Karlsruher Institut für Technologie

Exercise Sheet 7

Discussion: 7. February 2018

Exercise 1: Extended Canonical Ordering for 4-Connected Graphs ⋆⋆

A planar graph $G = (V, E)$ is called *proper triangular planar* (PTP, for short) if every interior face of G is a triangle and the exterior face of G is a quadrangle, and G has no separating triangles.

Let $G = (V, E)$ be a PTP graph with vertices a, b, c, d on the outer face. A labeling $v_1 = a, v_2 = c, v_3, \dots, v_n = d$ of the vertices of G is called an *extended canonical ordering* of G if for every $4 \leq k \leq n$:

(i) The subgraph G_{k-1} induced by $v_1, \dots, v_{k-1}$ is biconnected and the boundary C_{k-1} of G_{k-1} contains the edge (a, b), and

(ii) the vertex v_k is on the boundary of exterior face of G_{k-1}, and its neighbors in G_{k-1} form a subinterval of the path $C_{k-1} \setminus (a, b)$ with at least two elements. If $k \leq n - 2$, then v_k has at least two neighbors in $G \setminus G_{k-1}$.

Let $G = (V, E)$ be a PTP graph with vertices a, b, c, d on the outer face. Prove the following statements. We denote by G_C the graph that is induced by the vertices in the interior and on the boundary of a simple cycle C.

(a) The graph obtained from G by the removal of the vertices c, d and all edges incident to them is biconnected.

(b) Let $C = \{a = u_1, \dots, u_k = b, a\}$ be a simple cycle of G such that $c, d \notin C$. Let $u_i \in C$, $2 \leq i \leq k - 1$ such that no internal chord of C is incident to u_i. Then the graph $G_C \setminus \{u_i\}$ is biconnected.

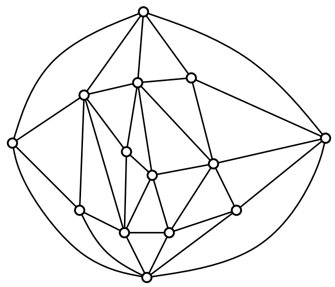
(c) Let C be as above and let (v_i, v_j), $1 \leq i < j \leq k$, be an internal chord of C. Then there exists a vertex v_l, $i < l < j$ that is adjacent to at least two vertices of $G \setminus G_C$.

Use the previous statements to prove the following lemma.

Lemma 1 *Every PTP graph G with four vertices a, b, c, d on the outer face has an extended canonical ordering such that $v_1 = a, v_2 = b, v_{n-1} = c, v_n = d$.*

Exercise 2: Construction of Rectangular Dual ⋆

Consider the graph G of the figure below. Check whether G satisfies the necessary conditions to have a rectangular dual. In affirmative, construct a rectangular dual of G.



Exercise 3: Contact Representation of Maximal Planar Graphs ⋆⋆

The figure below gives an example of contact representation of a planar graph with T-shapes. Prove the following Lemma.

Lemma 2 *Every maximal planar graph admits a contact representation with T-shapes.*

Hint: Use canonical ordering in the way similar to the construction of a visibility representation (Exercise Sheet 3).