Algorithmen zur Visualisierung von Graphen
Wintersemester 2018/2019
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Exercise Sheet 4

Discussion: 19. December 2017

Exercise 1: Feedback Arc Set ★★

In the lecture we introduced the Minimum Feedback Arc Set and the Minimum Feedback Set problems. Let $D = (V, A)$ be a directed graph and A' be a subset of A. The set A' is a *feedback arc set of* D if $D_f = (V, A \setminus A')$ is acyclic. If $D_r = (V, (A \setminus A') \cup \{vu \mid uv \in A'\})$ is acyclic, then A' is a *feedback set of* D. Every feedback set is a feedback arc set, the reverse is not necessarily true. Prove the following.

Lemma 1 *A set $A' \subset A$ is a minimum feedback arc set of D if and only if A' is a minimum feedback set of D.*

Exercise 2: Crossings in Layered Layouts ★

Prove the following.

Lemma 2 *The barycenter heuristic computes an optimal solution of the one-sided crossing minimization problem, if the instance admits a planar drawing.*

Exercise 3: Counting Crossings ★★

Prove the following lemmas.

Let $\pi : \{1, \ldots, n\} \to \{1, \ldots, n\}$ be a permutation. A pair (i, j) with $1 \leq i < j \leq n$ is an *inversion*, if $\pi(i) > \pi(j)$.

Lemma 3 *The number of inversions of a permutation π can be counted in $O(n \log n)$ time.*

Hint: Use an approach similar to merge sort.

Lemma 4 *Let Γ be a straight-line drawing of a bipartite graph $G = (A \dot\cup B, E)$ where the vertices of A and B are drawn on separate layers. Then the number of crossing in Γ can be counted in $O(|E| \log |V|)$ time.*

Can all crossings be reported in the same time?