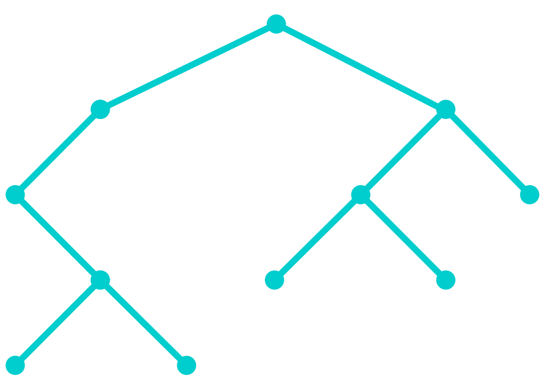
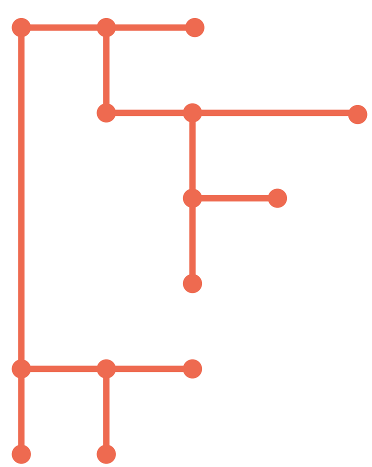
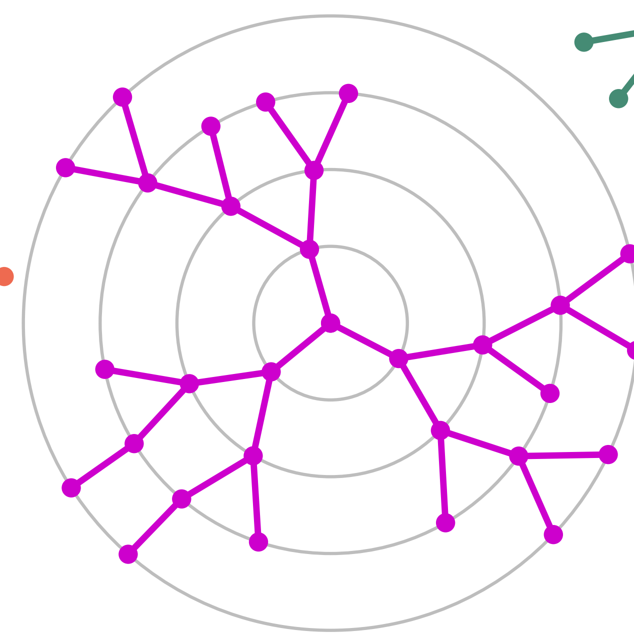
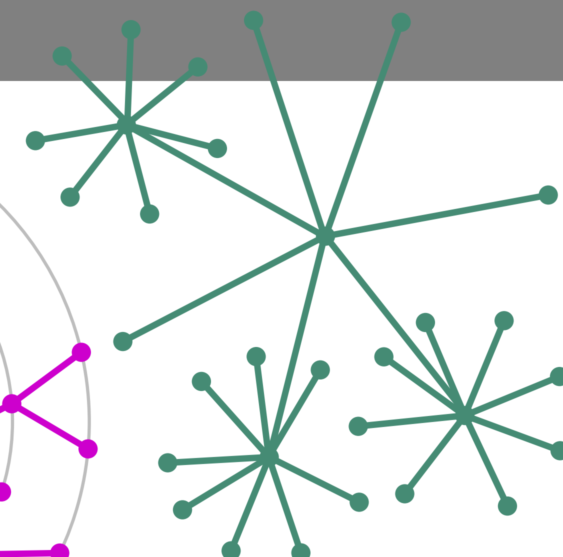
Algorithms for graph visualization

Divide and Conquer - Tree Layouts

WINTER SEMESTER 2017/2018

Tamara Mchedlidze

KIT – Universität des Landes Baden-Württemberg und
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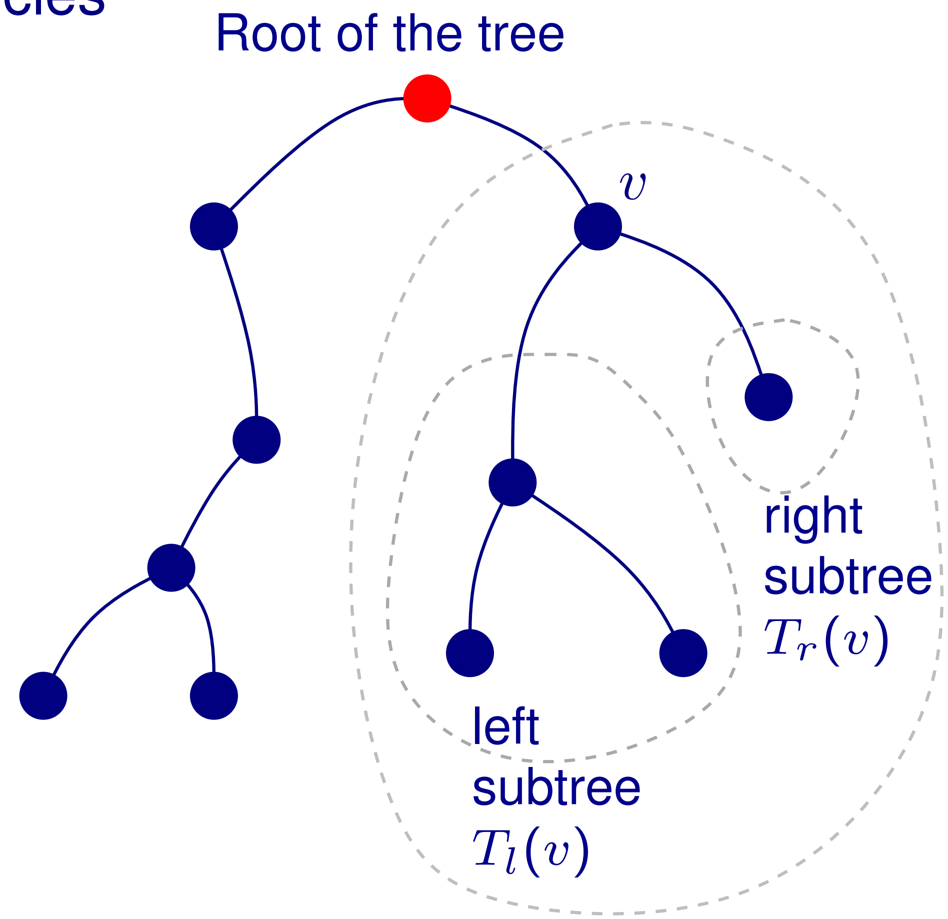
Overview

- Basic Definitions
- Level-based tree layout algorithm
- H(horizontal) V(vertical) tree layout algorithm
- Radial tree layout algorithm
- Other visualization styles

Basic Definitions

- Tree - connected graph without cycles
- Binary tree



3 - 1

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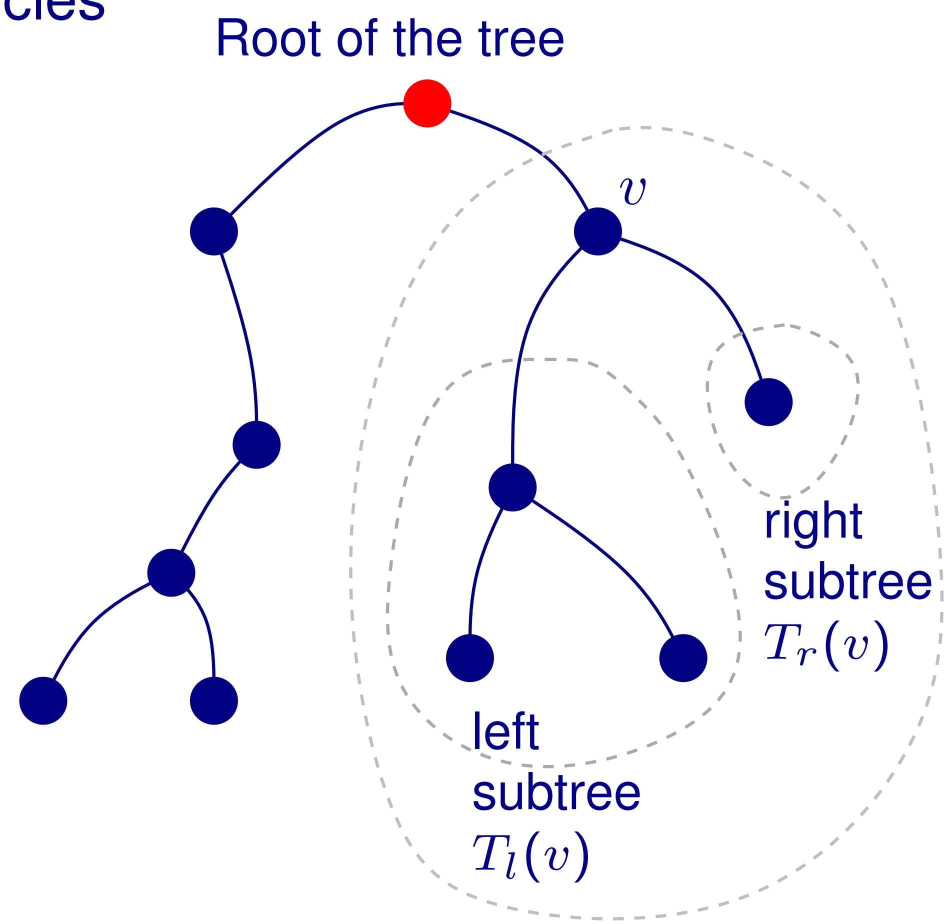
Tree traversals



3 - 2

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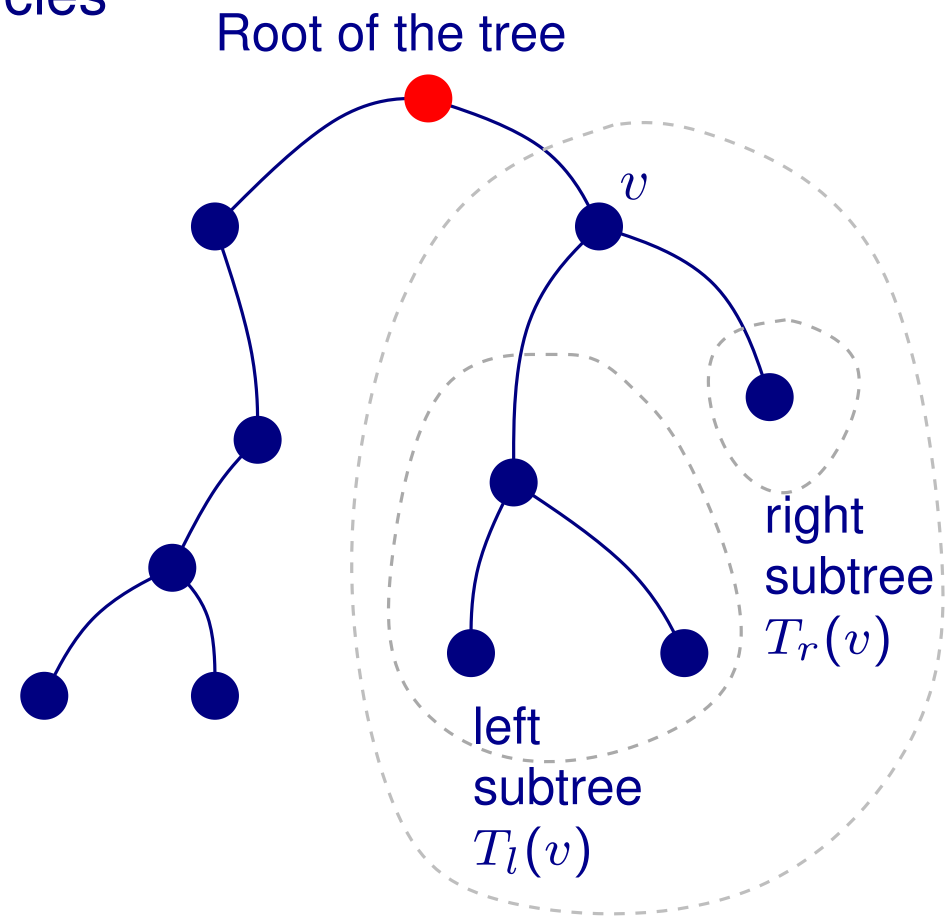
Depth-first search



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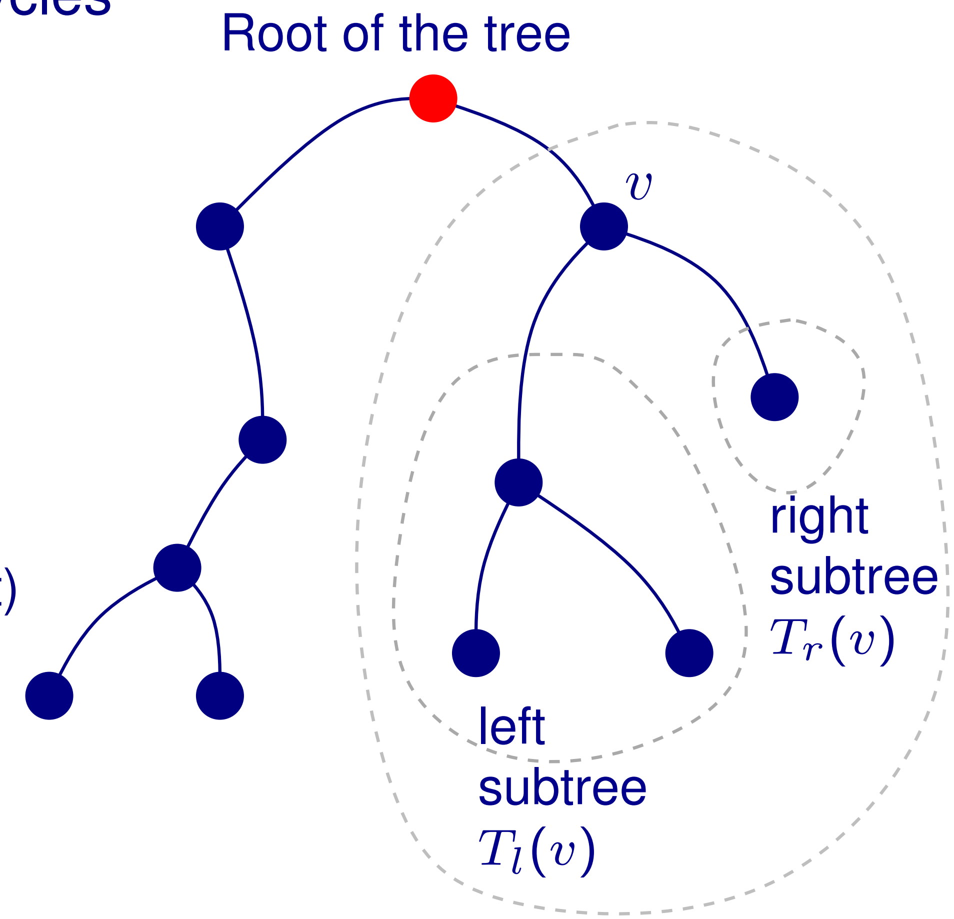
Tree traversals

Depth-first search

- Pre-order (First parent, then subtrees)
- In-order (Left child, parent, right child)
- Post-order (First subtrees, then parent)



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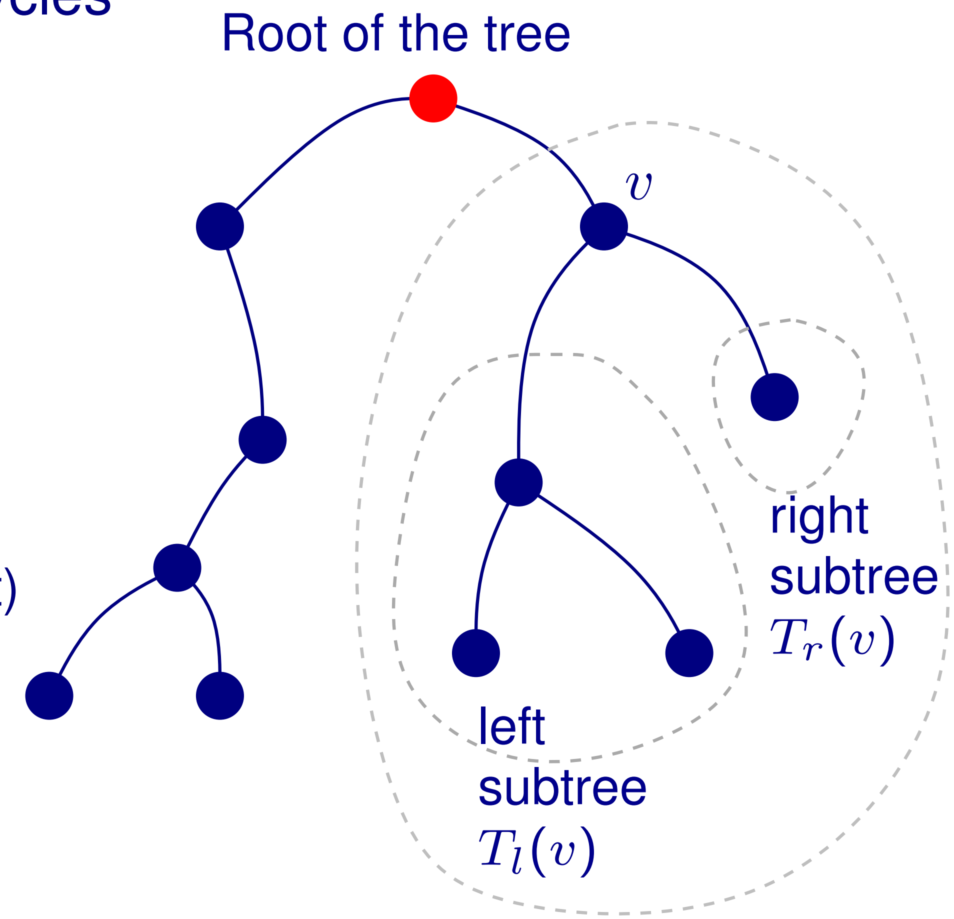
Breadth-first search



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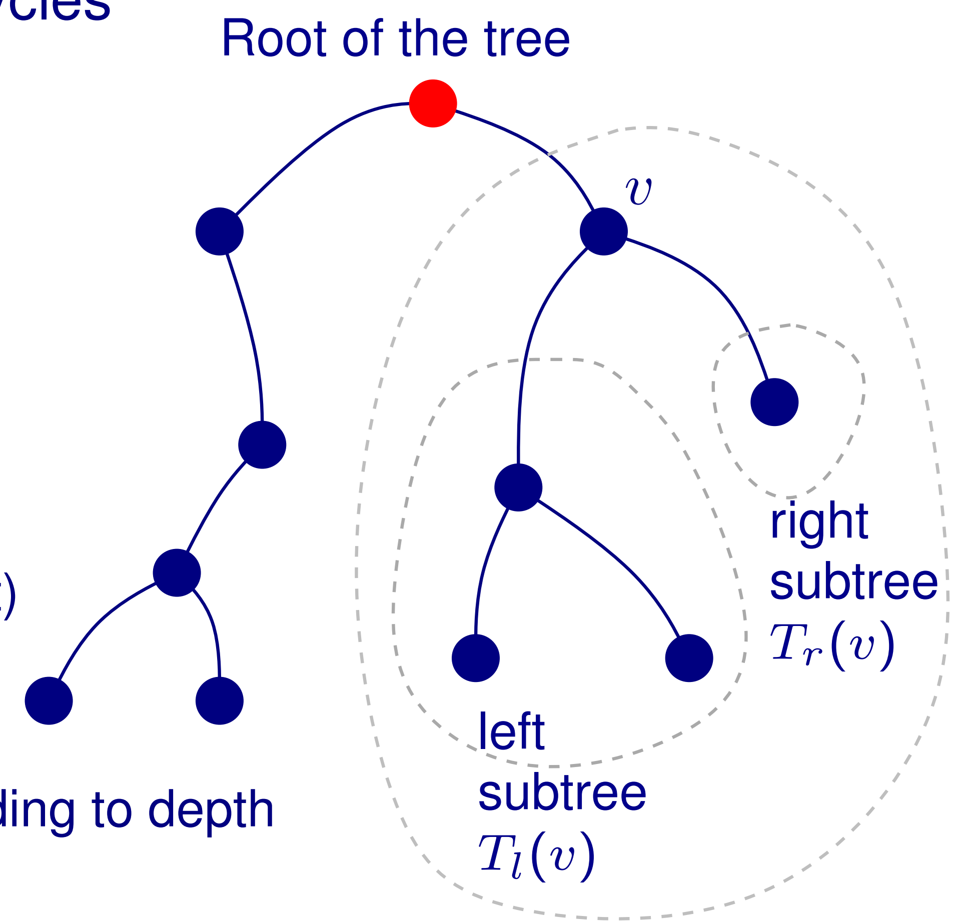
Breadth-first search

- Assignes vertices to levels corresponding to depth



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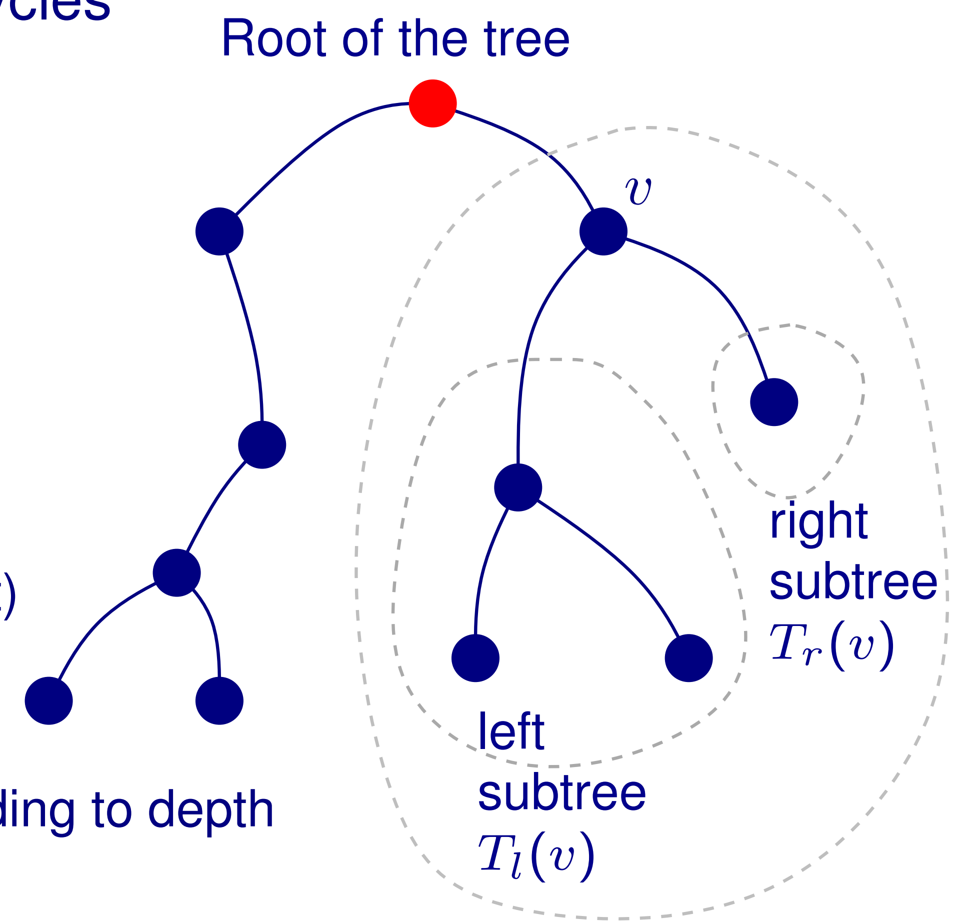

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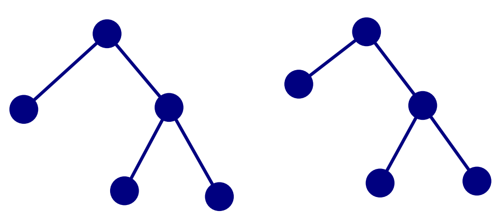
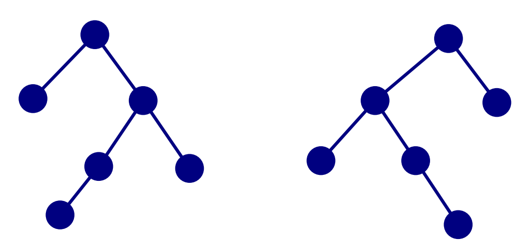
Isomorphism (of ordered trees)

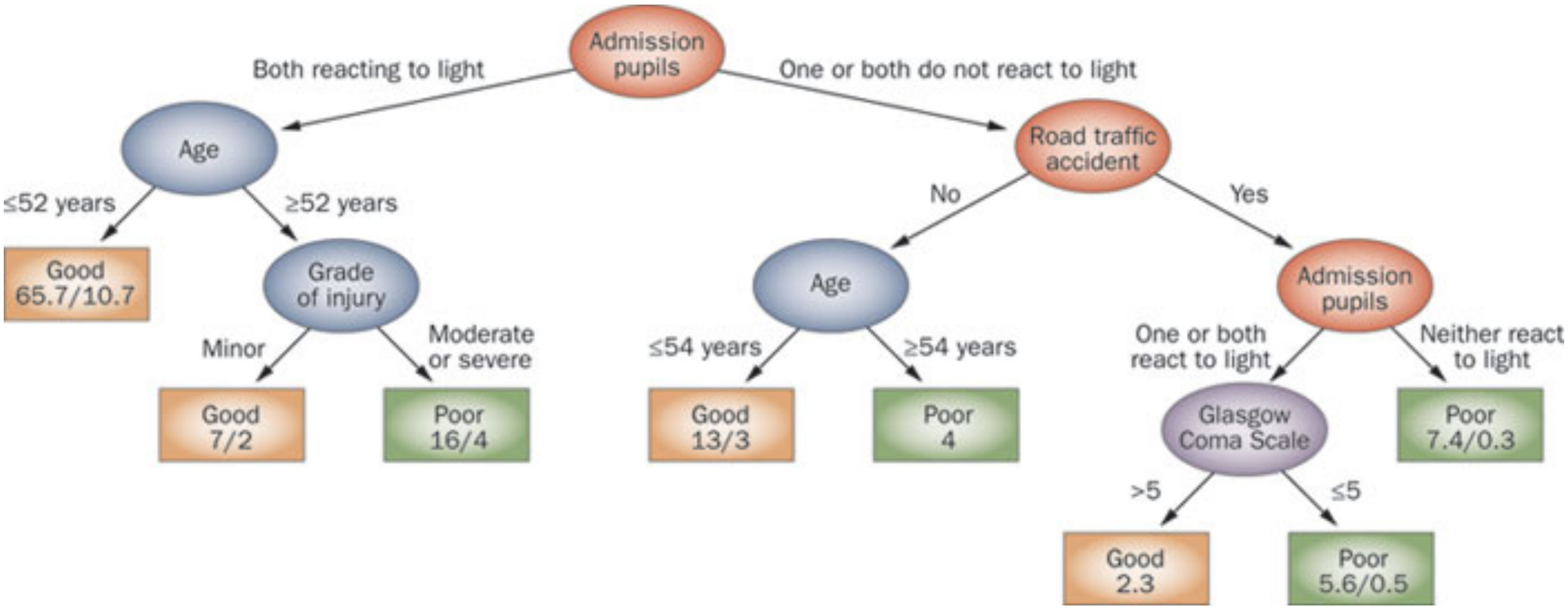
Simple



Axial



Applications



Decision tree analysis for prediction of outcome after traumatic brain injury
Nature Reviews Neurology

Applications

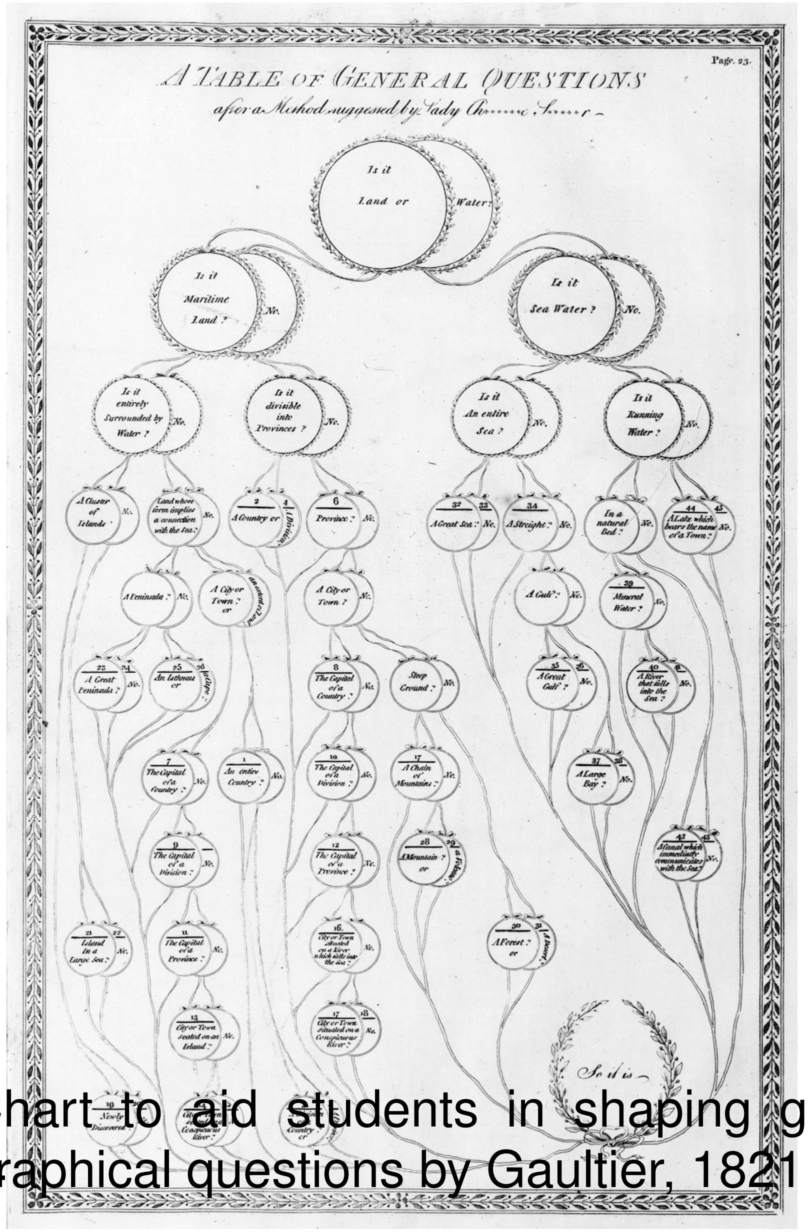


Chart to aid students in shaping geographical questions by Gaultier, 1821

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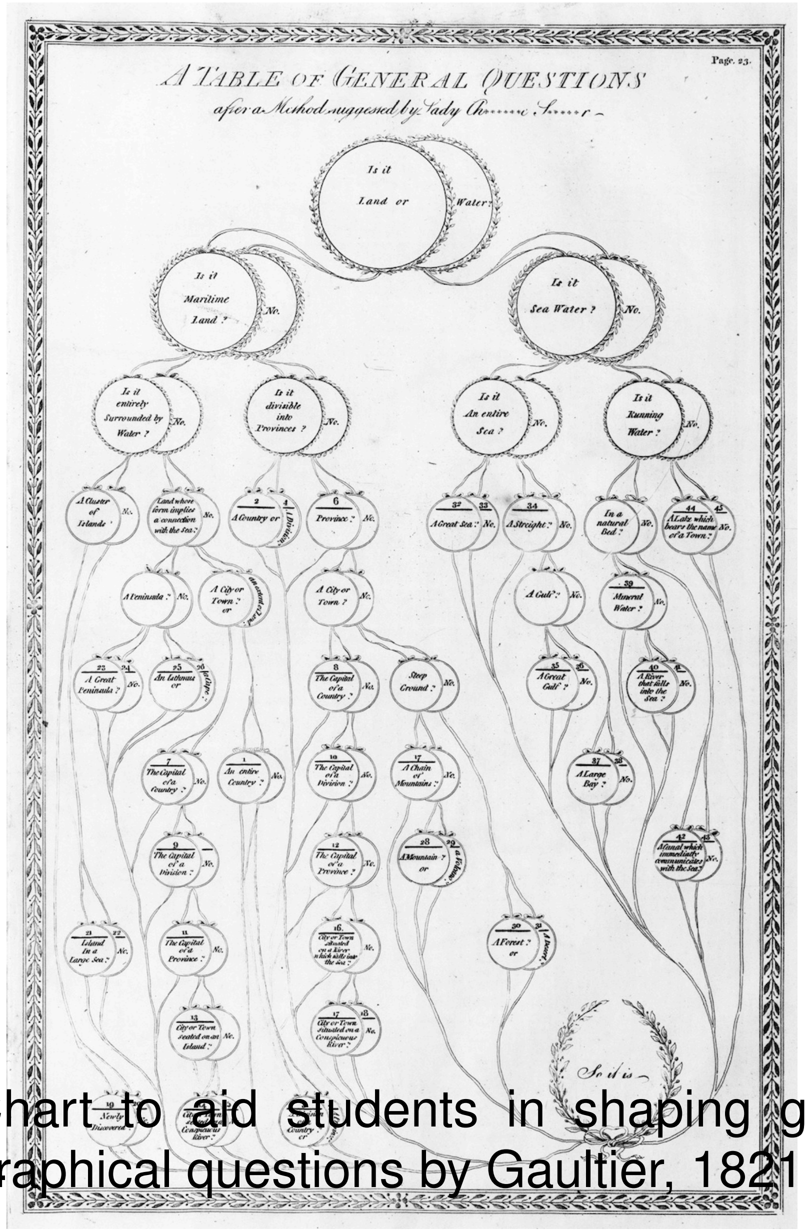

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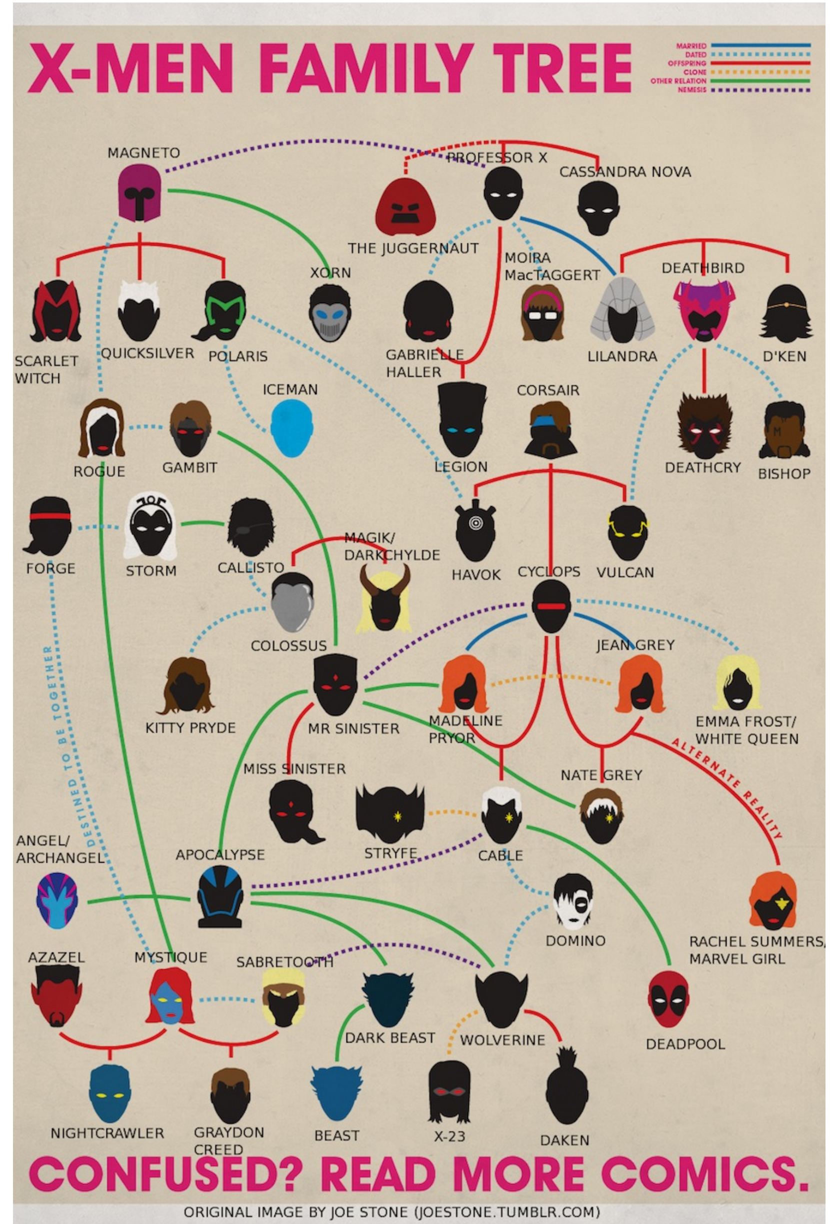
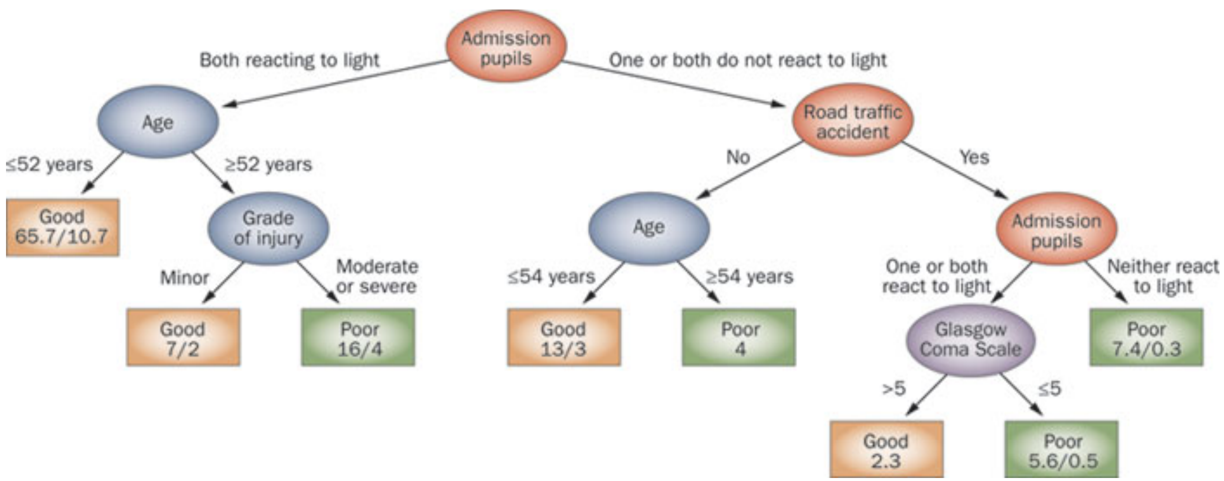


Level-Based Layout of a Tree

Discuss with your neighbour or in groups of three and write down

5 min

- What the properties of the layout?
- What are the drawing conventions and the aesthetics that we have to take into account?



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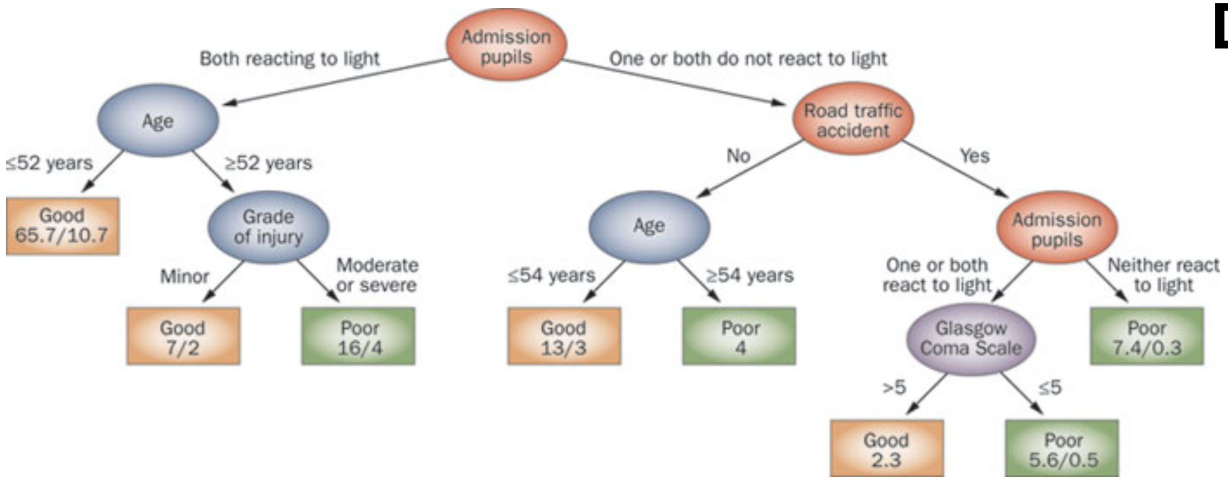

Drawing Conventions

- Vertices lie on layers
- Parent is above the children
- Edges are straight lines
- Parent is centred with respect to the children
- Isomorphic subtrees have identical drawings

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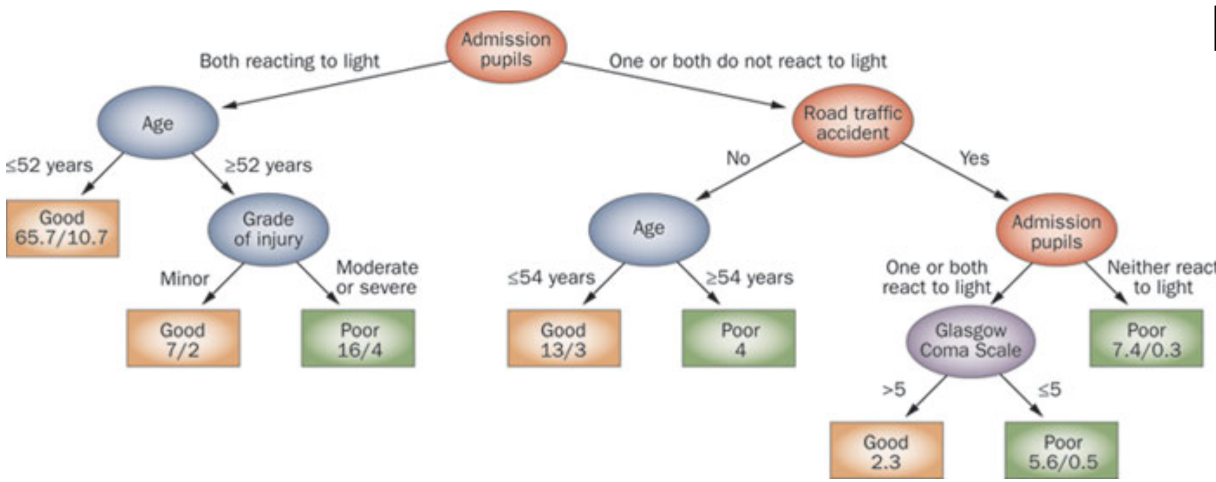

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Drawing Aesthetics

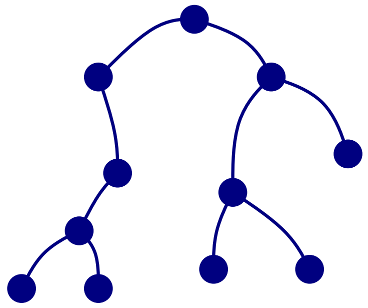
- Area

Level-based Layout

Algorithm Outline:

Input: A binary tree

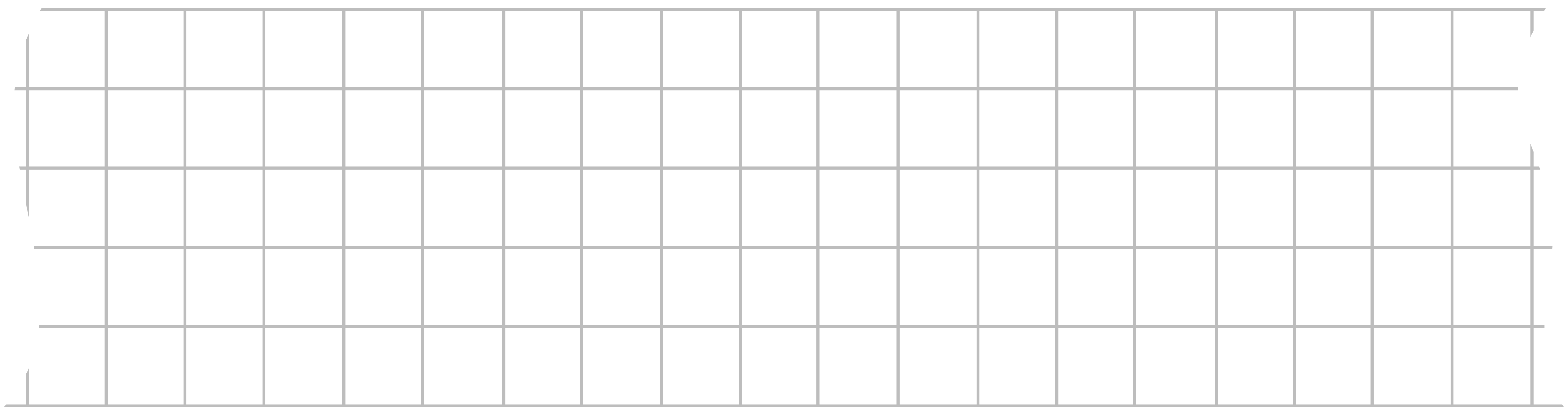
Output: A leveled drawing of T



Base case: A single vertex

Divide: Recursively apply the algorithm to draw the left and the right subtrees of T

Conquer:



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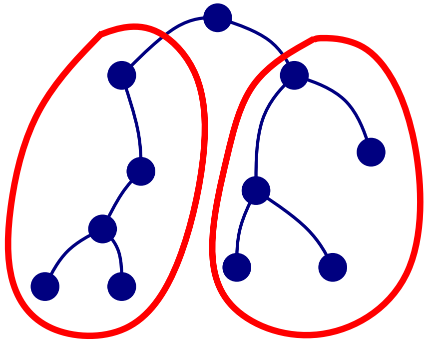

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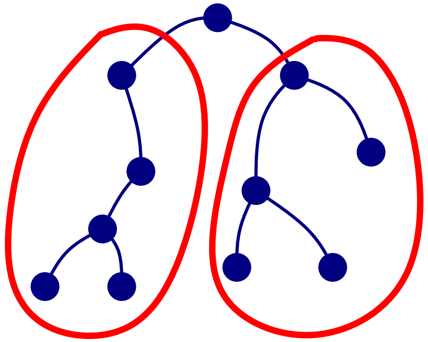

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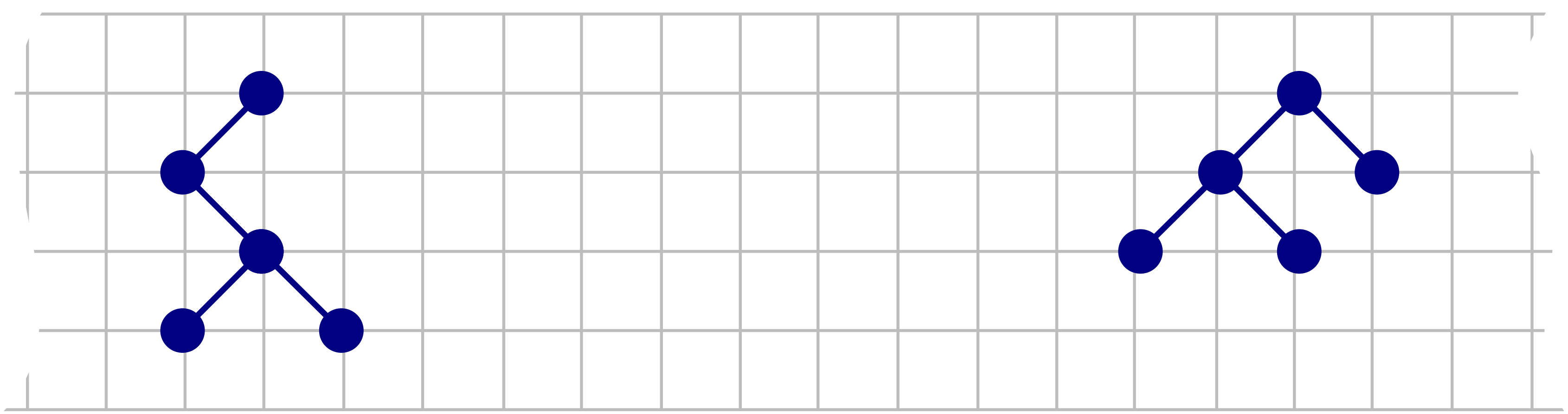

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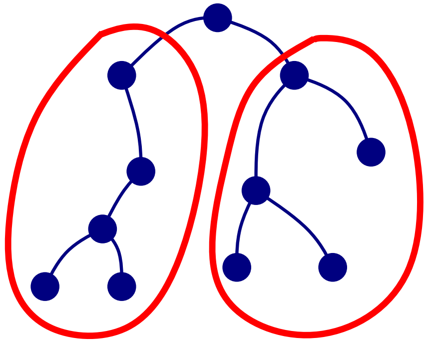

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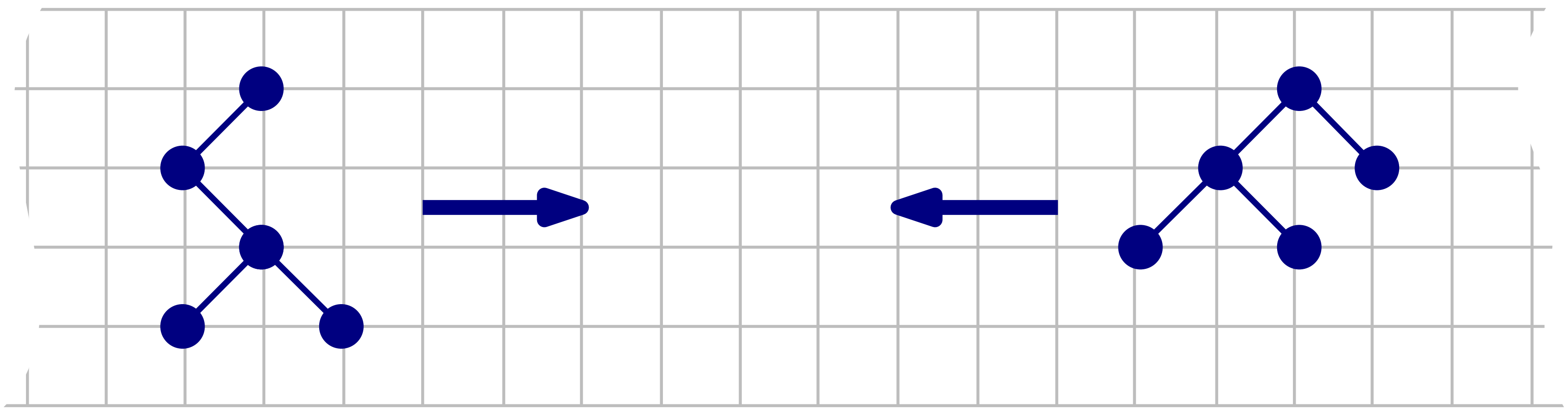

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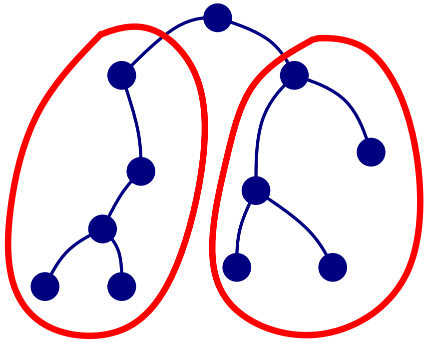

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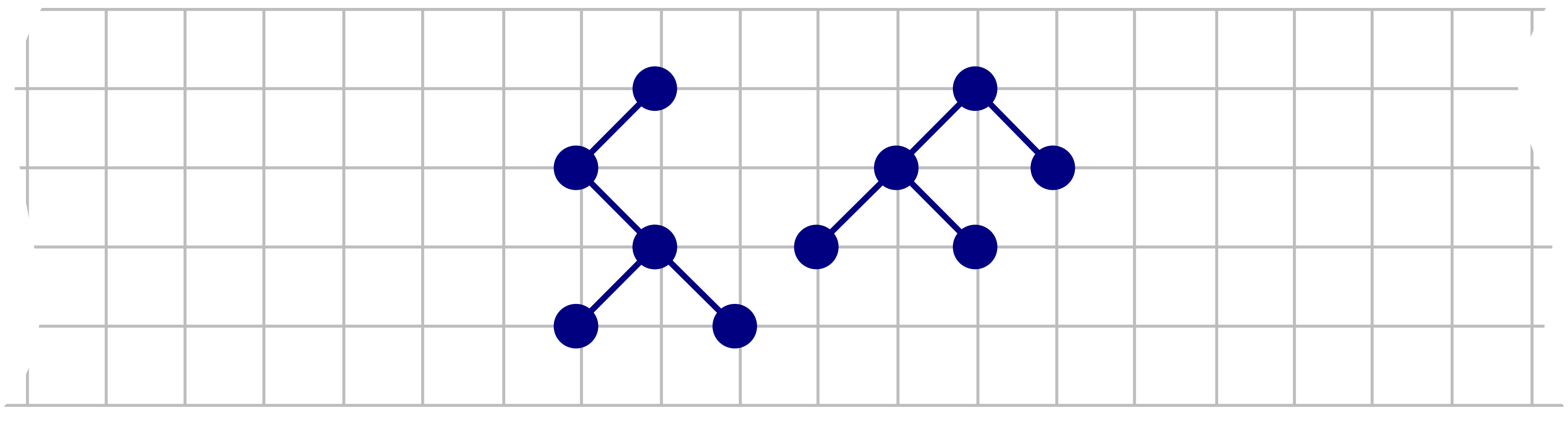

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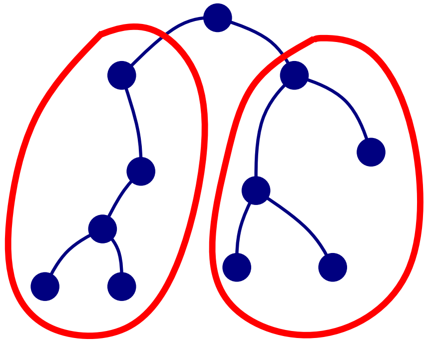

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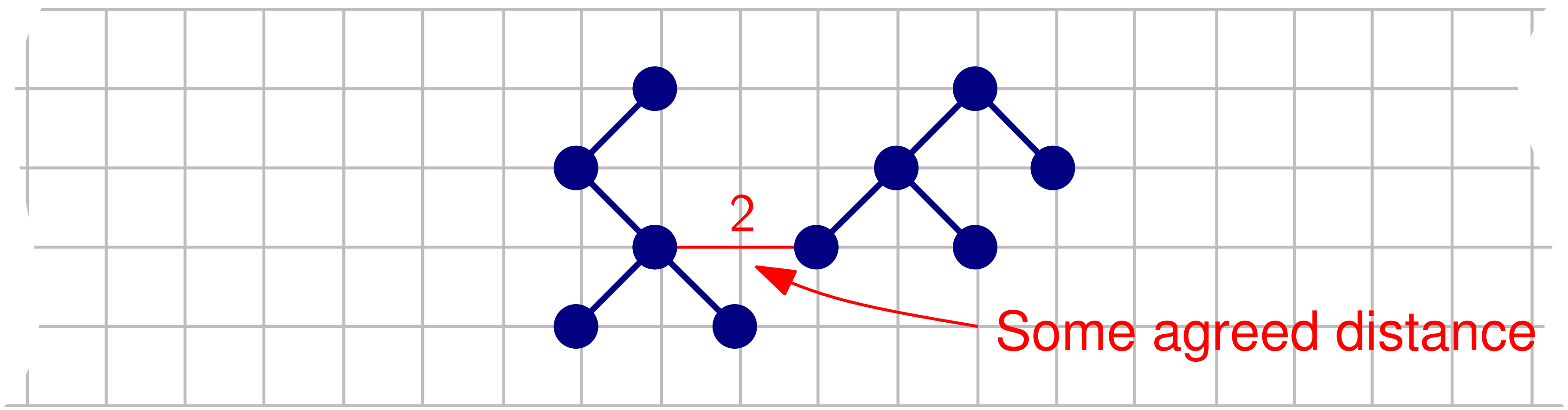

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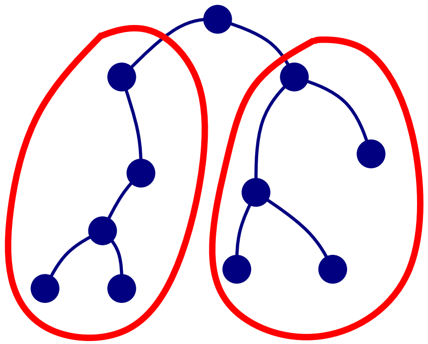

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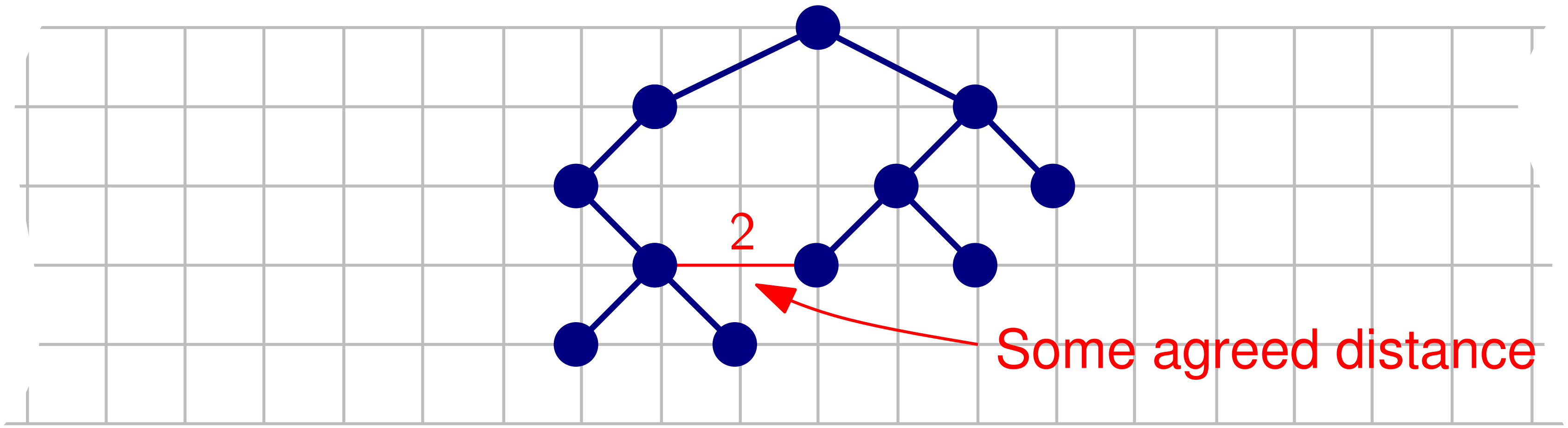

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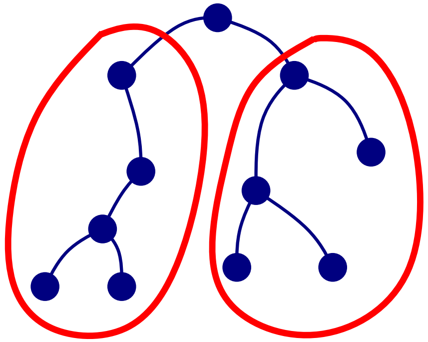

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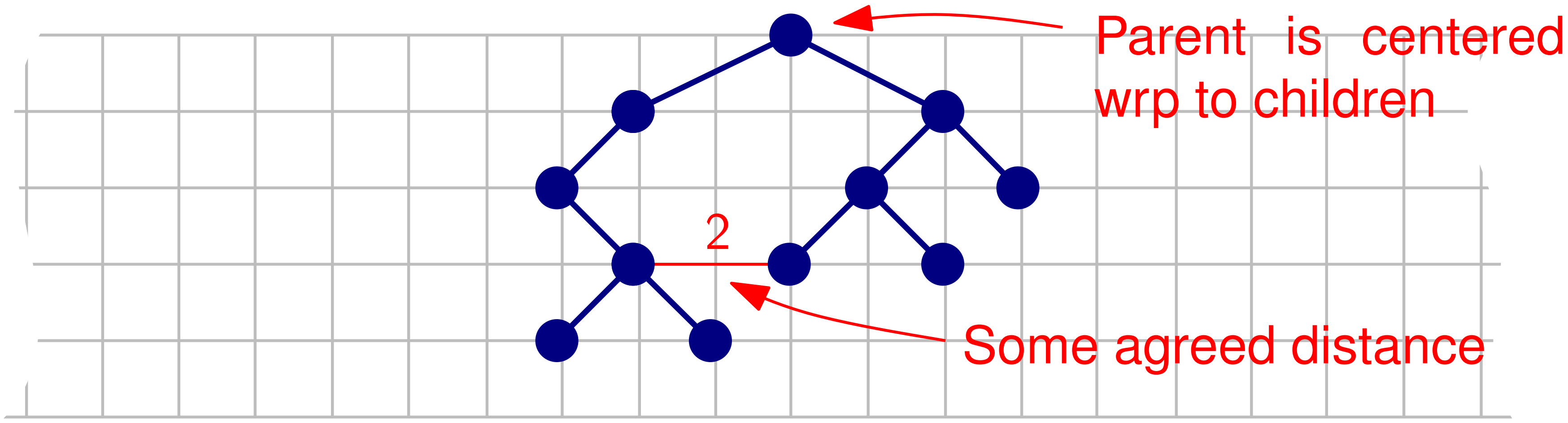
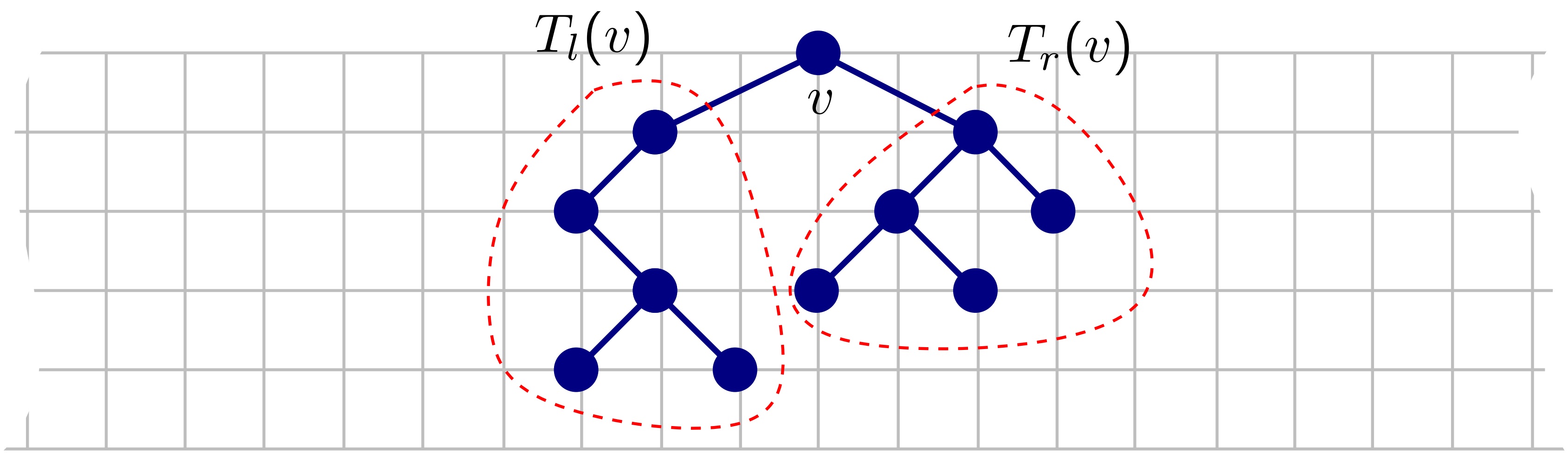


Level-based Layout

Implementation Details (postorder and preorder traversals)

Postorder traversal: For each vertex v compute horizontal displacement of the left and the right child



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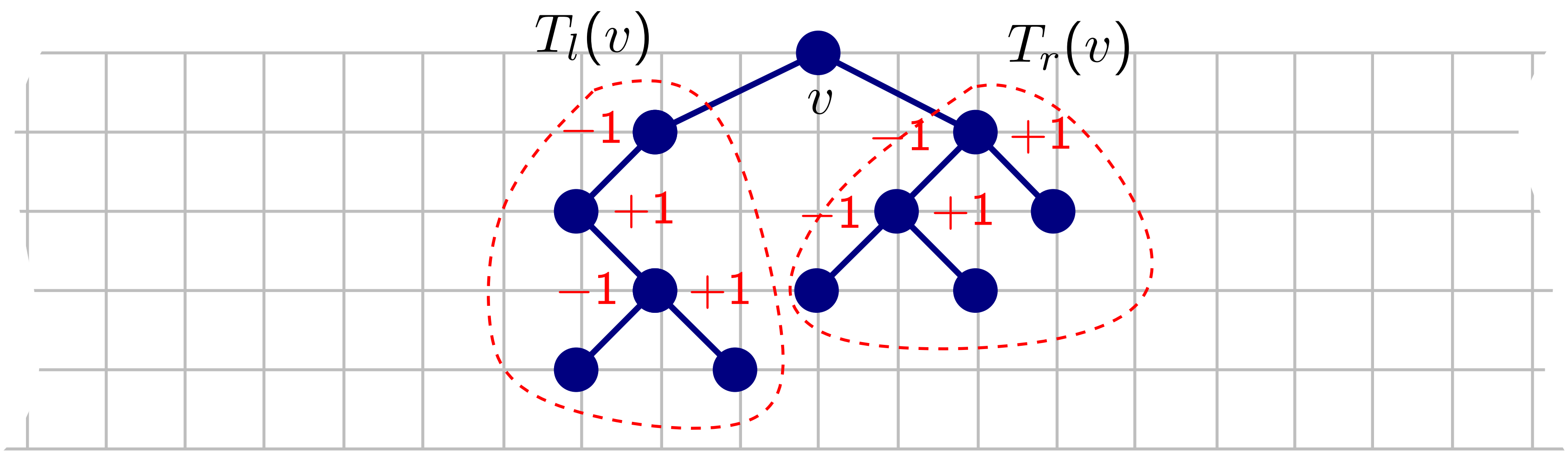

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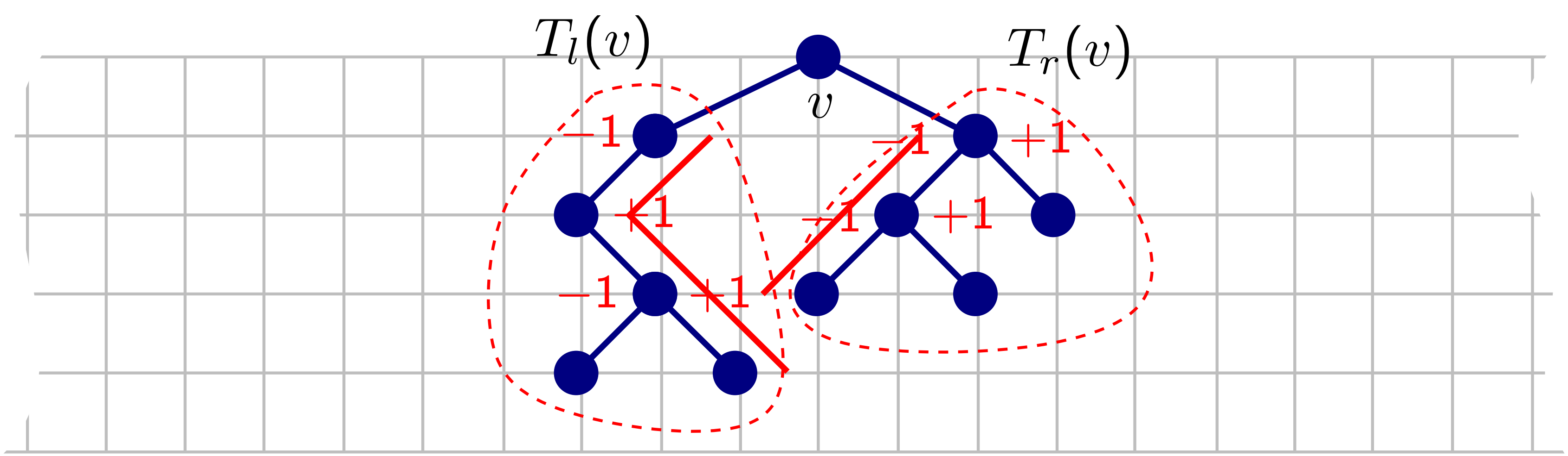

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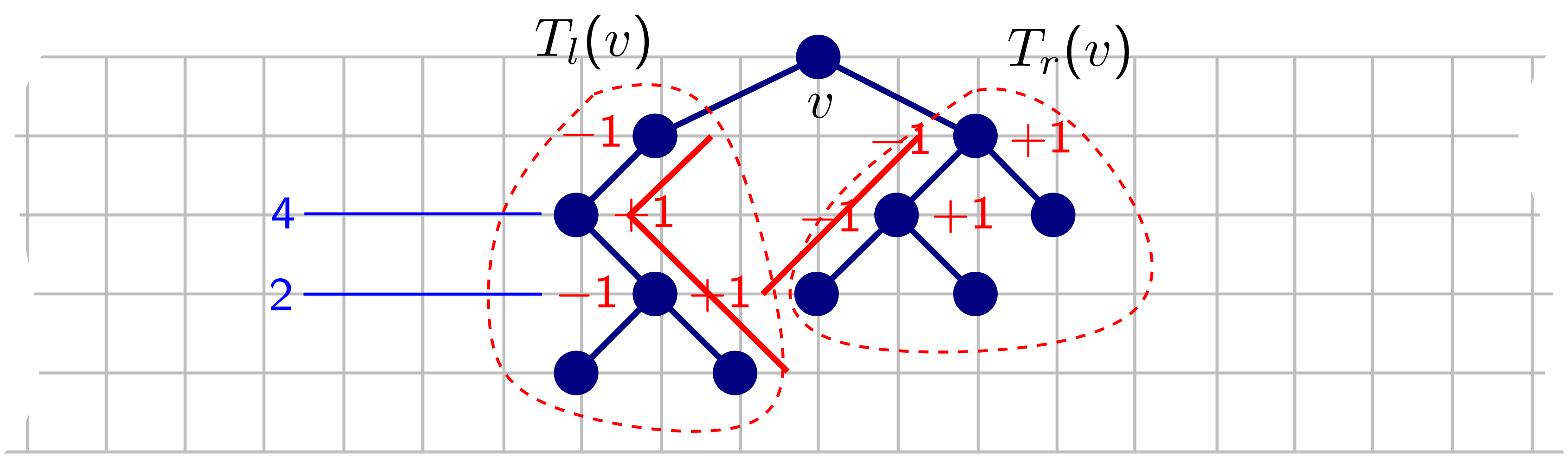

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Karlsruhe Institute of Technology

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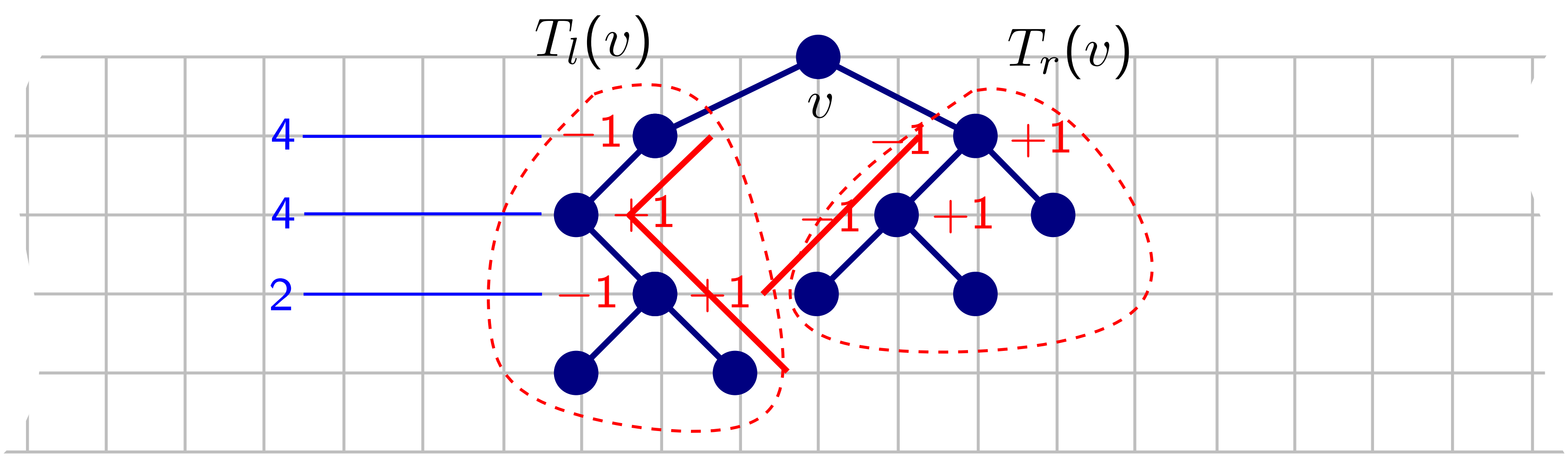

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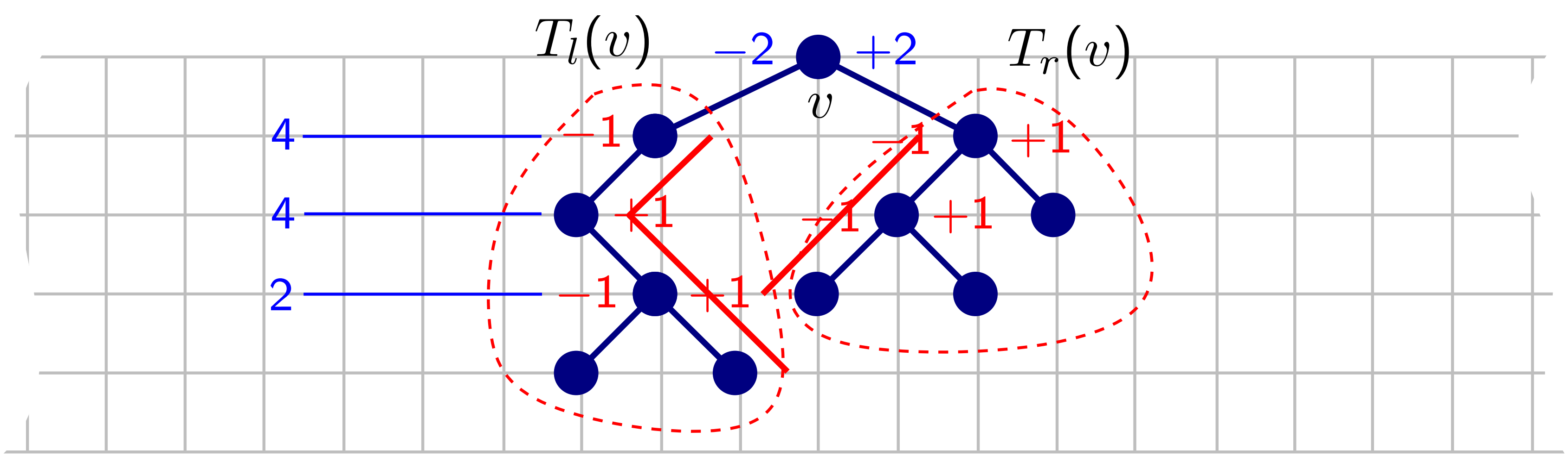

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- Assume at each vertex u (below v) we have stored the left and the right boundary of the subtree $T(u)$
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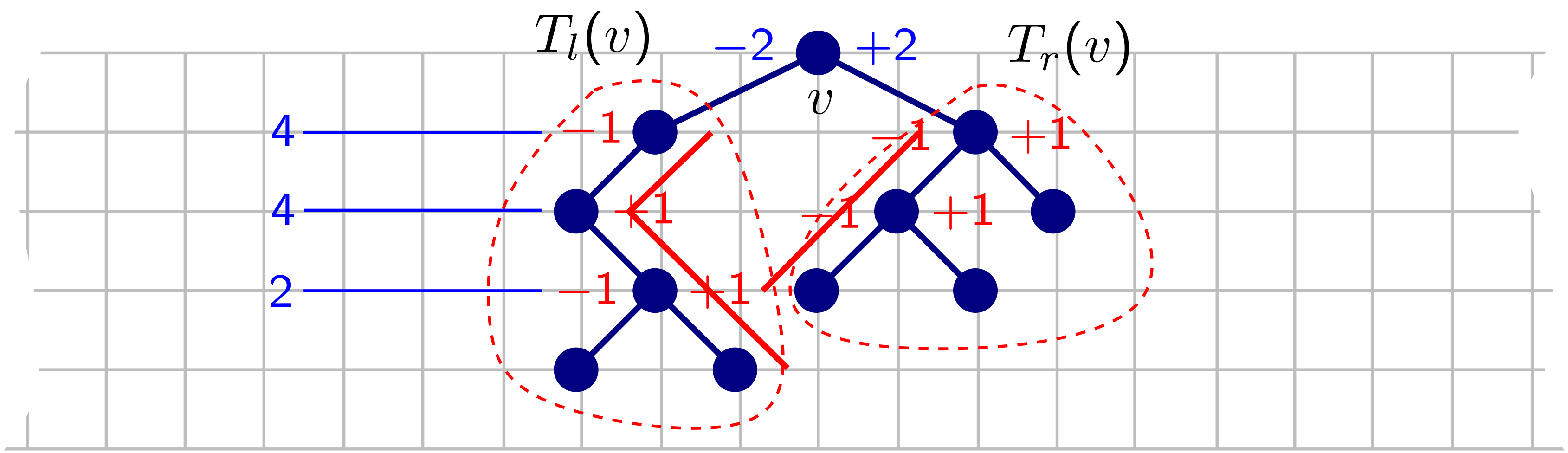

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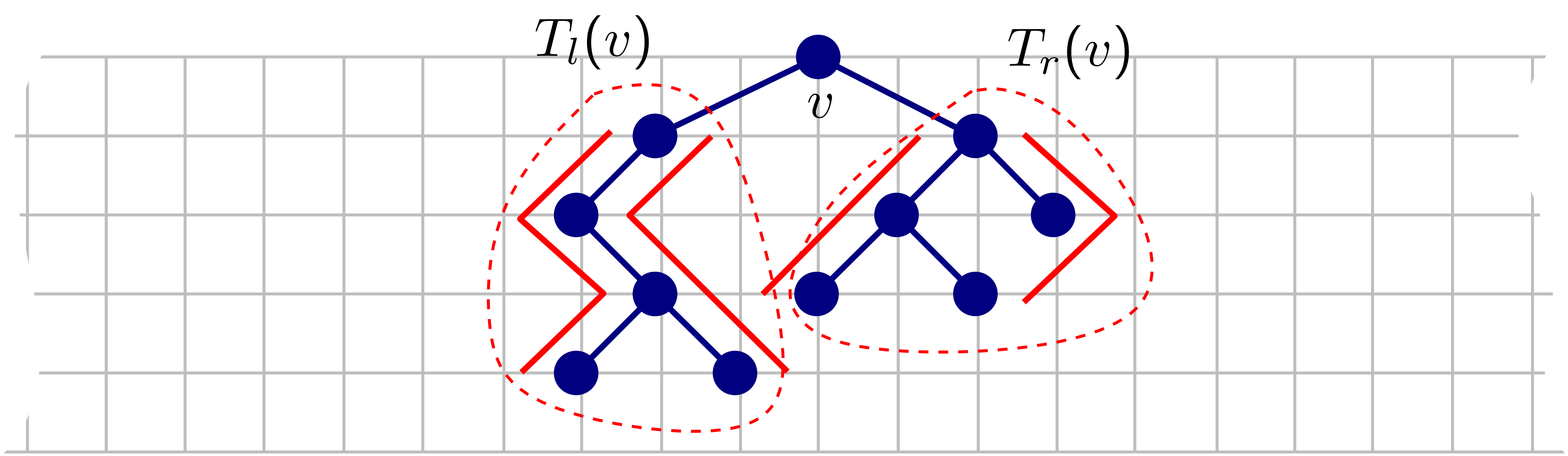

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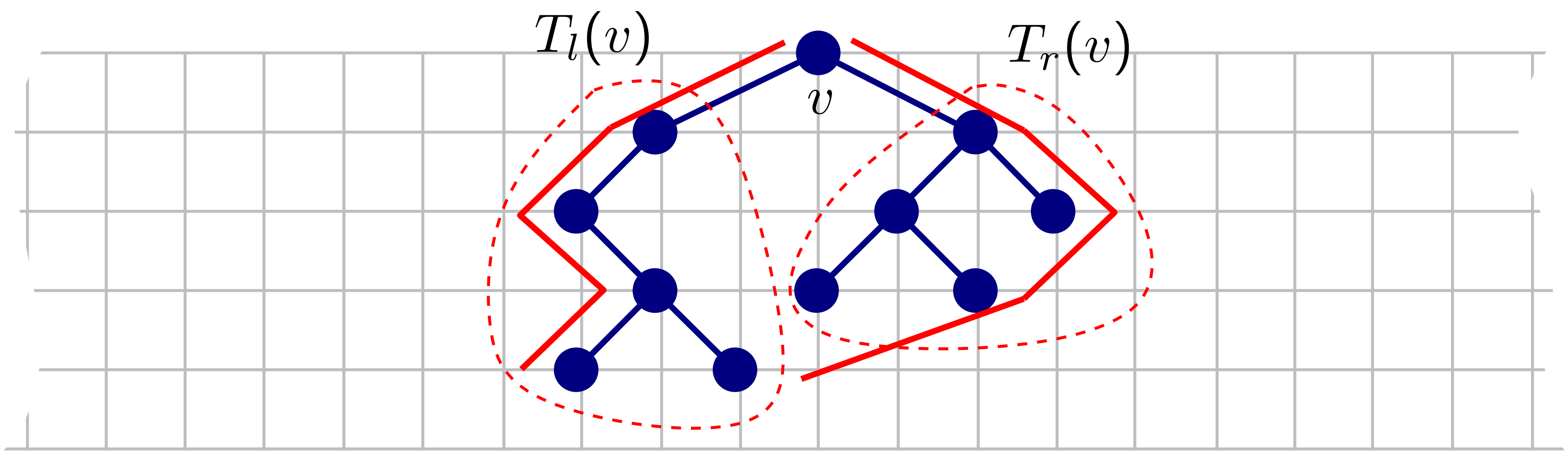

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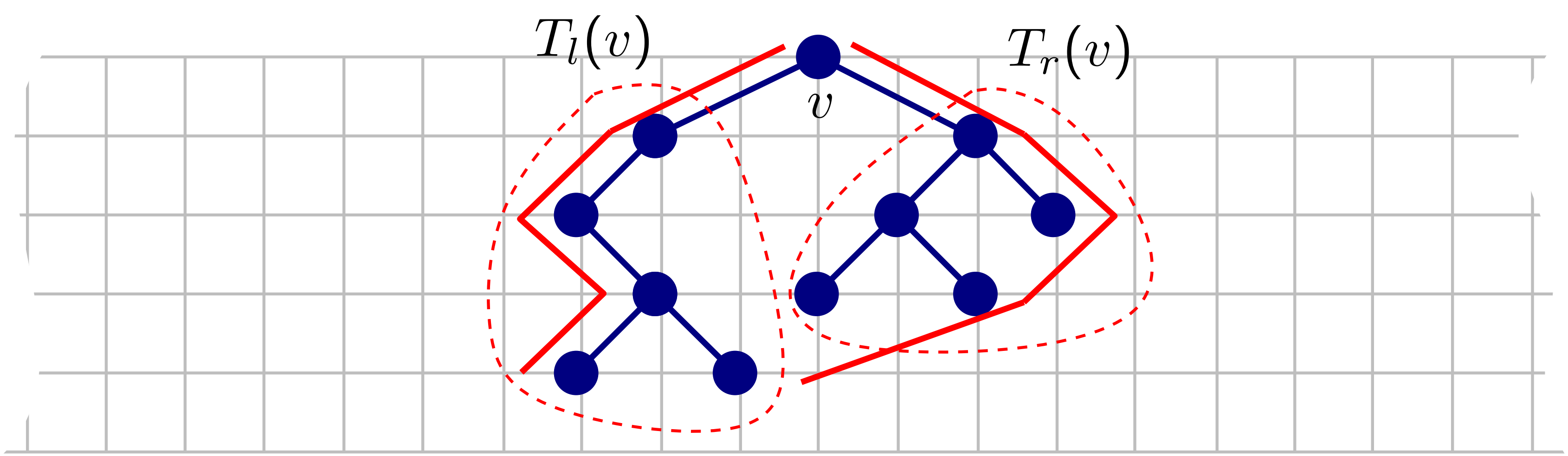
- Store at v the left and the right boundaries of $T(v)$



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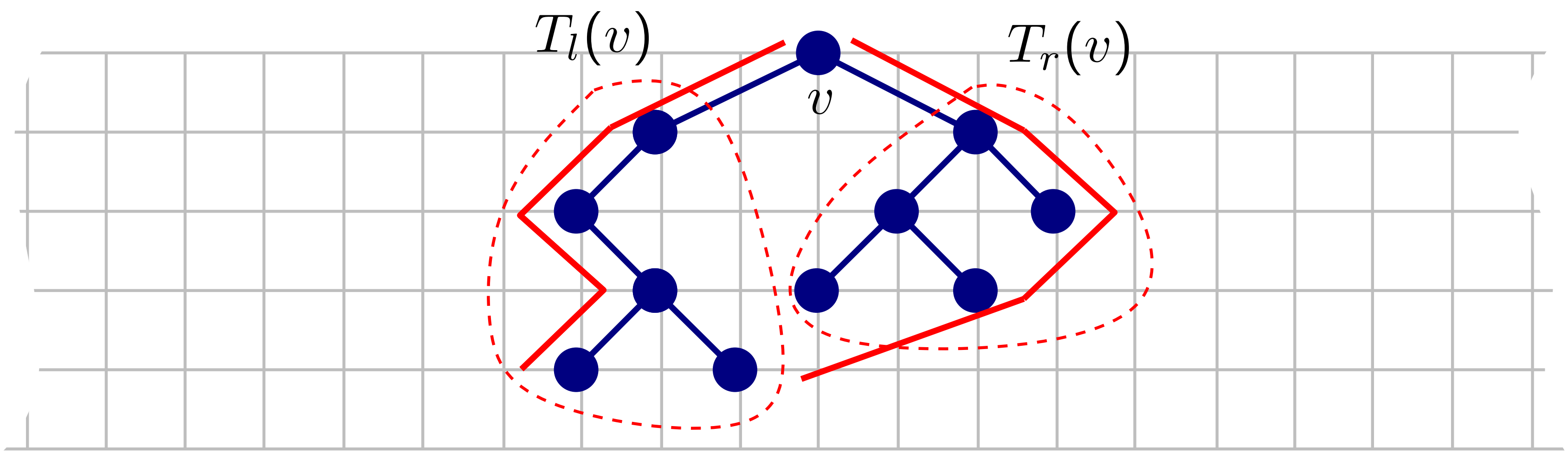
Implementation Details (postorder and preorder traversals)

Postorder traversal: For each vertex v compute horizontal displacement of the left and the right child

Preorder traversal: Compute x- and y-coordinates.



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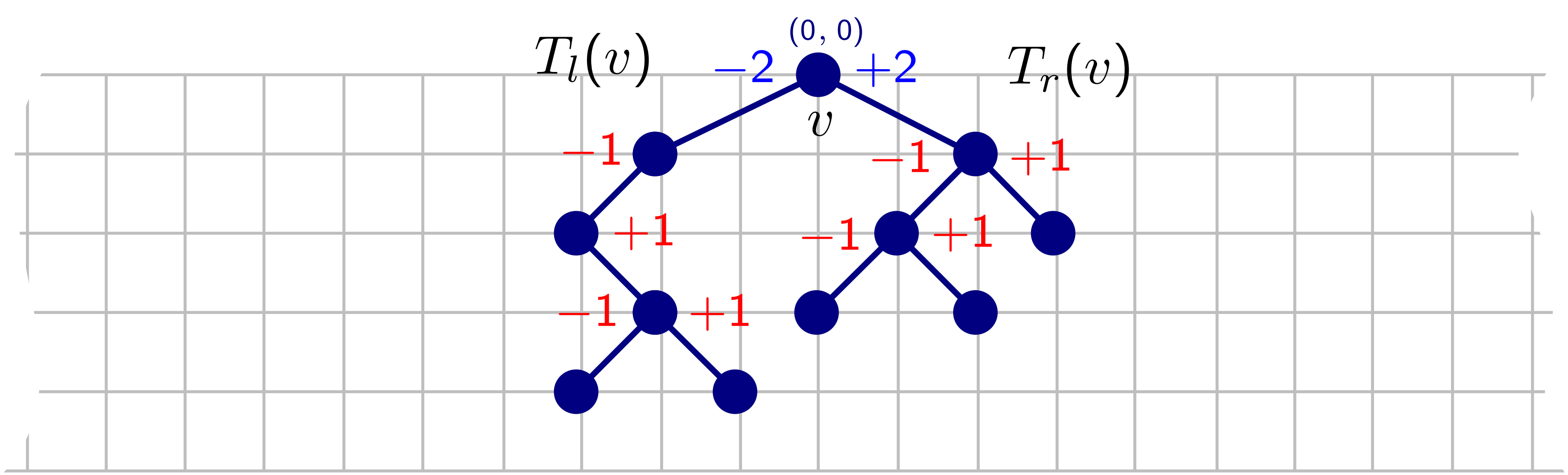

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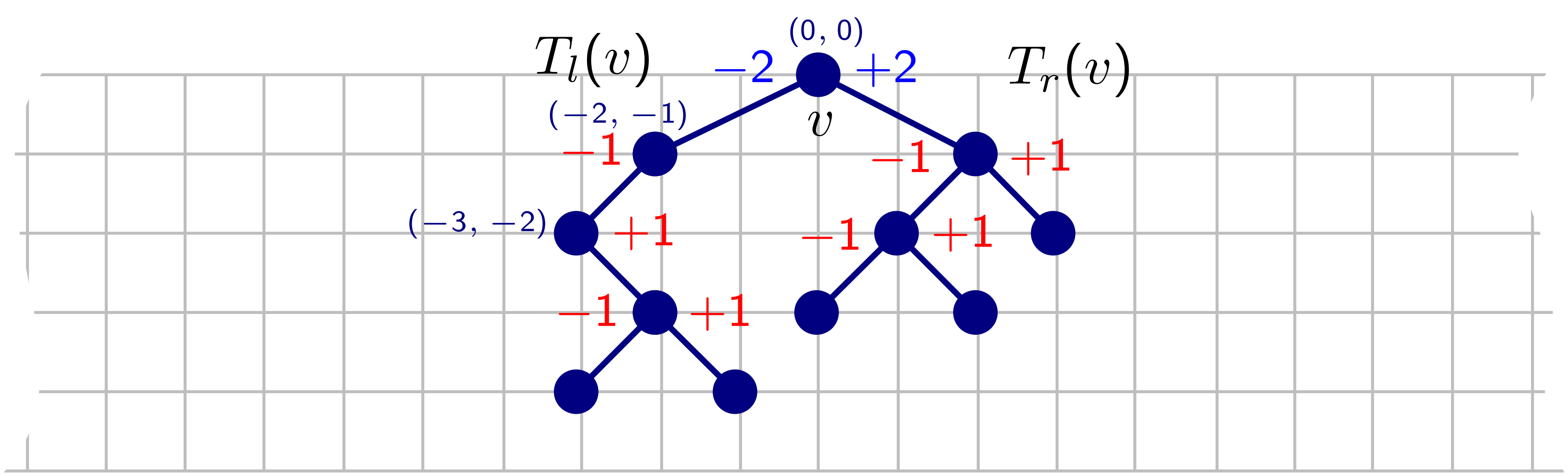

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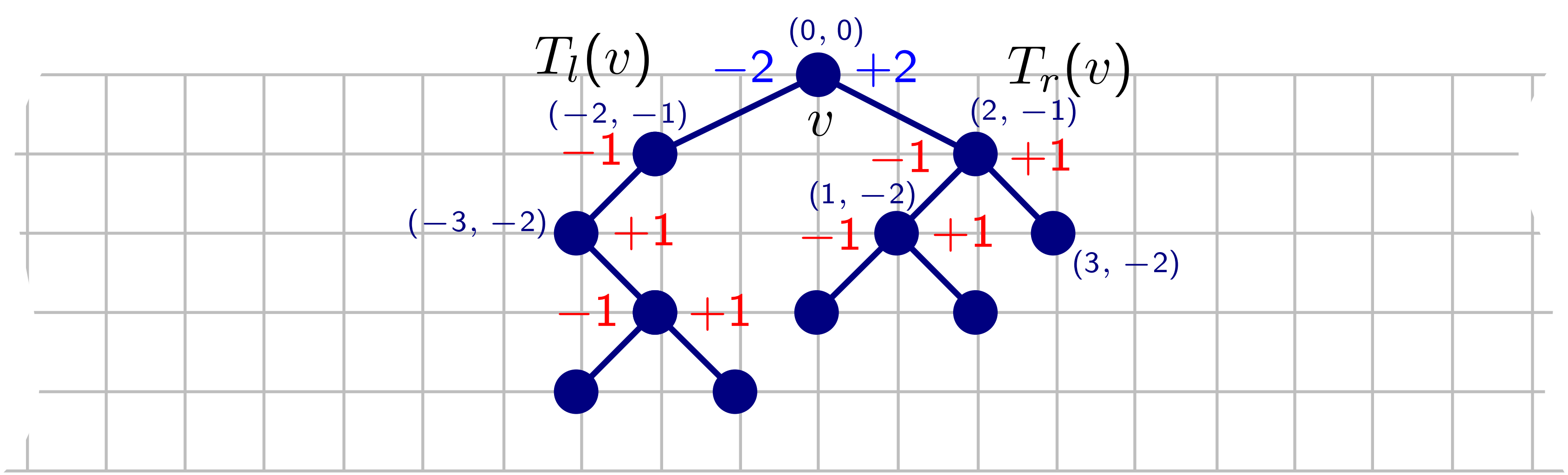

Level-based Layout: Time Complexity

Think and write down and then discuss with your neighbour(s)

5+5 min

- What is the time complexity of the algorithm?
- What should we keep in mind to achieve this time complexity?

Level-based Layout

Time Complexity

Postorder traversal: For each vertex v compute horizontal displacement of the left and the right child

- Assume at each vertex u (below v) we have stored the left and the right boundary of the subtree $T(u)$
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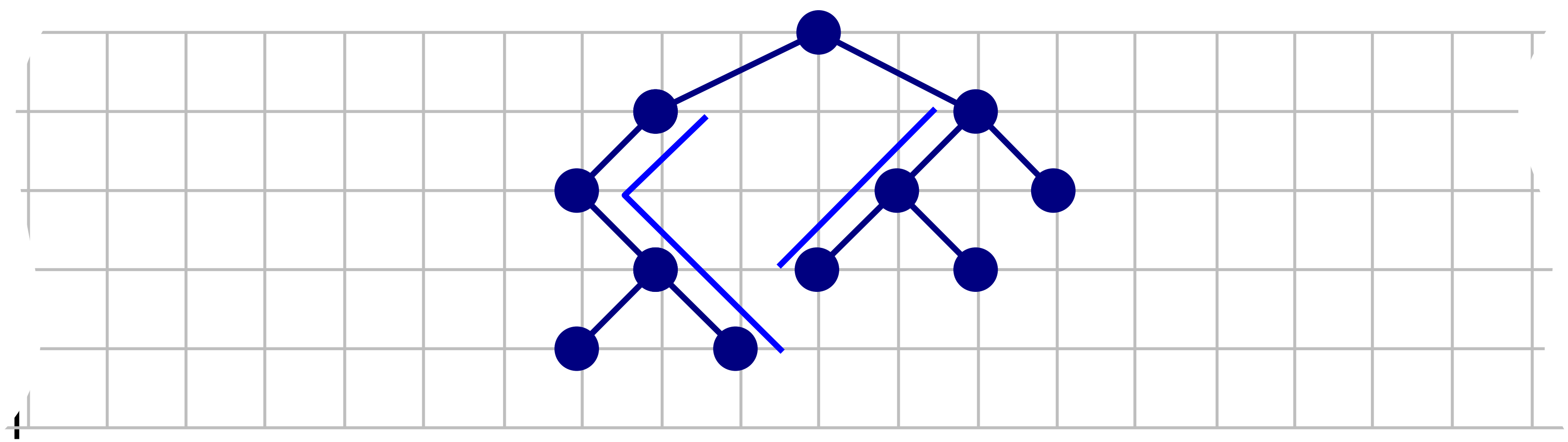
Preorder traversal: Compute x- and y-coordinates.



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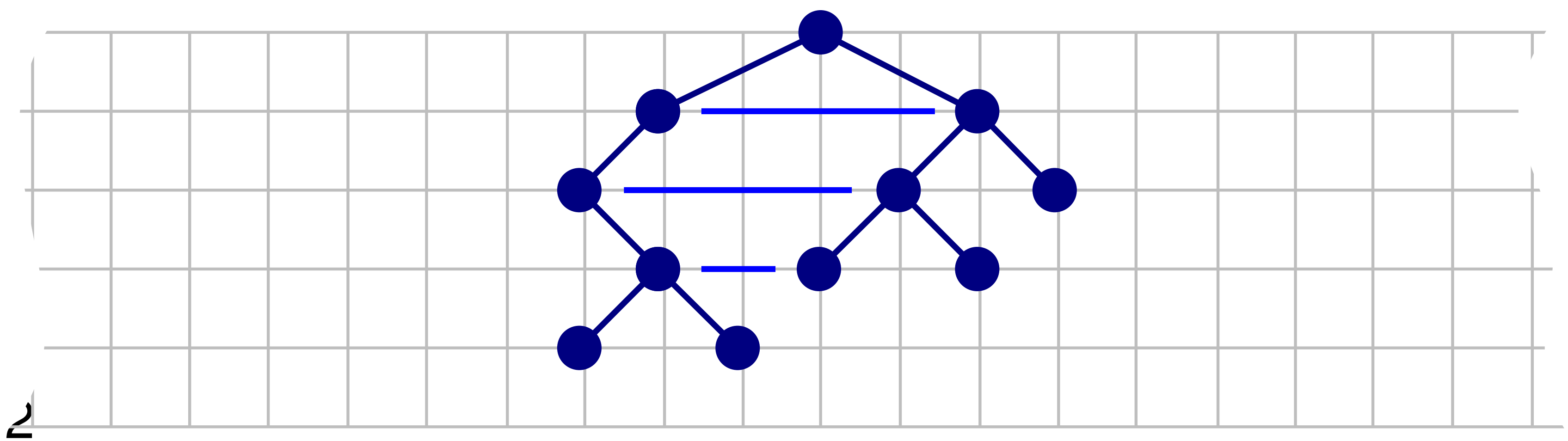

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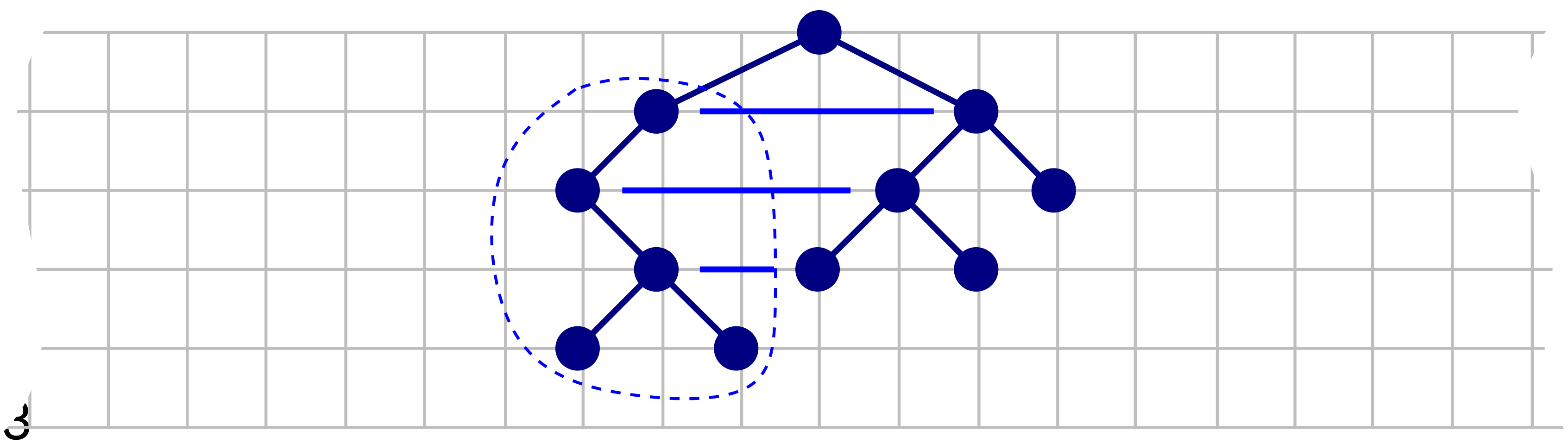

Karlsruhe Institute of Technology

Time Complexity

Postorder traversal: For each vertex v compute horizontal displacement of the left and the right child

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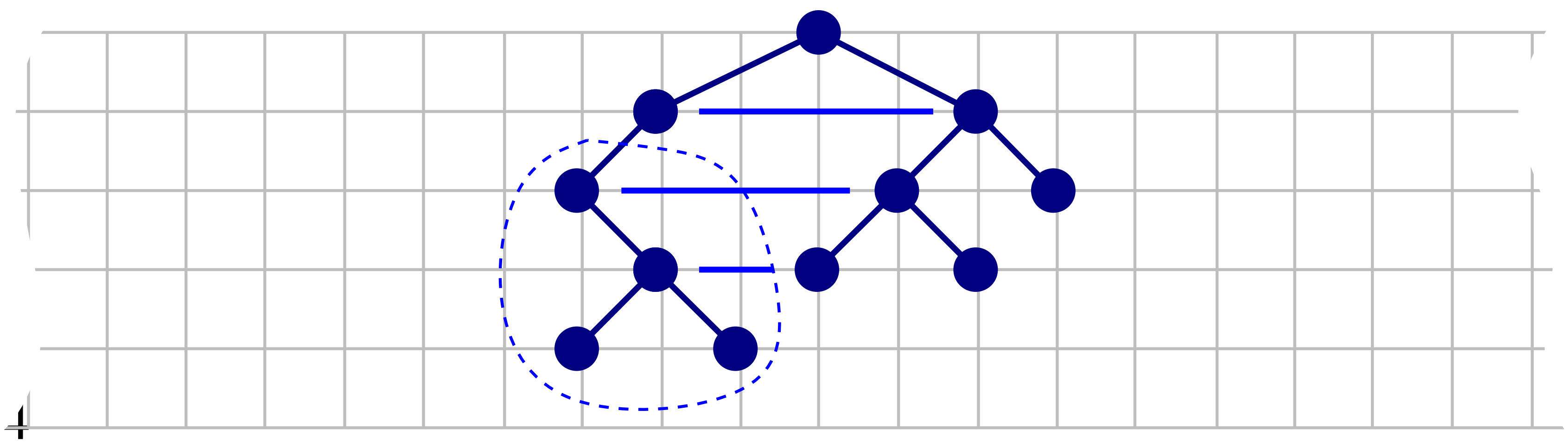

Level-based Layout

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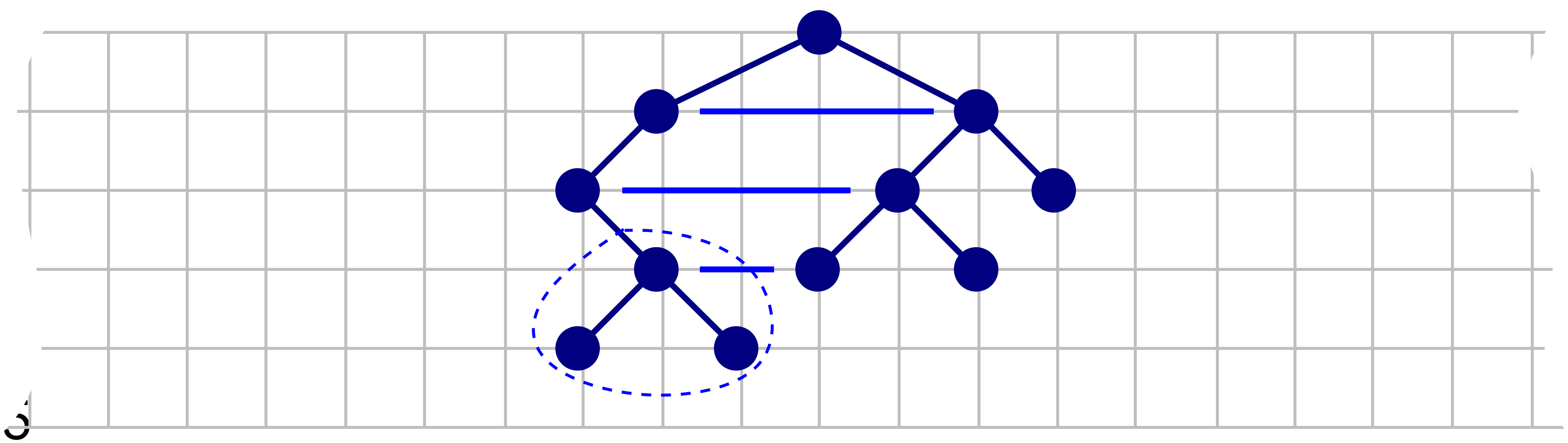

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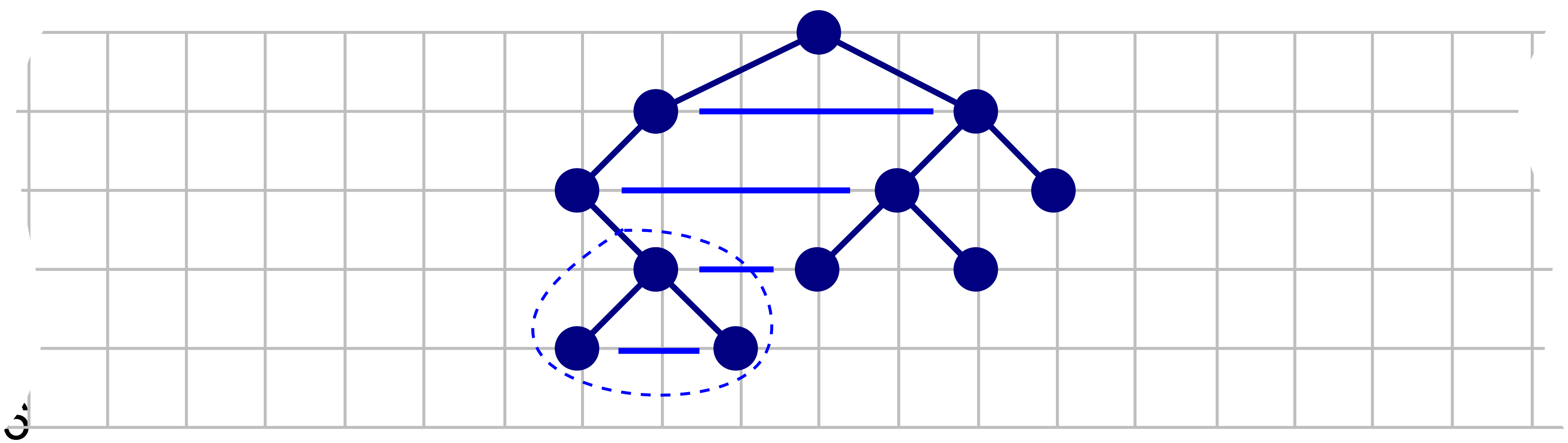

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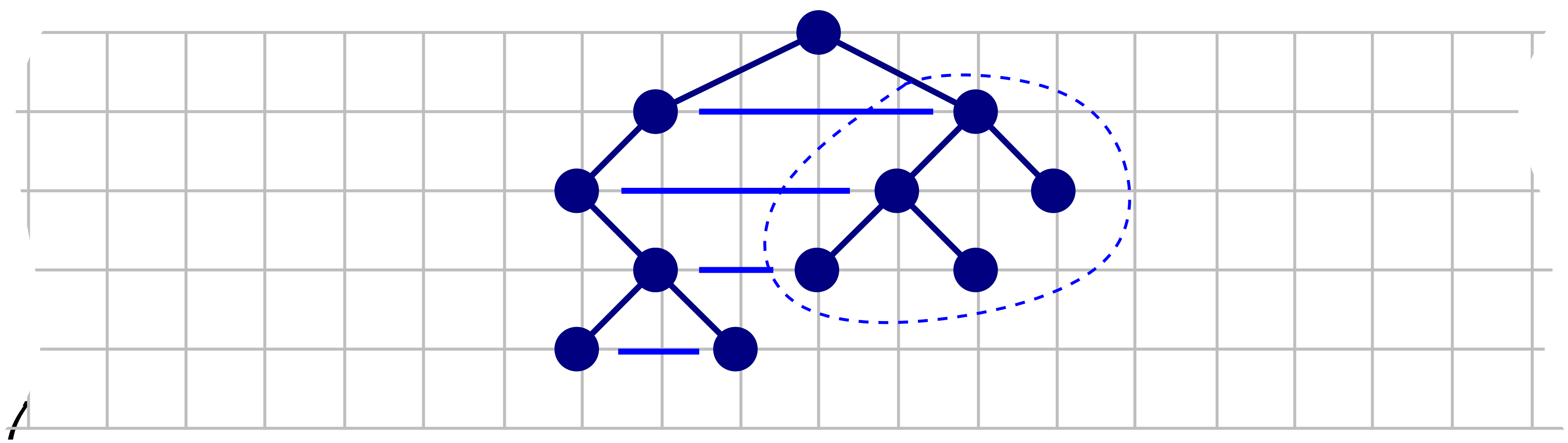

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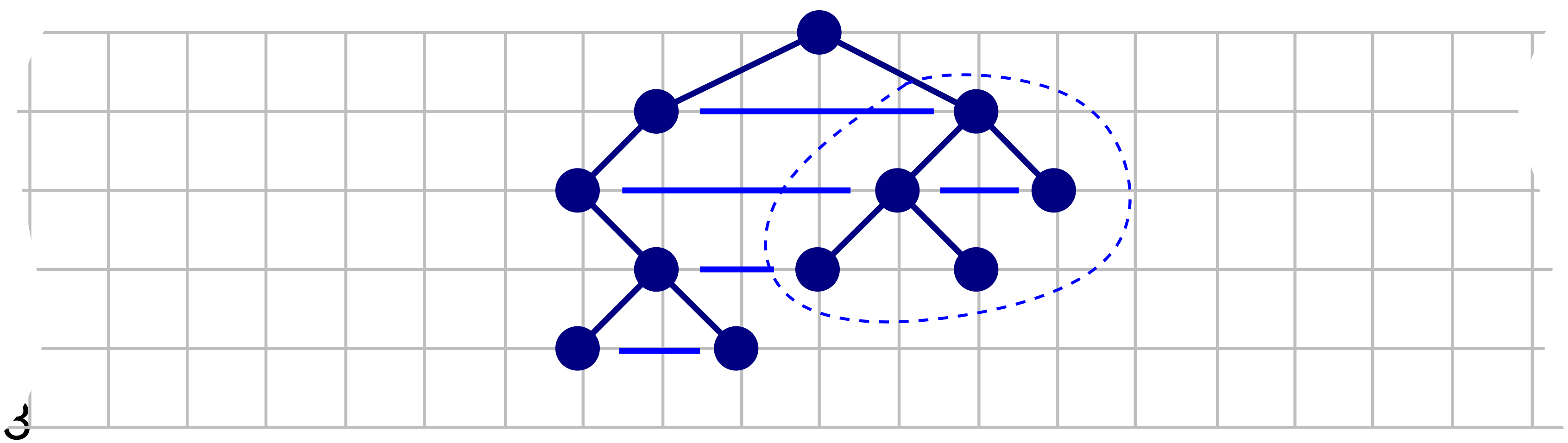

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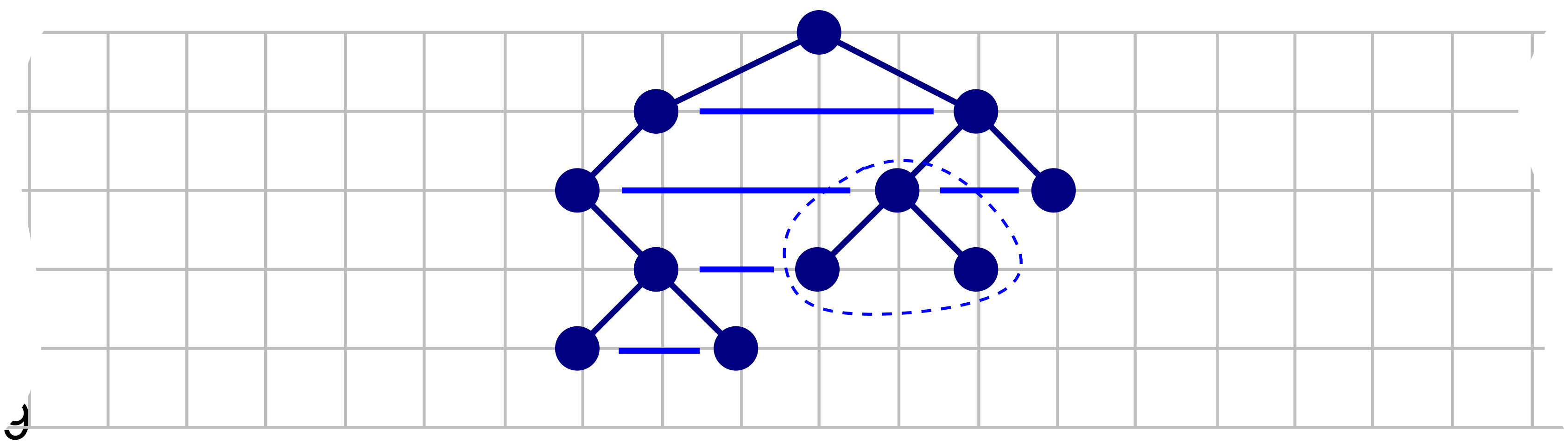

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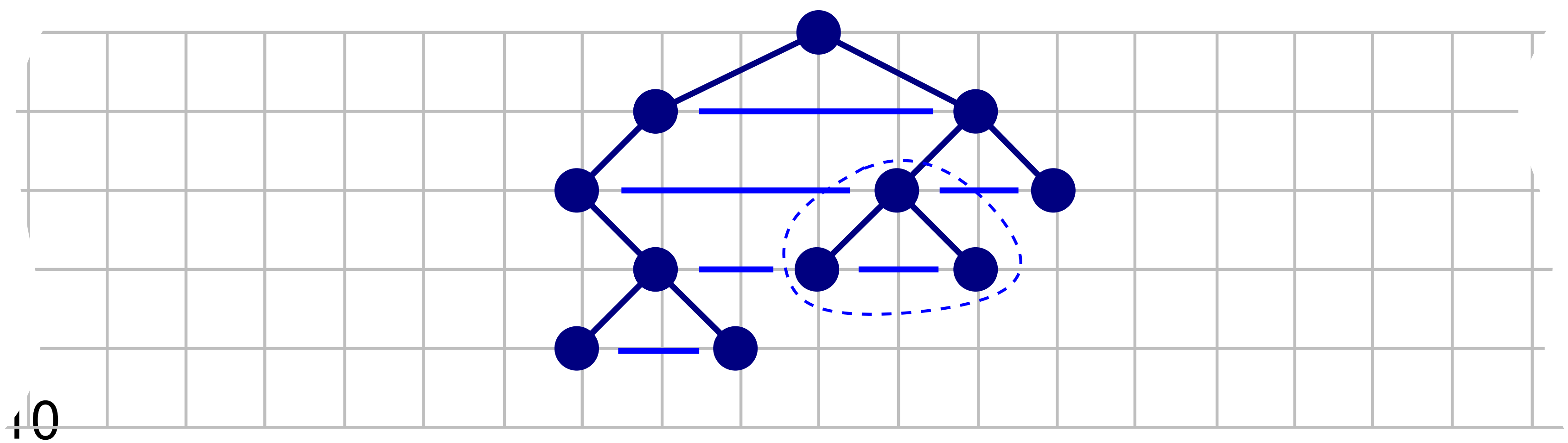

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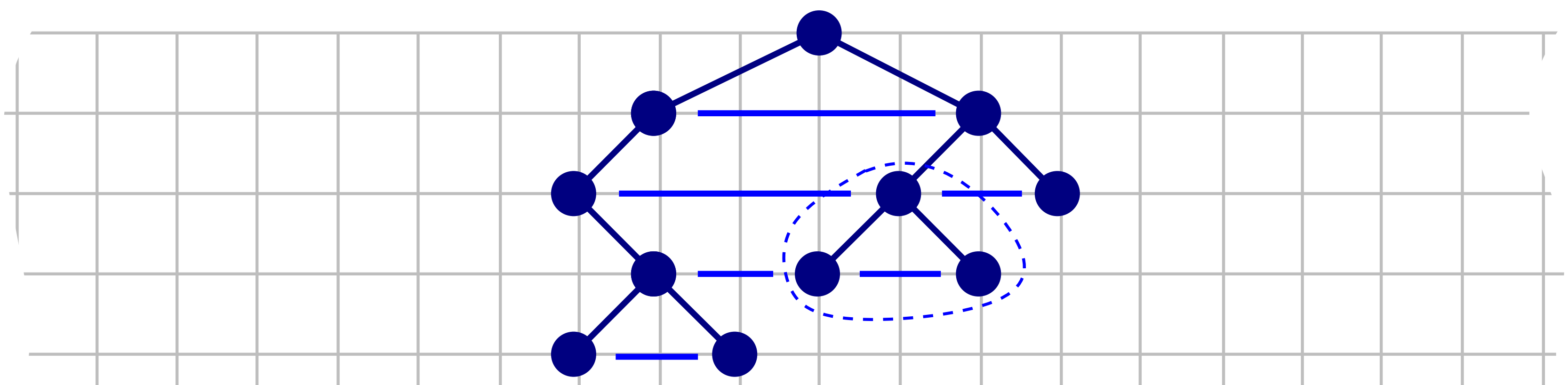

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$O(n)$

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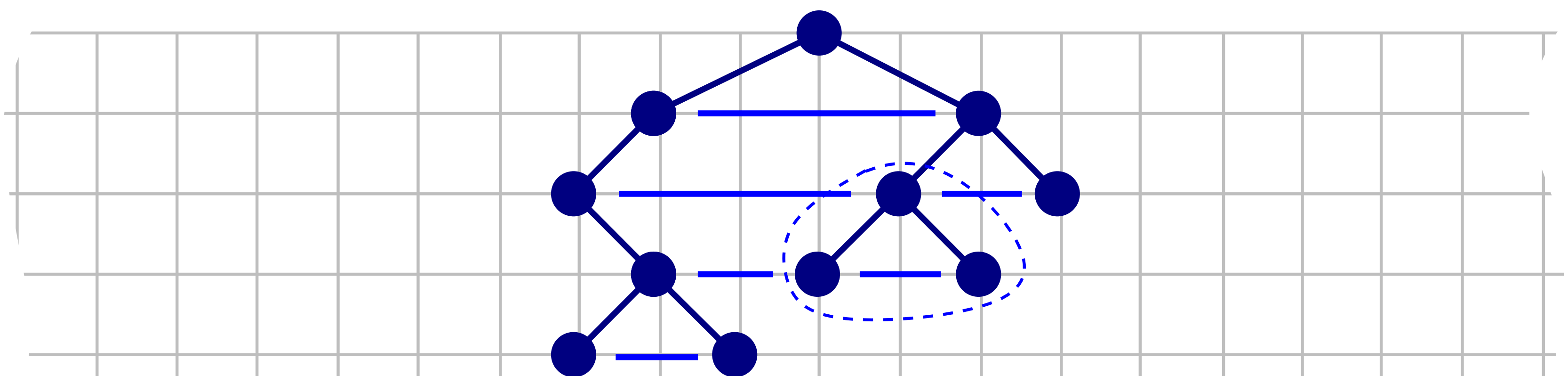

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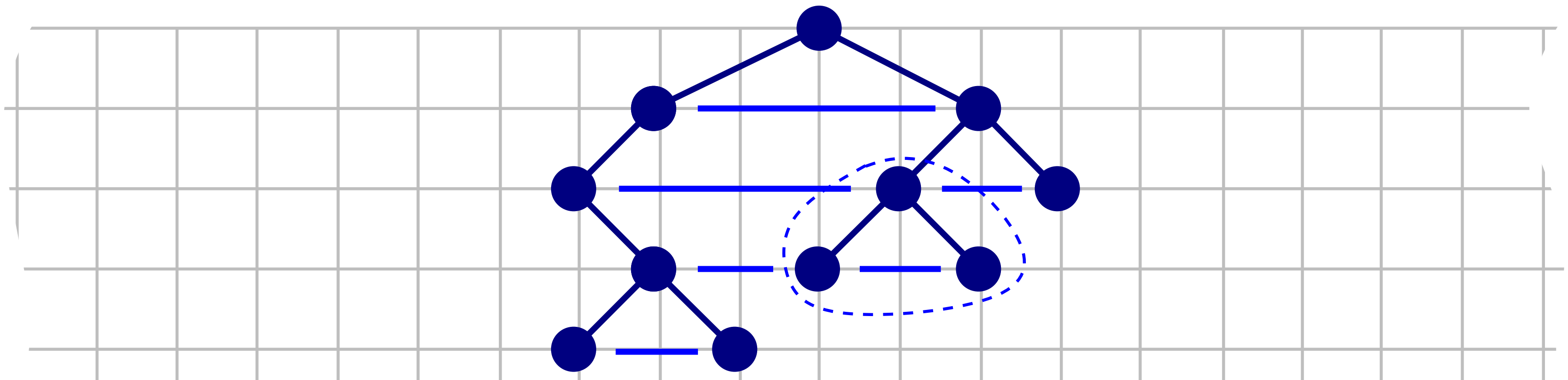

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Level-based Layout

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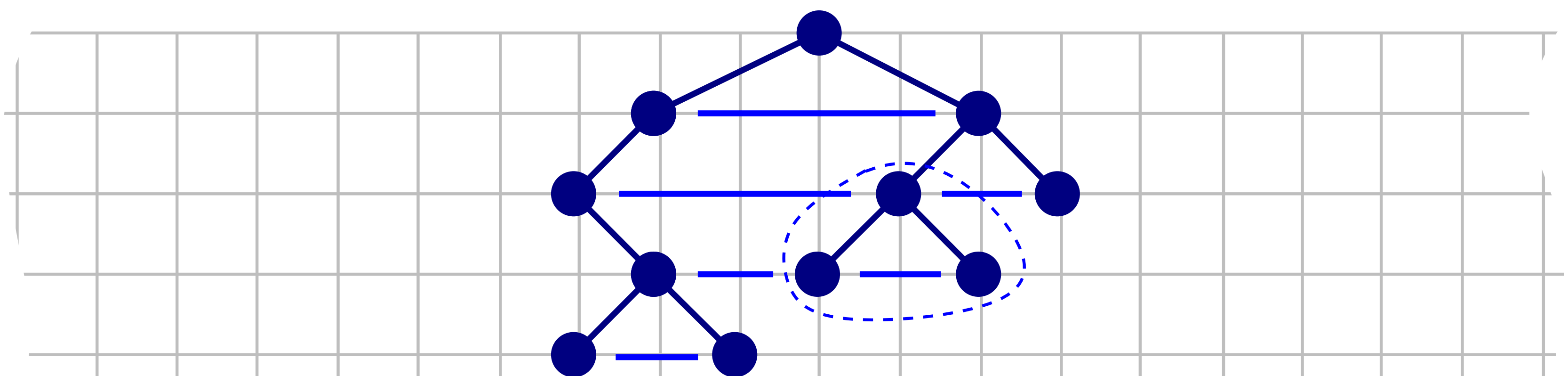

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Level-based Layout

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Theorem (Reingold & Tilford)

Let T be a binary tree with n vertices. Algorithm (R & T) constructs a drawing Γ of T in $O(n)$ time, such that:

- Γ is planar and straight-line
- $\forall v \in T$ y-coordinate of v is $-depth(v)$
- Vertical and horizontal distance is at least 1
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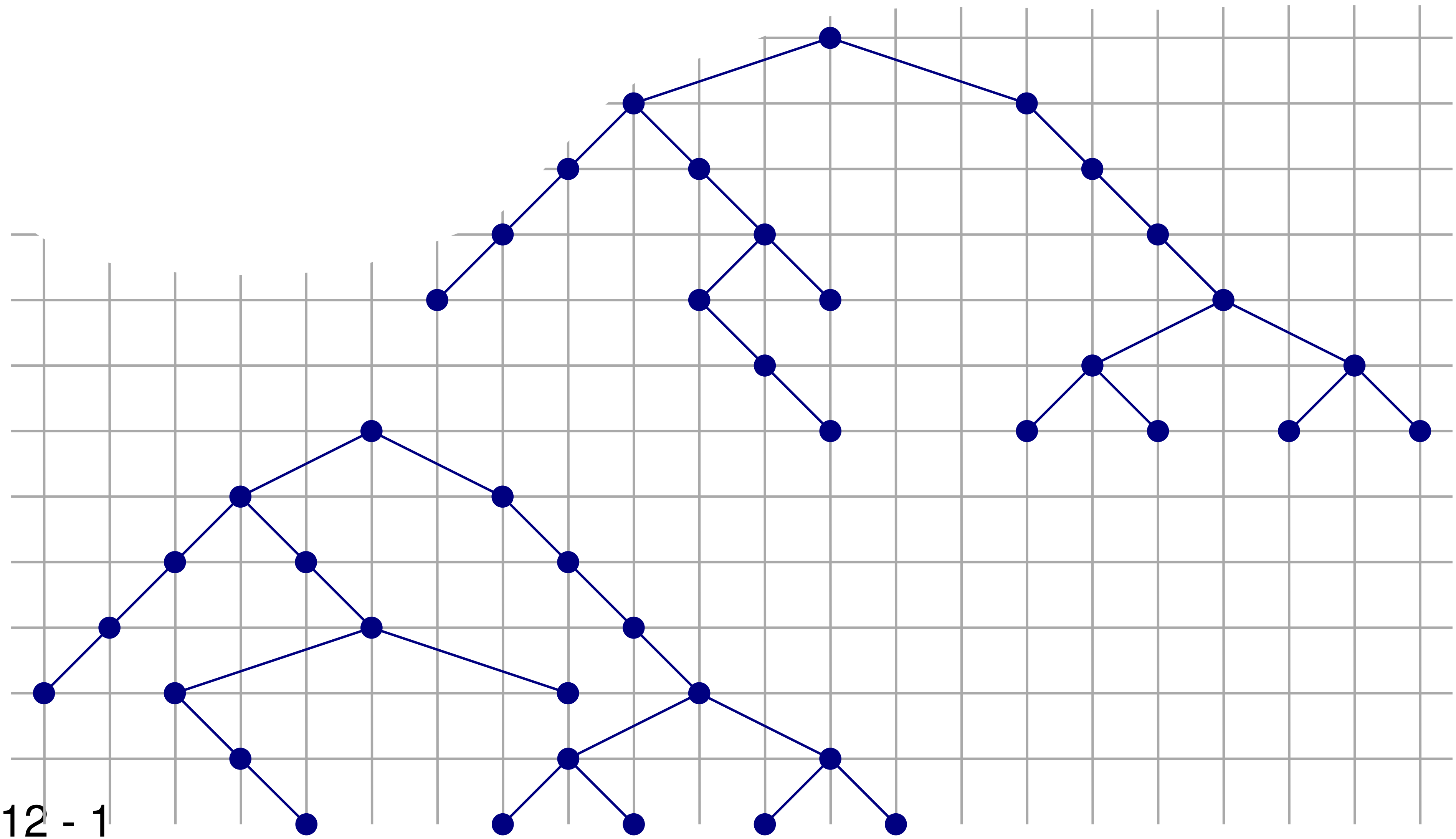
- Simply isomorphic subtrees have congruent (coincident) drawing, up to translation
- Axially isomorphic trees have congruent drawing, up to translation and reflection around y-axis

Level-based Layout

- The presented algorithm tries to minimize width



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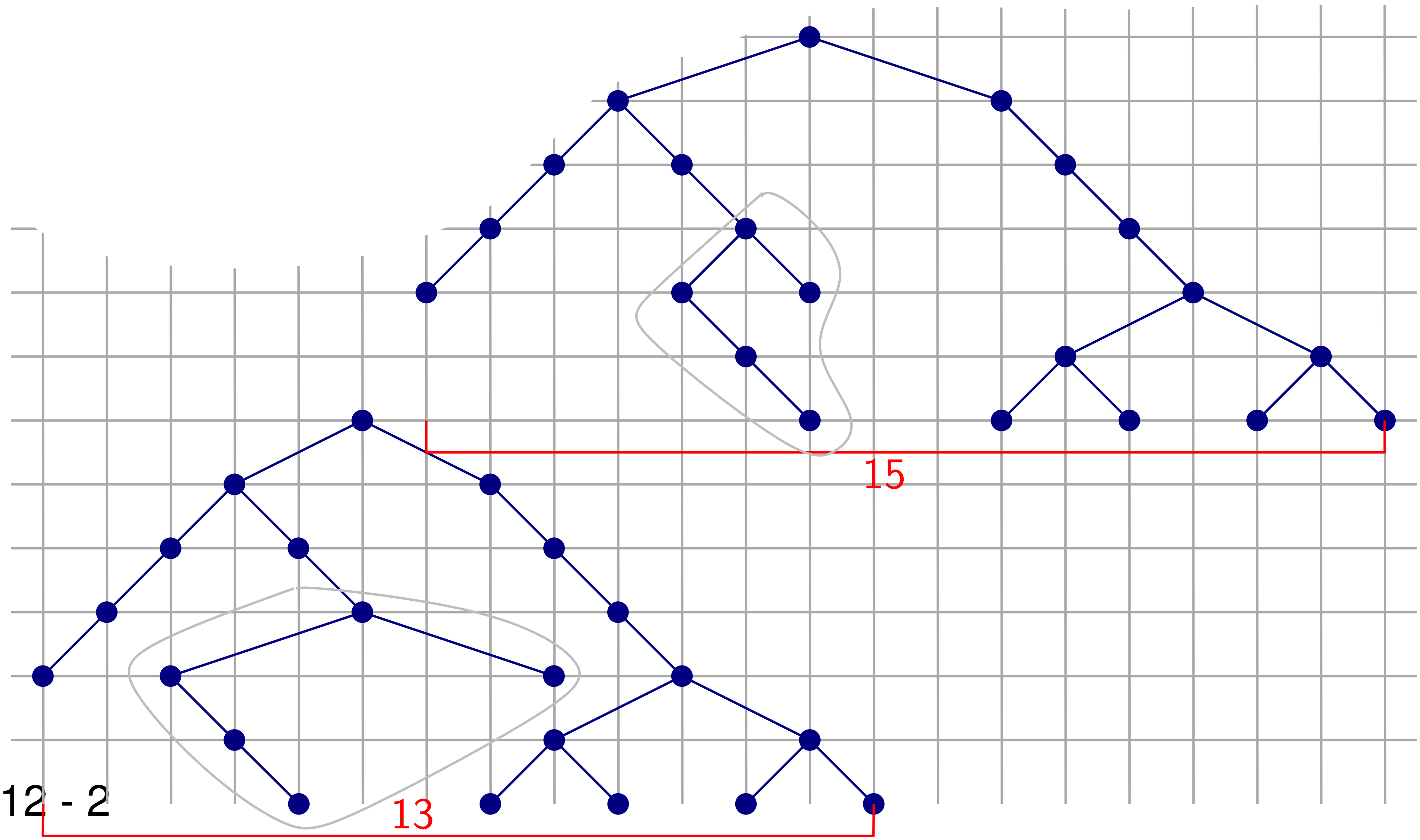

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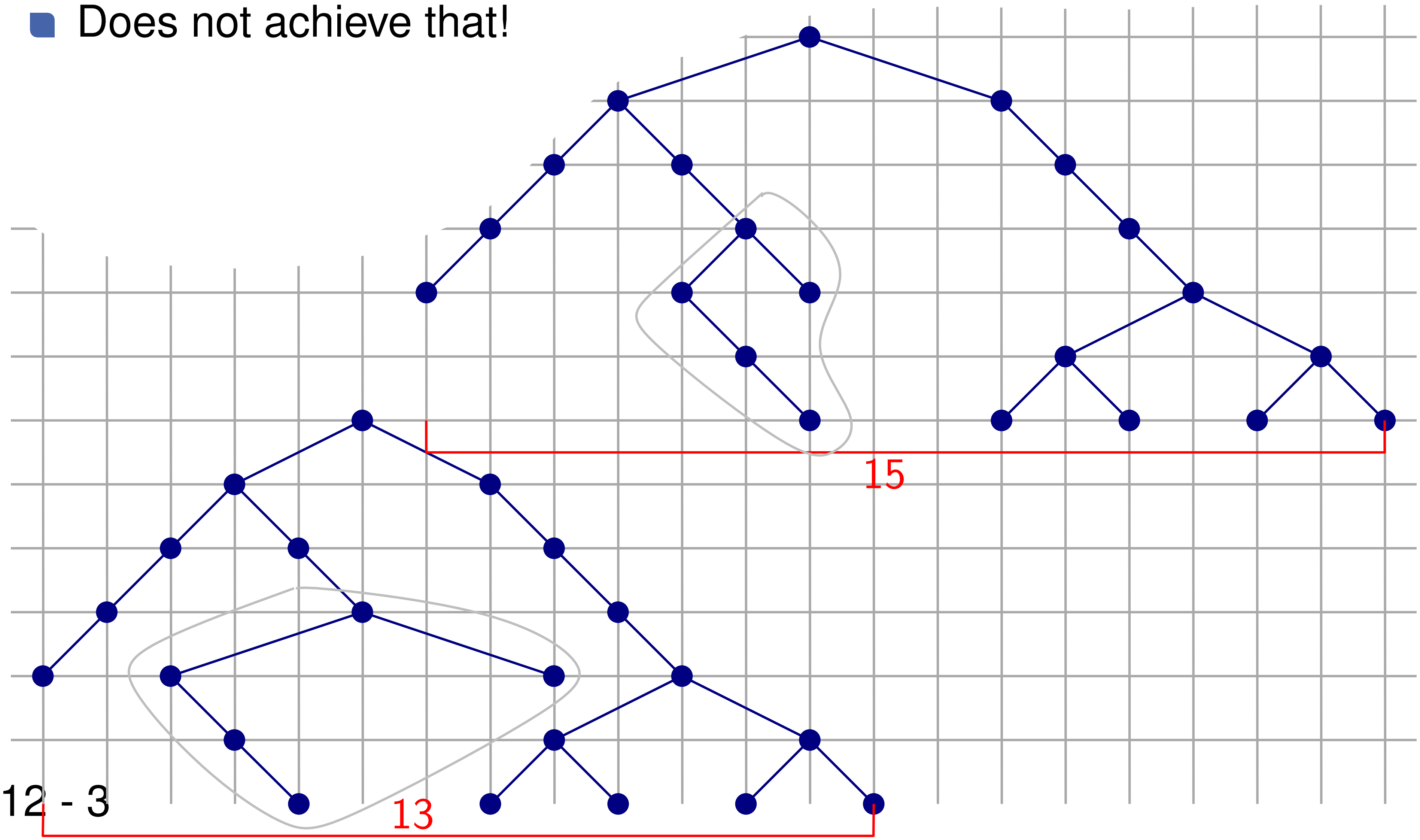

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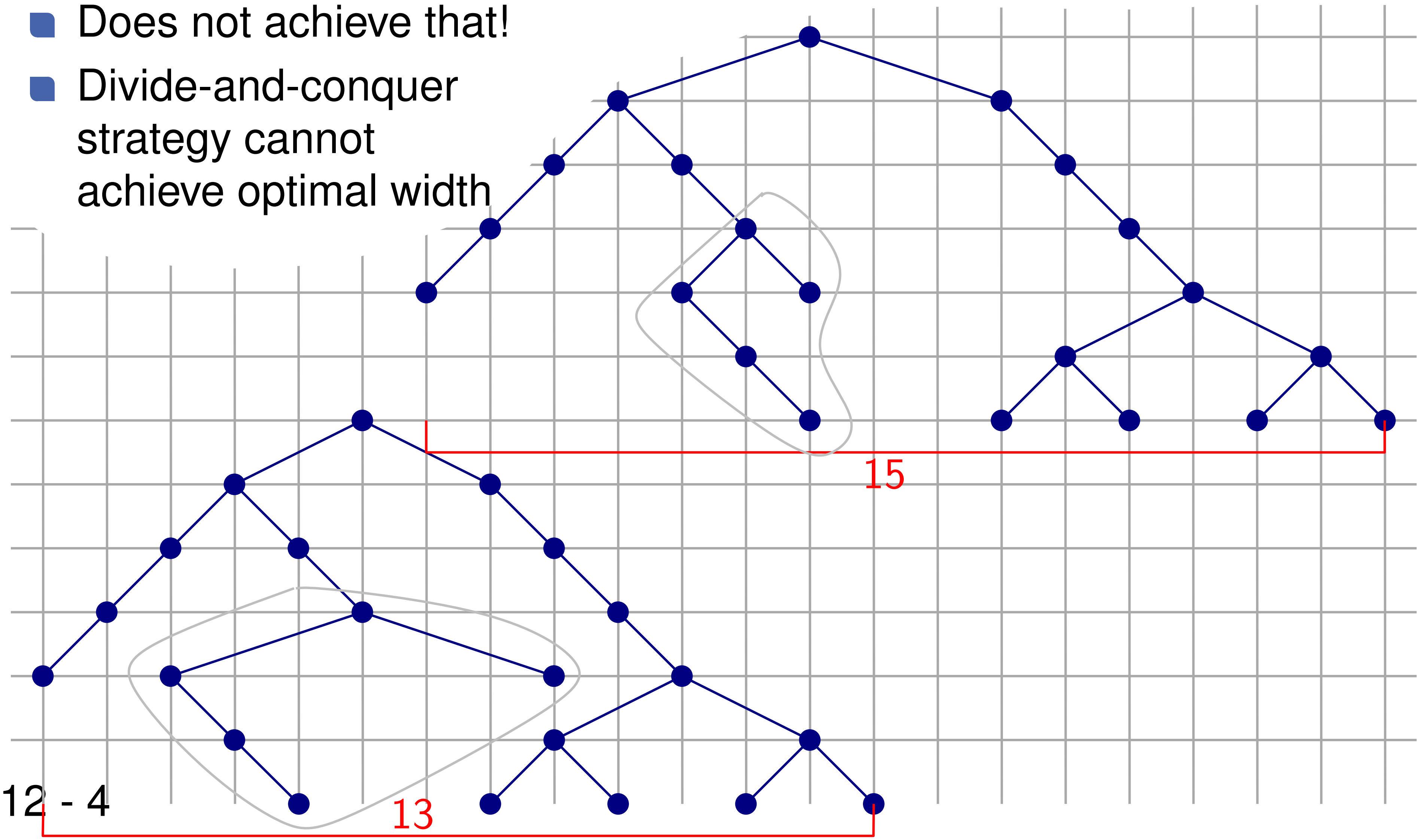

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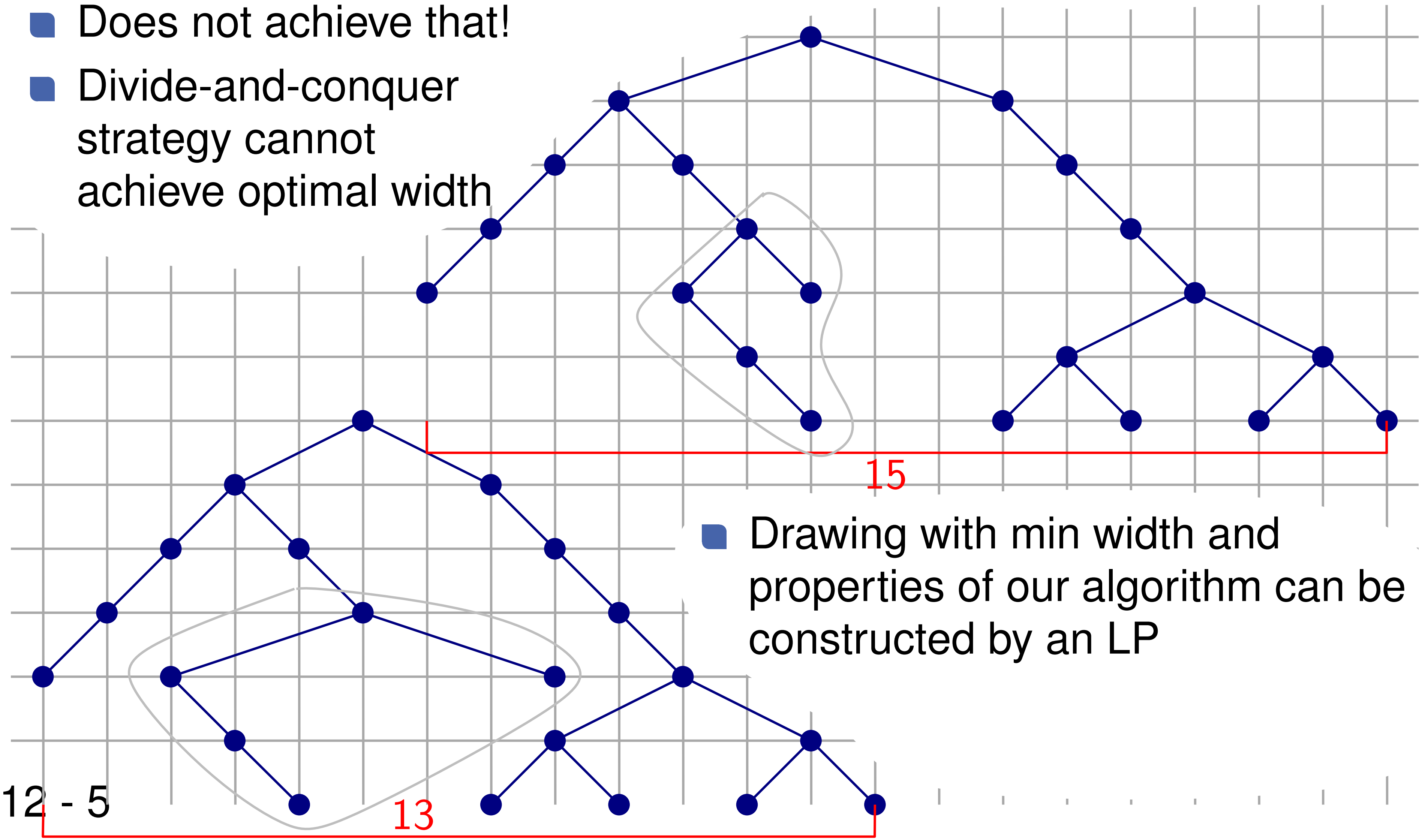

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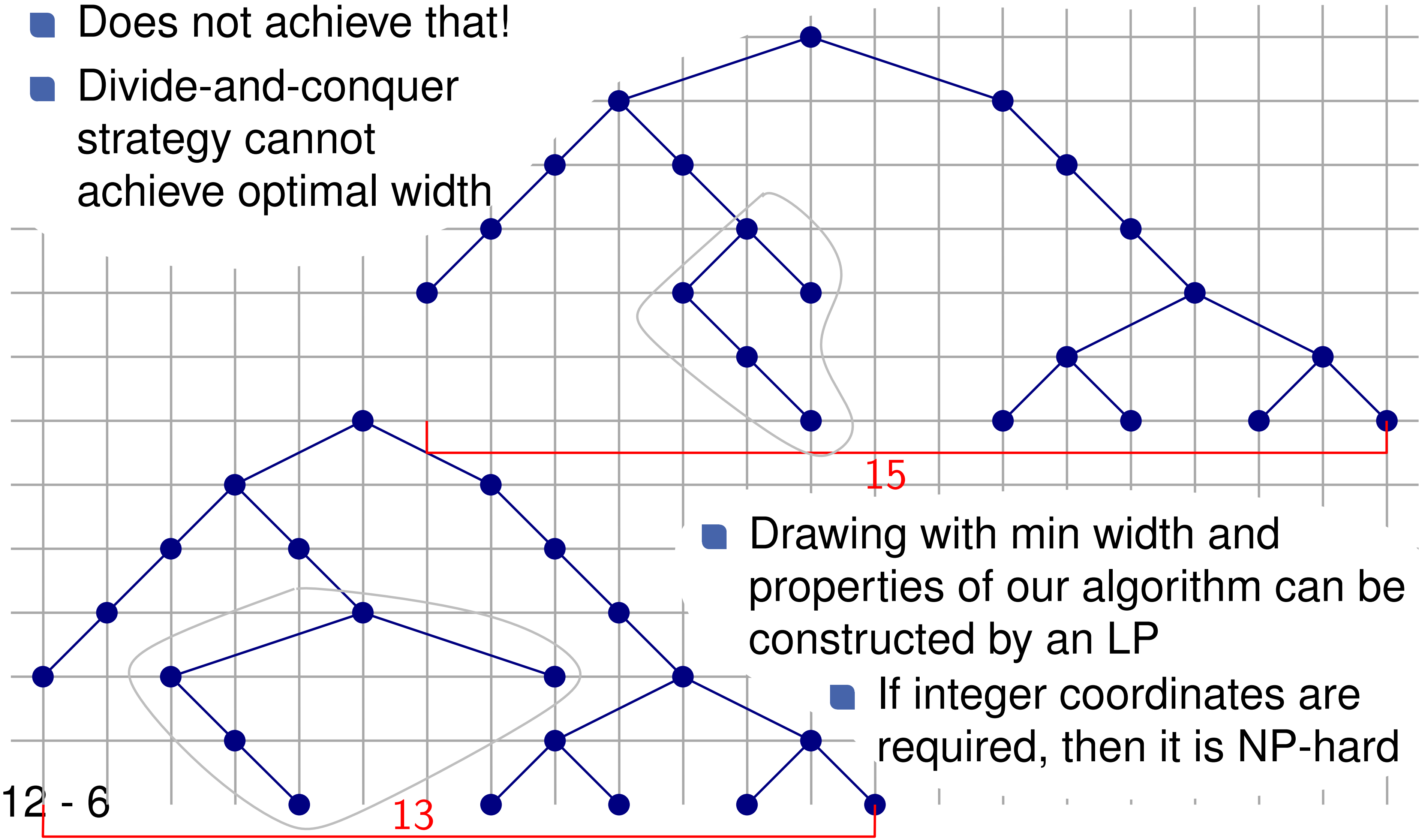

- Drawing with min width and properties of our algorithm can be constructed by an LP
  - If integer coordinates are required, then it is NP-hard

Literature

Level-based Layout for Trees

- Book Di Battista et al: Chapter 3.1.2
- Skript: Chapter 6.1