Karlsruhe Institute of Technology

The Maximum Transmission Switching Flow Problem

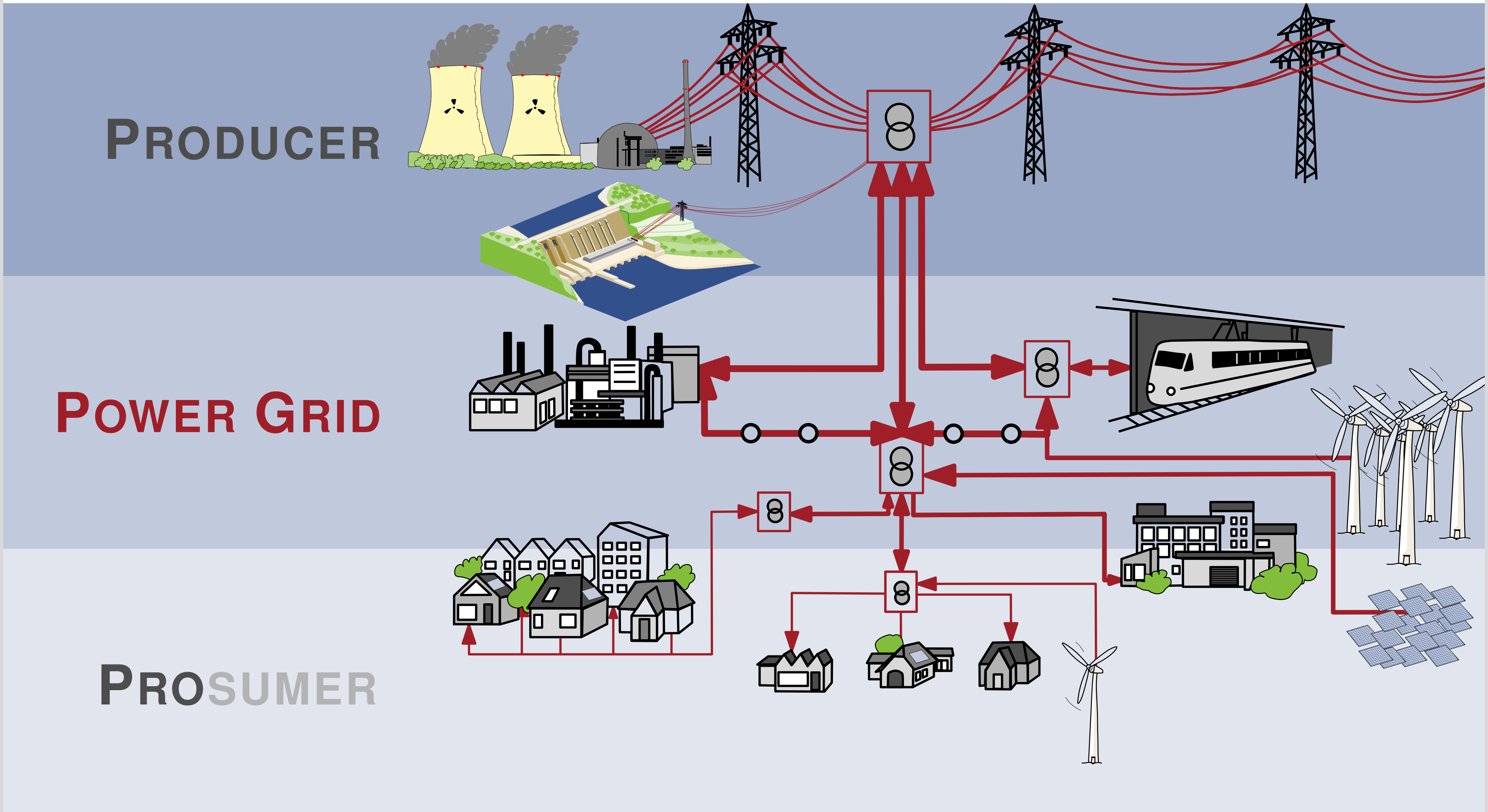
[Grastien et al., 2018]

Seminar · Energieinformatik · Winter Term 2018/19 · Nov. 06th, 2018

Franziska Wegner

INSTITUTE OF THEORETICAL INFORMATICS · ALGORITHMICS GROUP

Recent Development in Power Grids



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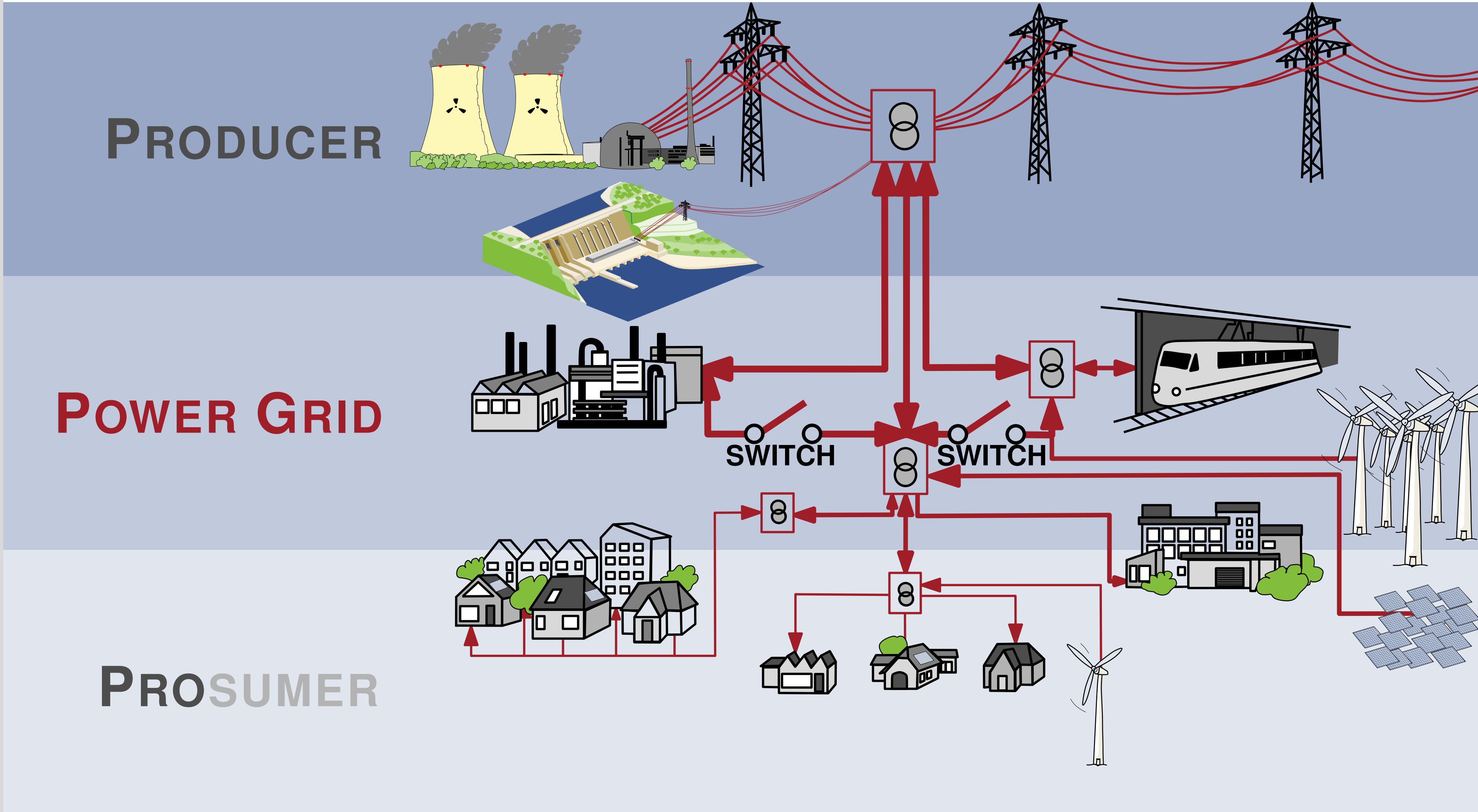

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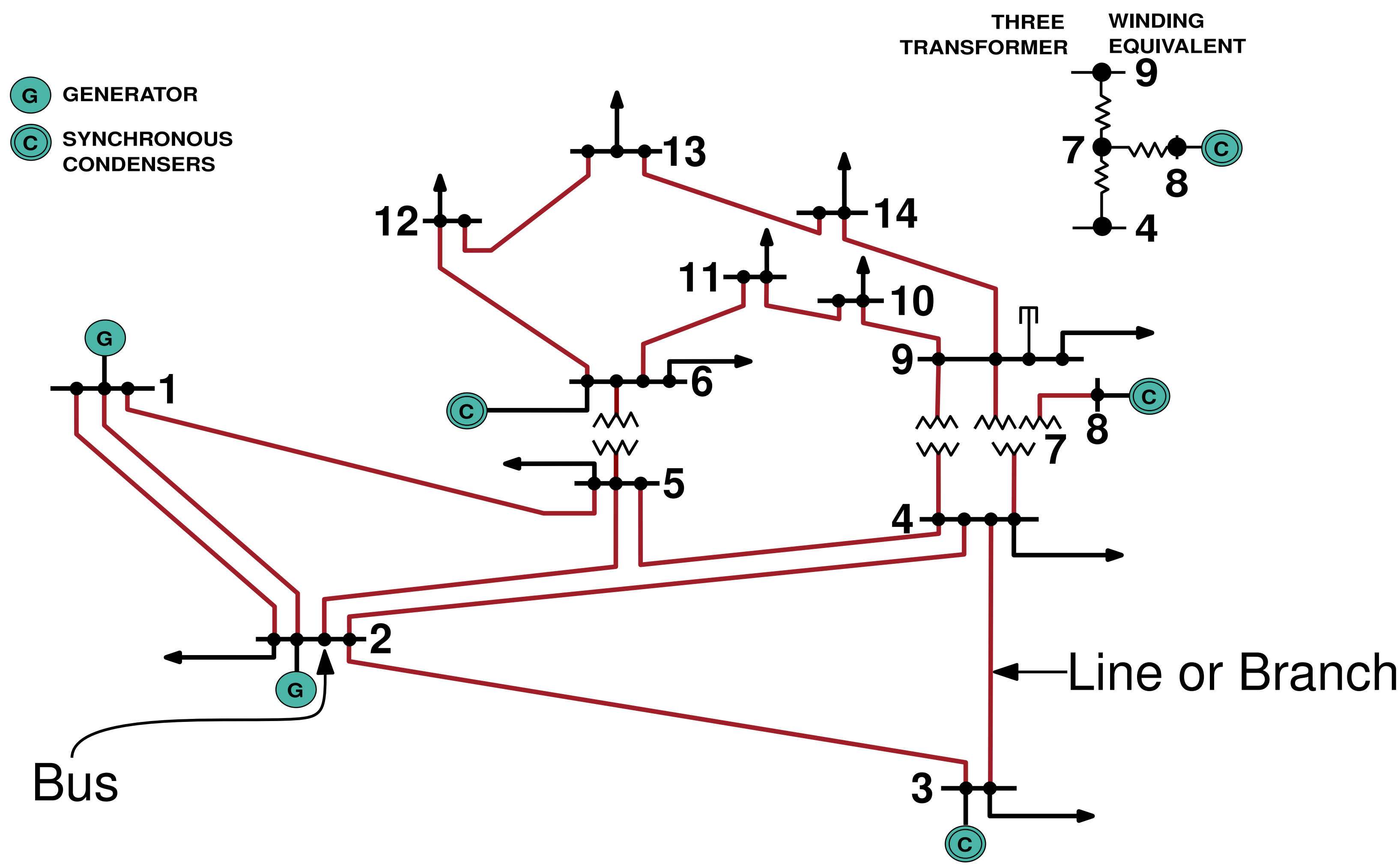
From a Transmission Network to a Graph

[University of Washington, 1999]

Graph $G = (V, E)$



AEP 14 BUS TEST SYSTEM BUS CODE DIAGRAM

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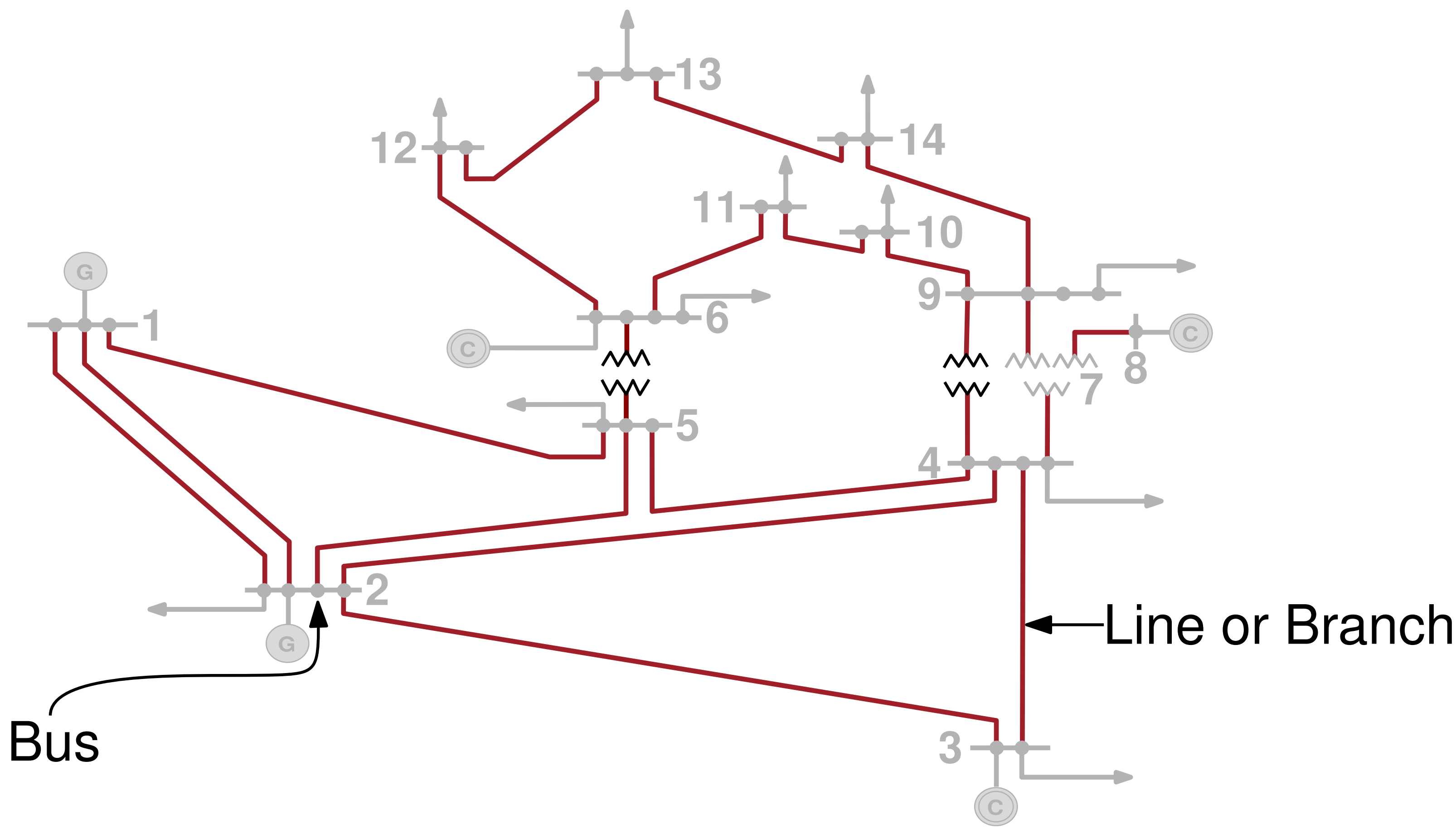

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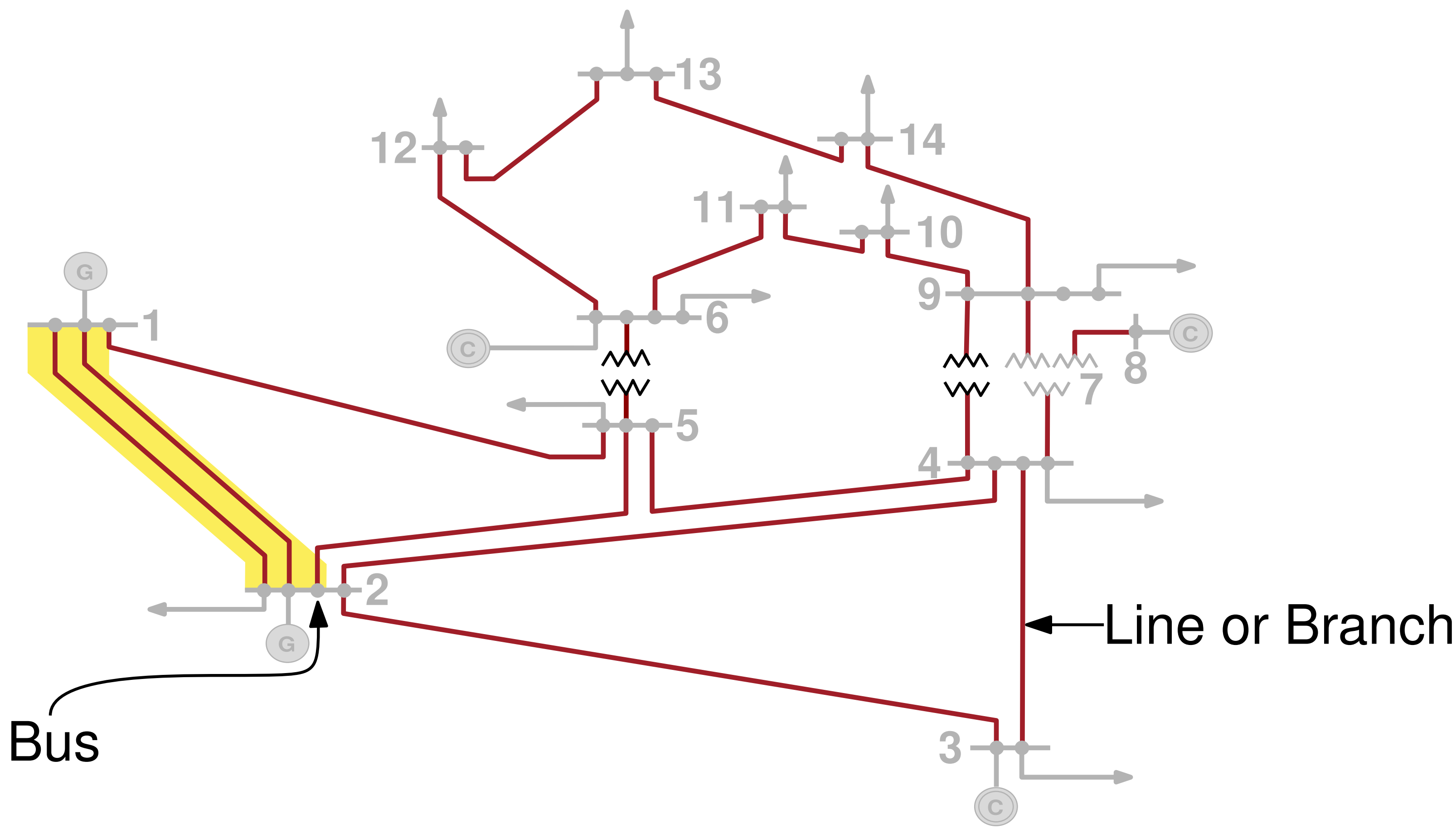

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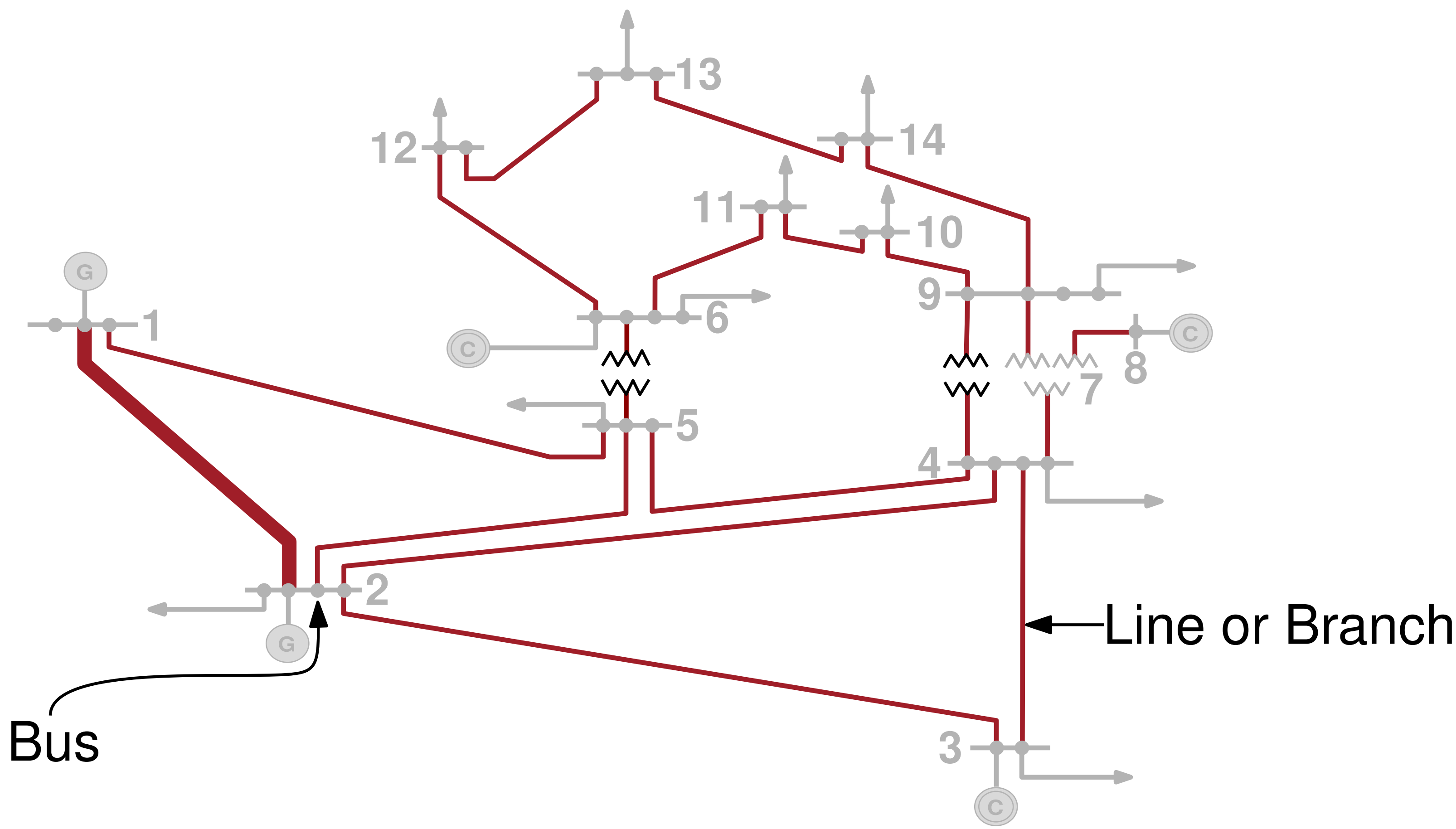

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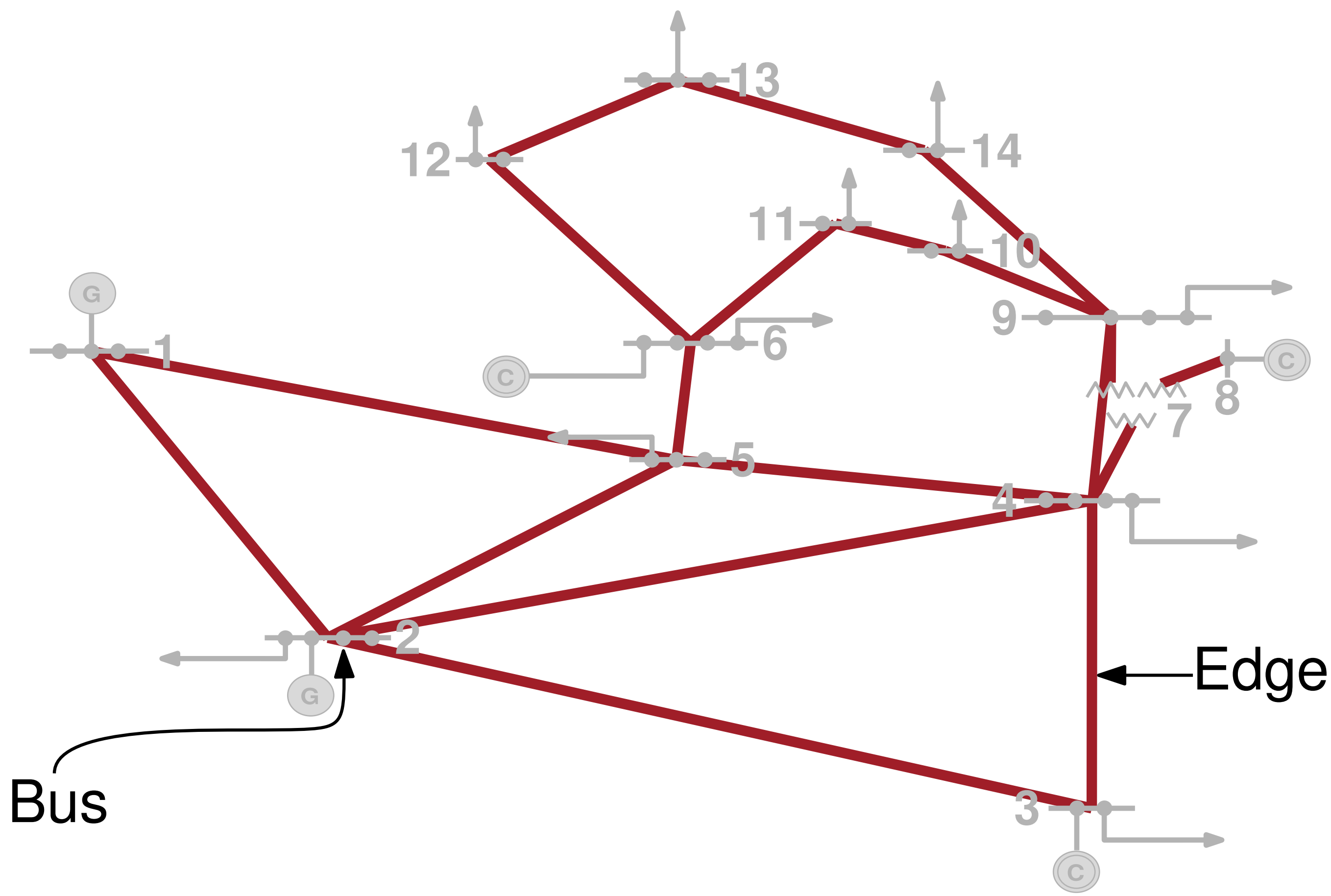

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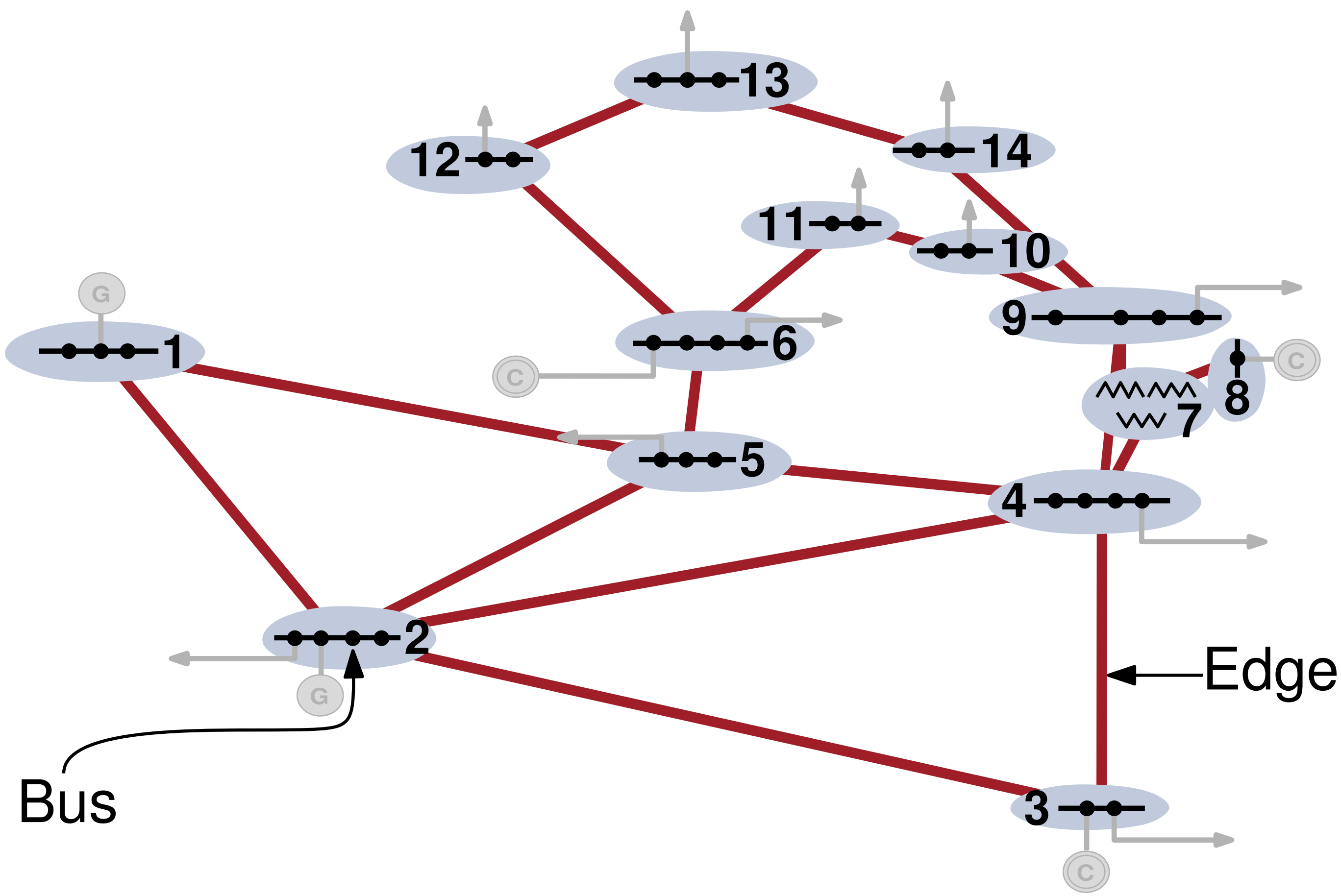

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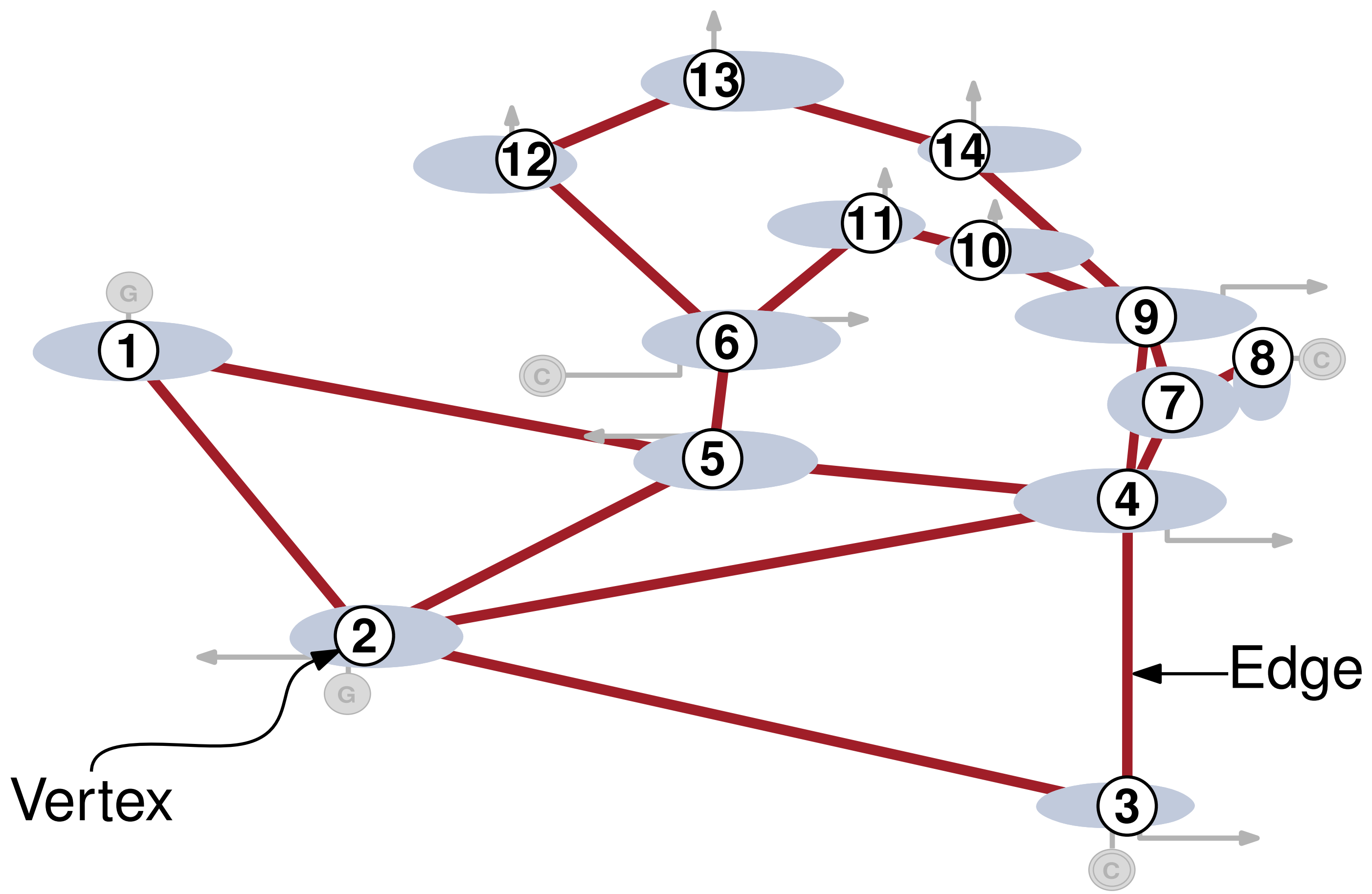

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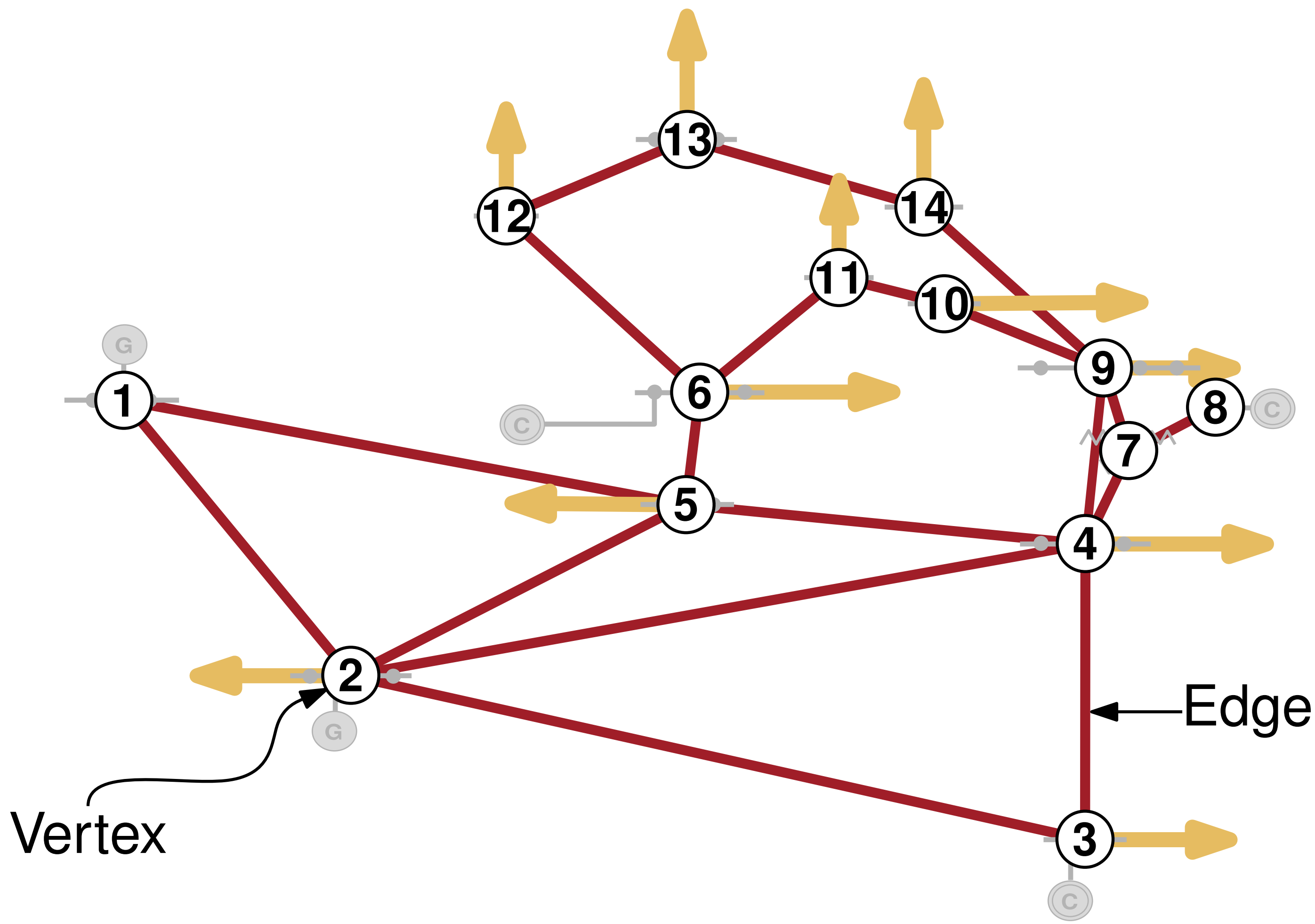

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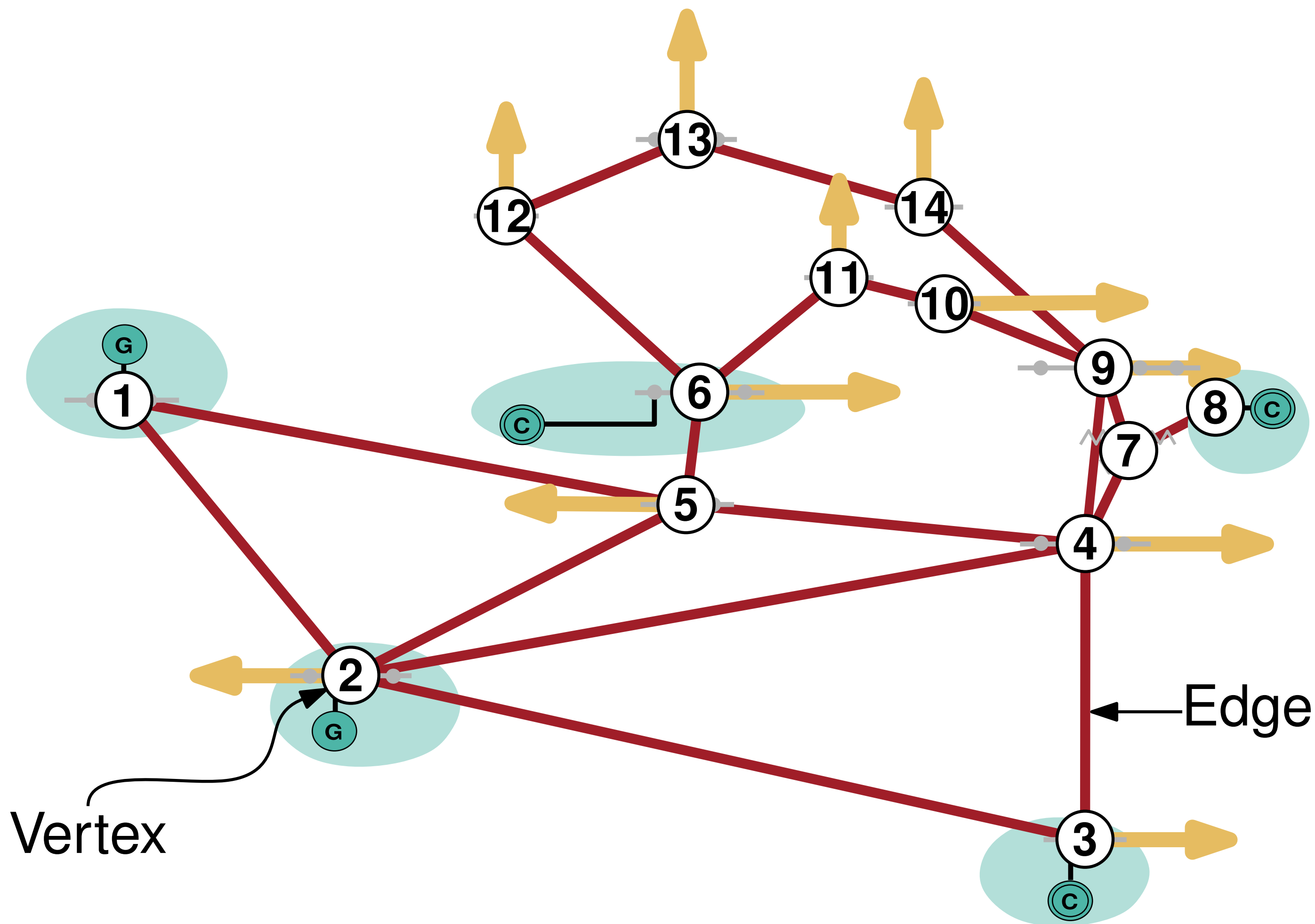

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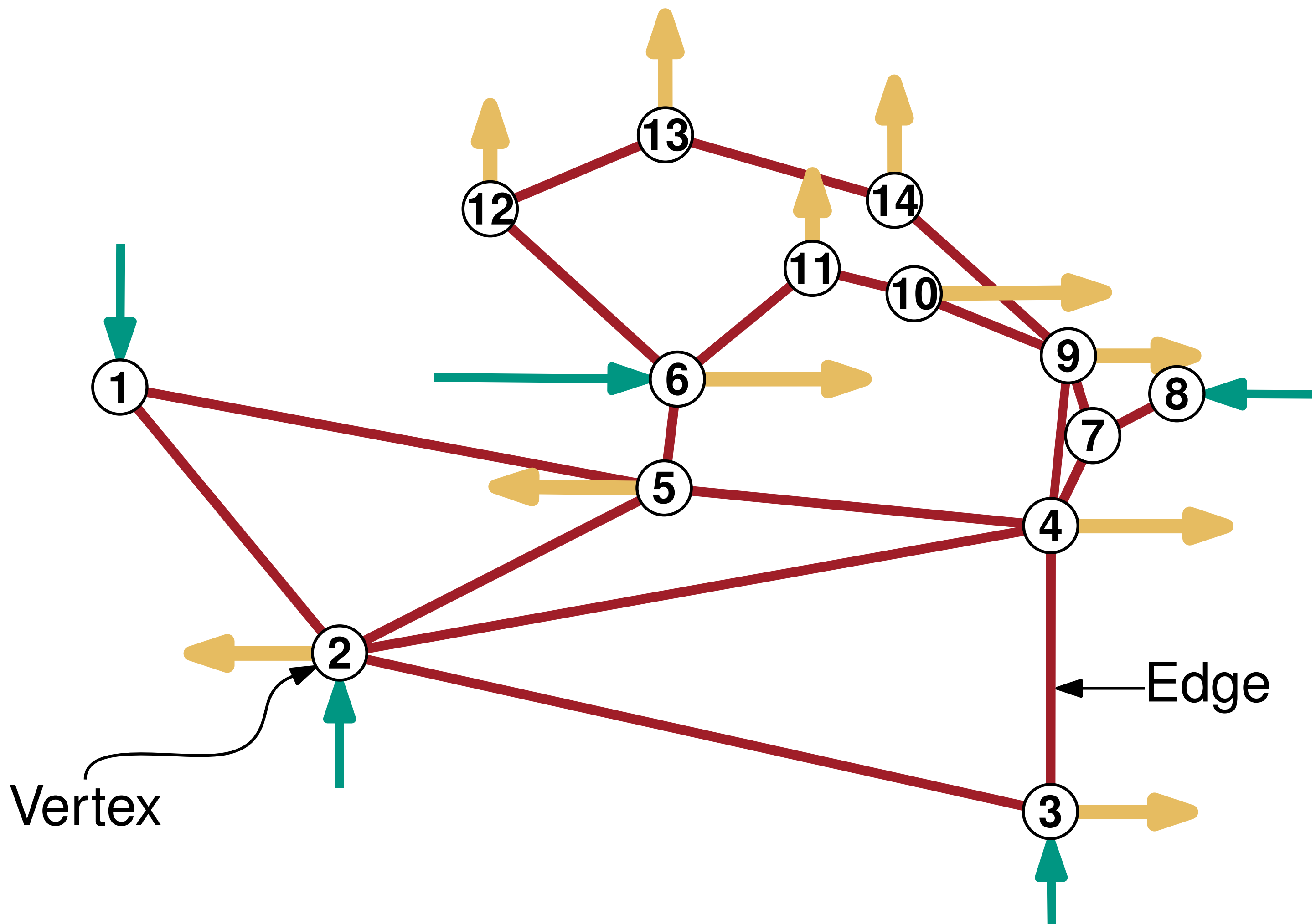

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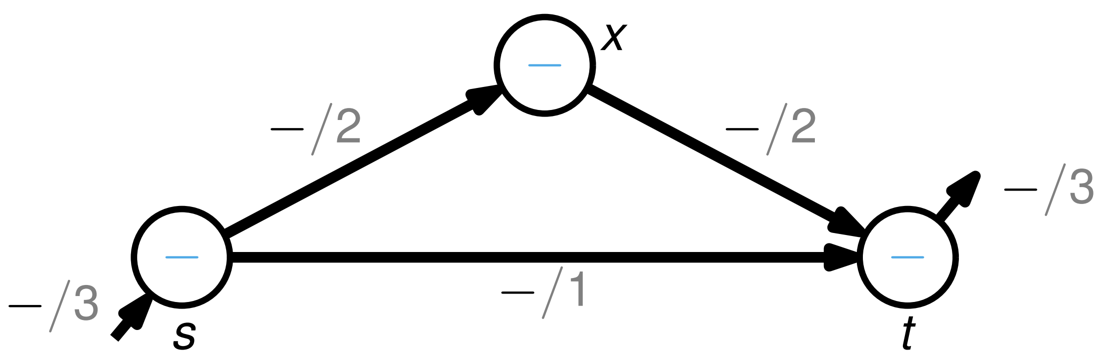

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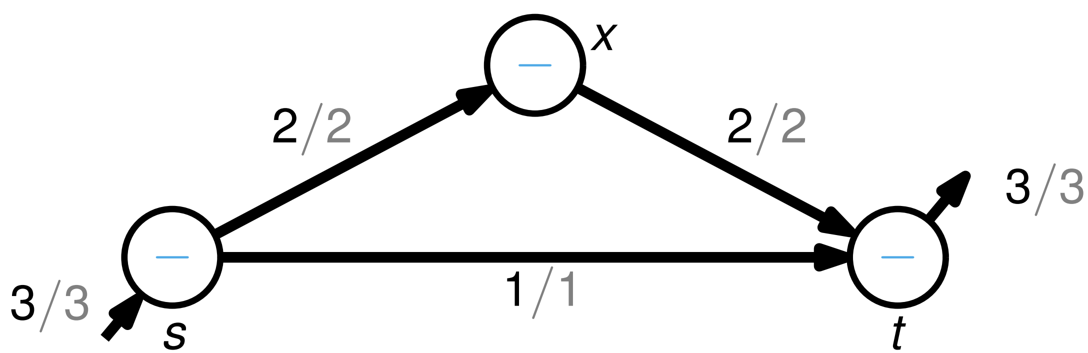

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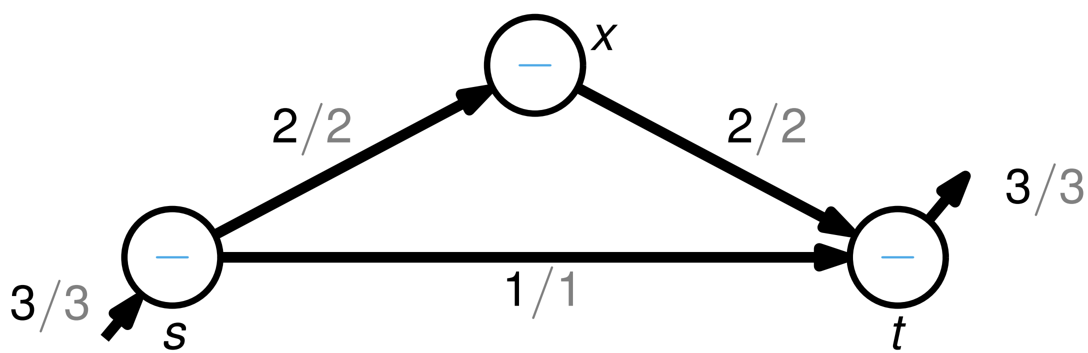

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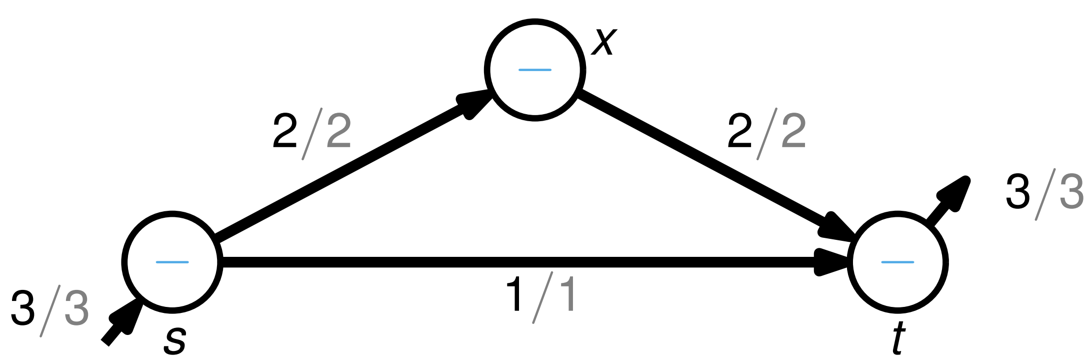

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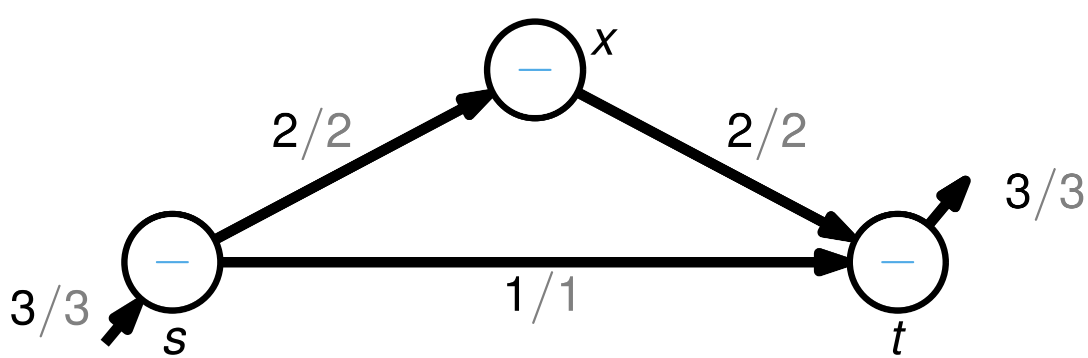

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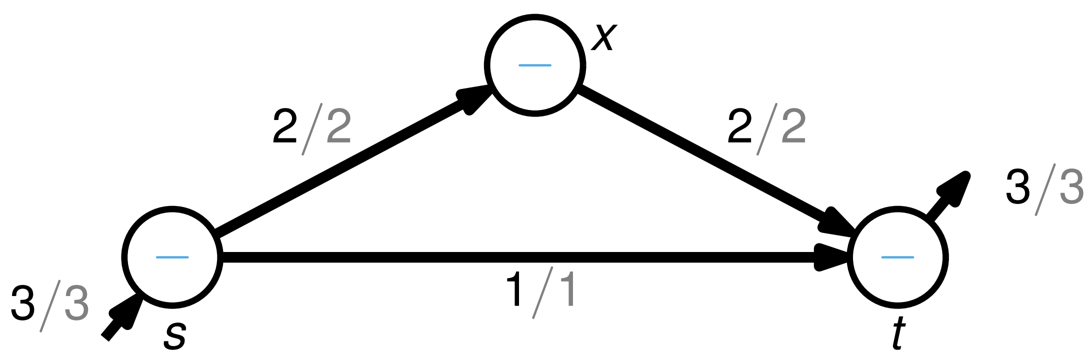

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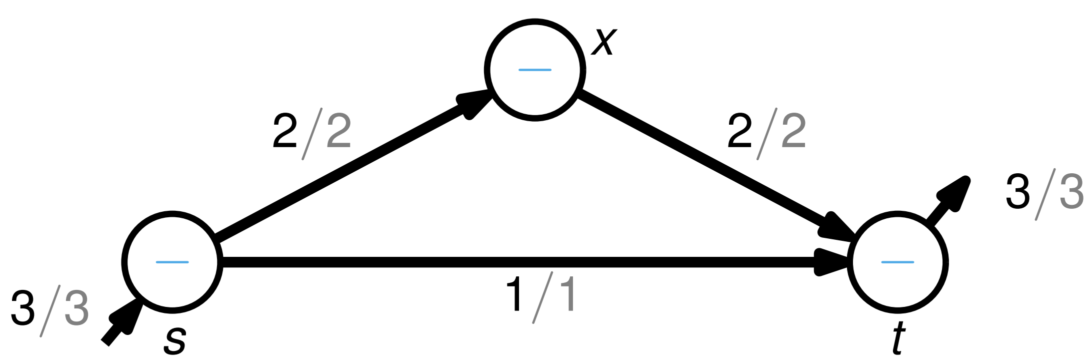

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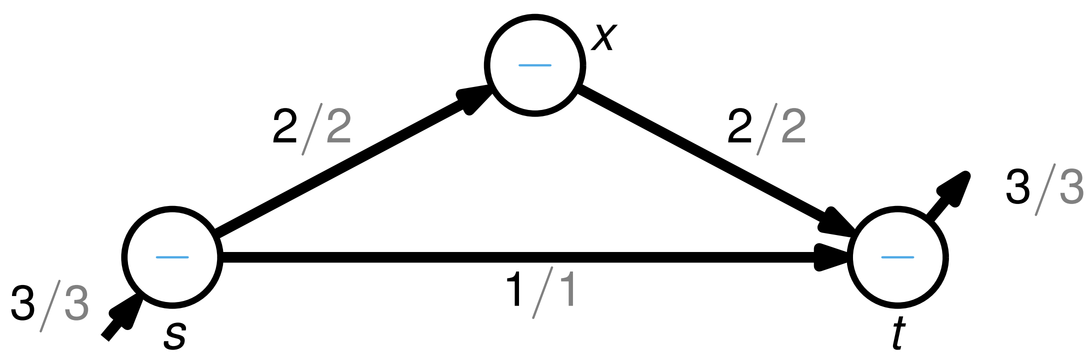

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Maximum Power Flow (MPF)

[Zimmerman et al., 2011]

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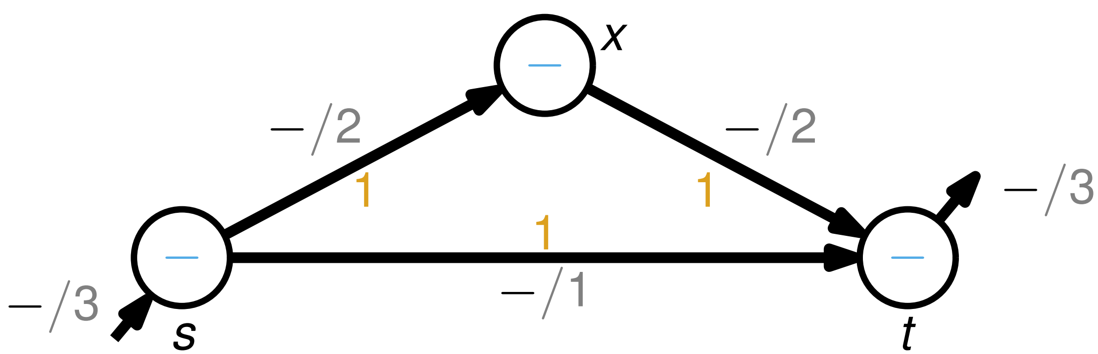

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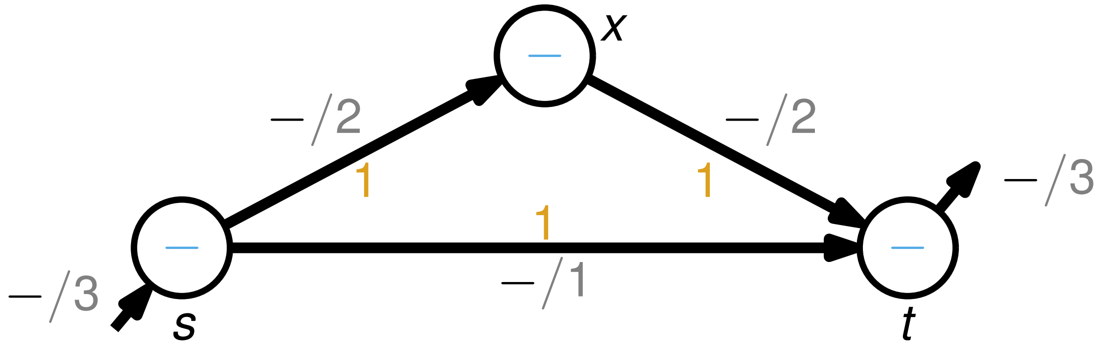

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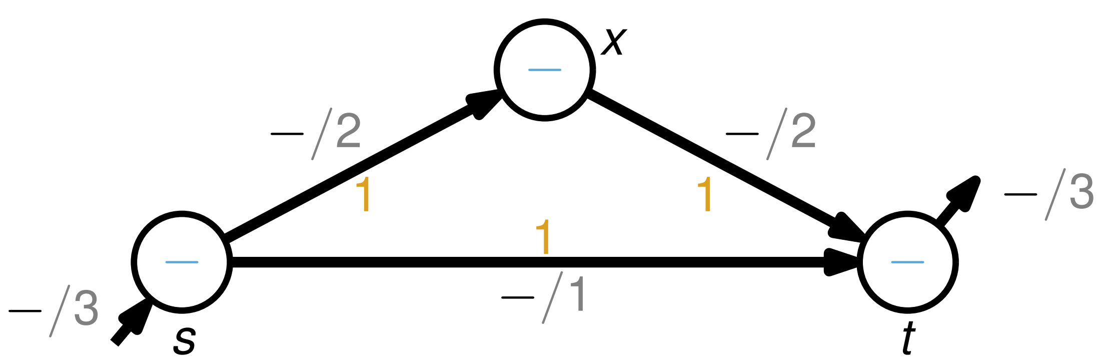

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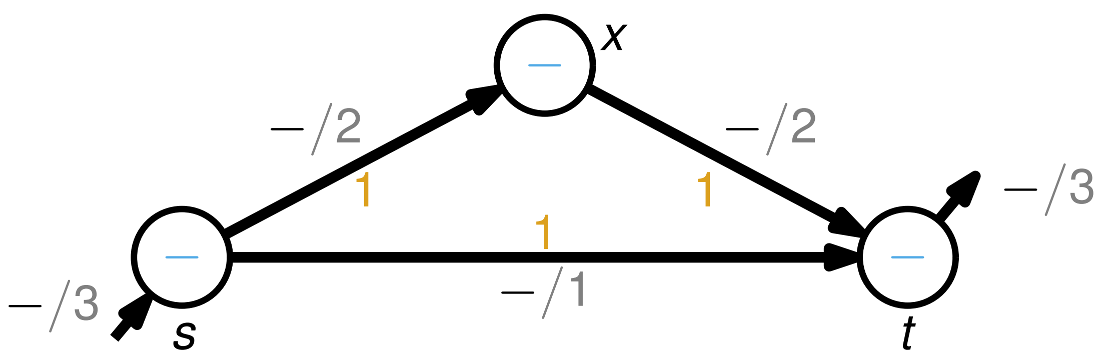

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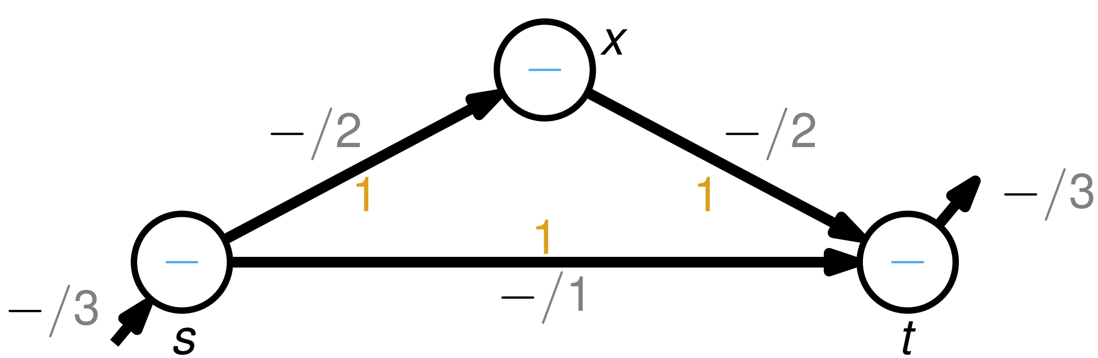

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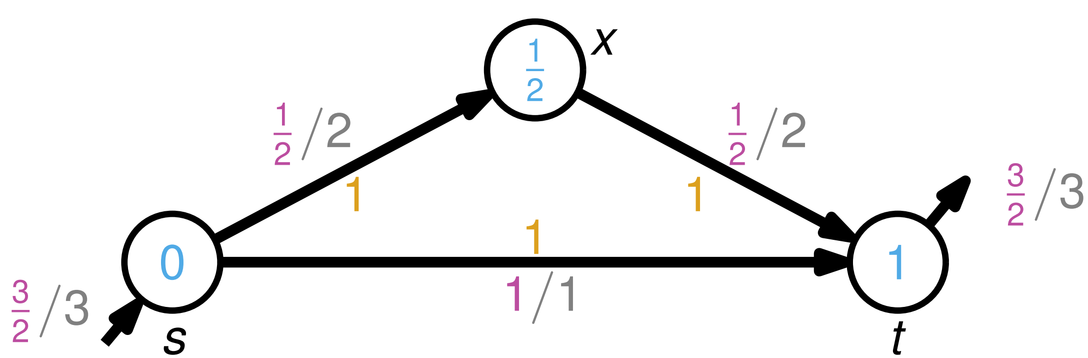

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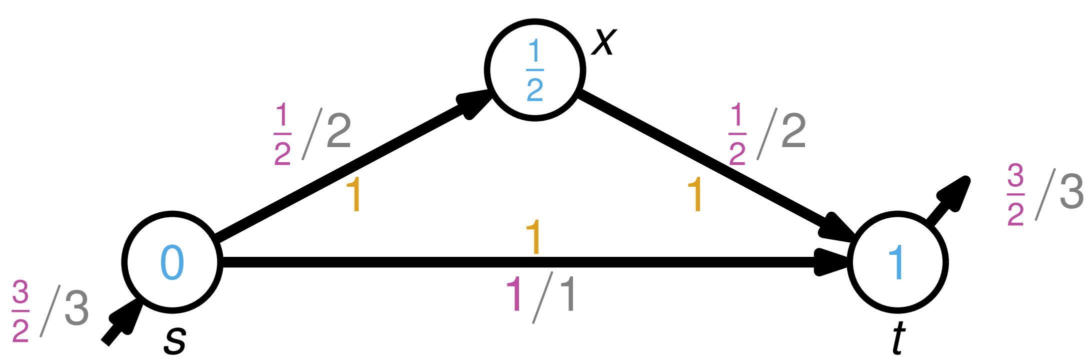

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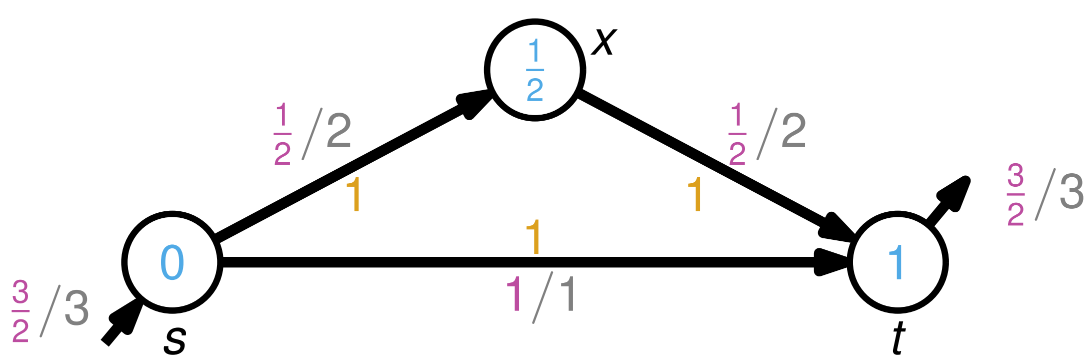

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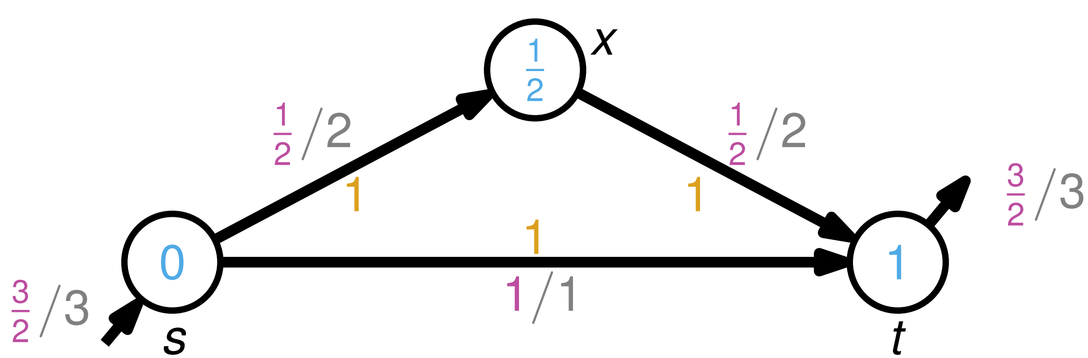

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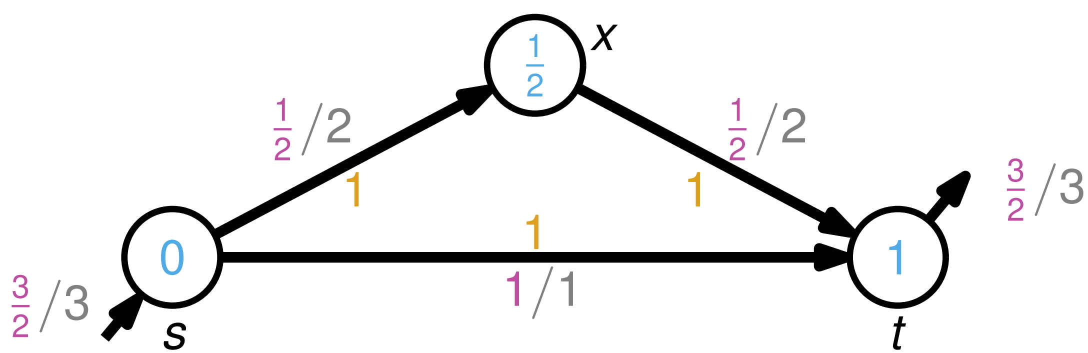

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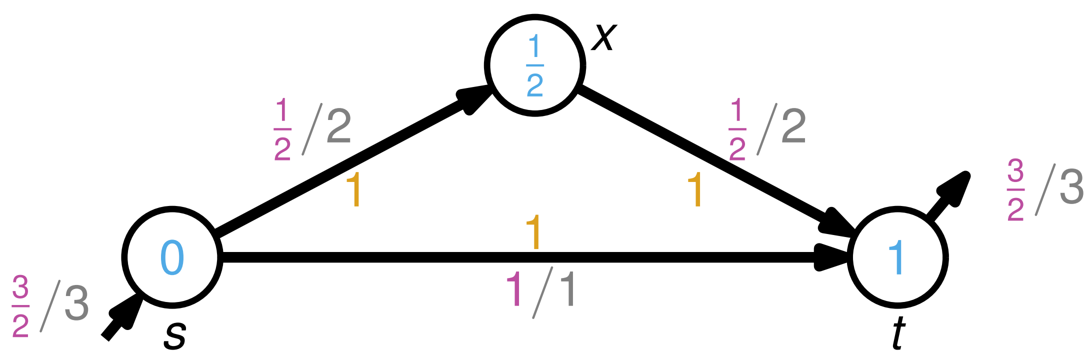

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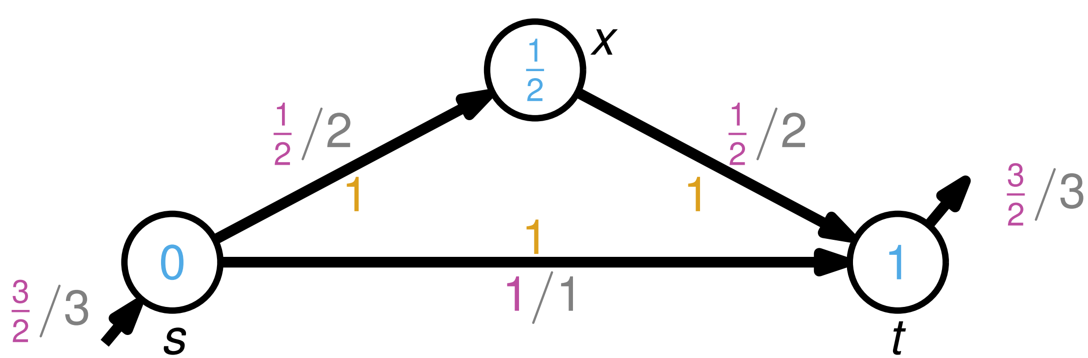

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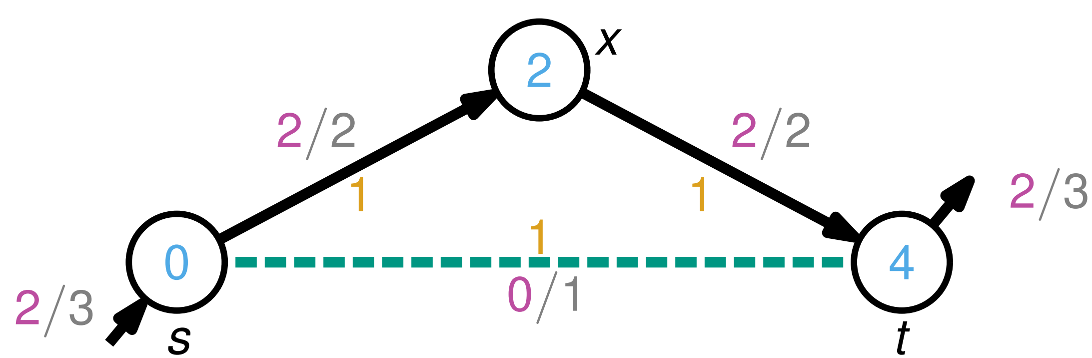

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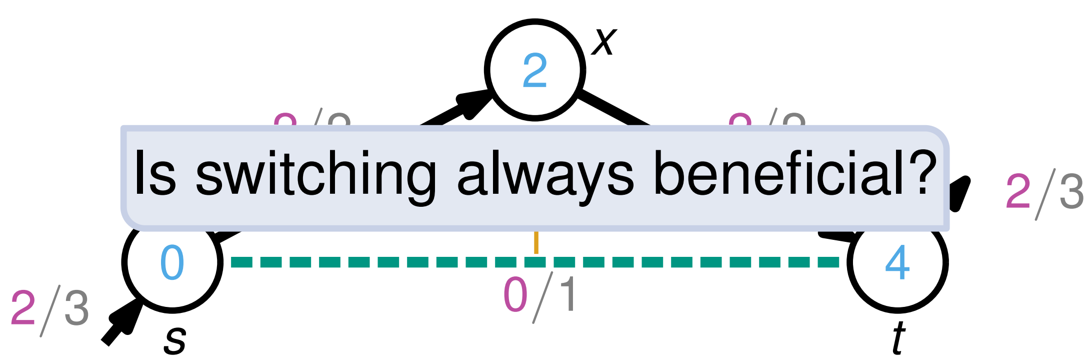

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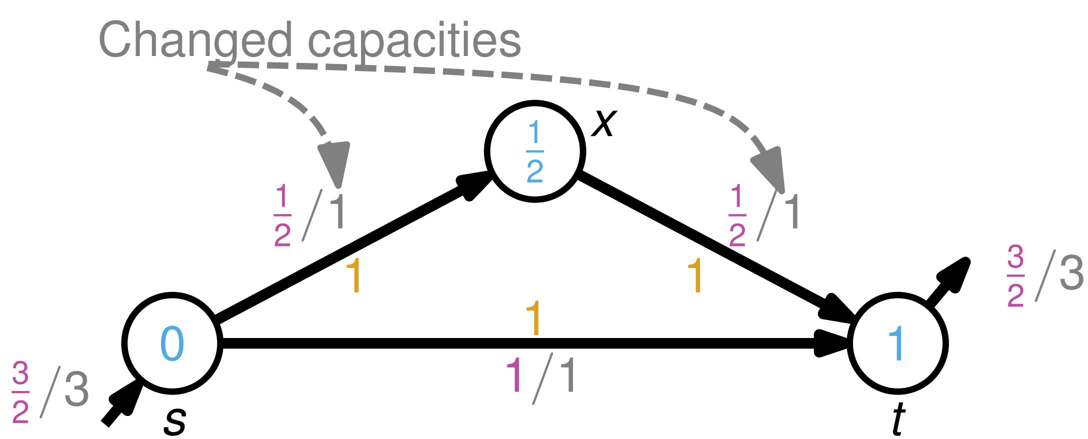

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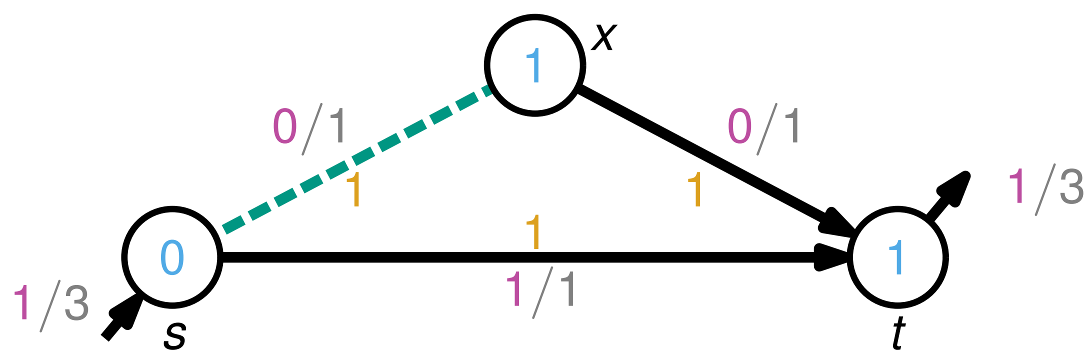
with value $\text{OPT}_{\text{MTSF}}(\mathcal{N})$ with f being a physical feasible flow meaning

$$|f(u,v)| \leq z(u,v) \cdot \text{cap}(u,v) \quad \forall (u,v) \in E$$

$$b(u,v) \cdot z(u,v) \cdot \big(\theta(v) - \theta(u)\big) = f(u,v) \quad \forall (u,v) \in E$$



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The MAXIMUM TRANSMISSION SWITCHING FLOW (MTSF) Problem [Fisher et al., 2008]

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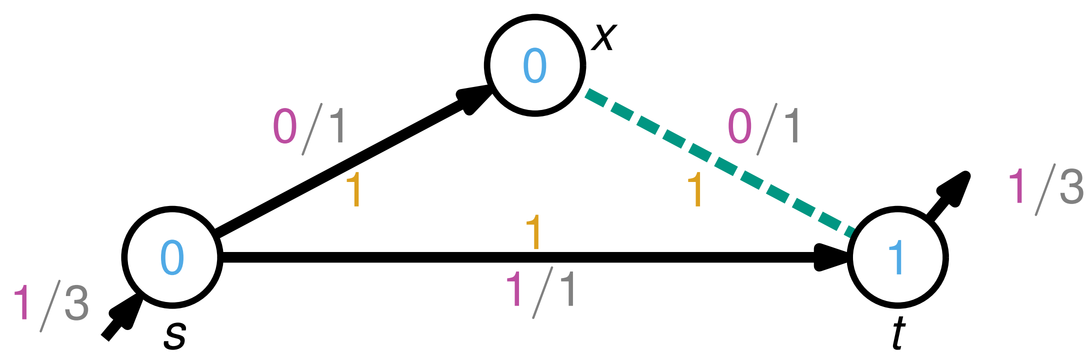

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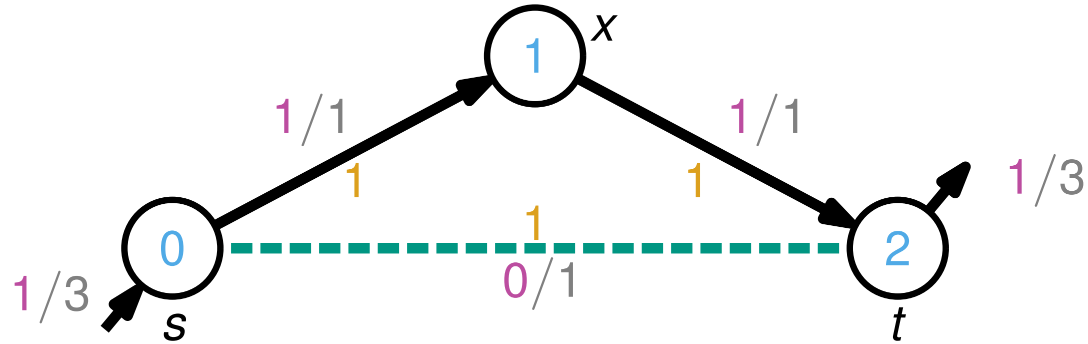

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The Maximum Transmission Switching Flow (MTSF) Problem [Fisher et al., 2008]

Optimization Problem MTSF

Instance: A power grid $\mathcal{N}$.

Objective: Find a set $S \subseteq \underline{E}$ of switched edges such that $\text{OPT}_{\text{MPF}}(\mathcal{N} - S)$ is maximum among all choices of switched edges S.

Decision Problem *k*-MTSF

Instance: A power grid $\mathcal{N}$ and $k \in \mathbb{Q}_{\geq 0}$.

Objective: Is it possible to remove a set of edges $S \subseteq \underline{E}$ such that there is an physical feasible flow f in $\mathcal{N} - S$ with flow value $F(\mathcal{N} - S, f) \geq k$?

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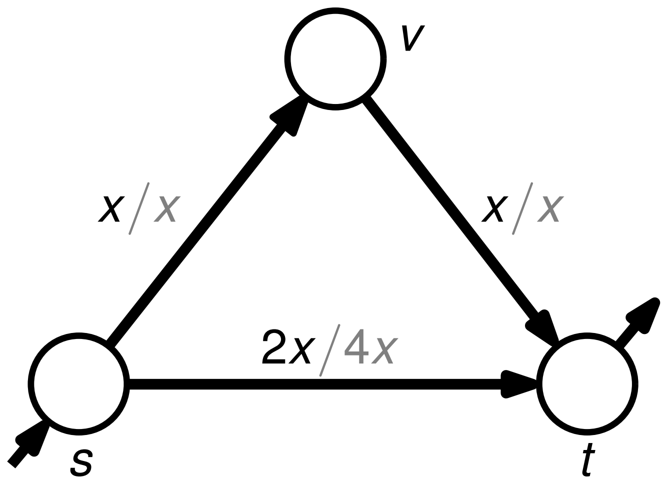
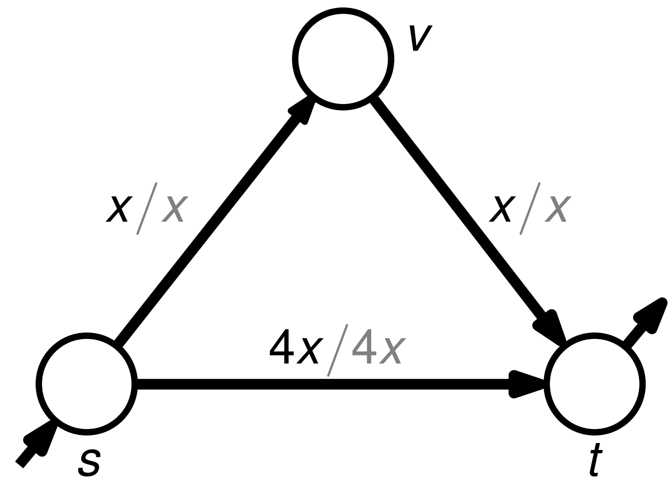
The MTSF Problem





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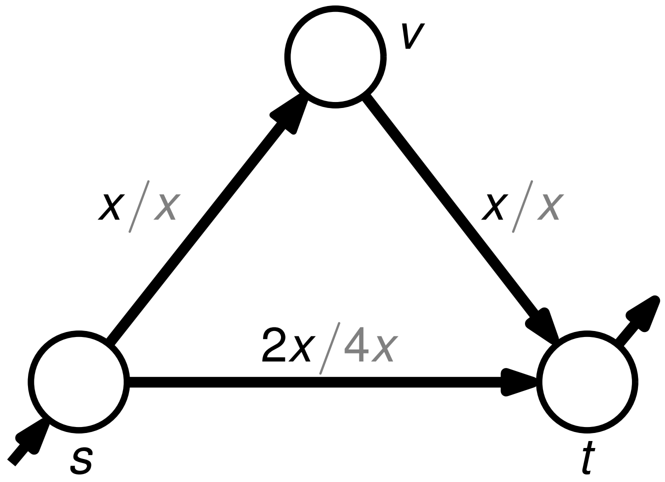
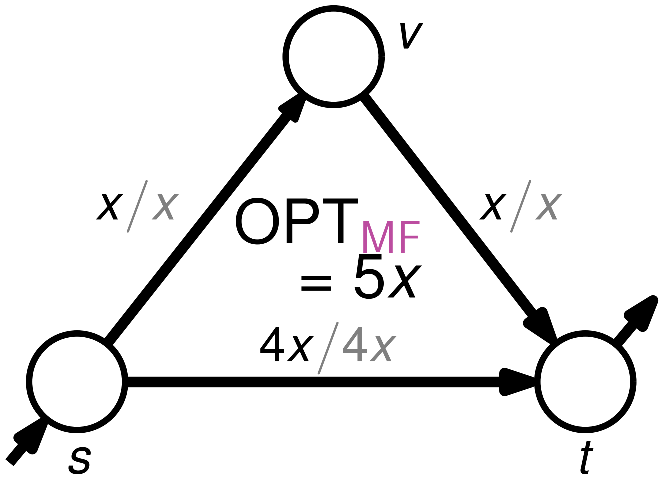
The MTSF Problem





flow model

upper bound

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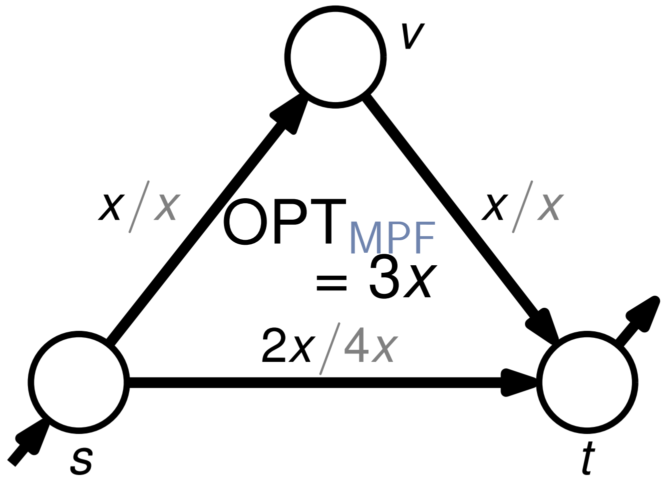
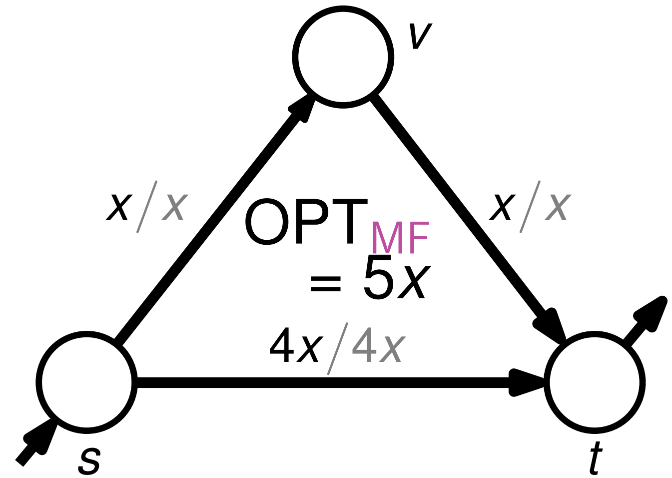
The MTSF Problem





physical model
(AC linearization)

lower bound

flow model

upper bound

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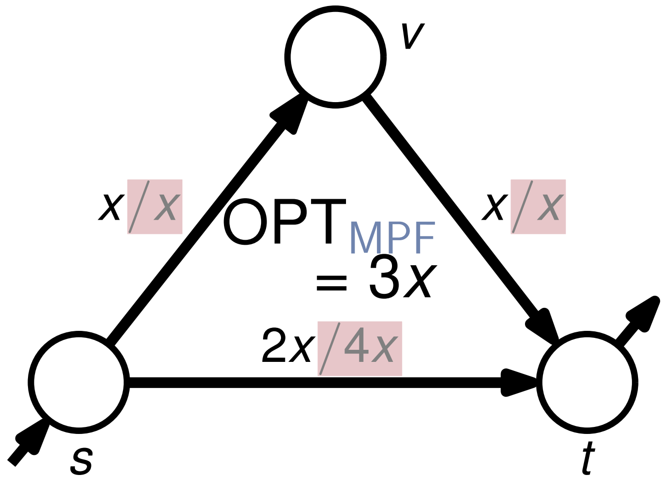
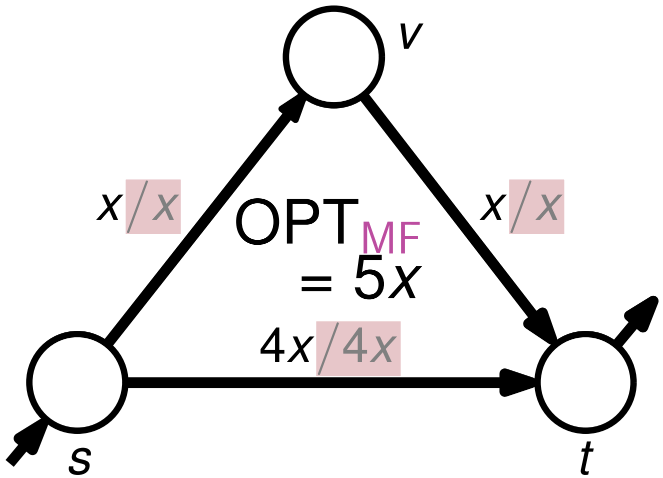
The MTSF Problem





physical model
(AC linearization)

lower bound

capacity constraints

flow model

upper bound

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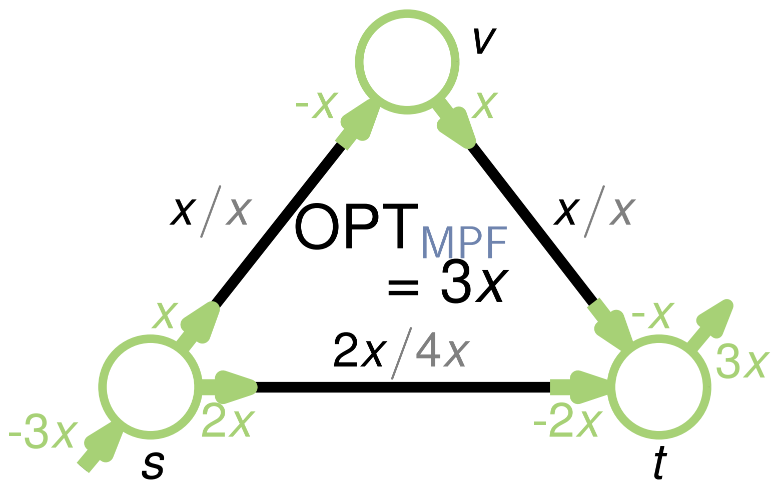
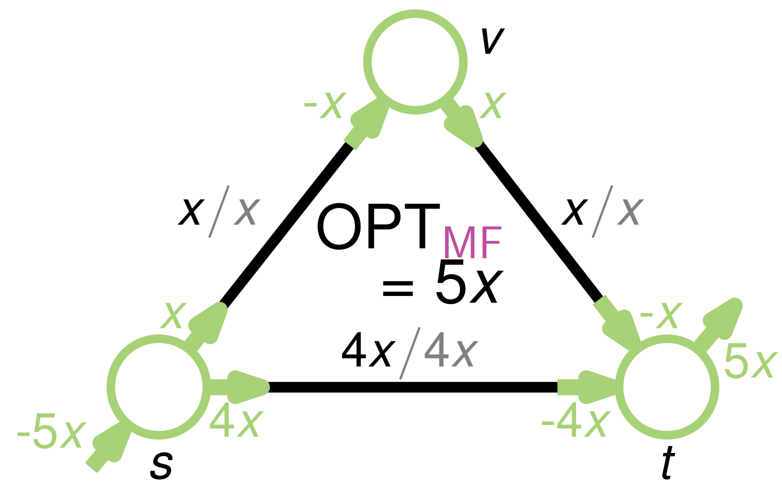
The MTSF Problem





physical model
(AC linearization)

lower bound

flow model

upper bound

capacity constraints

Kirchhoff's Current Law (KCL)

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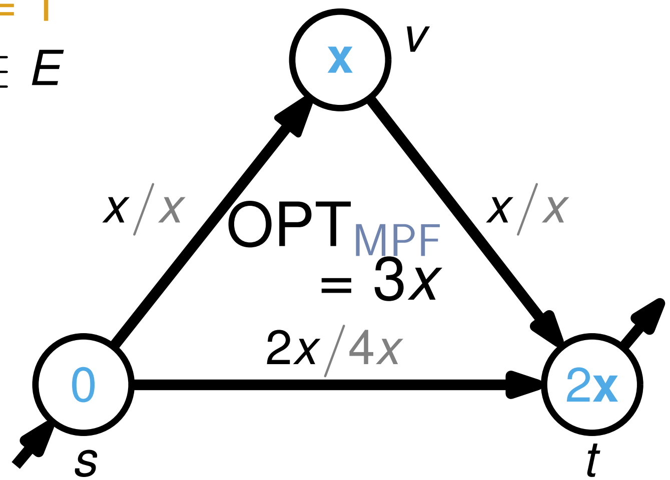
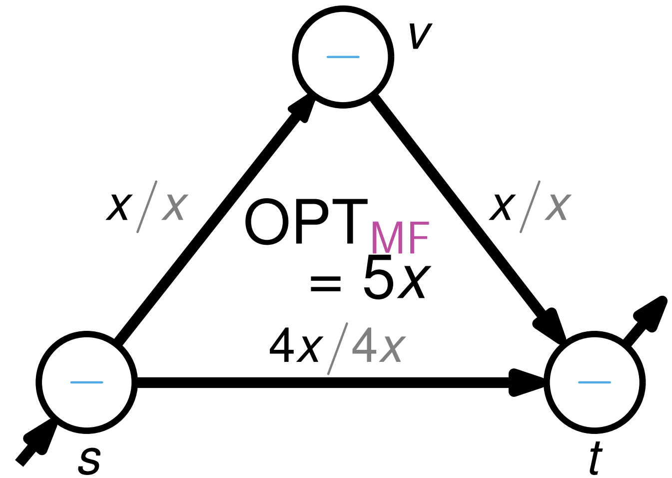
The MTSF Problem

$b(i, j) := 1$
$\forall (i, j) \in E$





physical model
(AC linearization)

lower bound

flow model

upper bound

capacity constraints

Kirchhoff's Current Law (KCL)

DC power flow constraints

$$\forall (u, v) \in E : f(u, v) = b(u, v) \big(\theta(v) - \theta(u)\big)$$

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The MTSF Problem

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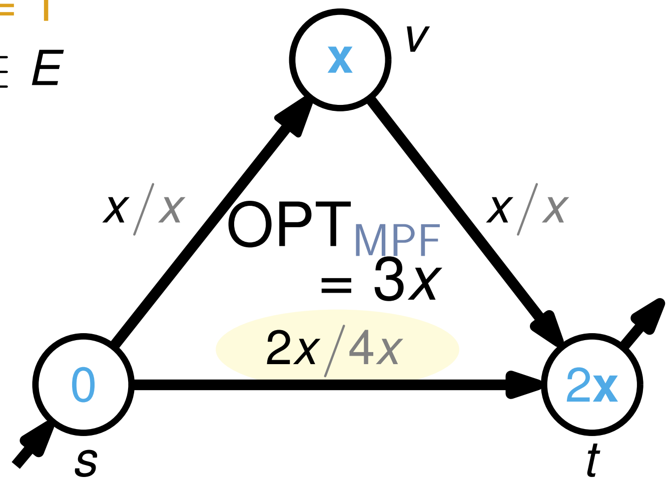

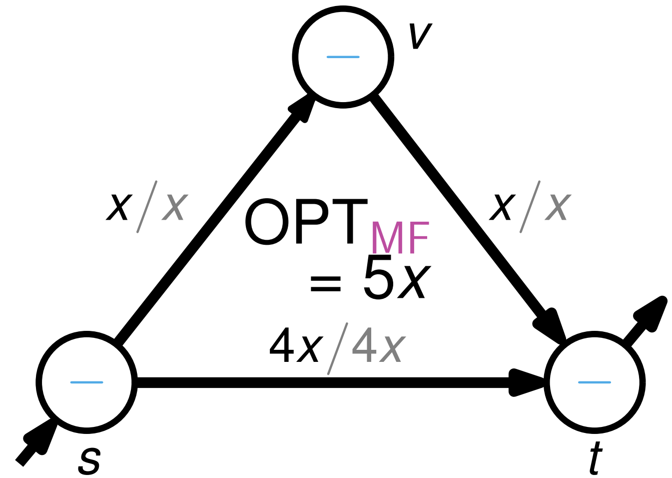

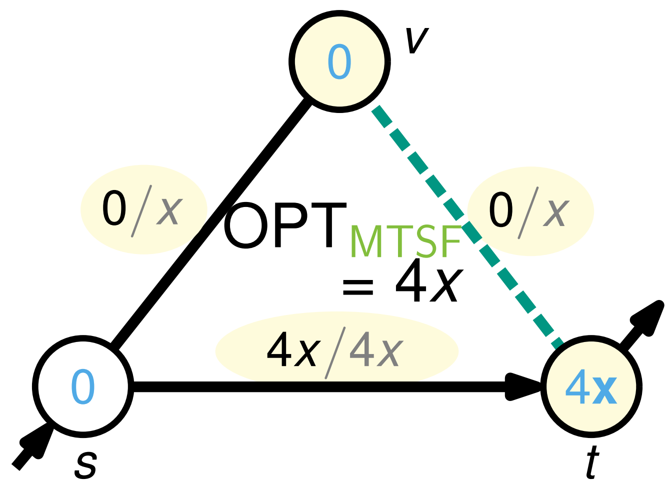

$$\forall (u, v) \in E\colon\ f(u, v) = b(u, v)\big(\theta(v) - \theta(u)\big)$$

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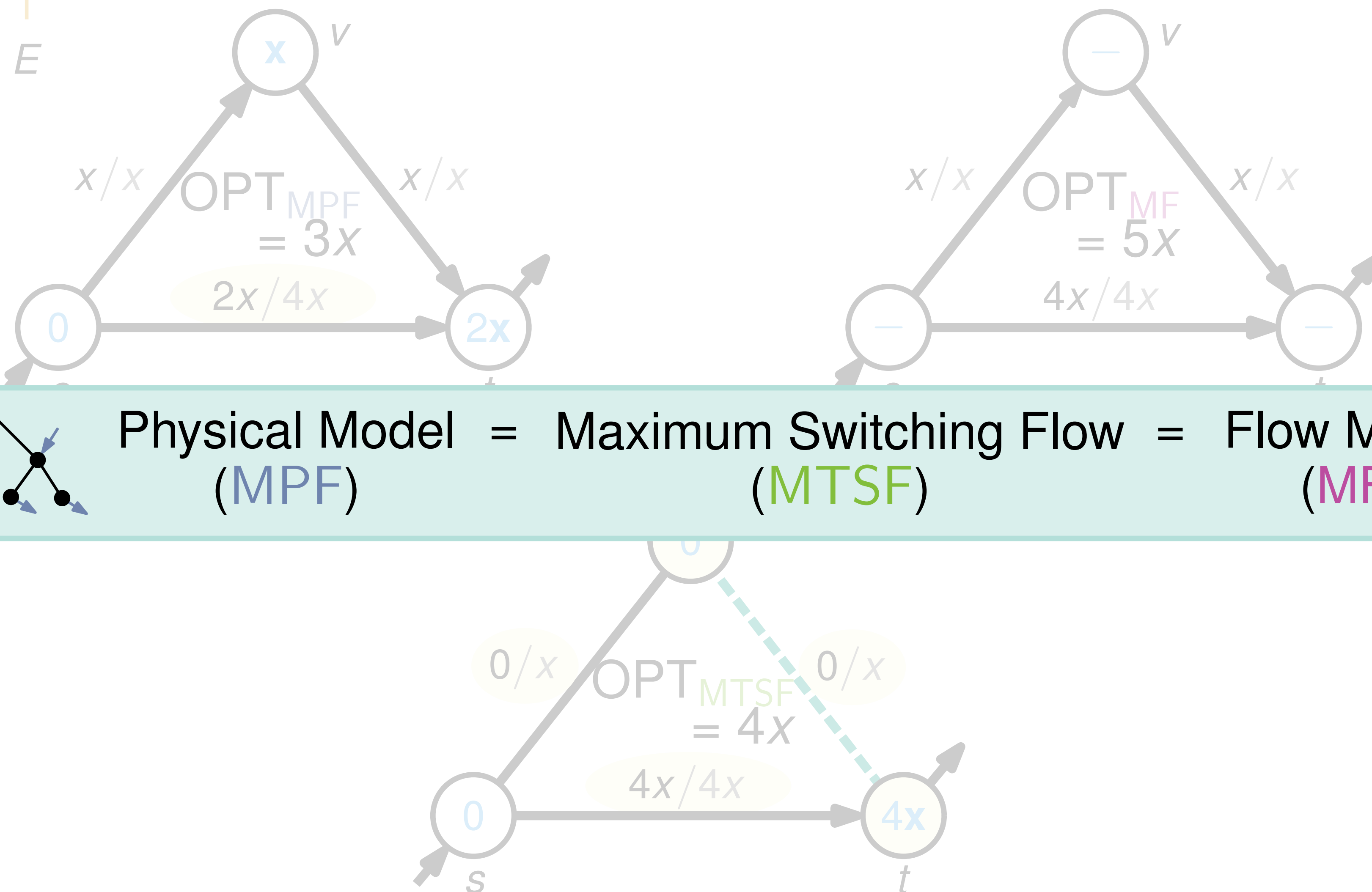

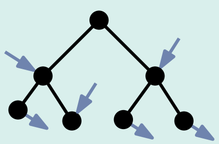

Physical Model (MPF) = Maximum Switching Flow (MTSF) = Flow Model (MF)

$$\forall (u,v) \in E\colon\ f(u,v) = b(u,v)\big(\theta(v) - \theta(u)\big)$$

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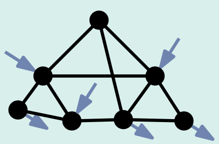
The MTSF Problem

$b(i, j) := 1$
$\forall (i, j) \in E$



Physical Model (MPF) $\leq$ Maximum Switching Flow (MTSF) $\leq$ Flow Model (MF)

$$\forall (u, v) \in E : f(u, v) = b(u, v)\big(\theta(v) - \theta(u)\big)$$

Overview of the MTSF Results

Graph Structure	Complexity	Algorithm

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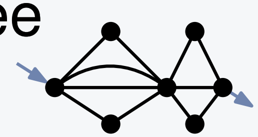
Overview of the MTSF Results

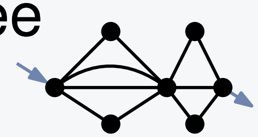
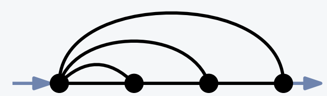
Karlsruhe Institute of Technology

easy

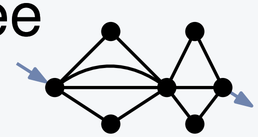
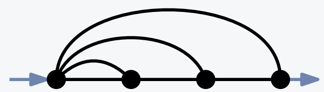
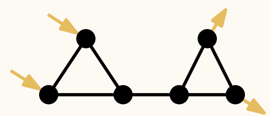
	Graph Structure	Complexity	Algorithm
one generator, one load	penrose-minor-free graphs 	polynomial-time solvable	DTP

Overview of the MTSF Results

complexity

	Graph Structure		Complexity	Algorithm
one generator, one load	penrose-minor-free graphs		polynomial-time solvable	DTP
	series-parallel graphs		NP-hard	✗

Overview of the MTSF Results

	Graph Structure	Complexity	Algorithm
one generator, one load	penrose-minor-free graphs 	polynomial-time solvable	DTP ✓
	series-parallel graphs 	NP-hard	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3 	NP-hard [Lehmann et al., 2014]	2-approx. ✓

complexity ↓

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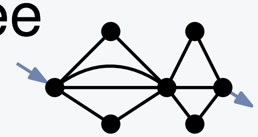
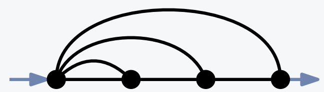
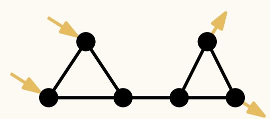
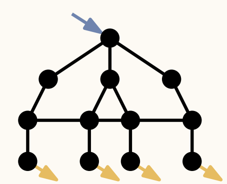
Overview of the MTSF Results

Karlsruhe Institute of Technology

complexity

	Graph Structure		Complexity	Algorithm
one generator, one load	penrose-minor-free graphs		polynomial-time solvable	DTP ✓
	series-parallel graphs		NP-hard	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	2-approx. ✓
	2-level trees		NP-hard [Lehmann et al., 2014]	✗

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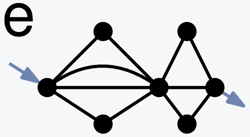
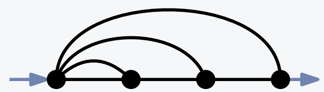
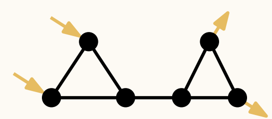
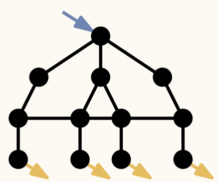
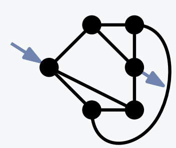
Overview of the MTSF Results

	Graph Structure		Complexity	Algorithm
one generator, one load	penrose-minor-free graphs		polynomial-time solvable	DTP ✓
	series-parallel graphs		NP-hard	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	2-approx. ✓
	2-level trees		NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗

complexity ↓

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Overview of the MTSF Results

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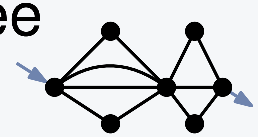
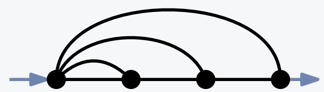
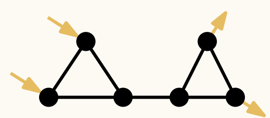
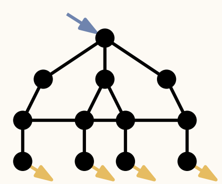
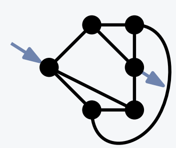
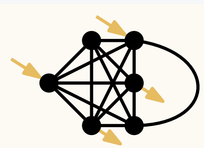
complexity

	Graph Structure		Complexity	Algorithm
one generator, one load	penrose-minor-free graphs		polynomial-time solvable	DTP ✓
one generator, one load	series-parallel graphs		NP-hard	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	2-approx. ✓
arbitrary generators, arbitrary loads	2-level trees		NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗
$\lvert V_G\rvert = 2$, $\lvert V_C\rvert = 2$	arbitrary graphs		non-APX [Lehmann et al., 2014]	✗

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Overview of the MTSF Results

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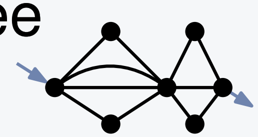
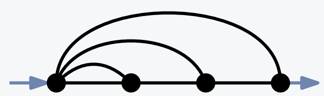
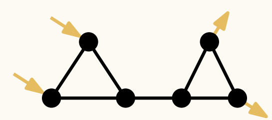
complexity

	Graph Structure		Complexity	Algorithm
one generator, one load	penrose-minor-free graphs		polynomial-time solvable	DTP
	series-parallel graphs		NP-hard	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	2-approx.
	2-level trees		NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗
$\|V_G\| = 2$, $\|V_C\| = 2$	arbitrary graphs		non-APX [Lehmann et al., 2014]	✗

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Overview of the MTSF Results

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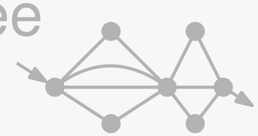
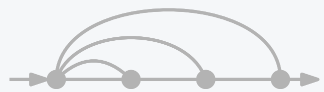
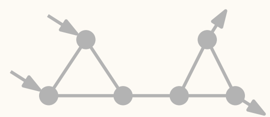
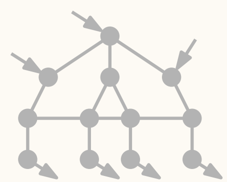
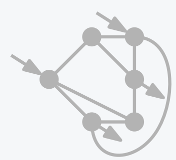
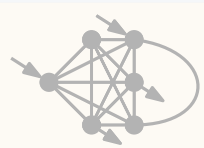
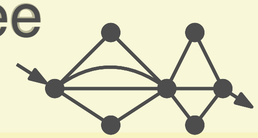
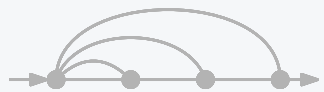
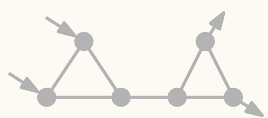
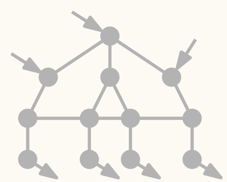
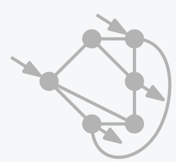
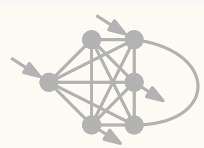
complexity

	Graph Structure		Complexity	Algorithm
one generator, one load	penrose-minor-free graphs		polynomial-time solvable	DTP ✓
	series-parallel graphs		NP-hard	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	2-approx. ✓
	2-level trees		NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗
$\|V_G\|=2$, $\|V_C\|=2$	arbitrary graphs		non-APX [Lehmann et al., 2014]	✗

Overview of the MTSF Results

	Graph Structure	Complexity	Algorithm
one generator, one load	penrose-minor-free graphs 	polynomial-time solvable	DTP ✓
one generator, one load	series-parallel graphs 	NP-hard	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3 	NP-hard [Lehmann et al., 2014]	2-approx. ✓
arbitrary generators, arbitrary loads	2-level trees 	NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3 	strongly NP-hard [Lehmann et al., 2014]	✗
$\lvert V_G\rvert = 2$, $\lvert V_C\rvert = 2$	arbitrary graphs 	non-APX [Lehmann et al., 2014]	✗

complexity

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Dominating Theta Path (DTP)

[Section 5; Grastien et al., 2018]

Fix $u, v \in V$ and a u-v-path π.

Susceptance Norm:

$$\|\pi\|_b := \sum_{e \in E(\pi)} \frac{1}{b(e)}$$

Minimum Capacity:

$$\underline{\mathrm{cap}}(\pi) := \min\{\mathrm{cap}(e) \mid e \in \pi\}$$

Dominating Theta Path (DTP)

[Section 5; Grastien et al., 2018]

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Angle Difference of π:

$$\Delta\theta(\pi) := \|\pi\|_b \cdot \underline{\text{cap}}(\pi)$$

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Dominating Theta Path (DTP)

[Section 5; Grastien et al., 2018]

Fix $u, v \in V$ and a u-v-path π.

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Angle Difference of π:

$$\Delta\theta(\pi) := \|\pi\|_b \cdot \underline{\text{cap}}(\pi)$$

Dominating Theta Path (DTP):

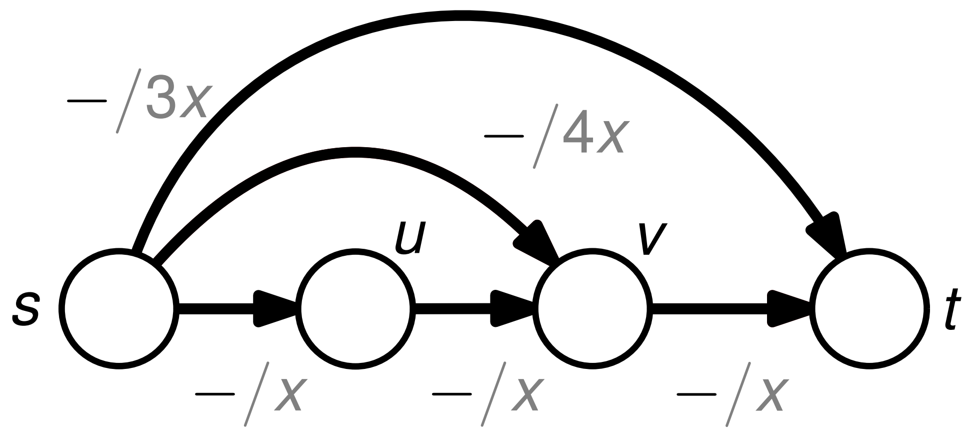
$$\Delta\theta_{\text{min}}(u, v) := \min\{\Delta\theta(\pi) \mid \pi \text{ is a } u\text{-}v\text{-path}\}$$

Computing DTP [Section 5; Grastien et al., 2018]

Description:

- Bicriterial Dijkstra with labels $(\|\pi\|_b, \underline{\text{cap}}(\pi))$
- at most $|E|$ labels per vertex

$b(i,j) := 1 \quad \forall (i,j) \in E$



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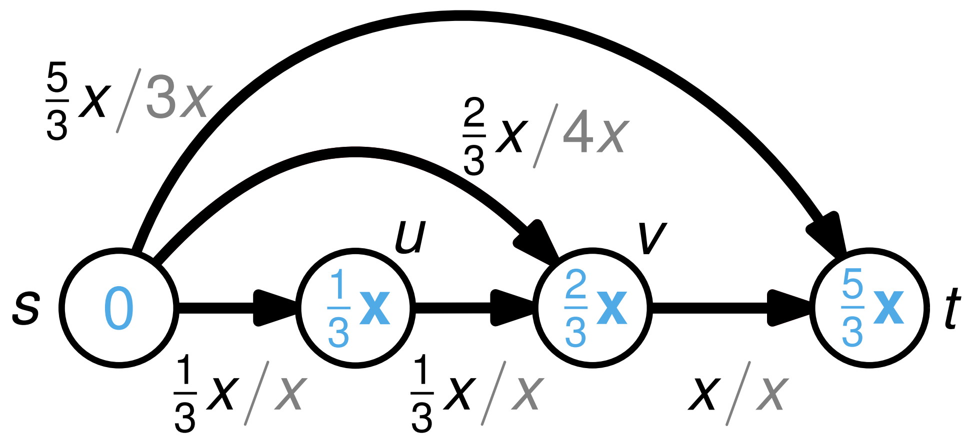

$\mathrm{OPT}_{\mathrm{MPF}} = \frac{8}{3}x$

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Computing DTP [Section 5; Grastien et al., 2018]

Description:

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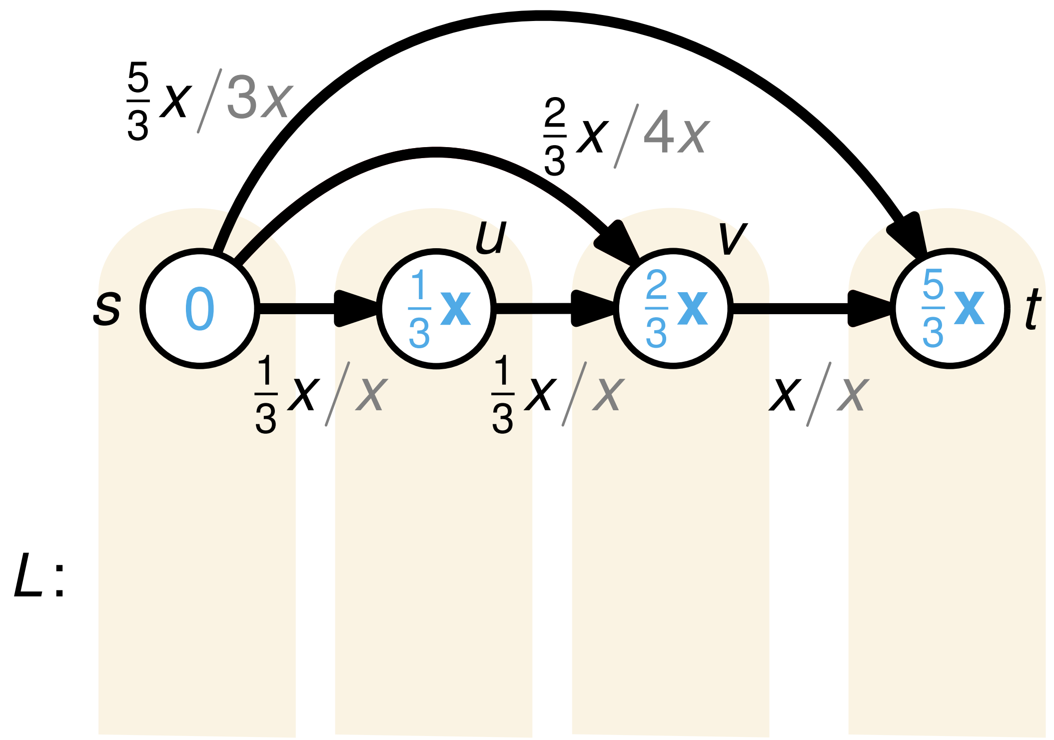

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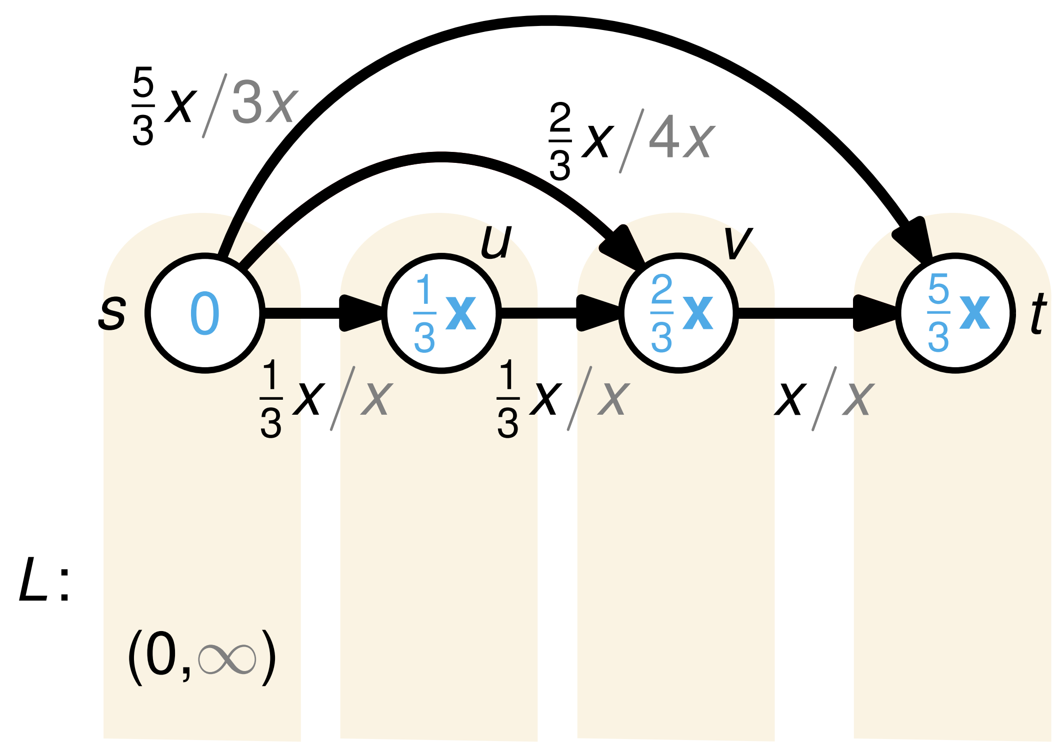

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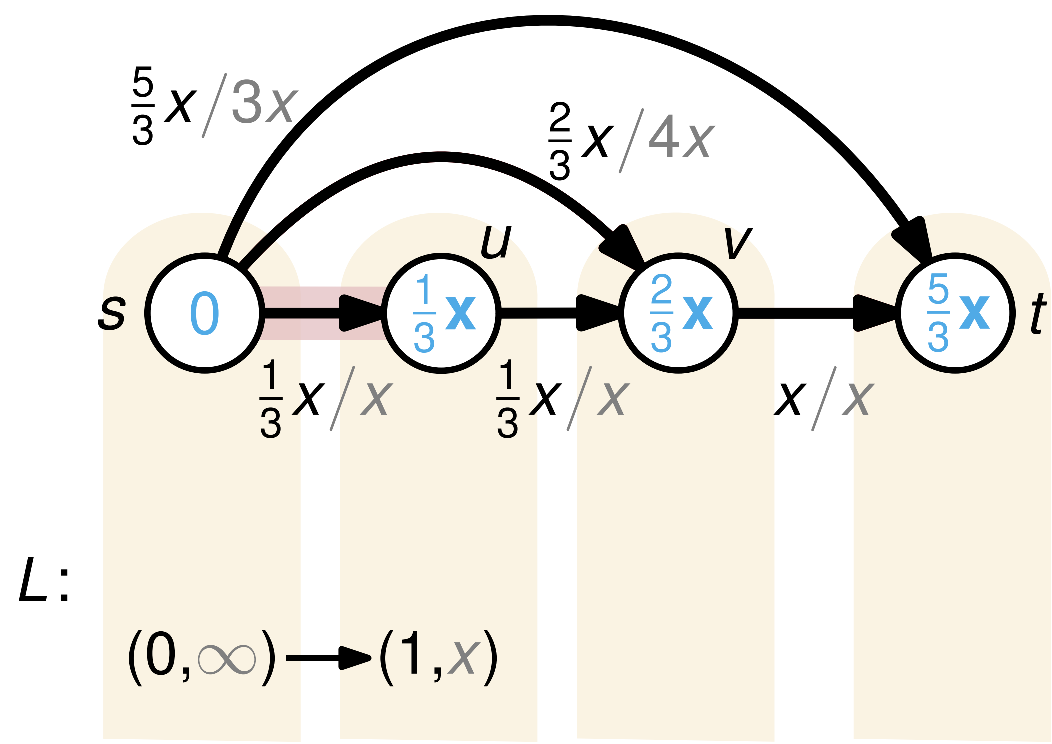

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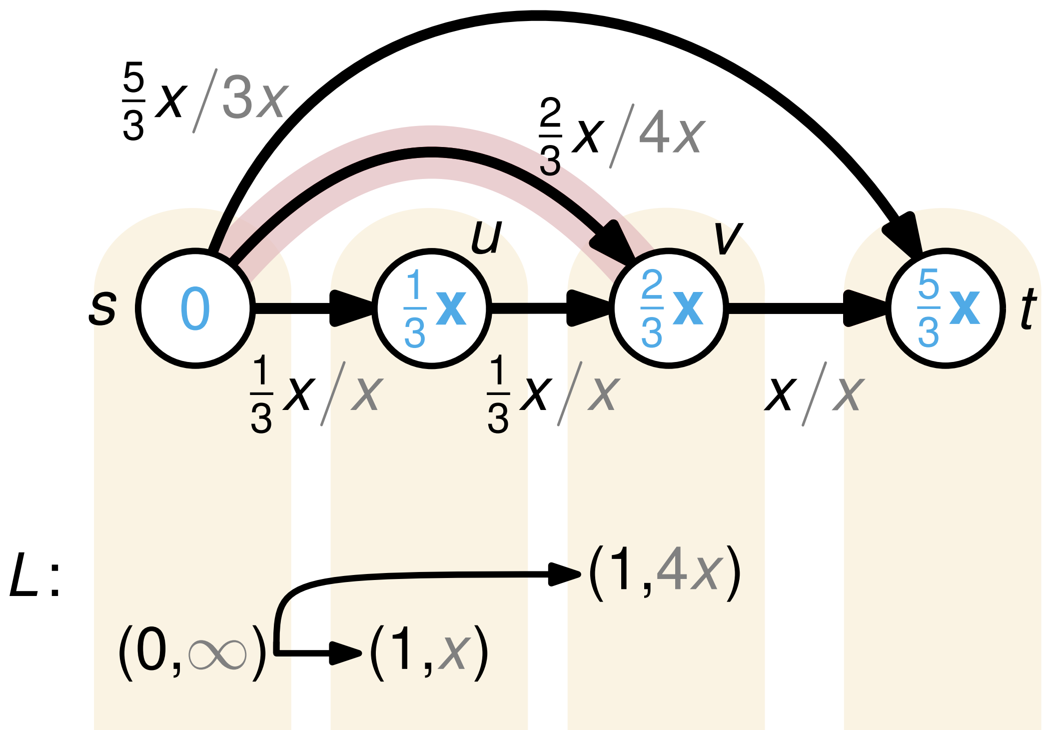

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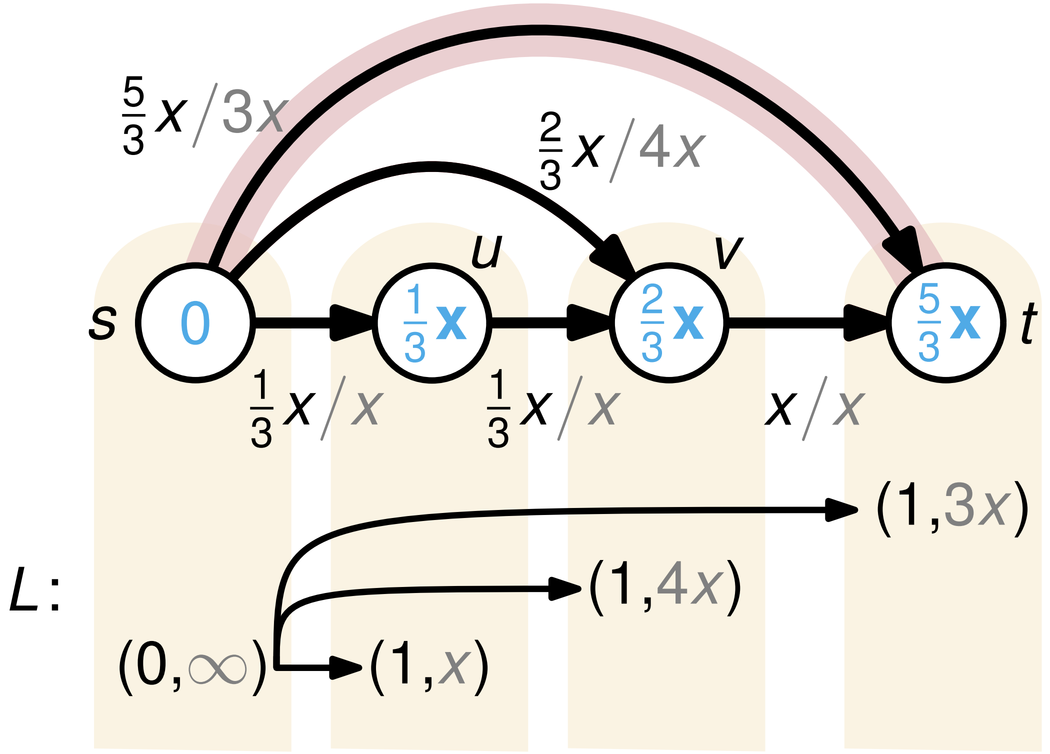

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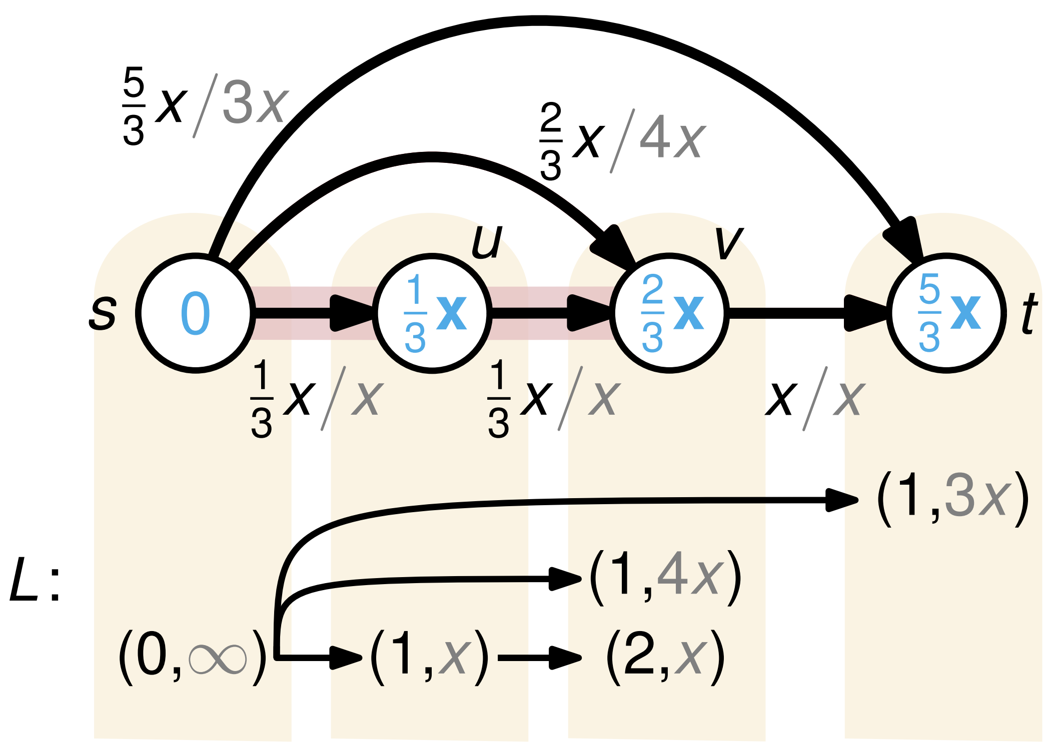

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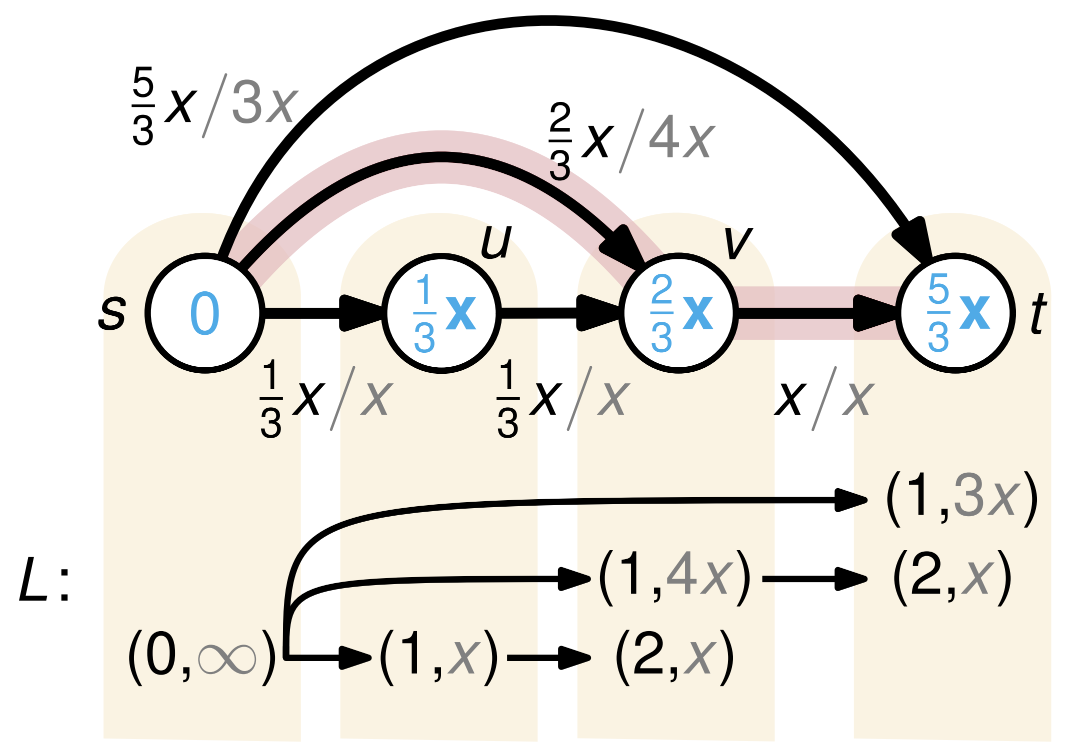

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Computing DTP [Section 5; Grastien et al., 2018]

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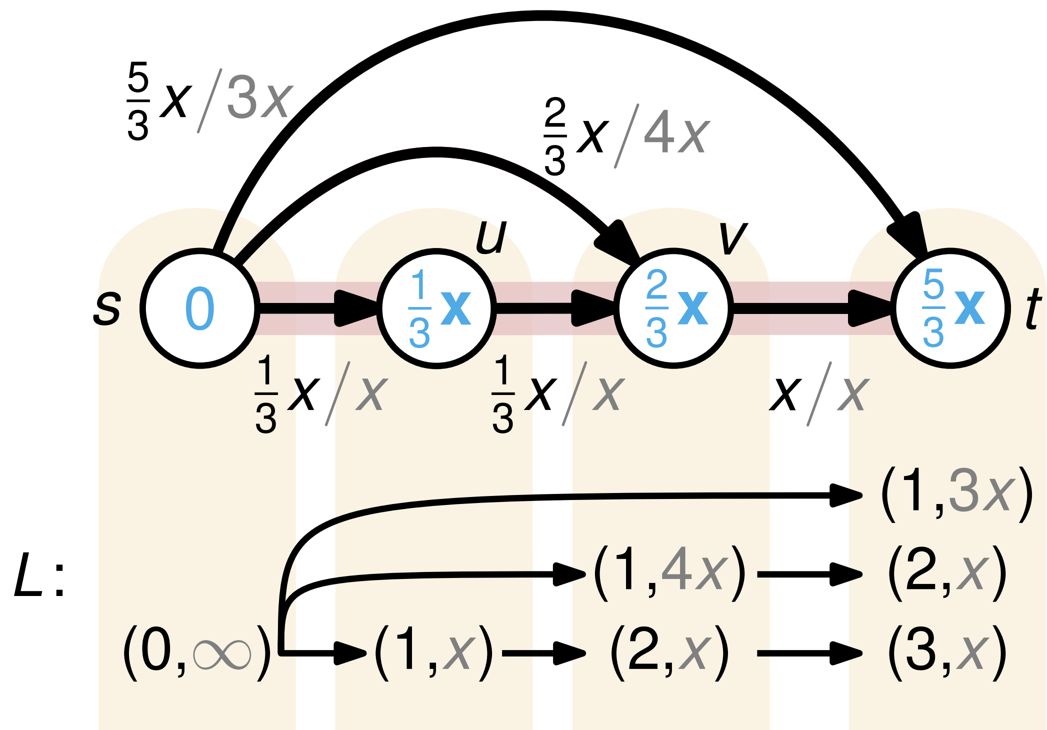

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Computing DTP [Section 5; Grastien et al., 2018]

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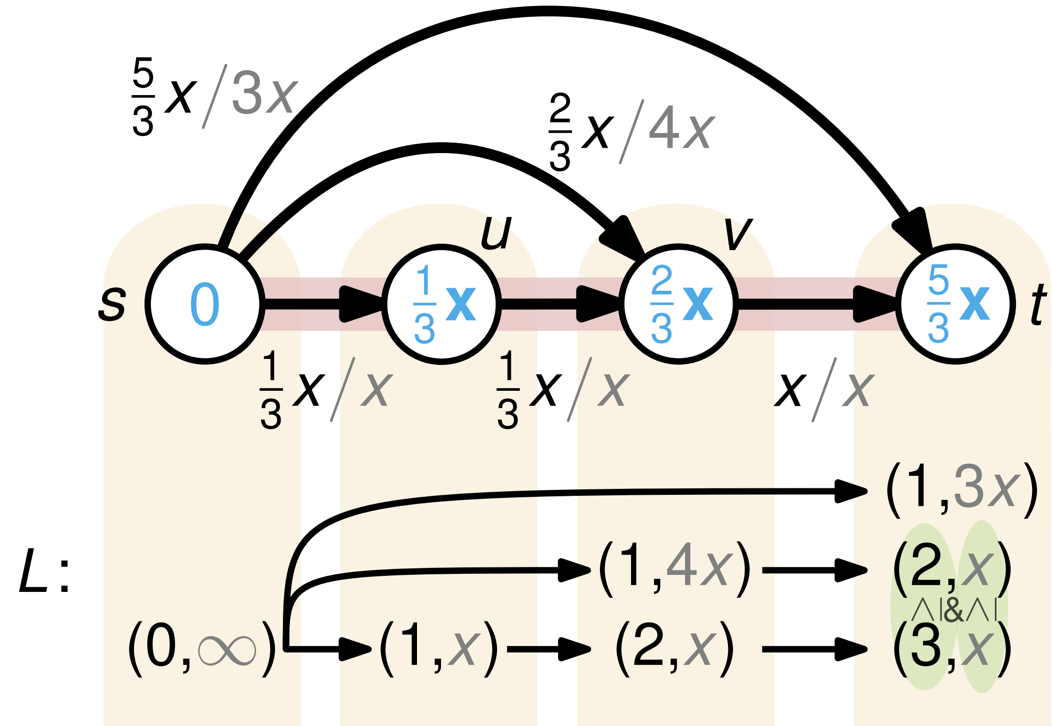

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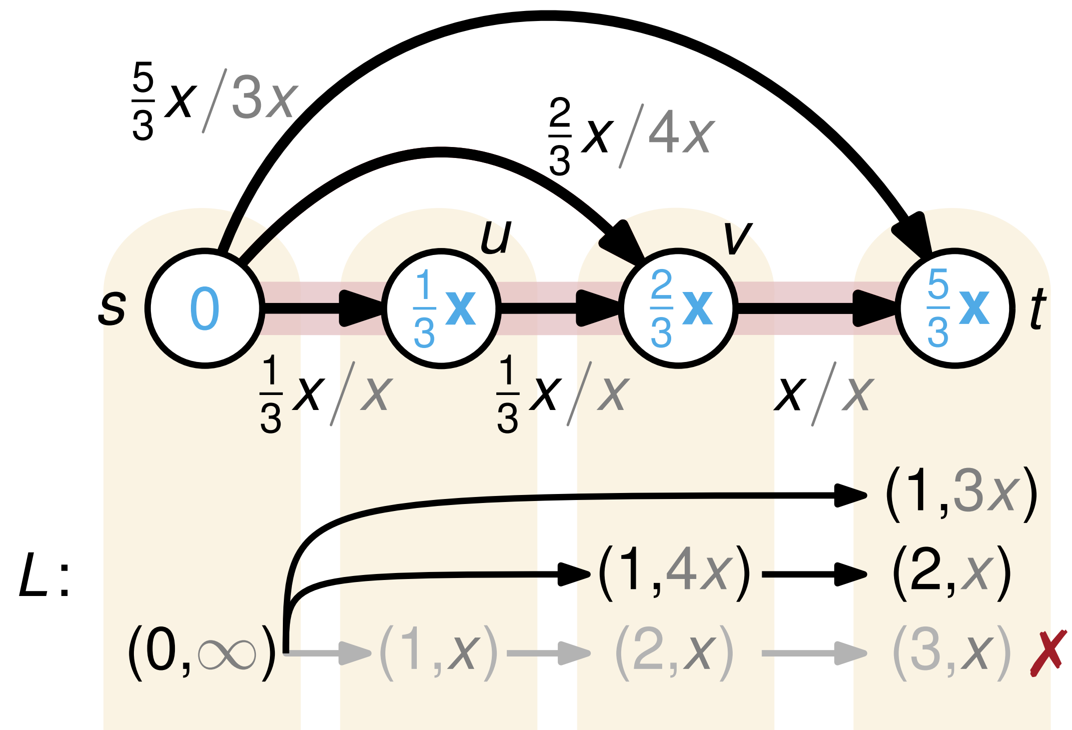

$\text{OPT}_{\text{MPF}} = \frac{8}{3}x$

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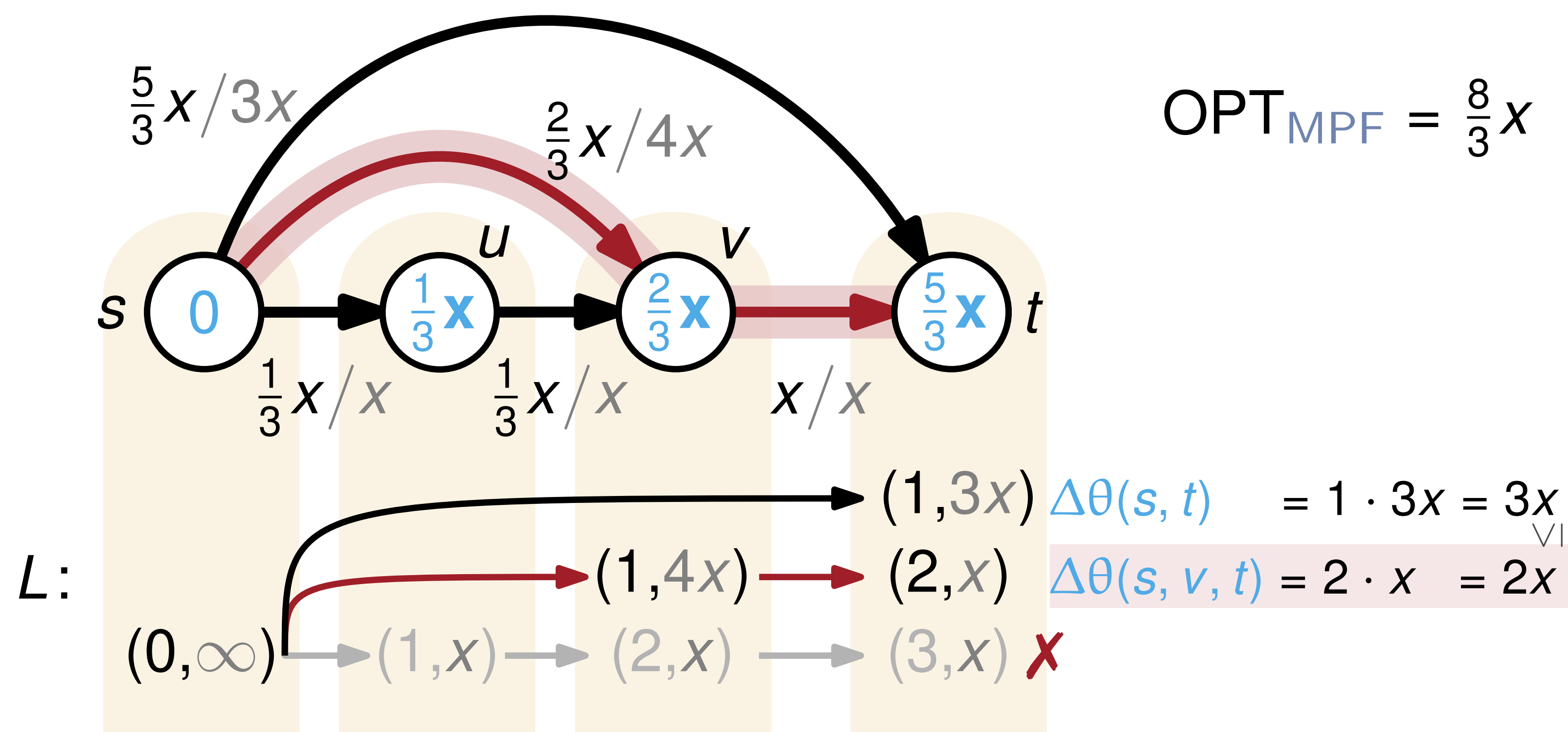
Computing DTP [Section 5; Grastien et al., 2018]

Karlsruhe Institute of Technology

Description:

- Bicriterial Dijkstra with labels $(\|\pi\|_b, \underline{\text{cap}}(\pi))$
- at most $|E|$ labels per vertex

$b(i,j) := 1 \quad \forall (i,j) \in E$

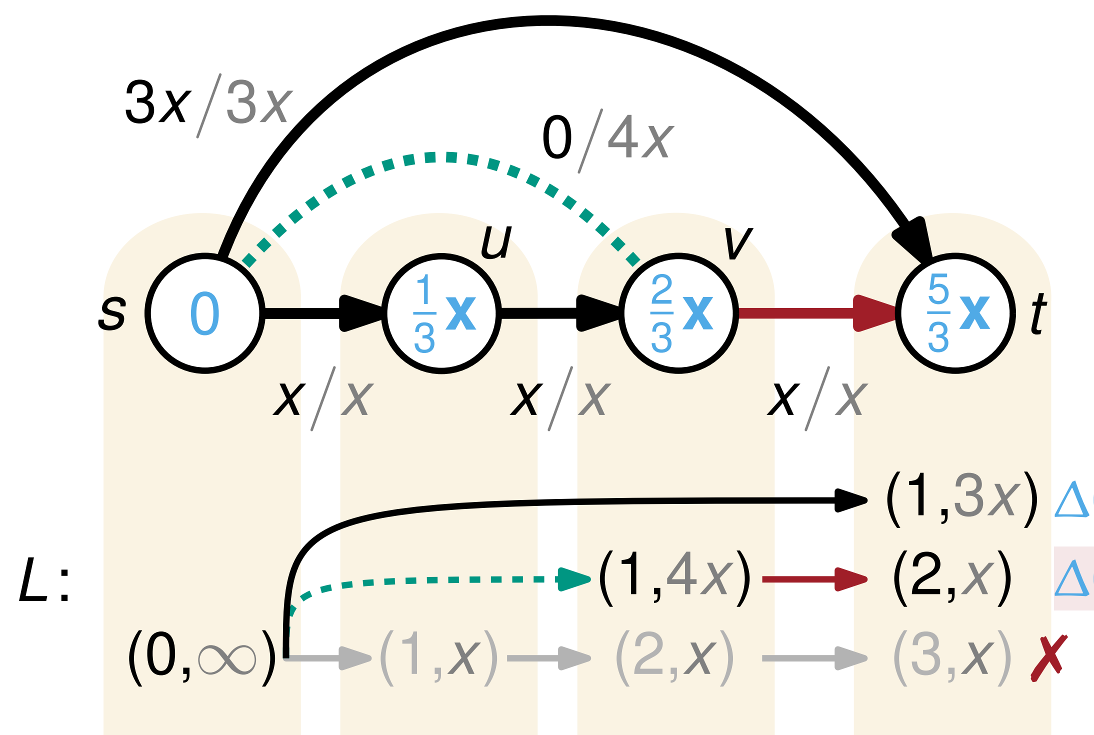


Computing **DTP** [Section 5; Grastien et al., 2018]

Description:

- Bicriterial Dijkstra with labels $(\|\pi\|_b, \underline{\text{cap}}(\pi))$
- at most $|E|$ labels per vertex

$b(i,j) := 1 \quad \forall (i,j) \in E$



$OPT_{MPF} = 4x$
$= OPT_{MTSF}$

$\Delta\theta(s,t) = 1 \cdot 3x = 3x$

$\vee|$

$\Delta\theta(s,v,t) = 2 \cdot x = 2x$

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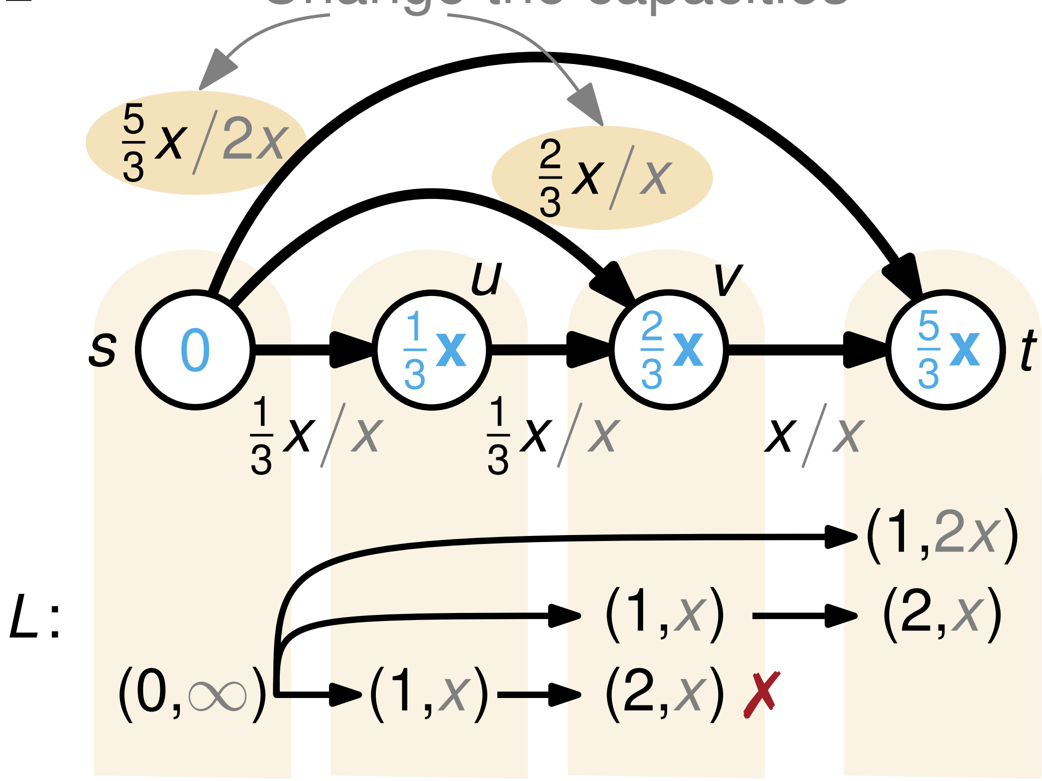
Computing DTP [Section 5; Grastien et al., 2018]

Description:

- Bicriterial Dijkstra with labels $(\|\pi\|_b, \underline{\mathrm{cap}}(\pi))$
- at most $|E|$ labels per vertex

$b(i,j) := 1 \quad \forall (i,j) \in E$

Change the capacities



$\mathrm{OPT}_{\mathrm{MPF}} = \frac{8}{3}x$

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Computing DTP [Section 5; Grastien et al., 2018]

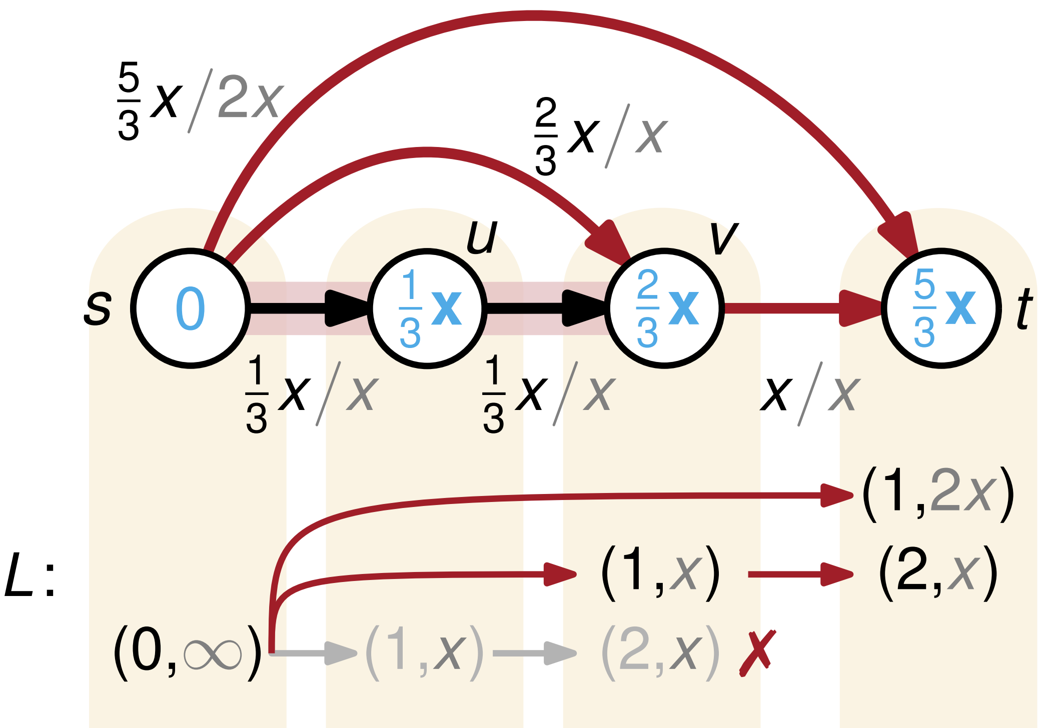
Karlsruhe Institute of Technology

Description:

- Bicriterial Dijkstra with labels ($\|\pi\|_b$, $\underline{\text{cap}}(\pi)$)
- at most $|E|$ labels per vertex

$b(i,j) := 1 \quad \forall (i,j) \in E$

- DTPs from s do not have to form a tree



$\text{OPT}_{\text{MPF}} = \frac{8}{3}x$

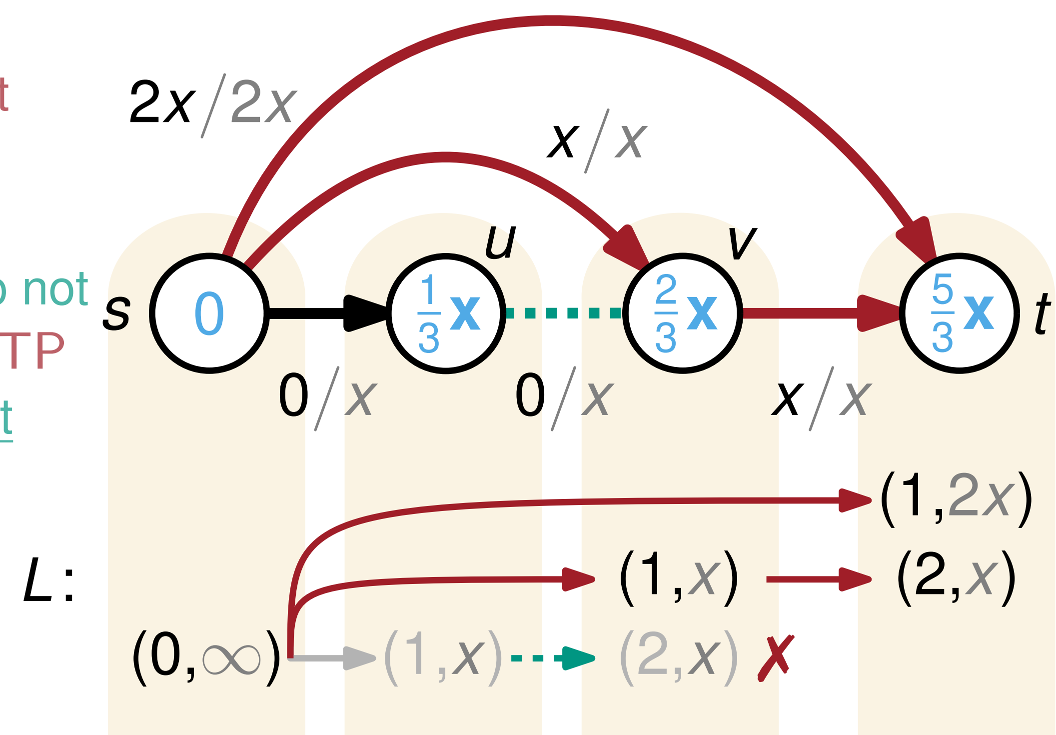
Computing DTP [Section 5; Grastien et al., 2018]

Description:

- Bicriterial Dijkstra with labels ($\|\pi\|_b$, cap(π))
- at most $|E|$ labels per vertex

$b(i,j) := 1 \quad \forall (i,j) \in E$

- DTPs from s do not have to form a tree
- Optimal switches do not have to lie on the DTP if the structure is not penrose-minor free



$OPT_{MPF} = 3x$
$= OPT_{MTSF}$

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Computing DTP [Section 5; Grastien et al., 2018]

Description:

- Bicriterial Dijkstra with labels $(\|\pi\|_b, \underline{\text{cap}}(\pi))$
- at most $|E|$ labels per vertex

$b(i,j) := 1 \quad \forall (i,j) \in E$

On general graphs the DTP algorithm runs in time $\mathcal{O}\left(2^{|V|}|V| \cdot |E|^3\right)$.

- Optimal switches do not have to lie on the DTP if the structure is not penrose-minor free

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Computing DTP [Section 5; Grastien et al., 2018]

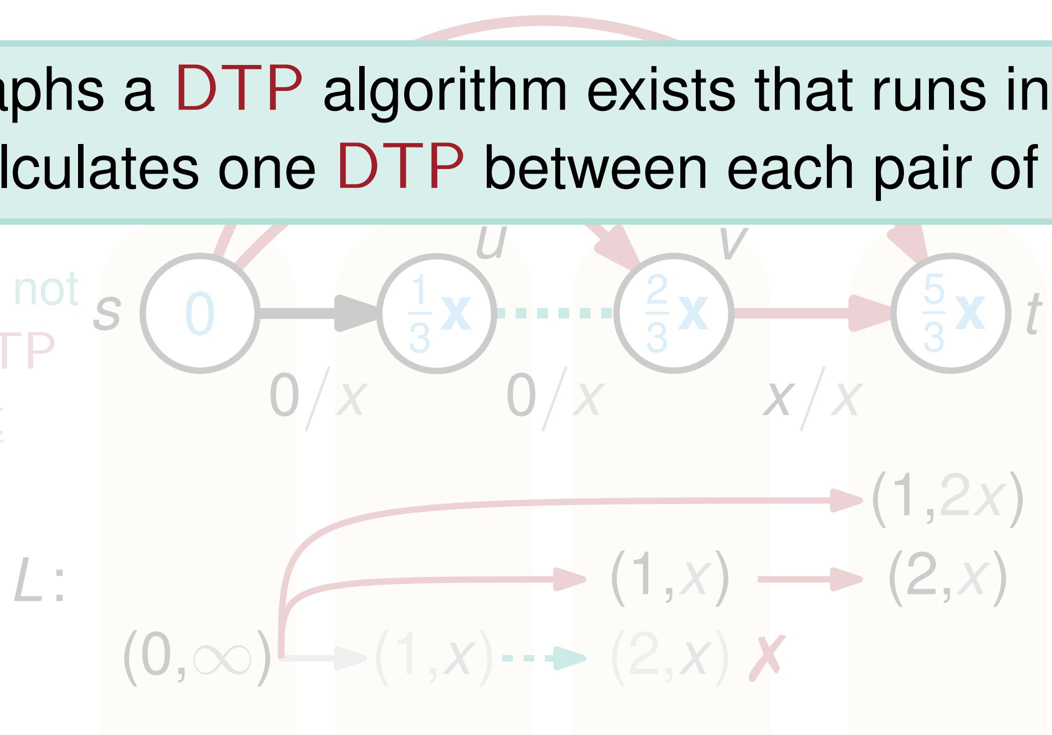
Description:

- Bicriterial Dijkstra with labels ($\|\pi\|_b$, $\underline{\text{cap}}(\pi)$)
- at most $|E|$ labels per vertex

$b(i,j) := 1 \quad \forall (i,j) \in E$

On general graphs a DTP algorithm exists that runs in polynomial time and calculates one DTP between each pair of u and v.

- Optimal switches do not have to lie on the DTP if the structure is not penrose-minor free



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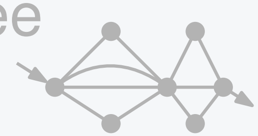
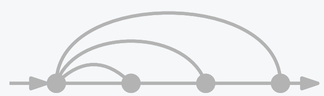
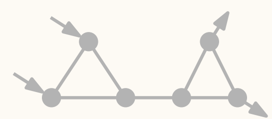
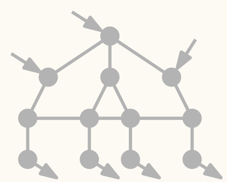
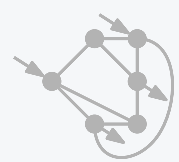
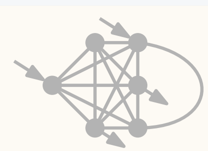
Overview of the MTSF Results

complexity

	Graph Structure		Complexity	Algorithm
one generator, one load	penrose-minor-free graphs		polynomial-time solvable	DTP ✓
	series-parallel graphs		NP-hard	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	2-approx. ✓
	2-level trees		NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗
$\lvert V_G \rvert = 2$, $\lvert V_C \rvert = 2$	arbitrary graphs		non-APX [Lehmann et al., 2014]	✗

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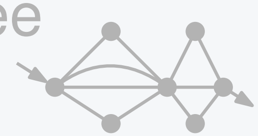
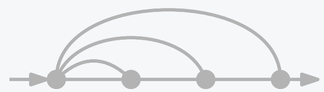
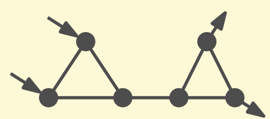
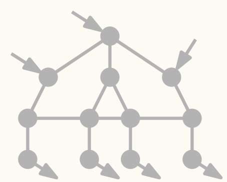
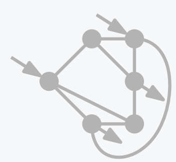
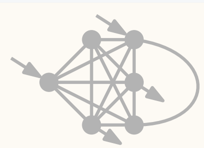
Overview of the MTSF Results

Karlsruhe Institute of Technology

	Graph Structure	Complexity	Algorithm
one generator, one load	penrose-minor-free graphs 	polynomial-time solvable	DTP ✓
	series-parallel graphs 	NP-hard	✗
arbitrary generators, arbitrary loads	**cacti with max degree of 3** 	**NP-hard** [Lehmann et al., 2014]	**2-approx.** ✓
	2-level trees 	NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3 	strongly NP-hard [Lehmann et al., 2014]	✗
$\lvert V_G \rvert = 2$, $\lvert V_C \rvert = 2$	arbitrary graphs 	non-APX [Lehmann et al., 2014]	✗

complexity

2-approximation on Cacti

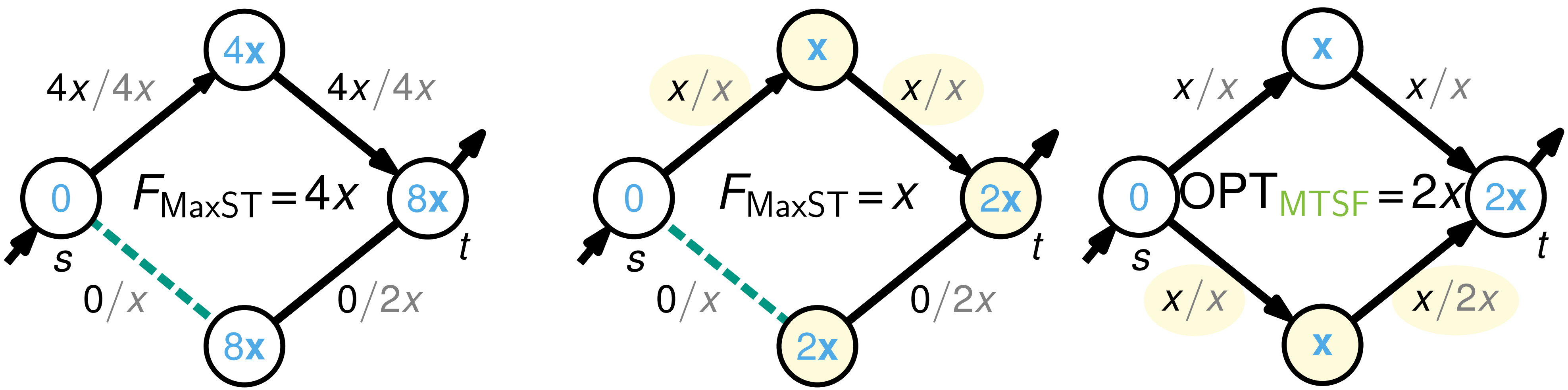
Description

- Remove from each cycle the edge with the smallest capacity

$\Leftrightarrow$ the MAXIMUM SPANNING TREE (MaxST)

MaxST on Cacti

- MTSF is NP-hard on cacti [Lehmann et al., 2014]



Theorem 1 [page 348; Grastien et al., 2018]

MaxST is a factor 2-approximation algorithm for the MF and MTSF problem on cacti.

2-approximation on Cacti

Description

- Remove from each cycle the edge with the smallest capacity

⇔ the MAXIMUM SPANNING TREE (MaxST)

MaxST on Cacti

- MTSF is NP-hard on cacti [Lehmann et al., 2014]

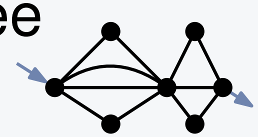
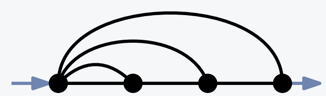
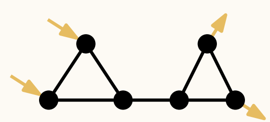
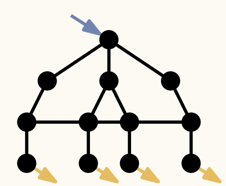
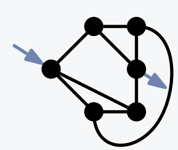
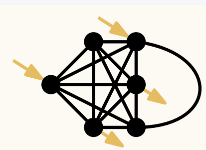
On cacti the MaxST algorithm runs in time $\mathcal{O}(|V|)$.

Theorem 1 [page 348; Grastien et al., 2018]

MaxST is a factor 2-approximation algorithm for the MF and MTSF problem on cacti.

Summary & Future Work

complexity

	Graph Structure		Complexity	Algorithm
one generator, one load	penrose-minor-free graphs		polynomial-time solvable	✓
	series-parallel graphs		NP-hard	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	✓
	2-level trees		NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗
$\lvert V_G \rvert = 2$, $\lvert V_C \rvert = 2$	arbitrary graphs		non-APX [Lehmann et al., 2014]	✗

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complexity

	Graph Structure		Complexity	Algorithm
one generator, one load	penrose-minor-free graphs		polynomial-time solvable	✓
	series-parallel graphs		NP-hard	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	✓
	2-level trees		NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗
$\|V_G\| = 2$, $\|V_C\| = 2$	arbitrary graphs		non-APX [Lehmann et al., 2014]	✗

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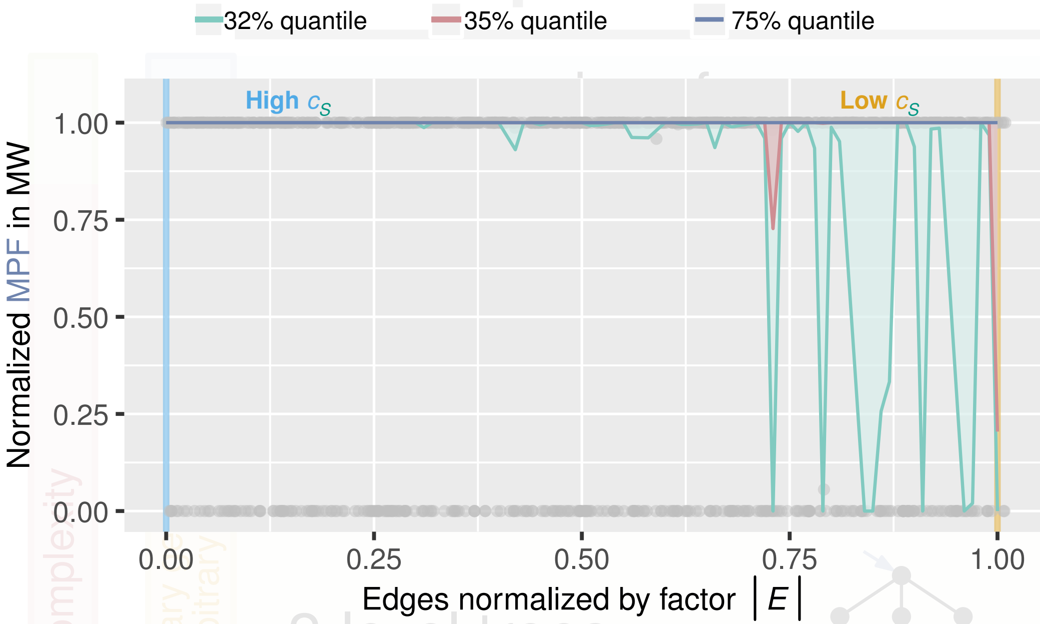
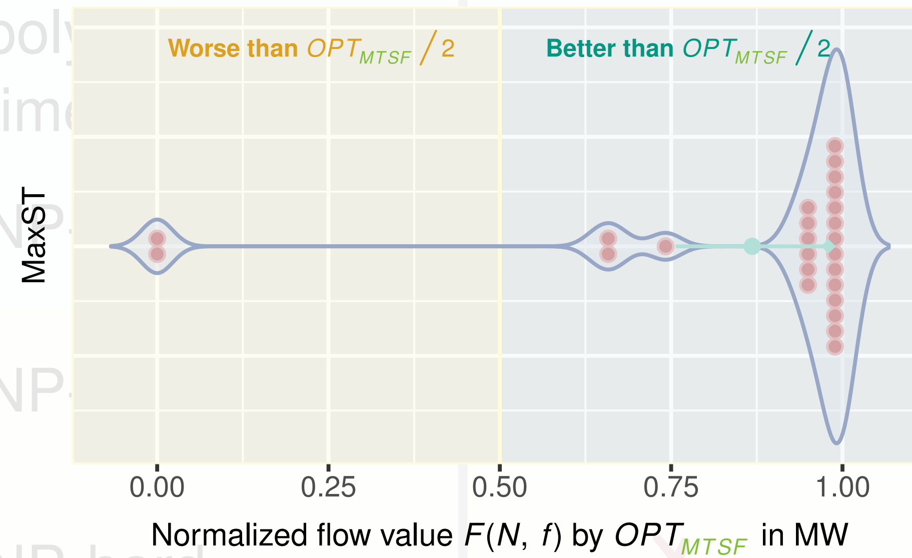
Summary & Future Work

[page 349; Grastien et al., 2018]

Results





Summary & Future Work

[page 349; Grastien et al., 2018]

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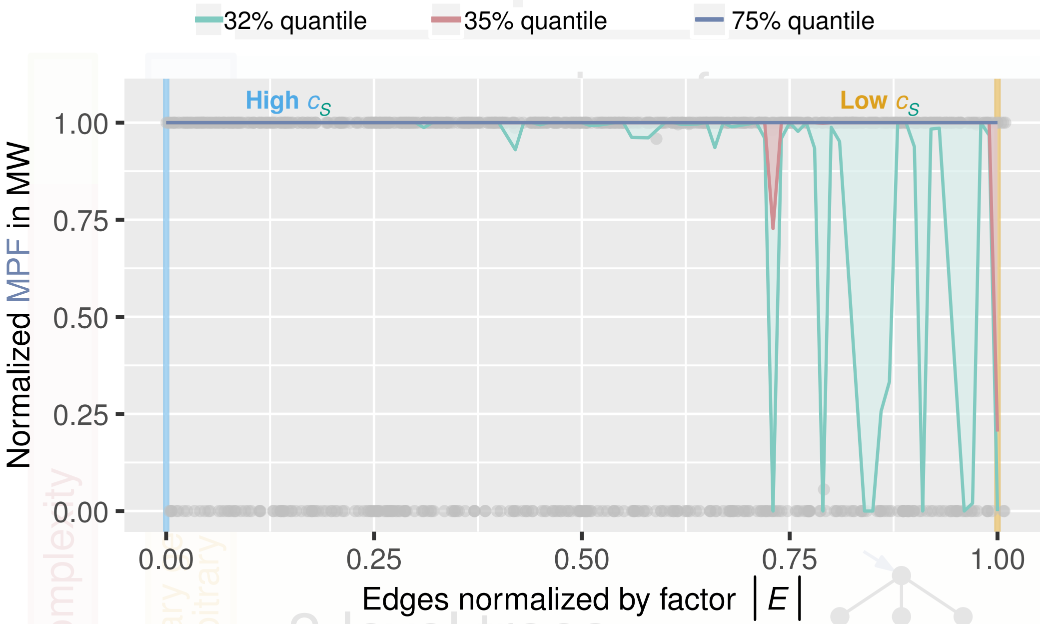
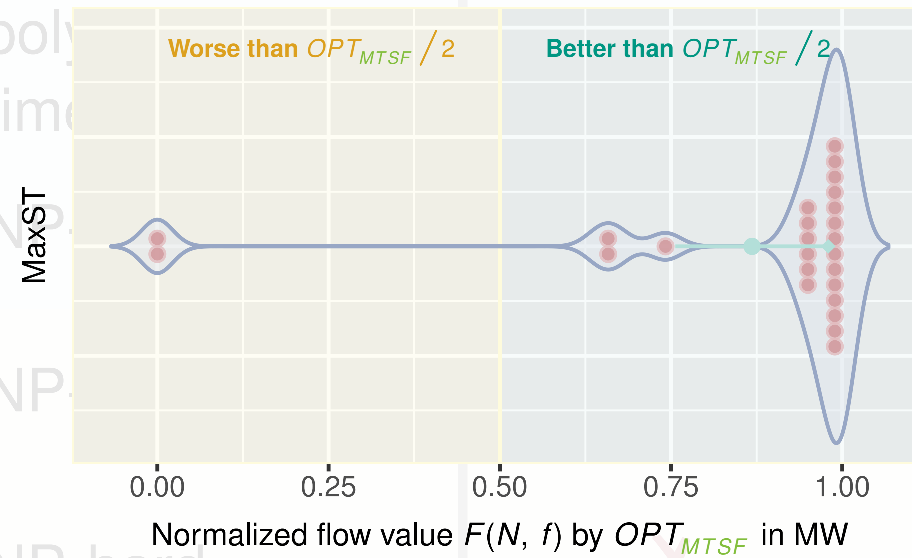
Results





Future Work

- What happens if we minimize the number of switches or fix a set of non-switchable edges?
- Is there a PTAS on cacti for MTSF?
- Replace ✗ by ✓

Summary & Future Work

Joint work with. . .

Alban Grastien

Ignaz Rutter

Dorothea Wagner

Matthias Wolf

Future Work

- What happens if we minimize the number of switches or fix a set of non-switchable edges?
- Is there a PTAS on cacti for MTSF?
- Replace ✗ by ✓

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