Algorithms for graph visualization

Divide and Conquer - Tree Layouts - Part 2
Overview

- H(horizontal) V(vertical) tree layout algorithm
- Radial tree layout algorithm
- Other visualization styles
Applications

Cons cell diagram in LISP.
Cons(constructs) are memory objects which hold two values or pointers to values.

![Cons cell diagram in LISP](http://gajon.org/)

**Figure 3:** Diagram of cons cells of the simple tree.

Discuss with your neighbour(s) and then share

2+3 min
Applications

Cons cell diagram in LISP.

*Cons*(constructs) are memory objects which hold two values or pointers to values.

![Diagram of cons cells of the simple tree.](http://gajon.org/)

**Figure 3:** Diagram of cons cells of the simple tree.

Discuss with your neighbour(s) and then share.

- What are the drawing conventions and aesthetics?
HV-Layout

Drawing Conventions:
- Children are vertically and horizontally aligned with the root
- The bounding boxes of the children do not intersect

Drawing Aesthetics:
- Height, width, area
HV-Layout

Divide & Conquer Approach:

HV-Layout
HV-Layout

Induction base: ⊣

Induction step: combine layouts

horizontal combination

vertical combination
Right-Heavy HV-Layout

**Right-Heavy approach:**
- At every induction step apply horizontal combination
- Place the larger subtree to the right
Right-Heavy HV-Layout

Right-Heavy approach:
- At every induction step apply horizontal combination
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Lemma
Let $T$ be a binary tree. The height of the drawing constructed by Right-Heavy approach is at most $\log n$. 
Right-Heavy HV-Layout

Right-Heavy approach:
- At every induction step apply horizontal combination
- Place the larger subtree to the right

Lemma
Let $T$ be a binary tree. The height of the drawing constructed by Right-Heavy approach is at most $\log n$.

Proof:
- Each vertical edge has length 1
- Let $w$ be the lowest node in the drawing
- Let $P$ be a path from $w$ to the root of $T$
- For every edge $(u, v)$ in $P$: $|T(v)| > 2|T(u)|$
- $\Rightarrow P$ contains at most $\log n$ edges
Right-Heavy HV-Layout

- At every induction step apply horizontal combination
- Place the larger subtree to the right

Think:
- What are the implementational details of the algorithm?
  How to compute the coordinates? Can we do it in $O(n)$ time?
Right-Heavy HV-Layout

**Theorem**

Let $T$ be a binary tree with $n$ vertices. The Right-Heavy algorithm constructs in $O(n)$ time a drawing $\Gamma$ of $T$ such that:
Right-Heavy HV-Layout

Theorem

Let $T$ be a binary tree with $n$ vertices. The Right-Heavy algorithm constructs in $O(n)$ time a drawing $\Gamma$ of $T$ such that:

- $\Gamma$ is HV-drawing (planar, orthogonal)
Right-Heavy HV-Layout

**Theorem**

Let $T$ be a binary tree with $n$ vertices. The Right-Heavy algorithm constructs in $O(n)$ time a drawing $\Gamma$ of $T$ such that:

- $\Gamma$ is HV-drawing (planar, orthogonal)
- The width of $\Gamma$ is at most $1$ min

Take a minute to think about the width of the layout
# Right-Heavy HV-Layout

## Theorem

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- $\Gamma$ is HV-drawing (planar, orthogonal)
- The width of $\Gamma$ is at most $n-1$
Right-Heavy HV-Layout

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Right-Heavy HV-Layout

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- The area is $O(n \log n)$
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- Simply and axially isomorphic subtrees have congruent drawings, up to translation
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**General rooted tree:**

```
  largest subtree
```

9 - 8
HV-Layout

Bad news We can not minimize the area by using divide & conquer approach
HV-Layout

**Bad news** We can not minimize the area by using divide & conquer approach

**Good news** We can compute minimum area using Dynamic Programming
HV-Layout

Bad news We can not minimize the area by using divide & conquer approach
Good news We can compute minimum area using Dynamic Programming

HV-Layout for Trees

- Book Di Battista et al: Chapter 3.1.4
- Skript: page 86

10 - 3
Applications

An unrooted phylogenetic tree for myosin, a superfamily of proteins. "A myosin family tree" Journal of Cell Science

Radial layout

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Applications

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"A myosin family tree“ Journal of Cell Science
Applications

Flare Visualization Toolkit code structure by Heer, Bostock and Ogievetsky, 2010

Greek Myth Family by Ribecca, 2011
Radial Layout

Drawing Conventions:
- Vertices lie on circular layers according to their depth
- Drawing is planar

Drawing Aesthetics:
- Distribution of the vertices
Radial Layout

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- Vertices lie on circular layers according to their depth
- Drawing is planar

Drawing Aesthetics:
- Distribution of the vertices

Take a minute to think about a possible algorithm to optimize the distribution of the vertices 1 min
Radial Layout

**Example:** Angle corresponding to the subtree rooted at \( u \): 

\[
\tau_u = \frac{\ell(u)}{\ell(v) - 1}
\]
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Radial Layout

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Radial Layout

How to avoid crossings:
Radial Layout

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How to avoid crossings:

- $\tau_u$ - angle of the wedge corresponding to vertex $u$
- $\rho_i$ - radius of layer $i$
- $\ell(v)$ - number of nodes in the subtree rooted at $v$
- $\cos \frac{\tau_u}{2} = \frac{\rho_i}{\rho_{i+1}}$
Radial Layout

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$$\tau_u = \min\left\{ \frac{\ell(u)}{\ell(v)-1}, 2 \arccos \frac{\rho_i}{\rho_{i+1}} \right\}$$
(correction)
Radial Layout

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\[ \tau_u = \min \left\{ \frac{\ell(u)}{\ell(v)} - 1, 2 \arccos \frac{\rho_i}{\rho_{i+1}} \right\} \]
  (correction)

Alternatively use number of leaves in the subtree to subdivide the angles [book Di Battista et al.]
Radial Layout

Discuss with your neighbour(s) and then share

- Why the produced drawing is planar?

- $\ell(v)$-number of nodes in the subtree rooted at $v$

- $\cos \frac{\tau_u}{2} = \frac{\rho_i}{\rho_{i+1}}$

- $\tau_u = \min\{\frac{\ell(u)}{\ell(v)} - 1, 2 \arccos \frac{\rho_i}{\rho_{i+1}}\}$ (correction)

- Alternatively use number of leaves in the subtree to subdivide the angles [book Di Battista et al.]
Radial Layout

Theorem

Let $T$ be a rooted tree with $n$ vertices. The radial algorithm constructs in $O(n)$ time a drawing $\Gamma$ of $T$ such that:

- $\Gamma$ is planar
- Each vertex lies on the radial layer equal to its height
- The area of the drawing is at most $O(h^2d_M^2)$, $h$-height, $d_M$-max number of children

Assuming that the radii of consecutive layers differ by the same number and the distance between the vertices on the layer is a constant
Radial Layout

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radius is at least $d_M$
radius is at least $hd_M$
Radial Layout for Trees

- Book Di Battista et al: Chapter 3.1.3
- Skript: Chapter 6.1.2
Other Visualization Styles

Writing Without Words: the project explores methods of visually-representing text and visualises the differences in writing styles when comparing different authors.
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similar to Ballon layout
Other Visualization Styles

A phylogenetically organised display of data for all placental mammal species.

Fractal tree layout
for more applications and layouts...