Karlsruhe Institute of Technology

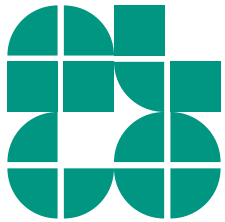
Algorithms for Graph Visualization

Layered Layout

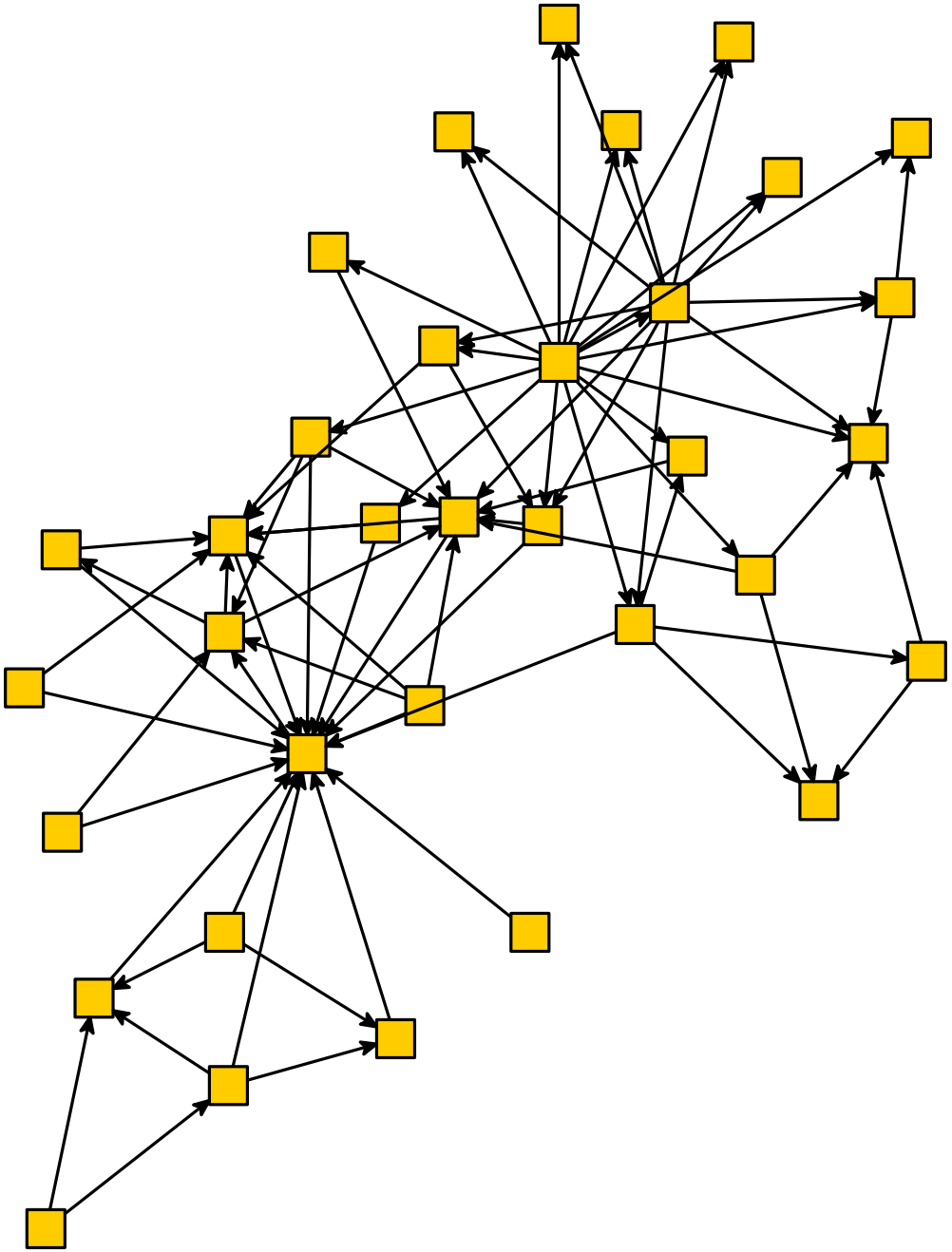
INSTITUT FÜR THEORETISCHE INFORMATIK · FAKULTÄT FÜR INFORMATIK

Tamara Mchedlidze

13.12.2017



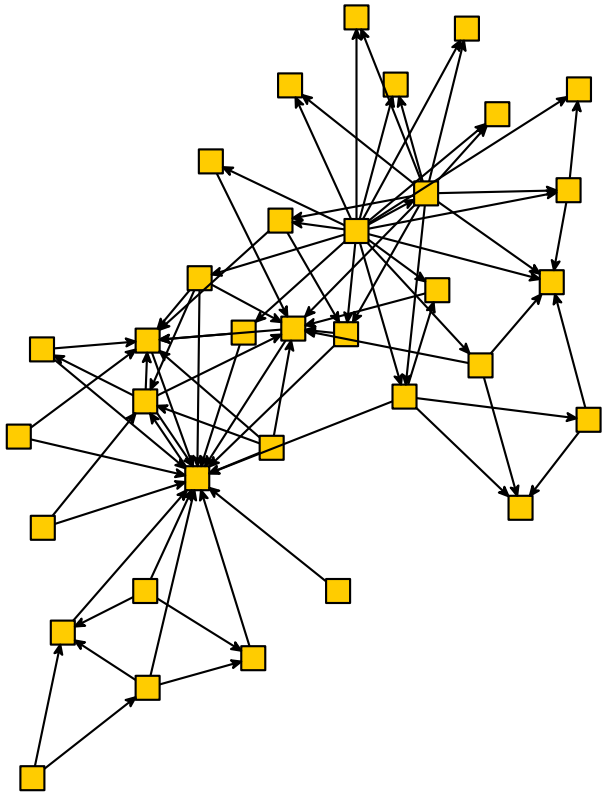
Example

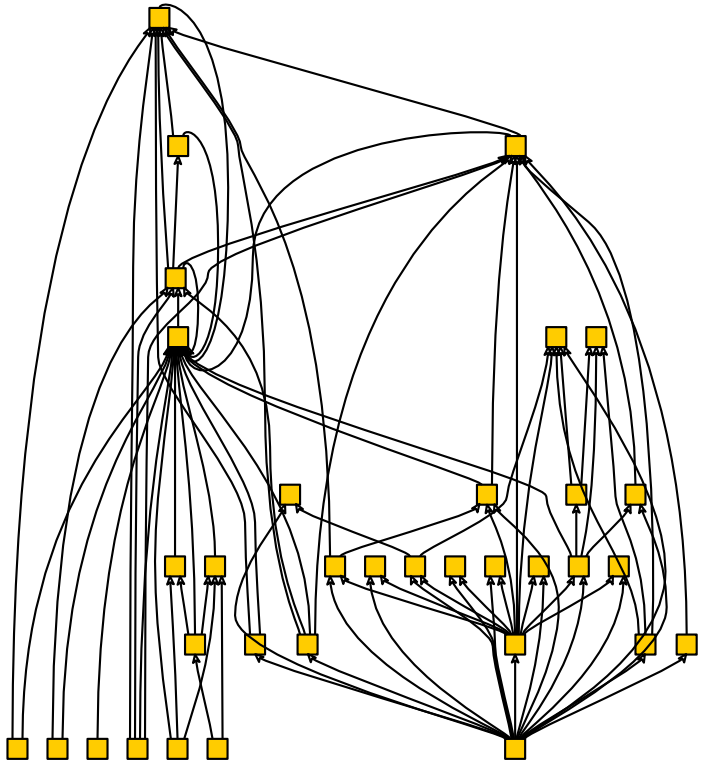


Layered Layout

Given: directed graph $D = (V, A)$

Find: drawing of D that emphasized the hierarchy





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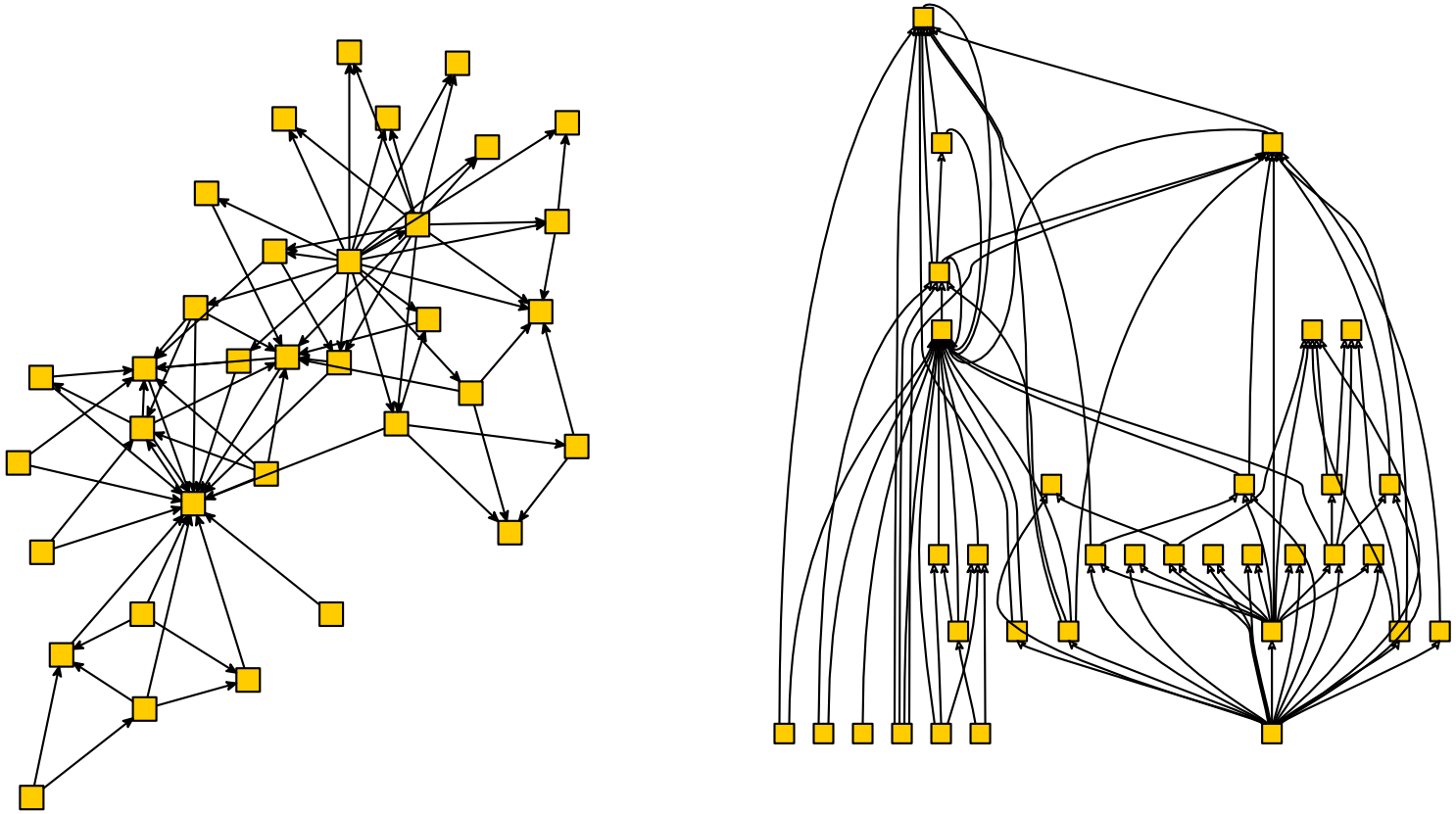

- many edges pointing to the same direction
- nodes lie on (few) horizontal lines

Layered Layout

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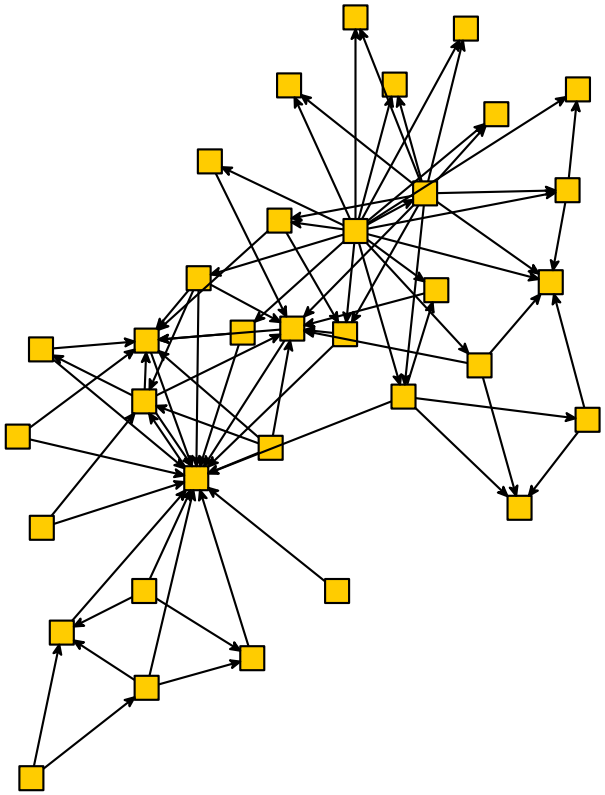
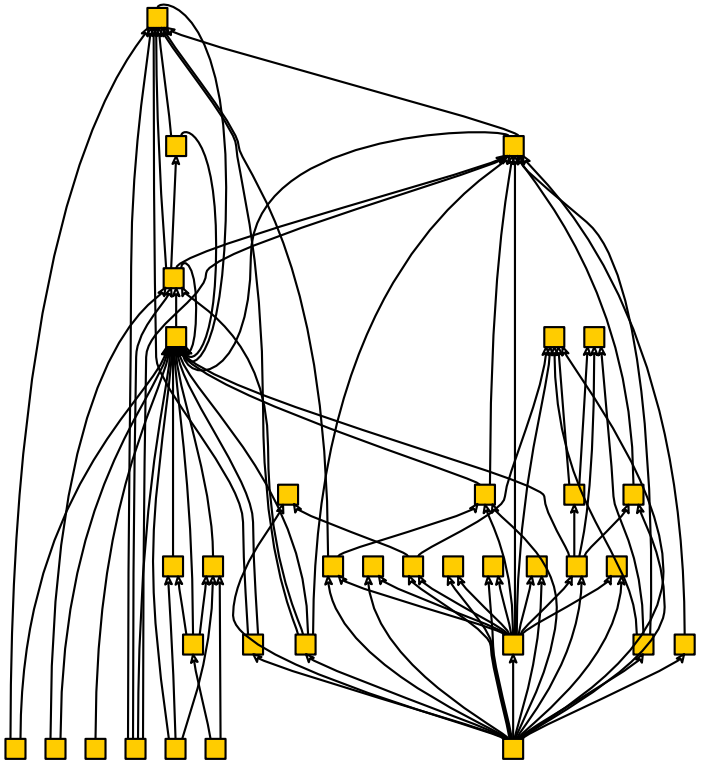
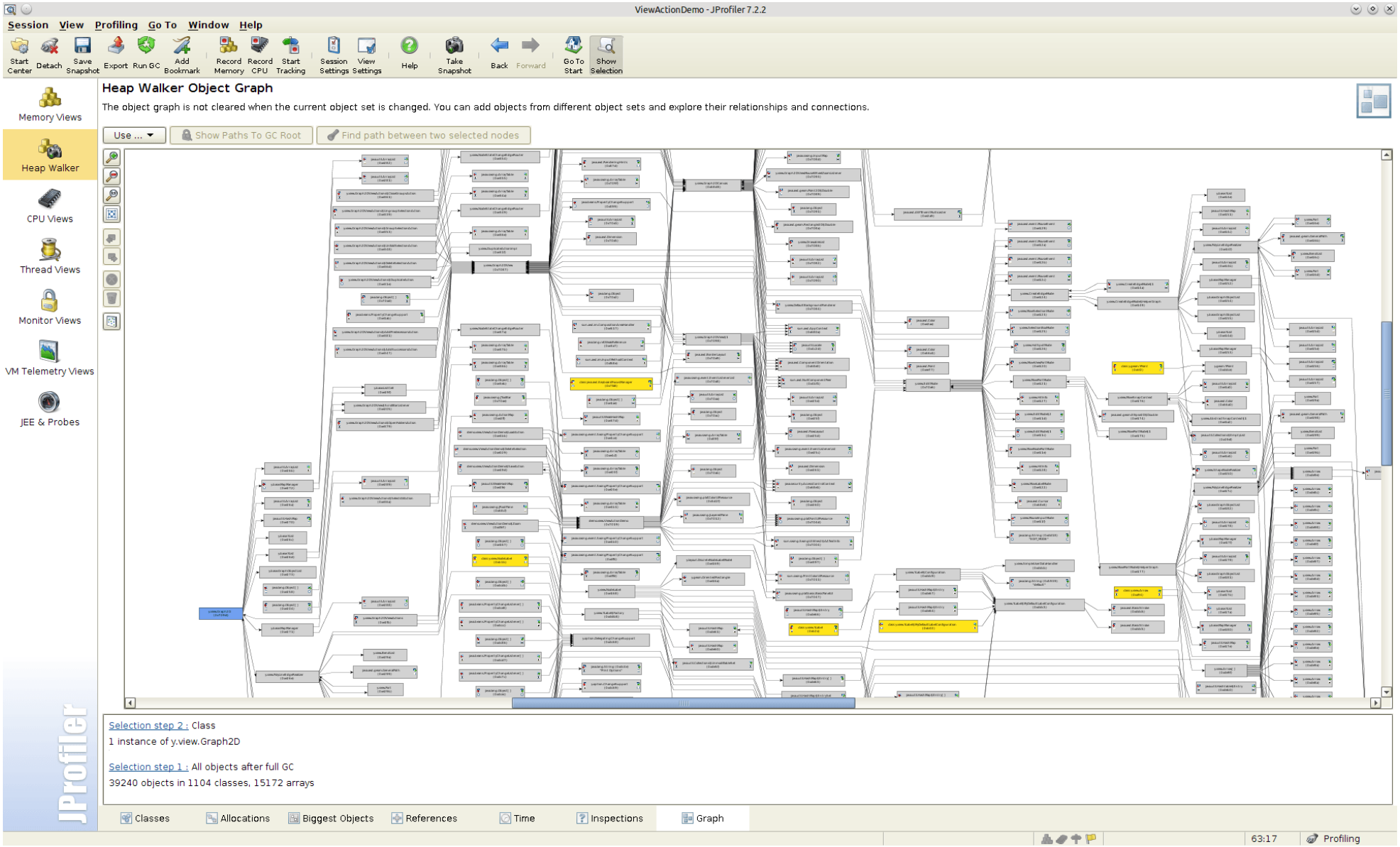




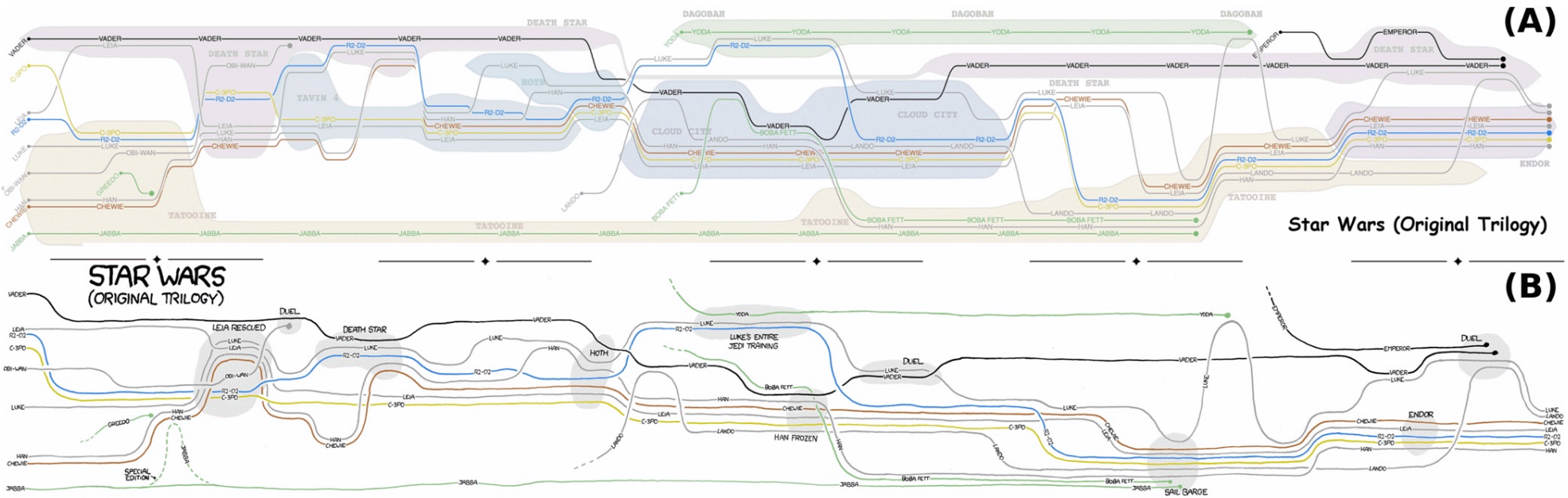
- edges as straight as possible and short
- few edge crossings
- nodes distributed evenly

Application: Java Profiler



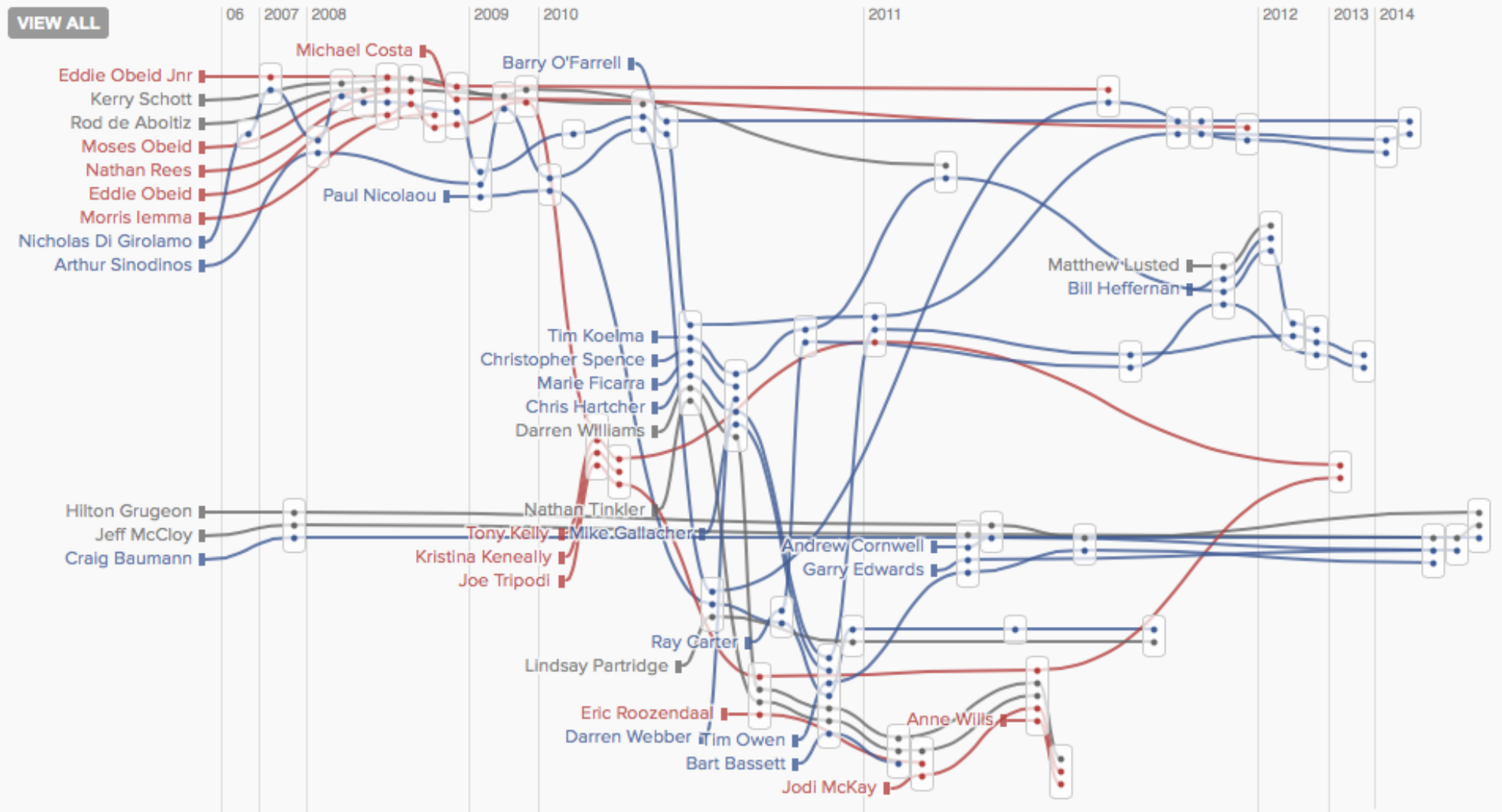
yEd Gallery: Java profiler JProfiler using yFiles

Application: Storylines



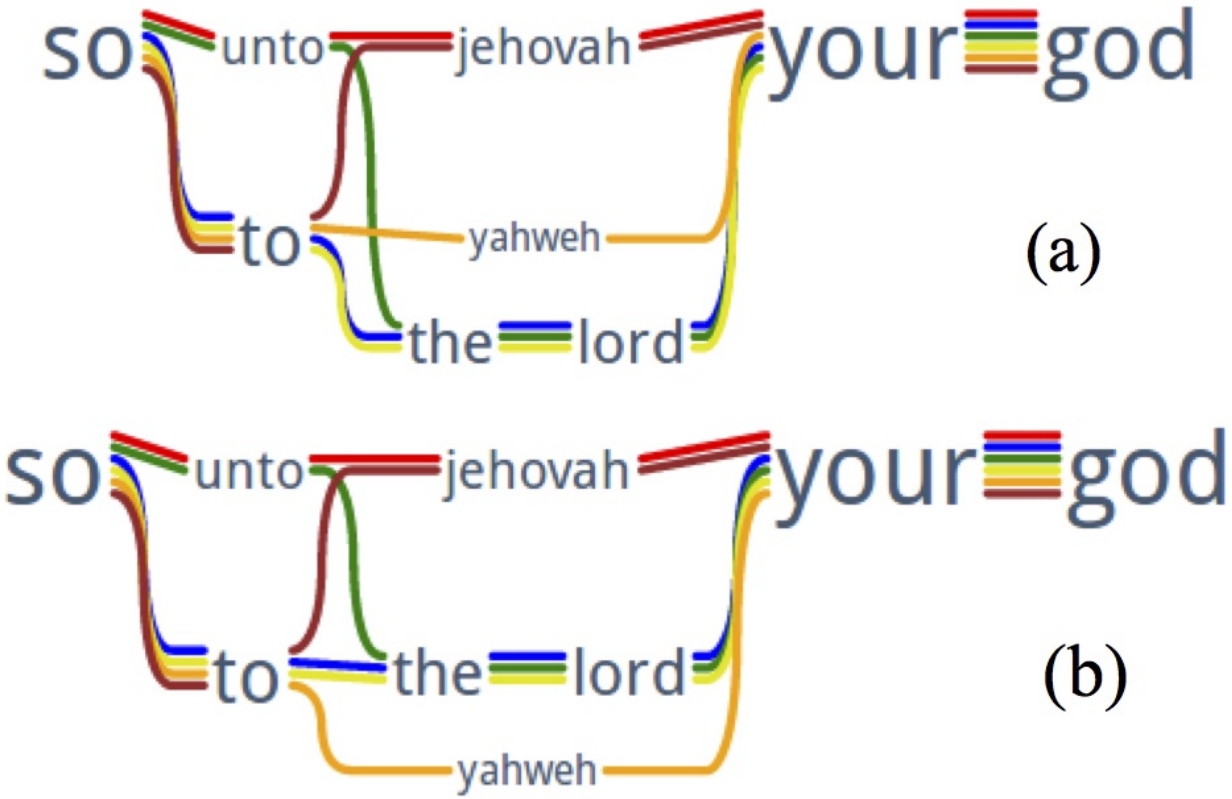
Source: "Design Considerations for Optimizing Storyline Visualizations" Tanahashi et al.

Application: Storylines



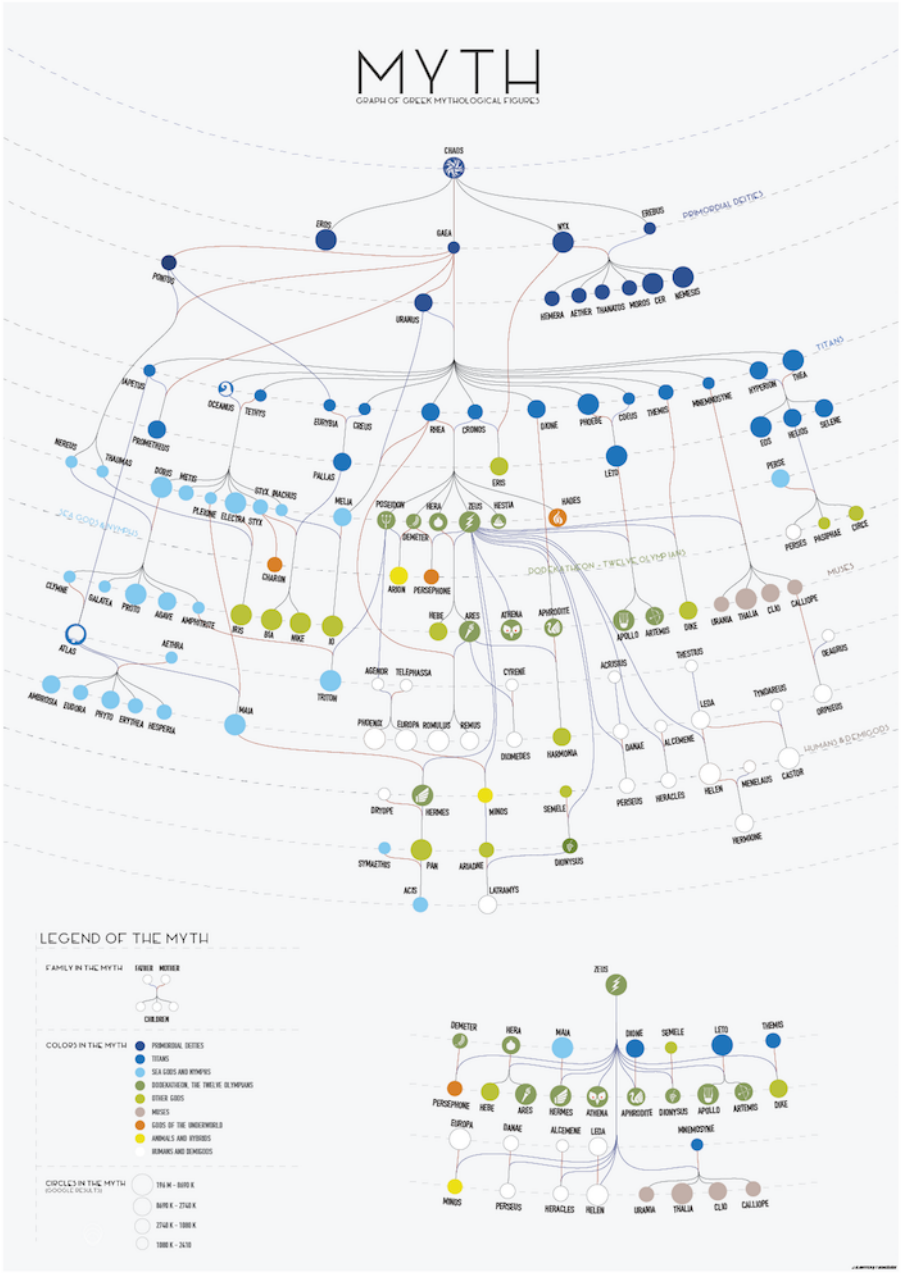
Source: ABC news, Australia

Application: Text-Variant graphs



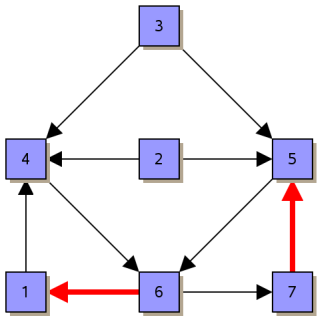
Source: Improving the Layout for Text Variant Graphs Jänicke et al.

Application: Mythological Creatures and Gods



Source: Visualization that won the Graph Drawing contest 2016. Klawitter&Mchedlidze

Sugiyama Framework (Sugiyama, Tagawa, Toda 1981)



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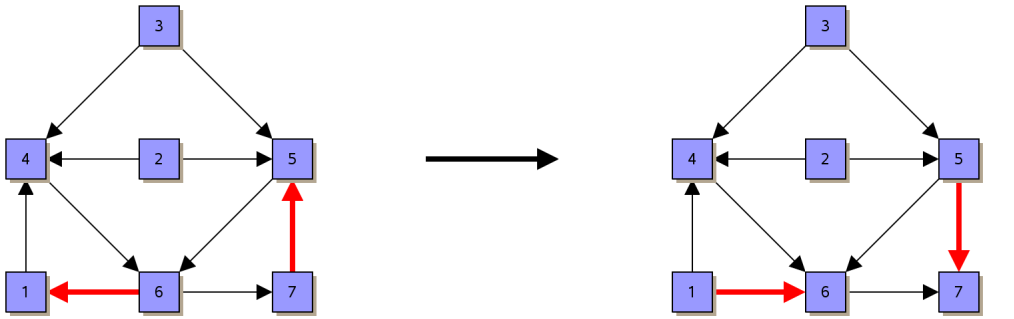

given resolve cycles

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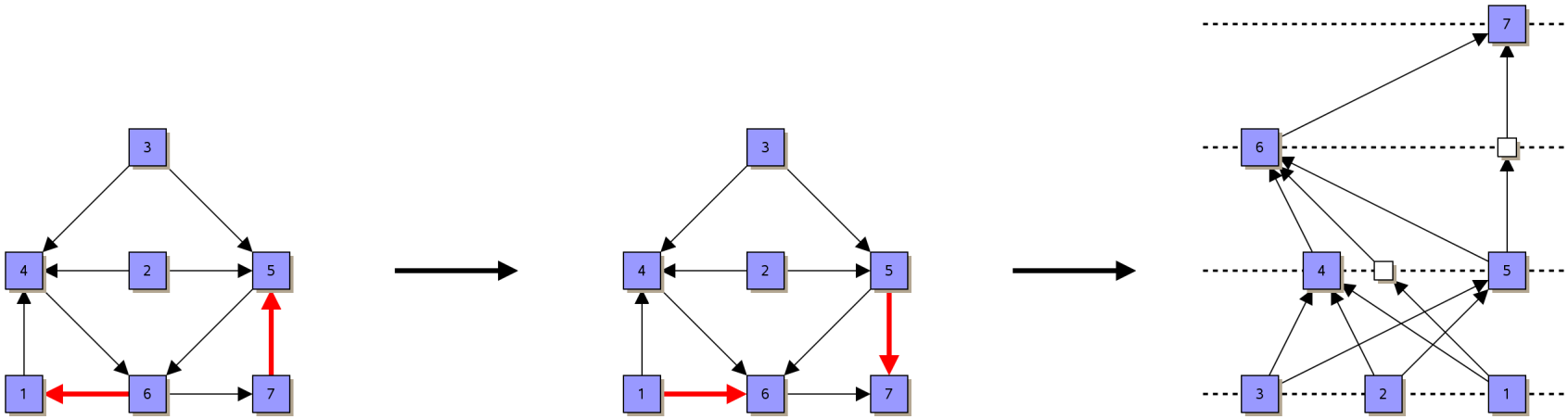

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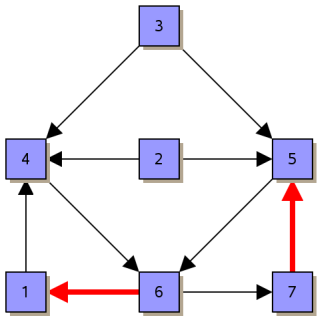

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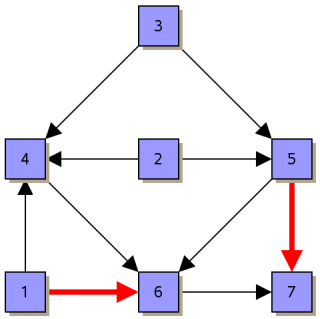

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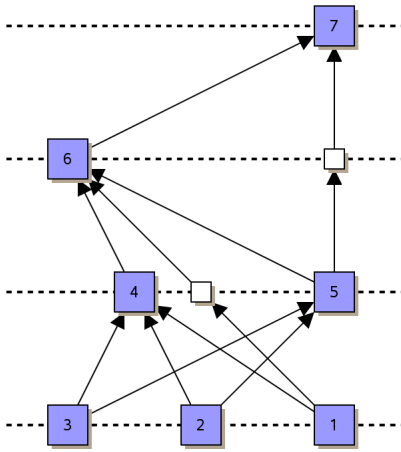
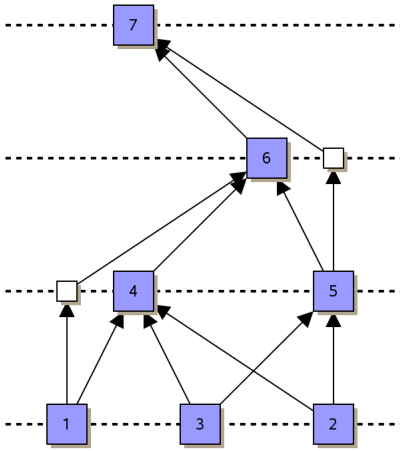


layer assignement



crossing minimization

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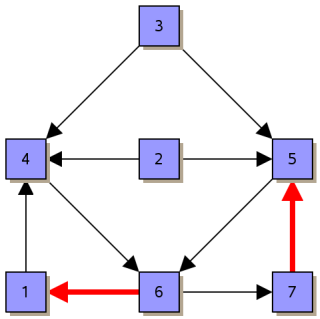

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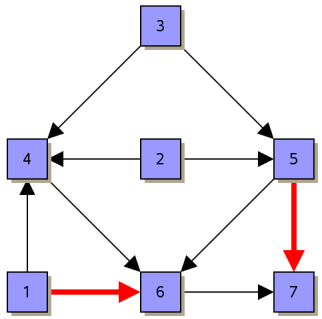

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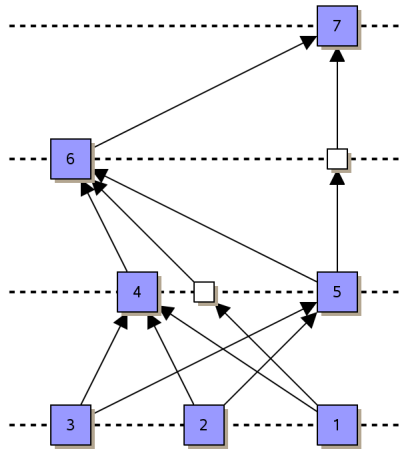

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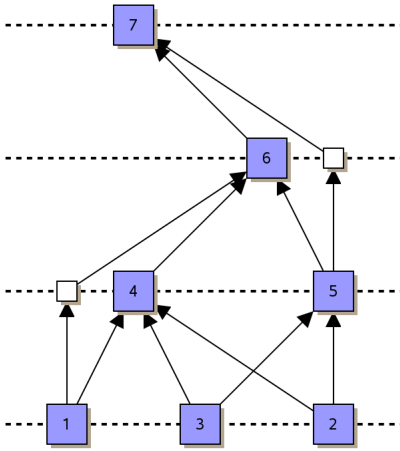

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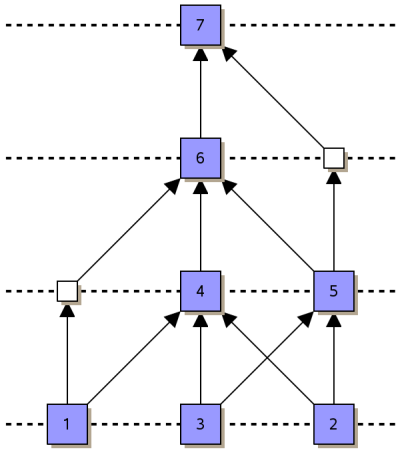

node positioning

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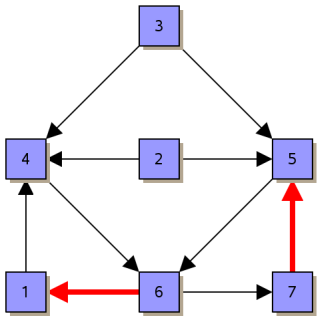

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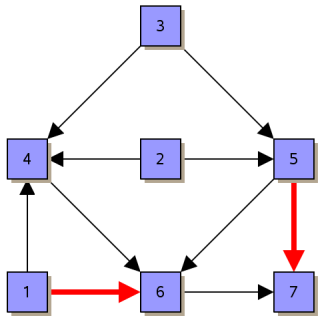

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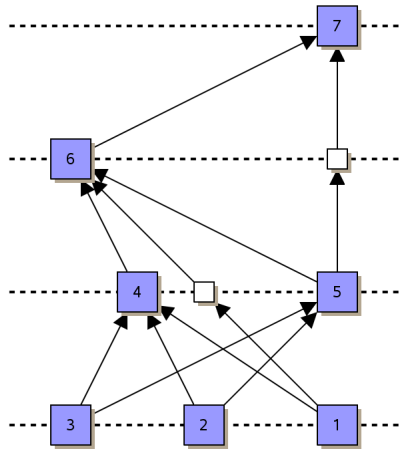

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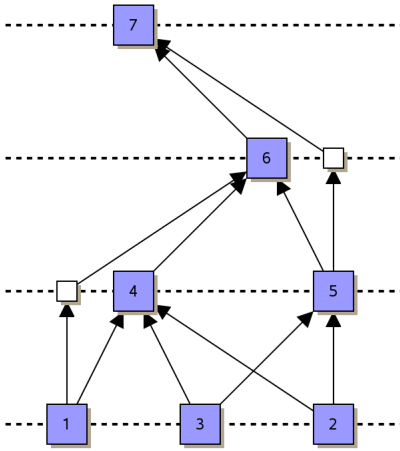

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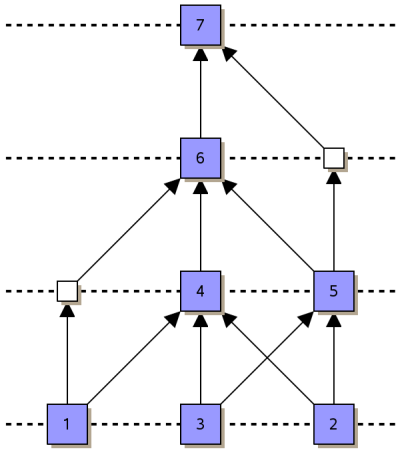

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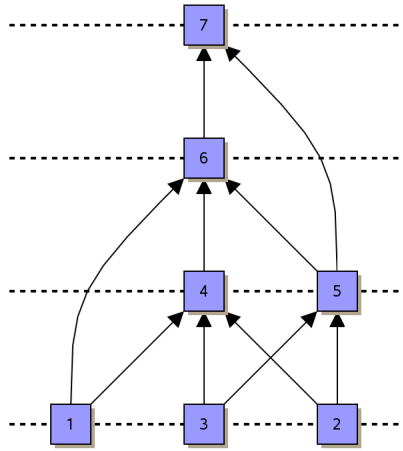

edge drawing

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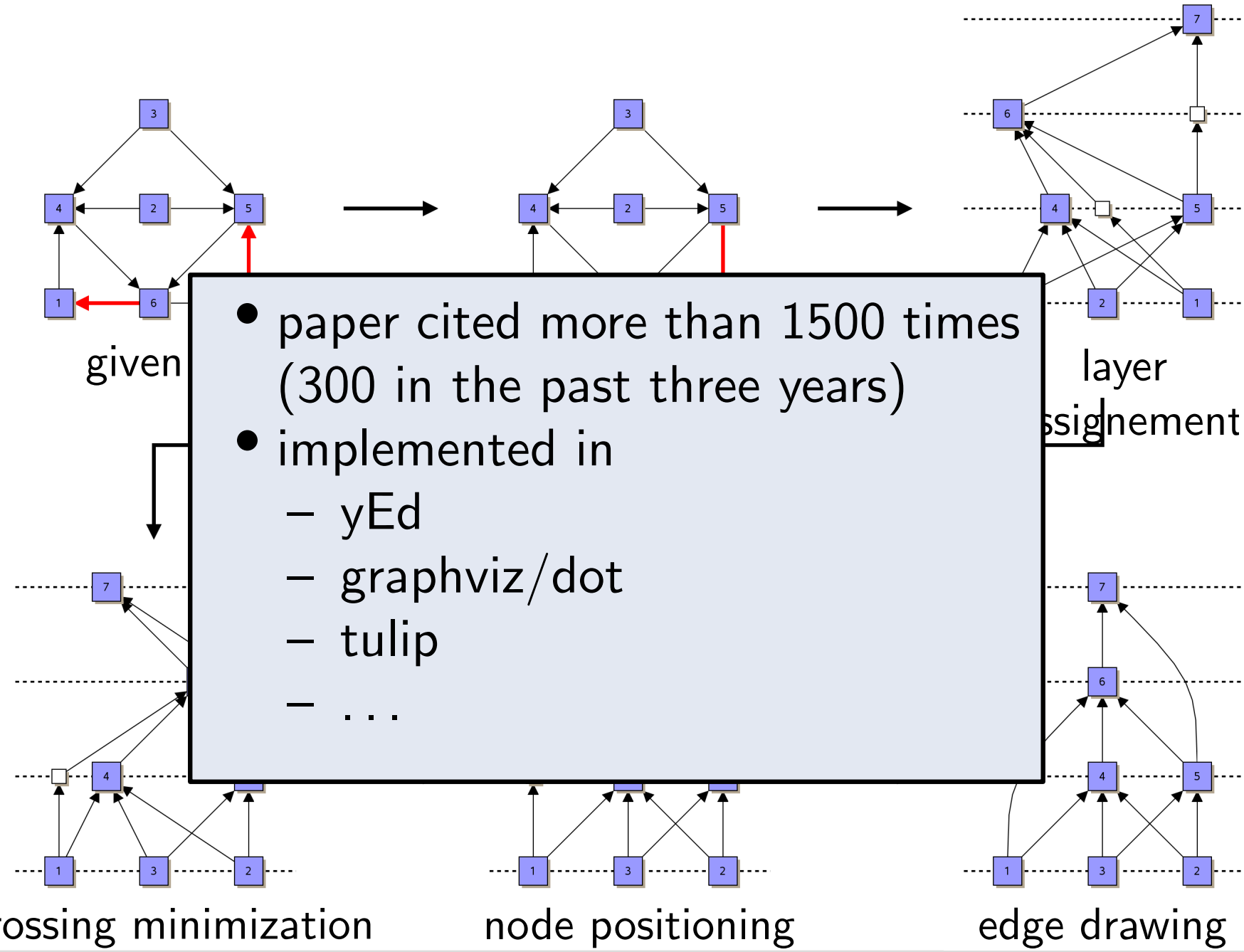
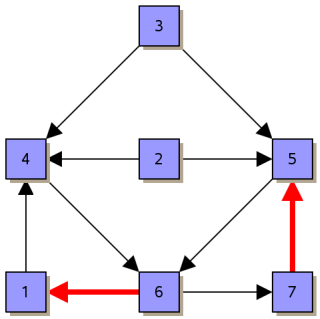


- paper cited more than 1500 times (300 in the past three years)
- implemented in
  - yEd
  - graphviz/dot
  - tulip
  - ...

Sugiyama Framework (Sugiyama, Tagawa, Toda 1981)



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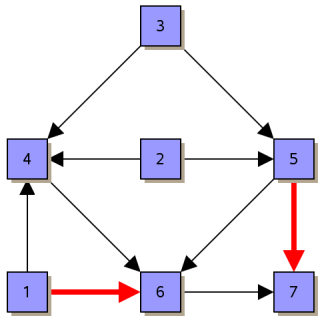

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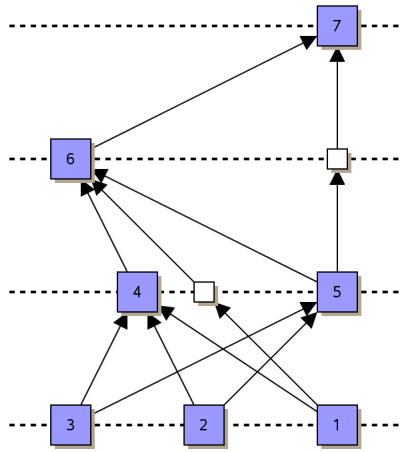

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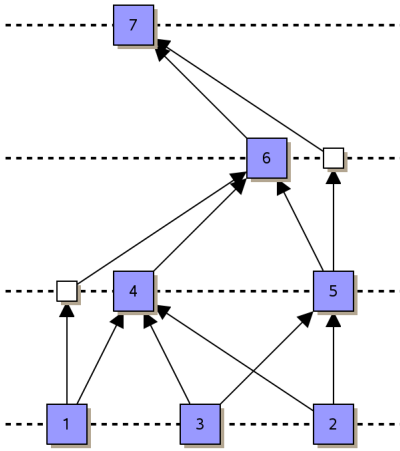

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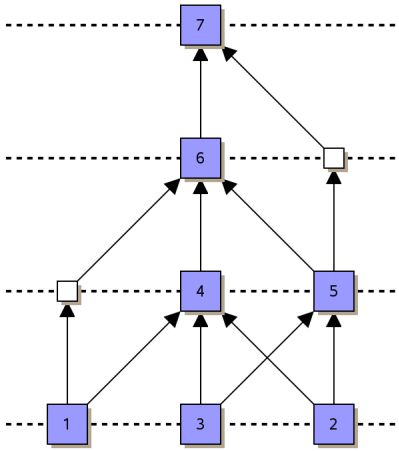

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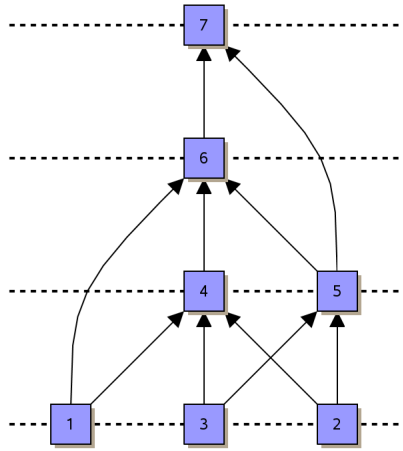

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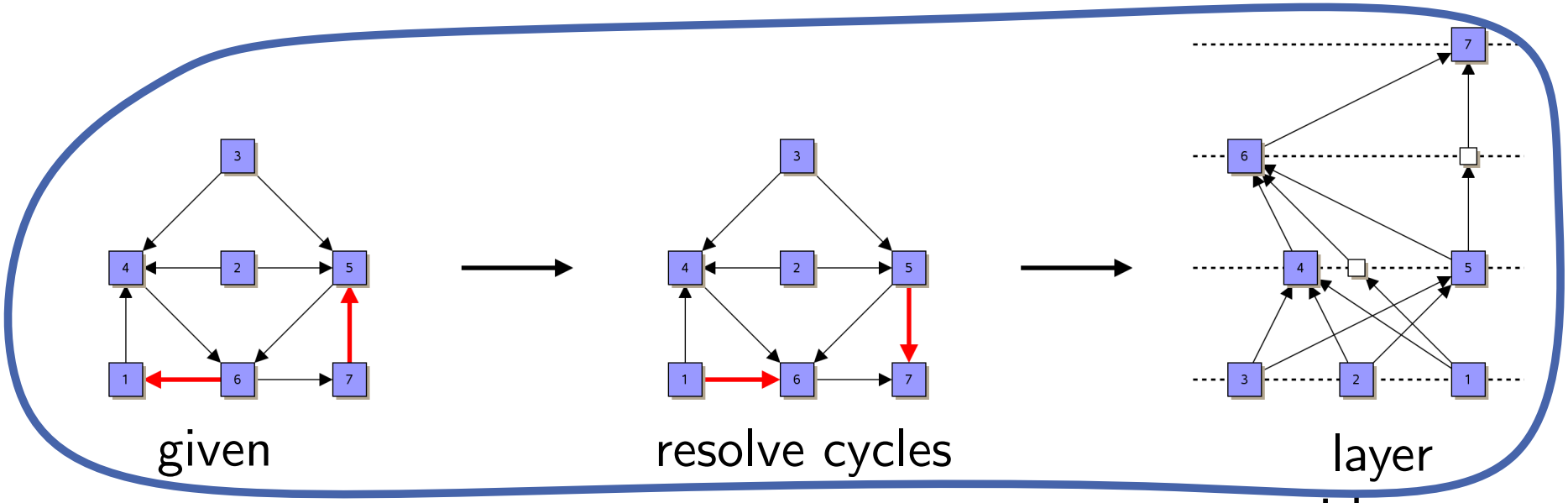
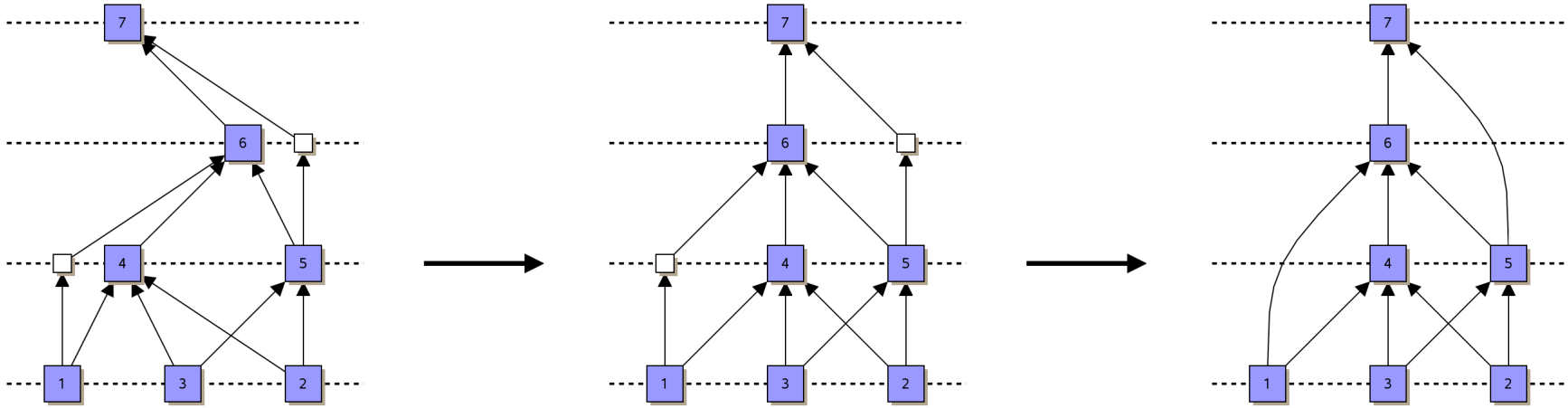
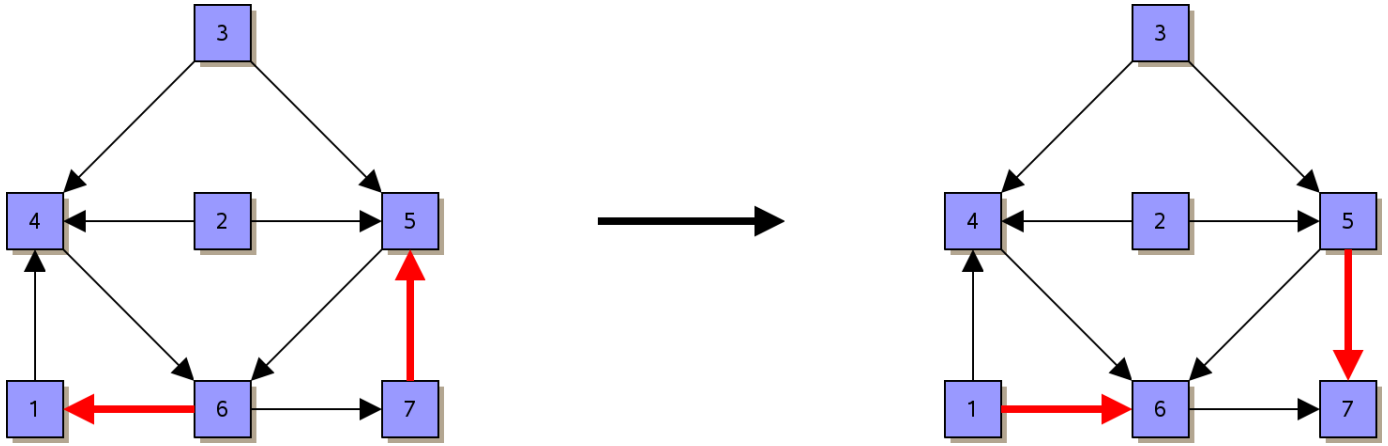




Step 1: Resolve Cycles



Feedback Arc Set

Idea:
- find maximum acyclic subgraph
- inverce the directions of the other edges

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Given: directed graph $D = (V, A)$

Find: acyclic subgraph $D' = (V, A')$ with maximum $|A'|$

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Minimum Feedback Arc Set (FAS)

Given: directed graph $D = (V, A)$

Find: $A_f \subset A$, with $D_f = (V, A \setminus A_f)$ acyclic with minimum $|A_f|$

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Minimum Feedback Set (FS)

Given: directed graph $D = (V, A)$

Find: $A_f \subset A$, with $D_f = (V, A \setminus A_f \cup \text{rev}(A_f))$ acyclic with minimum $|A_f|$

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All three problems are NP-hard!

Heuristic 1 (Berger, Shor 1990)

$A' := \emptyset$;

foreach $v \in V$ **do**

  if $|N^{\rightarrow}(v)| \geq |N^{\leftarrow}(v)|$ **then**

    $A' := A' \cup N^{\rightarrow}(v)$;

  else

    $A' := A' \cup N^{\leftarrow}(v)$;

  remove v and $N(v)$ from D.

return (V, A')

$$
\begin{aligned}
N^{\rightarrow}(v) &:= \{(v,u) : (v,u) \in A\} \\
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Work with your neighbour(s) and then share

Why D' does not contain cycles?

What one can say about $|A'|$ in terms of $|V|$?

7 min

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\end{array}
$$

Task togo:

- Is $D'' = (V, A' \cup \text{rev}(A \setminus A'))$ acyclic?
- What is the running time?

- $D' = (V, A')$ is a DAG
- $A \setminus A'$ is a feedback arc set

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Why D' does not contain cycles?
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Heuristic 2 (Eades, Lin, Smyth 1993)

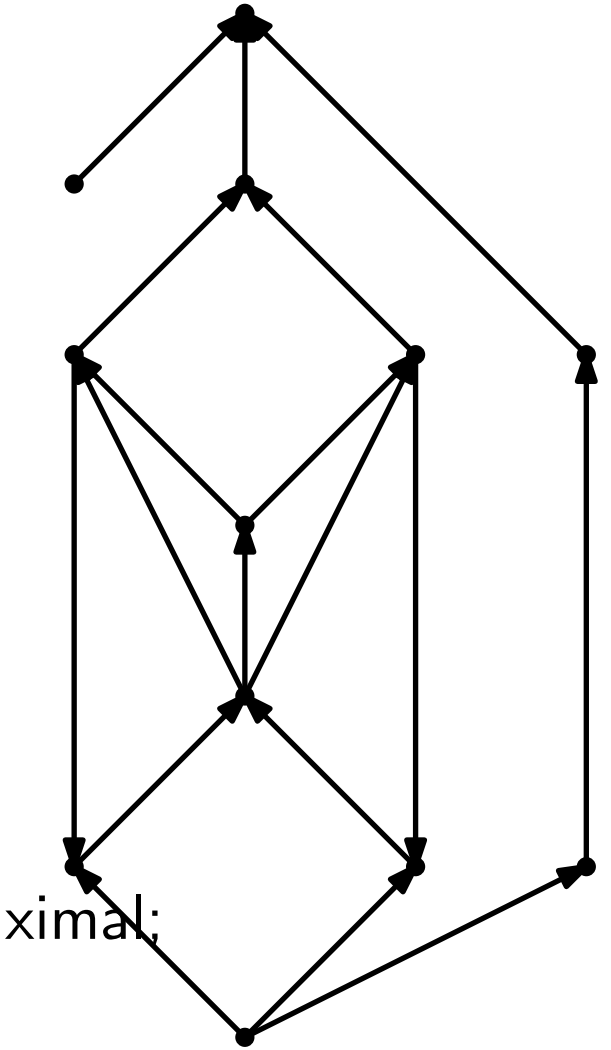
1 $A' := \emptyset$;

2 **while** $V \neq \emptyset$ **do**

3 | **while** in V exists a sink v **do**

4 | | $A' \leftarrow A' \cup N^{\leftarrow}(v)$

5 | | remove v and $N^{\leftarrow}(v)$: $\{V, n, m\}_{\text{sink}}$



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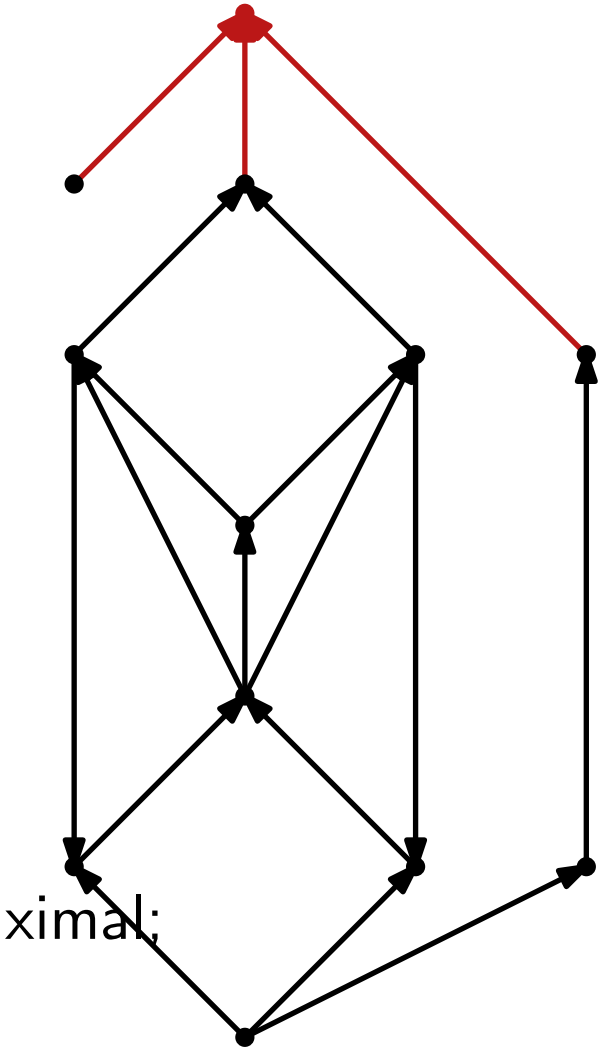

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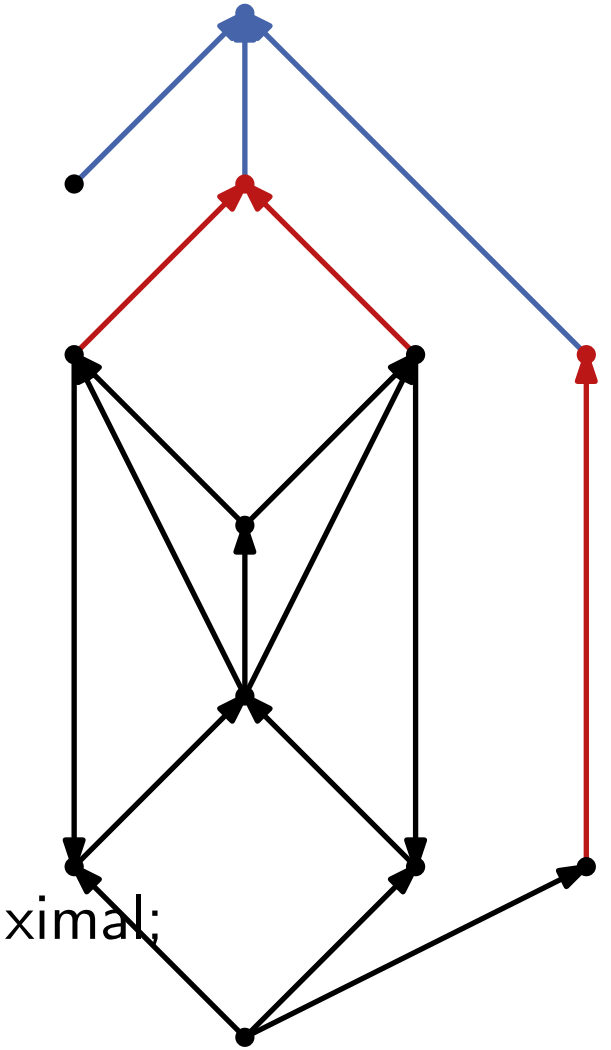

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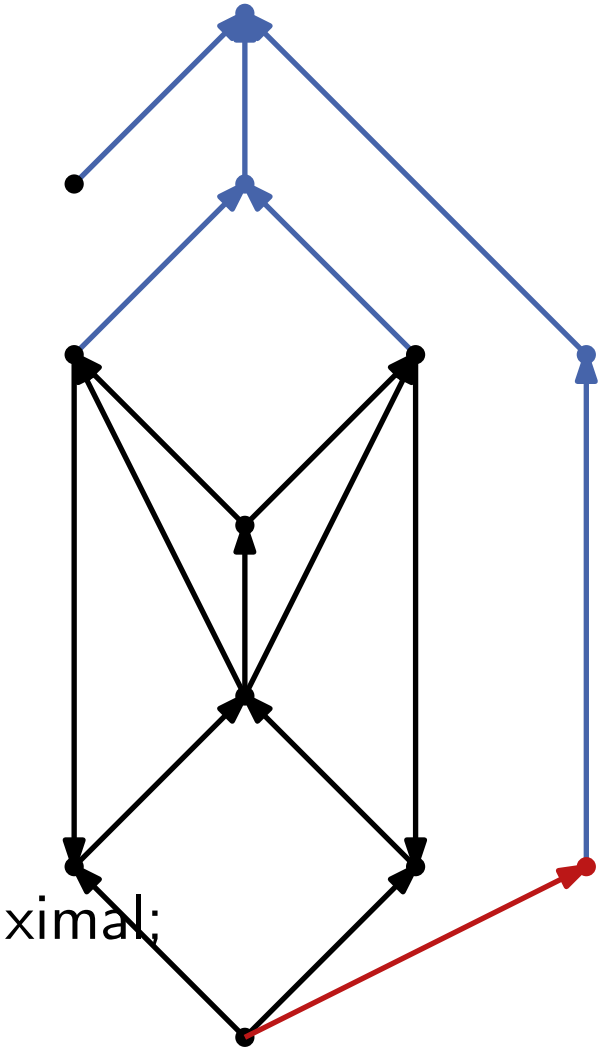

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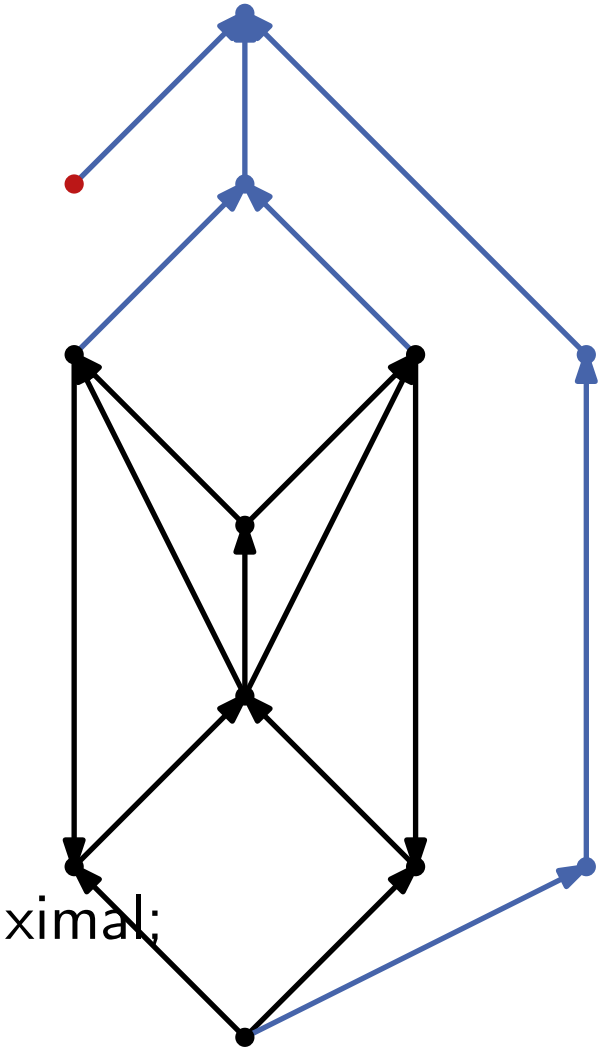

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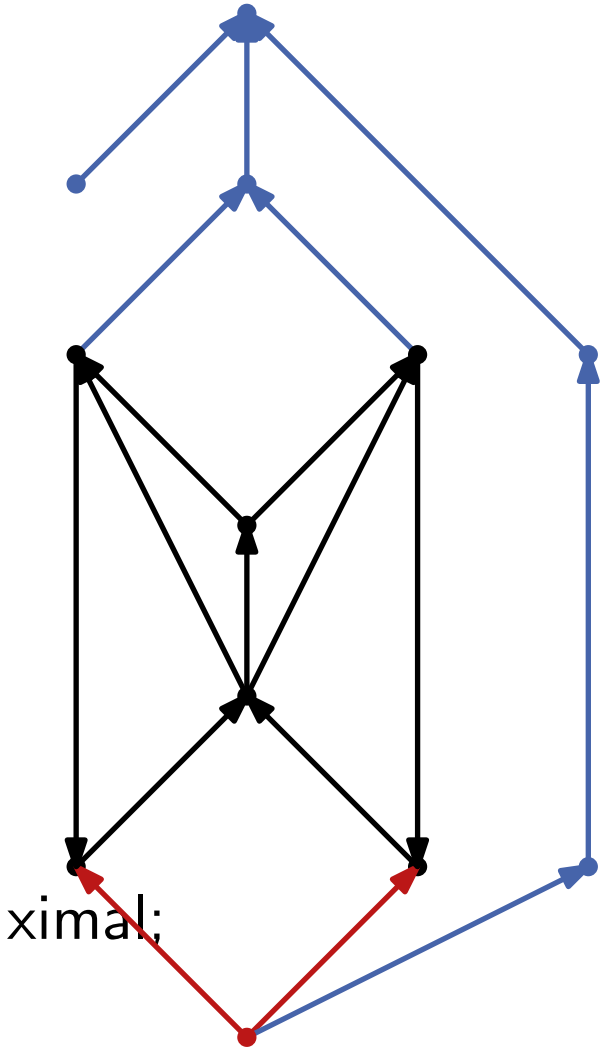

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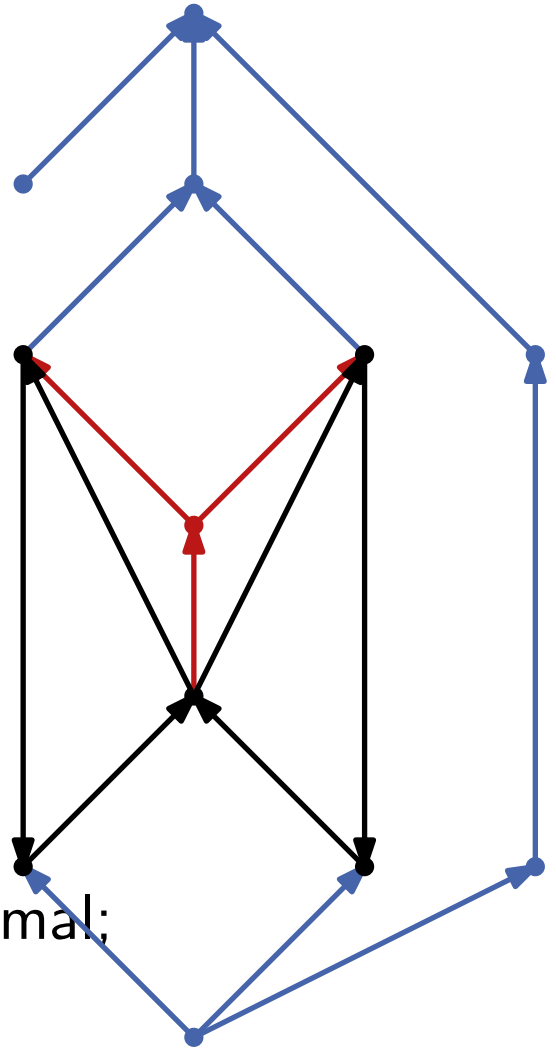

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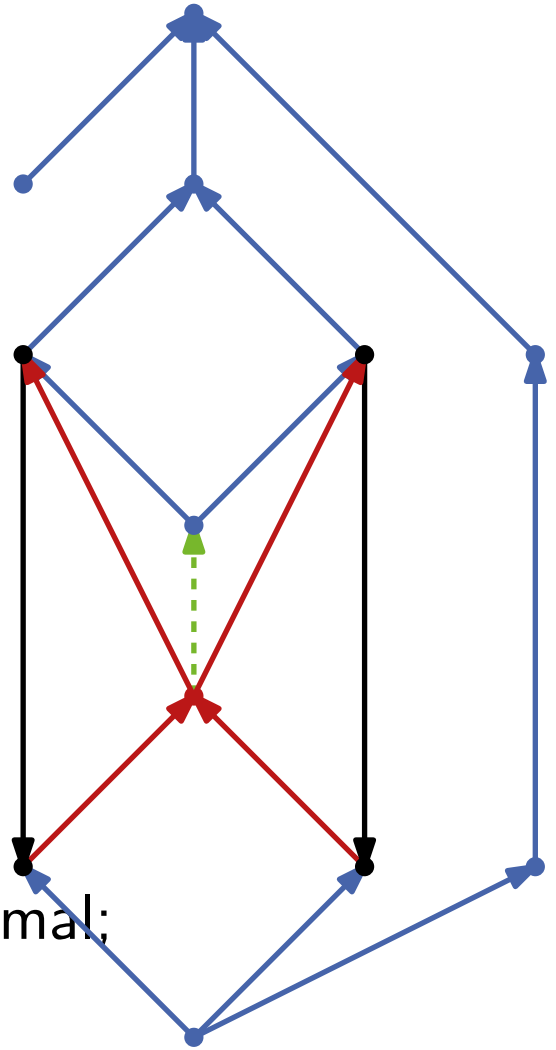

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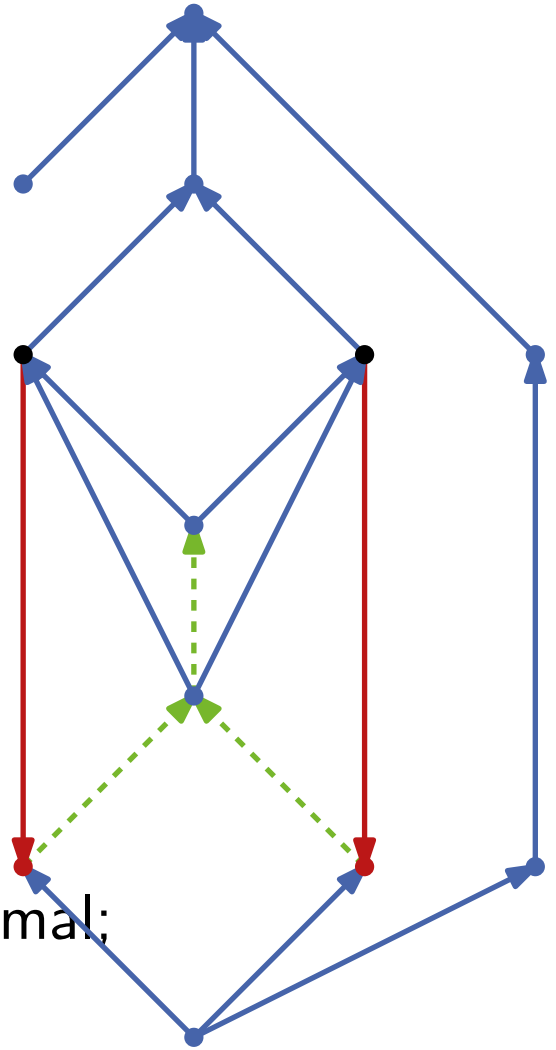

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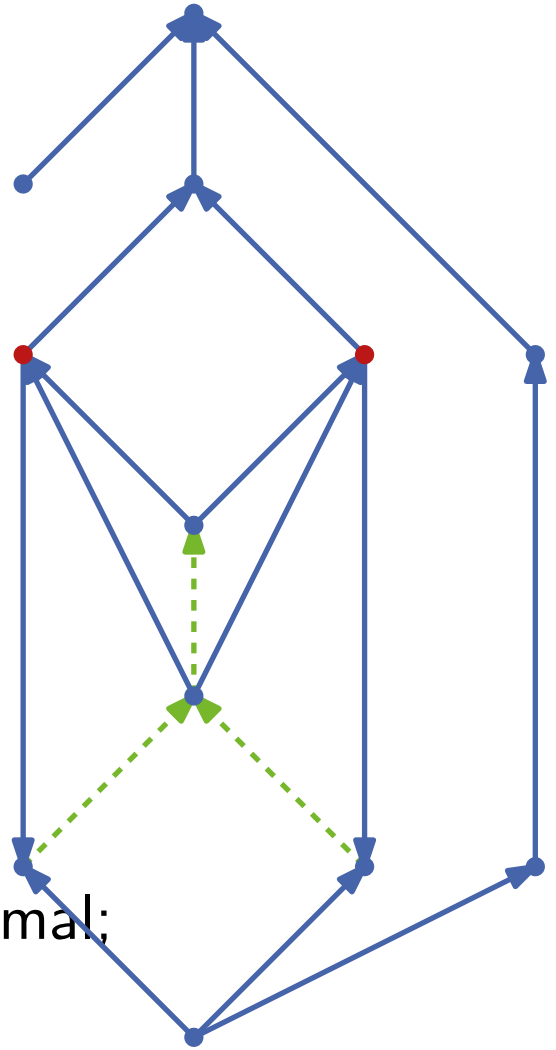

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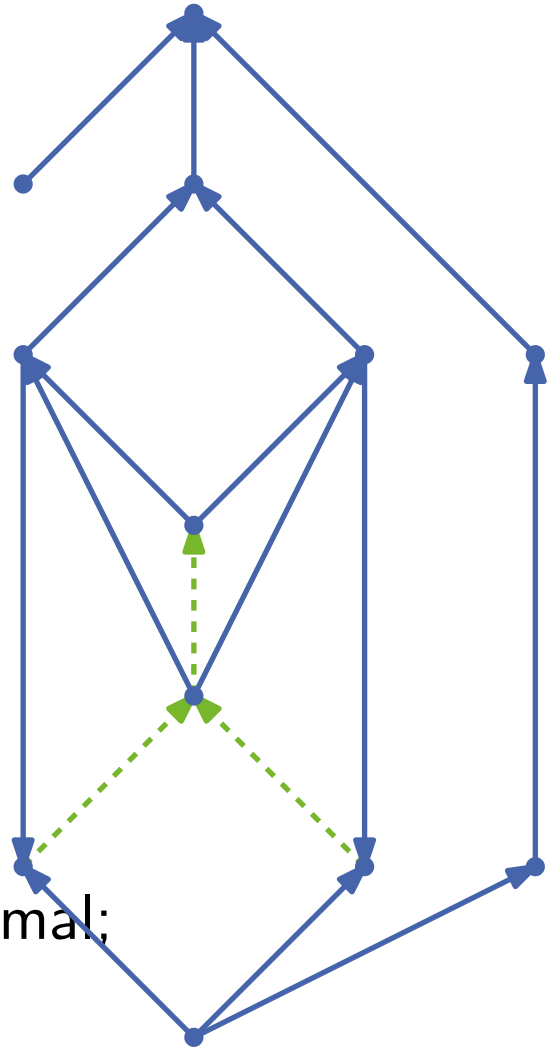

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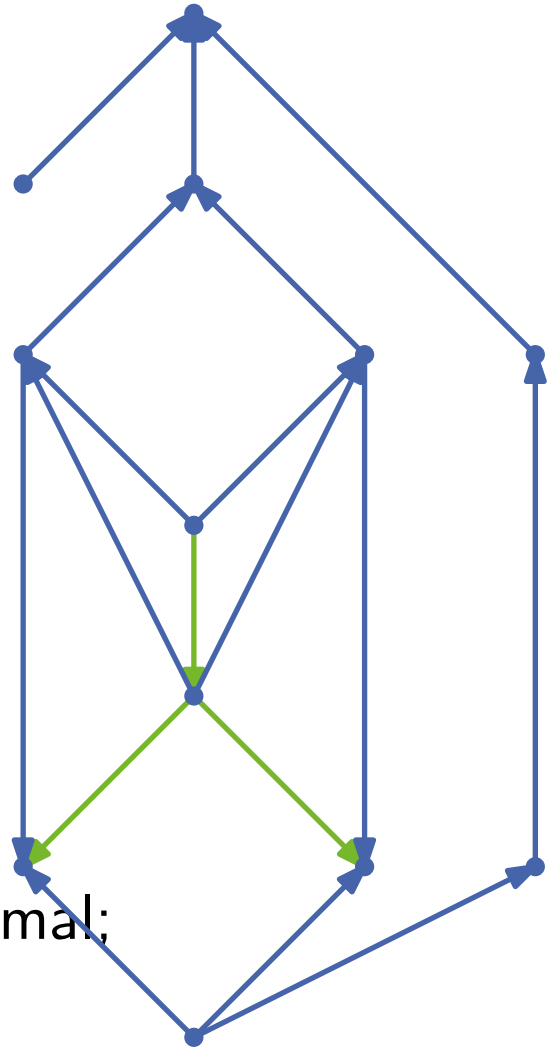

Heuristic 2 – Analysis

Theorem 1: Let $D = (V, A)$ be a connected, directed graph without 2-cycles. Heuristic 2 computes a set of edges A' with $|A'| \geq |A|/2 + |V|/6$.
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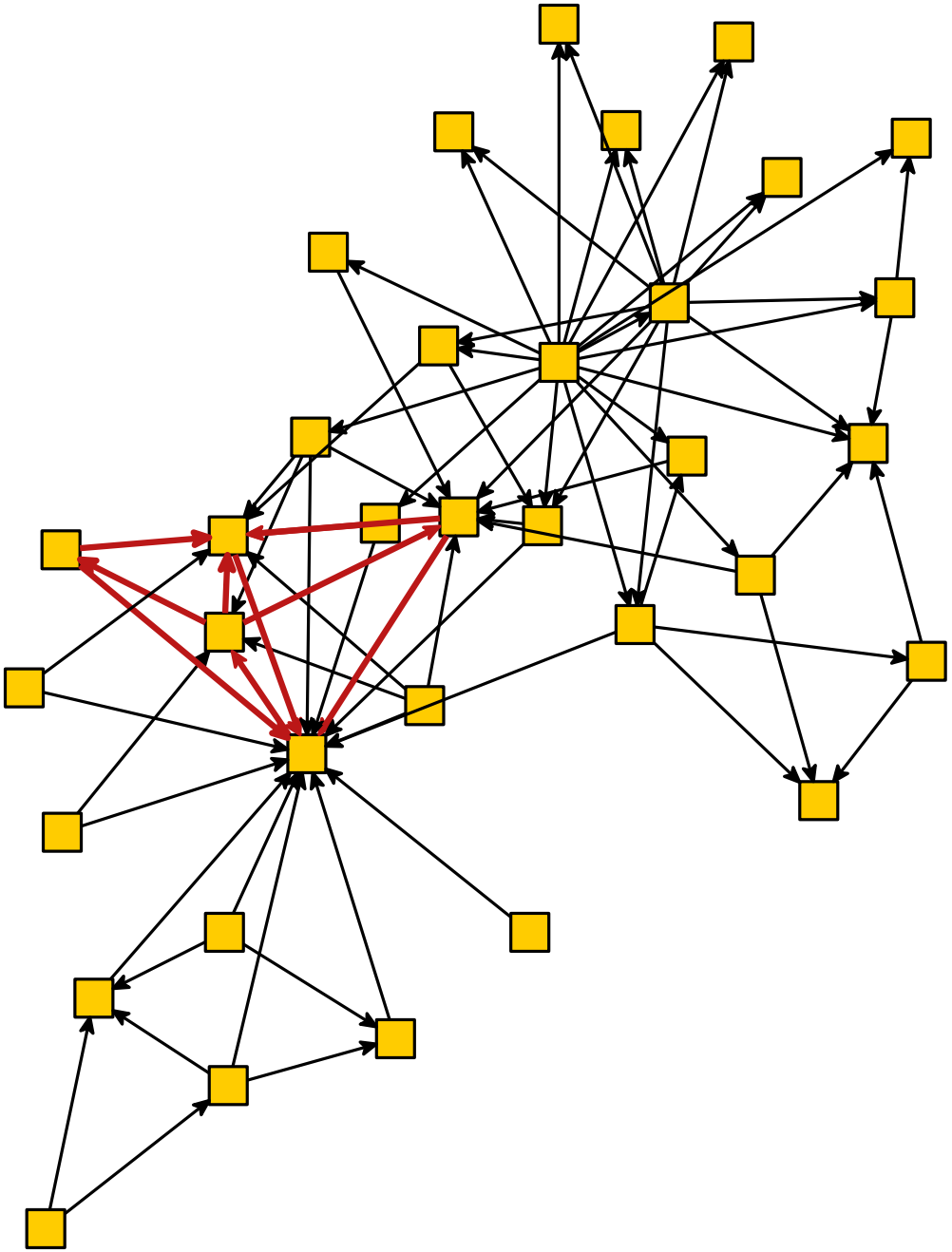
The running time is $O(|A|)$.

Exact Solution:

- integer linear programming, using branch-and-cut technique

  (Grötschel et al. 1985)

Example



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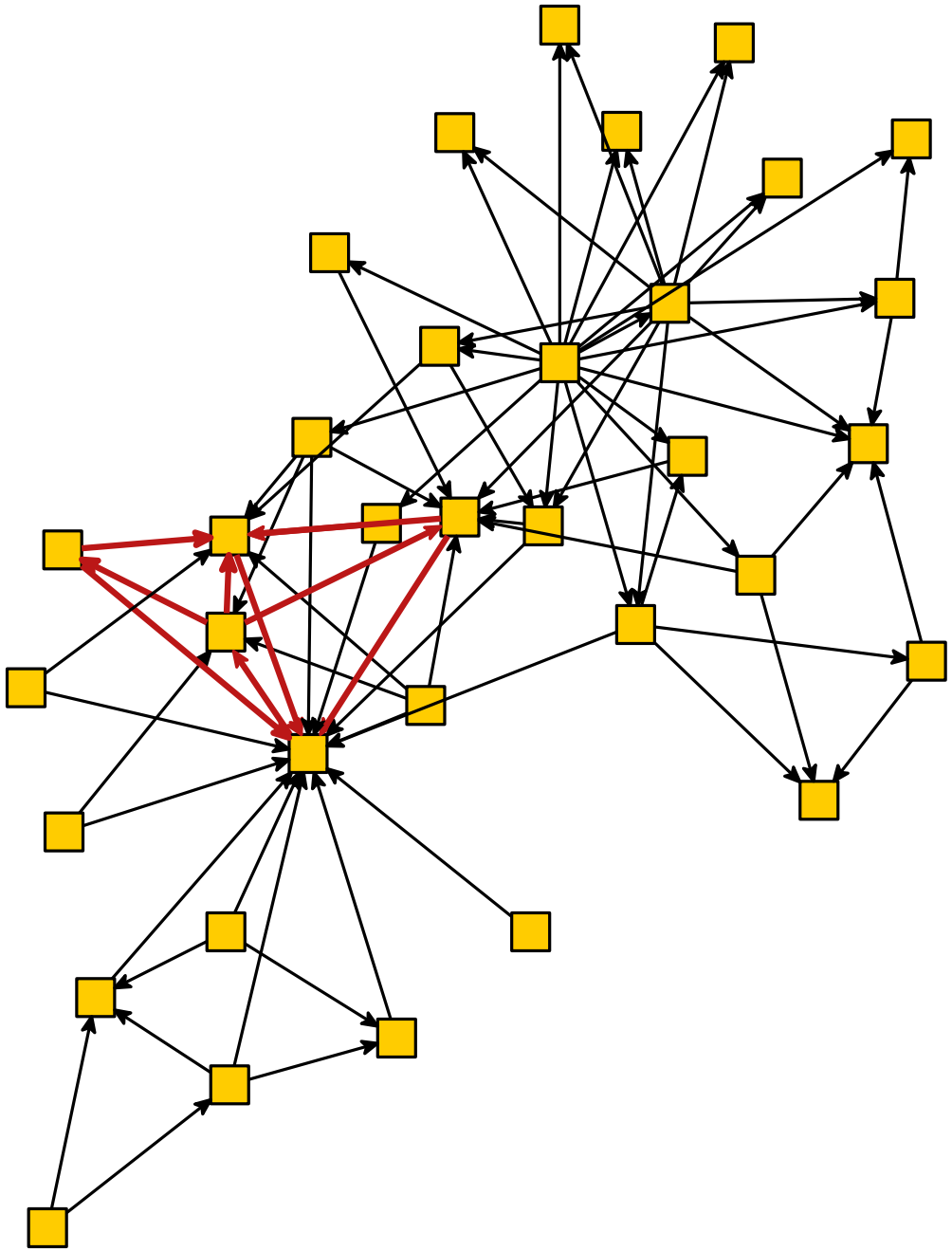
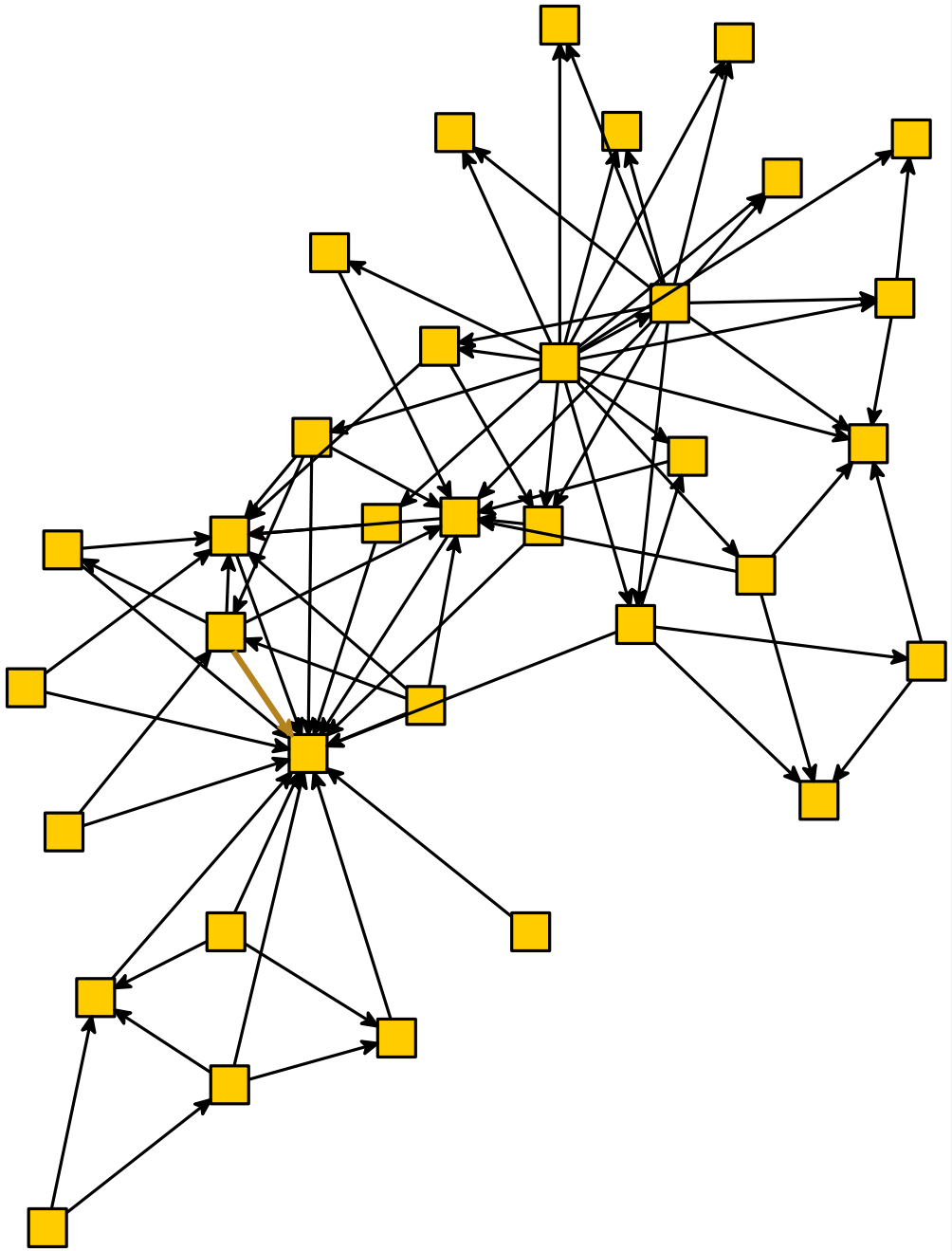
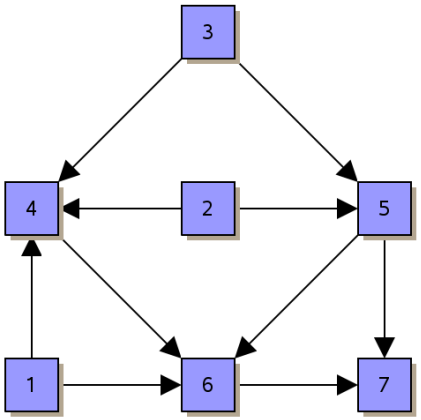
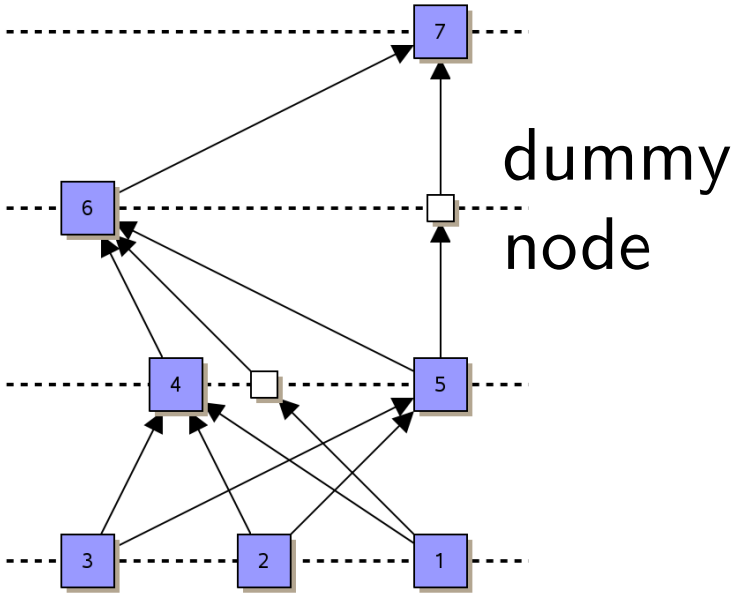




Step 2: Layer Assignement





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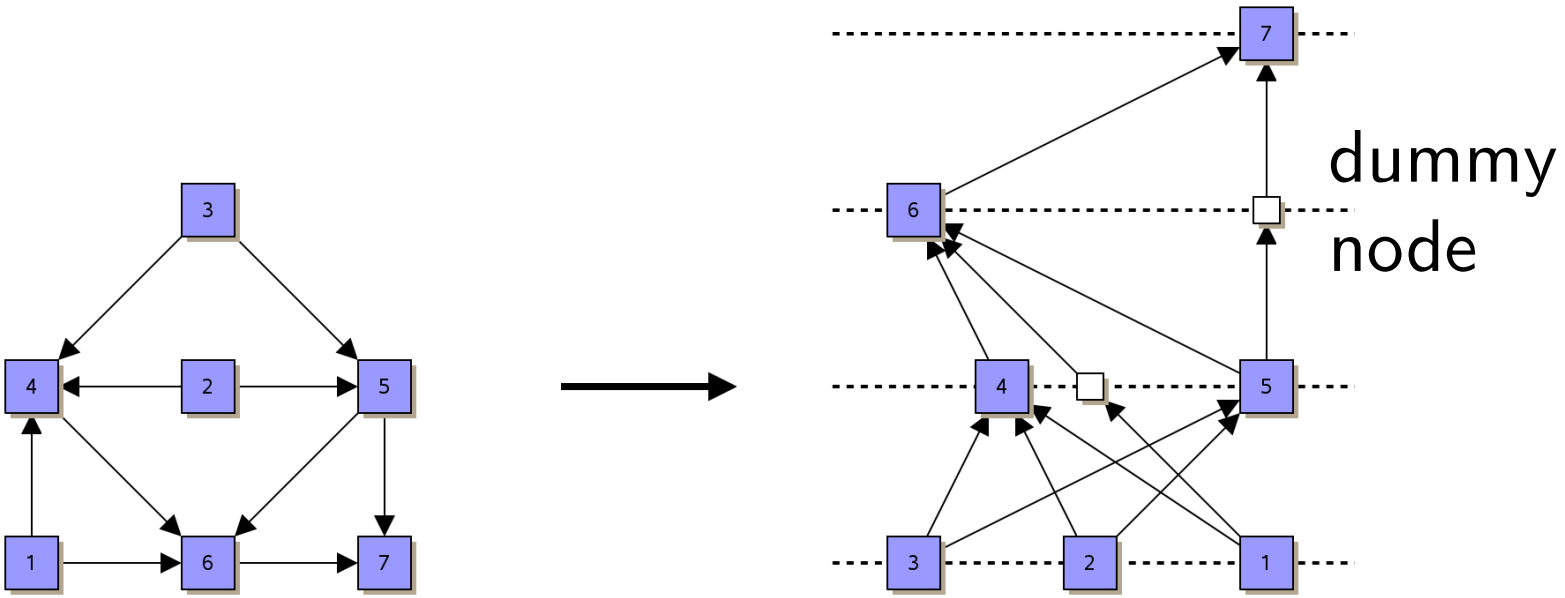

Think for a minute and then share

What could we optimize when doing the layer assignment?

1 min

Step 2: Layer Assignement

Given.: directed acyclic graph (DAG) $D = (V, A)$

Find: Partition the vertex set V into disjoint subsets (**layers**) $L_1, \ldots, L_h$ s.t. $(u, v) \in A$, $u \in L_i$, $v \in L_j \Rightarrow i < j$

Def: y-Coordinate $y(u) = i \Leftrightarrow u \in L_i$

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Criteria that we discuss

- minimize the number of layers h ($=$ height of the layouts)
- minimize the total length of edges ($\approx$ number of dummy nodes)
- minimize width, e.g. $\max\{|L_i| \mid 1 \leq i \leq h\}$

Height Optimization

Idea: assign each node v to the layer L_i, where i is the length of the longest simple path from a source to v

- all incomming neighbours lie below v
- the resulting height h is minimized

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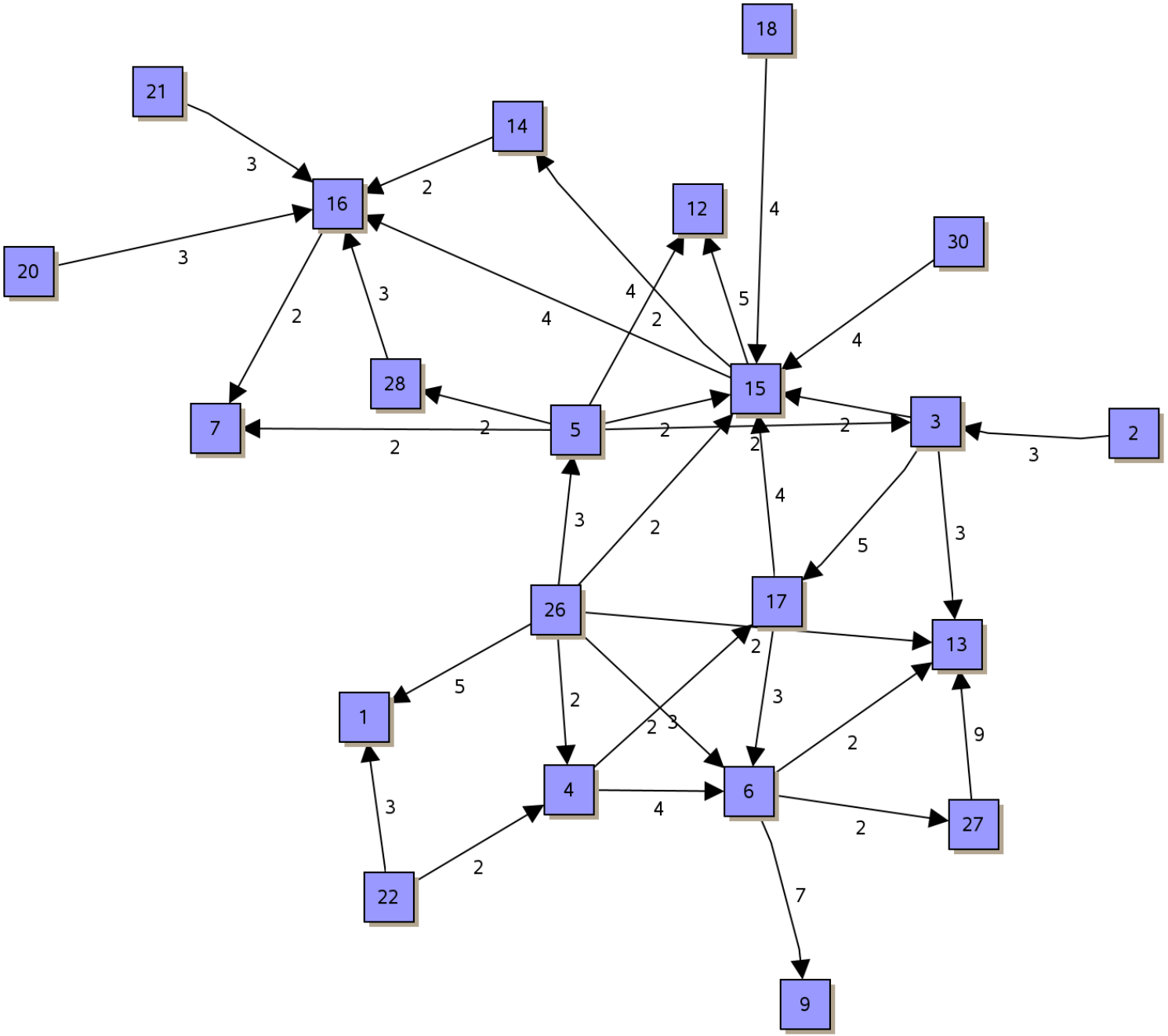
Algorithm

- $L_1 \leftarrow$ the set of sources in D
- set $y(u) \leftarrow \max_{v \in N^{\leftarrow}(u)}\{y(v)\} + 1$

Example


18
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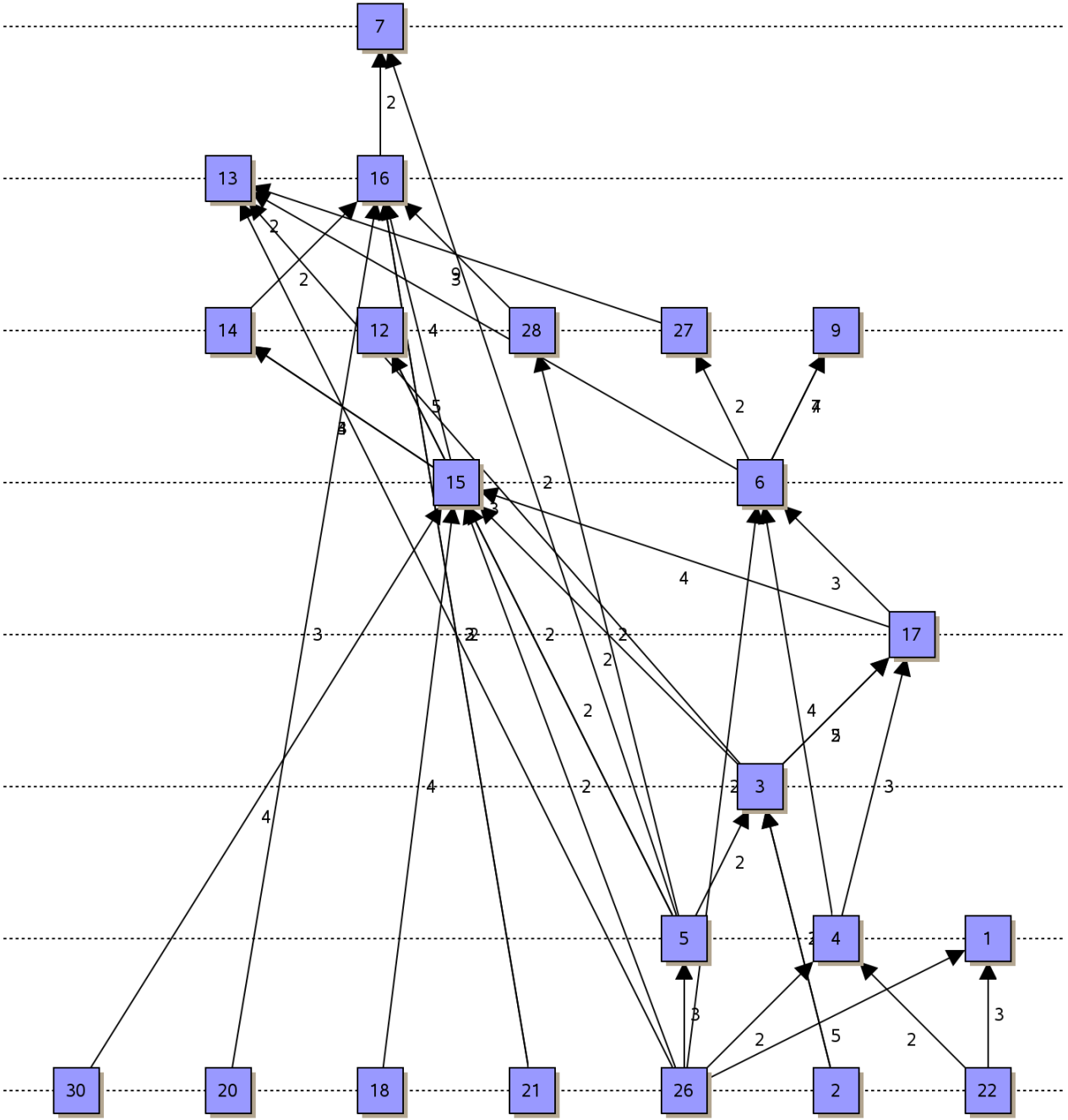

Total Edge Length

Karlsruhe Institute of Technology

Can be formulated as an integer linear program:

$$
\begin{aligned}
\text{min} \quad & \textstyle\sum_{(u,v)\in A}(y(v)-y(u)) & \\
\text{subject to} \quad & y(v)-y(u) \geq 1 & \forall (u,v) \in A \\
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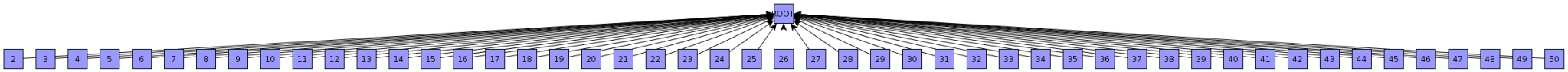
One can show that:

- Constraint-Matrix is **totally unimodular**
- $\Rightarrow$ Solution of the relaxed linear program is integer
- The total edge length can be minimized in a polynomial time

Width of the Layout



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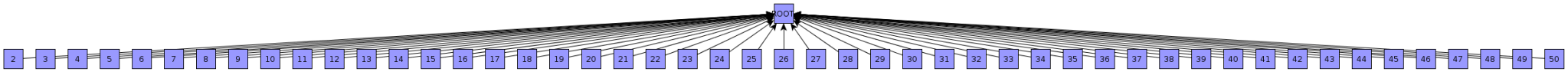


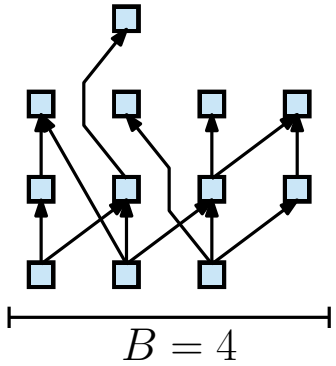
$\rightarrow$ bound the width!

Layer Assignement with Fixed Width

Fixed-Width Layer Assignment

Given: directed acyclic graph $D = (V, A)$, width B

Find: layer assignement $\mathcal{L}$ of minumum height with at most B nodes per layer



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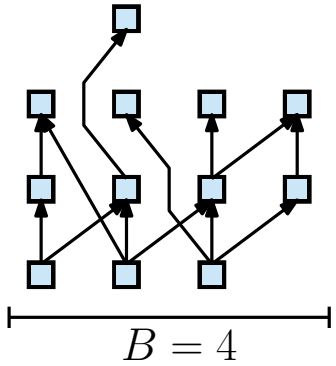

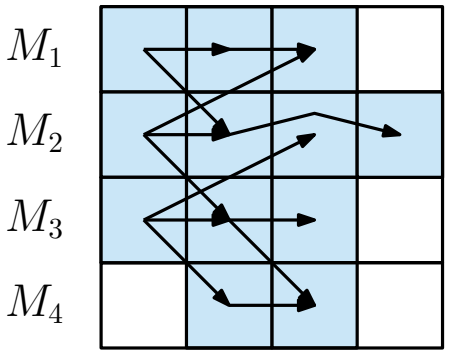

$\rightarrow$ equivalent to the following scheduling problem:

Minimum Precedence Constrained Scheduling (MPCS)

Given: n Jobs $J_1, \ldots, J_n$ with identical unit processing time, precedence constraints $J_i < J_k$, and B identical machines

Find: Schedule of minimum length, that satisfies all the precendence constraints

Fixed Width: Complexity

Theorem 2: It is NP-hard to decide, whether for n jobs $J_1, \ldots, J_n$ of identical length, given partial ordering constraints, and number of machinces B, there exists a schedule of height at most T, even if $T = 3$.

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Work with your neighbour(s) and then share

Why the corollary holds?

5 min

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Fixed Width: Algorithm

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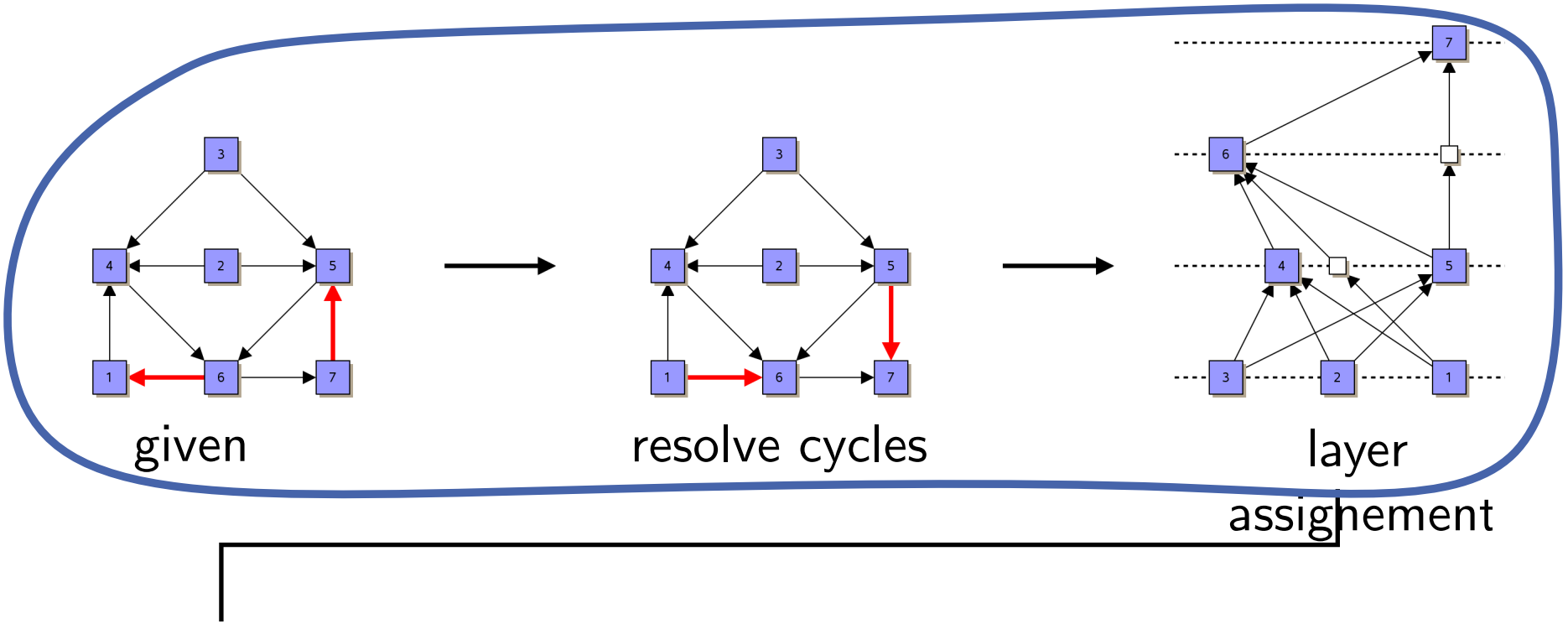
List-Scheduling-Algorithm:

- order jobs arbitrarily as a list $\mathcal{L}$
- when a machine is free, select an allowed job from $\mathcal{L}$; Machine is idle of there is no such job

Summary



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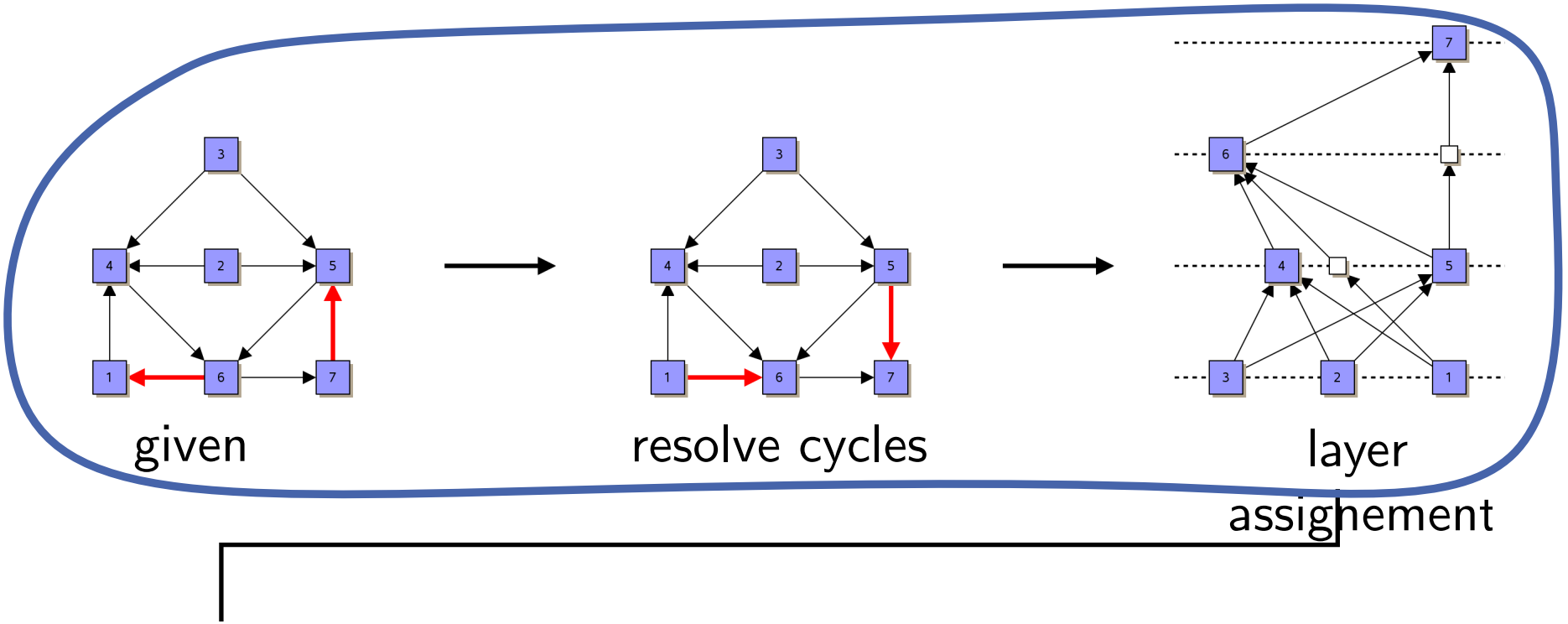
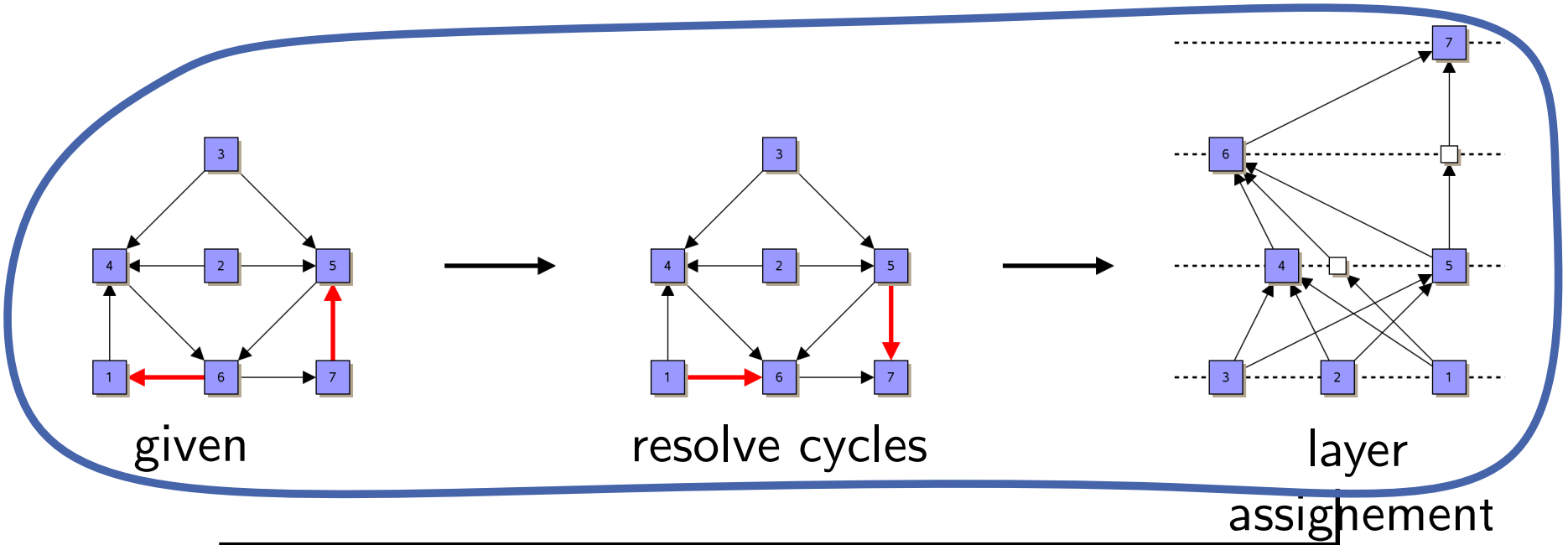


Resolve cycles

- equivalent to **minimum feedback set** problem, and is NP-hard
- Heuristic with $|A'| \geq |A|/2$
- Heuristic with $|A'| \geq |A|/2 + |V|/6$

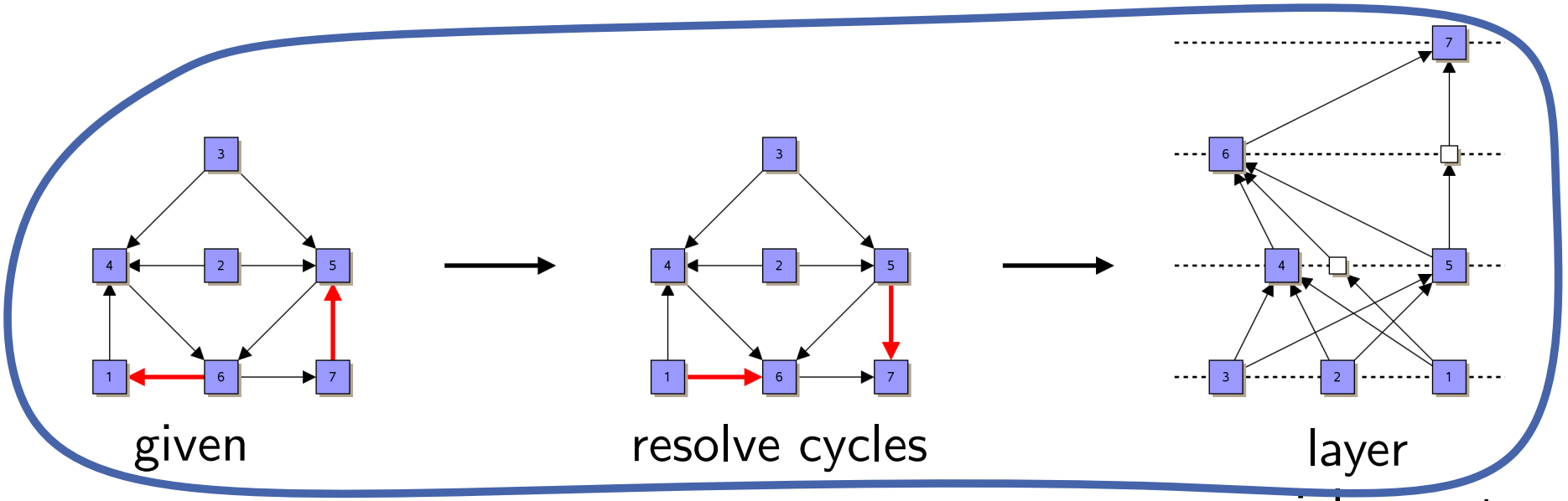
Summary

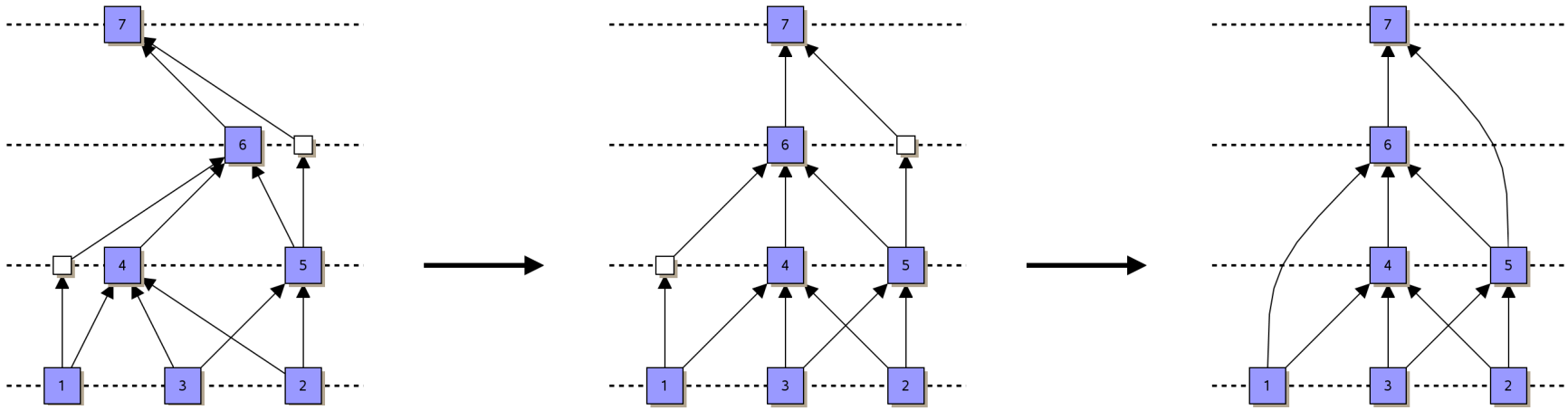


Layer assignement

- Height optimization: topological numbering
- Total edge length: polynomial alg. through integer linear program
- Height optimization with fixed width: equivalent to **MPCS**. NP-hard for 3 levels. Approximation algorithm with factor $\leq 2 - \frac{1}{B}$.

Summary


given
resolve cycles
layer assignement


crossing minimization
node positioning
edge drawing