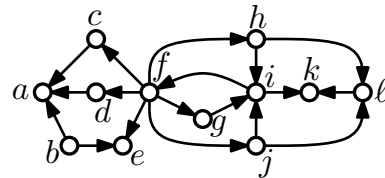


Exercise Sheet 5

Discussion: 17. January 2018

Exercise 1: Layered Layout ★

Execute each phase of the Sugiyama-Framework on the following graph. Remove all cycles by reversing a small set of edges and find a layer assignment of maximal width 4. Order the vertices on each layer such that the number of crossings is minimal.



Hint: Find independently an optimal solution of each subproblem.

Exercise 2: Crossings in Layered Layouts ★

Prove the following.

Lemma 1 *The barycenter heuristic computes an optimal solution of the one-sided crossing minimization problem, if the instances admits a planar drawing.*

Exercise 3: Counting Crossings ★★

Prove the following lemmas.

Let $\pi : \{1, \dots, n\} \rightarrow \{1, \dots, n\}$ be a permutation. A pair (i, j) with $1 \leq i < j \leq n$ is an *inversion*, if $\pi(i) > \pi(j)$.

Lemma 2 *The number of inversions of a permutation π can be counted in $O(n \log n)$ time.*

Hint: Use an approach similar to merge sort.

Lemma 3 *Let Γ be a straight-line drawing of a bipartite graph $G = (A \cup B, E)$ where the vertices of A and B are drawn on separate layers. Then the number of crossing in Γ can be counted in $O(|E| \log |V|)$ time.*

Can all crossings be reported in the same time?