Algorithms for graph visualization

Divide and Conquer - Tree Layouts - Part 2
Overview

- H(horizontal) V(vertical) tree layout algorithm
- Radial tree layout algorithm
- Other visualization styles
Applications

Cons cell diagram in LISP. *Cons*(constructs) are memory objects which hold two values or pointers to values.

![Cons cell diagram in LISP](http://gajon.org/)

**Figure 3:** Diagram of cons cells of the simple tree.  

Discuss with your neighbour(s) and then share

2+3 min

2 - 1
Applications

Cons cell diagram in LISP. *Cons*(constructs) are memory objects which hold two values or pointers to values.

![Cons cell diagram](http://gajon.org/)

**Figure 3**: Diagram of cons cells of the simple tree.

Discuss with your neighbour(s) and then share

- What are the drawing conventions and aesthetics?

2+3 min
HV-Layout

**Drawing Conventions:**
- Children are vertically and horizontally aligned with the root
- The bounding boxes of the children do not intersect

**Drawing Aesthetics:**
- Height, width, area
HV-Layout

Divide & Conquer Approach:
**HV-Layout**

**Induction base:** •

**Induction step:** combine layouts

- **Horizontal combination**
- **Vertical combination**
Right-Heavy HV-Layout

**Right-Heavy approach:**
- At every induction step apply horizontal combination
- Place the larger subtree to the right
Right-Heavy HV-Layout

**Right-Heavy approach:**
- At every induction step apply horizontal combination
- Place the larger subtree to the right

**Lemma**
Let $T$ be a binary tree. The height of the drawing constructed by Right-Heavy approach is at most $\log n$. 
Right-Heavy HV-Layout

**Right-Heavy approach:**
- At every induction step apply horizontal combination
- Place the larger subtree to the right

**Lemma**
Let $T$ be a binary tree. The height of the drawing constructed by Right-Heavy approach is at most $\log n$. 

**Proof:**
- Each vertical edge has length 1
- Let $w$ be the lowest node in the drawing
- Let $P$ be a path from $w$ to the root of $T$
- For every edge $(u, v)$ in $P$: $|T(v)| > 2|T(u)|$
- $\Rightarrow P$ contains at most $\log n$ edges
Right-Heavy HV-Layout

- At every induction step apply horizontal combination
- Place the larger subtree to the right

Discuss with your neighbour(s) and then share 10 min

- What are the implementational details of the algorithm?
- How to compute the coordinates? Can we do it in $O(n)$ time?
Right-Heavy HV-Layout

**Theorem**

Let $T$ be a binary tree with $n$ vertices. The Right-Heavy algorithm constructs in $O(n)$ time a drawing $\Gamma$ of $T$ such that:
Right-Heavy HV-Layout

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- $\Gamma$ is HV-drawing (planar, orthogonal)
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- $\Gamma$ is HV-drawing (planar, orthogonal)
- The width of $\Gamma$ is at most

Take a minute to think about the width of the layout 1 min

9 - 3
Right-Heavy HV-Layout

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Right-Heavy HV-Layout

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- The height is at most $\log n$
- The area is $O(n \log n)$
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- Simply and axially isomorphic subtrees have congruent drawings, up to translation
Right-Heavy HV-Layout

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**General rooted tree:**

```
  largest subtree
  /       \
```

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Bad news We can not minimize the area by using divide & conquer approach
HV-Layout

Bad news We can not minimize the area by using divide & conquer approach
Good news We can compute minimum area using Dynamic Programming
HV-Layout

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Good news We can compute minimum area using Dynamic Programming

HV-Layout for Trees

- Book Di Battista et al: Chapter 3.1.4
- Skript: page 86

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Applications

Radial layout


An Unrooted Phylogenetic Tree of the Myosin Superfamily
Tony Hodge, MRC-LMB
Jamie Cope, UC Berkeley
July 2000

Node found in >90% Bootstrap trials
— Partial Sequence
— Class uncertain by matrix analysis
Applications

Radial layout

An unrooted phylogenetic tree for myosin, a superfamily of proteins.
"A myosin family tree“ Journal of Cell Science
Applications

Flare Visualization Toolkit code structure by Heer, Bostock and Ogievetsky, 2010

Greek Myth Family by Ribecca, 2011
Radial Layout

Drawing Conventions:
- Vertices lie on circular layers according to their depth
- Drawing is planar

Drawing Aesthetics:
- Distribution of the vertices
Radial Layout

Drawing Conventions:
- Vertices lie on circular layers according to their depth
- Drawing is planar

Drawing Aesthetics:
- Distribution of the vertices

Take a minute to think about a possible algorithm to optimize the distribution of the vertices
Radial Layout

Example: Angle corresponding to the subtree rooted at $u$: $\tau_u = \frac{\ell(u)}{\ell(v)-1}$
Radial Layout

**Example:** Angle corresponding to the subtree rooted at $u$: $\tau_u = \frac{\ell(u)}{\ell(v) - 1}$
Radial Layout

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Radial Layout

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**Example:** Angle corresponding to the subtree rooted at $u$: 
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(example diagram with nodes and angles labeled)
Radial Layout

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Radial Layout

How to avoid crossings:
Radial Layout

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Radial Layout

How to avoid crossings:

16 - 3
Radial Layout

How to avoid crossings:

16 - 4
Radial Layout

How to avoid crossings:
Radial Layout

How to avoid crossings:

- $\tau_u$ - angle of the wedge corresponding to vertex $u$
- $\rho_i$ - radius of layer $i$
- $\ell(v)$ - number of nodes in the subtree rooted at $v$
- $\cos \frac{\tau_u}{2} = \frac{\rho_i}{\rho_{i+1}}$
Radial Layout

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\[ \tau_u = \min\left\{ \frac{\ell(u)}{\ell(v)} - 1, 2 \arccos \frac{\rho_i}{\rho_{i+1}} \right\} \] (correction)
Radial Layout

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Alternatively use number of leaves in the subtree to subdivide the angles [book Di Battista et al.]
Radial Layout

Discuss with your neighbour(s) and then share

- Why the produced drawing is planar?

- \( \ell(v) \)-number of nodes in the subtree rooted at \( v \)

- \( \cos \frac{\tau_u}{2} = \frac{\rho_i}{\rho_{i+1}} \)

- \( \tau_u = \min\left\{ \frac{\ell(u)}{\ell(v)} - 1, 2 \arccos \frac{\rho_i}{\rho_{i+1}} \right\} \)
  (correction)

- Alternatively use number of leaves in the subtree to subdivide the angles [book Di Battista et al.]
Radial Layout

Theorem

Let $T$ be a rooted tree with $n$ vertices. The radial algorithm constructs in $O(n)$ time a drawing $\Gamma$ of $T$ such that:

- $\Gamma$ is planar
- Each vertex lies on the radial layer equal to its height
- The area of the drawing is at most $O(h^2d_M^2)$, $h$-height, $d_M$-max number of children

Assuming that the radii of consecutive layers differ by the same number and the distance between the vertices on the layer is at least one
Radial Layout

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radius is at least $d_M$

radius is at least $hd_M$
Radial Layout for Trees

- Book Di Battista et al: Chapter 3.1.3
- Skript: Chapter 6.1.2
Other Visualization Styles

Writing Without Words: the project explores methods of visually-representing text and visualises the differences in writing styles when comparing different authors.
Other Visualization Styles

Writing Without Words: the project explores methods of visually-representing text and visualises the differences in writing styles when comparing different authors.

similar to Ballon layout
Other Visualization Styles

A phylogenetically organised display of data for all placental mammal species.

Fractal tree layout
for more applications and layouts...