Exercise Sheet 1
Discussion: 24. October 2018

Exercise 1: Tree Layouts

Let $T = (V, E)$ be a rooted binary tree. For a vertex $v \in V$, we denote its $x$-coordinate by $x(v)$ and its $y$-coordinate by $y(v)$.

(a) We draw the tree $T$ as follows. For each vertex $v$ of $T$, we set $x(v)$ equal to the rank of $v$ in a post-order traversal of $T$, and $y(v)$ equal to its depth in $T$.
   (i) Show that the resulting straight-line drawing is planar.
   (ii) What is the area of the drawing?
   (iii) What happens if instead of a post-order traversal we use a pre-order traversal?
   (iv) Can the algorithm be extended to rooted ordered trees?

(b) We draw the tree $T$ as follows. For each vertex $v$ of $T$, we set $x(v)$ equal to the rank of $v$ in a pre-order traversal of $T$, and $y(v)$ equal to the rank of $v$ in a post-order traversal of $T$.
   (i) Show that the resulting straight-line drawing is planar and strictly downward (for each edge $(u, v)$, with $\text{depth}(u) < \text{depth}(v)$, it holds that $y(u) > y(v)$).
   (ii) Show that a vertex $v$ is in the subtree rooted at vertex $u$ if and only if $x(v) > x(u)$ and $y(v) < y(u)$.
   (iii) Do isomorphic subtrees have congruent drawings?

Exercise 2: HV-Layouts

Give an algorithm that for a given $n$-vertex binary tree constructs an HV-layout with minimum area in $O(n^2)$ time. Consider both ordered and non-ordered trees.
Exercise 3: Layouts of General Trees

Let $T = (V, E)$ be an arbitrary rooted tree (i.e., not necessarily binary). Prove that a planar straight-line drawing $\Gamma$ of $T$ such that siblings (vertices with the parent) have the same $y$-coordinate, parent-vertices are centered with respect to their children, and the area of $\Gamma$ is in $O(n^2)$, can be computed in $O(n)$ time.

Exercise 4: Minimal-Width Layout

Let $T = (V, E)$ be a rooted binary tree with a BFS-ordering and let $\text{depth}(v)$ be the respective BFS-level of a vertex $v \in V$. Formulate a linear program (LP) that computes a planar straight-line drawing $\Gamma$ of $T$ with minimal width such that $\Gamma$ respects the BFS-ordering, parent nodes are centered with respect to its children, and each vertex $v$ has $-\text{depth}(v)$ as $y$-coordinate. Is the running time of the resulting algorithm polynomial in the size of $T$?