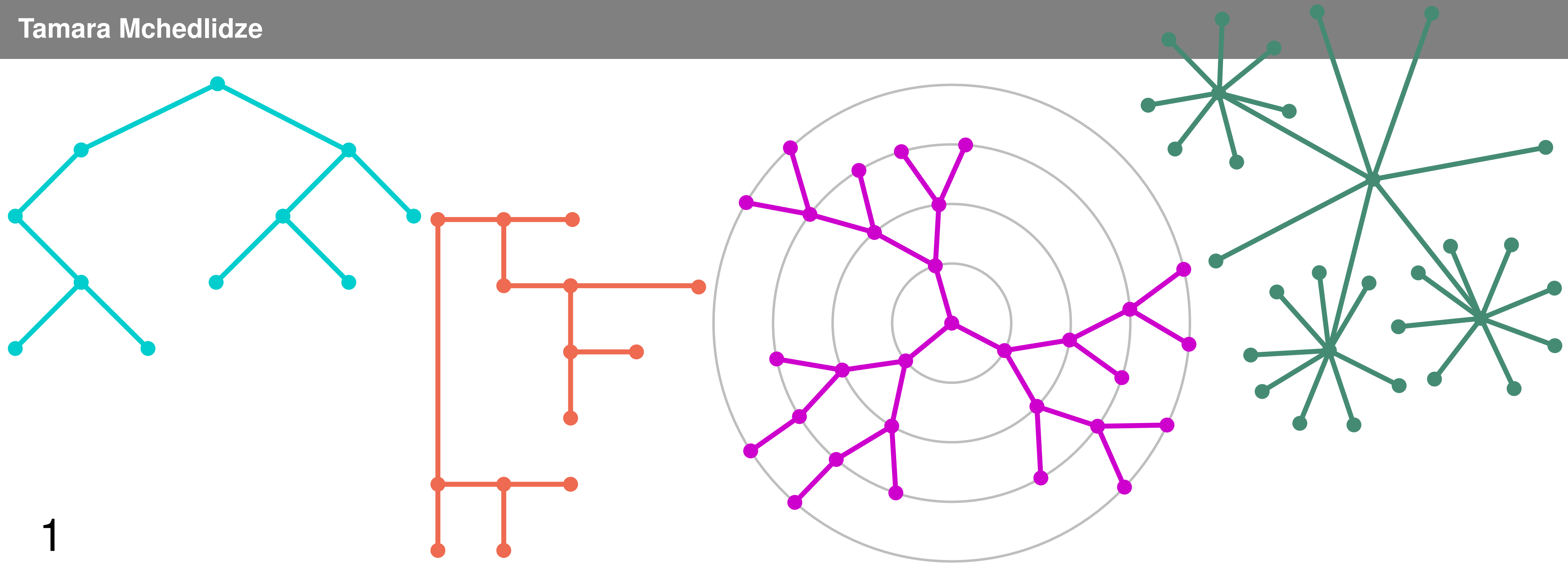
Algorithms for graph visualization

Divide and Conquer - Tree Layouts - Part 2

WINTER SEMESTER 2018/2019

Tamara Mchedlidze



1

Overview

- H(horizontal) V(vertical) tree layout algorithm
- Radial tree layout algorithm
- Other visualization styles

Applications

Cons cell diagram in LISP.
Cons(constructs) are memory objects which hold two values or pointers to values.

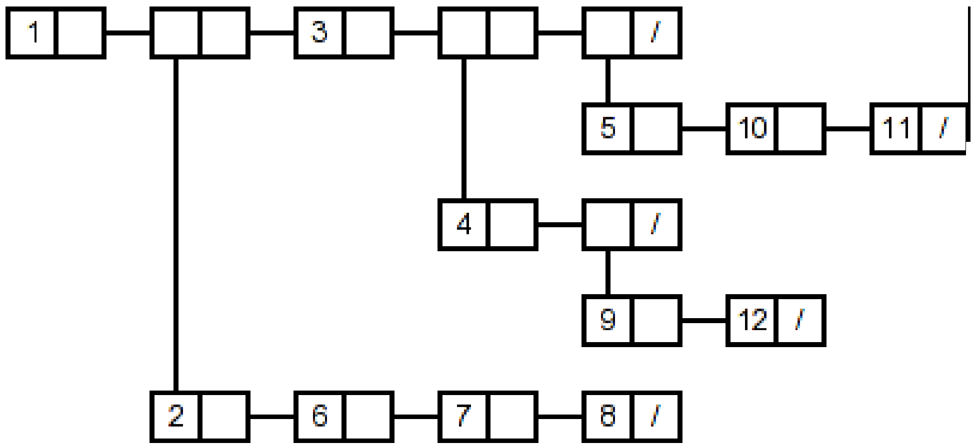


Figure 3: Diagram of cons cells of the simple tree. http://gajon.org/

Discuss with your neighbour(s) and then share

2+3 min

3 - 1

Applications

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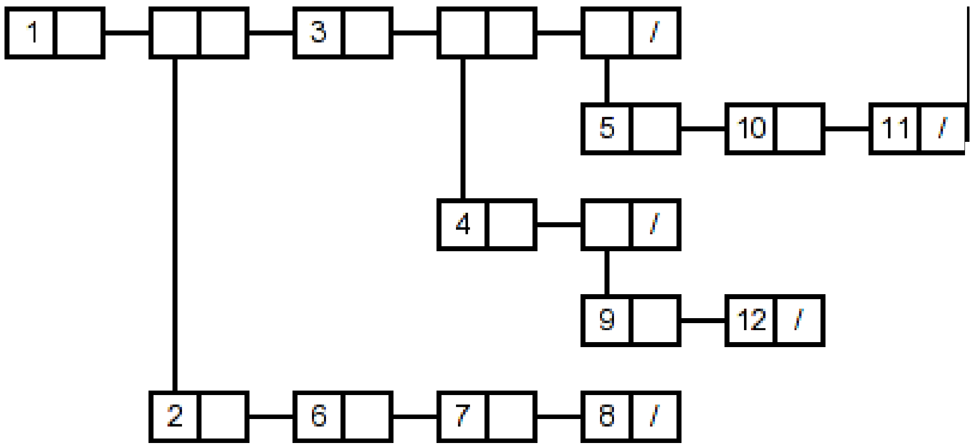


Figure 3: Diagram of cons cells of the simple tree. `http://gajon.org/`

3 - 2

Discuss with your neighbour(s) and then share

2+3 min

- What are the drawing conventions and aesthtetics?

HV-Layout

Drawing Conventions:

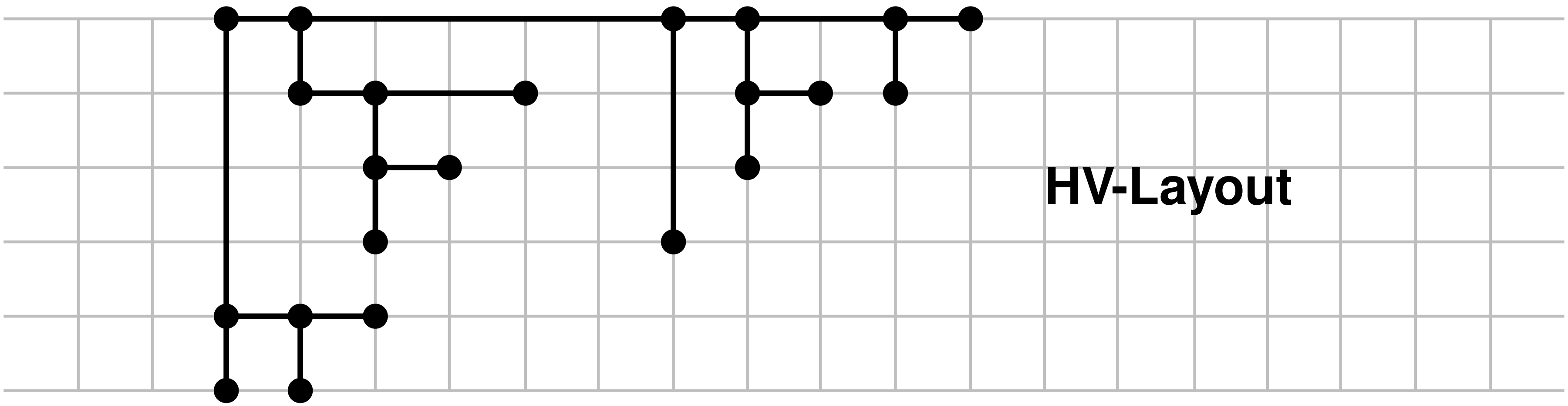
- Children are vertically and horizontally aligned with the root
- The bounding boxes of the children do not intersect

Drawing Aesthetics:

- Height, width, area

HV-Layout

Divide & Conquer Approach:

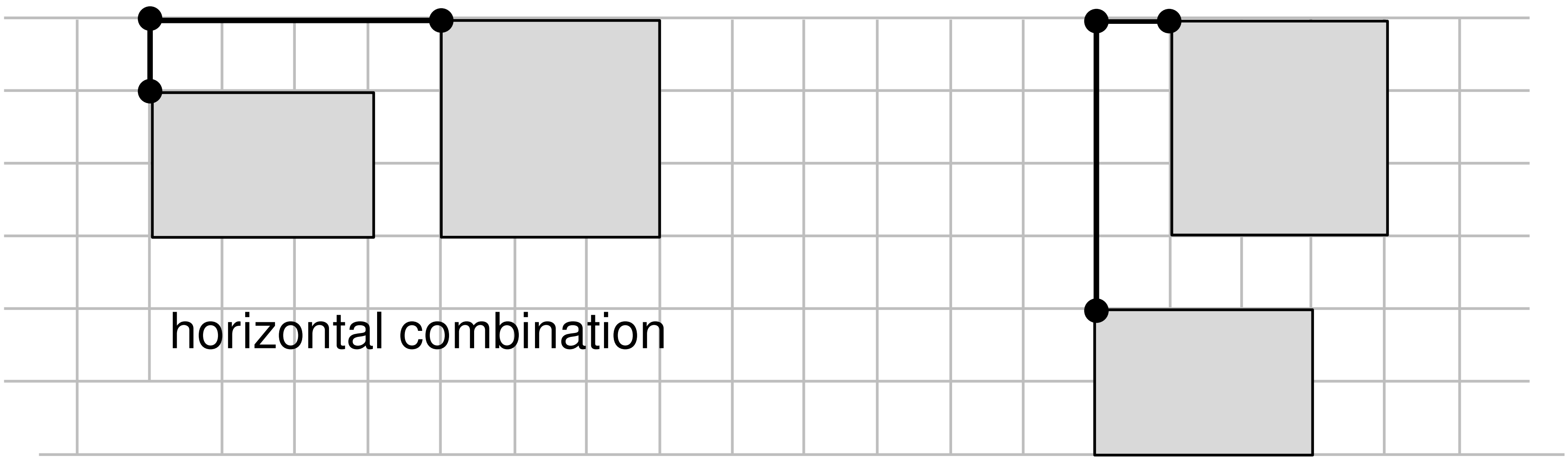


HV-Layout

HV-Layout

Induction base: 

Induction step: combine layouts



vertical combination

Right-Heavy HV-Layout

Right-Heavy approach:

- At every induction step apply horizontal combination
- Place the larger subtree to the right

Right-Heavy HV-Layout

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Lemma

Let T be a binary tree. The hight of the drawing constructed by Right-Heavy approach is at most $\log n$.

Right-Heavy HV-Layout

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Lemma

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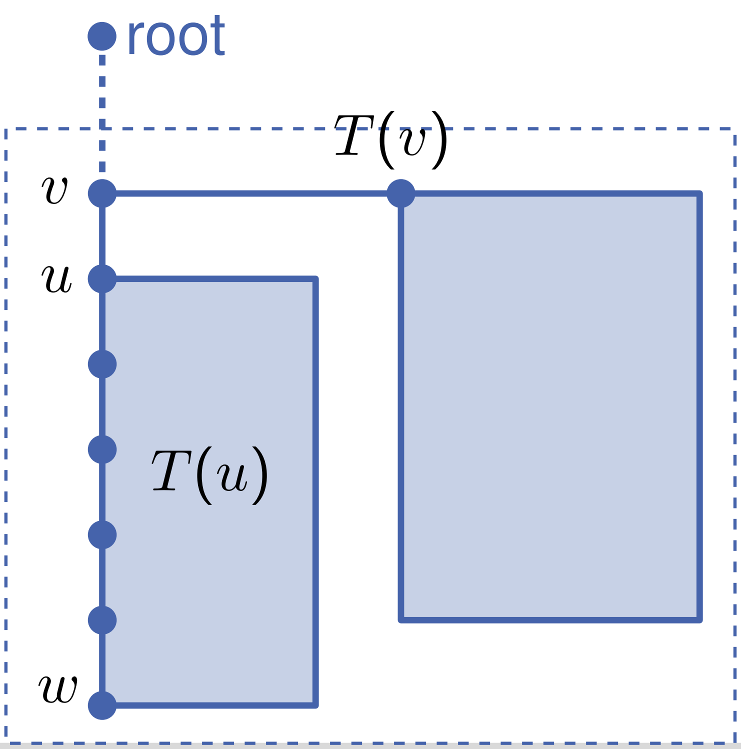
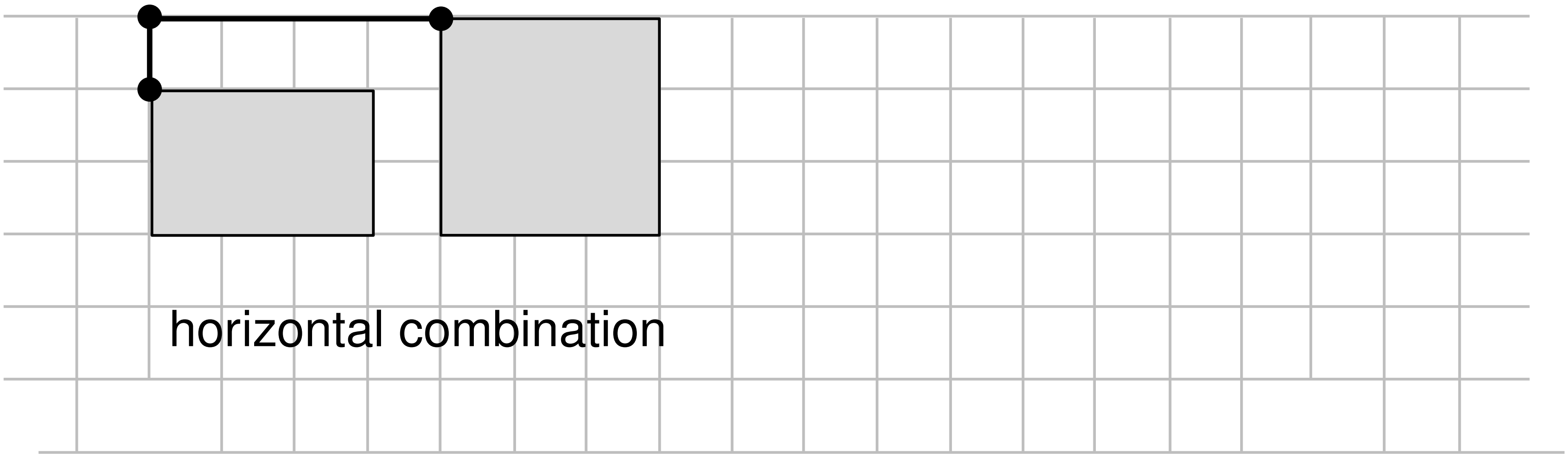


Proof:

- Each vertical edge has length 1
- Let w be the lowest node in the drawing
- Let P be a path from w to the root of T
- For every edge (u, v) in P: $|T(v)| > 2|T(u)|$
- $\Rightarrow P$ contains at most $\log n$ edges

Right-Heavy HV-Layout

- At every induction step apply horizontal combination
- Place the larger subtree to the right



Think:

- What are the implementational details of the algorithm? How to compute the coordinates? Can we do it in $O(n)$ time?

Right-Heavy HV-Layout

Theorem

Let T be a binary tree with n vertices. The Right-Heavy algorithm constructs in $O(n)$ time a drawing Γ of T such that:

Right-Heavy HV-Layout

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Let T be a binary tree with n vertices. The Right-Heavy algorithm constructs in $O(n)$ time a drawing Γ of T such that:

- Γ is HV-drawing (planar, orthogonal)

Right-Heavy HV-Layout

Karlsruhe Institute of Technology

Theorem

Let T be a binary tree with n vertices. The Right-Heavy algorithm constructs in $O(n)$ time a drawing Γ of T such that:

- Γ is HV-drawing (planar, orthogonal)
- The width of Γ is at most

Take a minute to think about the width of the layout

1 min

Right-Heavy HV-Layout

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Let T be a binary tree with n vertices. The Right-Heavy algorithm constructs in $O(n)$ time a drawing Γ of T such that:

- Γ is HV-drawing (planar, orthogonal)
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Right-Heavy HV-Layout

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- The area is $O(n \log n)$

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- Simply and axially isomorphic subtrees have congruent drawings, up to translation

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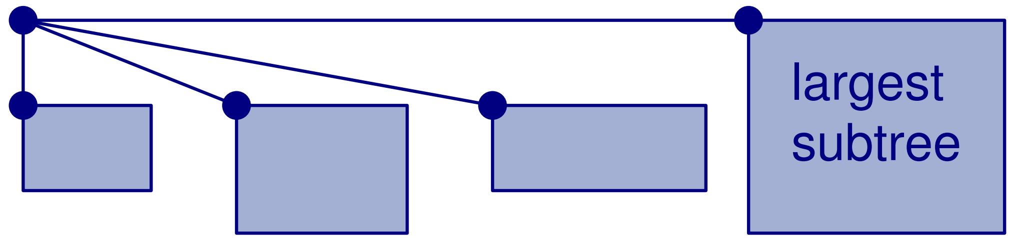
General rooted tree:



HV-Layout

Bad news We can not minimize the area by using divide & conquer approach

HV-Layout

Bad news We can not minimize the area by using divide & conquer approach

Good news We can compute minimum area using Dynamic Programming

HV-Layout

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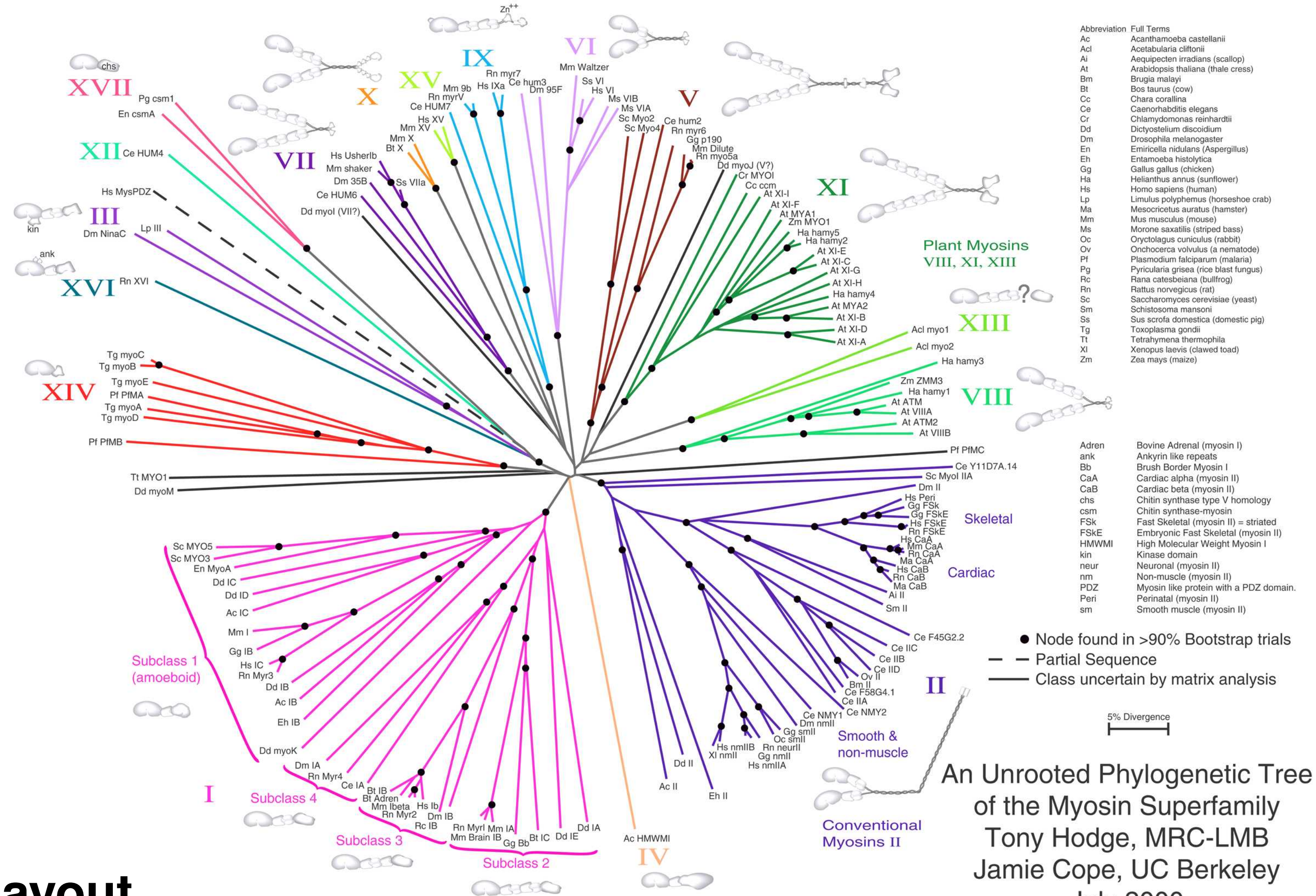
Good news We can compute minimum area using Dynamic Programming

HV-Layout for Trees

- Book Di Battista et al: Chapter 3.1.4
- Skript: page 86

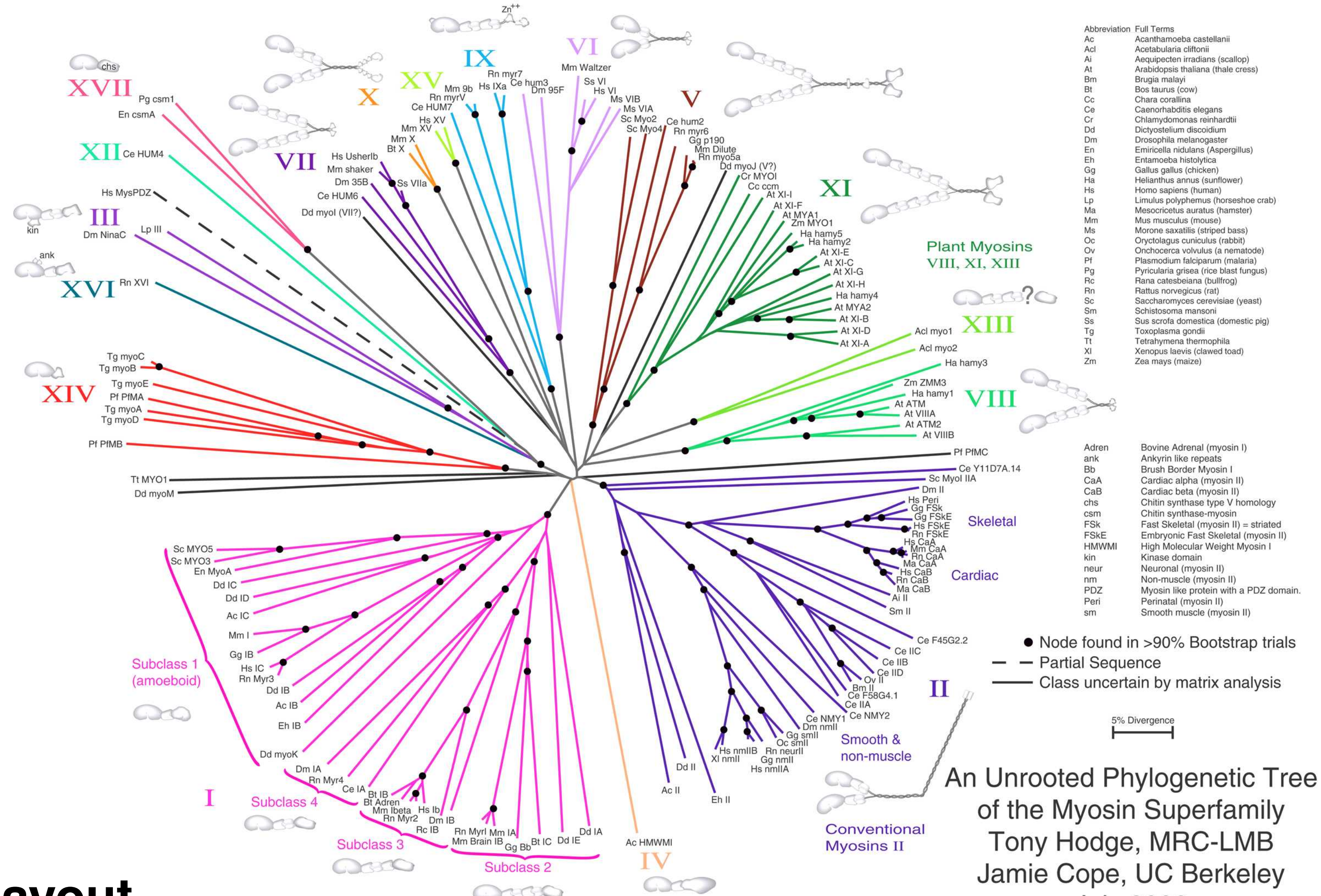
Applications



Radial layout

An unrooted phylogenetic tree for myosin, a superfamily of proteins.
”A myosin family tree“ *Journal of Cell Science*

Applications



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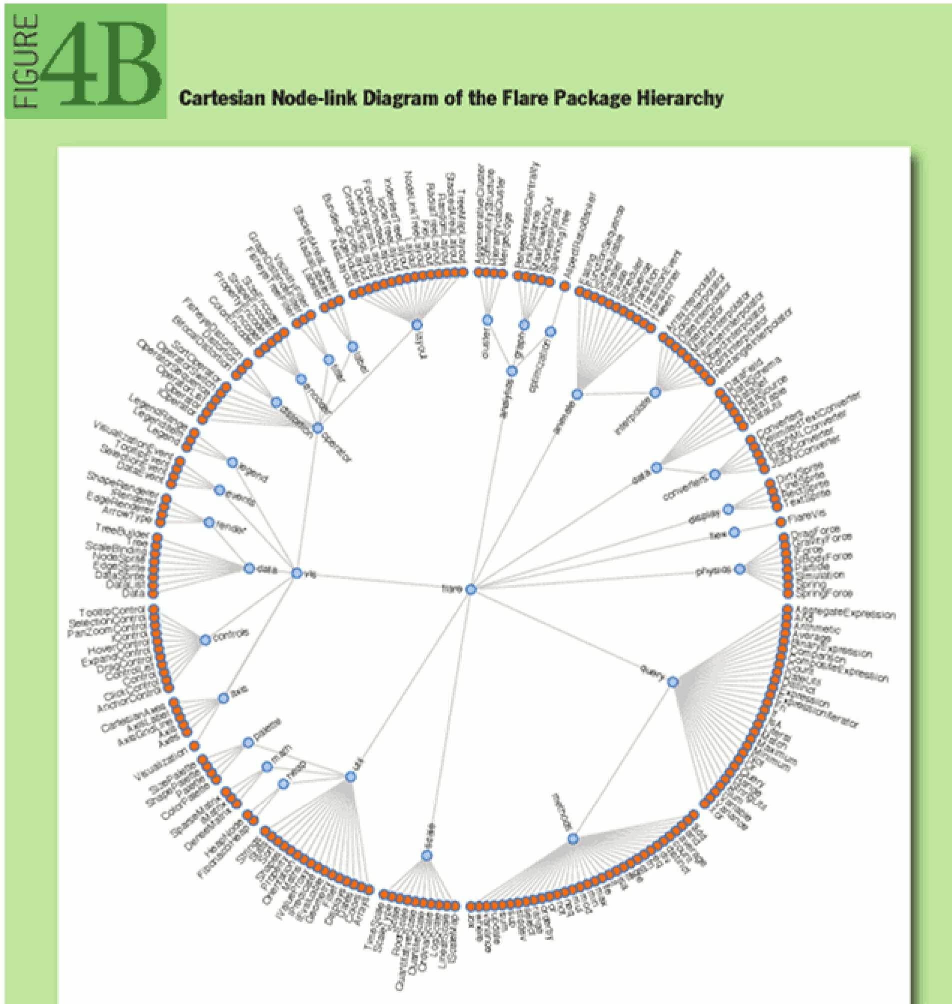
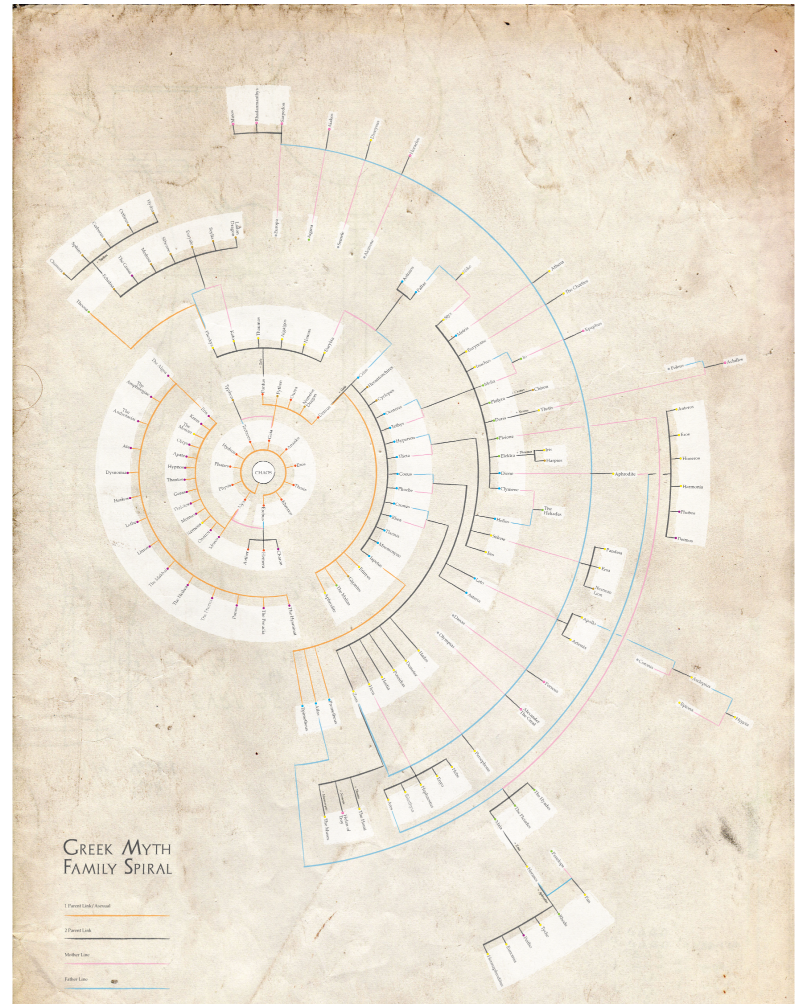
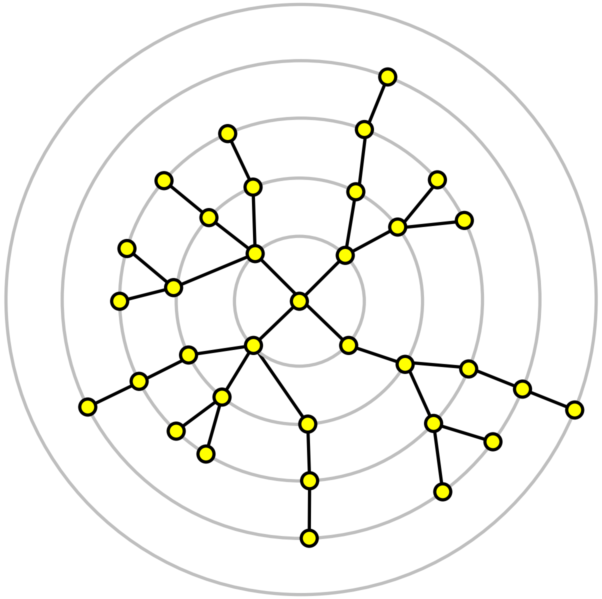


Flare Visualization Toolkit code structure by Heer, Bostock and Ogievetsky, 2010



Greek Myth Family by Ribecca, 2011

Radial Layout



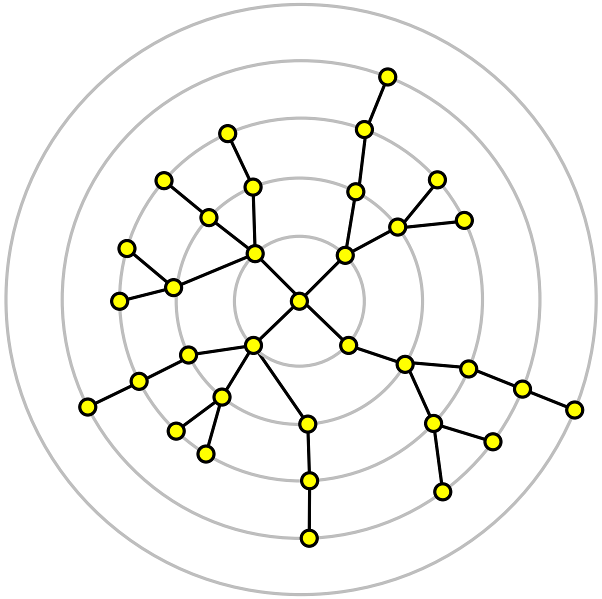
Drawing Conventions:

- Vertices lie on circular layers according to their depth
- Drawing is planar

Drawing Aesthetics:

- Distribution of the vertices

Radial Layout



Drawing Conventions:

- Vertices lie on circular layers according to their depth
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Drawing Aesthetics:

- Distribution of the vertices

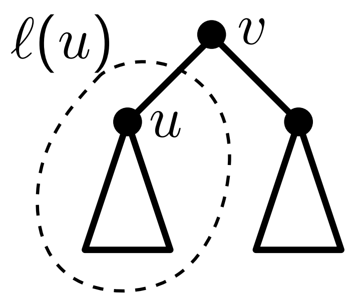
Take a minute to think about a possible algorithm to optimize the distribution of the vertices

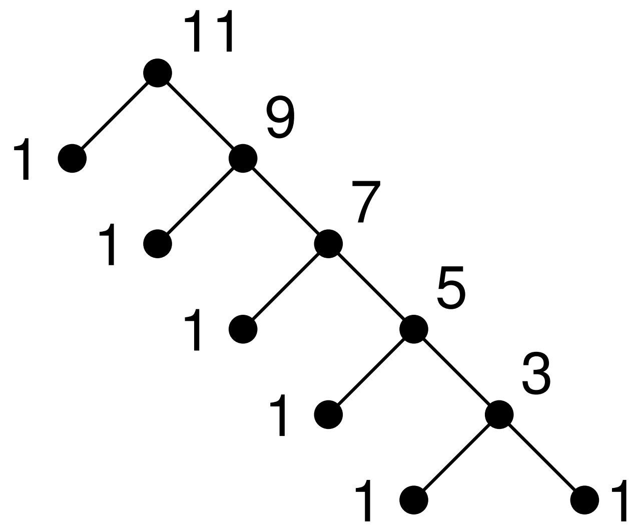
1 min

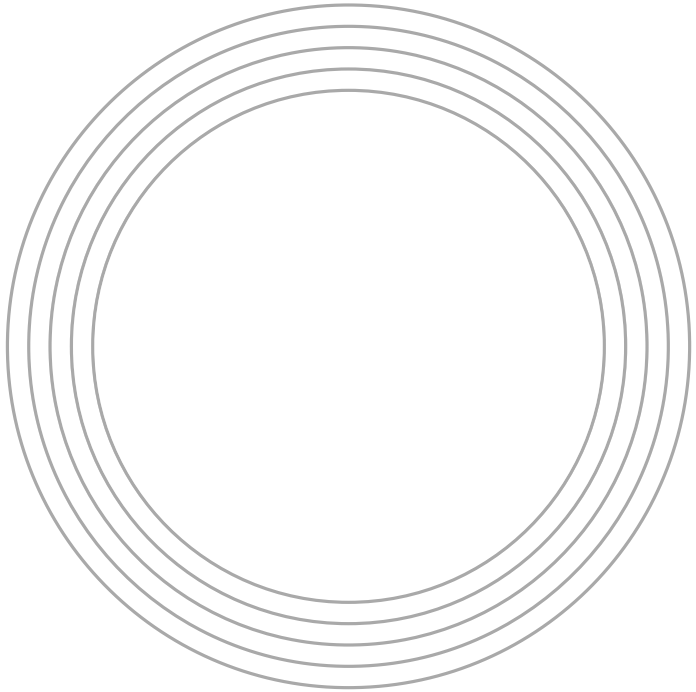
Radial Layout

Example:

- Angle corresponding to the subtree rooted at u: $\tau_u = \frac{\ell(u)}{\ell(v)-1}$







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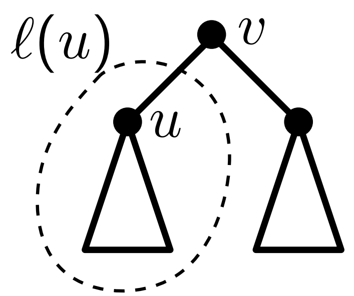

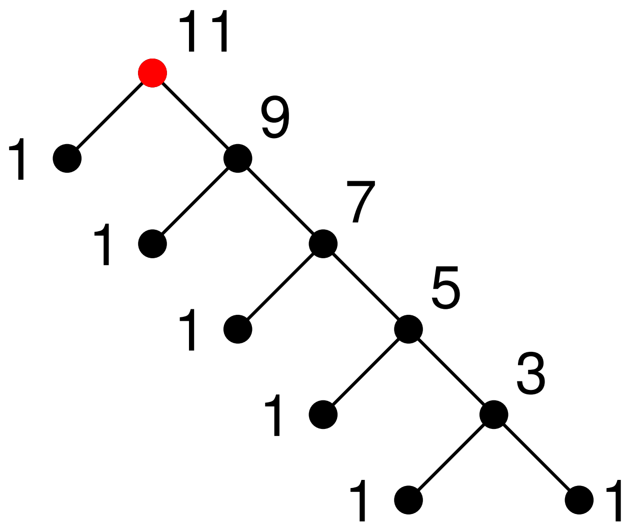

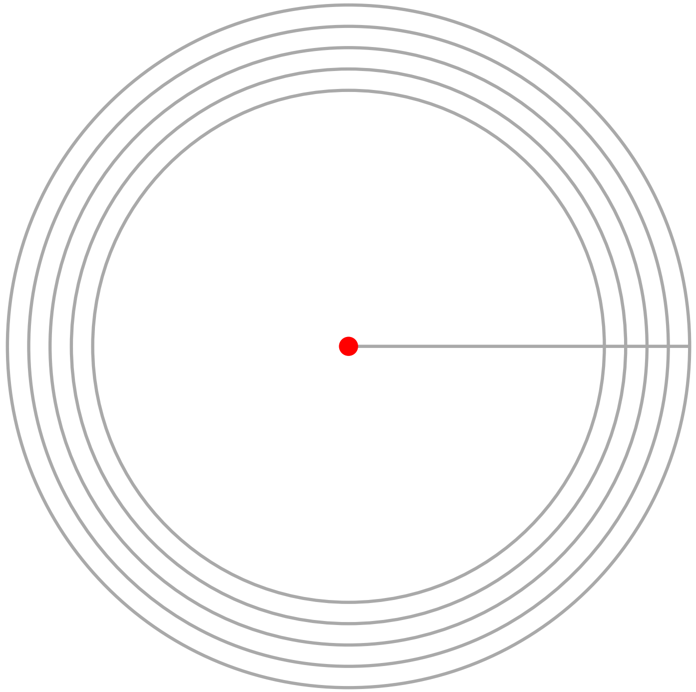

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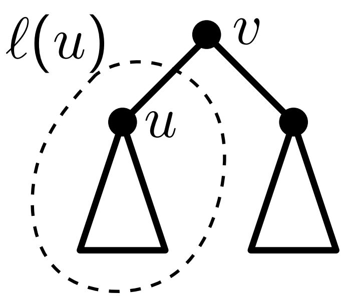

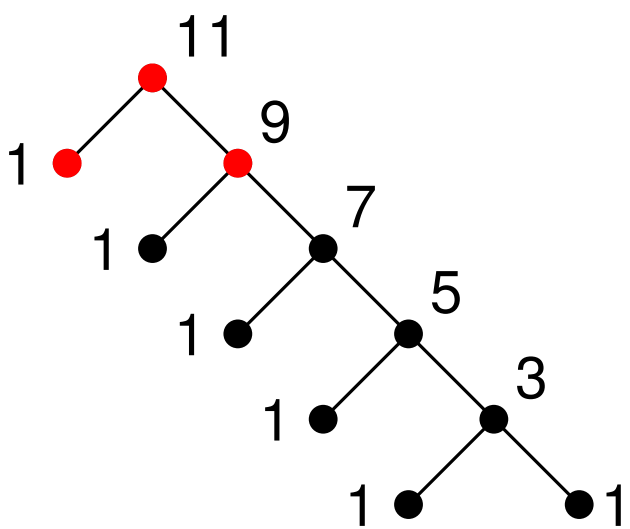

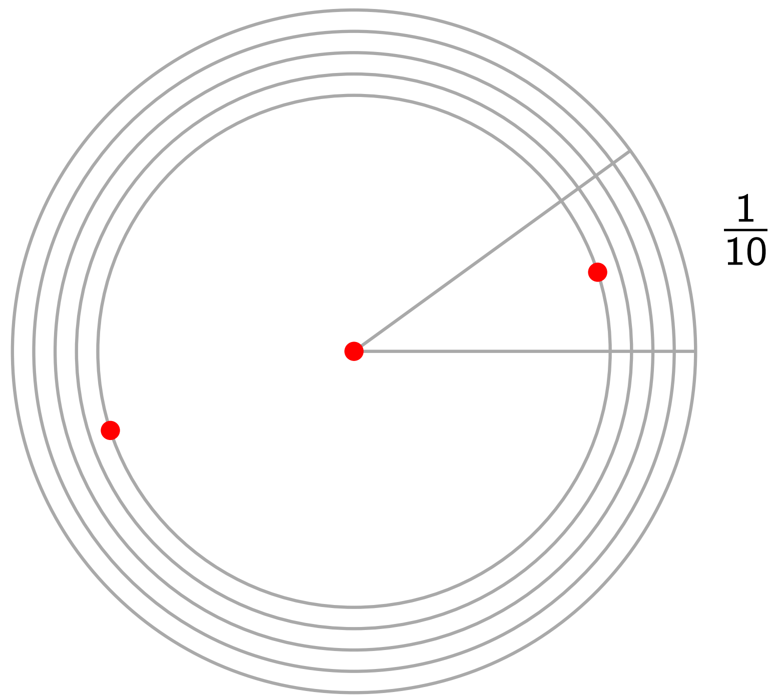

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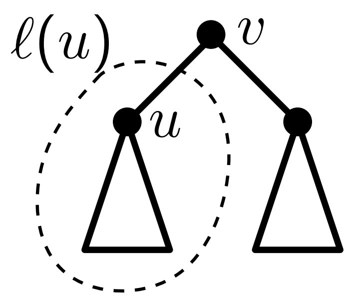

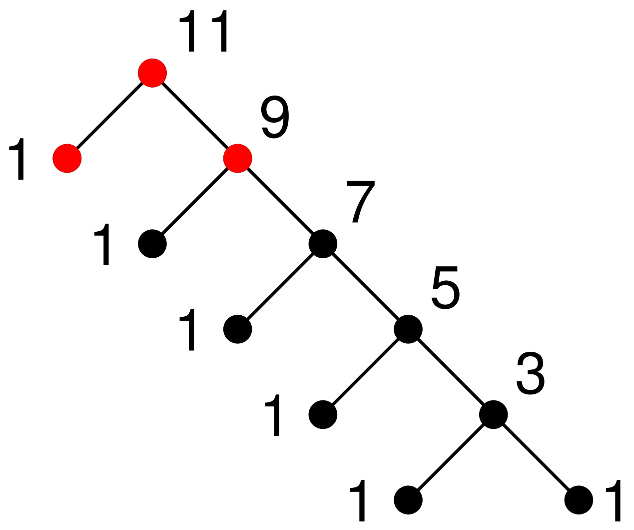

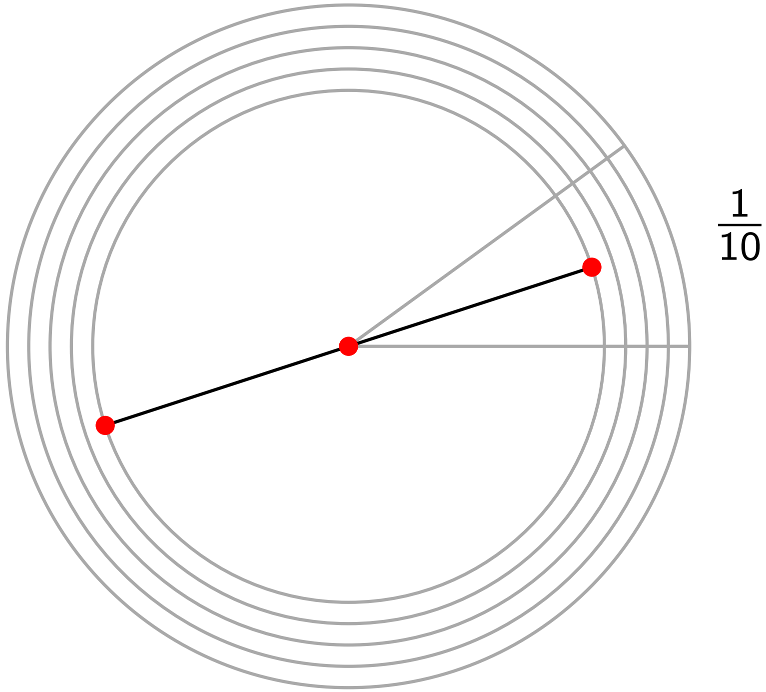

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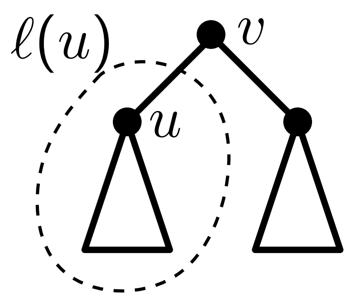

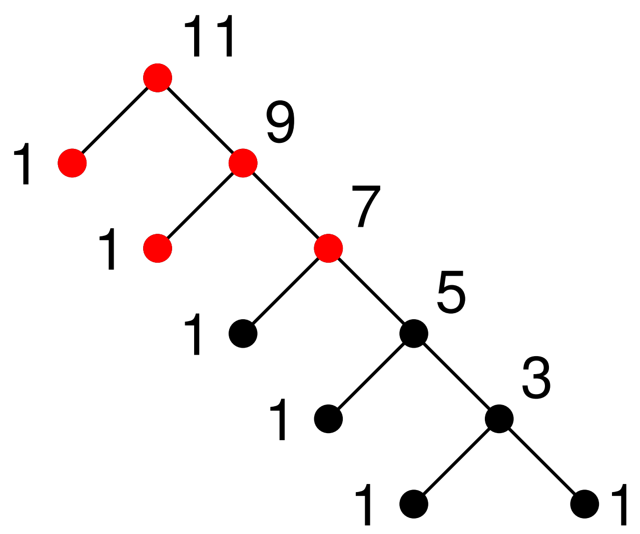

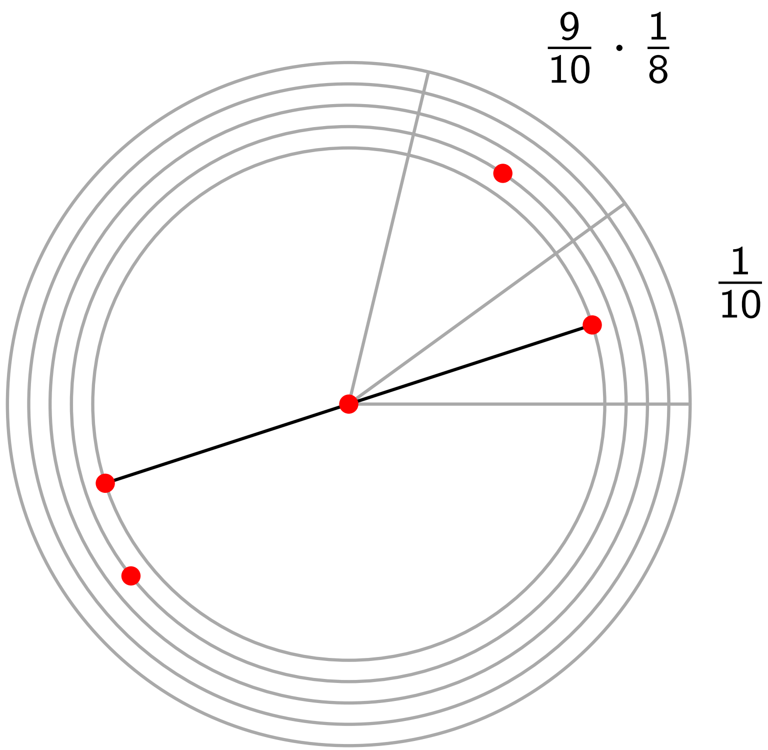

15 - 5

Radial Layout

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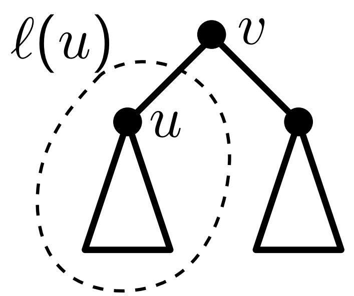

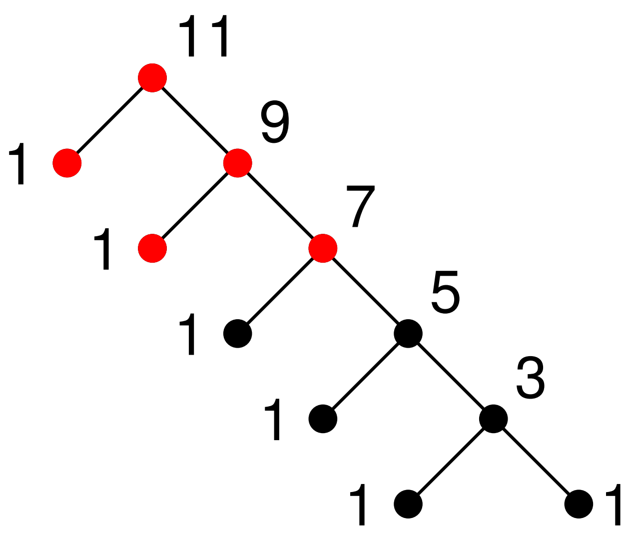

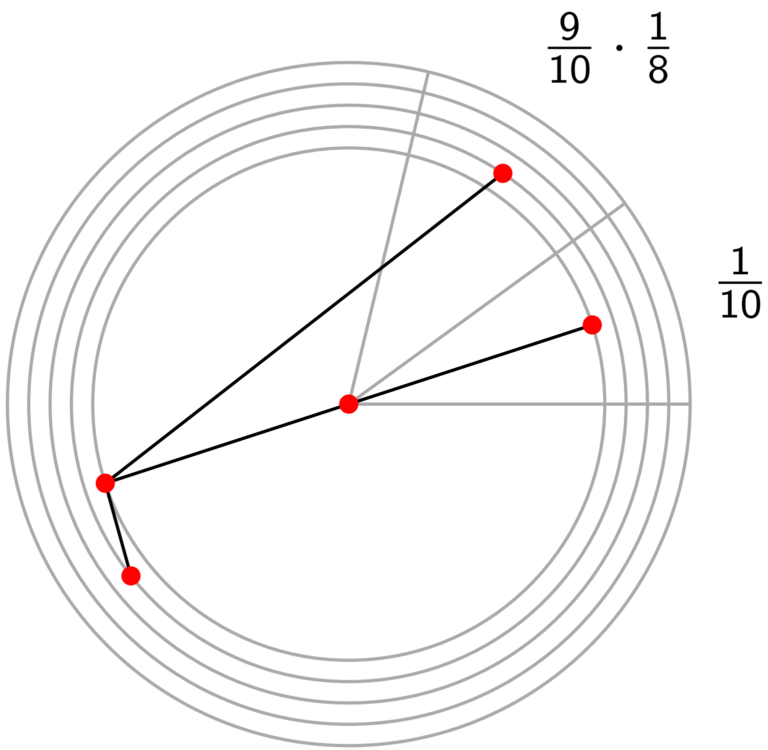

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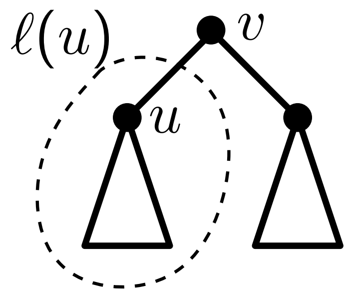

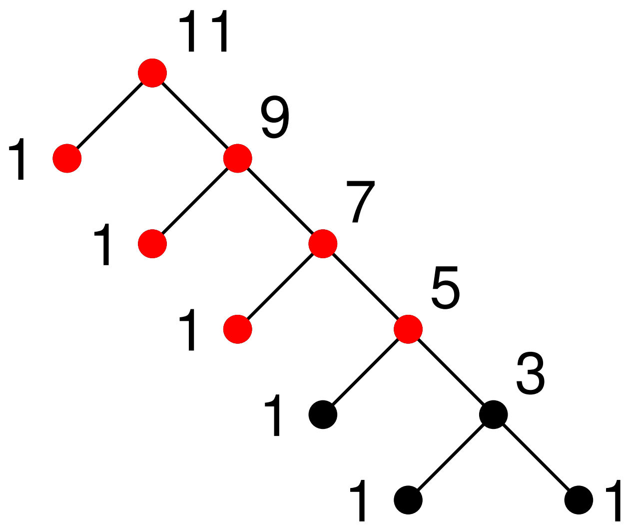

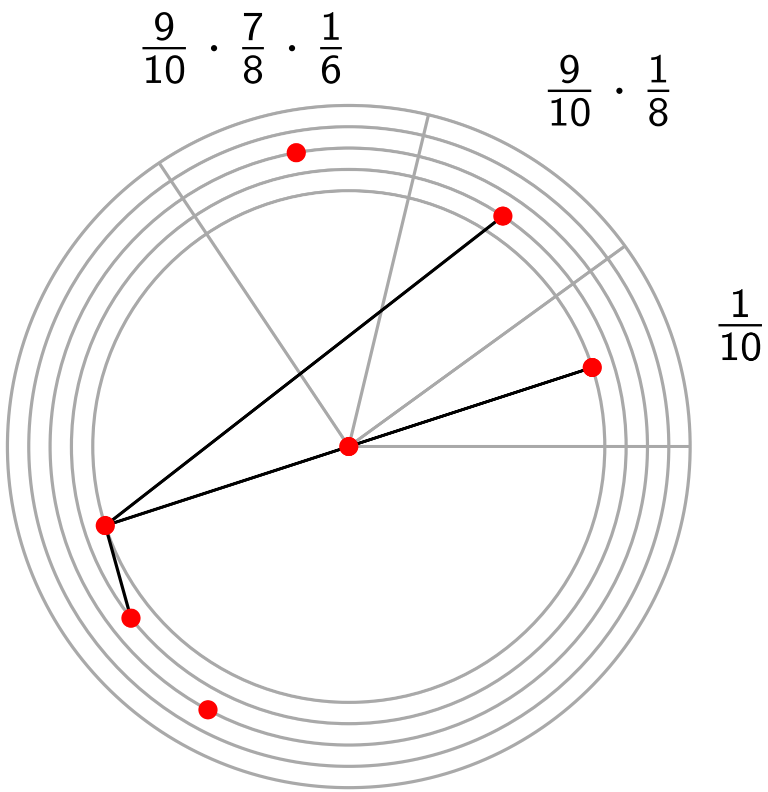

15 - 7

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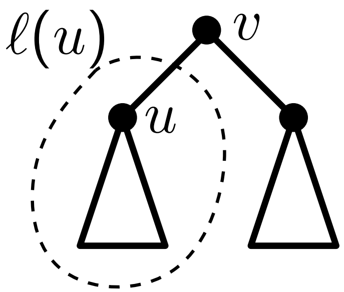
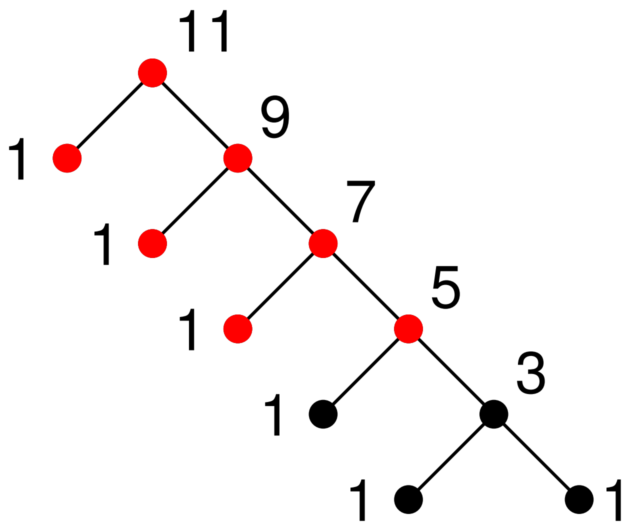
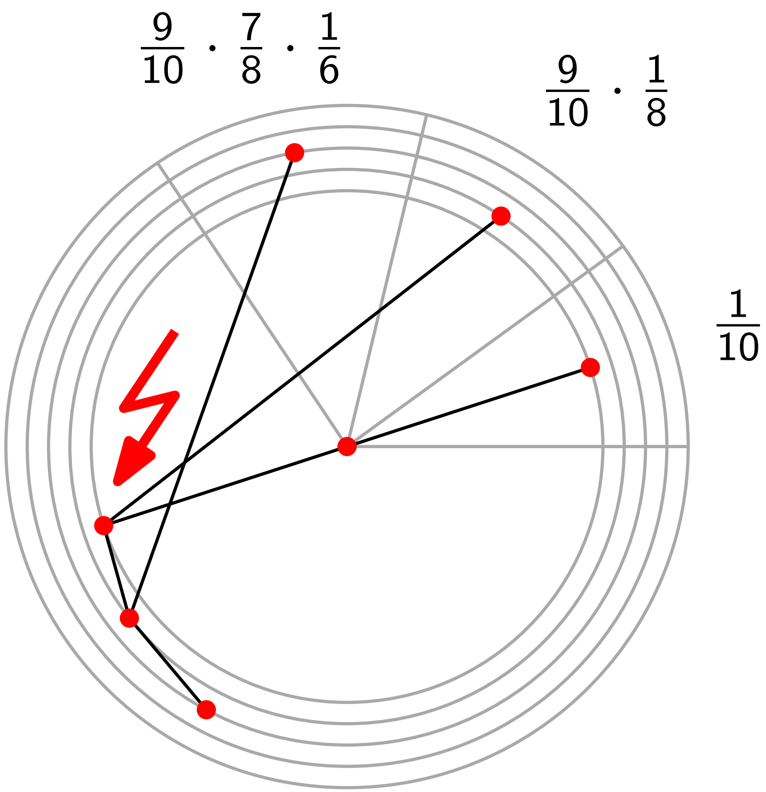


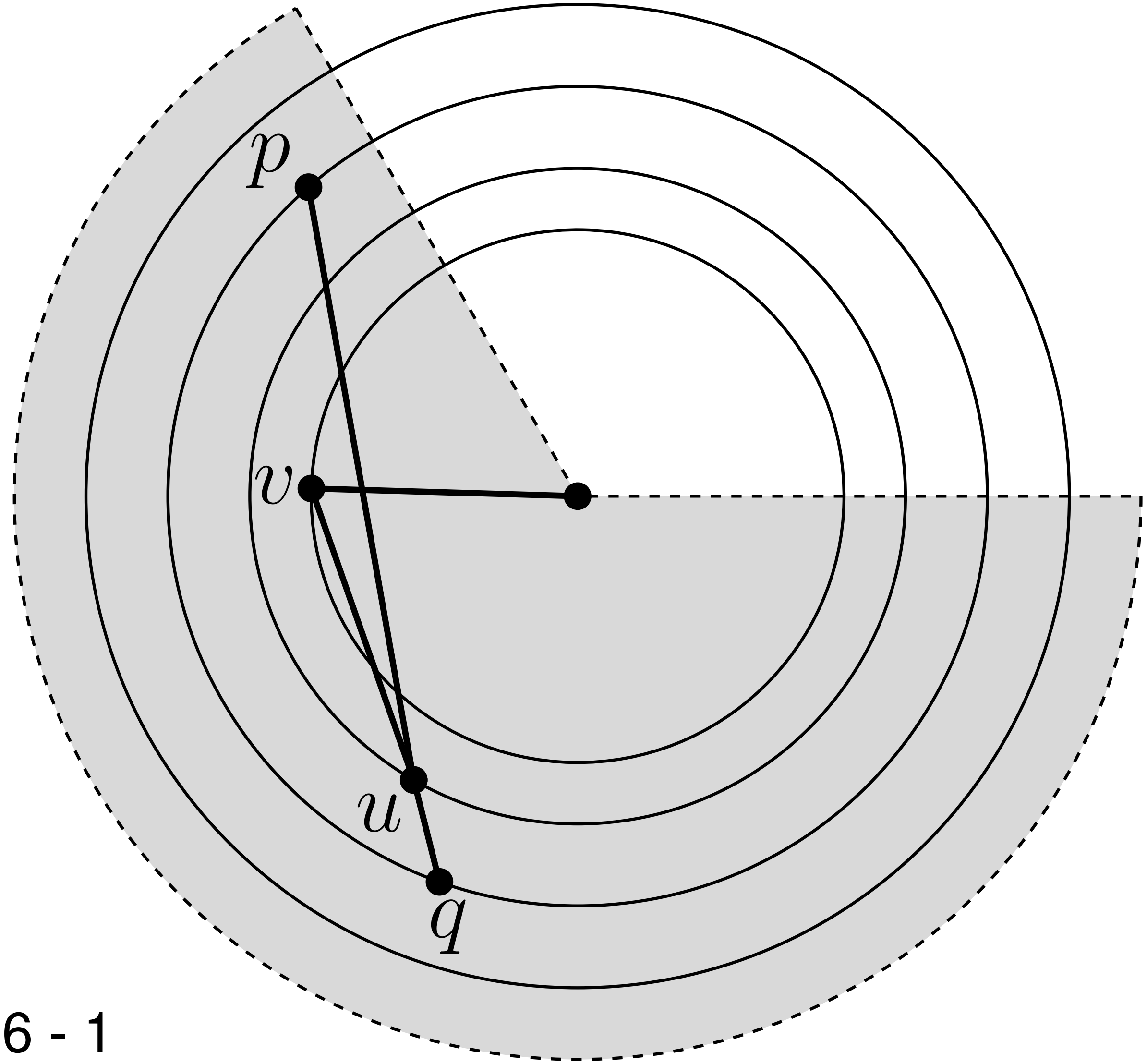




15 - 8

Radial Layout

How to avoid crossings:



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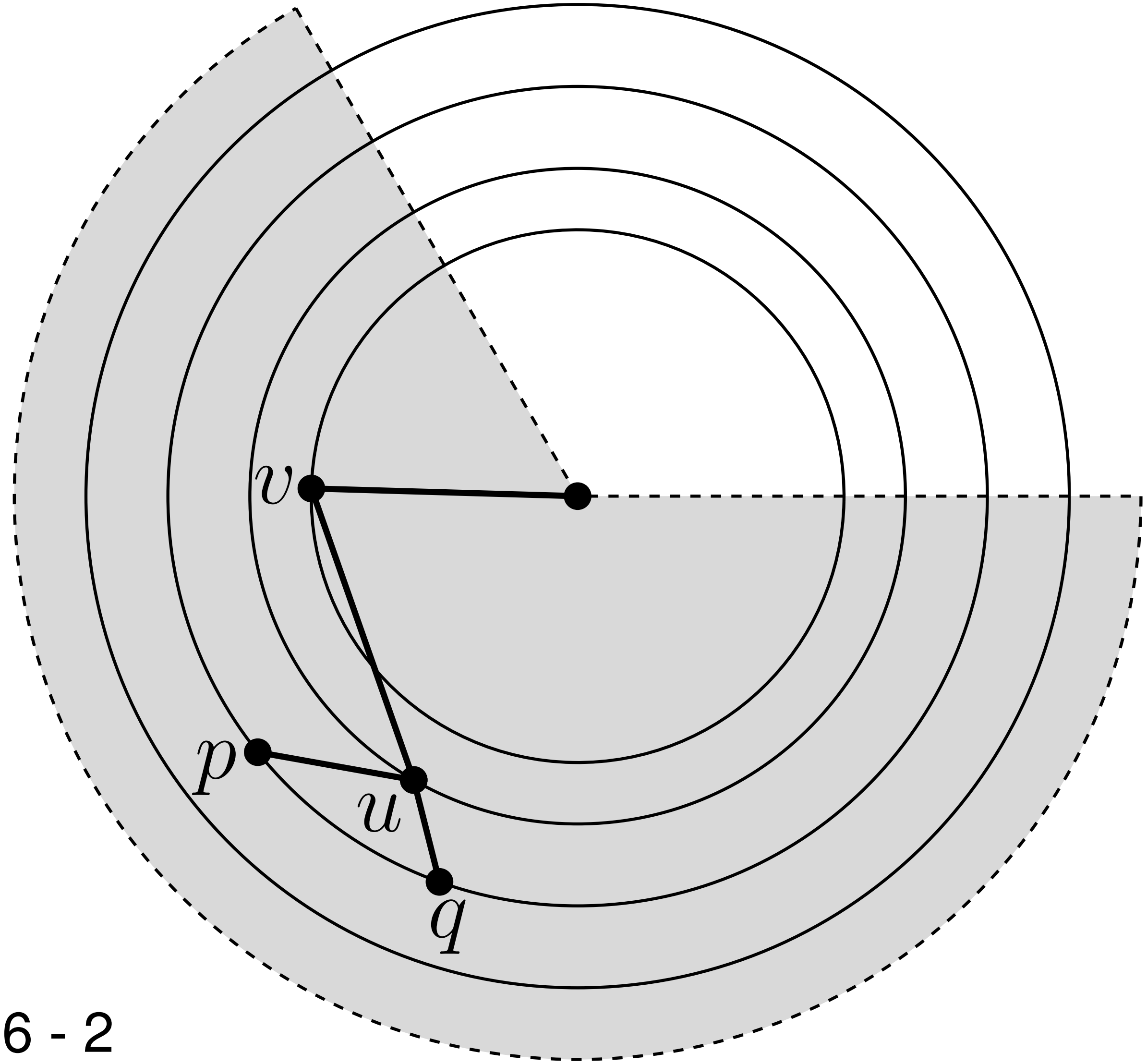

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Lehrstuhl Algorithmik I

Radial Layout

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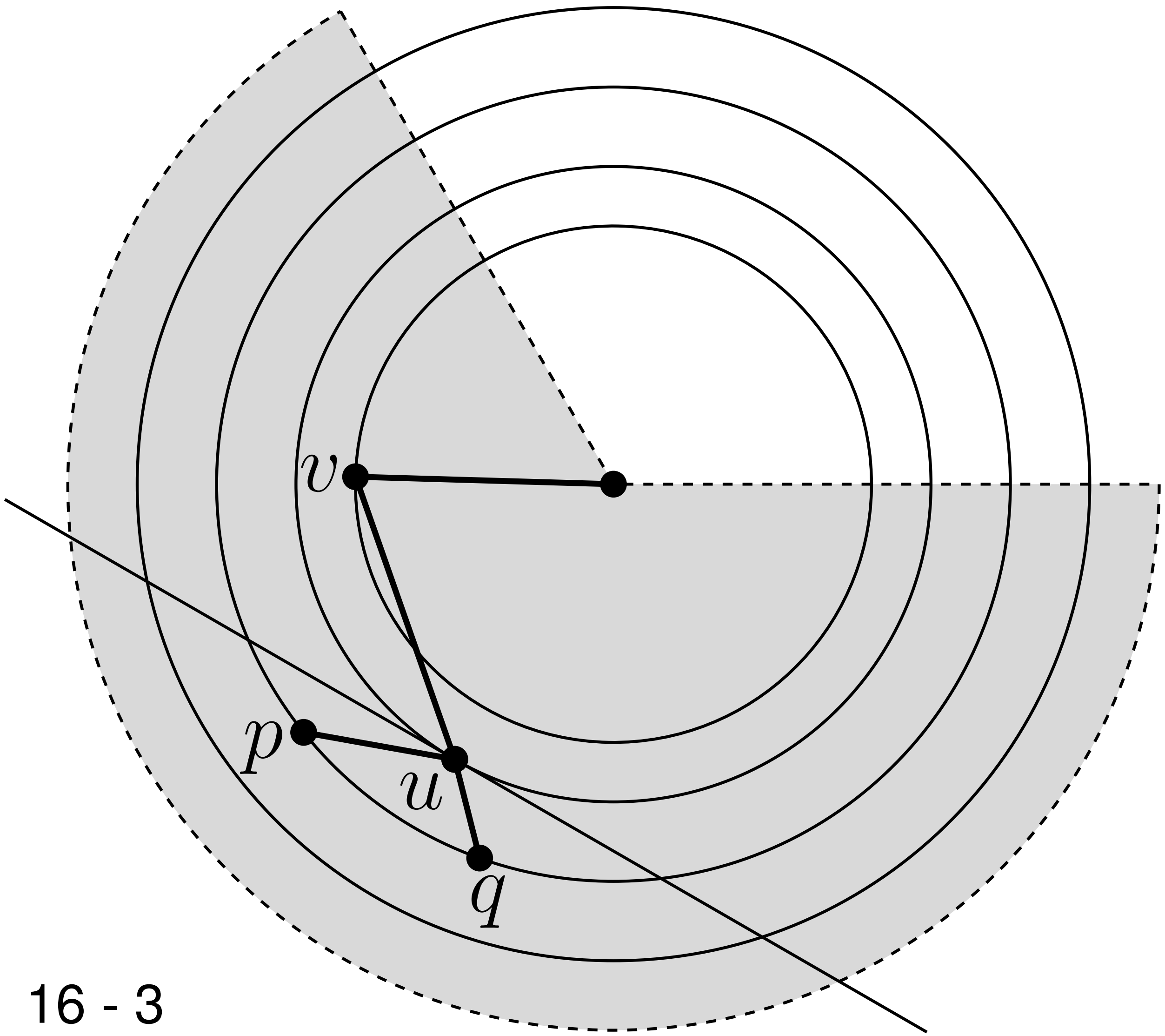

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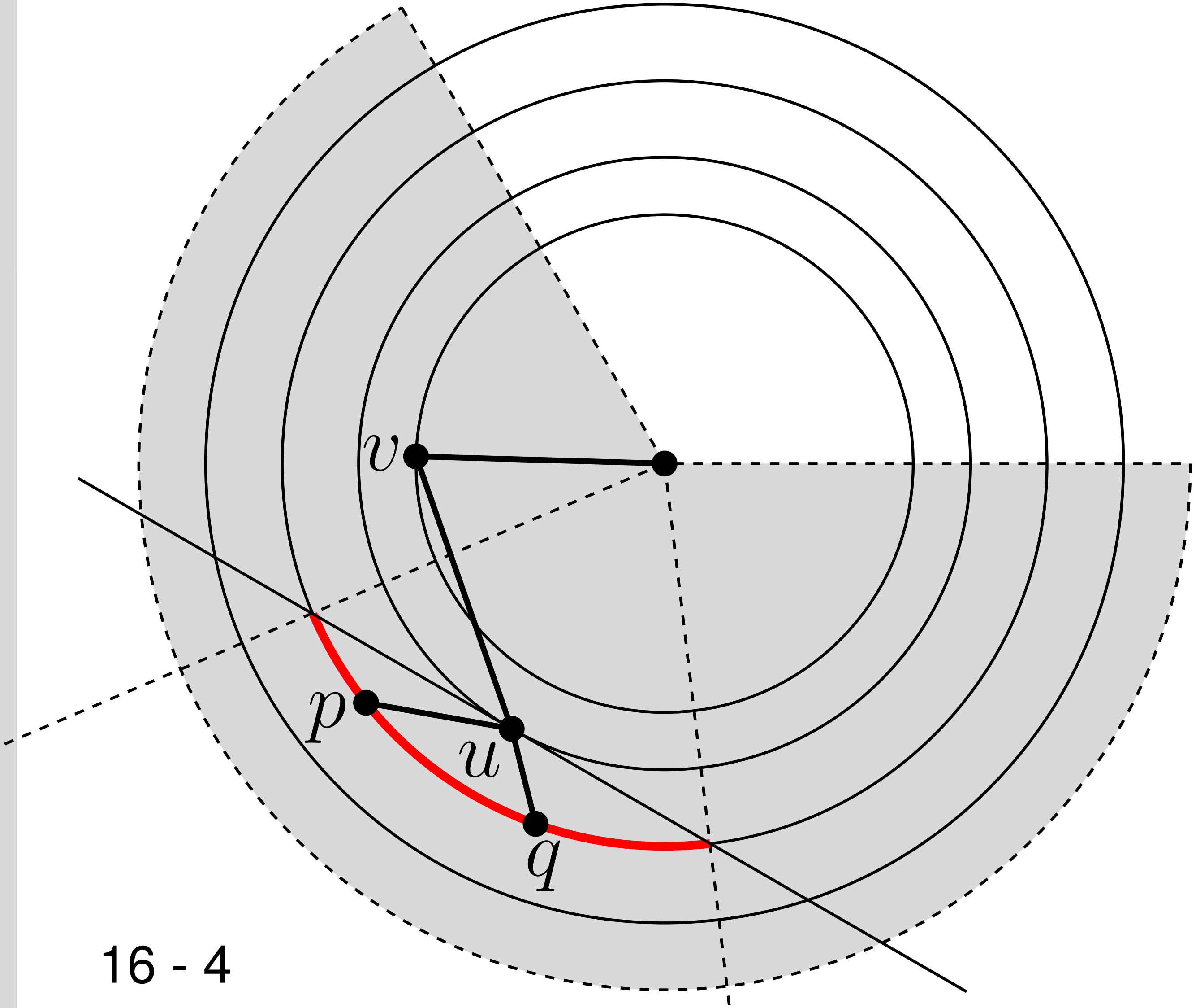

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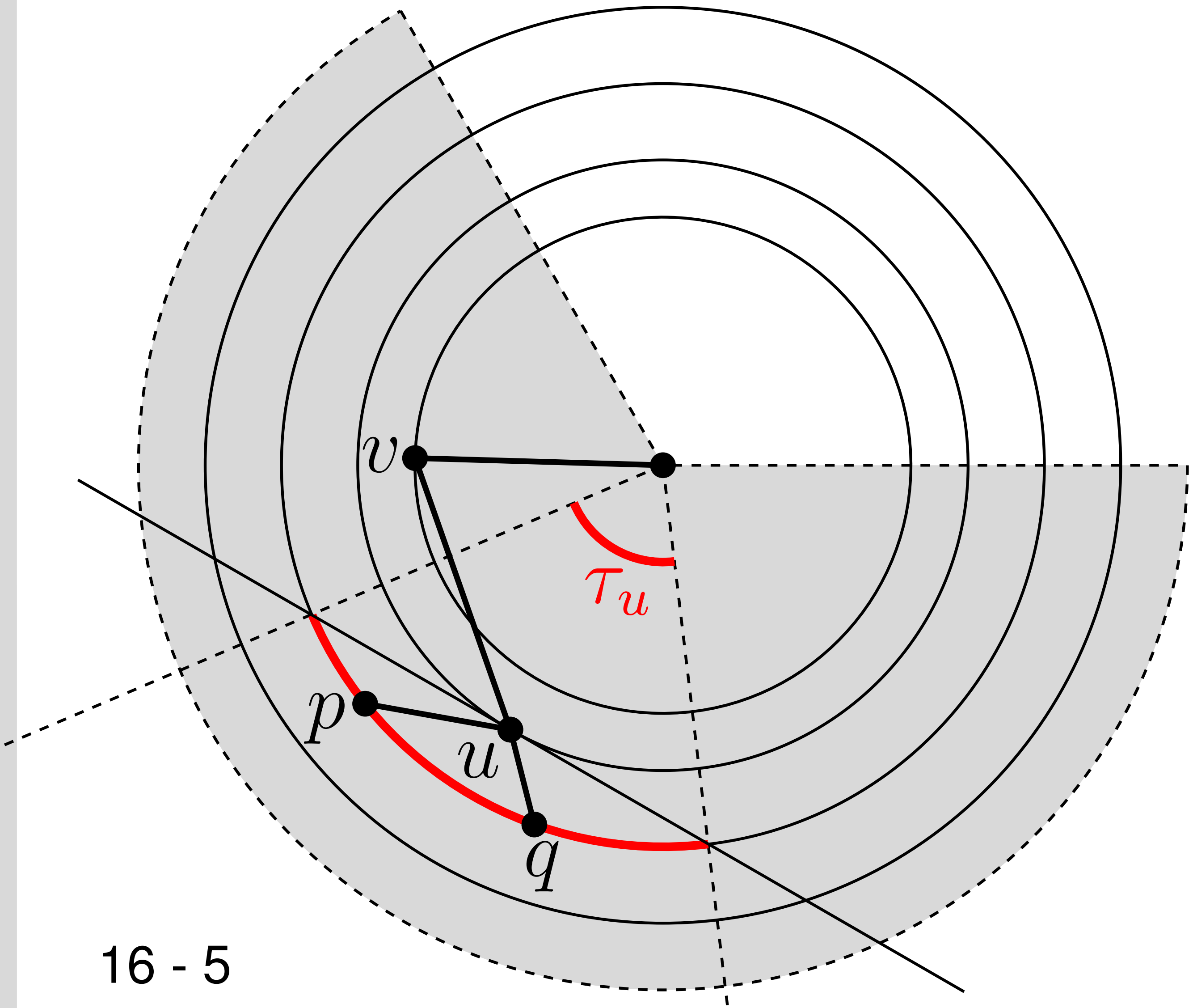

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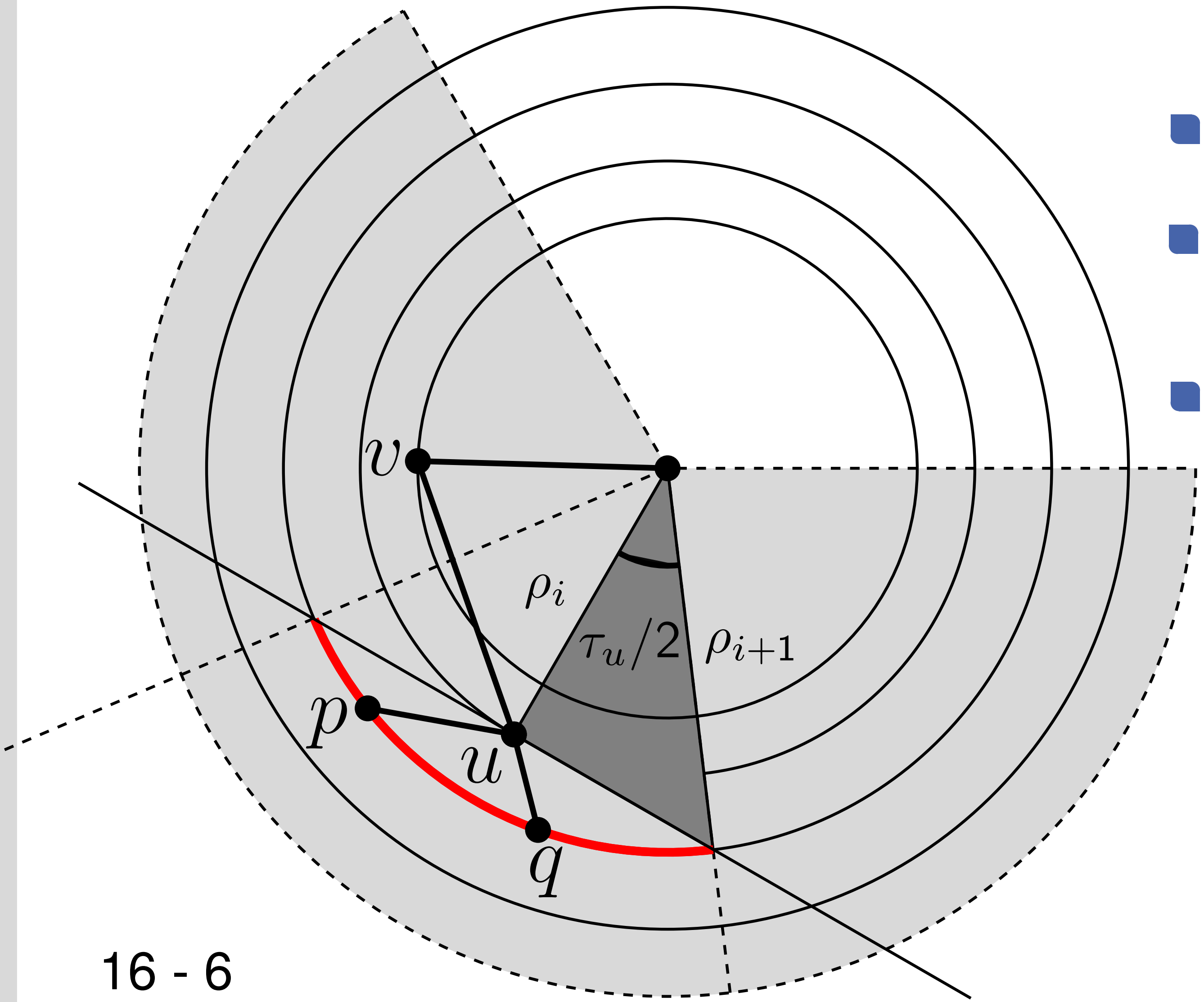

- τ_u - angle of the wedge corresponding to vertex u
- ρ_i - raduis of layer i
- $\ell(v)$-number of nodes in the subtree rooted at v
- $\cos \frac{\tau_u}{2} = \frac{\rho_i}{\rho_{i+1}}$

Radial Layout

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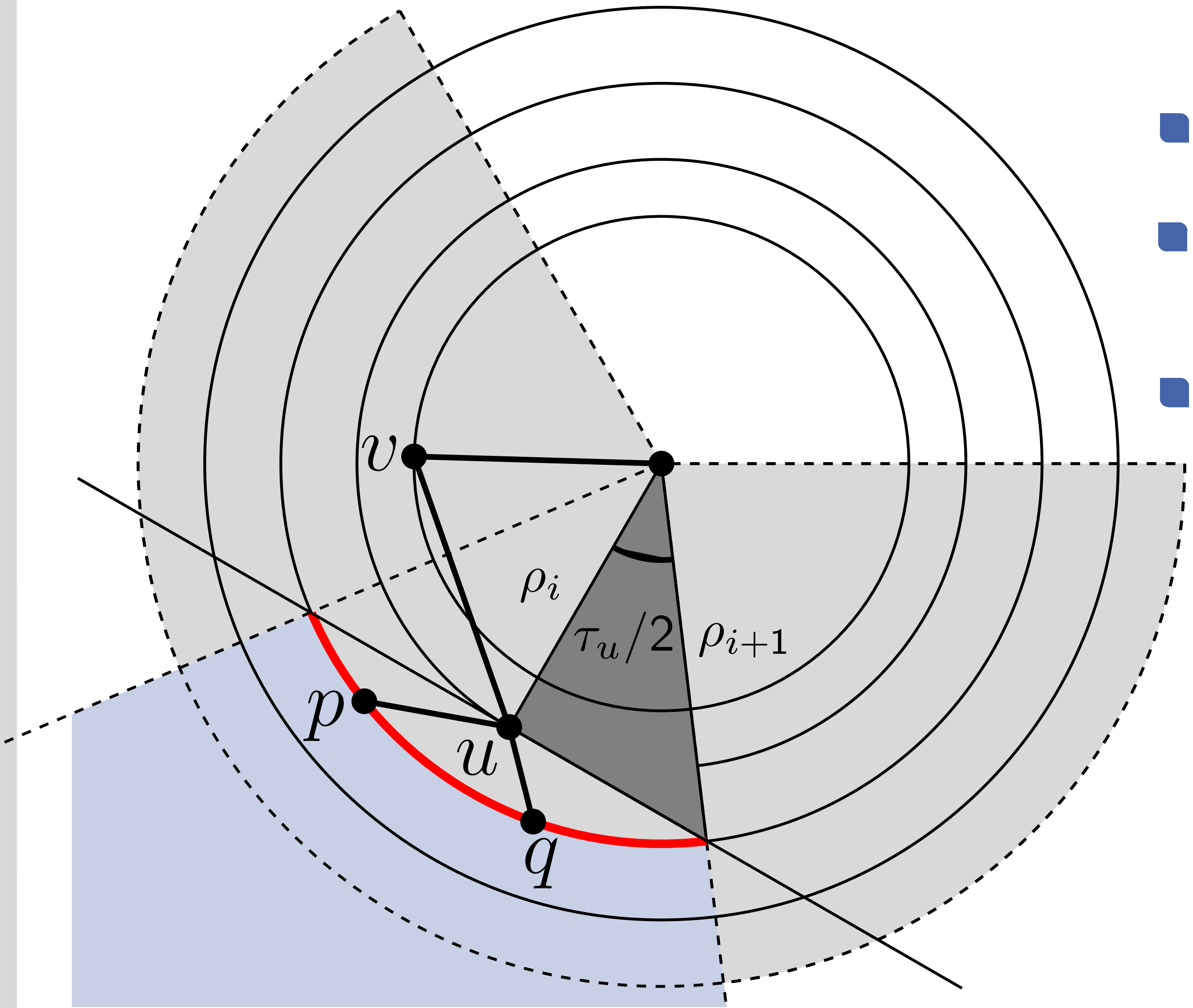

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Radial Layout

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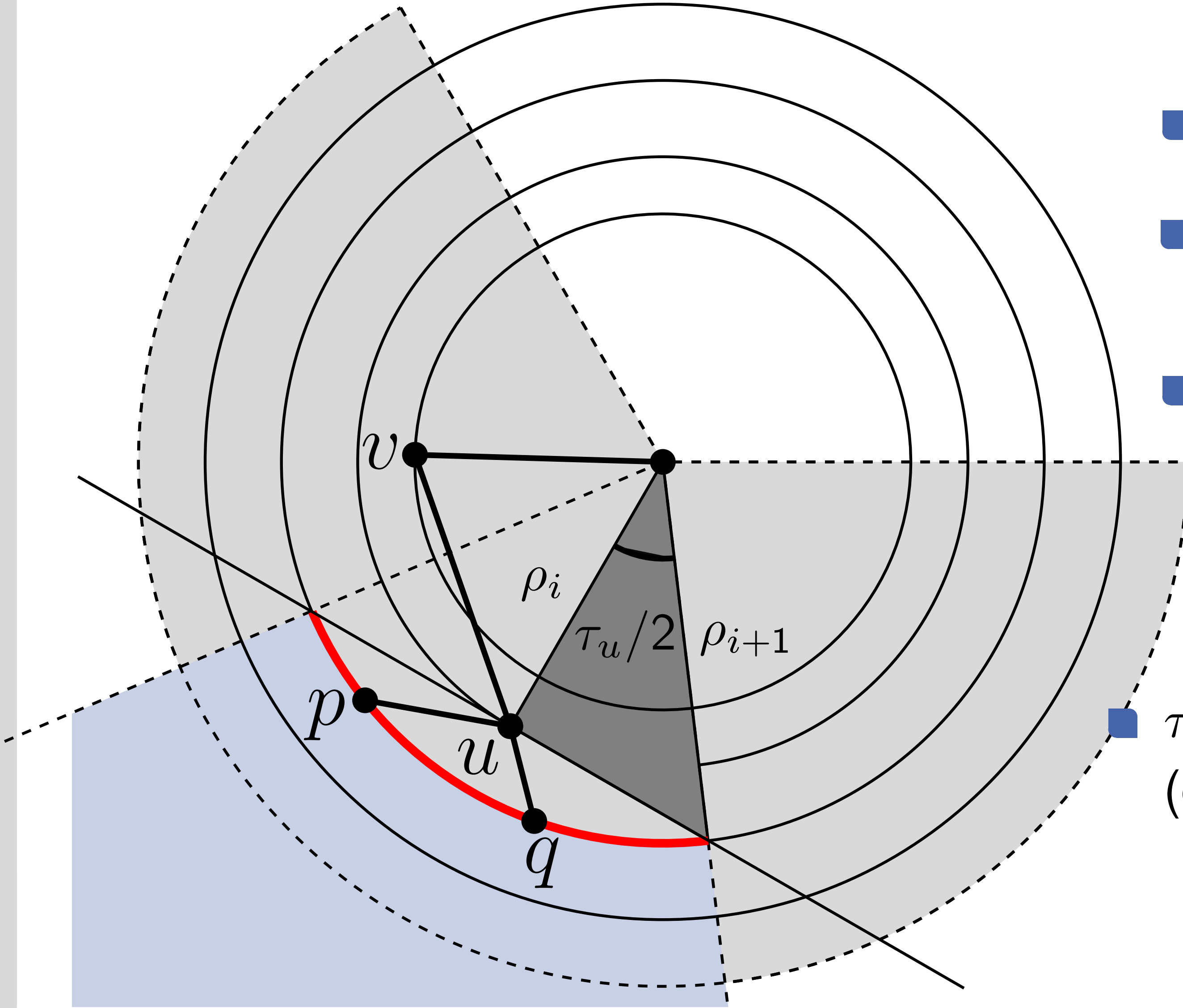

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- $\tau_u = \min\{\frac{\ell(u)}{\ell(v)-1}, 2\arccos\frac{\rho_i}{\rho_{i+1}}\}$ (correction)

Radial Layout

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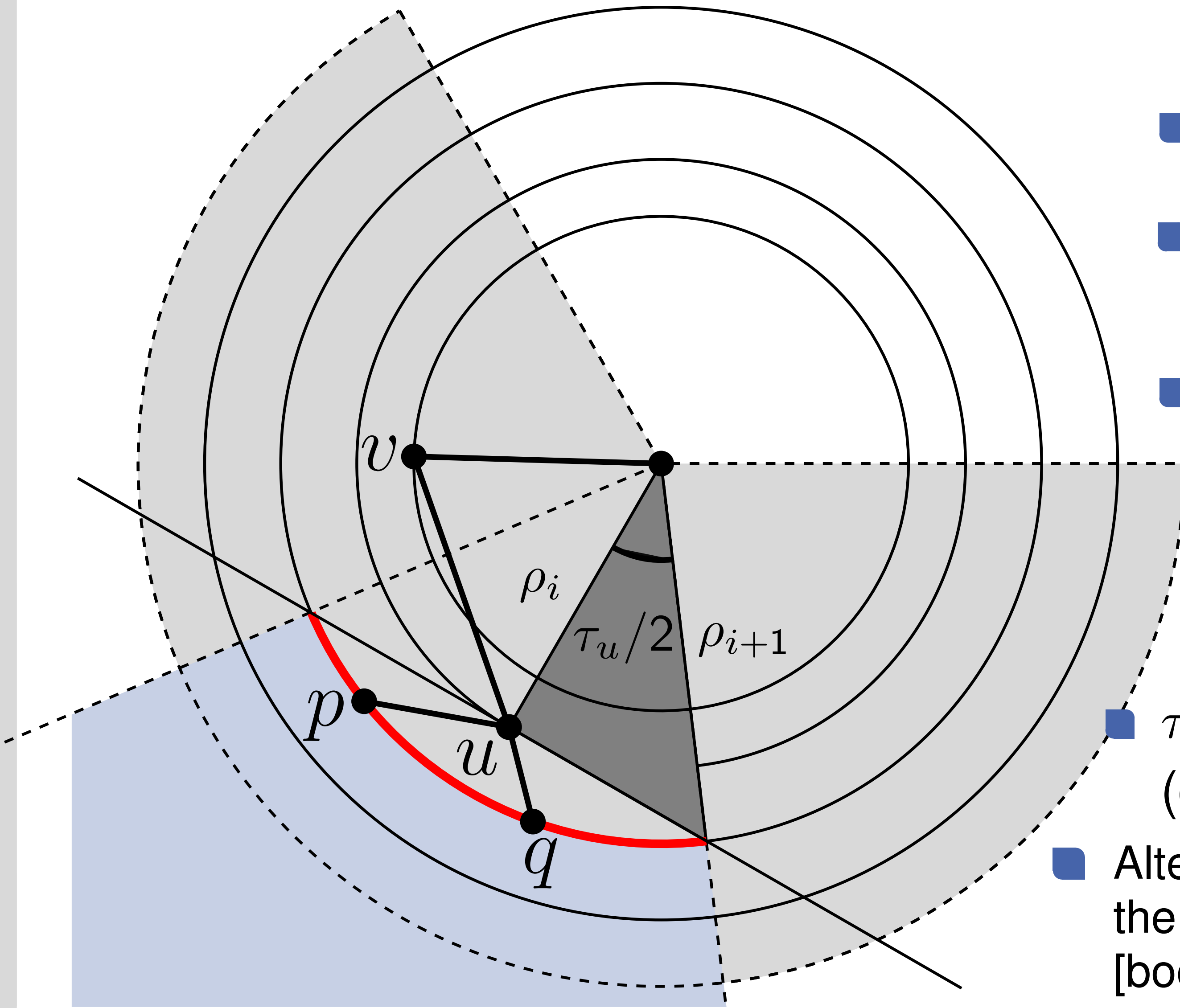

- τ_u - angle of the wedge corresponding to vertex u
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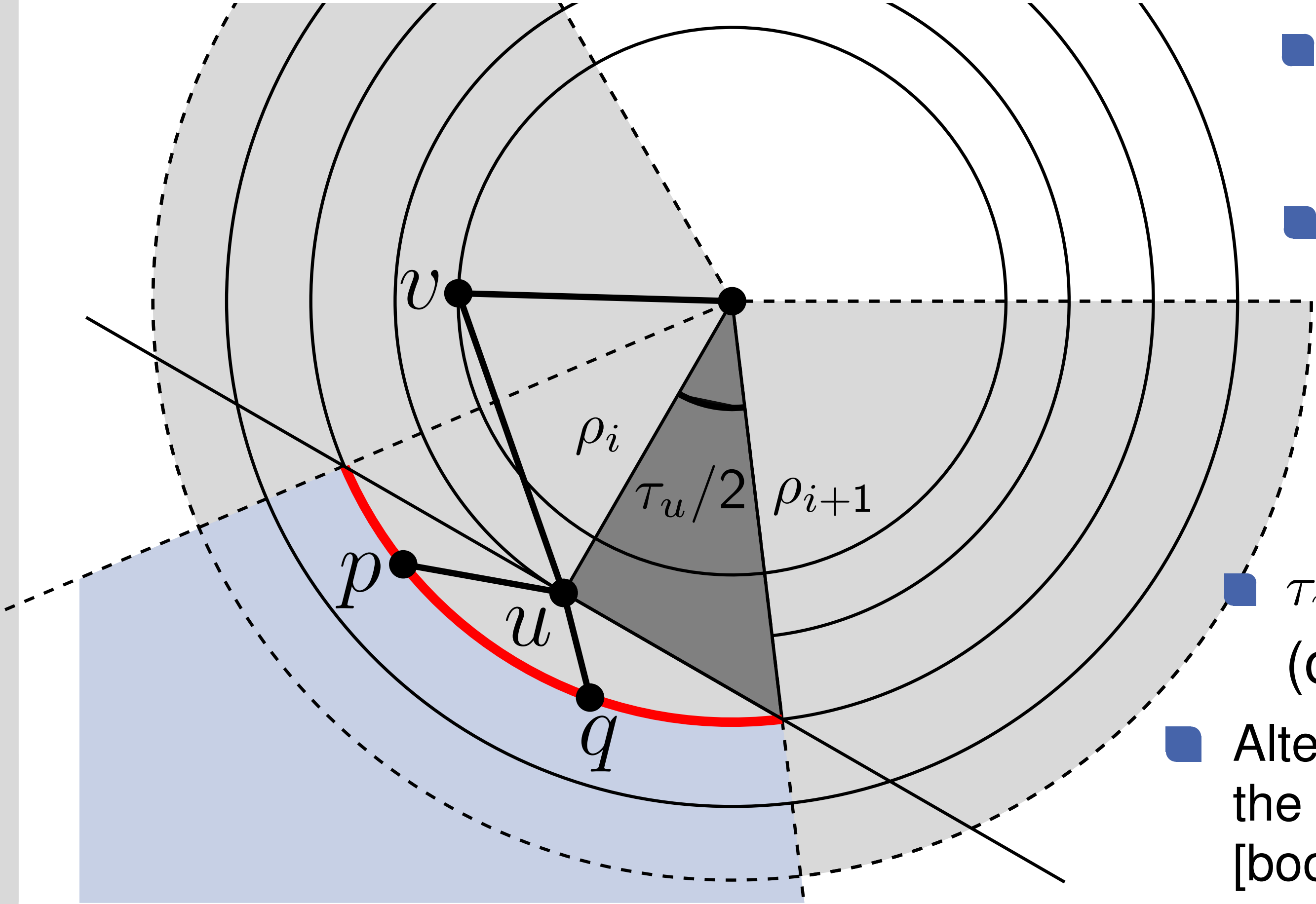
- $\tau_u = \min\{\frac{\ell(u)}{\ell(v)-1}, 2 \arccos \frac{\rho_i}{\rho_{i+1}}\}$ (correction)
- Alternatively use number of leaves in the subtree to subdivide the angles [book Di Battista et al.]

Discuss with your neighbour(s) and then share

5 min

- Why the produced drawing is planar?



- $\ell(v)$-number of nodes in the subtree rooted at v
- $\cos\frac{\tau_u}{2} = \frac{\rho_i}{\rho_{i+1}}$
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Radial Layout

Theorem

Let T be a rooted tree with n vertices. The radial algorithm constructs in $O(n)$ time a drawing Γ of T such that:

- Γ is planar
- Each vertex lies on the radial layer equal to its height
- The area of the drawing is at most $O(h^2 d_M^2)$, h-height, d_M-max number of children

Assuming that the radii of consecutive layers differ by the same number and the distance between the vertices on the layer is a constant

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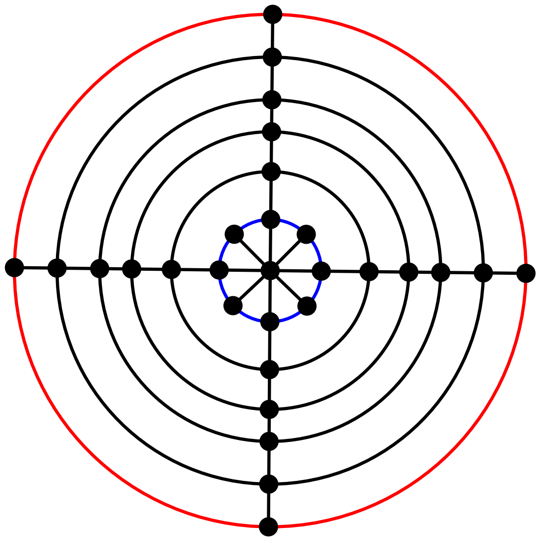

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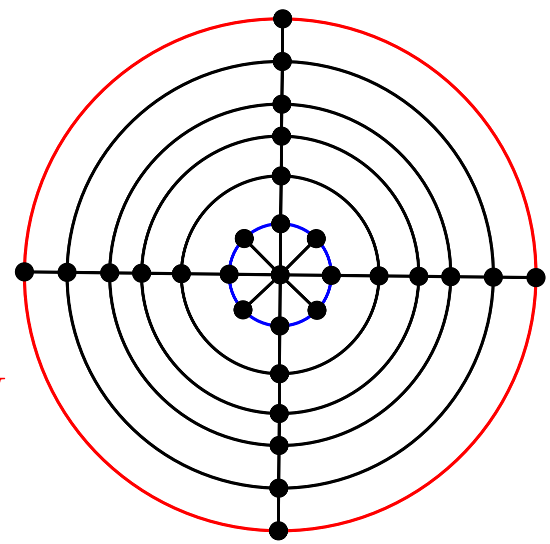
- Γ is planar
- Each vertex lies on the radial layer equal to its height
- The area of the drawing is at most $O(h^2 d_M^2)$, h-height, d_M-max number of children

Assuming that the radii of consecutive layers differ by the same number and the distance between the vertices on the layer is a constant



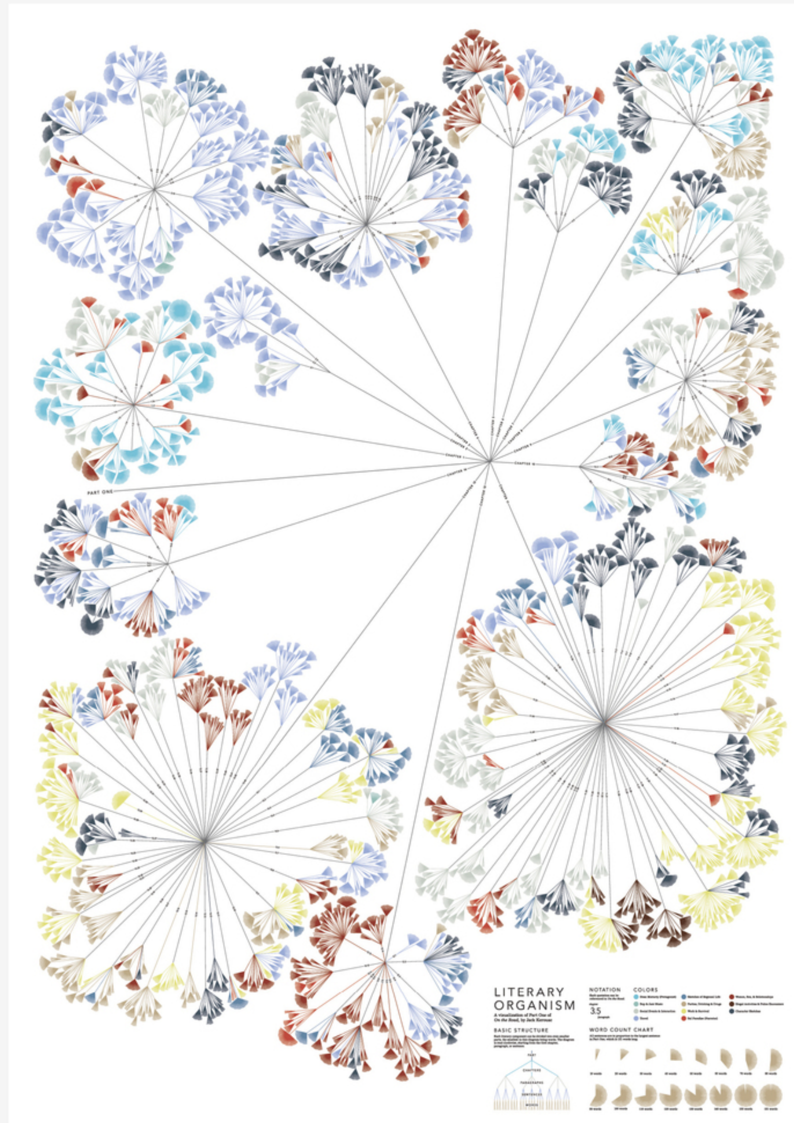
Reading

Radial Layout for Trees

- Book Di Battista et al: Chapter 3.1.3
- Skript: Chapter 6.1.2

Other Visualization Styles

Writing Without Words: the project explores methods of visually-representing text and visualises the differences in writing styles when comparing different authors.



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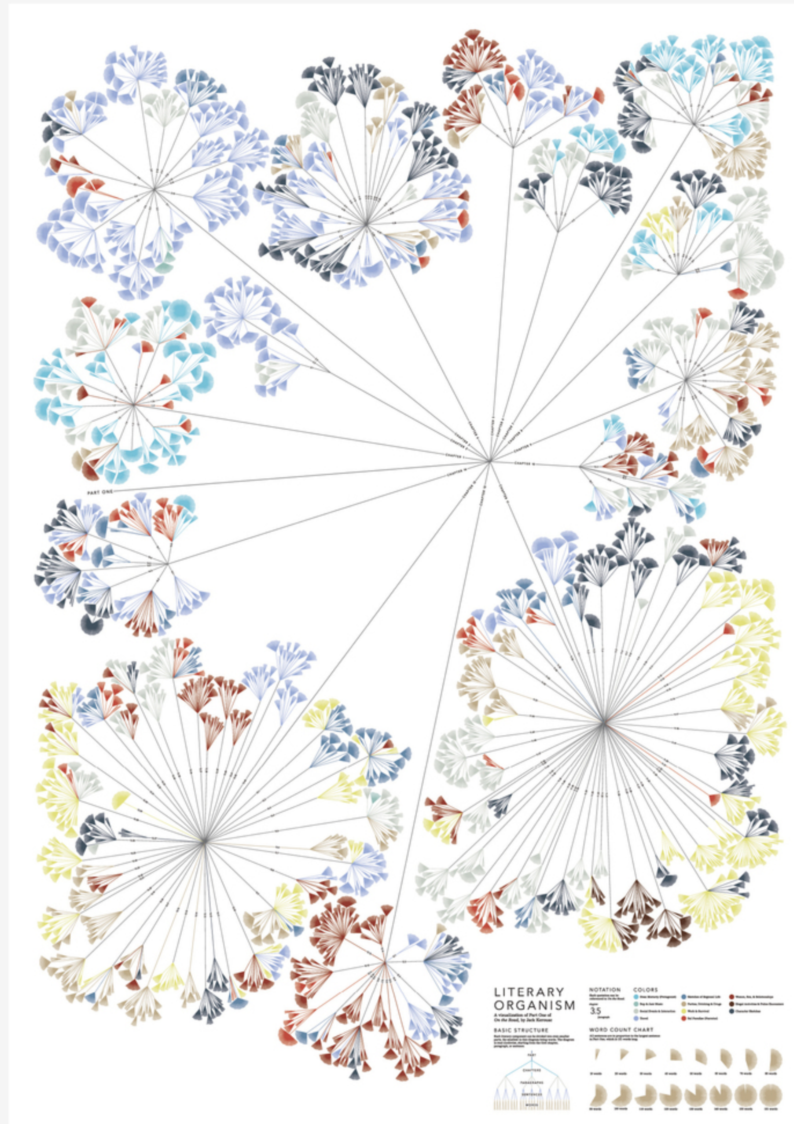
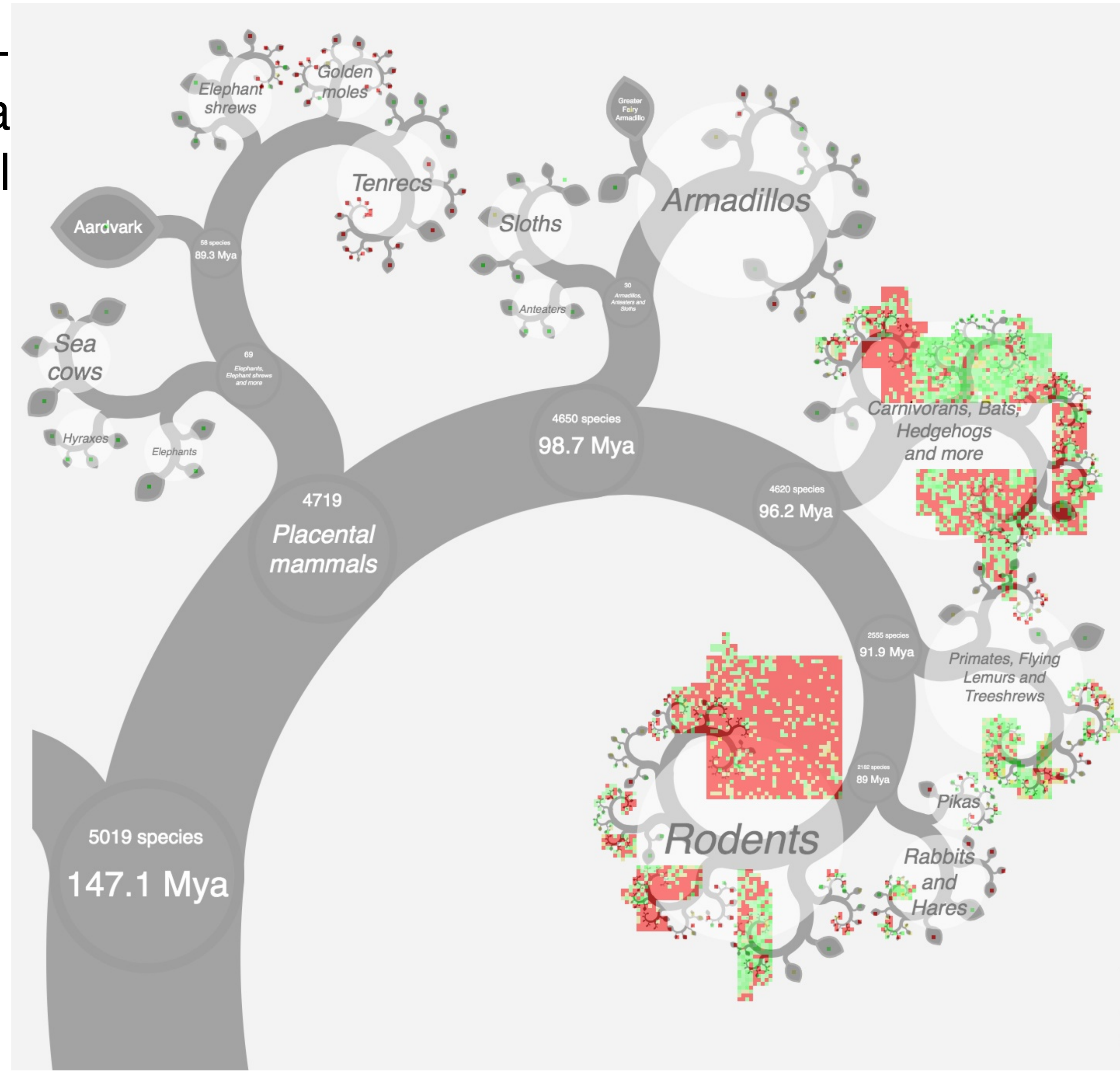


similar to Ballon layout

Other Visualization Styles

A phylogenetically organised display of data for all placental mammal species.



Fractal tree layout

for more applications and layouts...