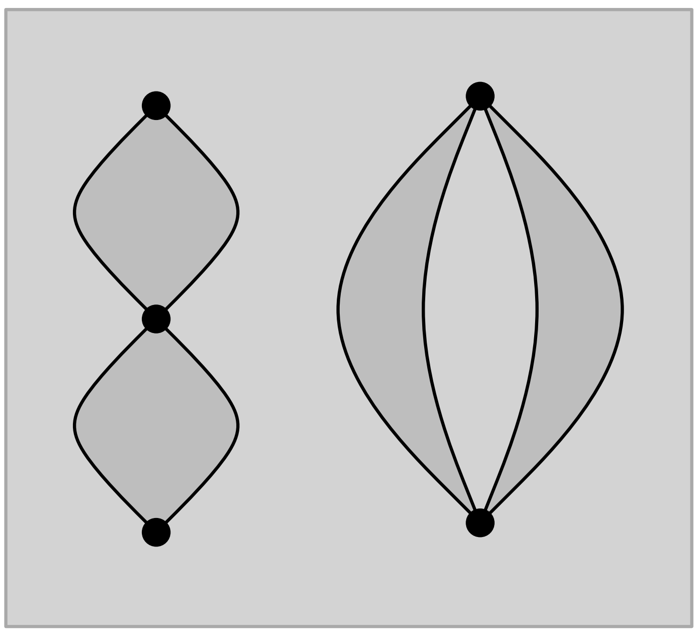
Karlsruhe Institute of Technology

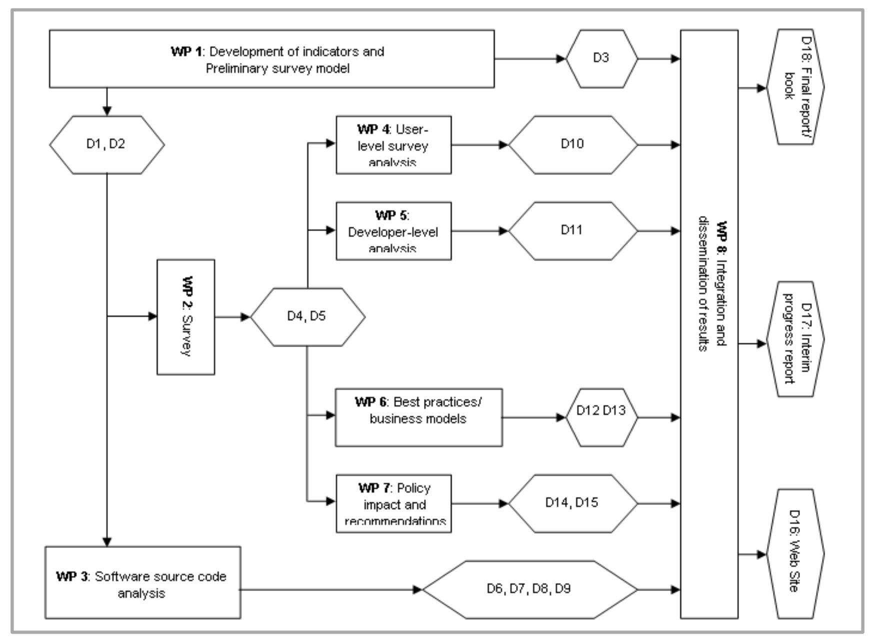
Algorithms for graph visualization

Divide and Conquer - Series-Parallel Graphs

WINTER SEMESTER 2017/2018

Tamara Mchedlidze





Overview

Series - parallel graph. Definition and Decomposition.

Algorithm for upward straight-line drawing.

Lower bound on the area.

Symmetry display for series-parallel graphs.

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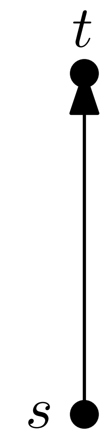
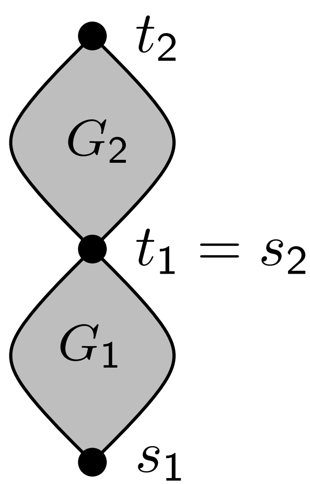
Series-parallel Graphs

Graph G is **series-parallel**, if

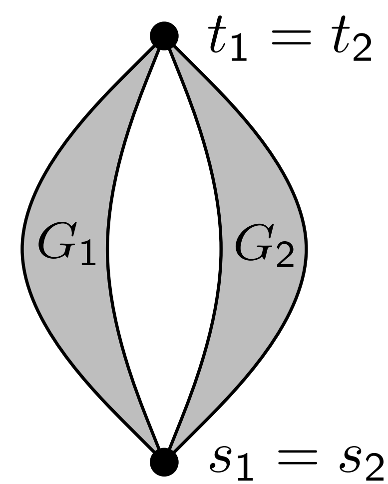
- It contains a single edge (s, t) (s-source, t-sink)
- It consists of two series-parallel graphs G_1, G_2 with sources s_1, s_2 and sinks t_1, t_2 which are combined using one of the following rules:



Series composition:
Identify t_1 and s_2,
s_1 is the source of G, t_2 is the sink of G



Parallel composition:
Identify s_1, s_2 and set it to be source of G
Identify t_1, t_2 and set it to be sink of G



A **decomposition tree** of G, which is a binary tree T with nodes of three types: S,P and Q-type.

Series-parallel Graphs. Decomposition Tree.

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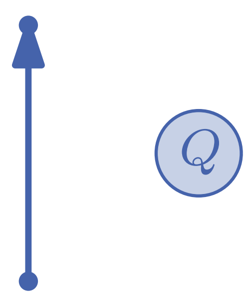

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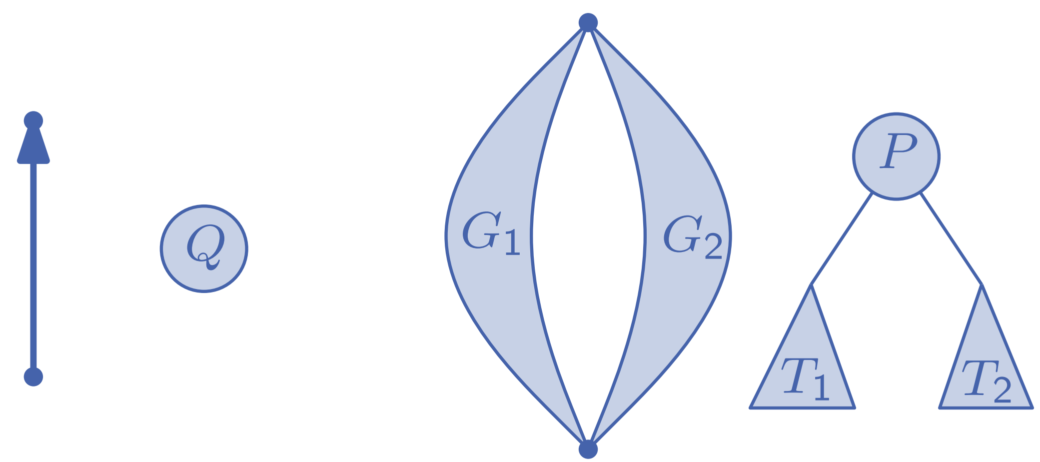

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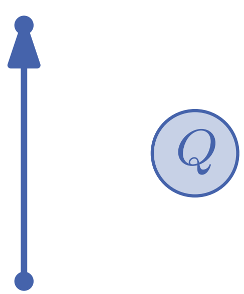
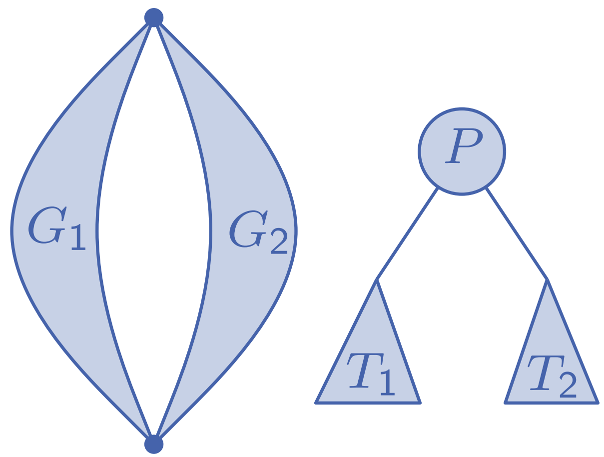
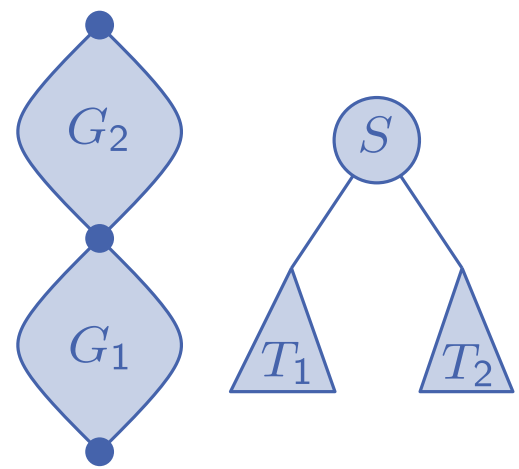
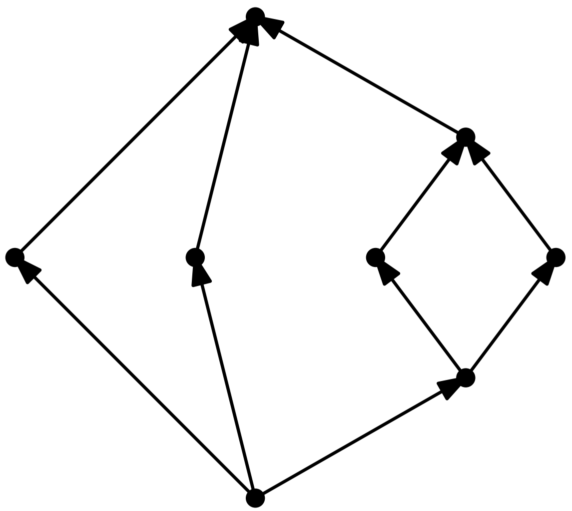






Series-parallel Graphs. Decomposition Example.



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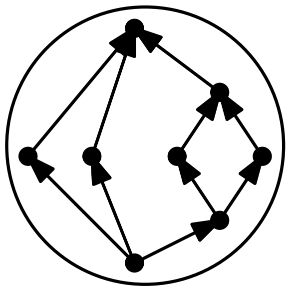

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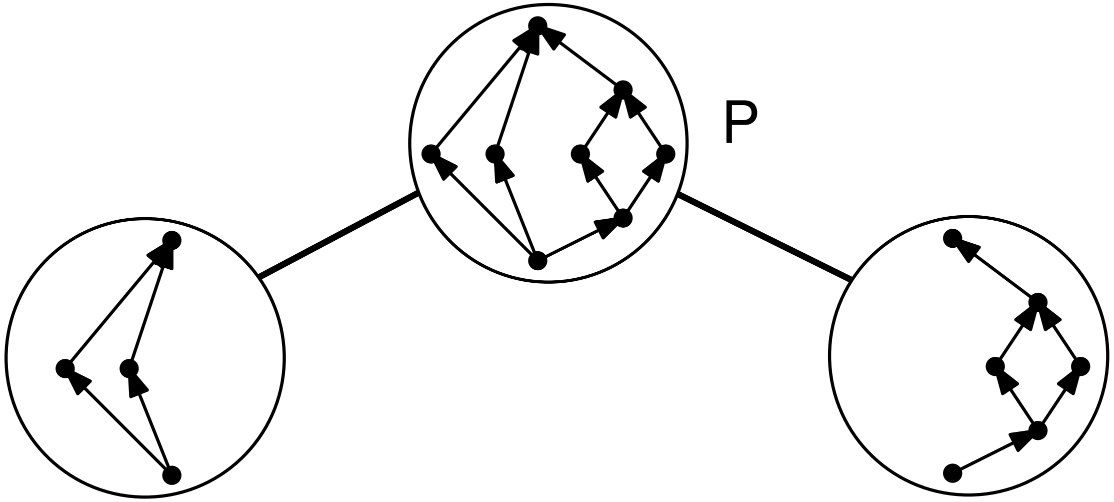

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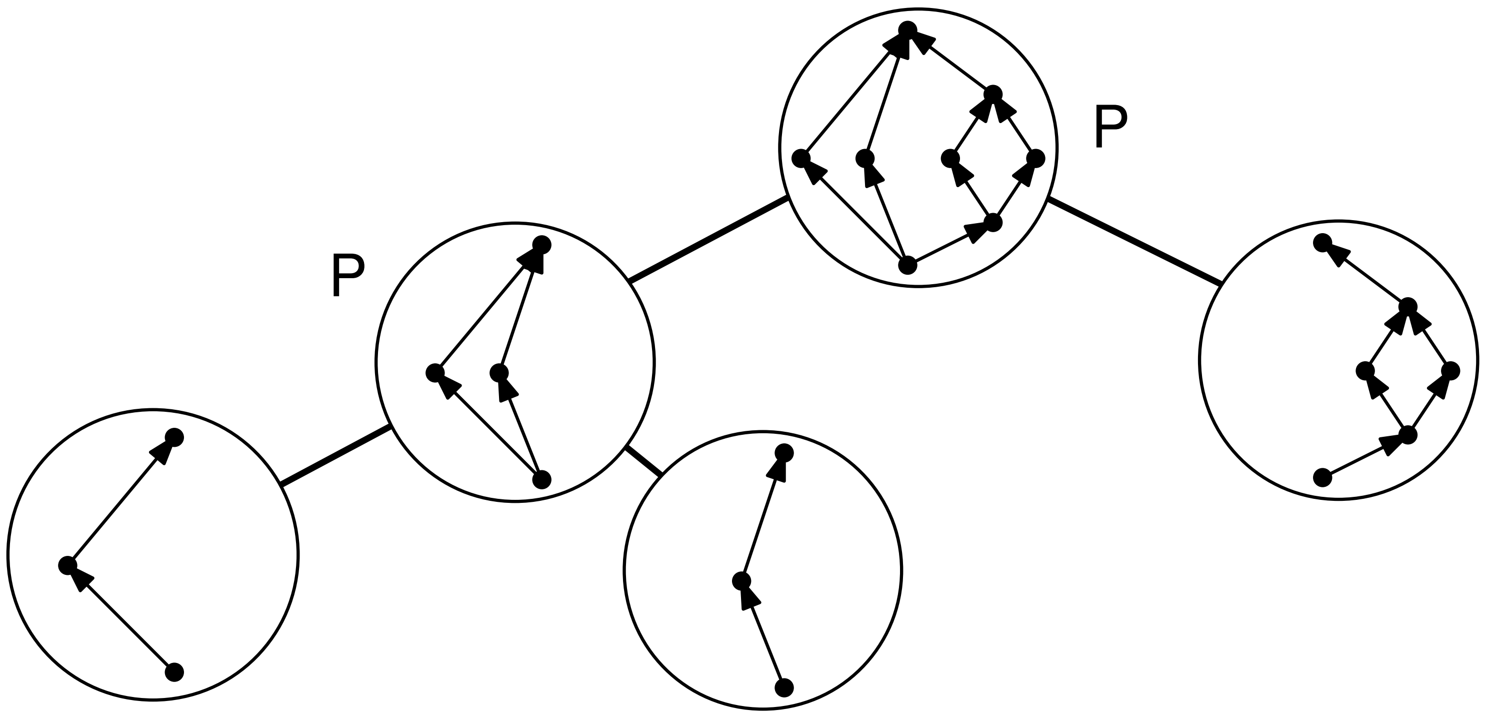

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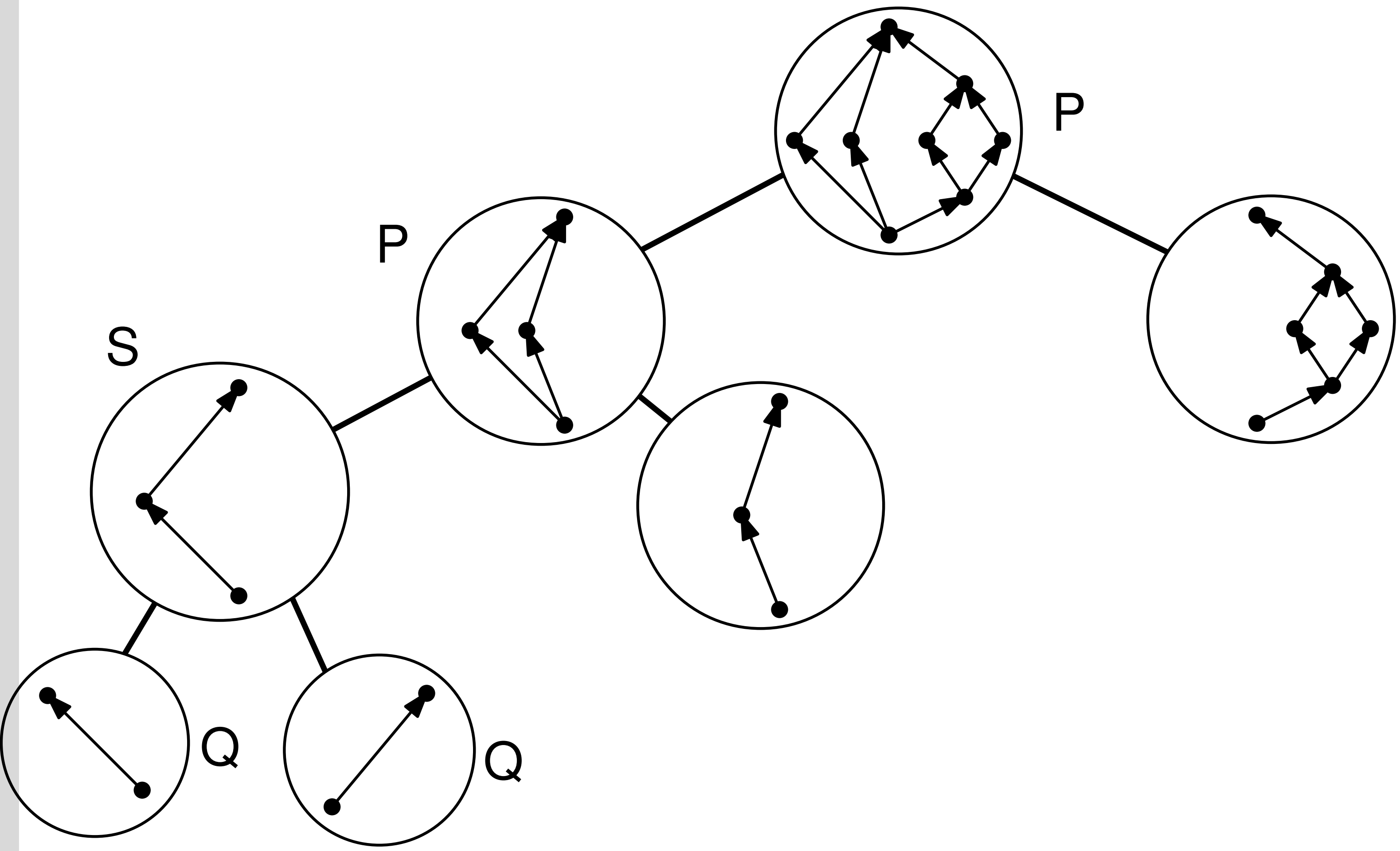

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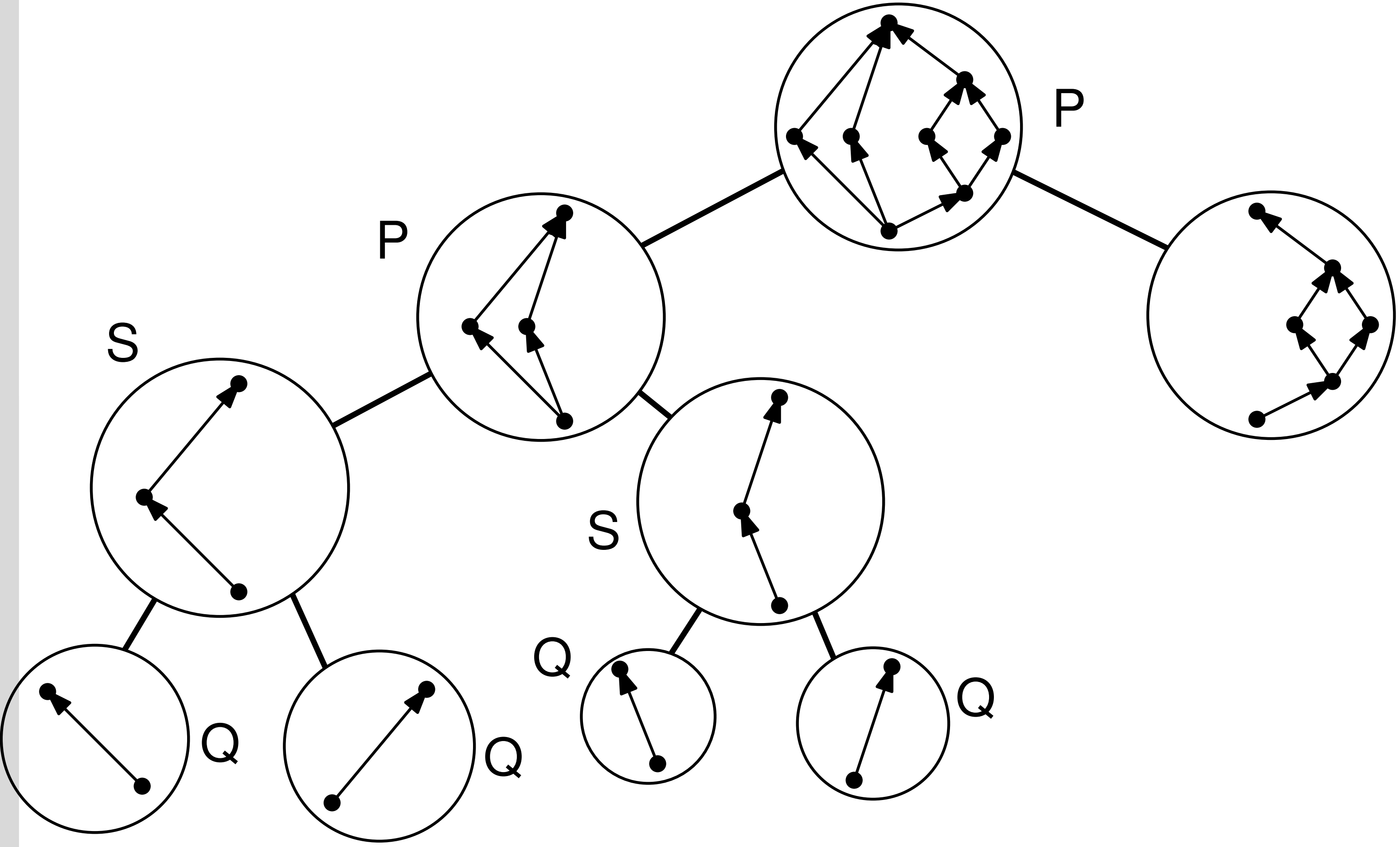

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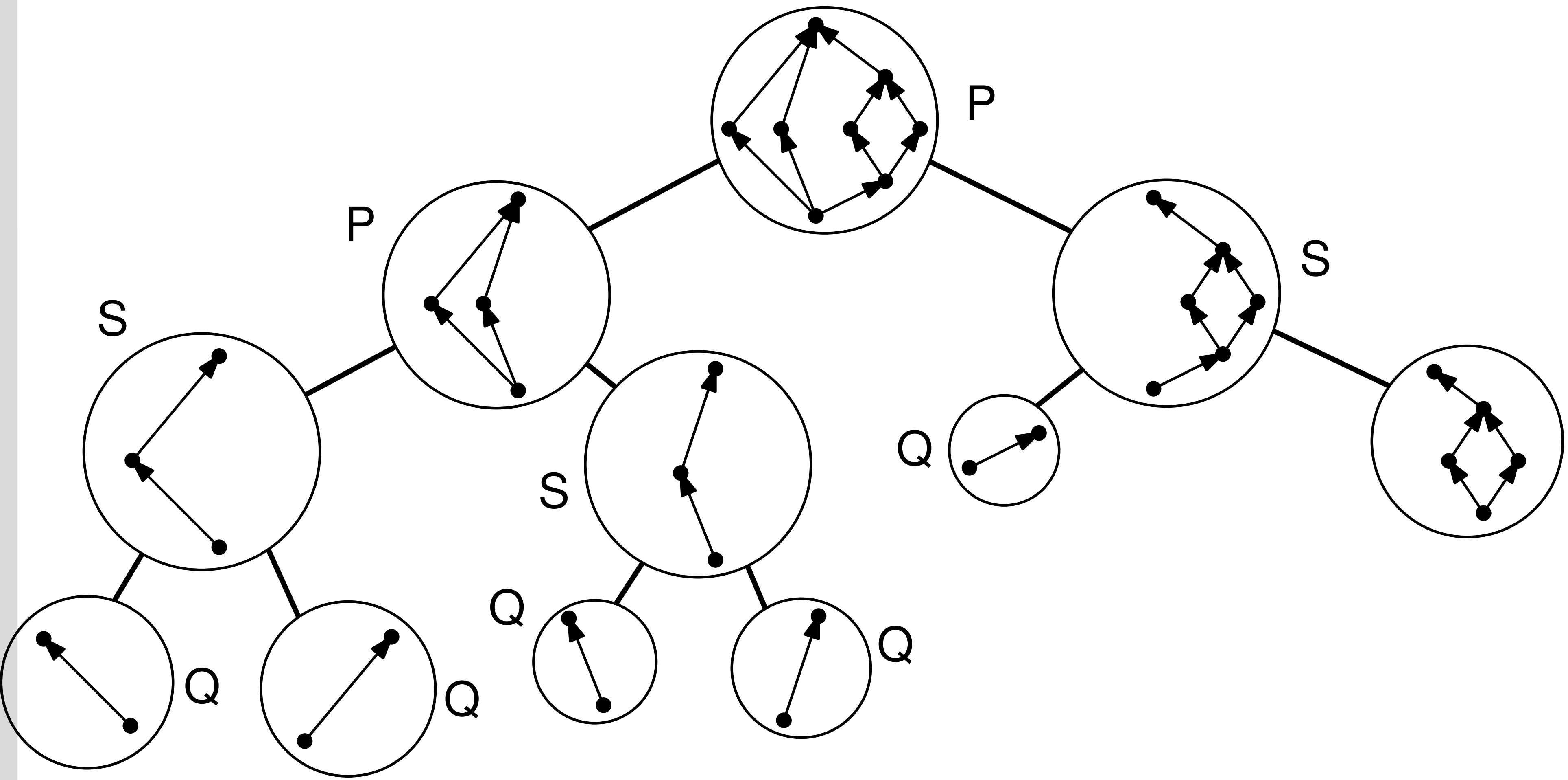

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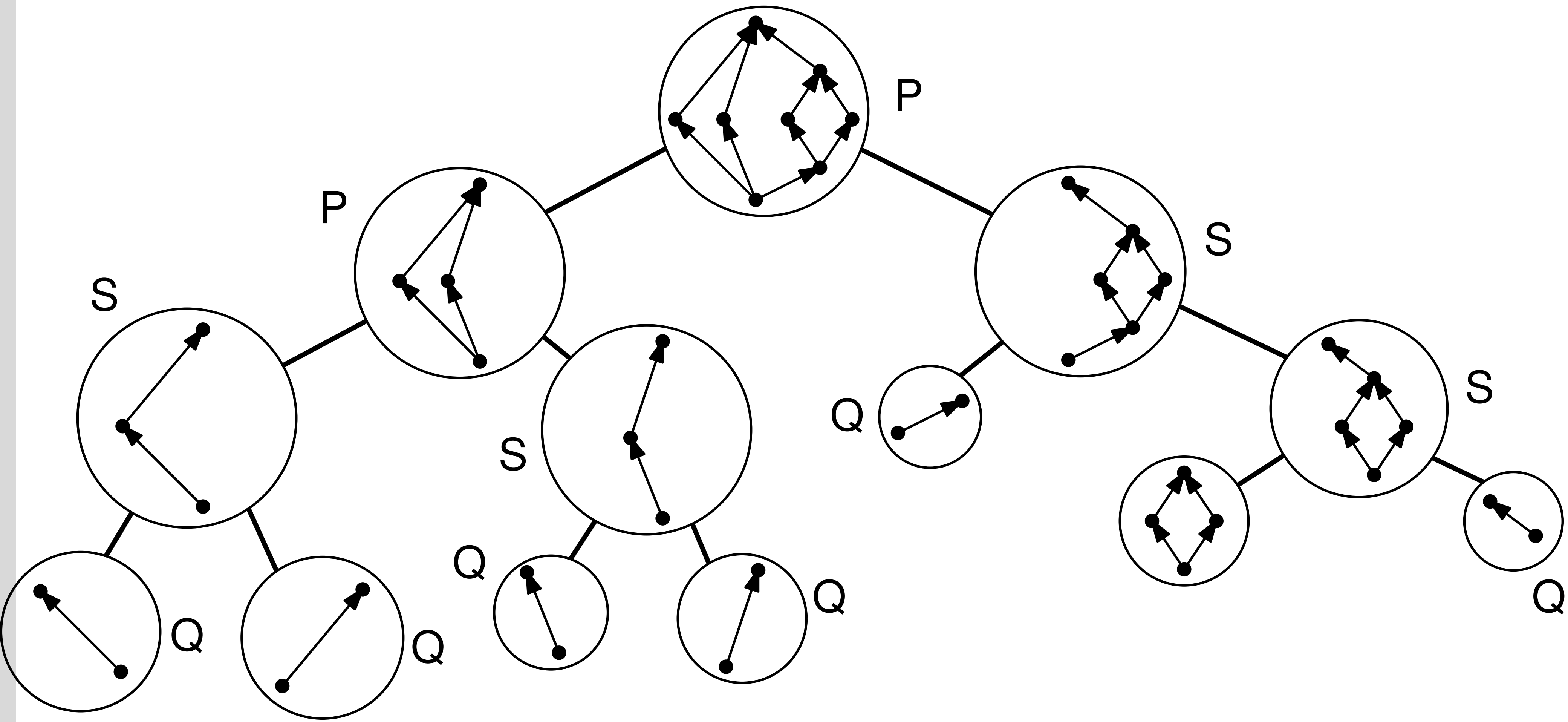

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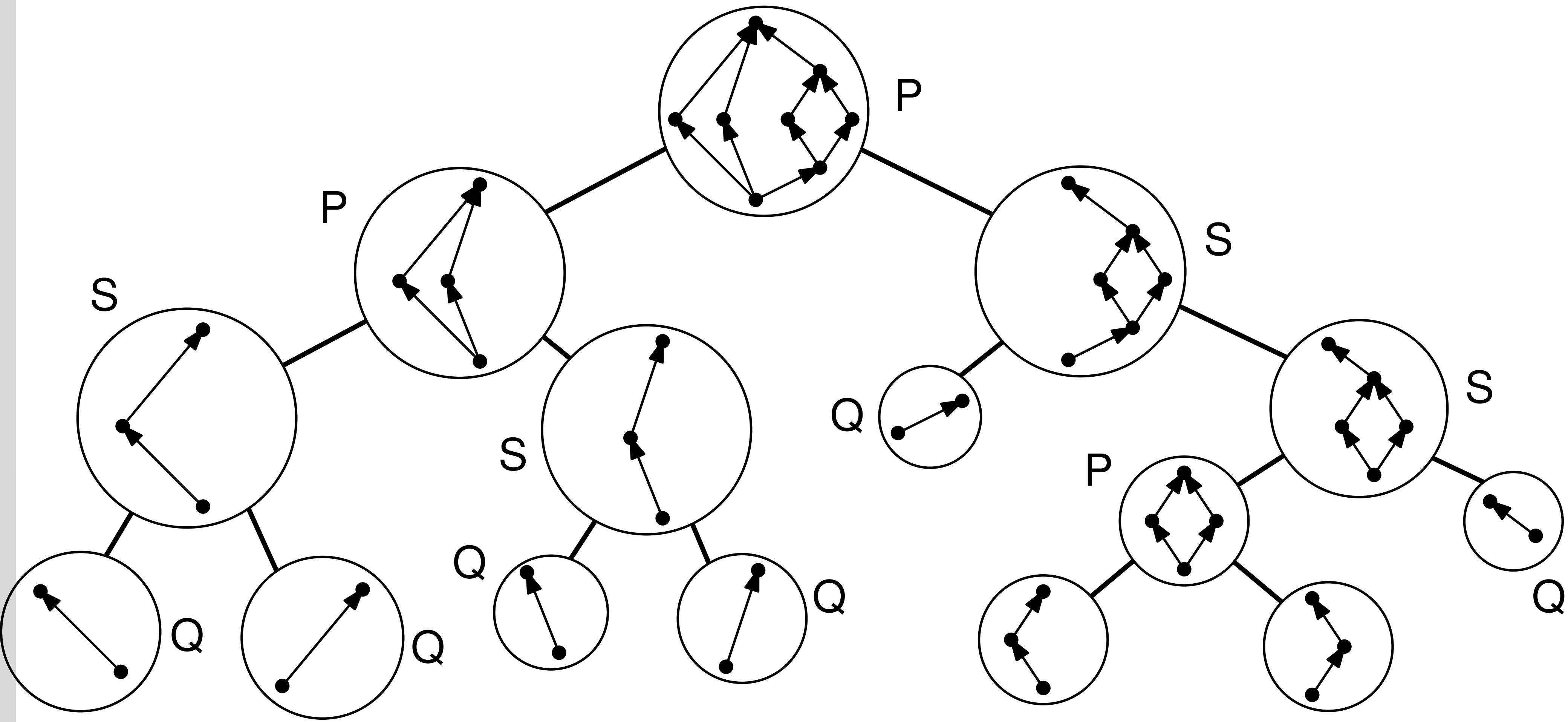

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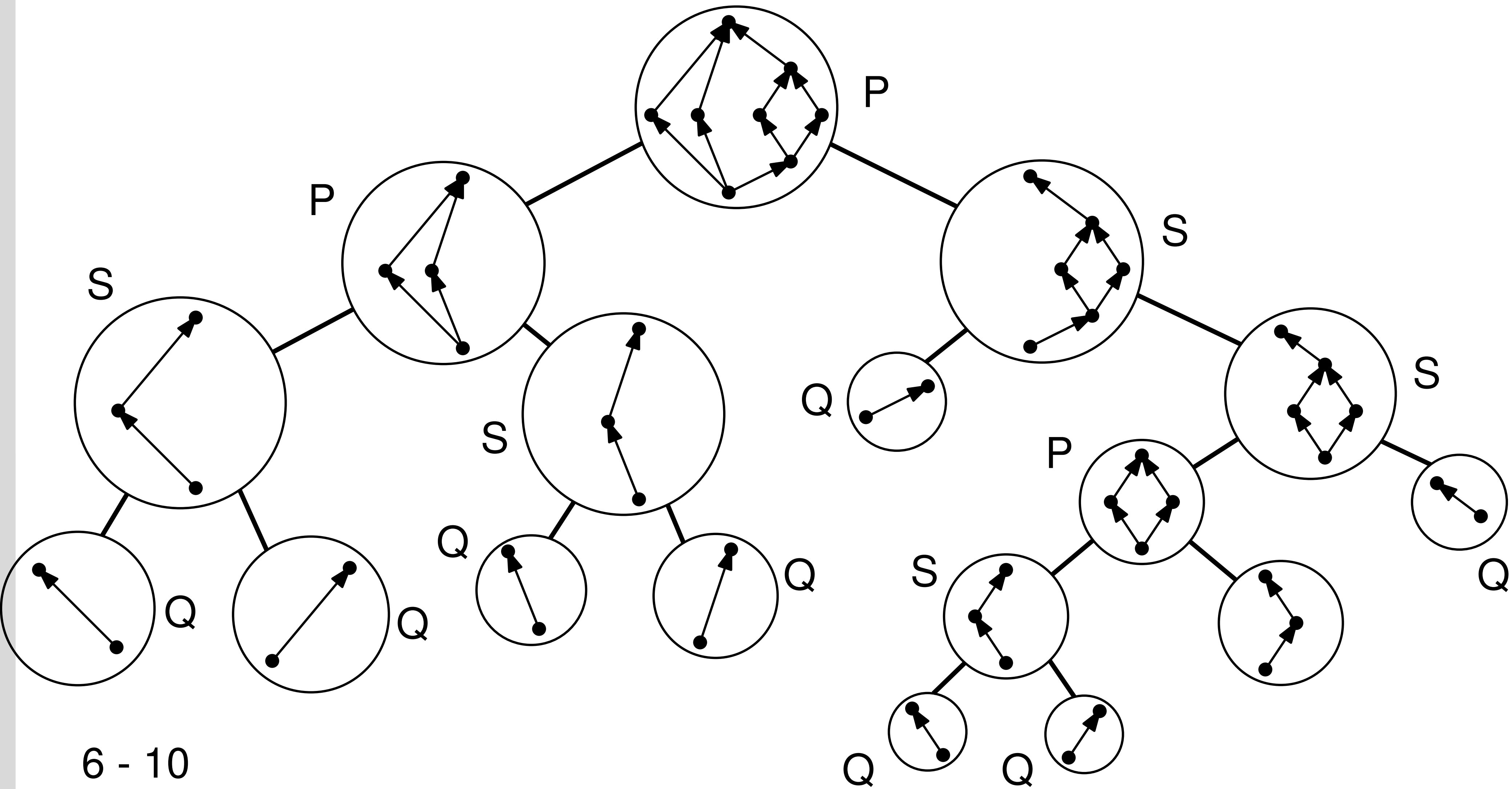

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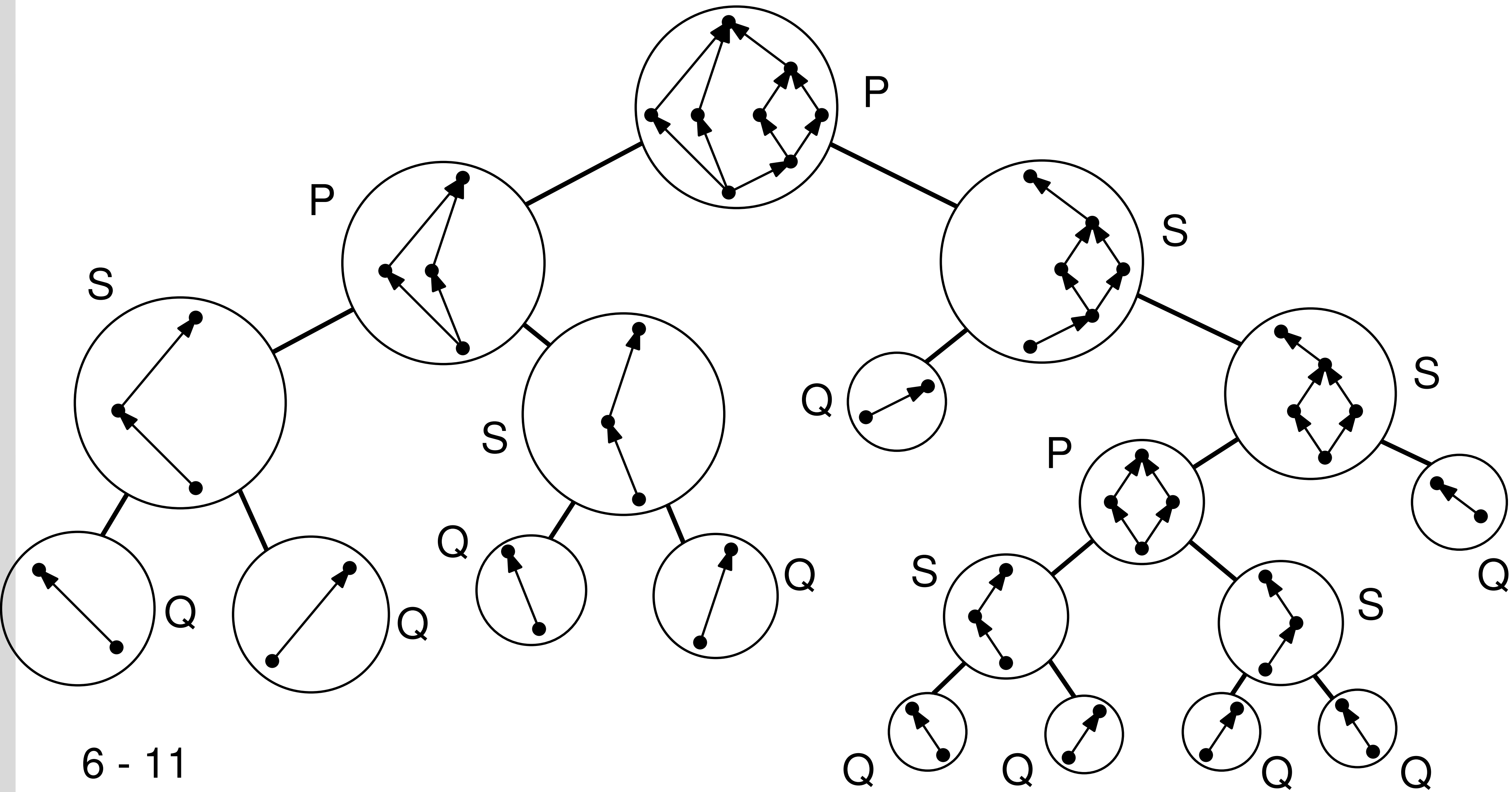

Series-parallel Graphs. Decomposition Tree.

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Lemma

Series-parallel graphs are acyclic and planar.

Lemma

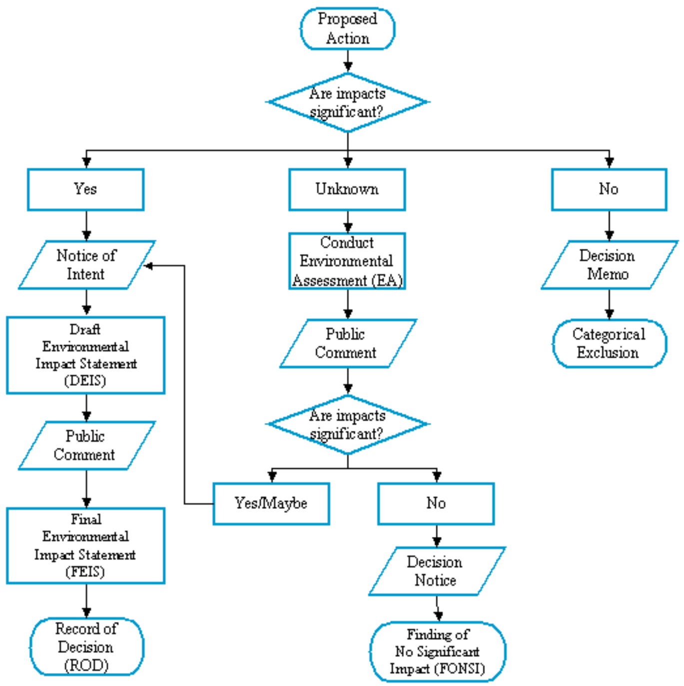
Series-parallel graphs are acyclic and planar.

Work with your neighbour(s) and then share

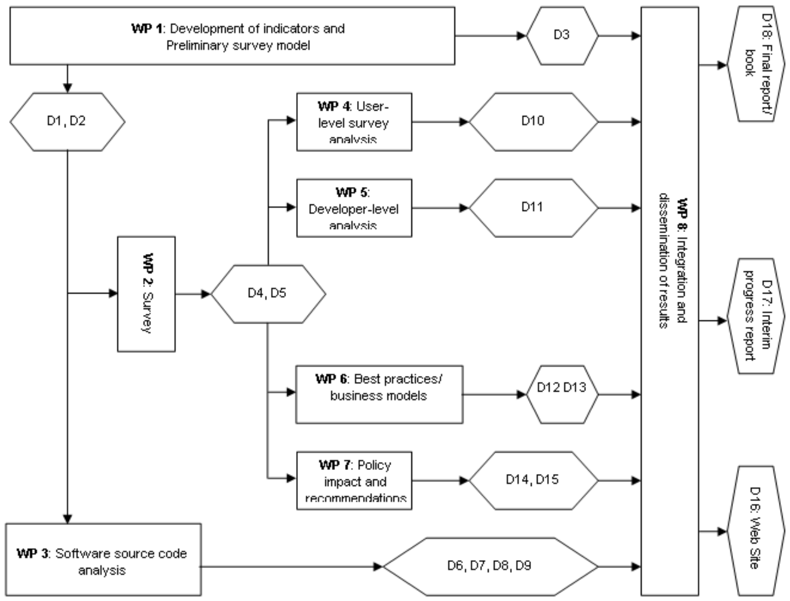
5 min

- Using the induction on the decomposition prove that a series-parallel graph is planar

Series-parallel Graphs. Applications.



Flowcharts



PERT-Diagrams

(Program Evaluation and Review Technique)

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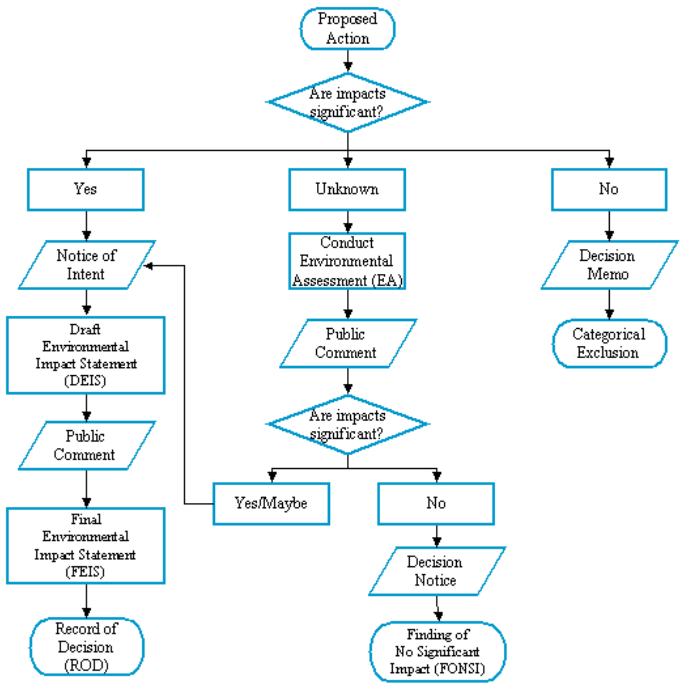

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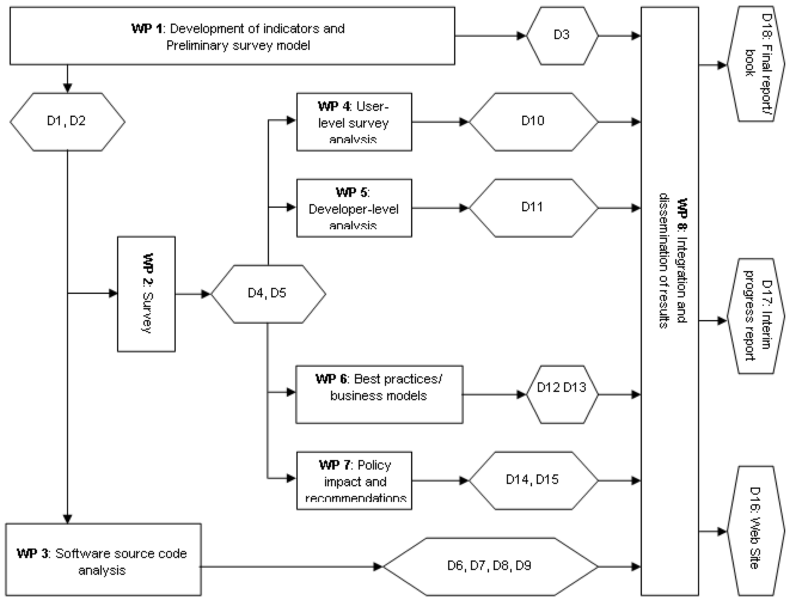

PERT-Diagrams

(Program Evaluation and Review Technique)

Computational Complexity: Linear time algorithms for $\mathcal{NP}$-hard problems (e.g. Maximum Matching, Maximum Independent Set, Hamiltonian Completion)

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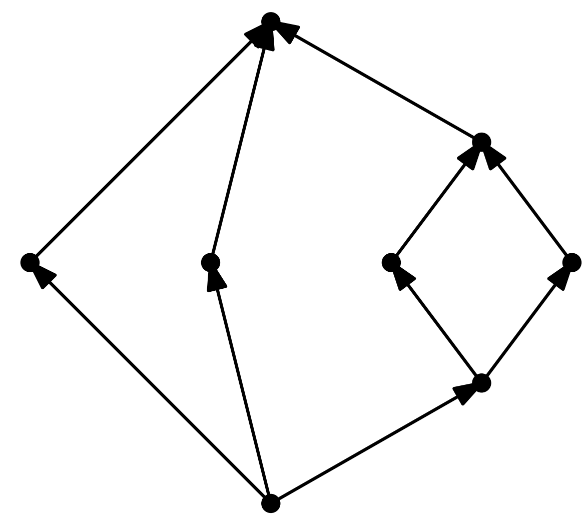
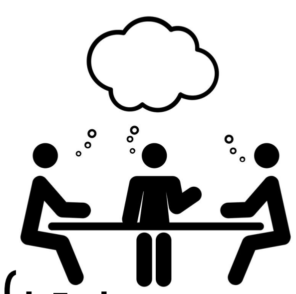
Conventions & Aesthetics for SP-graphs

Drawing Conventions



Drawing Aesthetics

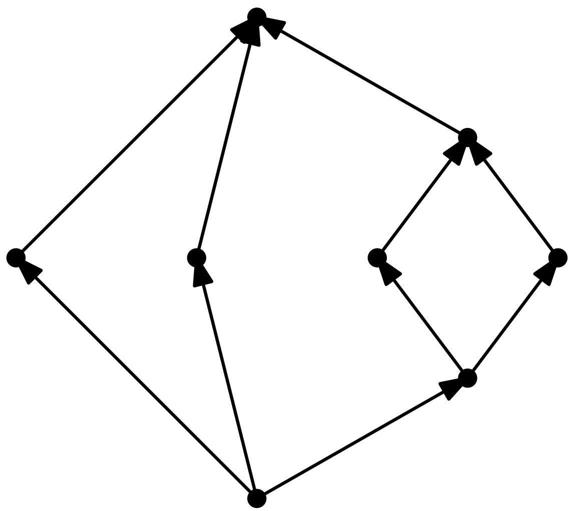


Let’s brainstorm!

Conventions & Aesthetics for SP-graphs

Drawing Conventions

- Planarity
- Straight-line edges
- Upward



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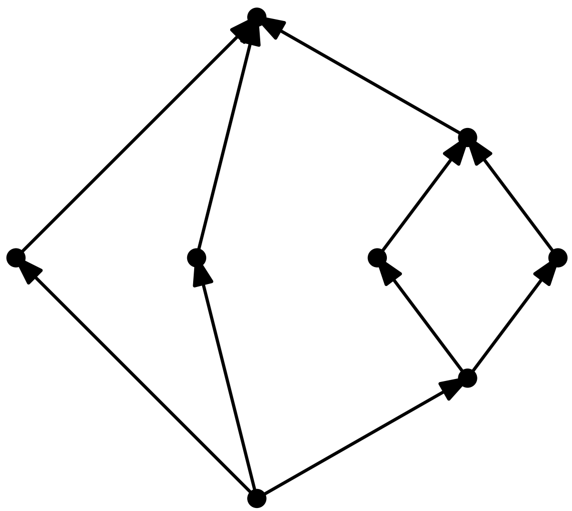


Drawing Aesthetics

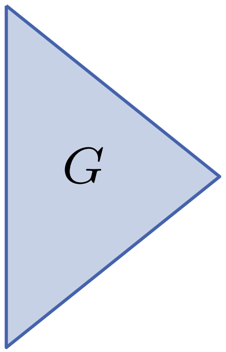
- Area
- Symmetry

Let's brainstorm!

Straight-line Drawing of SP-Graphs

Divide & Conquer Algorithm, using the decomposition tree

- Draw graph G inside a right-angled isosceles bounding triangle $\Delta(G)$



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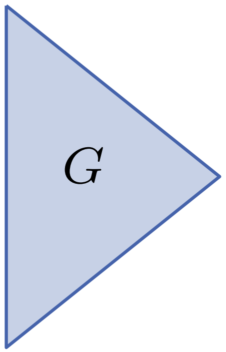

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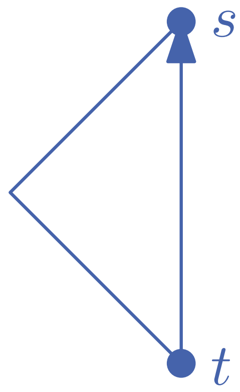

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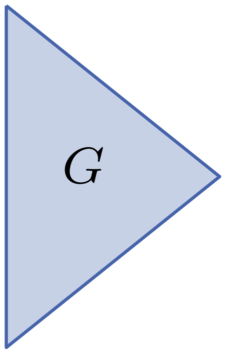

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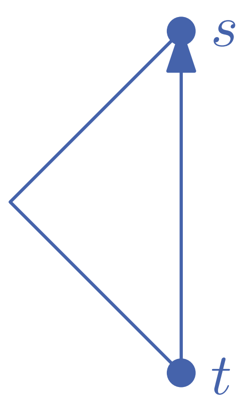

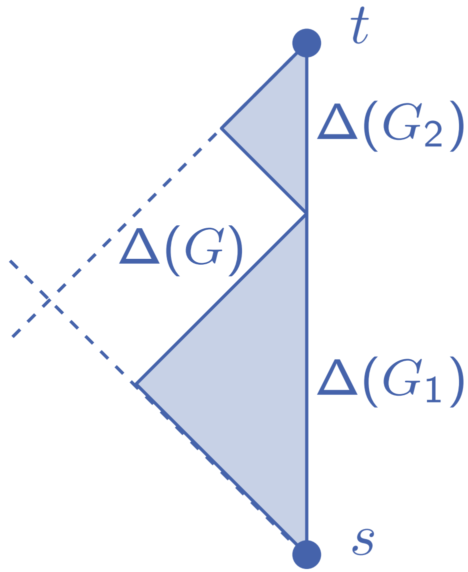

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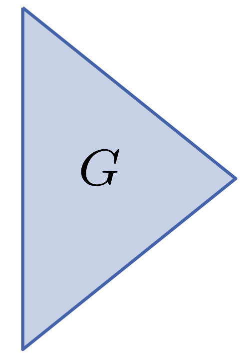

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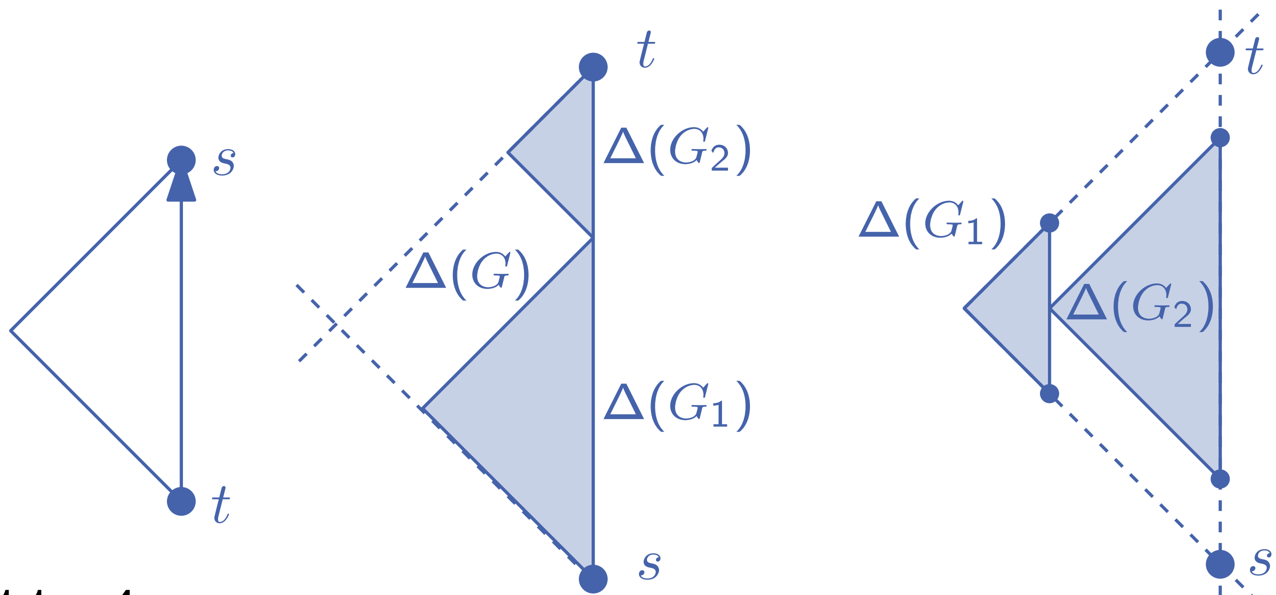

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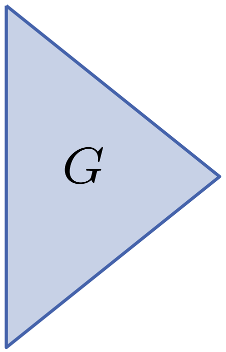

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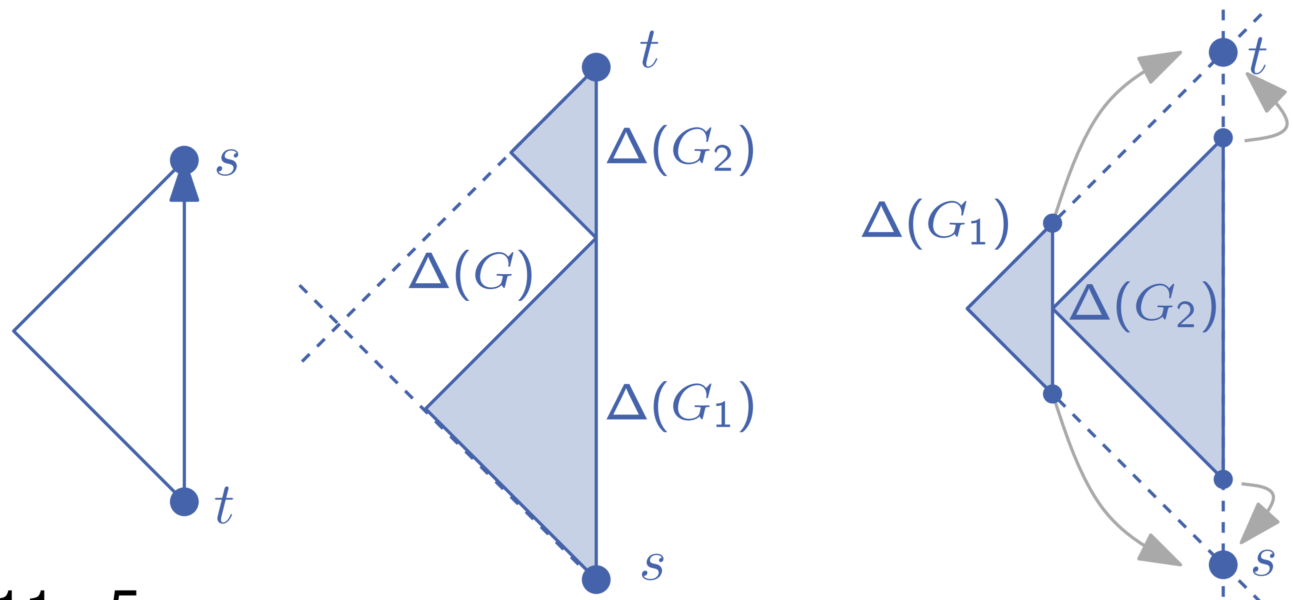

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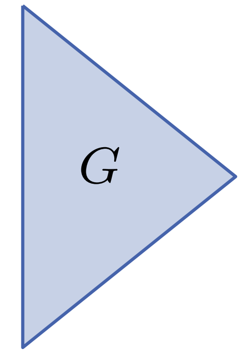

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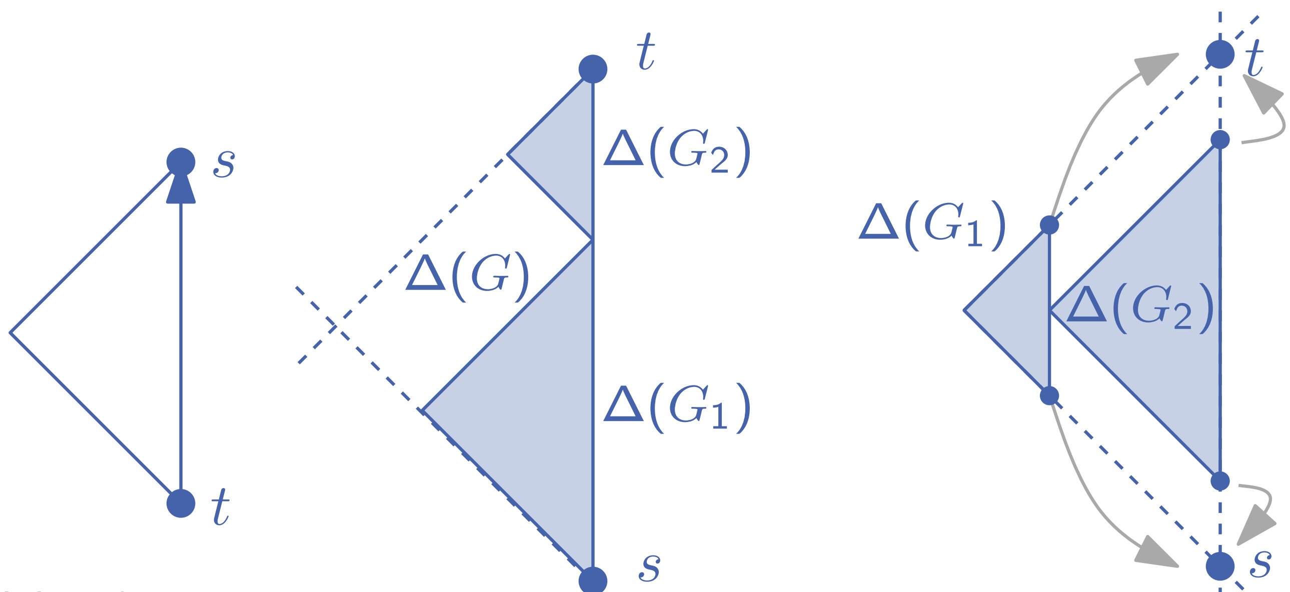

Do you see any problem?

Straight-line Drawing of SP-Graphs

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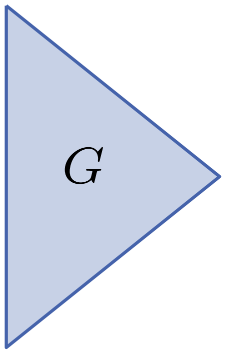

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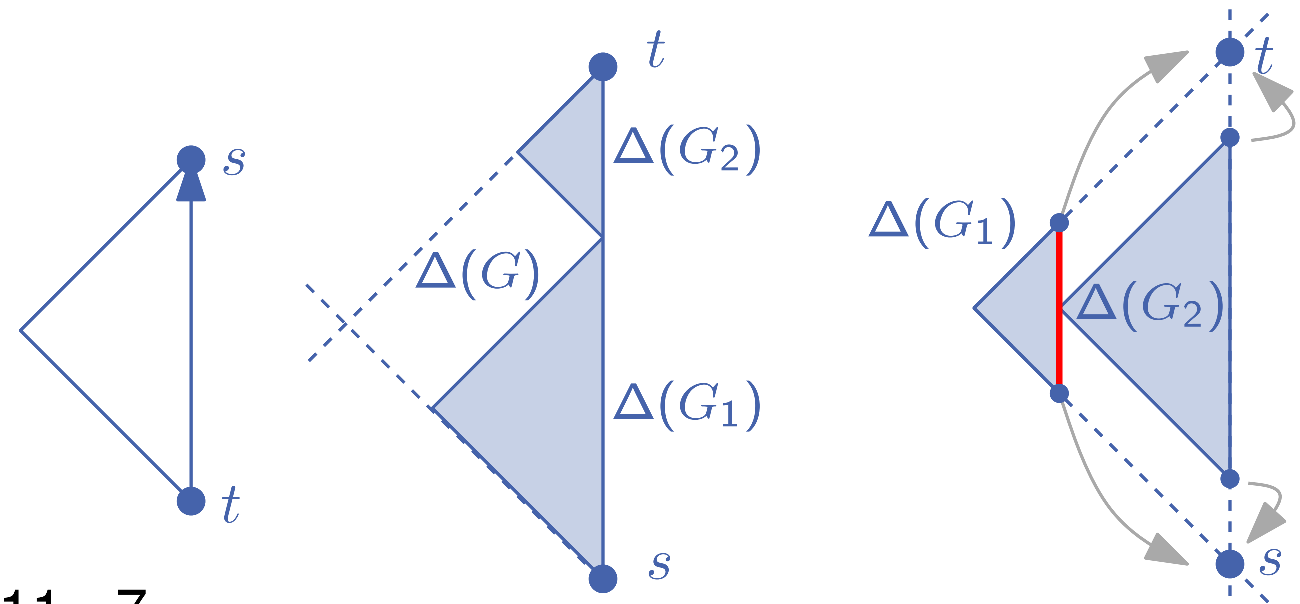

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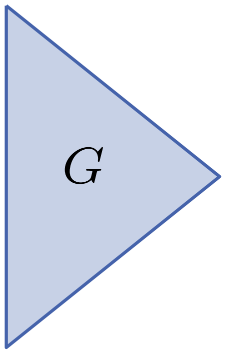

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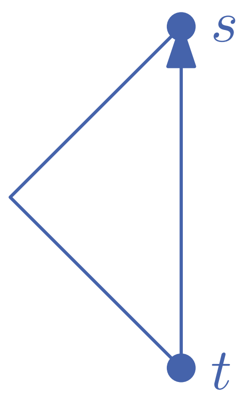

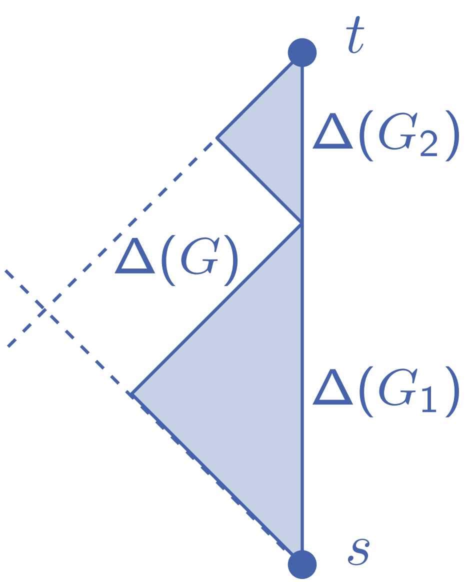

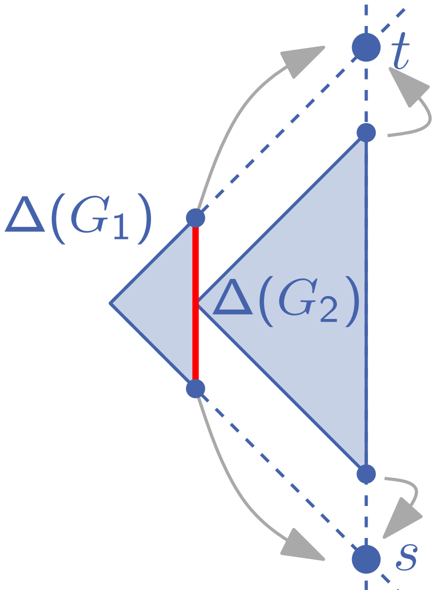

change embedding!

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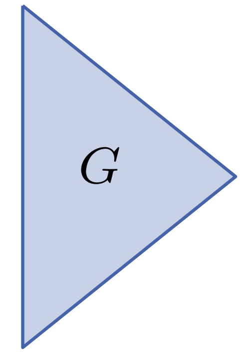

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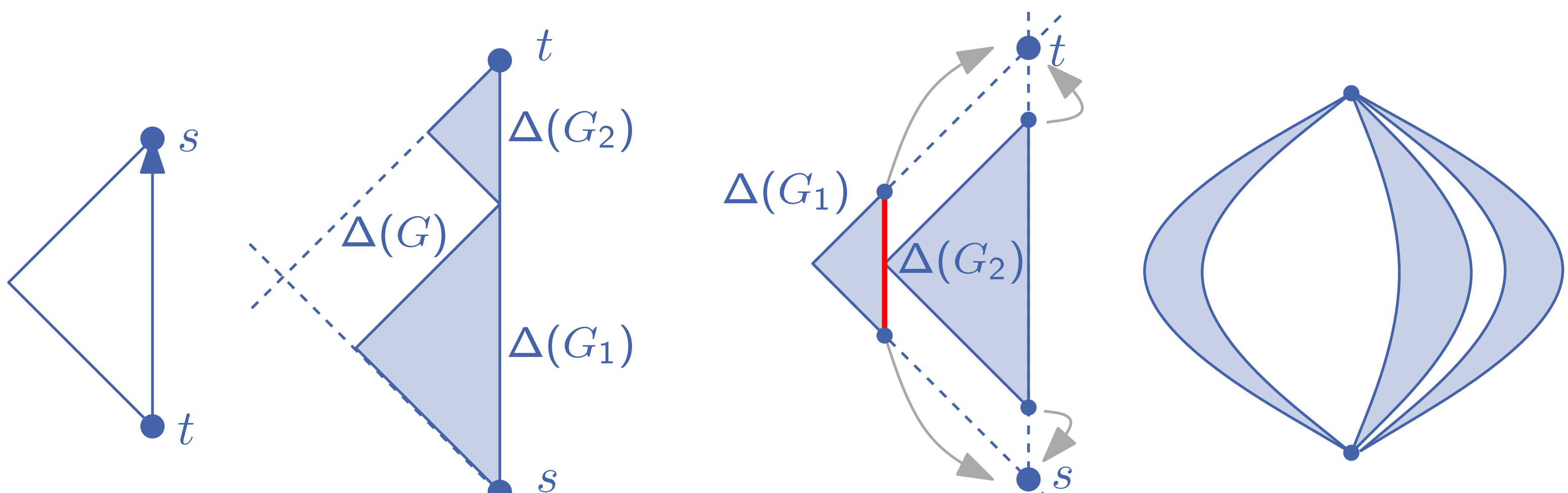

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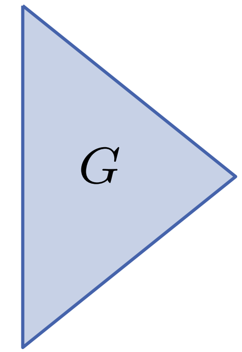

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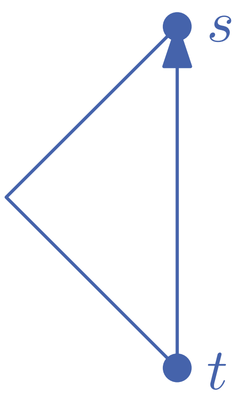

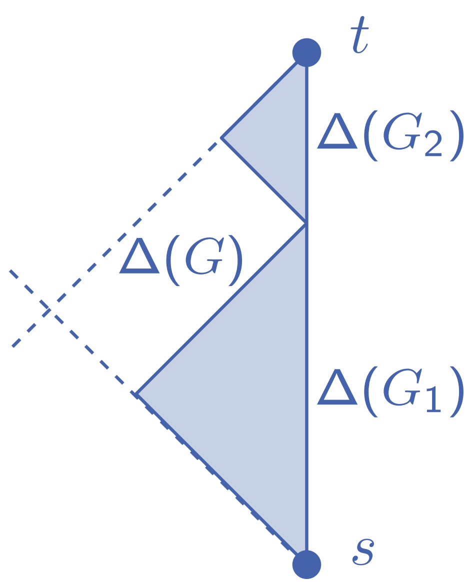

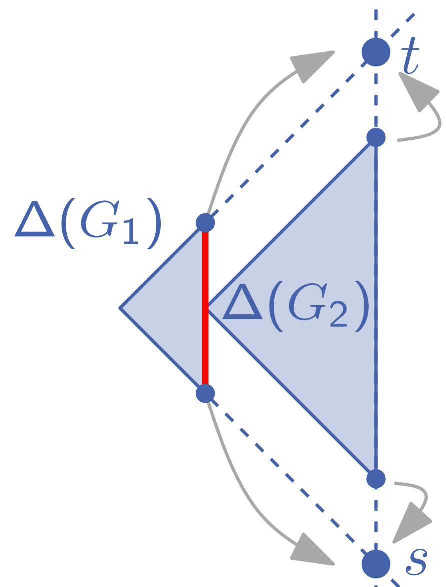

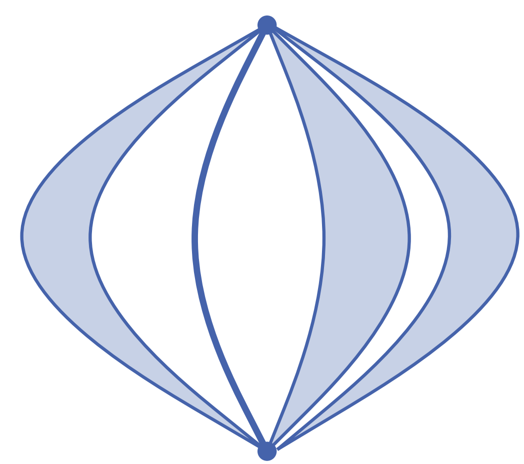

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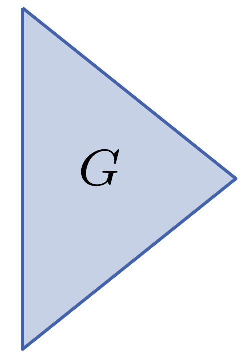

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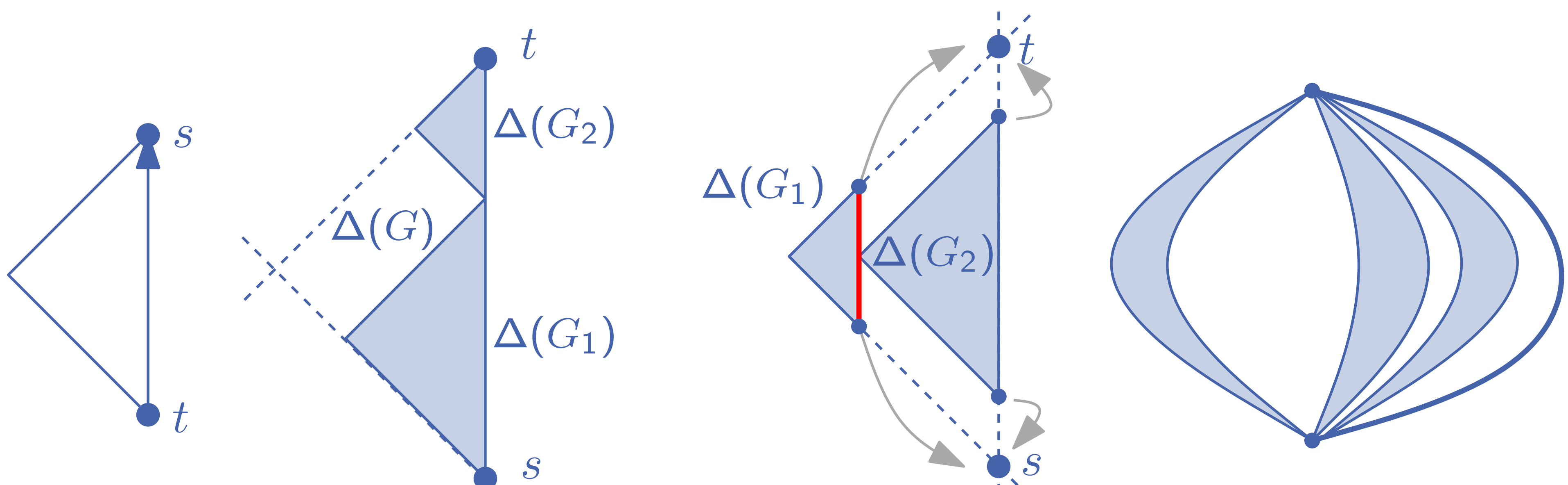

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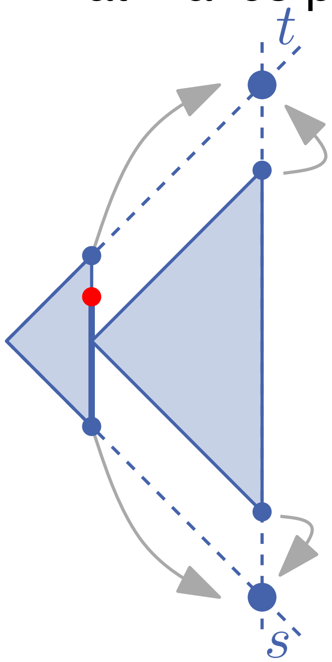
Straight-line Drawing of SP-Graphs

- What makes parallel composition possible without creating crossings?



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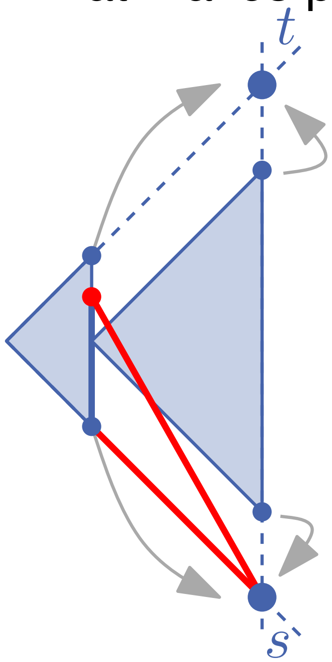

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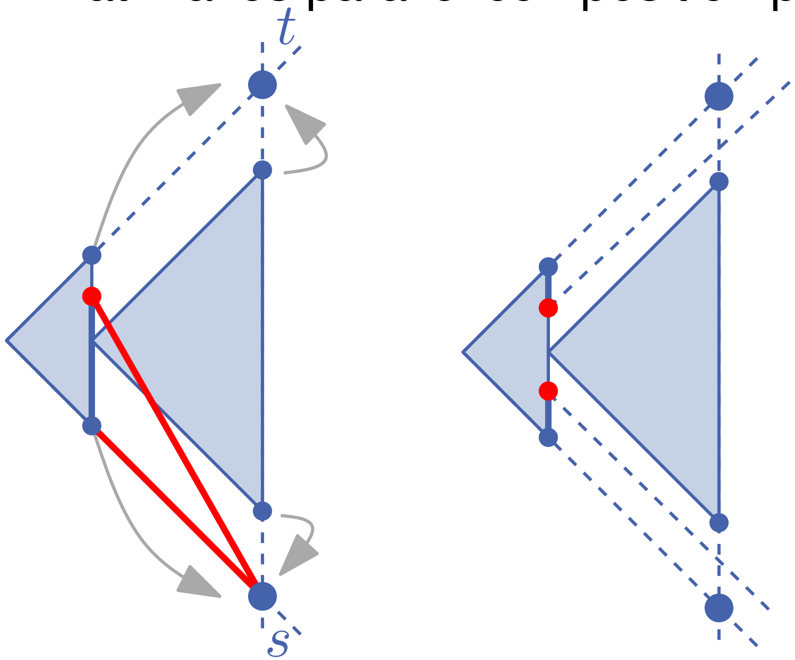

Straight-line Drawing of SP-Graphs

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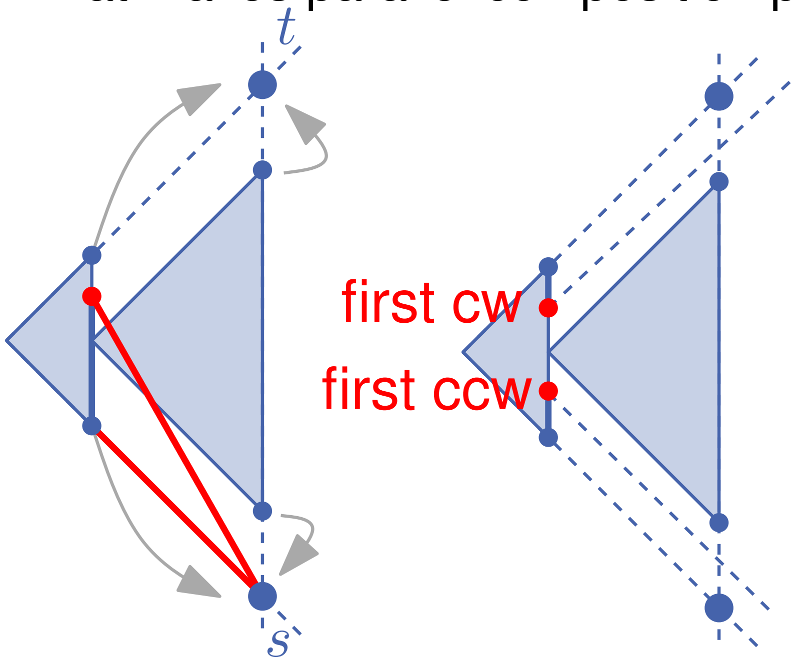

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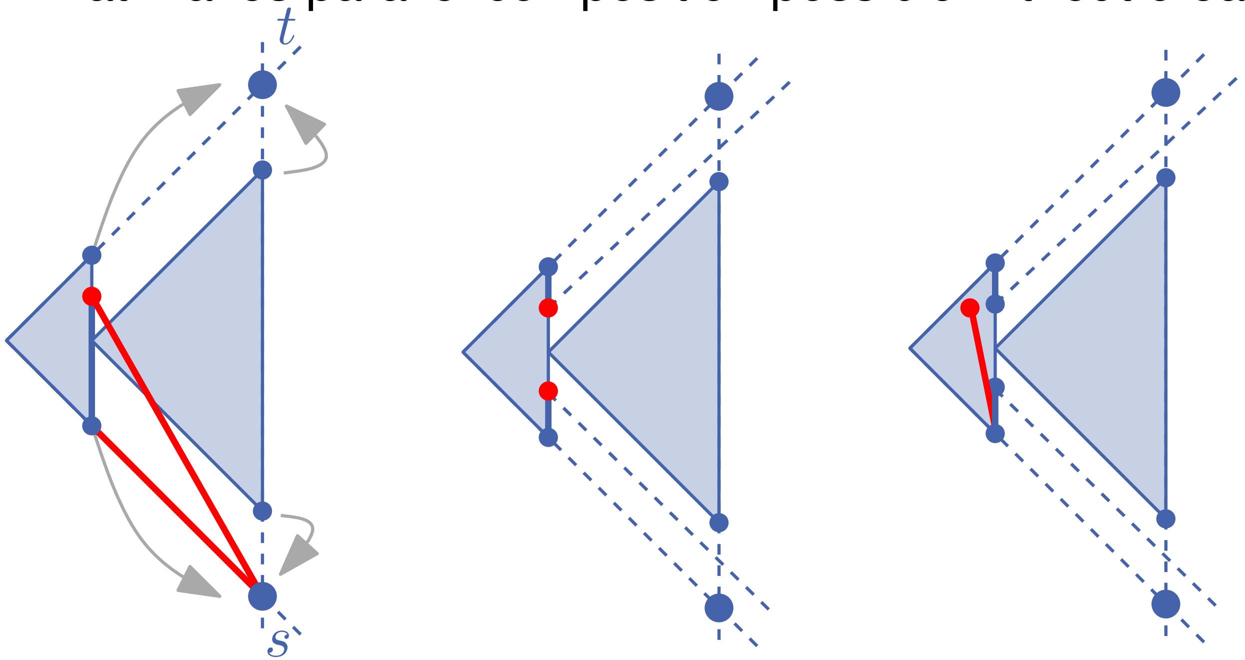

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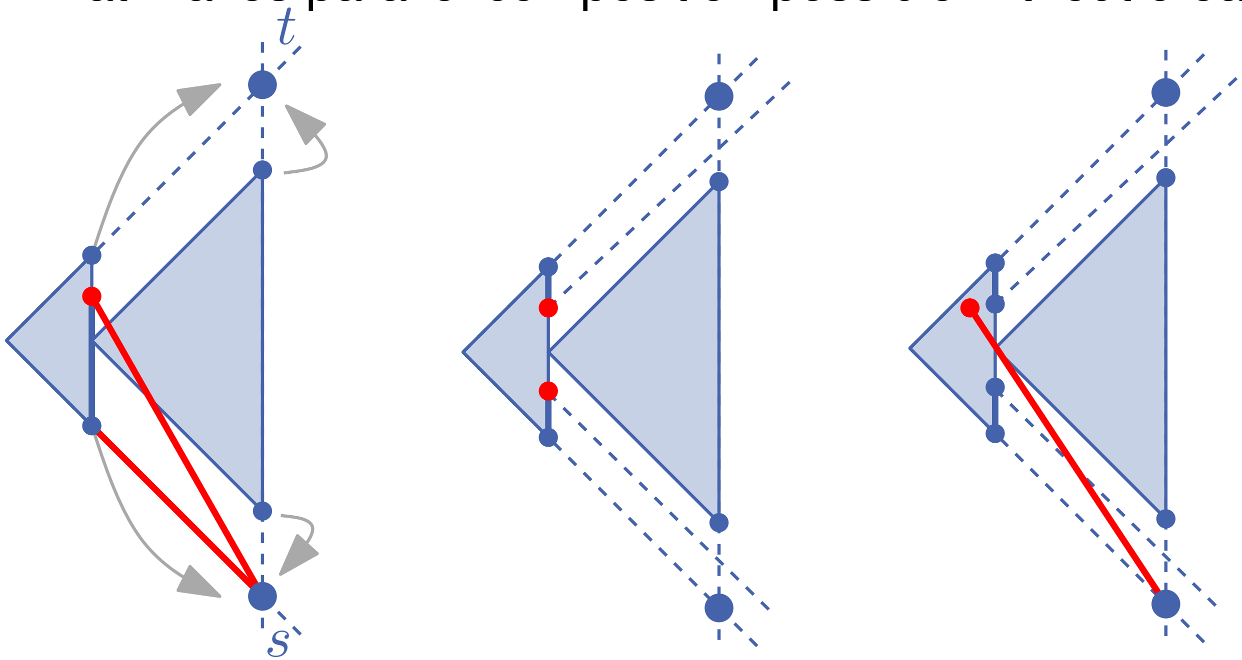

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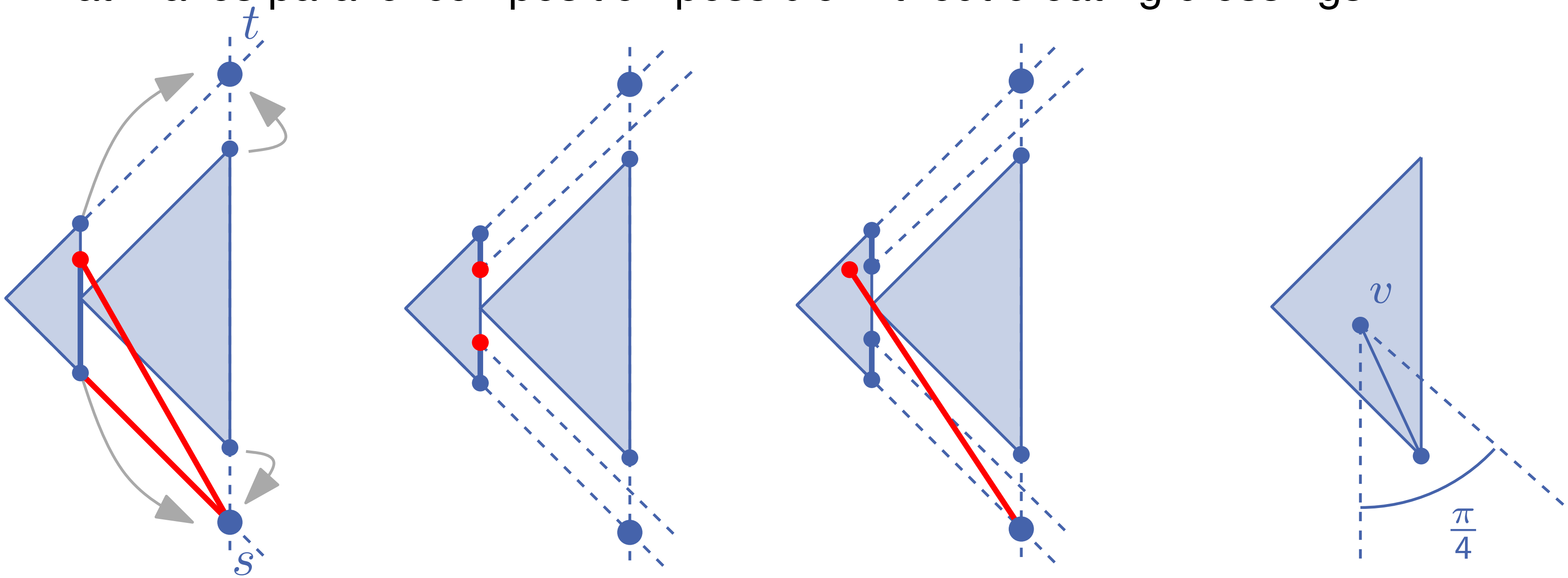

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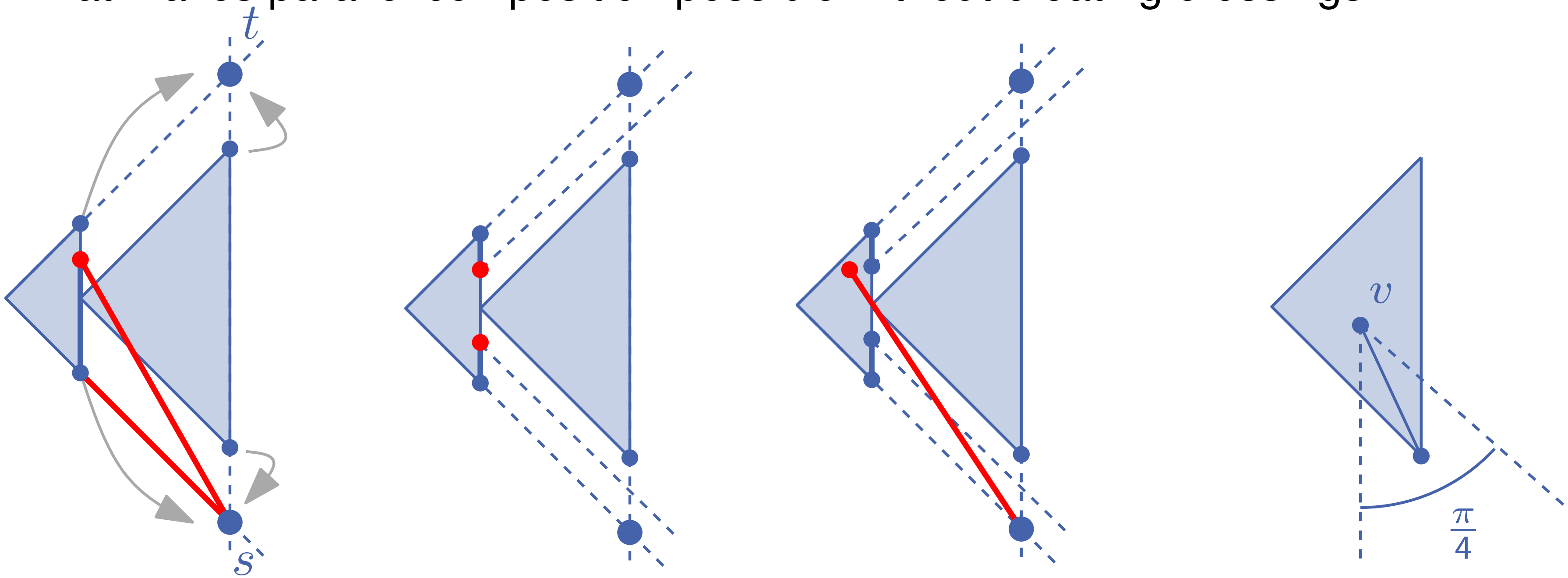

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Straight-line Drawing of SP-Graphs

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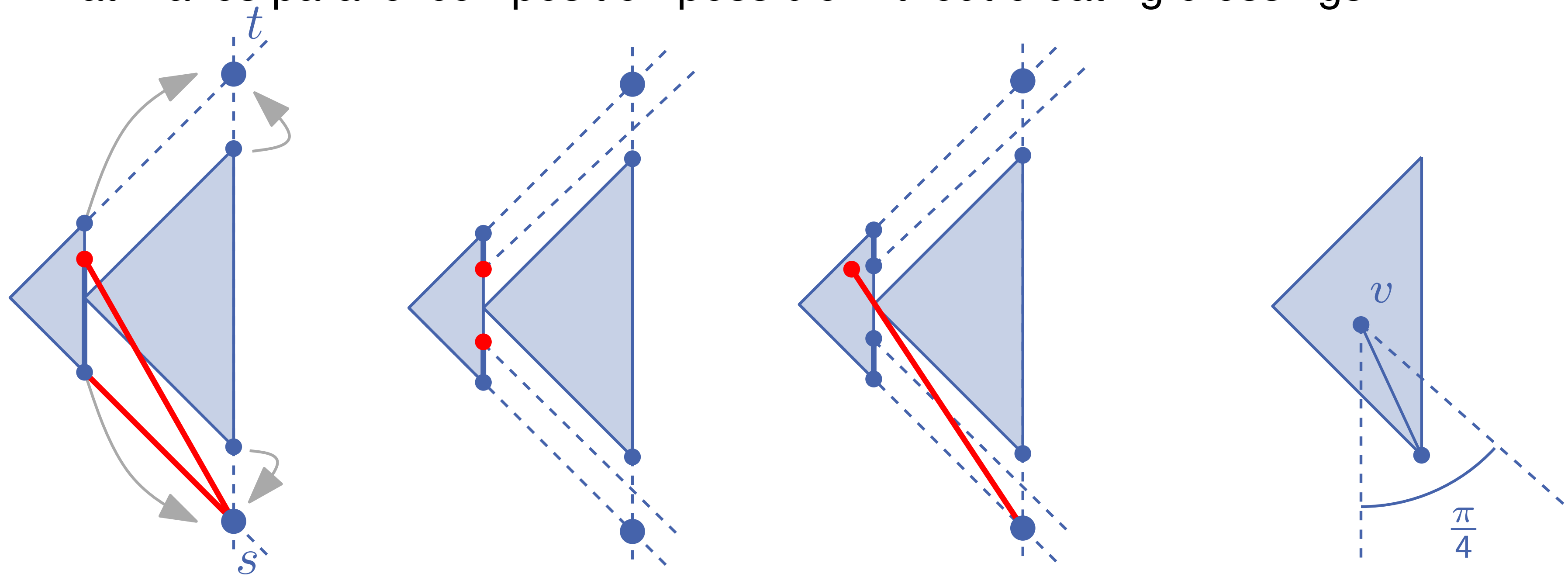

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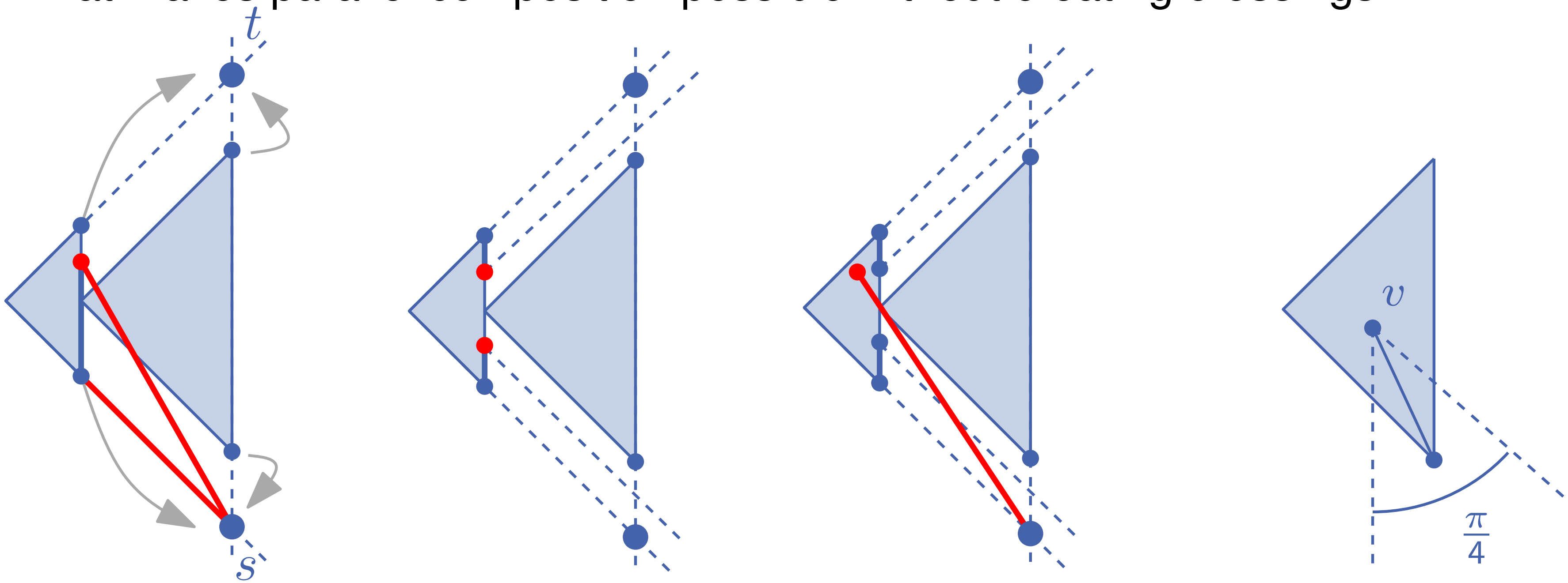


Assume the following holds: angle(v) does not contain any vertex

- This condition **is** preserved during the induction step.

Lemma

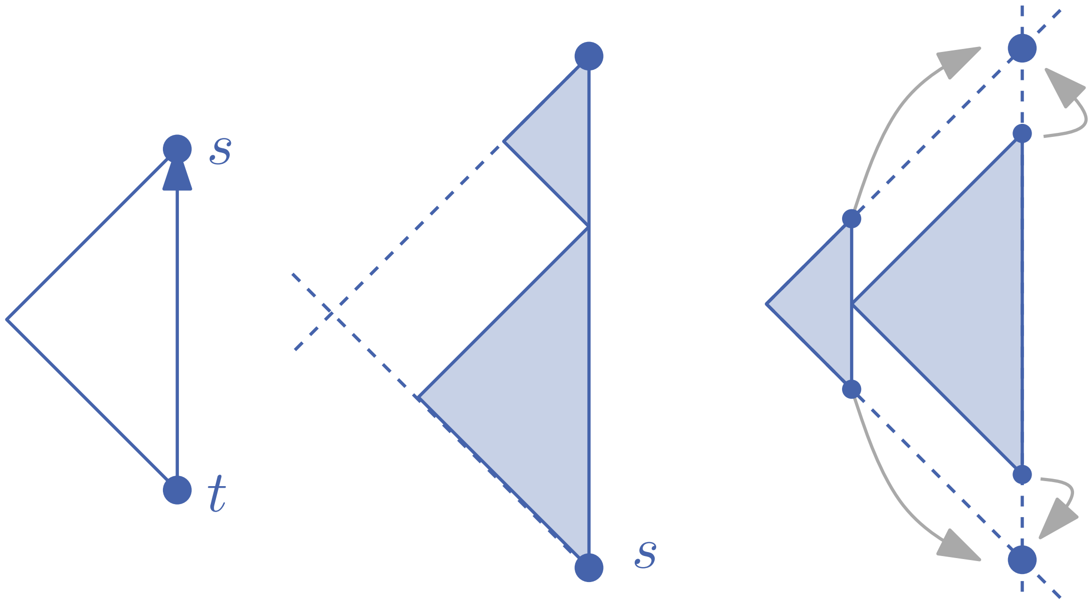
The drawing produced by the algorithm is planar.

Straight-line Drawing of SP-Graphs

Work with your neighbour(s) and then share

3 min

- What is the asymptotic upper bound on the area of the drawing? **Hint:** Think in terms of edges.



Straight-line Drawing of SP-Graphs

Work with your neighbour(s) and then share

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- What is the asymptotic upper bound on the area of the drawing? **Hint:** Think in terms of edges.

Lemma

The area of the produced drawing is $O(m^2)$, m is the number of edges.

Theorem

A series-parallel graph G (**with variable embedding**) admits an **upward** planar straigh-line drawing with $O(n^2)$ area. The isomorphic componencts of G have congruent drawings up to a translation.

Overview

Series - parallel graph. Definition and Decomposition.

Algorithm for upward straight-line drawing.

Lower bound on the area.

Symmetry display for series-parallel graphs.

Lower Bound for the Area

Theorem [Bertolazzi et al. 94]

There exists a $2n$-vertex series-parallel graph G_n such that any upward planar drawing of G_n respecting embedding requires area $\Omega(4^n)$.

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Lehrstuhl Algorithmik I

Lower Bound for the Area

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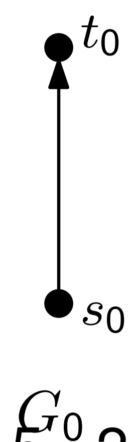

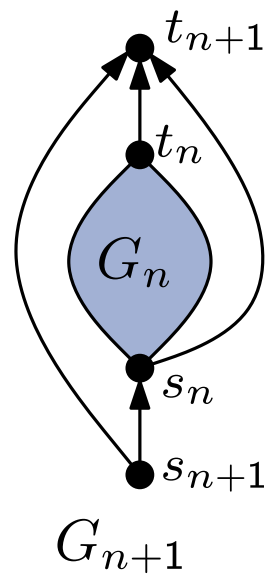

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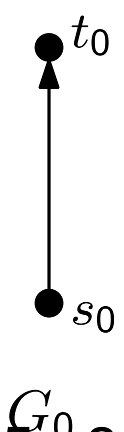

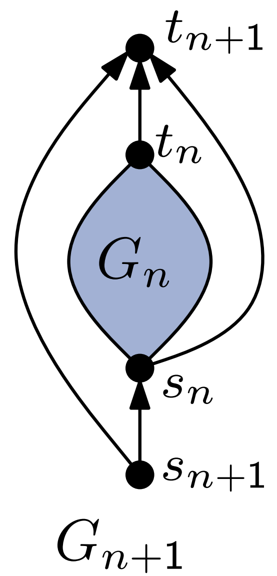

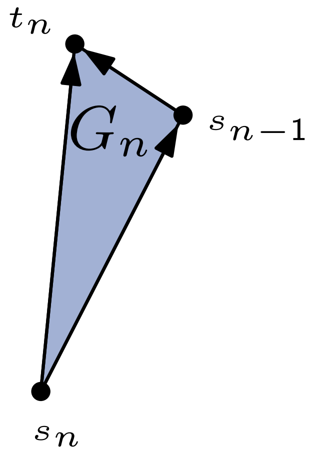

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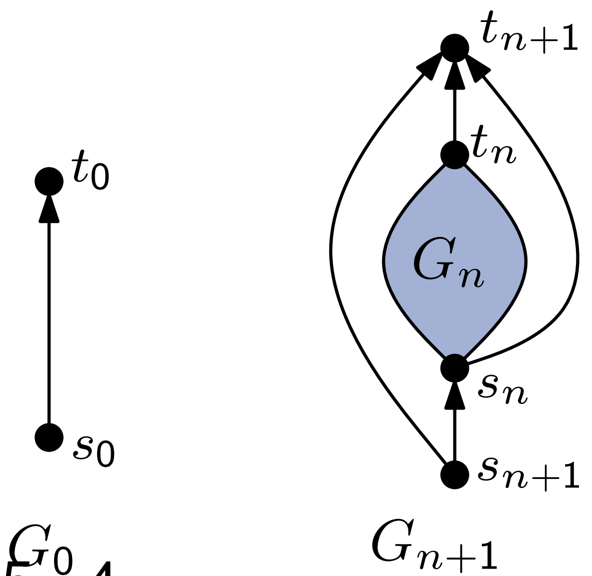

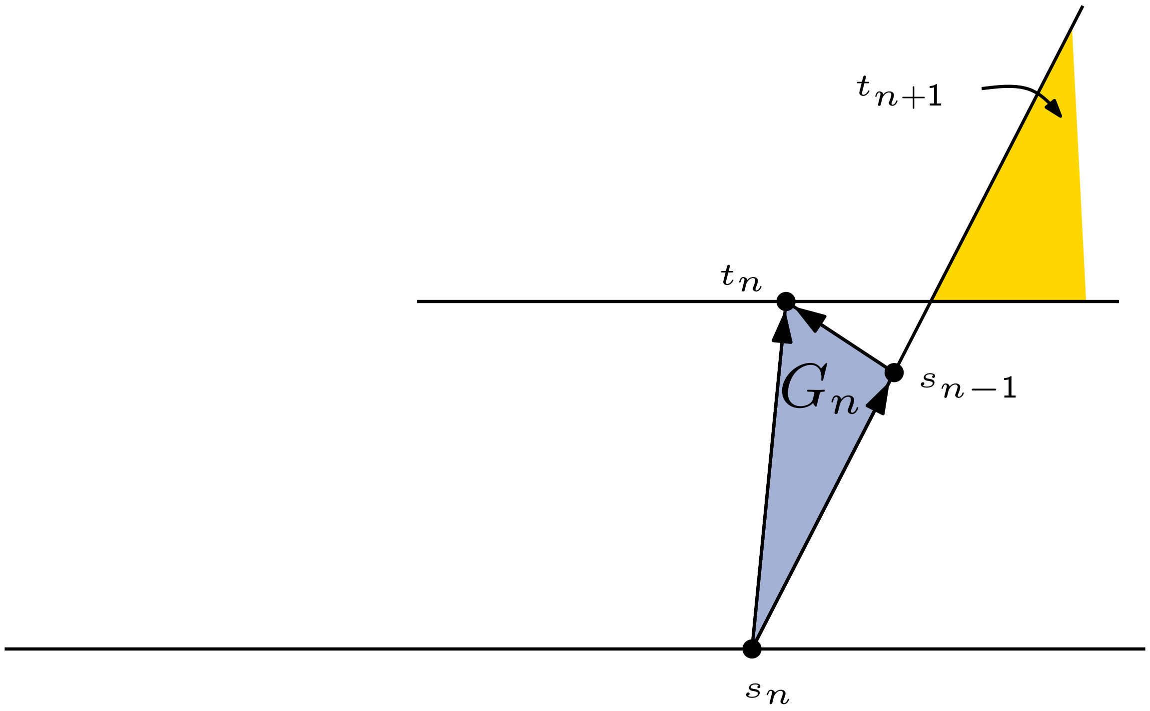

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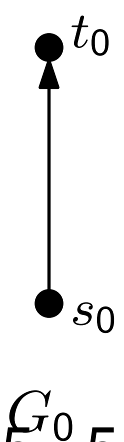

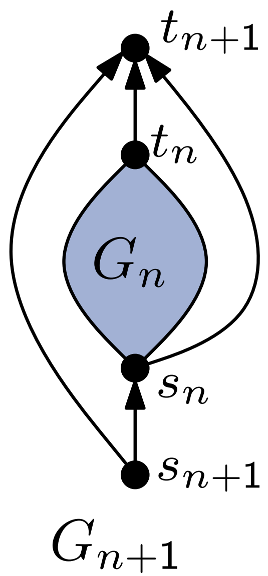

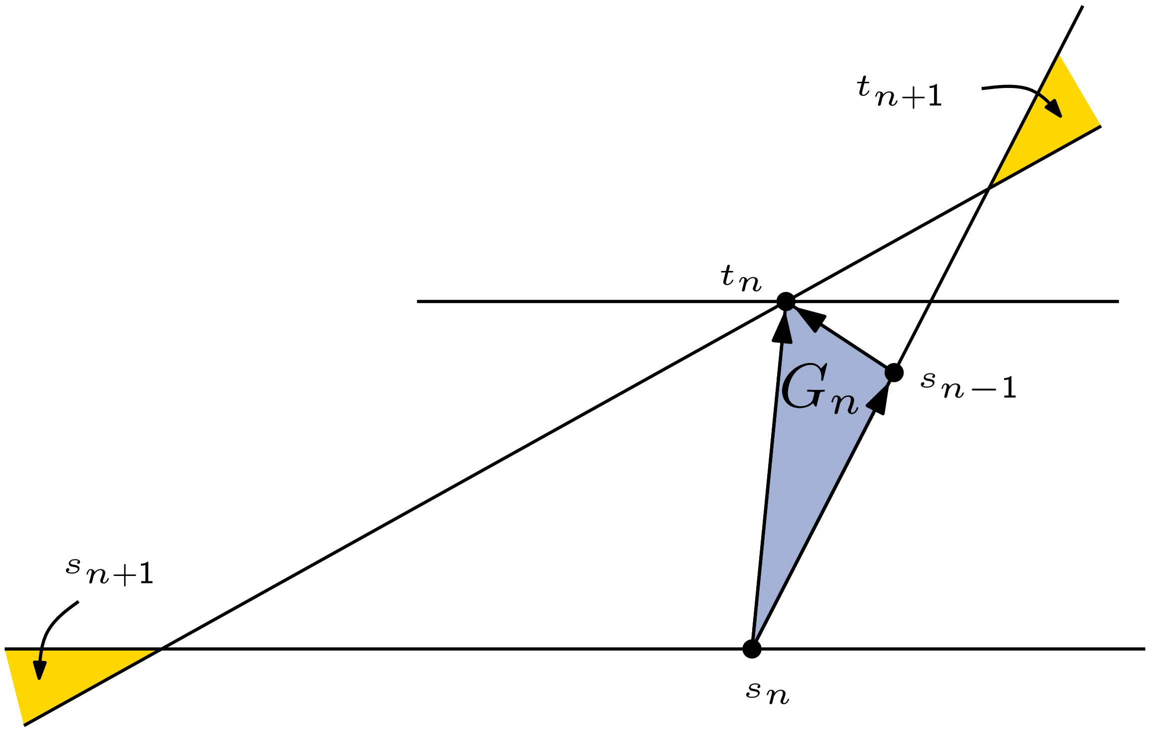

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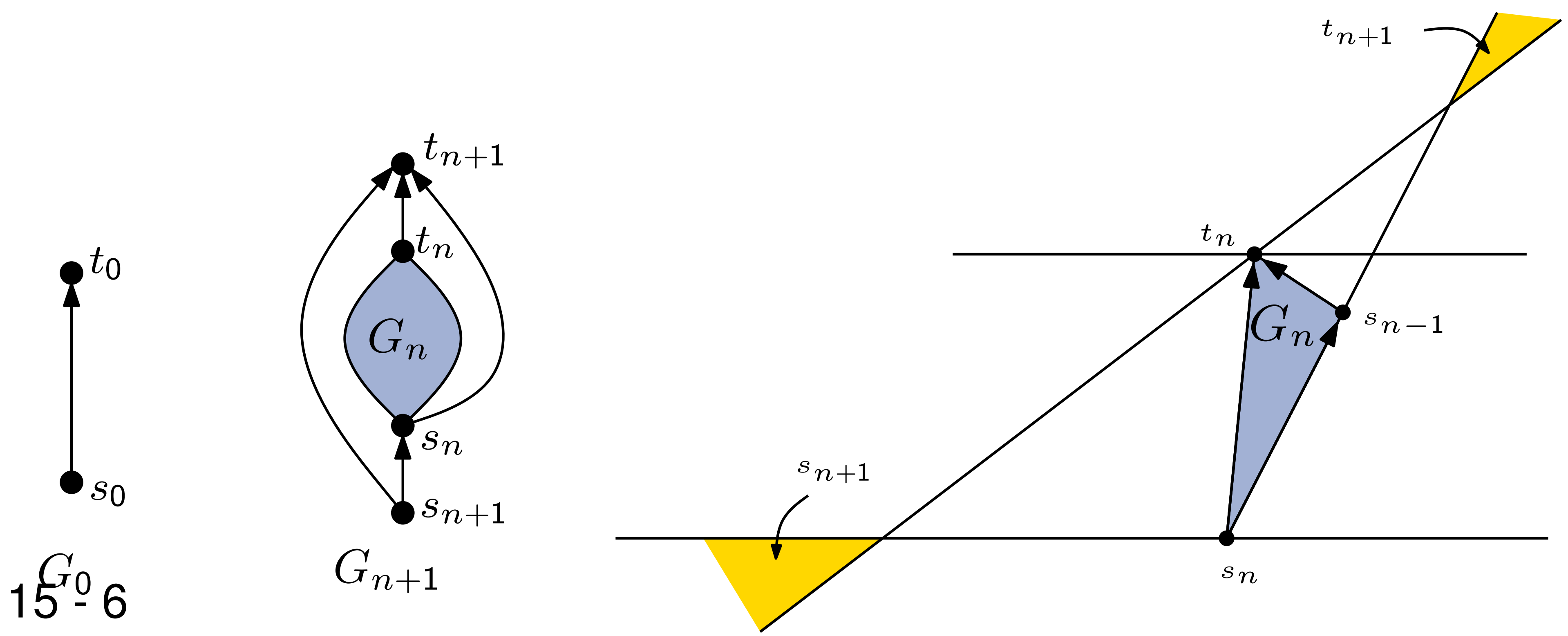

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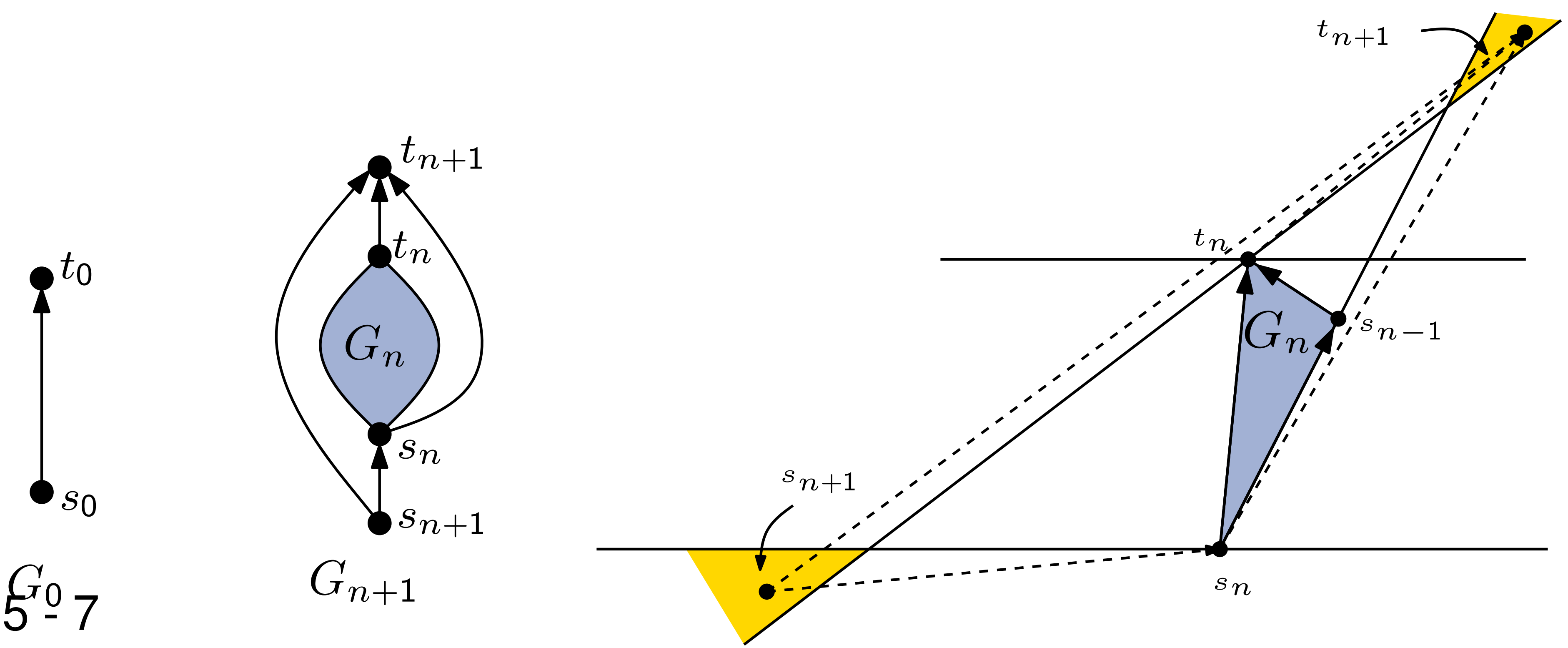

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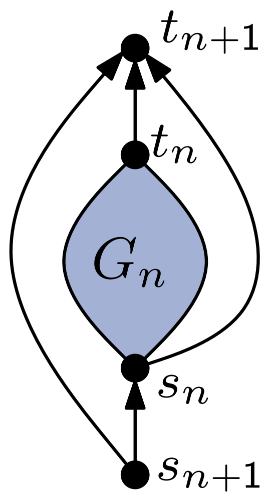
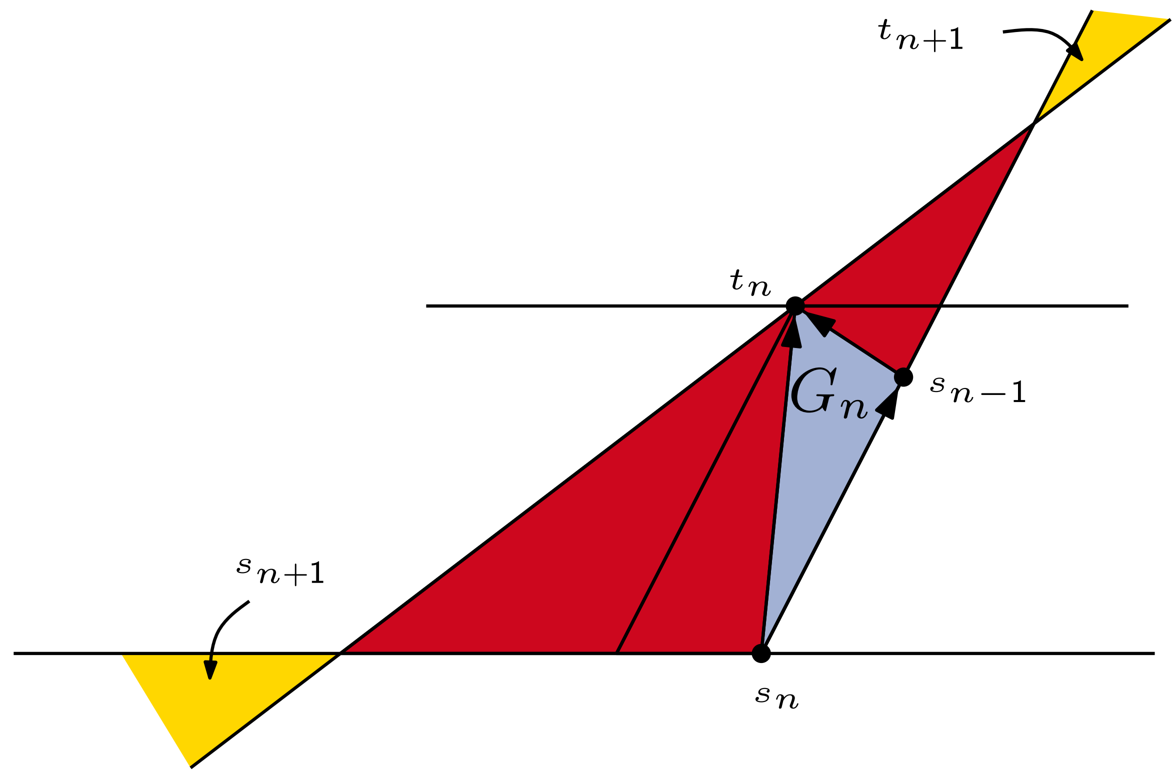


G_{n+1}



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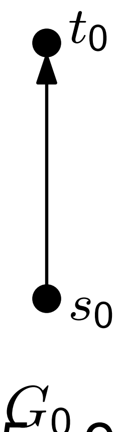

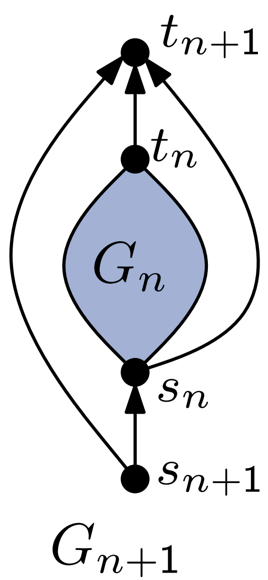

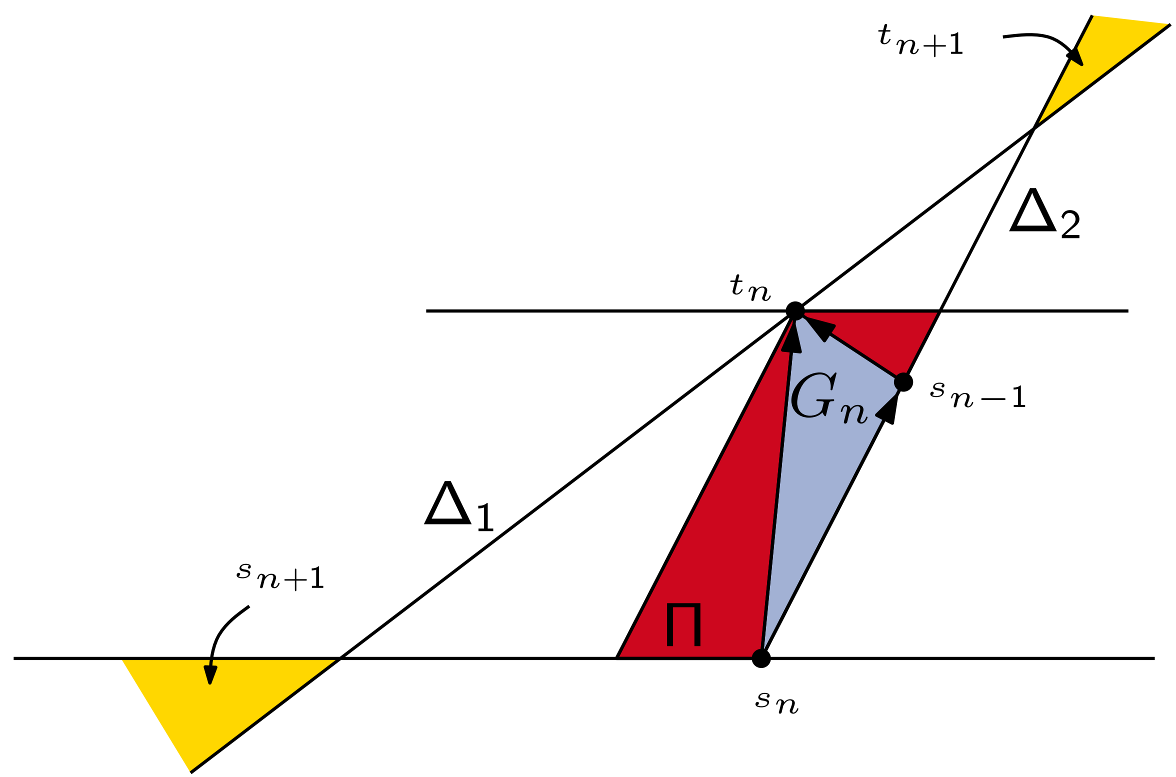

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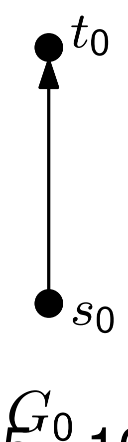
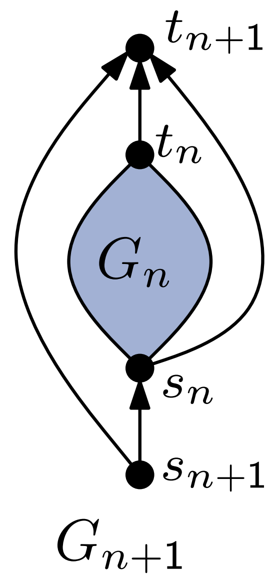
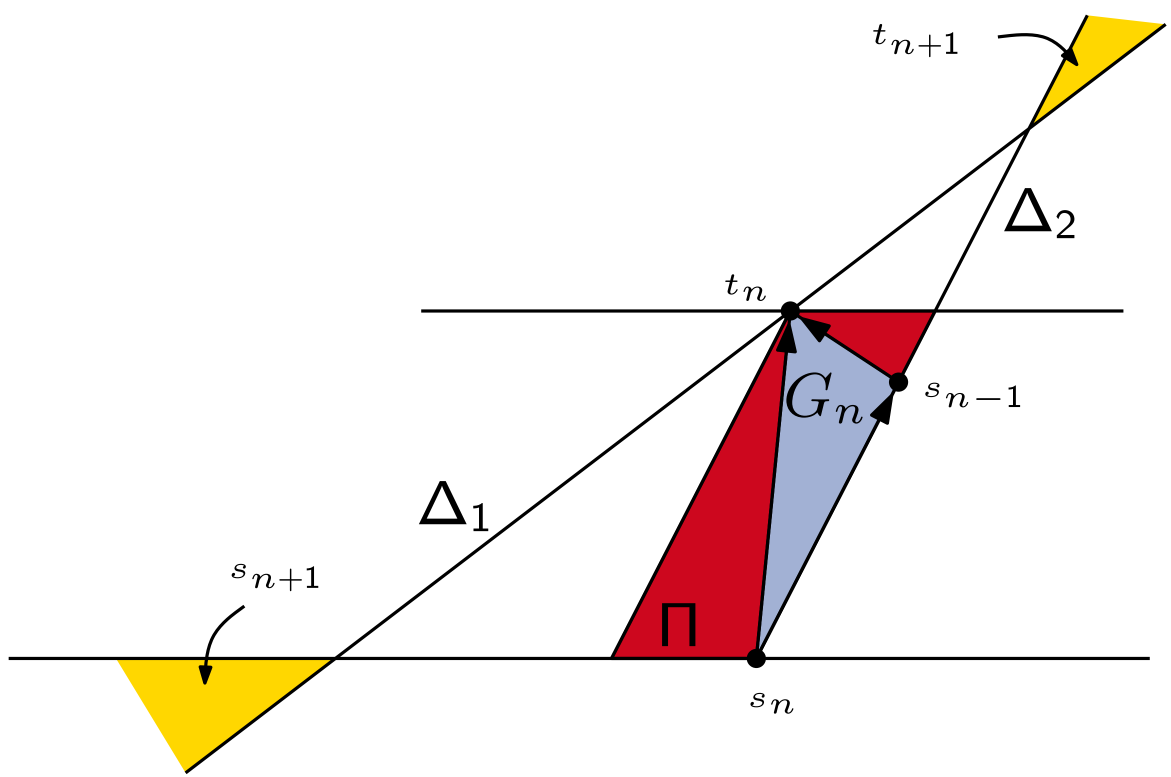
- We have that: $Area(\Pi) > 2 \cdot Area(G_n)$







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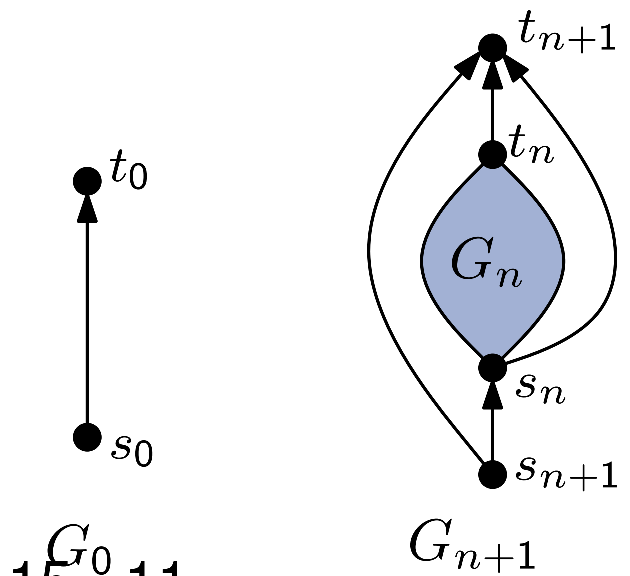

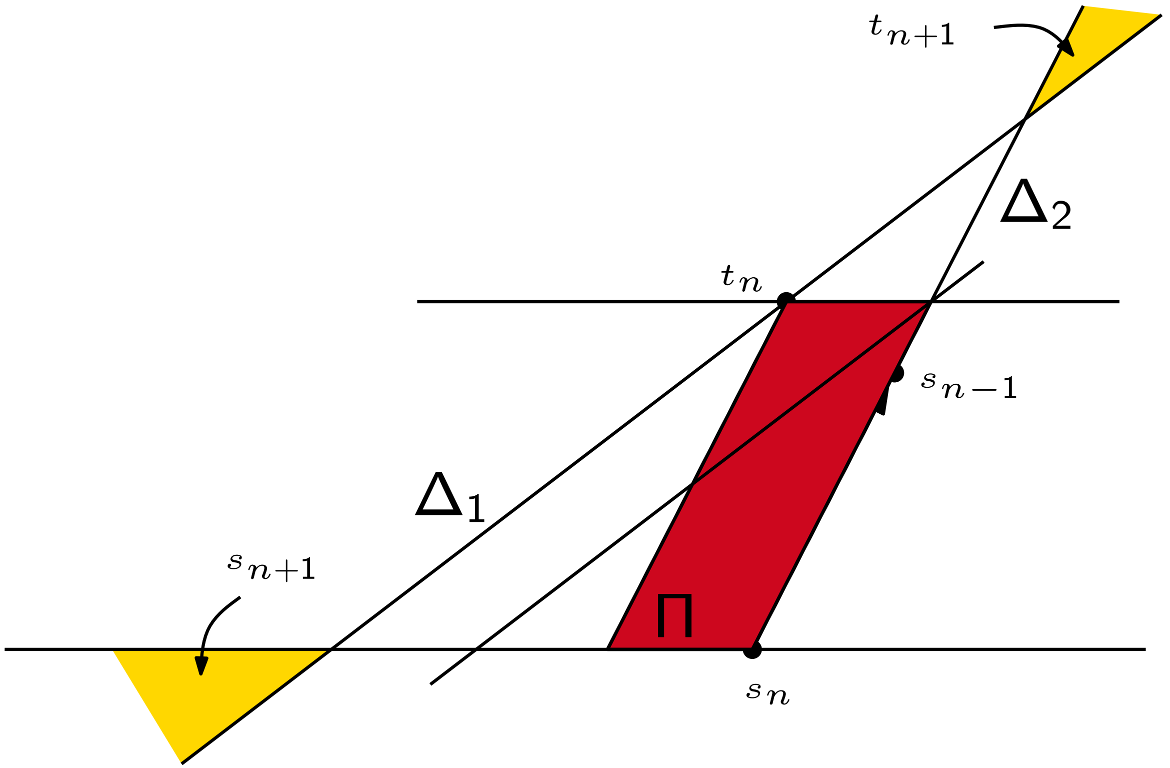

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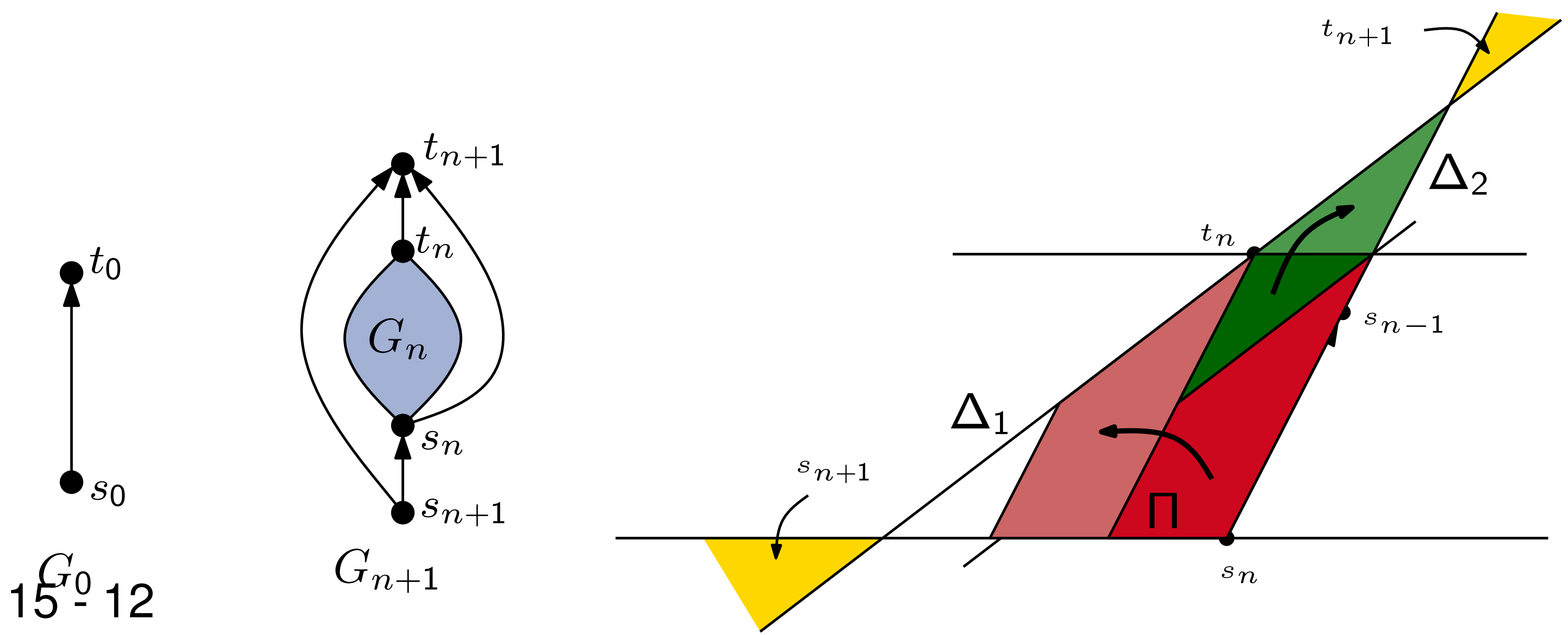

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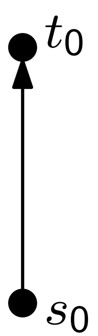

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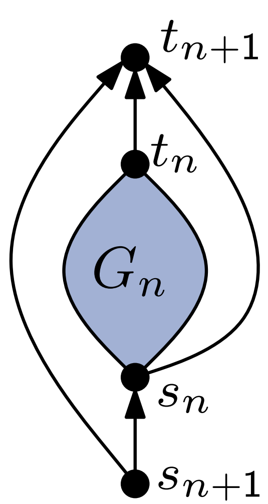

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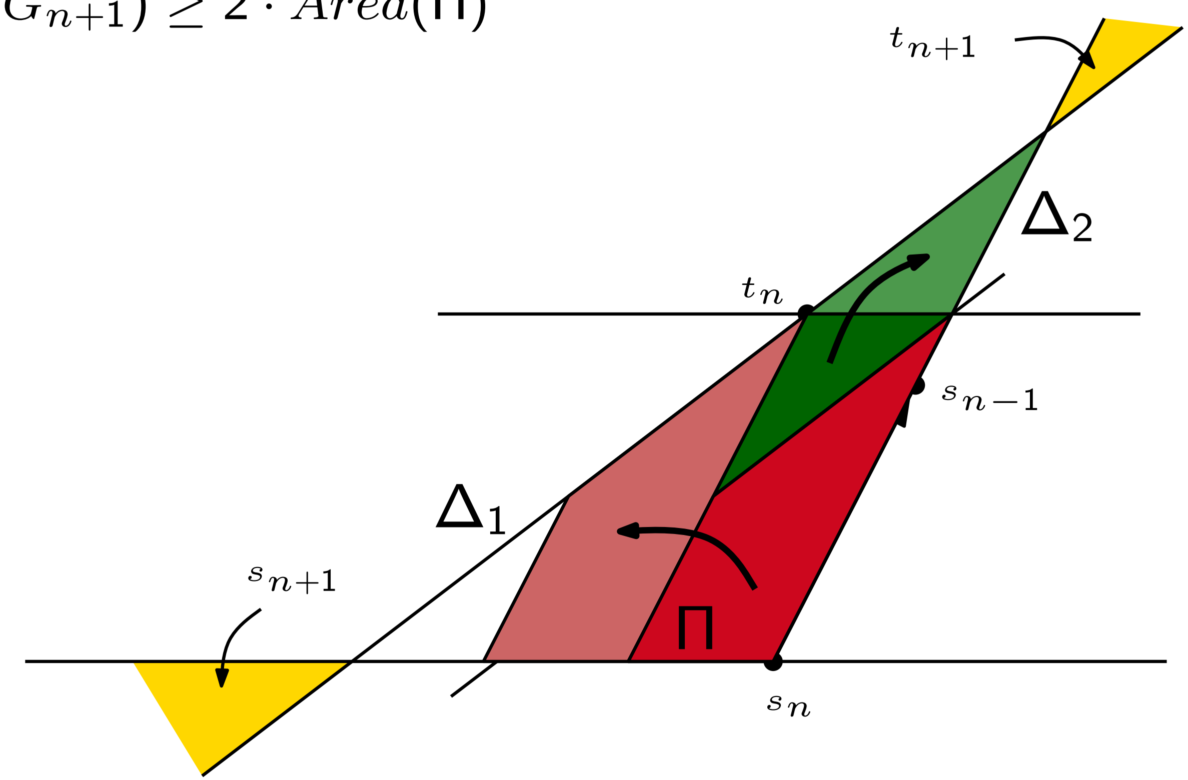

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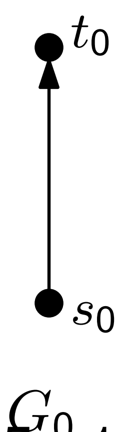

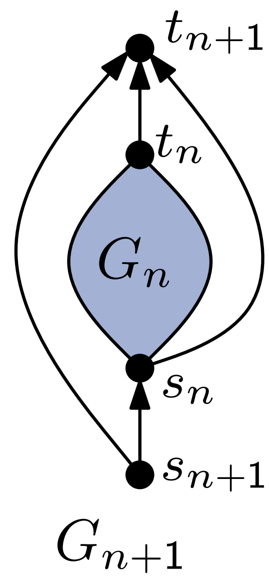

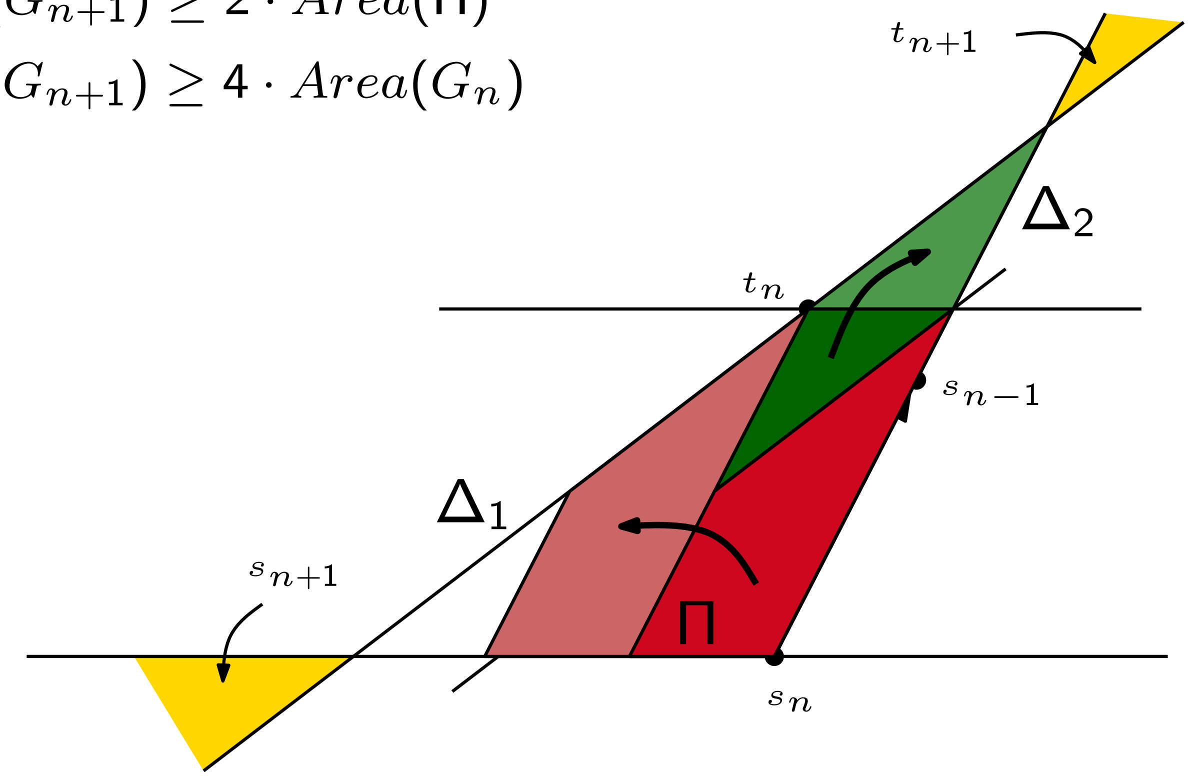

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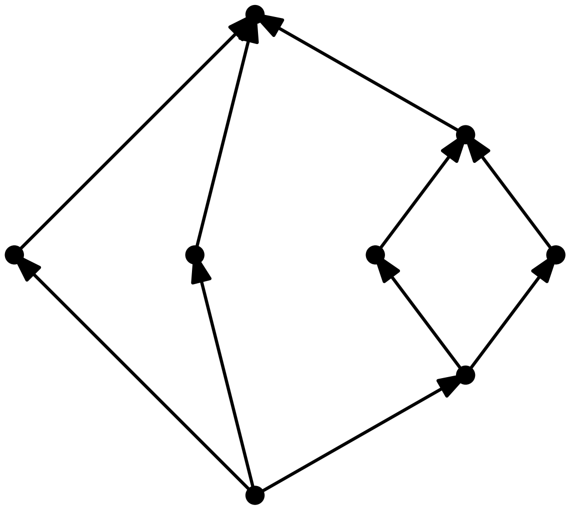

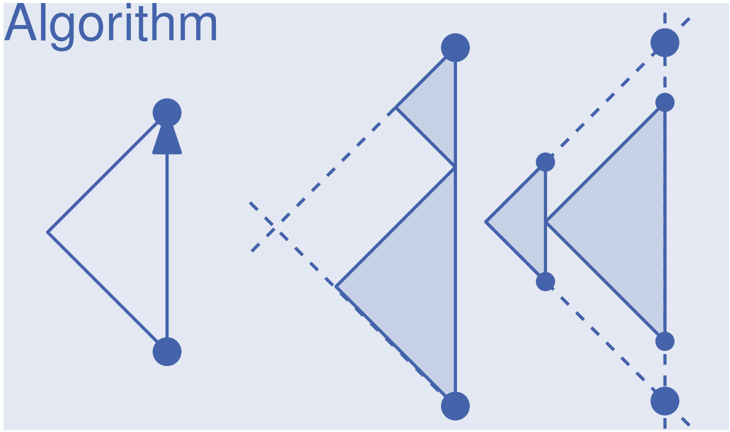

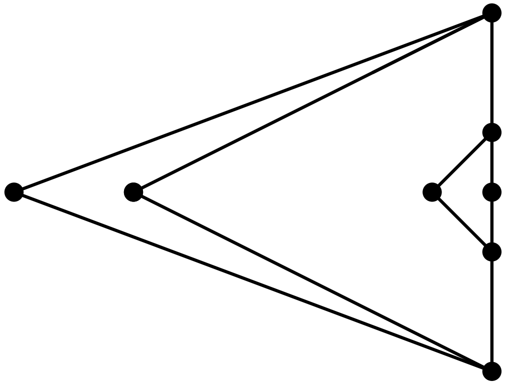

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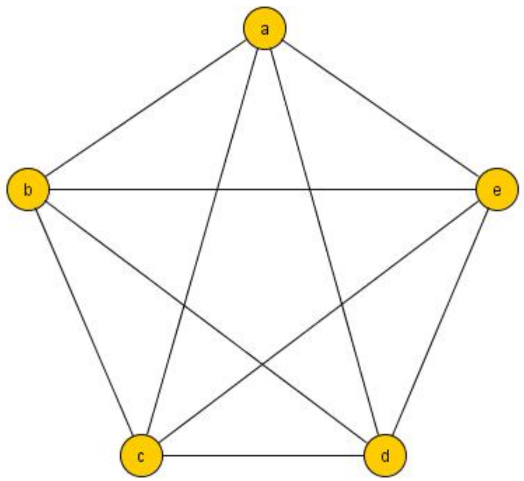
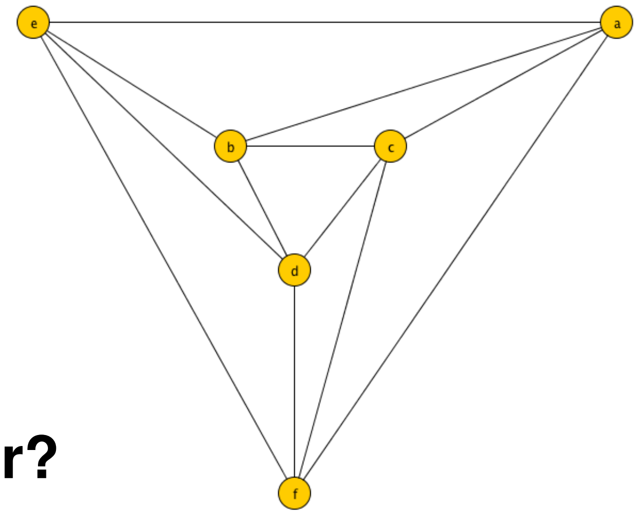
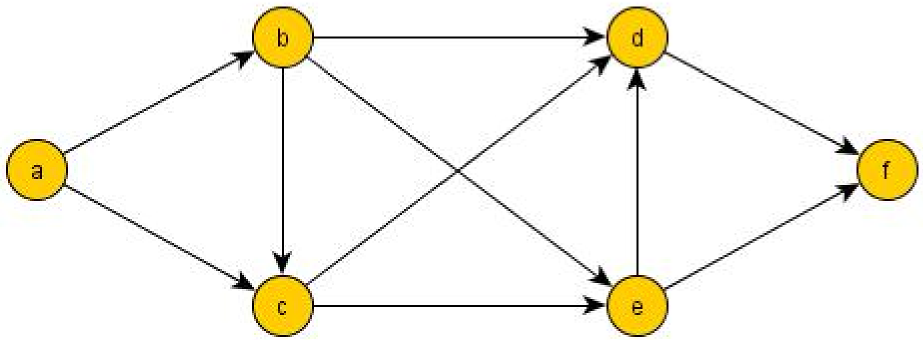
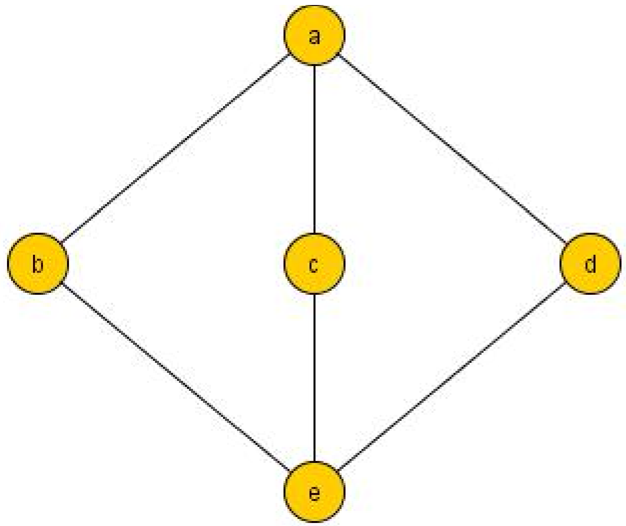




Do You Remember?





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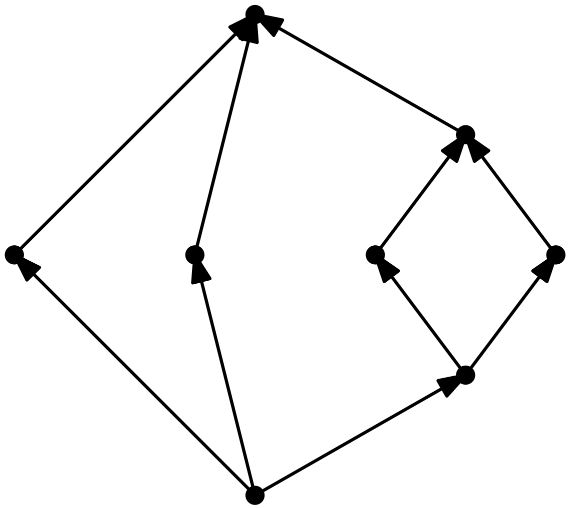

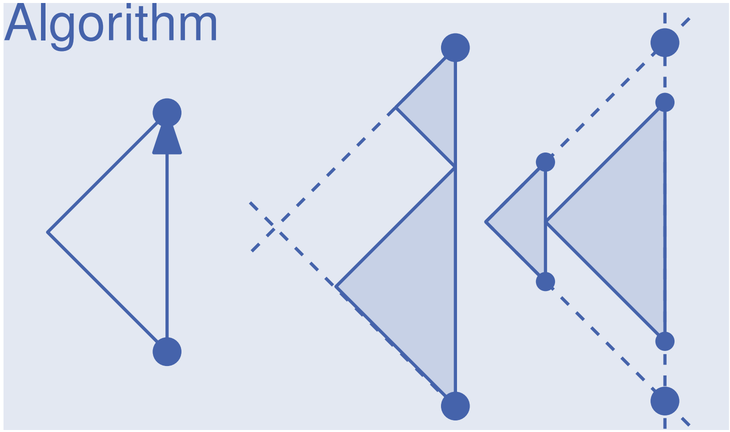

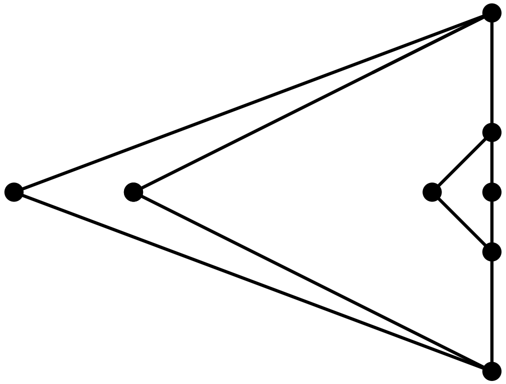

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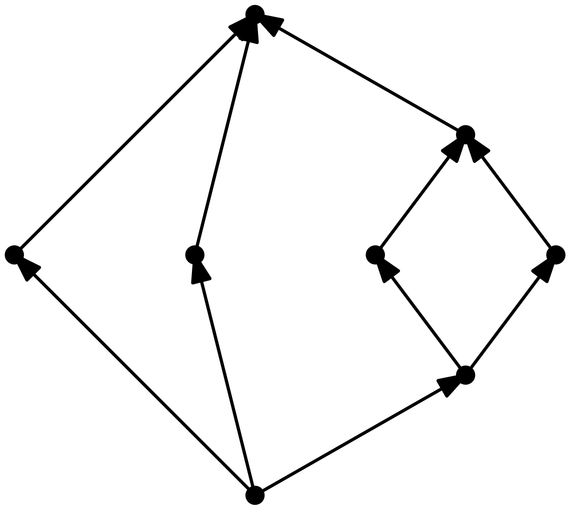

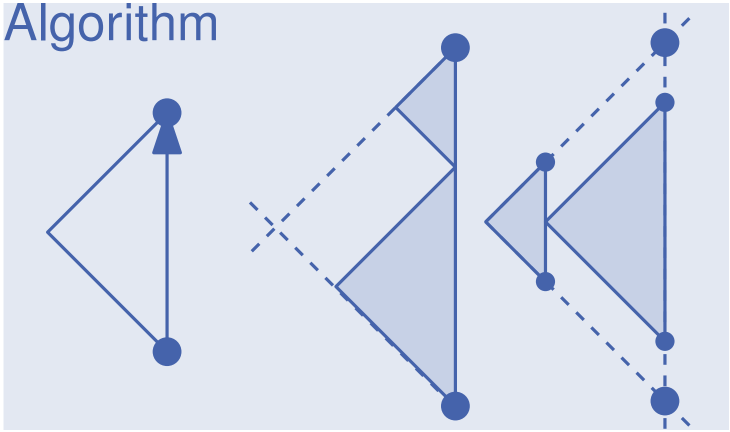

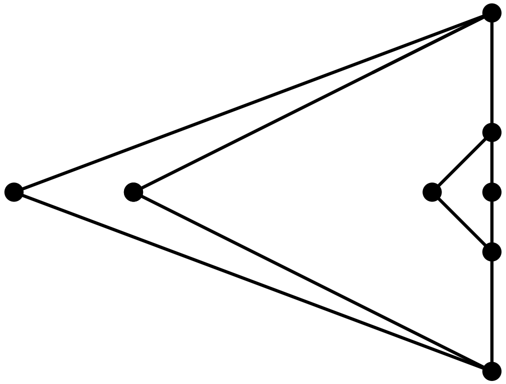

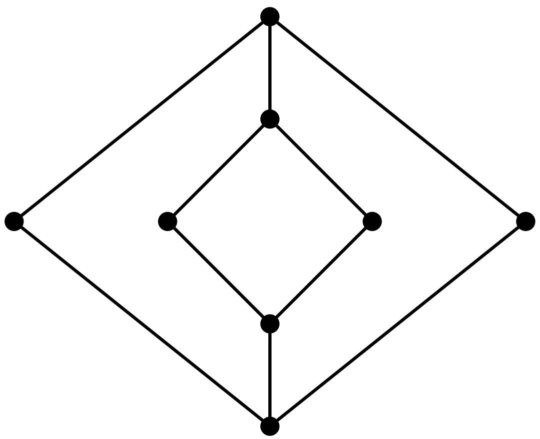

nicer???

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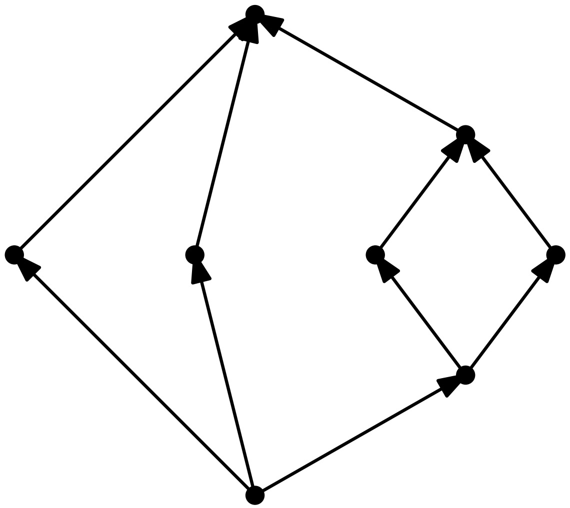

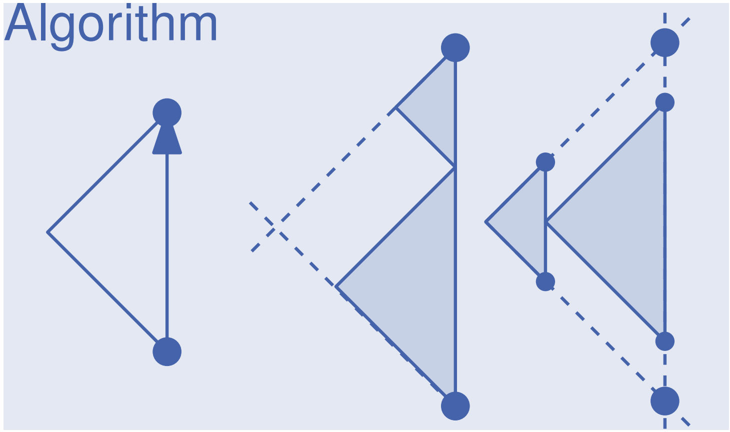

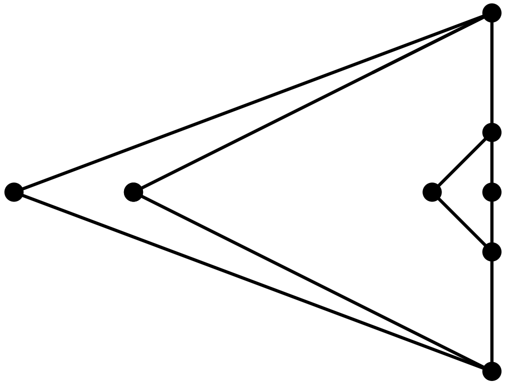

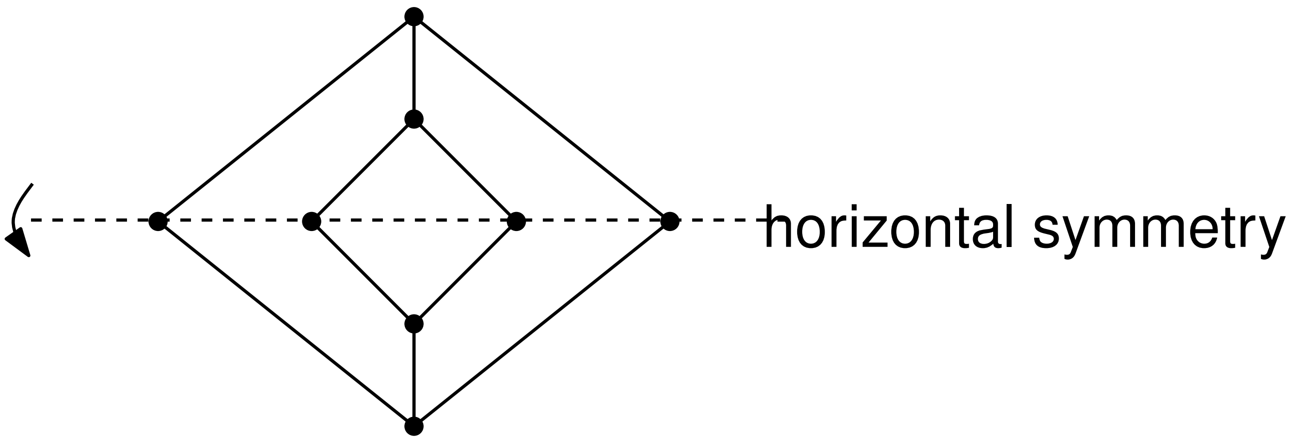

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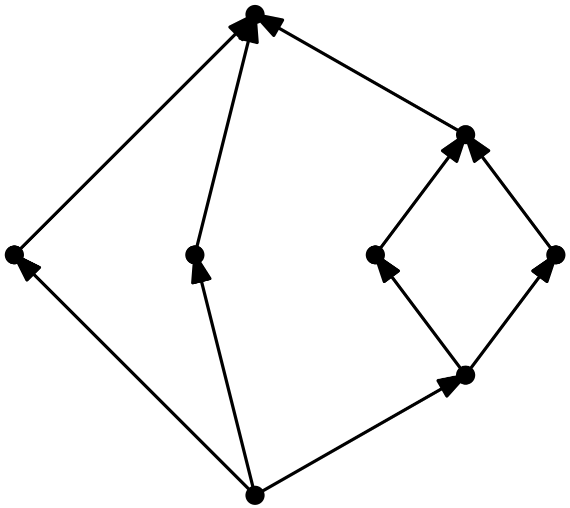

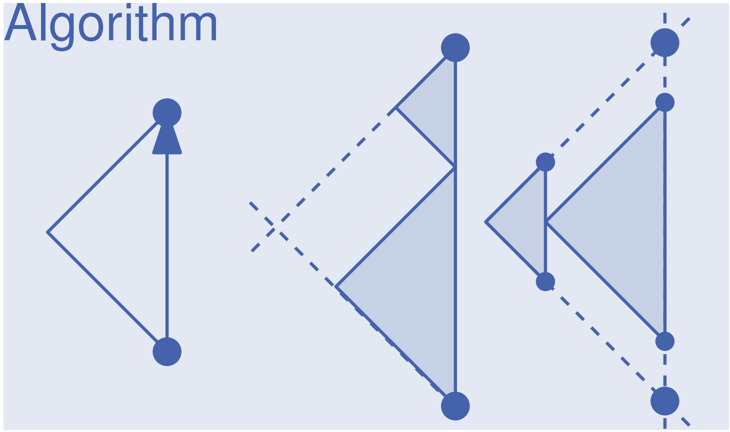

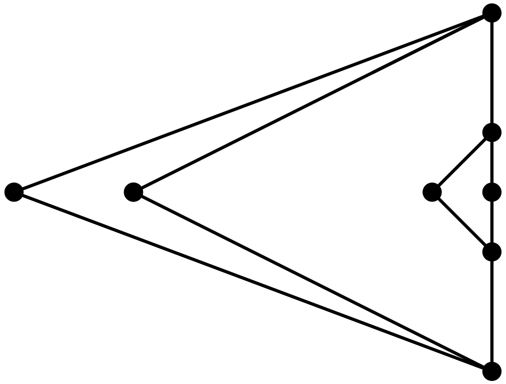

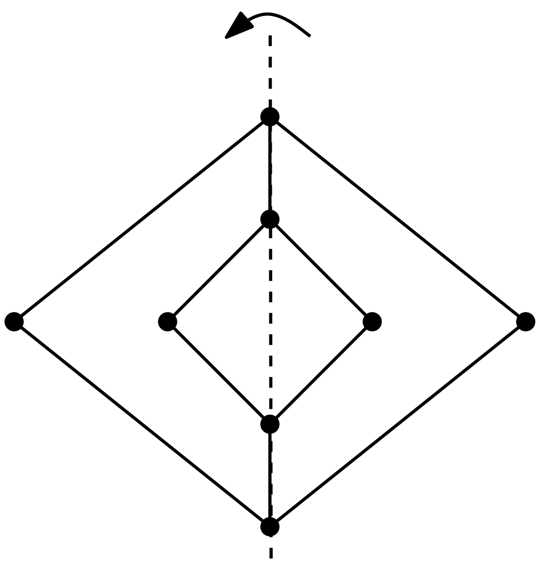

vertical symmetry

nicer???

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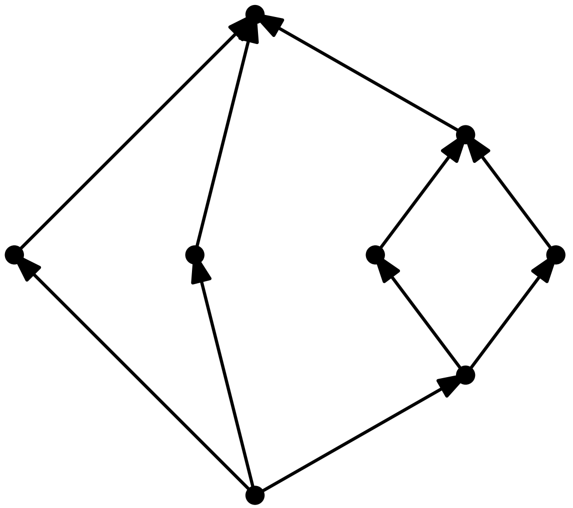

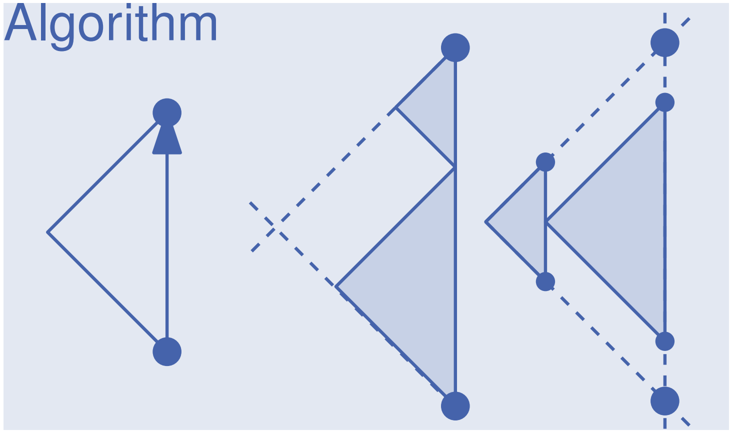

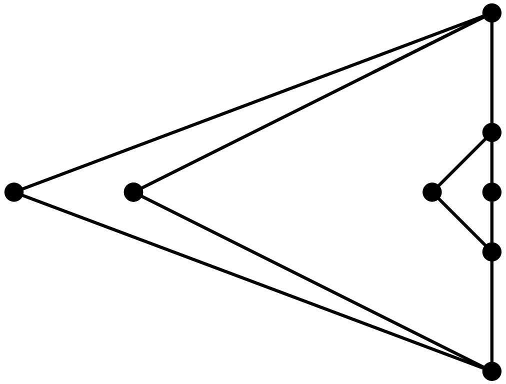

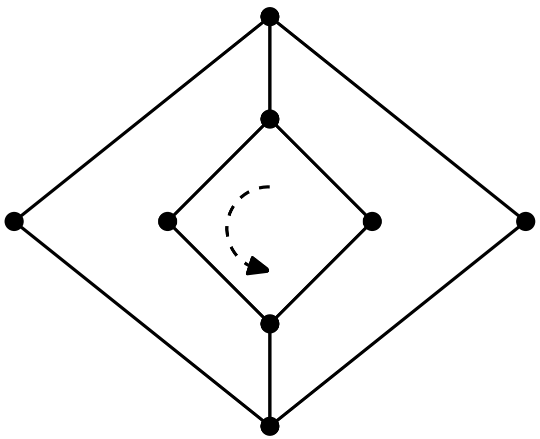

rotational symmetry

nicer???

17 - 7

Graph Automorphism

Definition: Automorphism of a digraph

An **automorphism** of a directed graph $G = (V, E)$ is a permutation of the vertex set which preserves adjacency of the vertices and either preserves or reverses all the directions of the edges:

- $(u, v) \in E \Leftrightarrow (\pi(u), \pi(v)) \in E$, or
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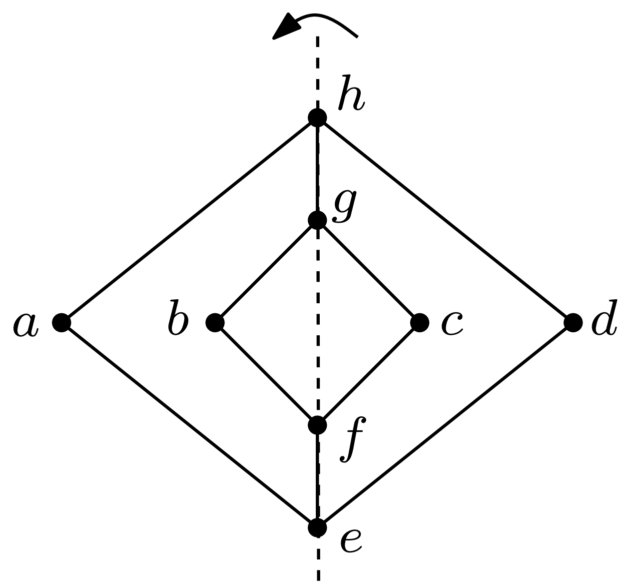
Example of automorphism is a permutation of vertex set a, b, c, d, e, f, h, g defined by the rules $b \to c \to b$, $a \to d \to a$, so d, c, b, a, e, f, h, g



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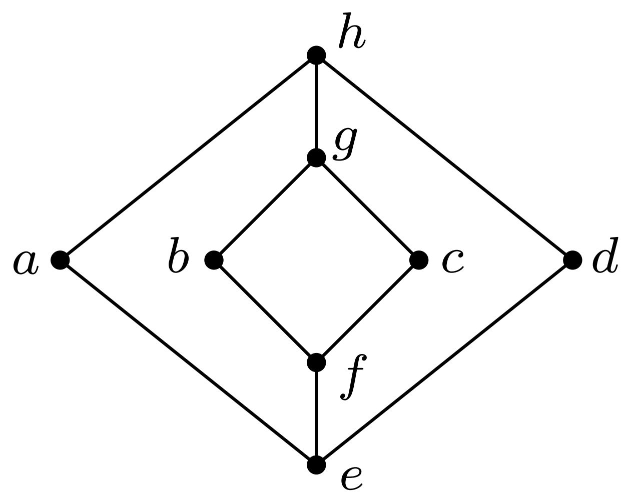

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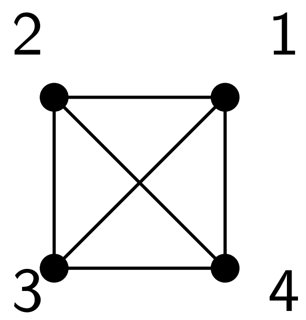
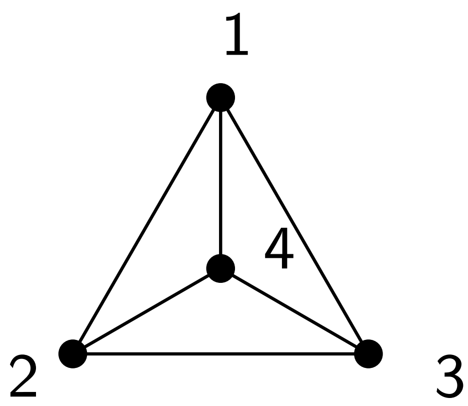
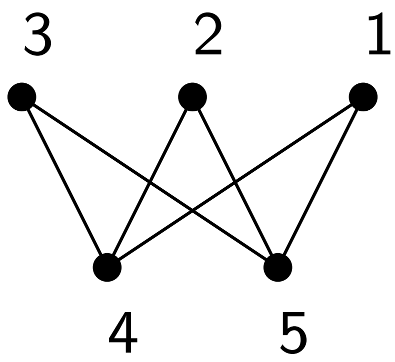
- Finding an automorphism group of a graph is *isomorphism complete*, that is equivalent to testing whether two graphs are isomorphic.
- For planar graphs, graphs with bounded degree isomorphism problem has polynomial-time algorithms (for more see citations in [HEL00]).

Geometric Automorphism

- Geometric realizability of automorphisms:







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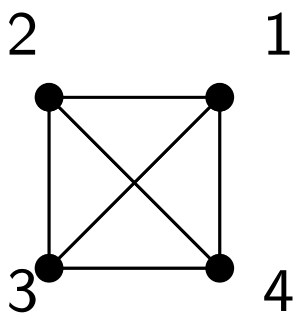

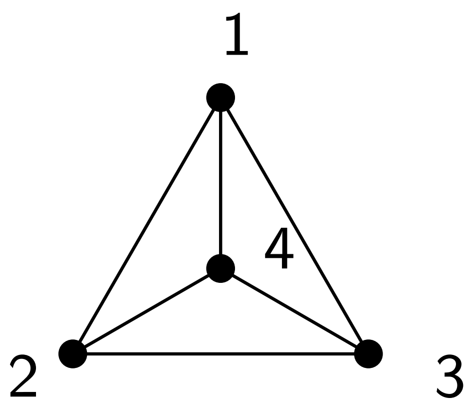

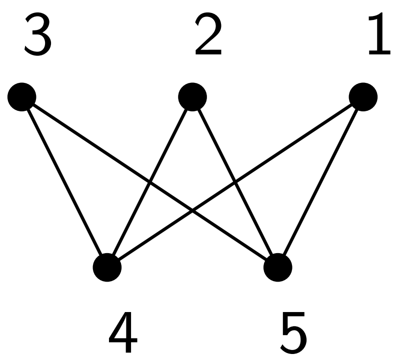

Pick one of the graphs above and try to list all its automorphisms. Which of them are represented as symmetries?

2 min

Geometric Automorphism

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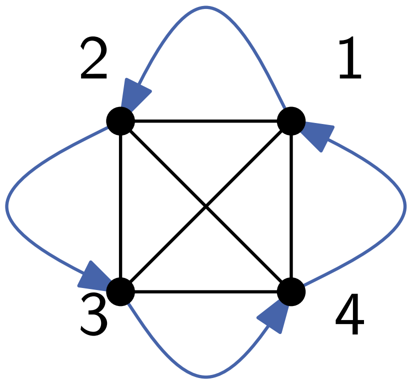
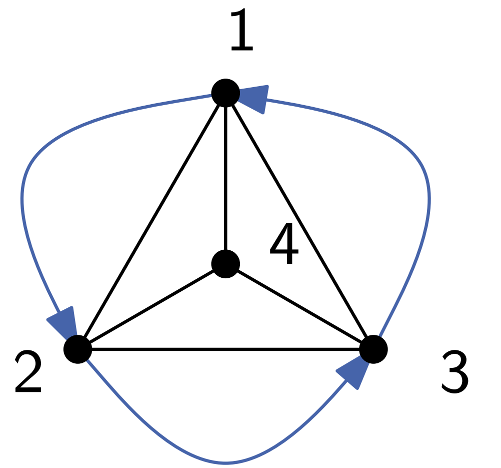


This drawing displays the automorphism $1 \to 2 \to 3 \to 4 \to 1$ as rotational symmetry. But does not shows the $1 \to 2 \to 3 \to 1$



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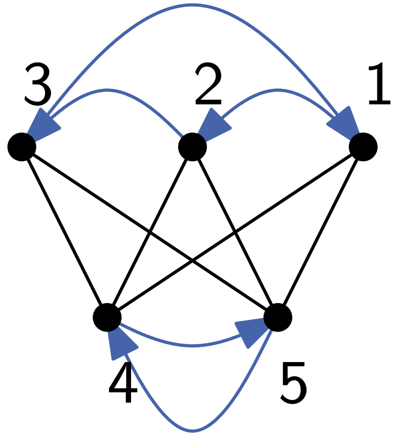

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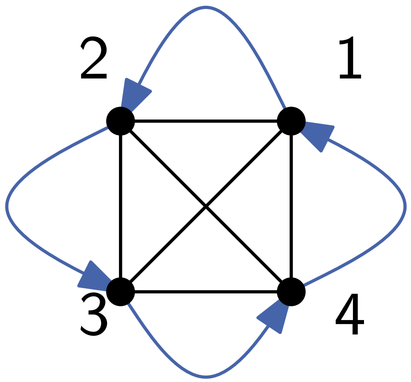

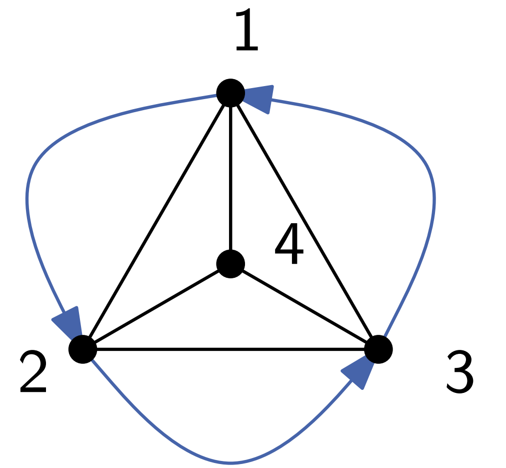

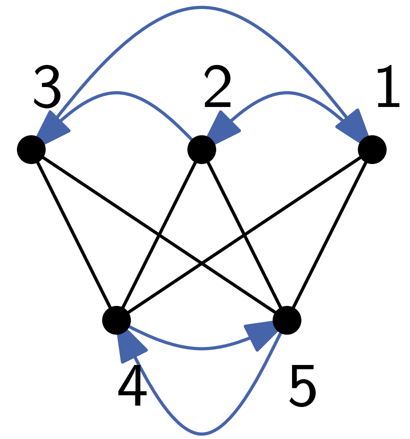

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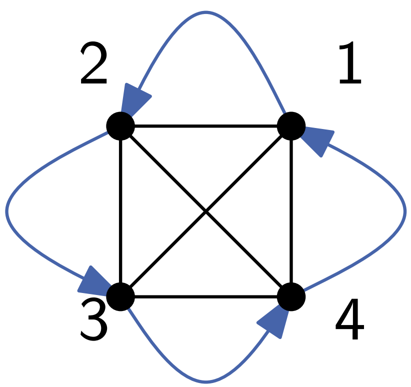

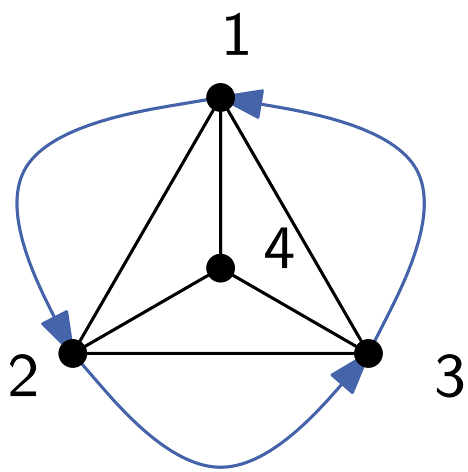

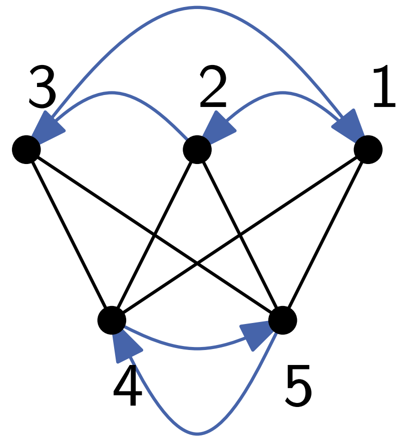

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Geometric Automorphism

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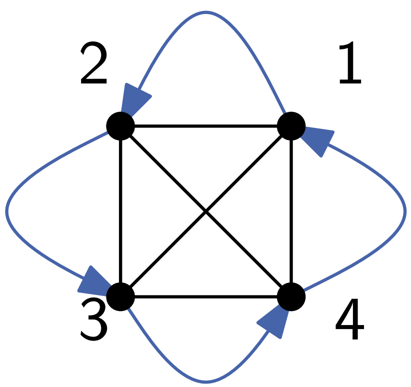

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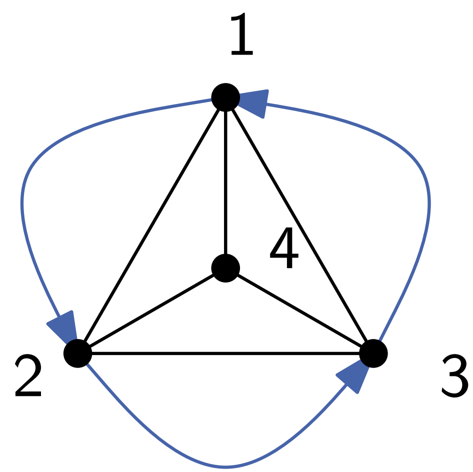

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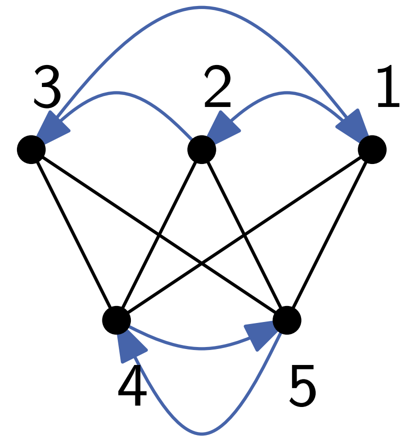
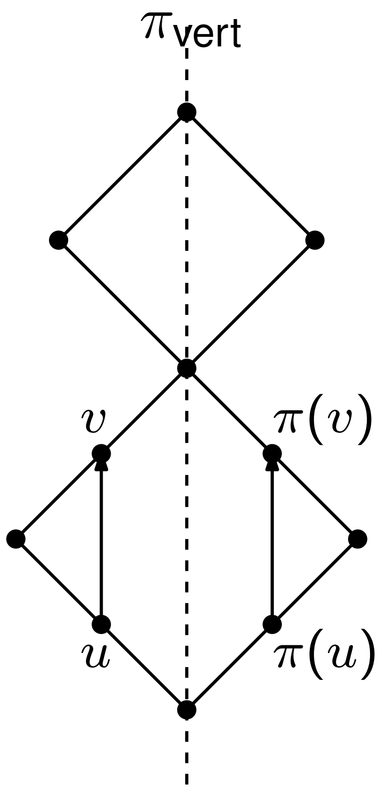


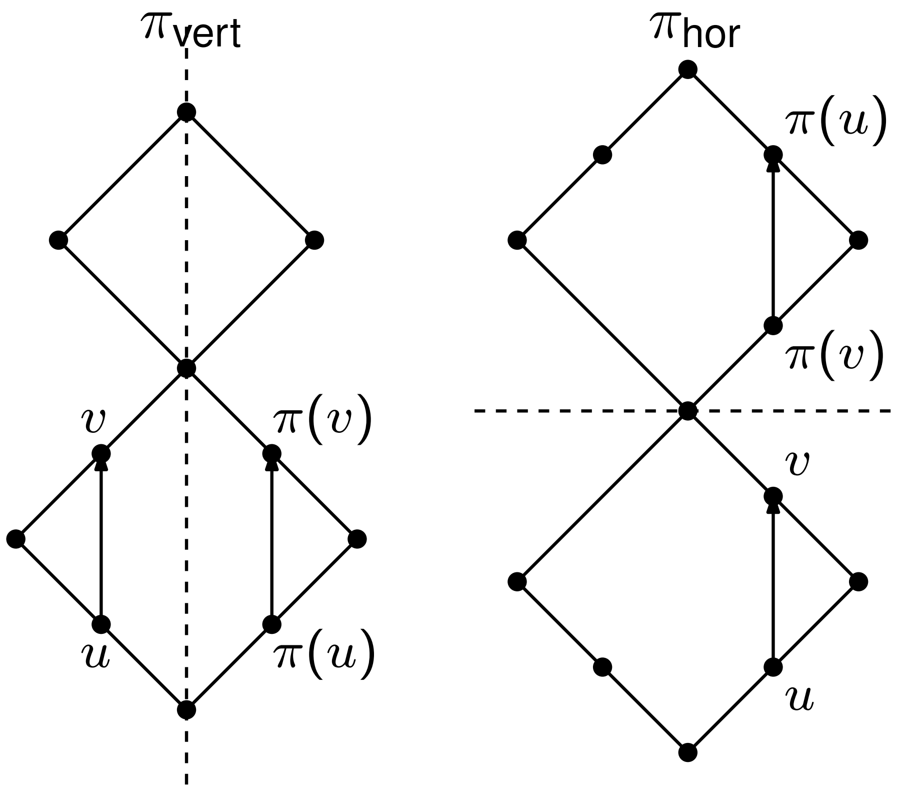
Automorphism $1 \to 2 \to 3 \to 1$, $4 \to 5 \to 4$ is not geometrically representable. But $1 \to 3 \to 1$, $4 \to 5 \to 4$ is representable as vertical symmetry.

- An automorphism group P of a graph is geometric, if there exists a drawing of G that displays each element of P as a symmetry.
- For general graphs it is $\mathcal{NP}$-hard to find a geometric automorphism of a graph.
- For planar graphs, planar geometric automorphisms can be found in polynomial time. For outerplanar graphs and trees in linear time.

Symmetries in SP-Graphs



Symmetries in SP-Graphs



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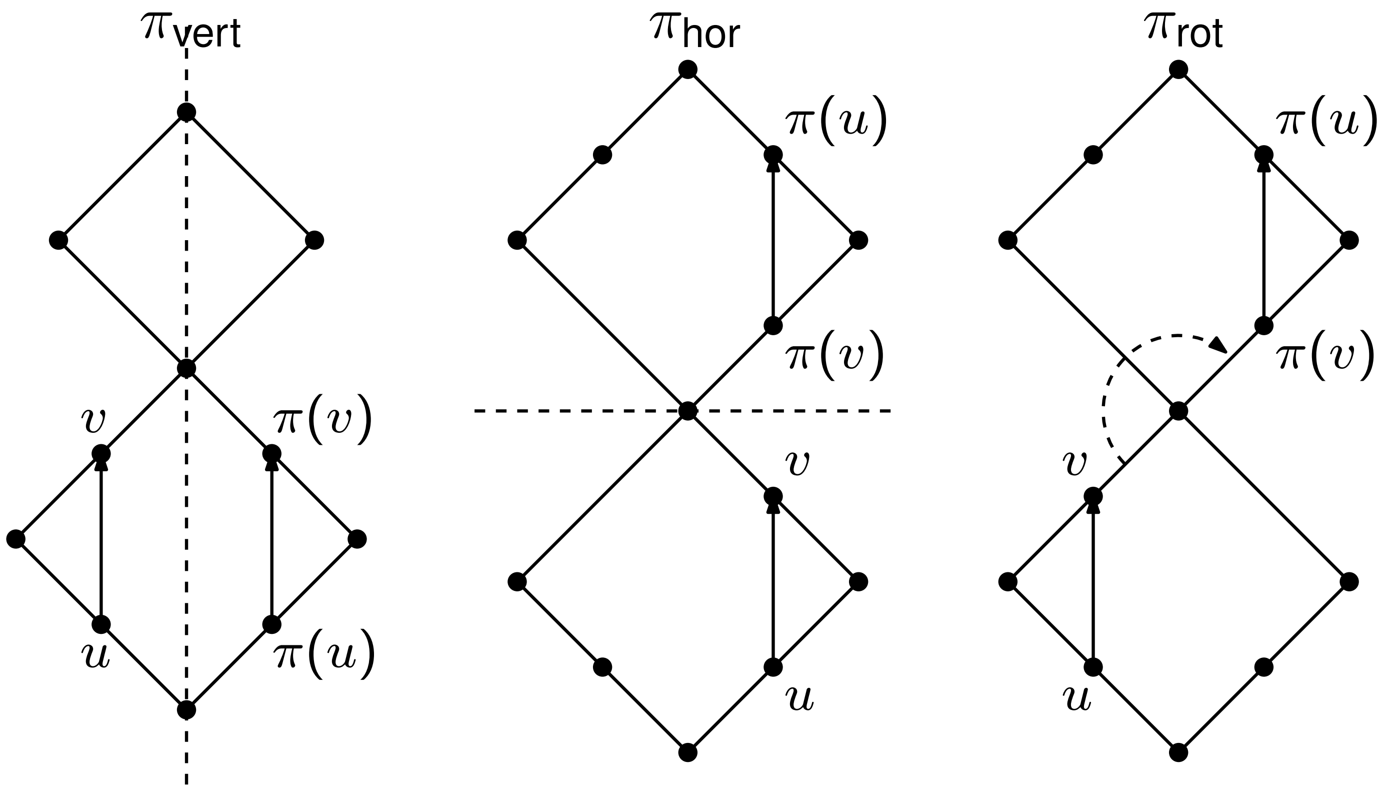

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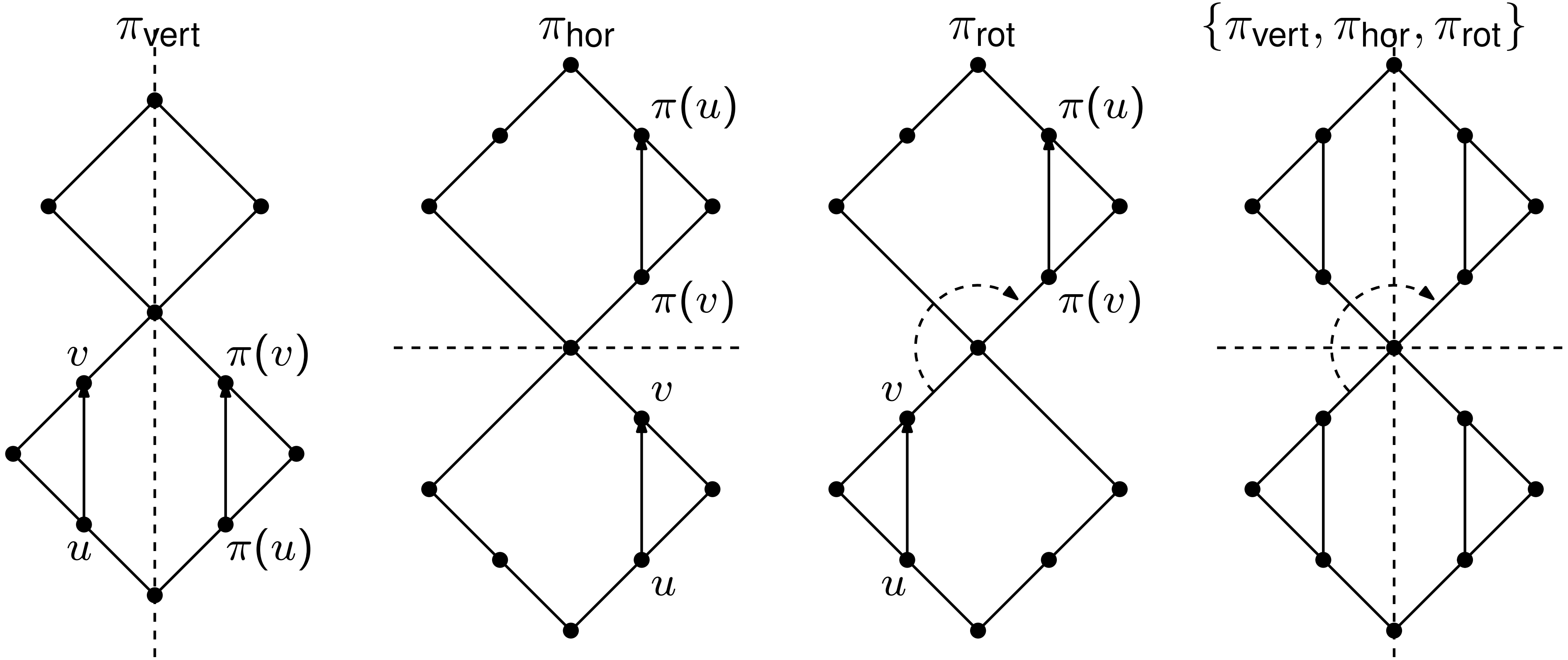

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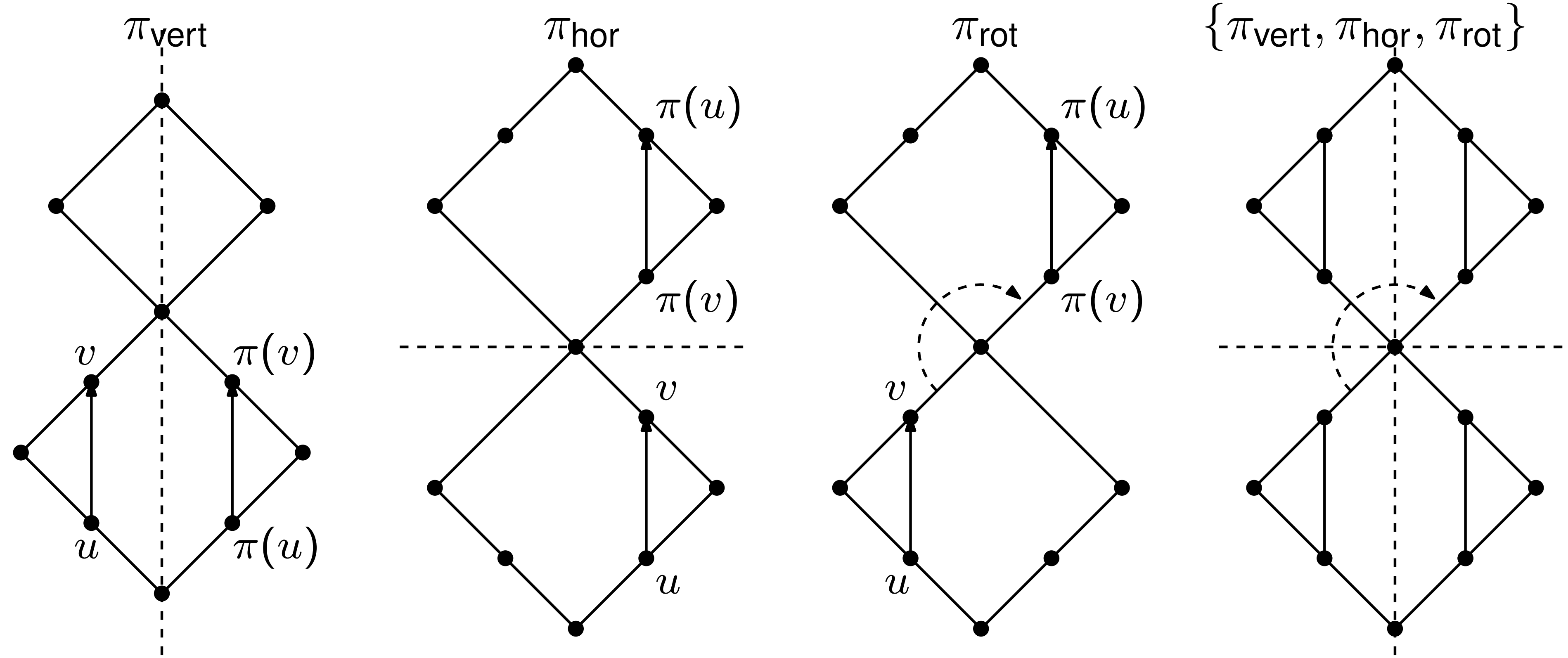
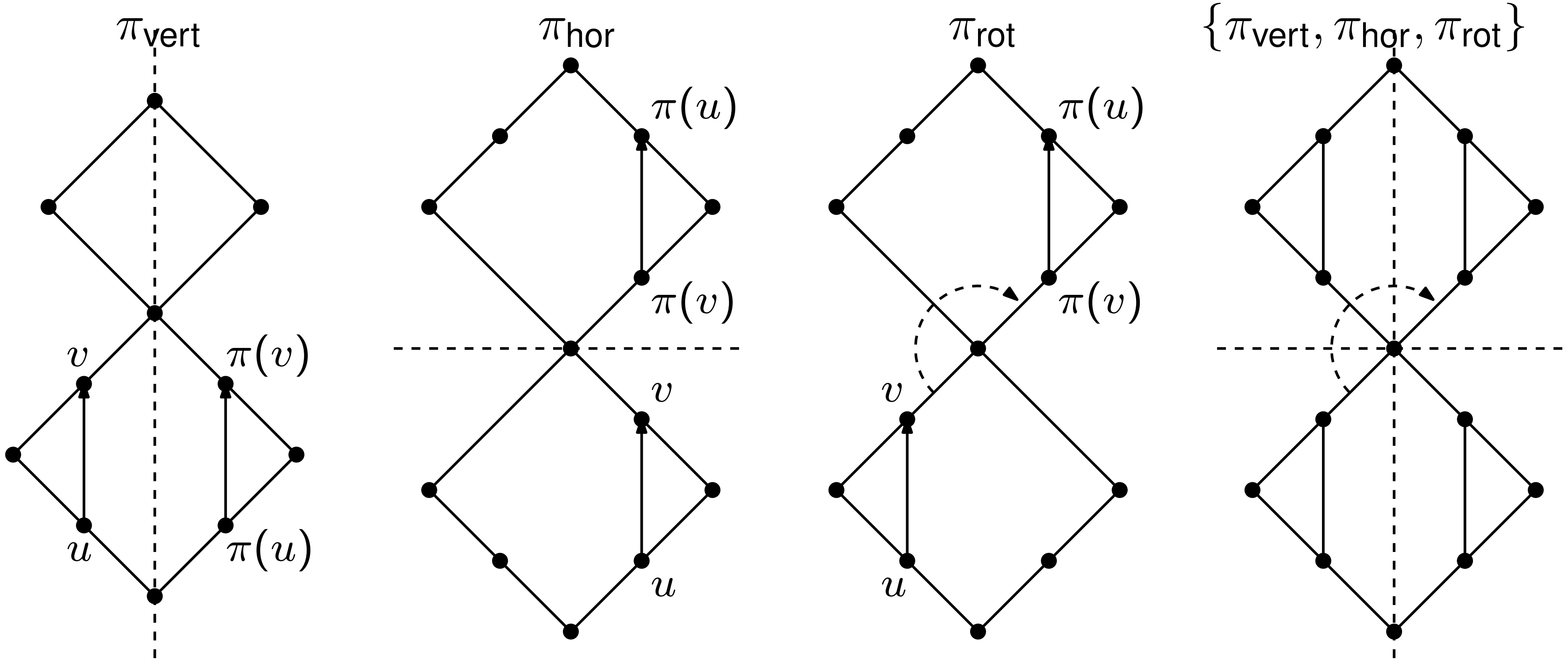


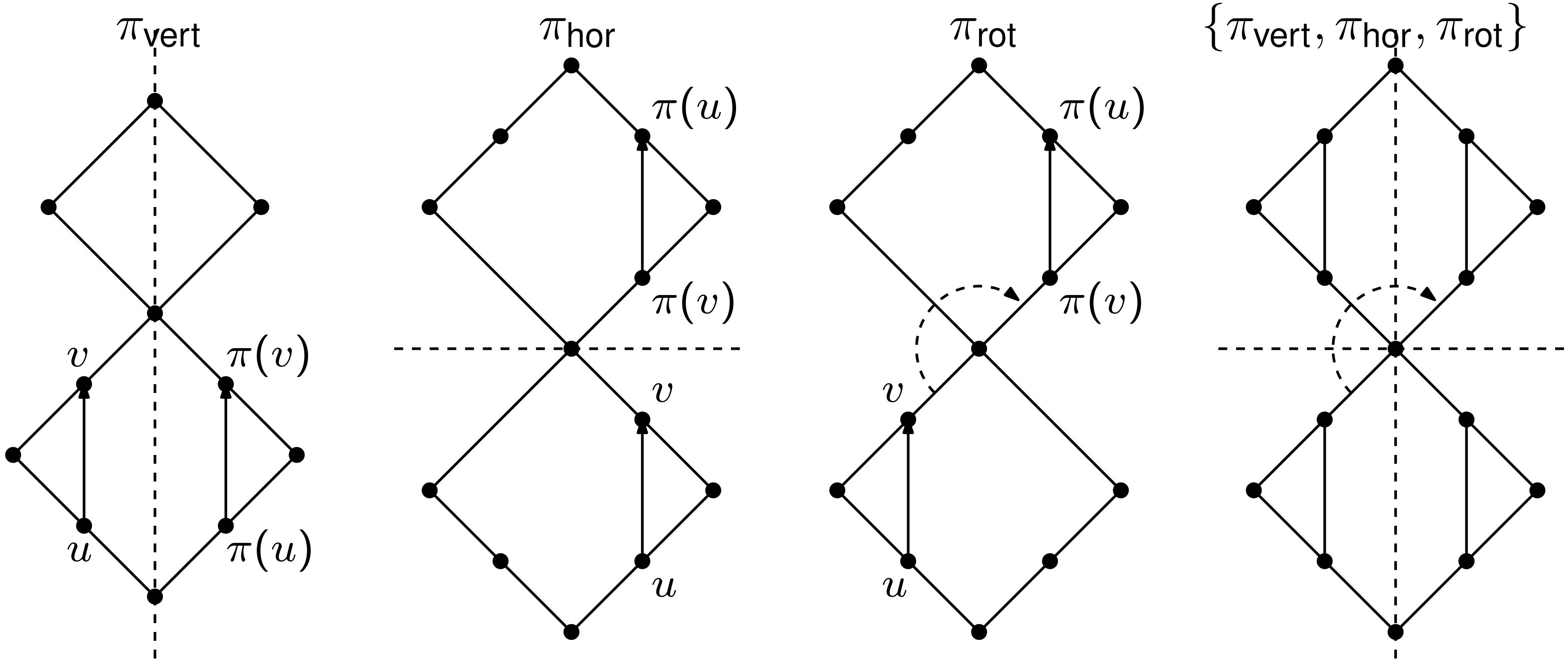
- A geometric automorphism group P of a graph G is upward planar, if there exists an upward planar drawing of G that displays each element of P as a symmetry.

Symmetries in SP-Graphs



- A geometric automorphism group P of a graph G is upward planar, if there exists an upward planar drawing of G that displays each element of P as a symmetry.
- How does a geometric automorphism group for a series-parallel graph look like?

Symmetries in SP-Graphs

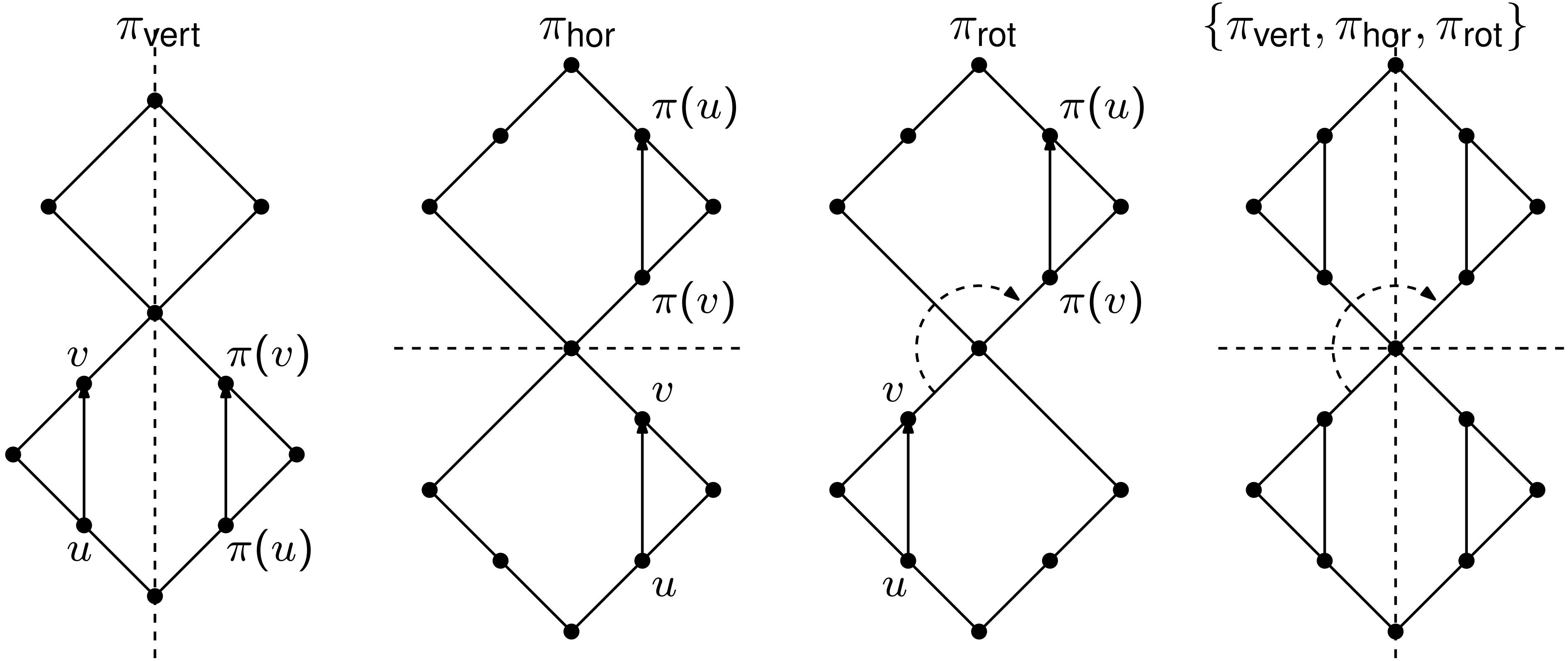


Theorem (Hong, Eades, Lee ’00) [HEL00]

An upward planar automorphism group of a series-parallel digraph is either

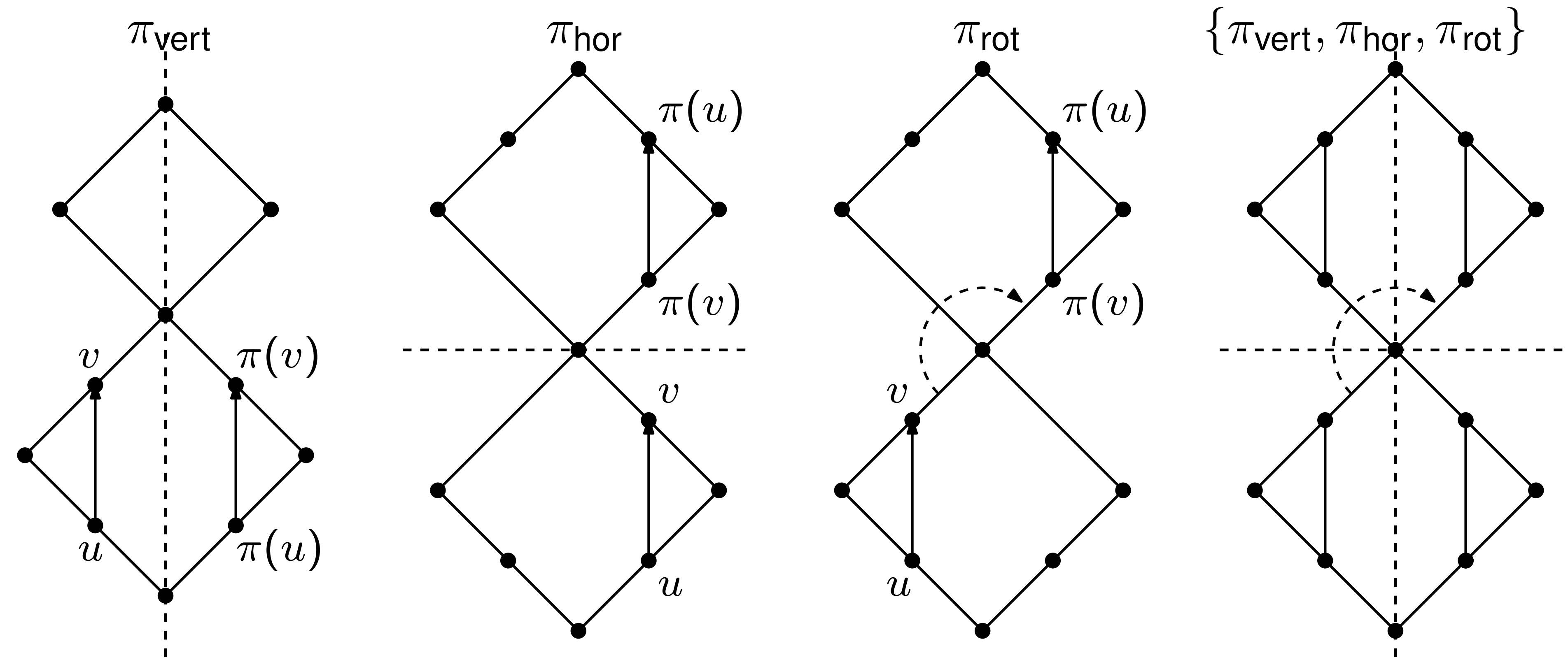
- $\{\text{id}\}$
- $\{\text{id}, \pi\}$ with $\pi \in \{\pi_{\text{vert}}, \pi_{\text{hor}}, \pi_{\text{rot}}\}$
- $\{\text{id}, \pi_{\text{vert}}, \pi_{\text{hor}}, \pi_{\text{rot}}\}$.

Symmetries in SP-Graphs



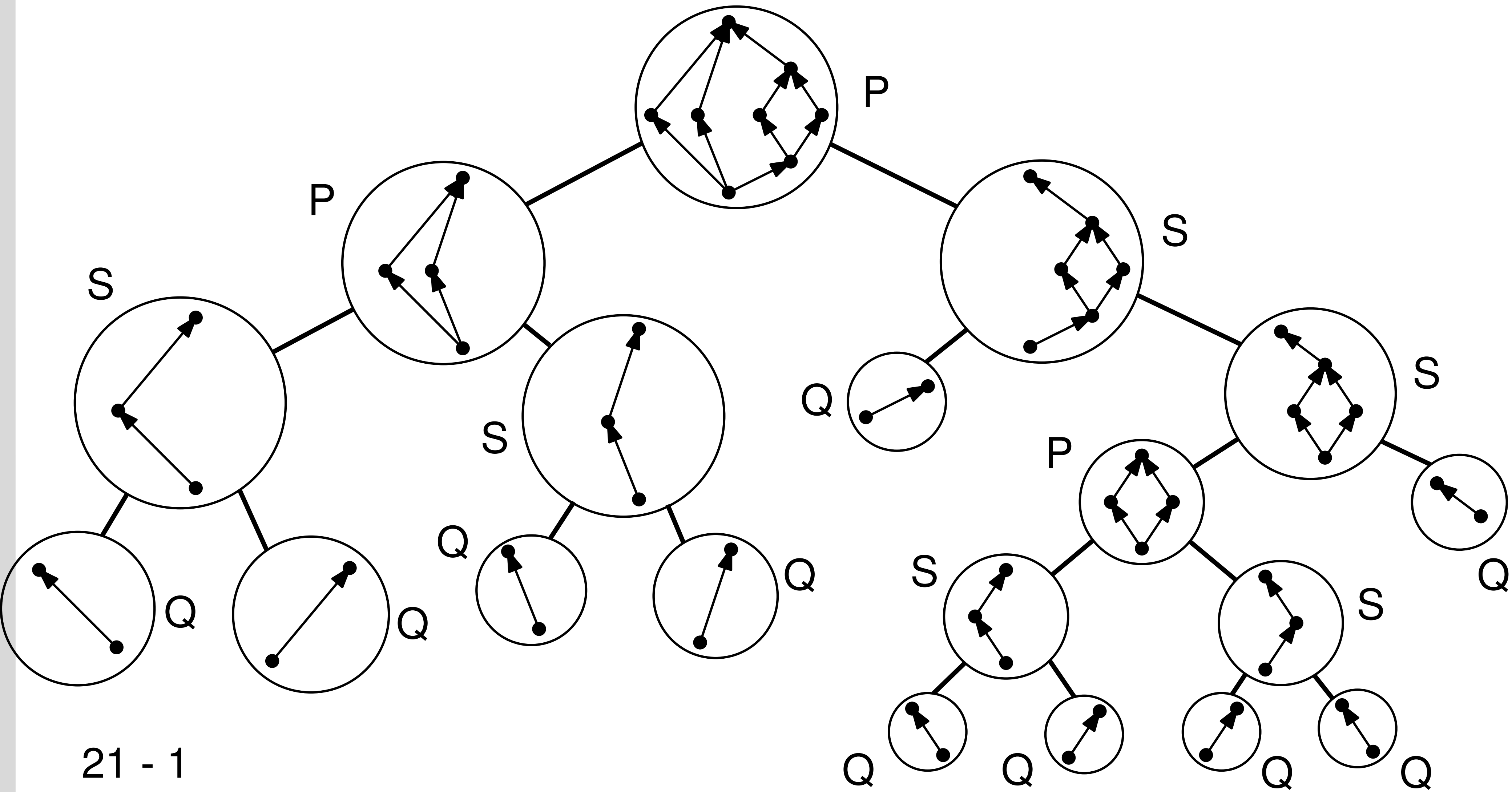
- The automorphism group of maximum size can be found in linear time.

Symmetries in SP-Graphs



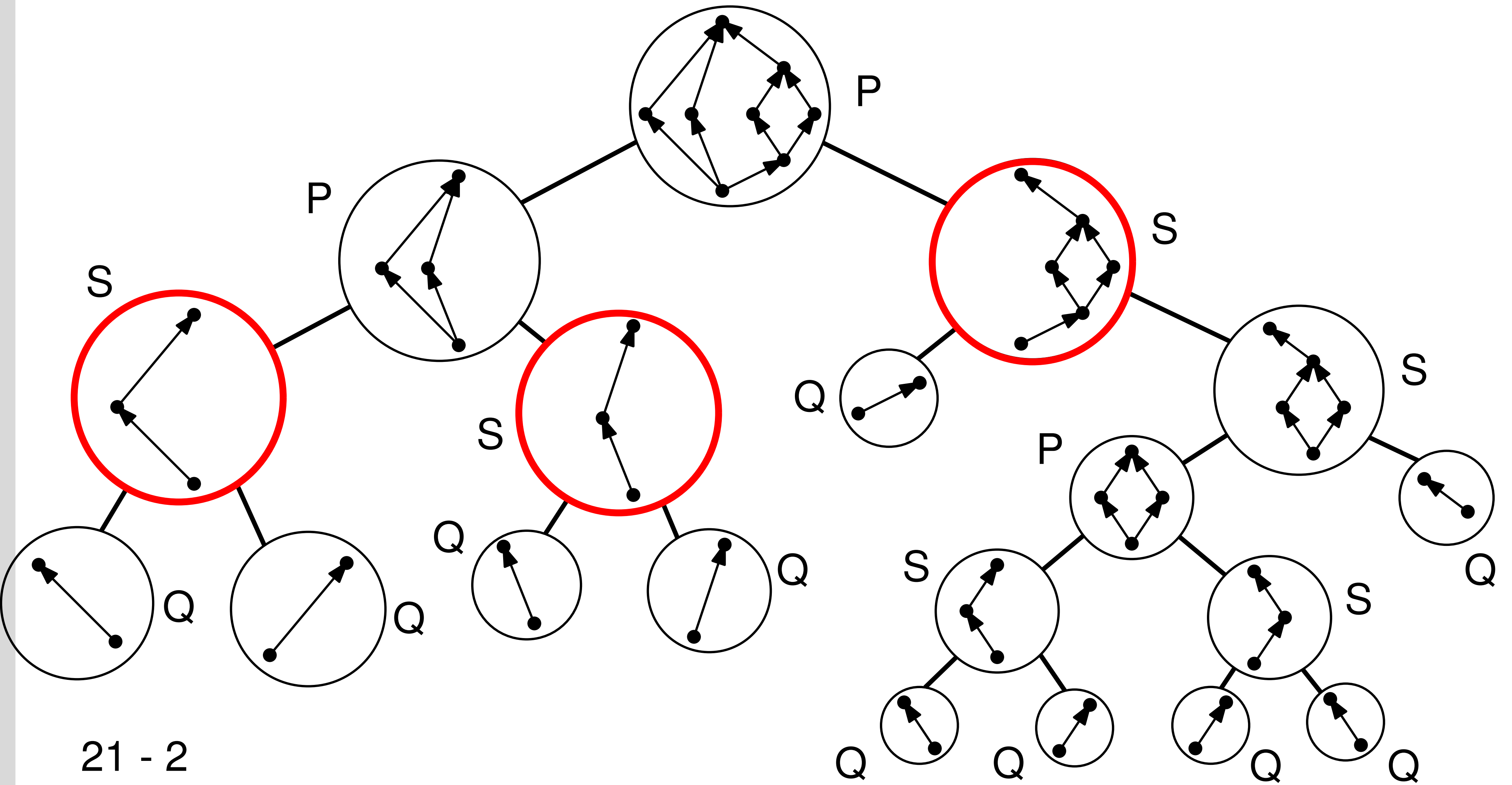
- The automorphism group of maximum size can be found in linear time.
- Given a maximum size automorphism group of a series-parallel graph, a polyline upward planar drawing that displays this automorphism can be constructed in linear time as well.

Vertical Automorphism



21 - 1

Vertical Automorphism



21 - 2

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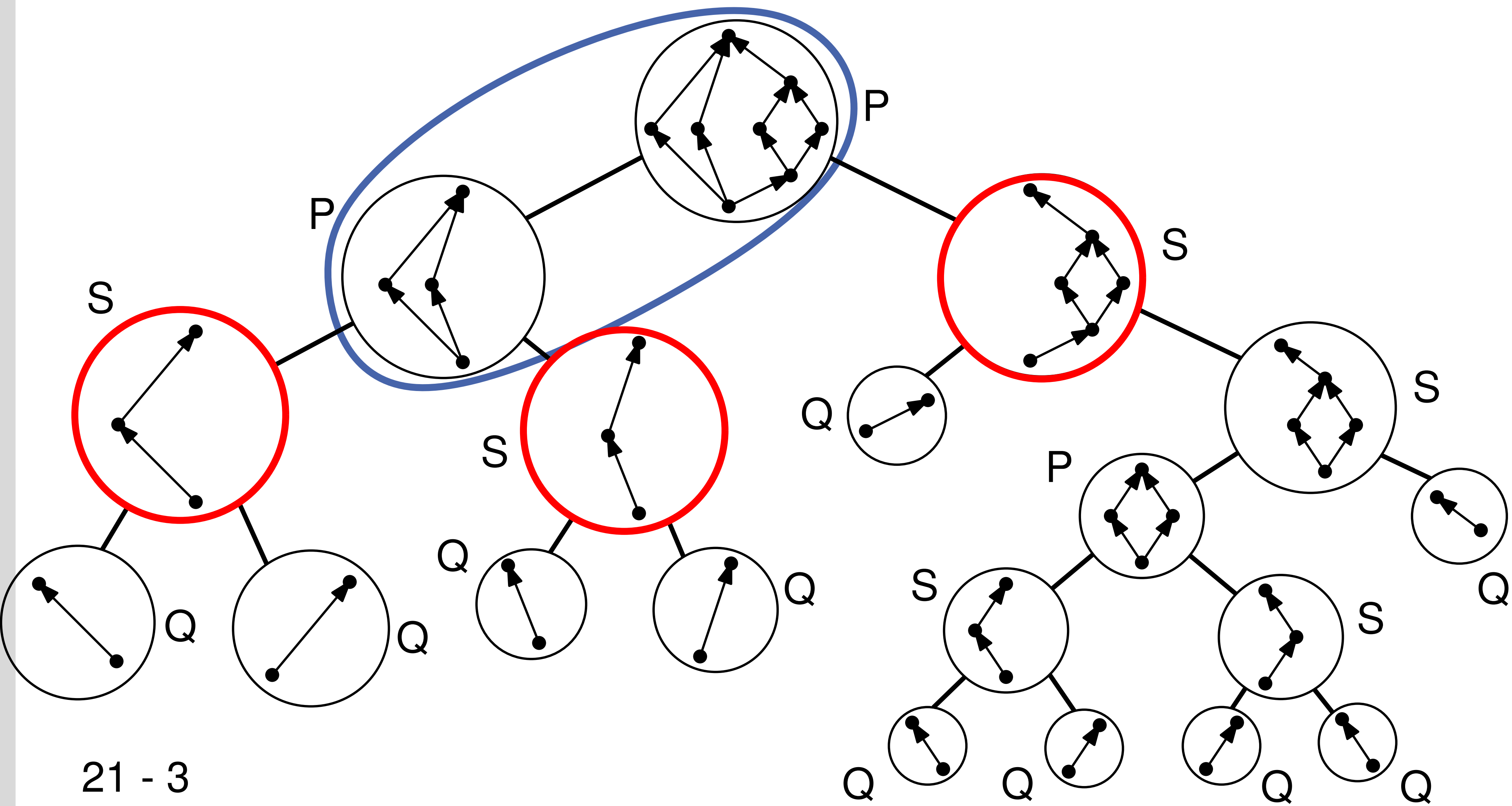
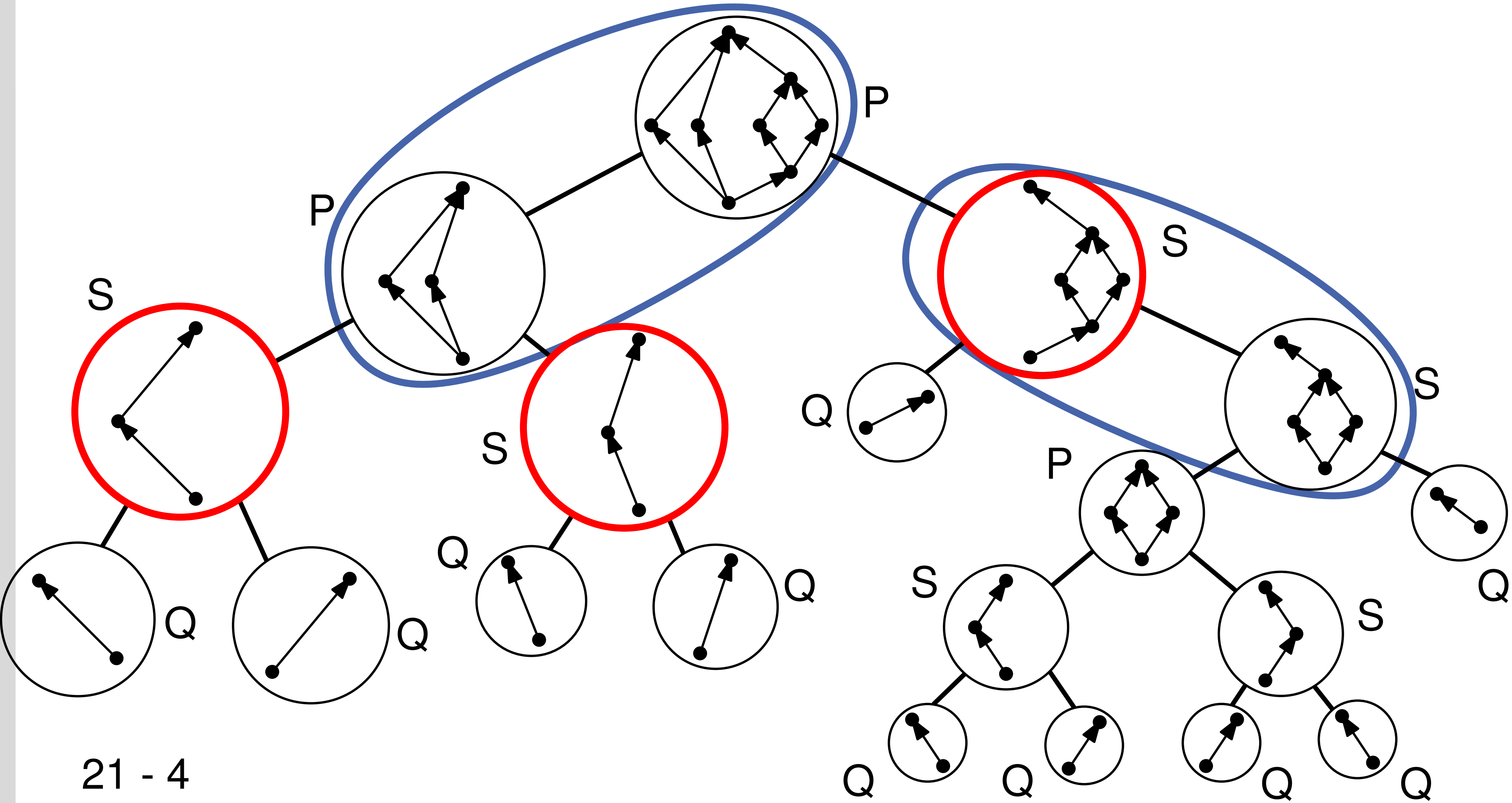


21 - 3

Vertical Automorphism



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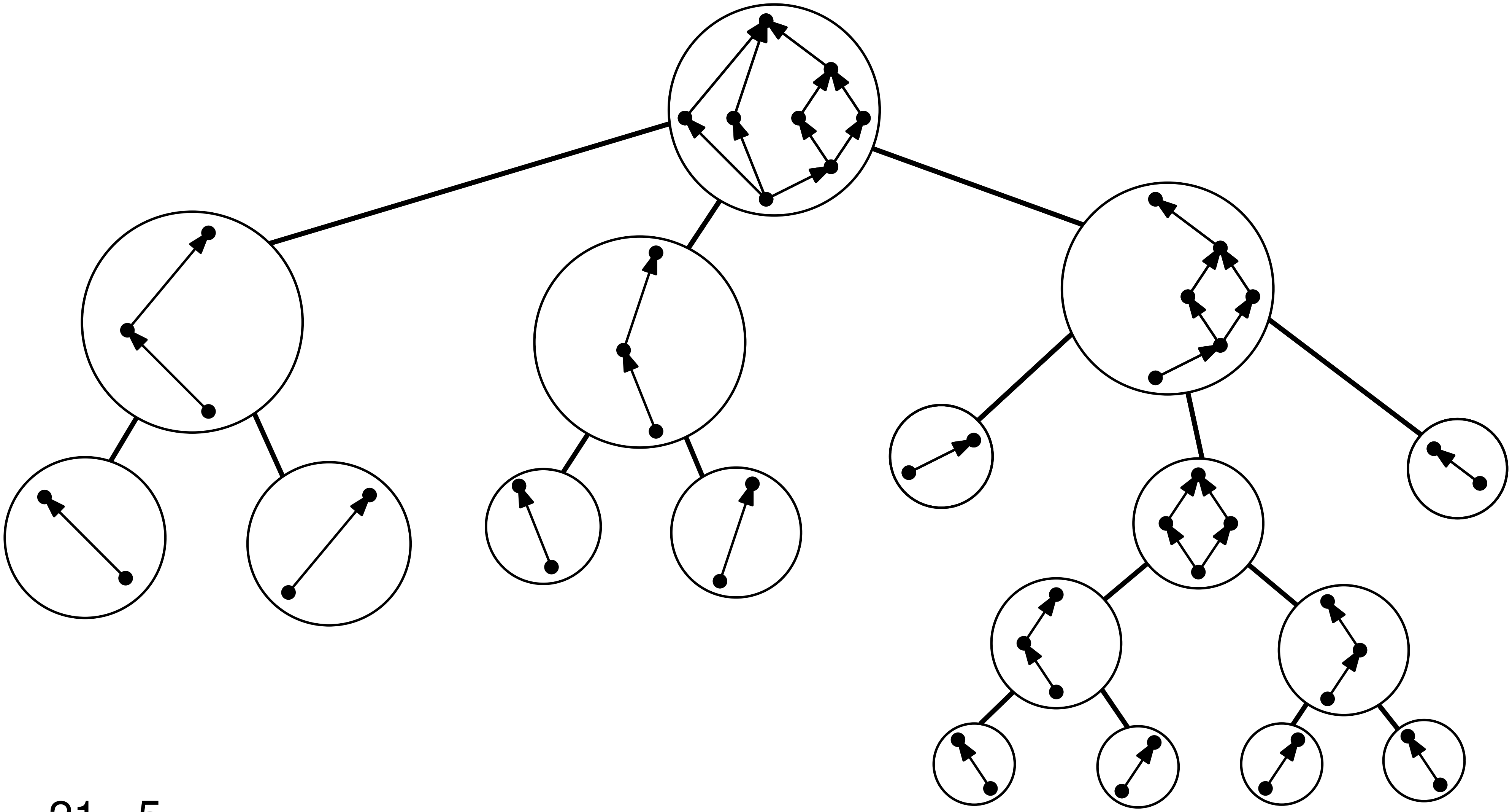
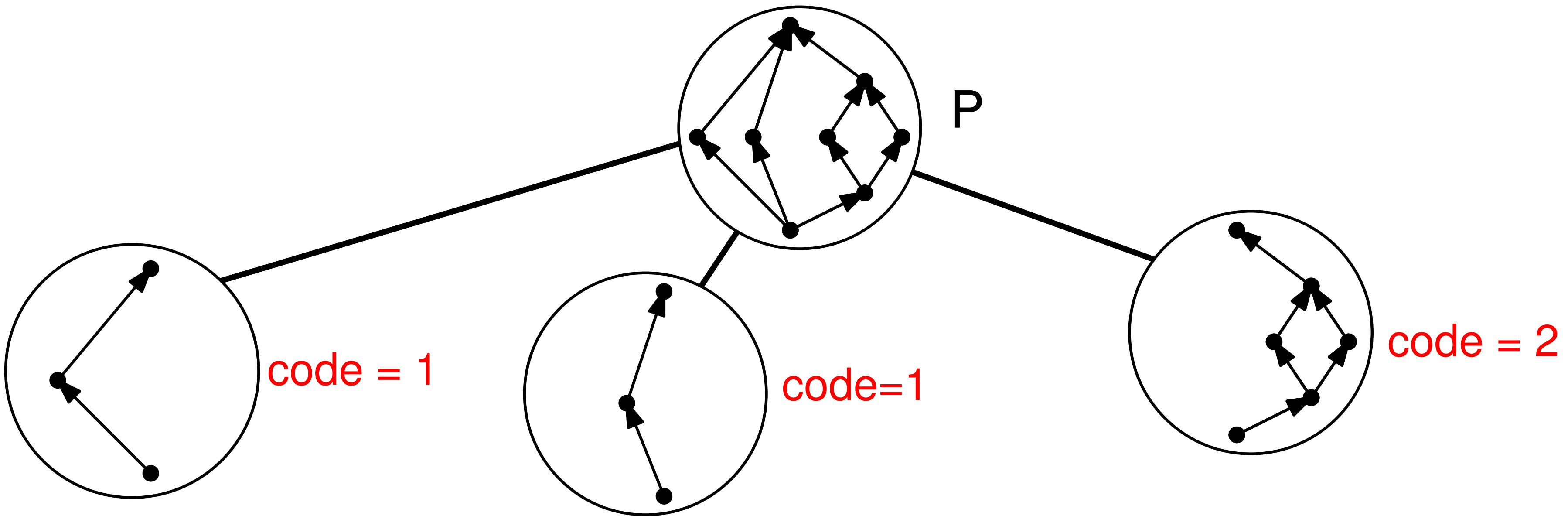


21 - 5

Vertical Automorphism

- $code(G)$ - two graphs at the same level have the same code iff they are isomorphic
- $tuple(G)$ - set of the codes of the children (sorted for a P-node)



21 - 6

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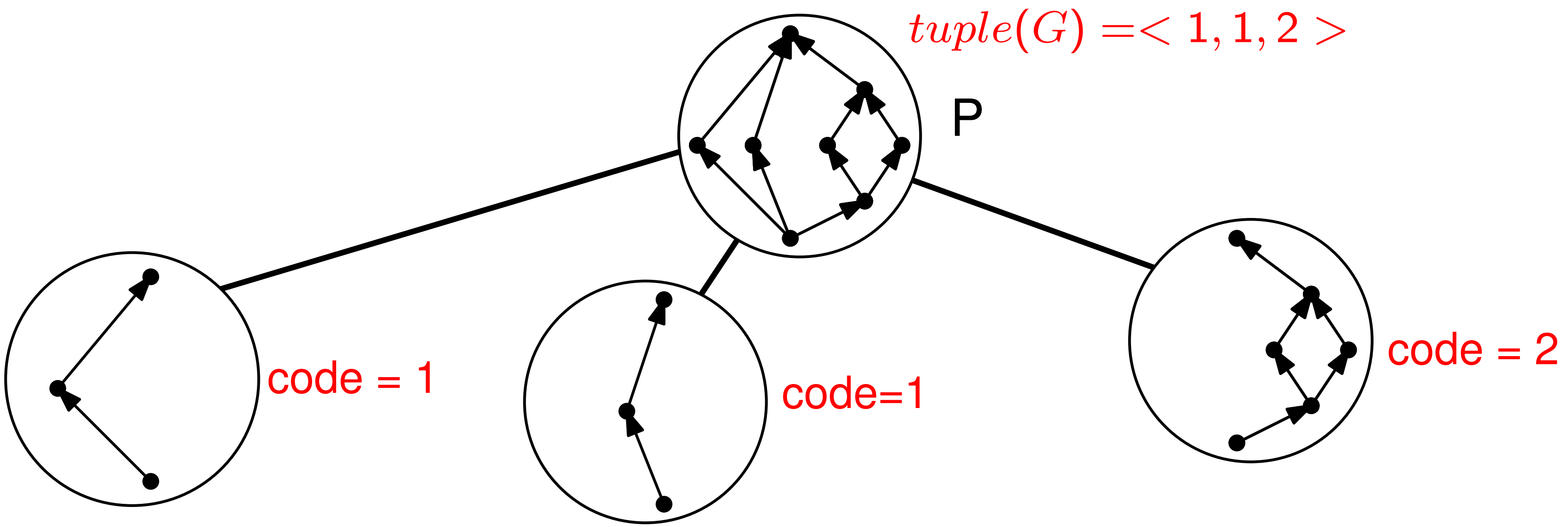
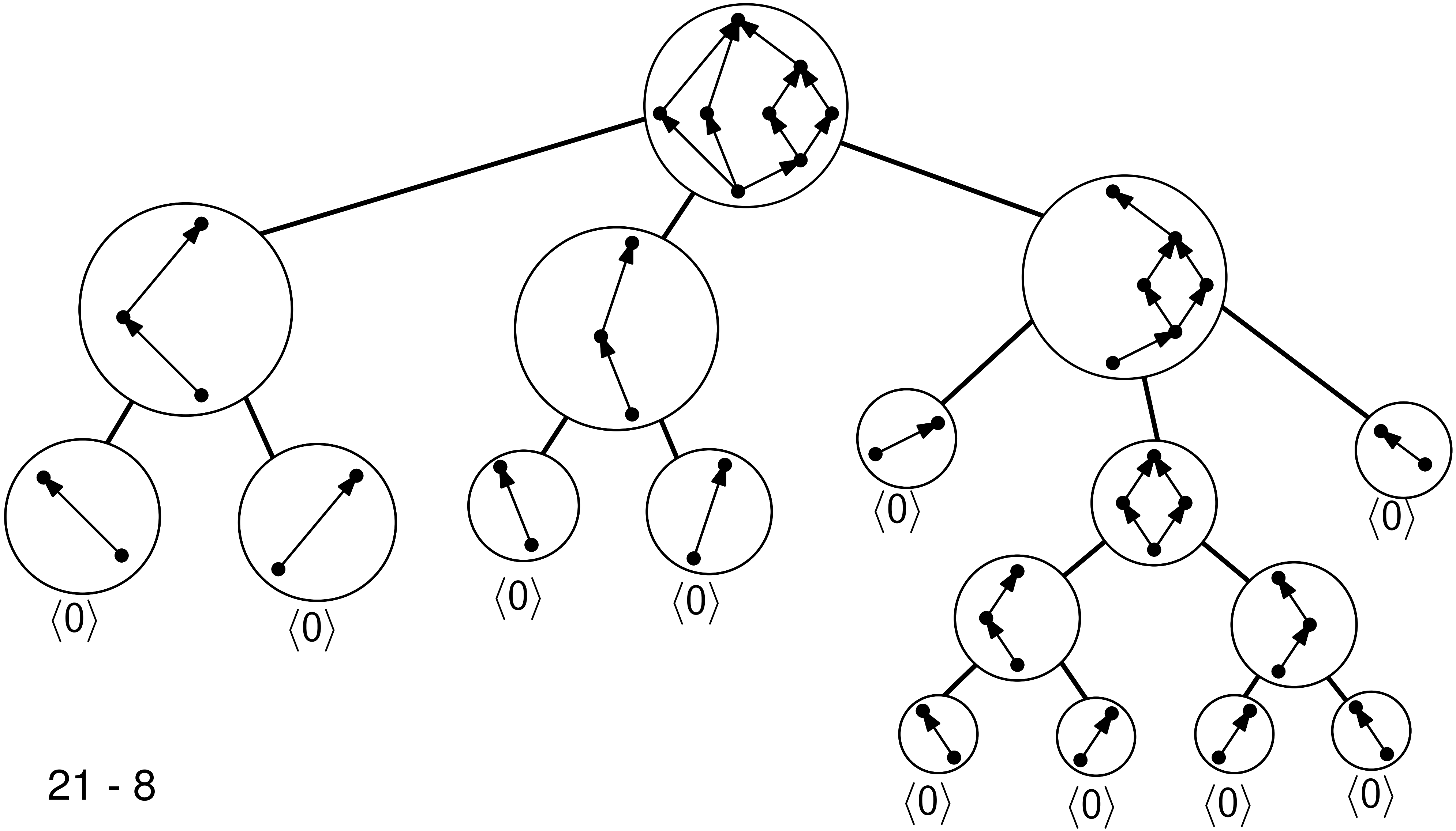


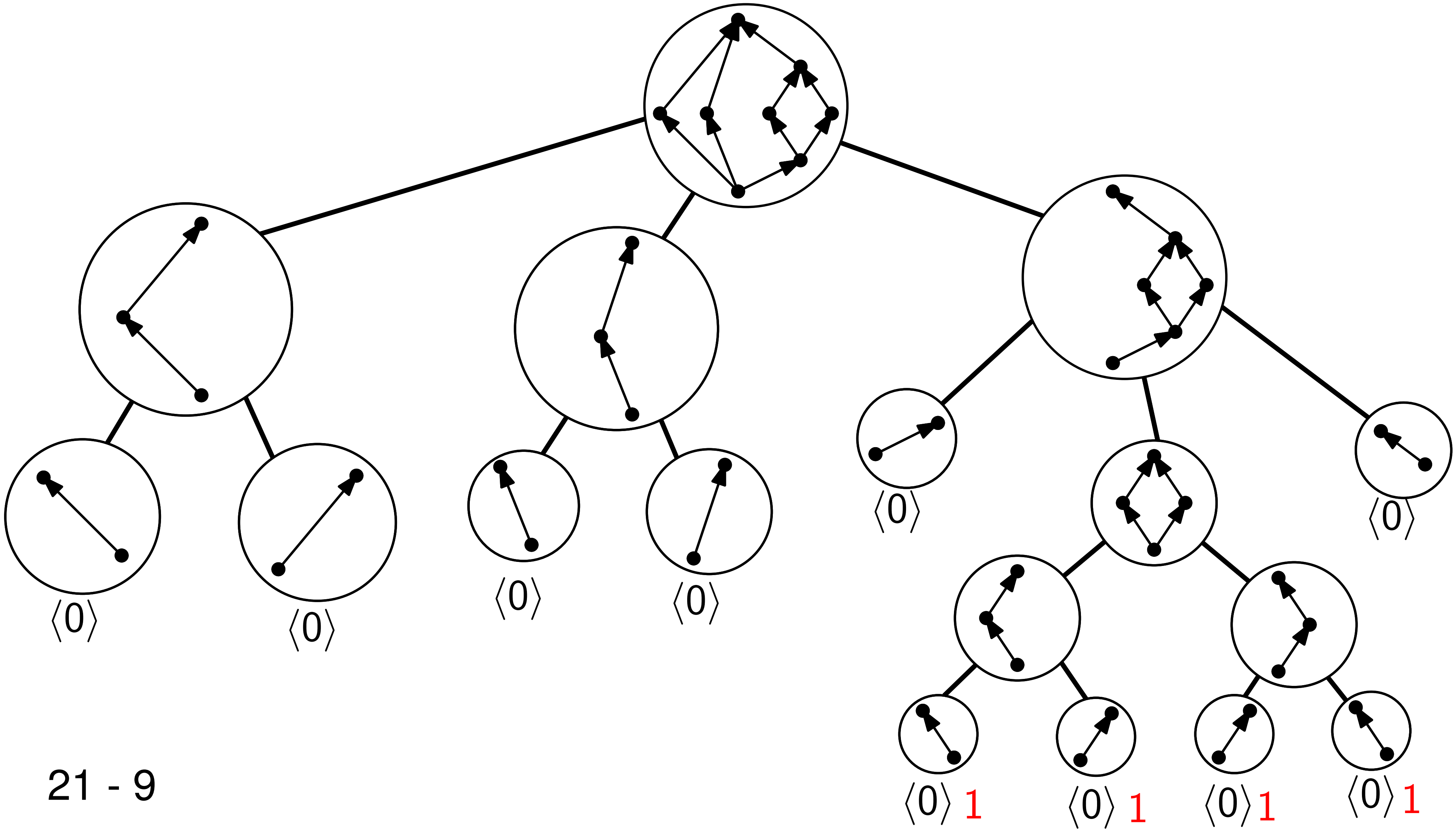
21 - 7

Vertical Automorphism



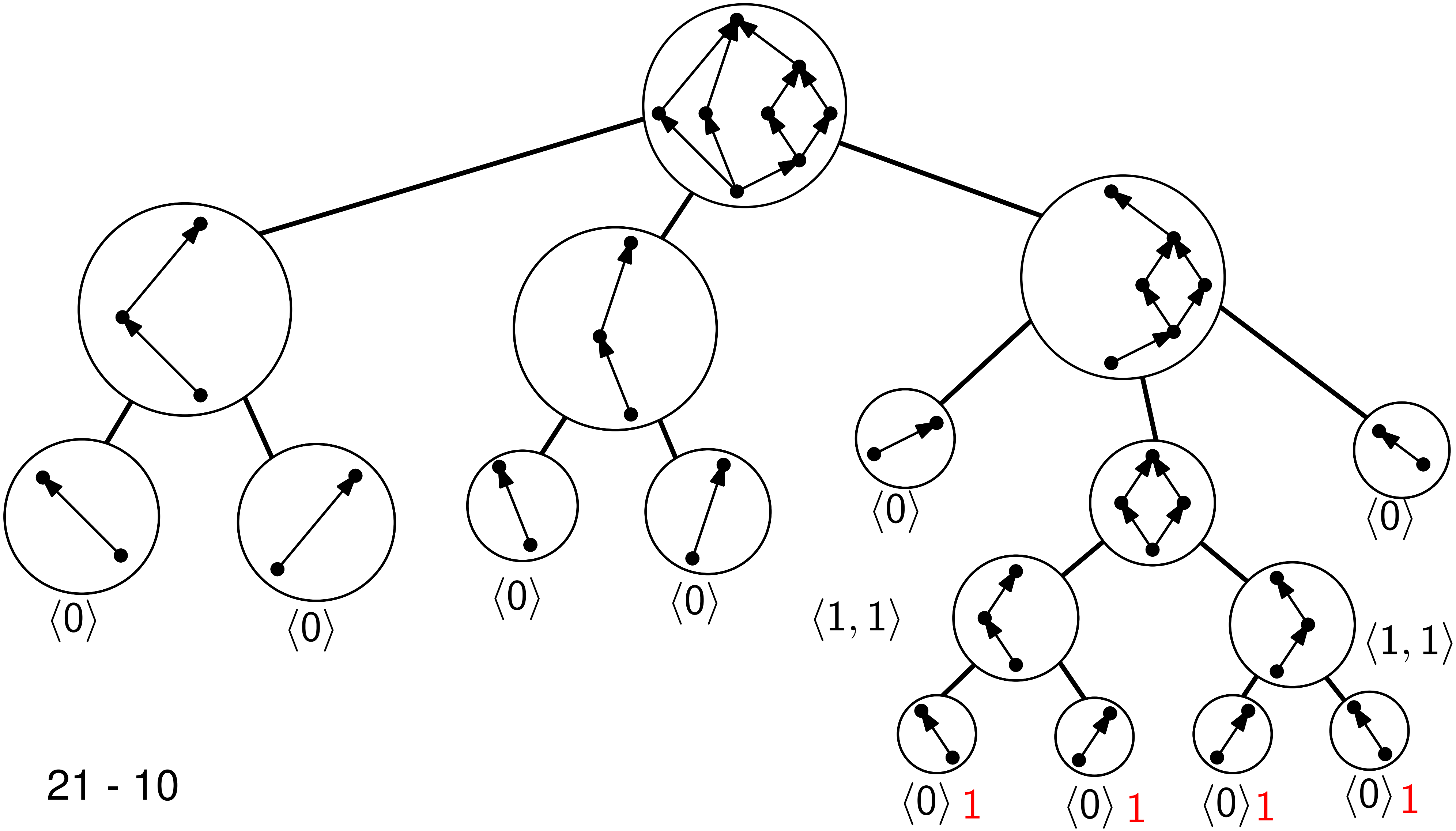
21 - 8

Vertical Automorphism



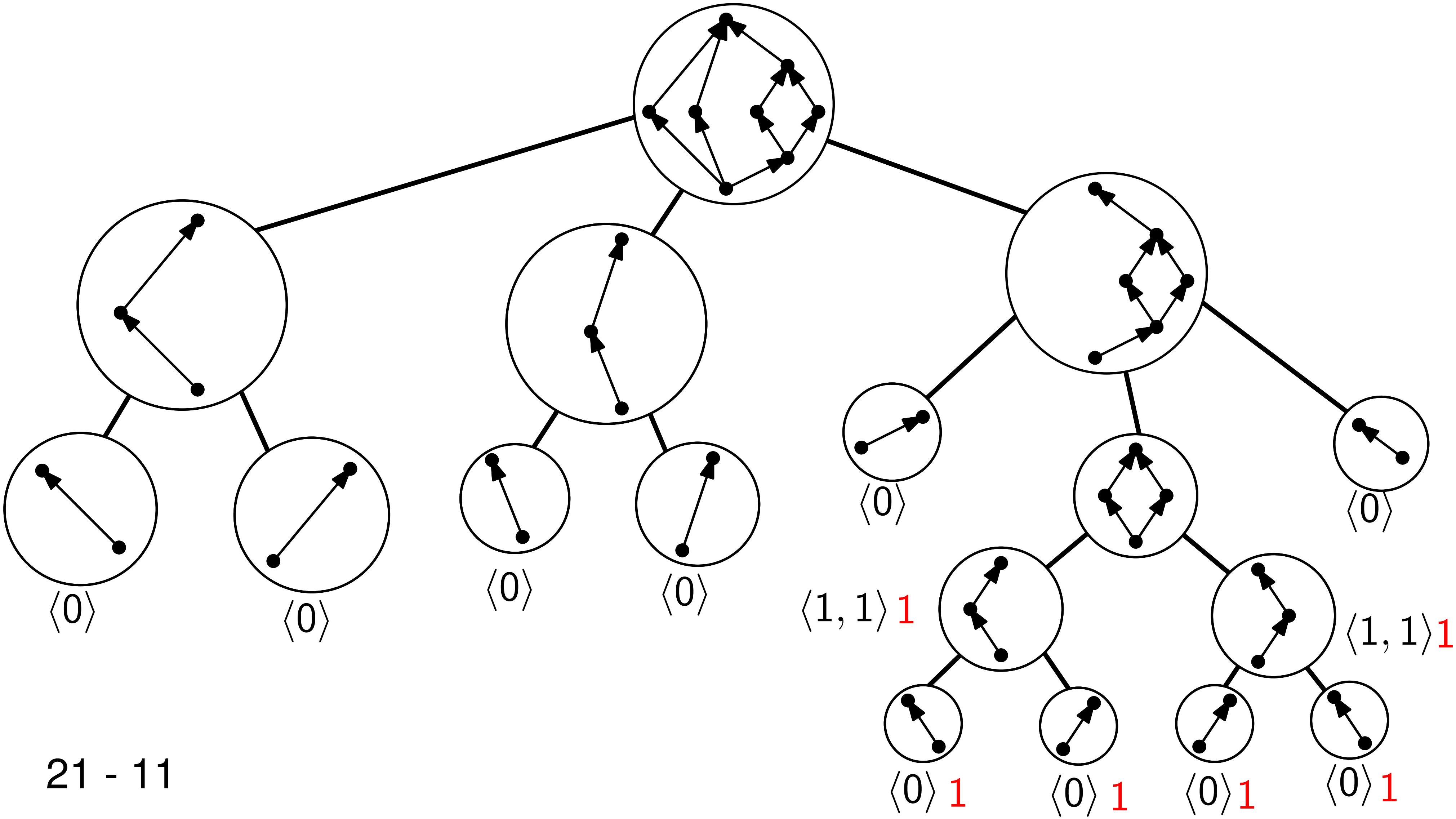
21 - 9

Vertical Automorphism



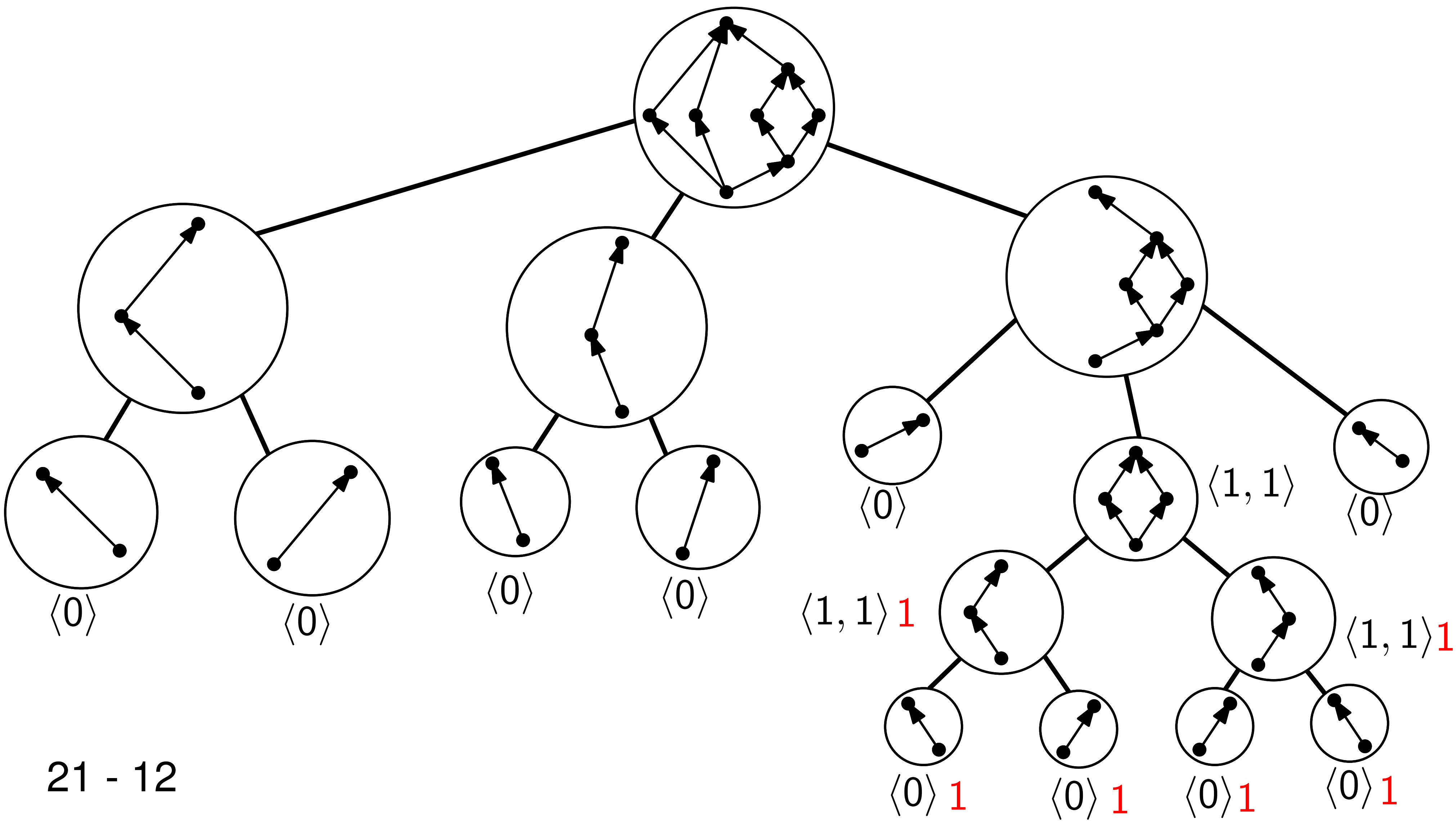
21 - 10

Vertical Automorphism

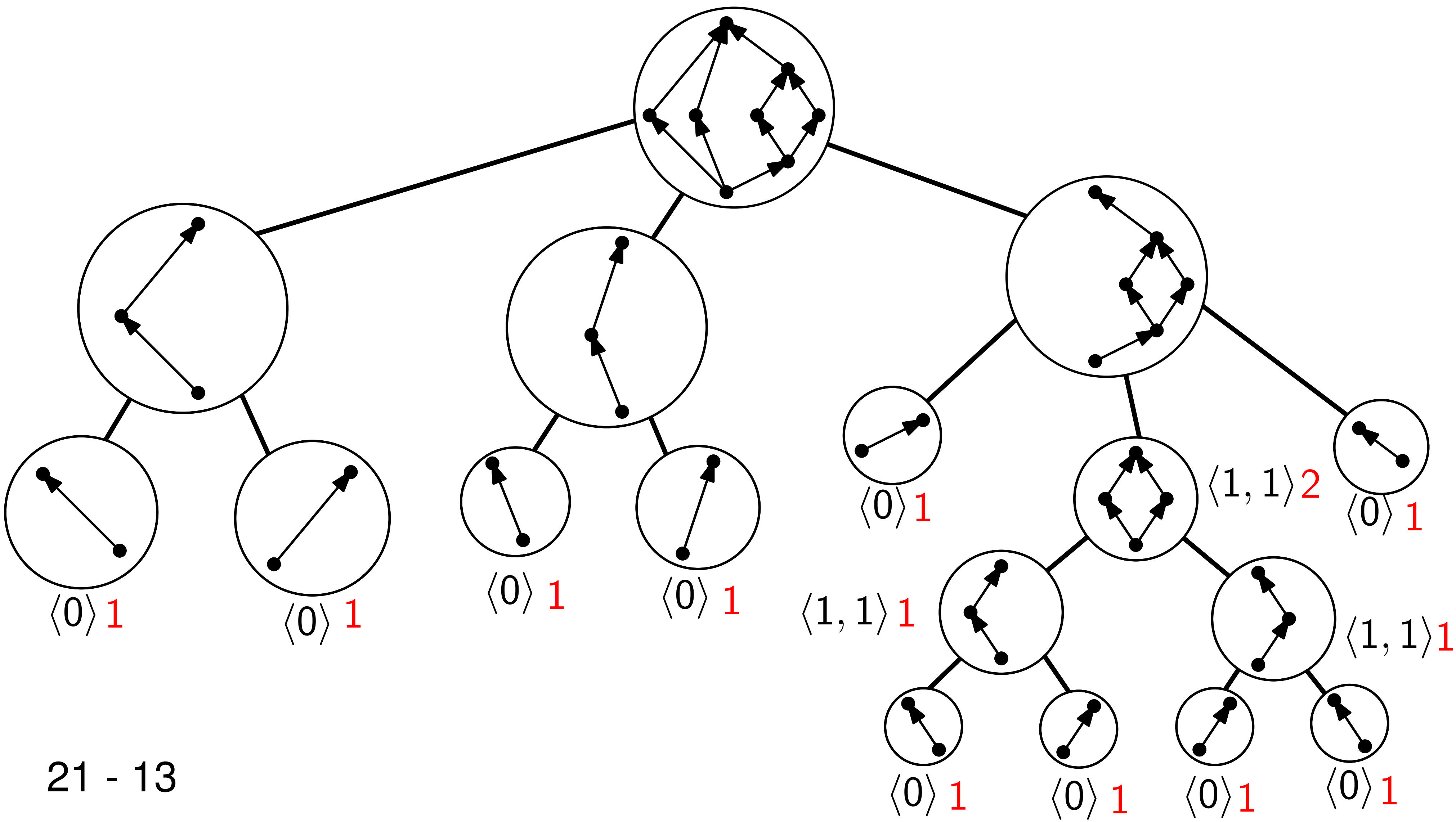


21 - 11

Vertical Automorphism

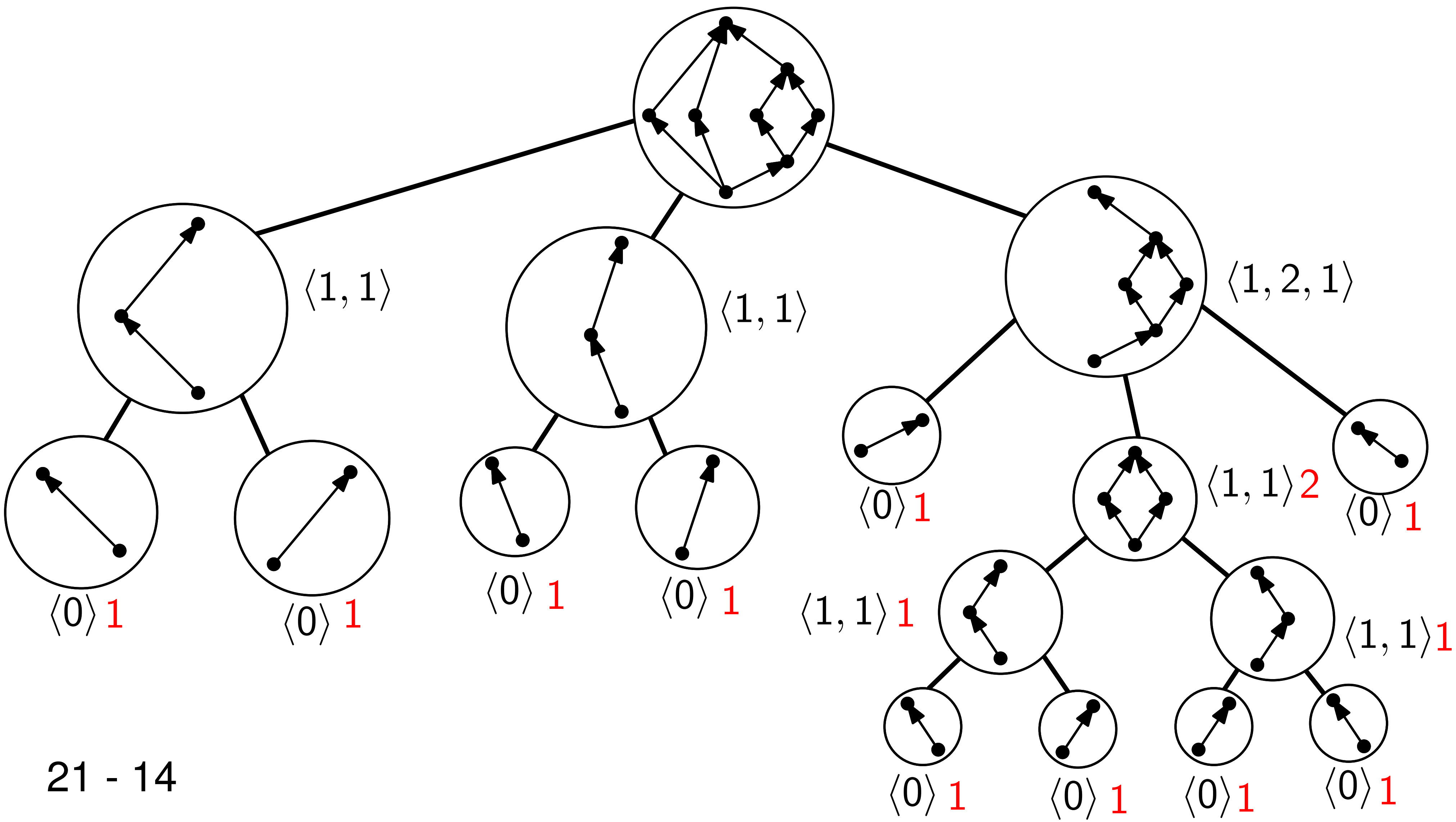


Vertical Automorphism



21 - 13

Vertical Automorphism



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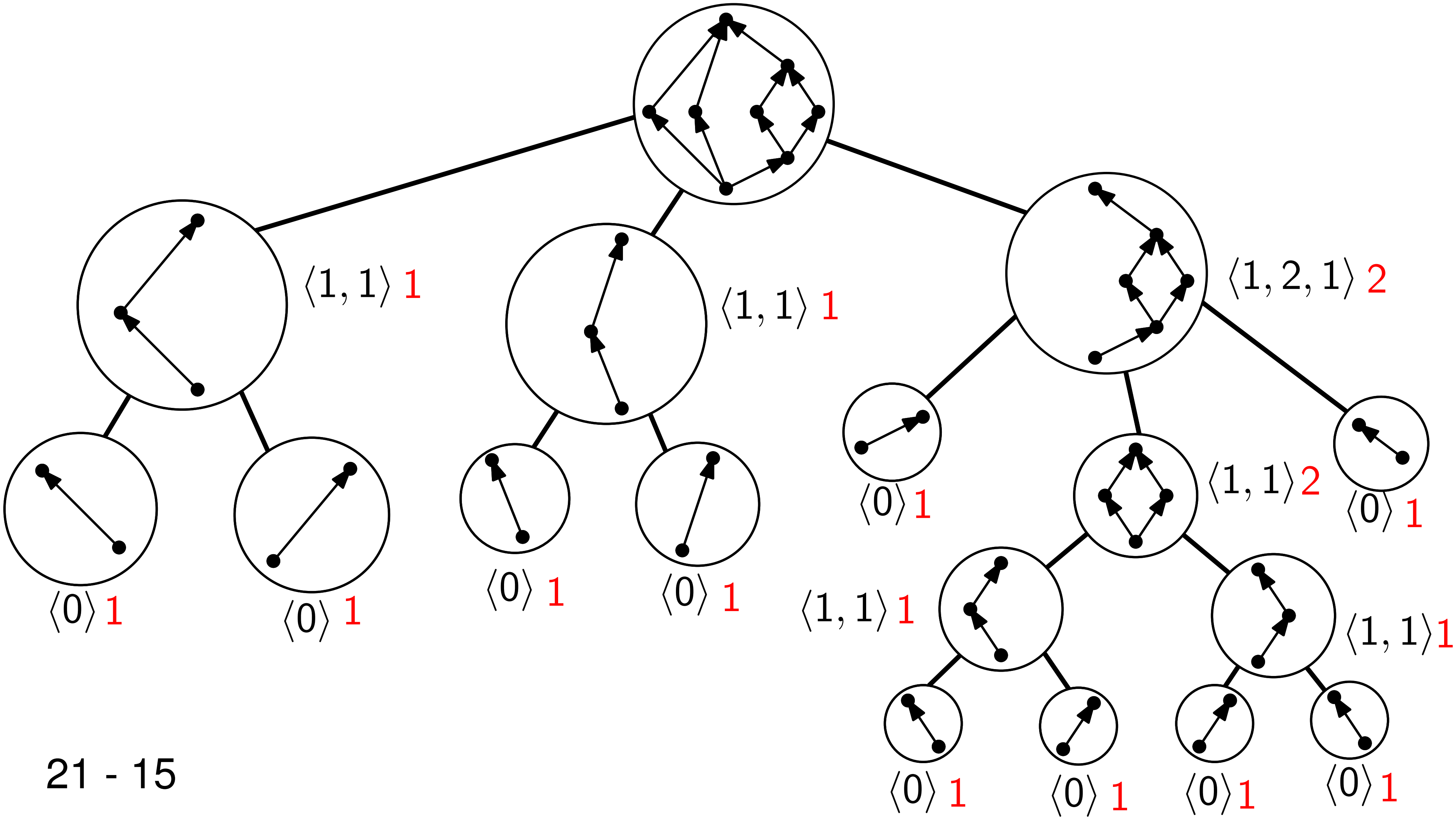

21 - 15

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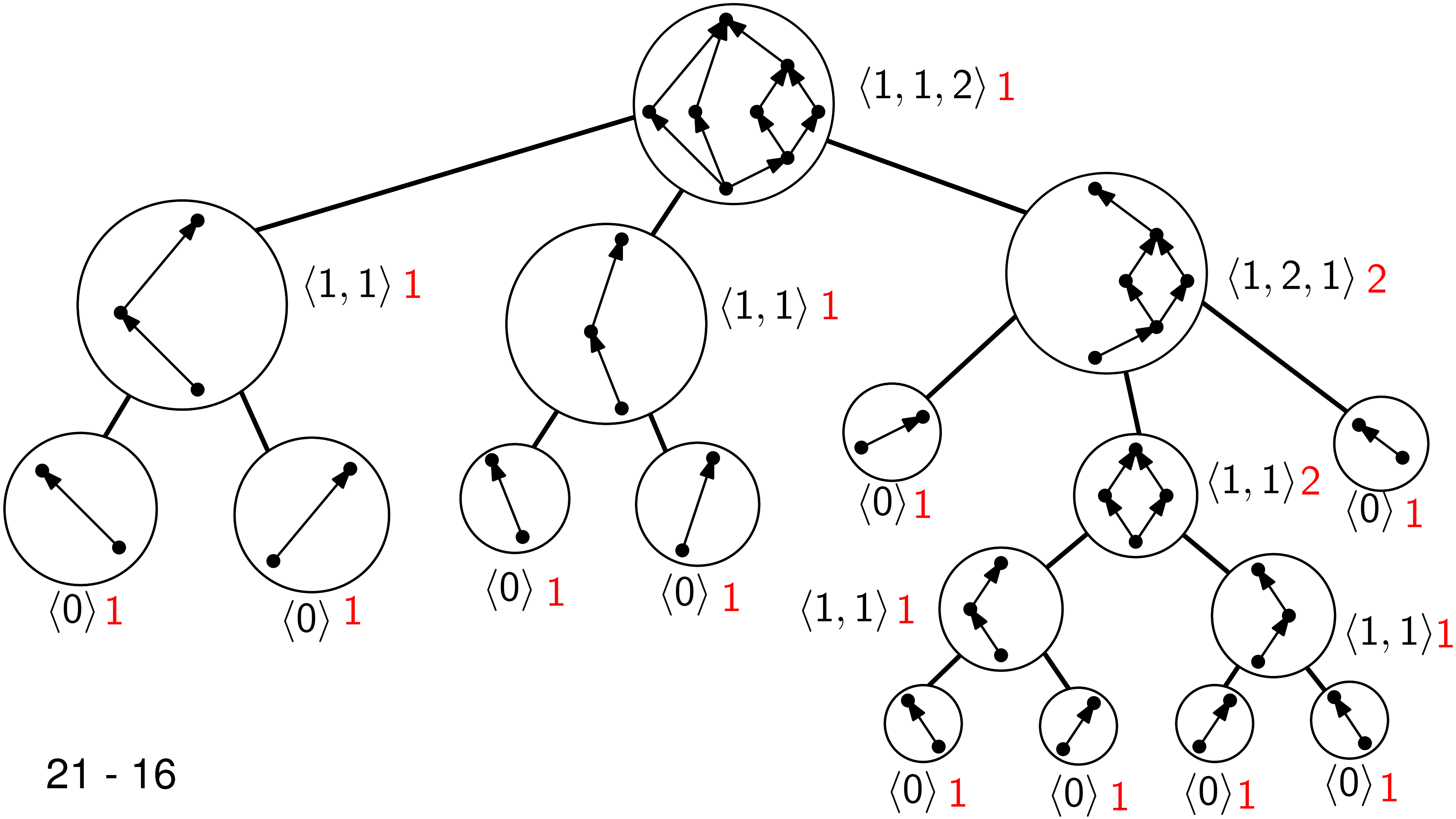

Vertical Automorphism

Algorithm constructing codes

- Set $tuple(G_i) = \langle 0 \rangle$ for all Q-nodes G_i of G.

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- For each $t = \max \text{depth}(G), \ldots, 0$
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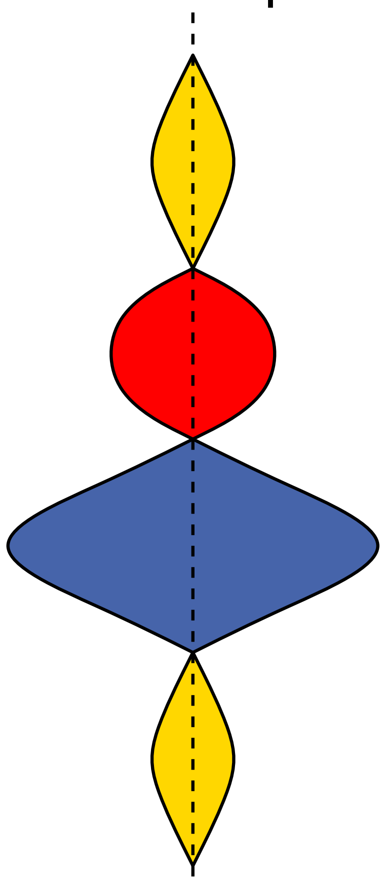
Lemma

Two nodes u and v at the same depth of the decomposition tree of G represent isomorphic subgraphs of G iff $code(u) = code(v)$.

Vertical Automorphism

- Let G be composed out of $G_1 \ldots G_n$ through series or parallel composition, $tuple(G)$ contains the codes of $G_1, \ldots, G_n$.
- How can we use $tuple(G)$ do decide whether G has a vertical automorphism?



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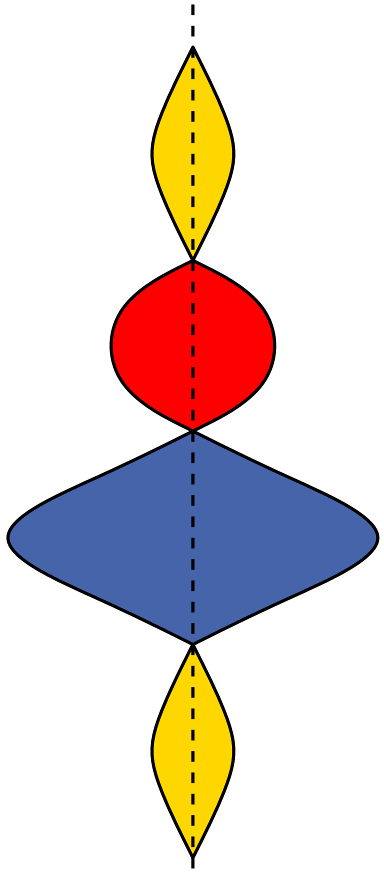

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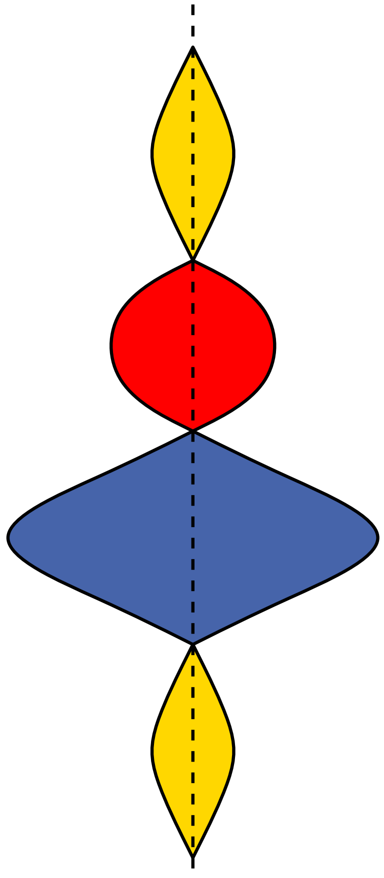

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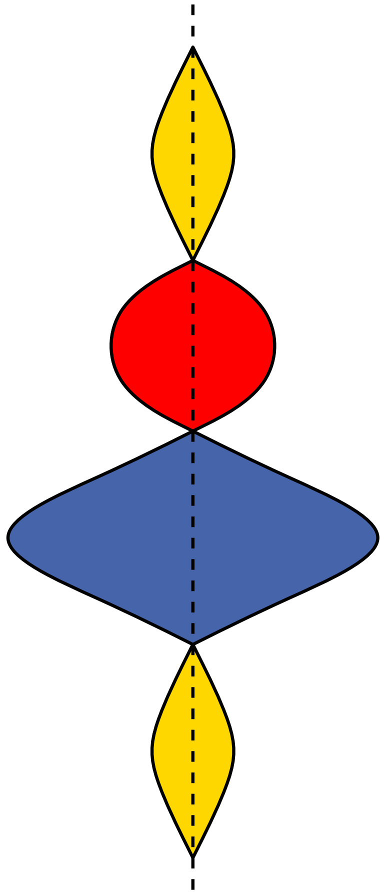

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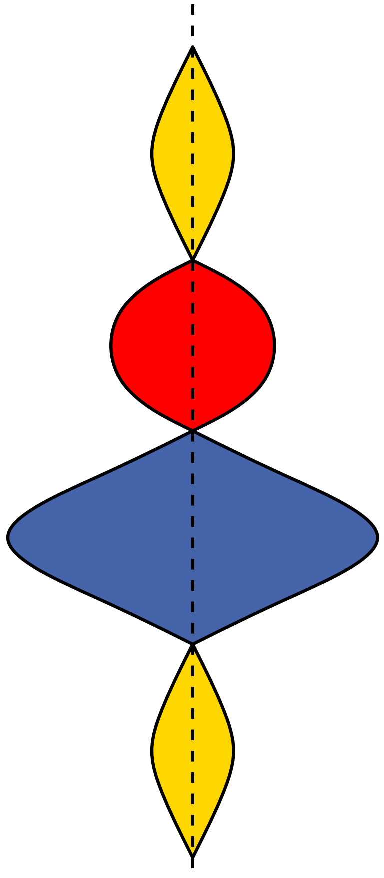

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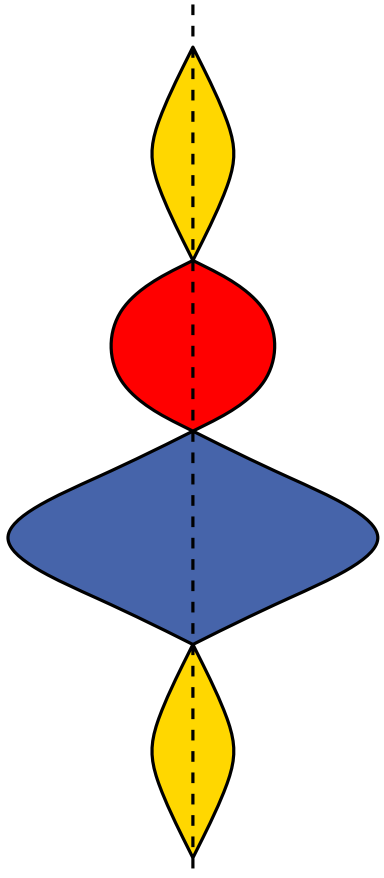

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- If each of $G_1, \dots, G_n$ has a vertical isomorphism, arrange them as in Figure.

Vertical Automorphism

Lemma (Hong, Eades, Lee ’00) [HEL00]

If G is a P-node, consider a partition of $\mathcal{C}_j = \{G_i : 1 \leq i \leq k, code(G_i) = j\}$, $j = 1, \ldots, k$ into classes of isomorphic graphs.

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- If there exists a unique j, such that $|\mathcal{C}_j|$ is odd $\Rightarrow$ G has a vertical automorphism iff graphs of $\mathcal{C}_j$ have a vertical automorphism.
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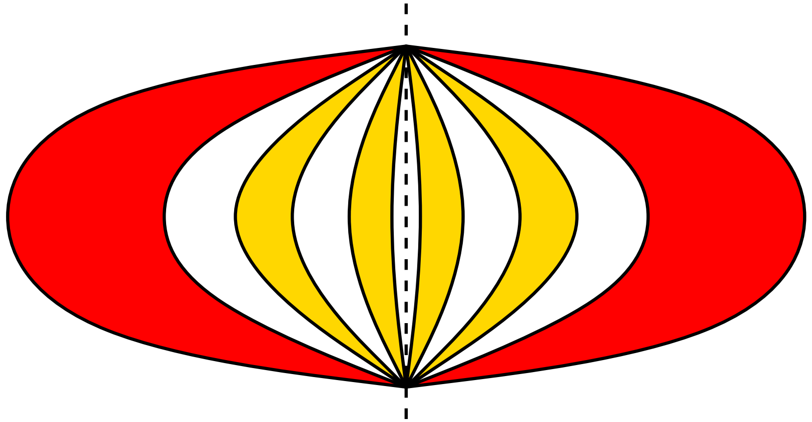

Proof:

- Arrange components as in Figure.

G is P-node, $tuple(G) = < \underbrace{1 \ldots 1}_{even}, \underbrace{2 \ldots 2}_{even}, \cdots >$

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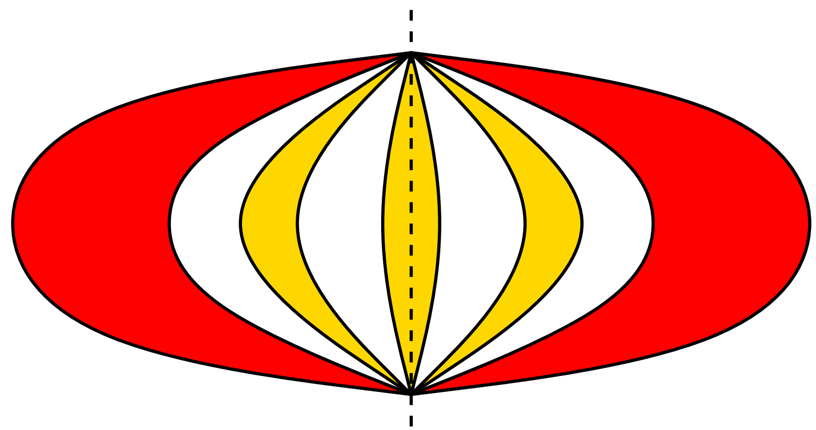

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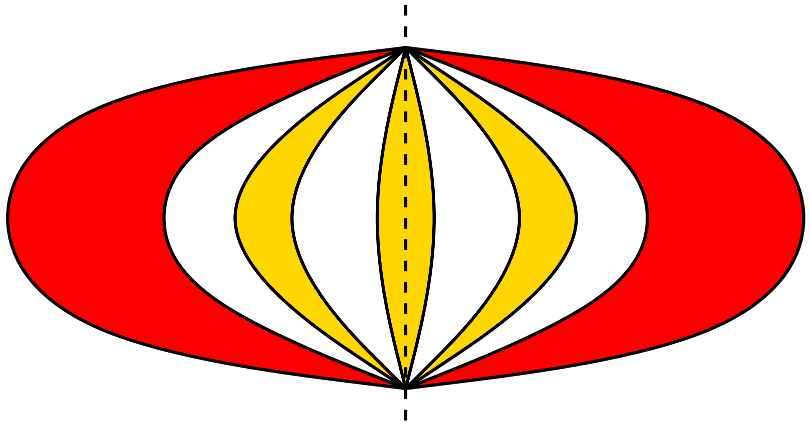

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- Any vertical automorphism “fixes” a member of $\mathcal{C}_j$, therefore it has a vertical automorphism.

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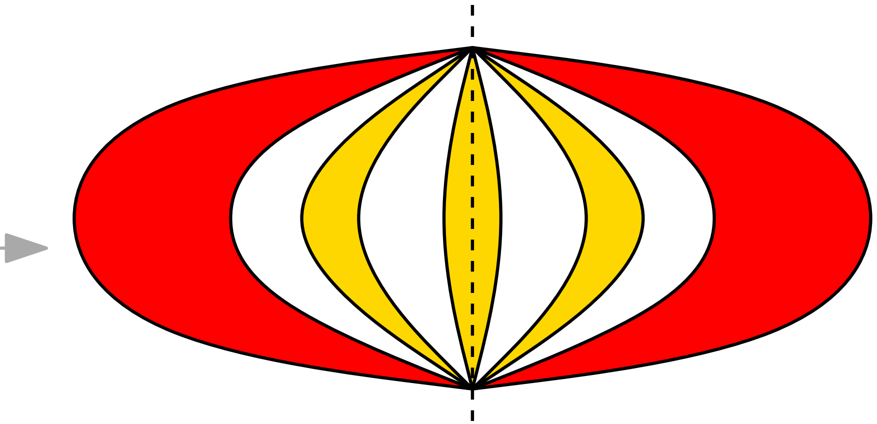

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- Conversly, arrange as in figure.

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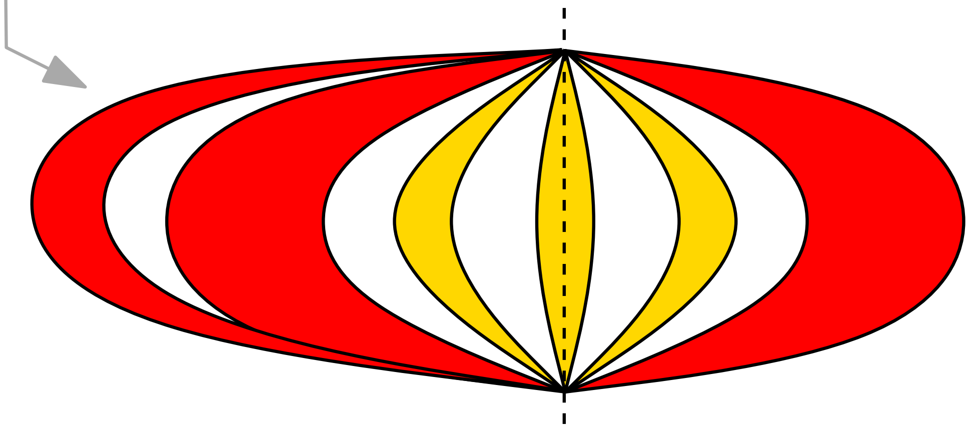

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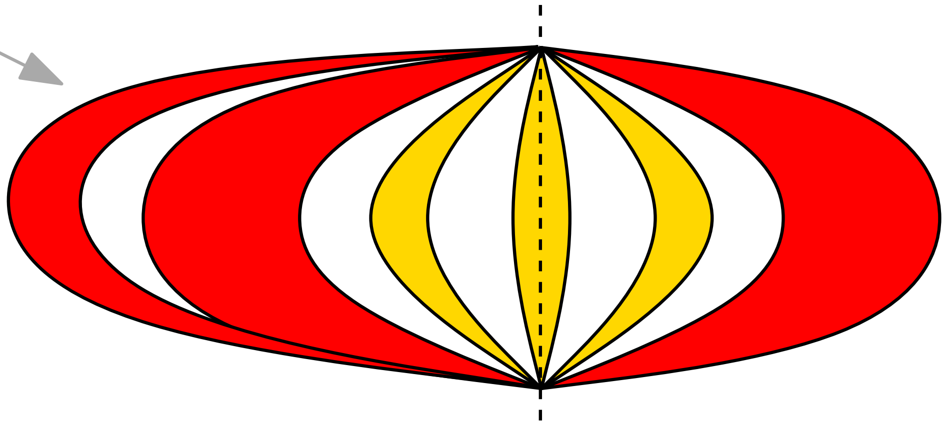

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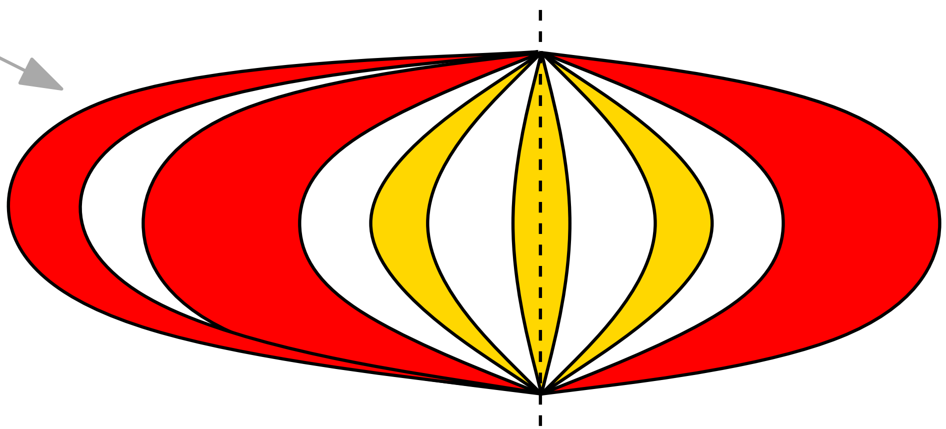

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Proof:

- Any vertical automorphism has to "fix" two distinct components.
- In both components we can find a path on which some vertices are aligned on the axis. Contradicts planarity.

Series-parallel graphs

- Book Di Battista et al: Chapter 3.2, 11.1
- Skript: Chapter 6.2
- [HEL00] Hong, Eades, Lee **Drawing series parallel digraphs symetrically** CGTA 2000