Algorithms for Graph Visualization

Introduction

INSTITUT FÜR THEORETISCHE INFORMATIK · FAKULTÄT FÜR INFORMATIK

Tamara Mchedlidze, Benjamin Niedermann

17.10.2016

Organizational

Lectures

- Tamara Mchedlidze
- `mched@iti.uka.de`
- Office 307
- Office hours: request by email

Exercises

- Benjamin Niedermann
- `benjamin.niedermann@kit.edu`
- Office 321
- Office hours: request by email

Organizational

Lectures

- Tamara Mchedlidze
- `mched@iti.uka.de`
- Office 307
- Office hours: request by email

Exercises

- Benjamin Niedermann
- `benjamin.niedermann@kit.edu`
- Office 321
- Office hours: request by email

Meetings

- Monday 09:45 – 11:15 Uhr, Room 301
- Wednesday 9:45 – 11:15 Uhr, Room -119
- exact plan on the web-page

Organizational

Webseite

`i11www.iti.kit.edu/teaching/winter2016/graphvis/`

- Latest news
- Lecture slides
- Exercise sheets
- Literature & Additional material
- Lecture notes (skript)

Organizational

Webseite

i11www.iti.kit.edu/teaching/winter2016/graphvis/

- Latest news
- Lecture slides
- Exercise sheets
- Literature & Additional material
- Lecture notes (skript)

Graph Visualization in Master's Studies

Bachelor

Algorithms 1 & 2

Basic Theory of Inf.

Algorithms for Planar Graphs

Master

Algorithms for Graph Visualization

Algorithms for Graph Visualization

Learning goals: At the end of the semester you:

- Know terms and problem definitions
- Know the introduced algorithms, understand how they work, can analyse them
- Can select appropriate algorithms and data structures
- Can analyse a new graph drawing problem and construct an efficient algorithm/prove hardness

Algorithms for Graph Visualization

Learning goals: At the end of the semester you:

- Know terms and problem definitions
- Know the introduced algorithms, understand how they work, can analyse them
- Can select appropriate algorithms and data structures
- Can analyse a new graph drawing problem and construct an efficient algorithm/prove hardness

Prerequisites: Algorithms 1 & 2, Theoretical Basics of inf.

Helpful: Algorithms for Planar Graphs

Algorithms for Graph Visualization

Learning goals: At the end of the semester you:

- Know terms and problem definitions
- Know the introduced algorithms, understand how they work, can analyse them
- Can select appropriate algorithms and data structures
- Can analyse a new graph drawing problem and construct an efficient algorithm/prove hardness

Prerequisites: Algorithms 1 & 2, Theoretical Basics of inf.

Helpful: Algorithms for Planar Graphs

Suggested time requirements: 5LP = 150h

- Attending Lecture and Exercises: ca. 35h
- Preparation/post-processing ca. 35h
- Work on the exercises ca. 40h
- Preparation for the exam ca. 40h

Examination

Master Informatics

- Algorithms for Graph Visualization (IN4INALGVG)
- **new Module:** Graph visualization+ (more about this later)

Examination

Master Informatics

- Algorithms for Graph Visualization (IN4INALGVG)
- **new Module:** Graph visualization+ (more about this later)

Examination procedure:

- In order to take an exam you need to participate actively in the exercise sessions
  (e.g. present your own solutions on the board)
- oral exam(app. 20 Minutes)

Graph Visualization+

Background: International Symposium on Graph Drawing (GD) and Graph Drawing Challenge

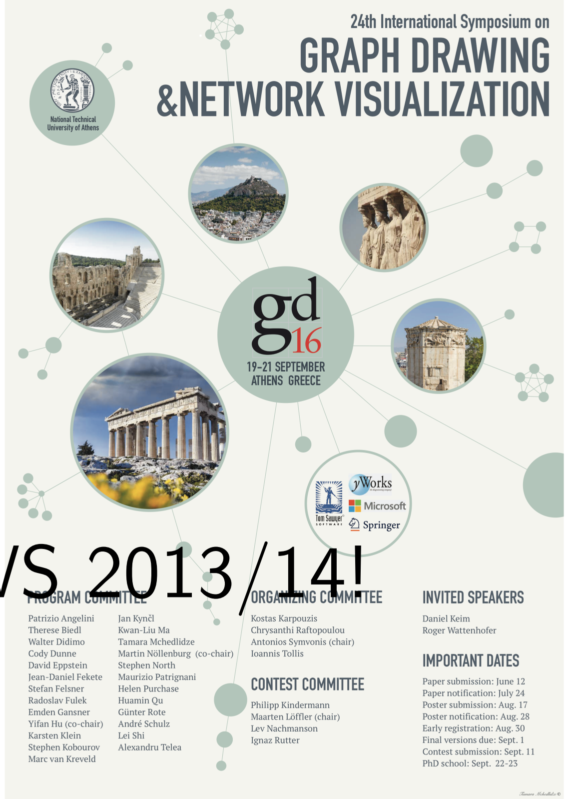
Graph Visualization+

Background: International Symposium on Graph Drawing (GD) and Graph Drawing Challenge

Task: develop software for a given graph visualization problem

Graph Visualization+

Background: International Symposium on Graph Drawing (GD) and Graph Drawing Challenge



Task: develop software for a given graph visualization problem

Winner 2014: KIT-Team from the lecture WS 2013/14!

Graph Visualization+

Karlsruhe Institute of Technology

Background: International Symposium on Graph Drawing (GD) and Graph Drawing Challenge

Task: develop software for a given graph visualization problem

Winner 2014: KIT-Team from the lecture WS 2013/14!

Second place 2015: KIT-Team from the lecture SS 2015!

Best poster award 2016: One of the students who continued working on the 2015's topic!

Graph Visualization+

Background: International Symposium on Graph Drawing (GD) and Graph Drawing Challenge

Task: develop software for a given graph visualization problem

2013/14:
- Software development was part of the lecture
- Lots of fun but time-consuming

2015:
- as a practical course SS 2015
- combined modul with 10 LP
- time-wise worked very well
- **evaluation:** excelent practical course award

Structure of the Lecture

Media:

- **Slides** & Blackboard
- Exercise sheets are provided a week before the execise session
- (incomplete) Lecture notes/Books
- Original literature (papers)

Structure of the Lecture

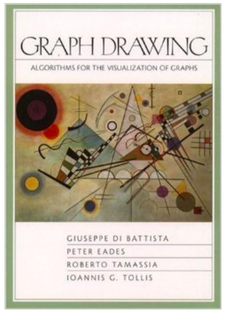
Media:

- **Slides** & Blackboard
- Exercise sheets are provided a week before the execise session
- (incomplete) Lecture notes/Books
- Original literature (papers)

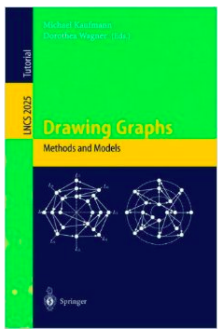
Content:

- We look at the **algorithmic core** of visualization problems
- **not:** graphical Rendering
- Modelling, Algorithms, formal Analysis
  - Divide & Conquer / Recursion
  - combinatorial optimization (ILP)
  - incremental algorithms
  - algorithms for special graph classes
  - drawing techniques using physical analogies

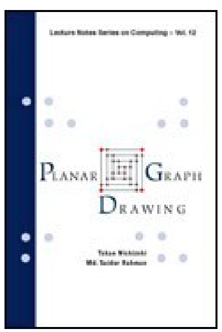
Literature (available in the library)



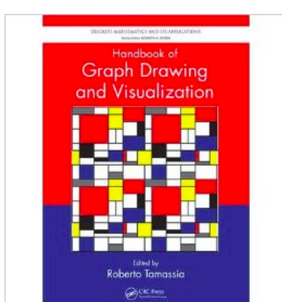
G. di Battista, P. Eades, R. Tamassia, I. Tollis:
Graph Drawing
Prentice Hall, 1998



M. Kaufmann, D. Wagner:
Drawing Graphs: Methods and Models
Springer, 2001



T. Nishizeki, Md. S. Rahman:
Planar Graph Drawing
World Scientific, 2004



R. Tamassia:
Handbook of Graph Drawing and Visualization
CRC Press, 2013

http://cs.brown.edu/~rt/gdhandbook/

Usefull Knowledge

Basic knowledge of Graph Theory:

- Graph, Nodes/Vertices, Edges
- Node degree, Neighbourhood, Adjacent, Incident
- Connectivity, Tree, Cycle, Path
- ...

Usefull Knowledge

Basic knowledge of Graph Theory:

- Graph, Nodes/Vertices, Edges
- Node degree, Neighbourhood, Adjacent, Incident
- Connectivity, Tree, Cycle, Path
- ...

Basic knowledge in Algorithms:

- Asymptotic running time, O-notation
- Complexity, NP-Hardness
- Network flow
- Linear Programming
- Recursion
- Divide & Conquer
- Approximation
- ...

Introduction to Graph Visualization

Graph and its Representation

What is a Graph?

Graph and its Representation

What is a Graph?

Tuple $G = (V, E)$
Set of nodes $V = \{v_1, \ldots, v_n\}$
Set of edges $E = \{e_1, \ldots, e_m\}$,
$e_i = \{v_j, v_k\},\ 1 \leq i \leq m,\ 1 \leq j, k \leq n$

Graph and its Representation

What is a Graph?

Tuple $G = (V, E)$
Set of nodes $V = \{v_1, \ldots, v_n\}$
Set of edges $E = \{e_1, \ldots, e_m\}$,
$e_i = \{v_j, v_k\},\ 1 \leq i \leq m,\ 1 \leq j, k \leq n$

Representations?

Graph and its Representation

What is a Graph?

Tuple $G = (V, E)$
Set of nodes $V = \{v_1, \dots, v_n\}$
Set of edges $E = \{e_1, \dots, e_m\}$,
$e_i = \{v_j, v_k\},\ 1 \le i \le m,\ 1 \le j, k \le n$

Representations?

Set representation:

$$
\begin{aligned}
V &= \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\} \\
E &= \{\{v_1, v_2\}, \{v_1, v_8\}, \{v_2, v_3\}, \{v_3, v_5\}, \{v_3, v_9\}, \\
&\quad \{v_3, v_{10}\}, \{v_4, v_5\}, \{v_4, v_6\}, \{v_4, v_9\}, \{v_5, v_8\}, \\
&\quad \{v_6, v_8\}, \{v_6, v_9\}, \{v_7, v_8\}, \{v_7, v_9\}, \{v_8, v_{10}\}, \\
&\quad \{v_9, v_{10}\}\}
\end{aligned}
$$

Graph and its Representation

What is a Graph?

Tuple $G = (V, E)$
Set of nodes $V = \{v_1, \ldots, v_n\}$
Set of edges $E = \{e_1, \ldots, e_m\}$,
$e_i = \{v_j, v_k\},\ 1 \leq i \leq m,\ 1 \leq j,k \leq n$

Representations?

Set representation
Adjacency list

$$
\begin{array}{ll}
v_1: & v_2, v_8 \\
v_2: & v_1, v_3 \\
v_3: & v_2, v_5, v_9, v_{10} \\
v_4: & v_5, v_6, v_9 \\
v_5: & v_3, v_4, v_8 \\
v_6: & v_4, v_8, v_9 \\
v_7: & v_8, v_9 \\
v_8: & v_1, v_5, v_6, v_7, v_9, v_{10} \\
v_9: & v_3, v_4, v_6, v_7, v_8, v_{10} \\
v_{10}: & v_3, v_8, v_9
\end{array}
$$

Graph and its Representation

What is a Graph?

Tuple $G = (V, E)$
Set of nodes $V = \{v_1, \ldots, v_n\}$
Set of edges $E = \{e_1, \ldots, e_m\}$,
$e_i = \{v_j, v_k\},\ 1 \leq i \leq m,\ 1 \leq j, k \leq n$

Representations?

- Set representation
- Adjacency list
- Adjacency matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

Graph and its Representation

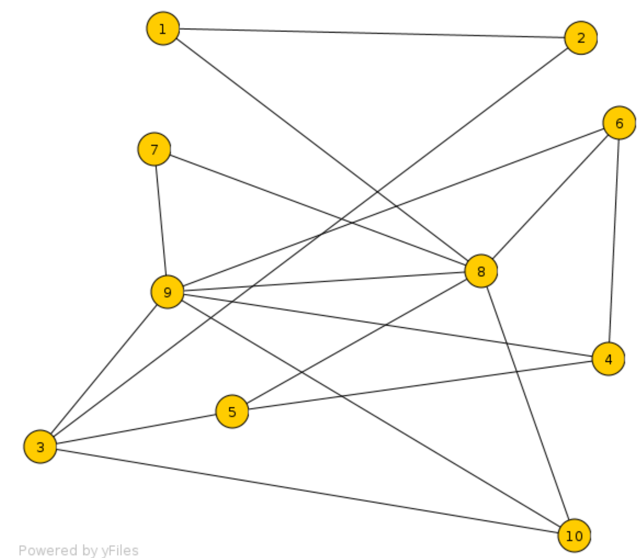
Karlsruhe Institute of Technology

What is a Graph?

Tuple $G = (V, E)$
Set of nodes $V = \{v_1, \ldots, v_n\}$
Set of edges $E = \{e_1, \ldots, e_m\}$,
$e_i = \{v_j, v_k\},\ 1 \leq i \leq m,\ 1 \leq j, k \leq n$

Representations?

Set representation
Adjacency list
Adjacency matrix
Drawing



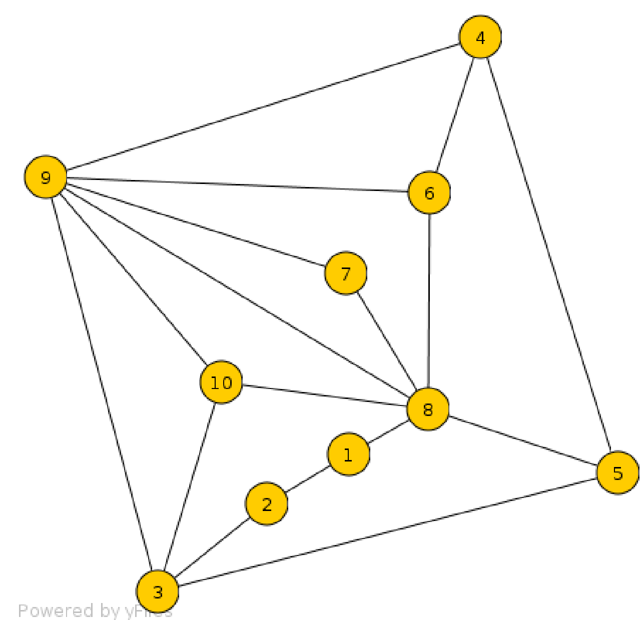
Graph and its Representation

What is a Graph?

Tuple $G = (V, E)$
Set of nodes $V = \{v_1, \ldots, v_n\}$
Set of edges $E = \{e_1, \ldots, e_m\}$,
$e_i = \{v_j, v_k\},\ 1 \leq i \leq m,\ 1 \leq j, k \leq n$

Representations?

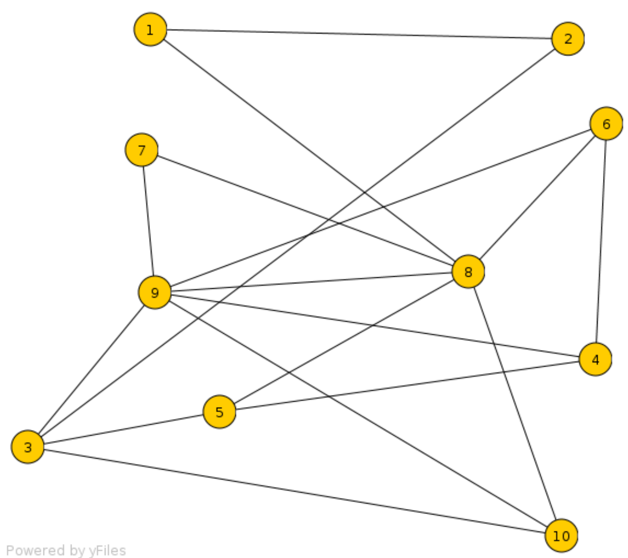
Set representation
Adjacency list
Adjacency matrix
Drawing



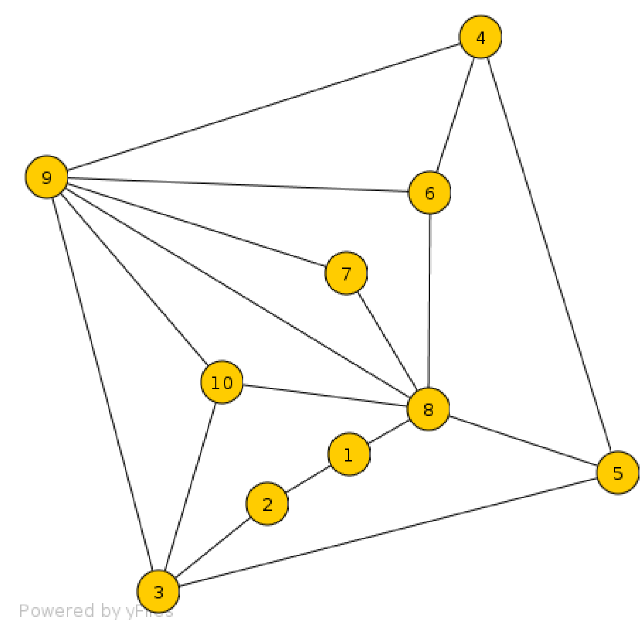
Graph and its Representation

$$
\begin{aligned}
V &= \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}\\
E &= \{\{v_1, v_2\}, \{v_1, v_8\}, \{v_2, v_3\}, \{v_3, v_5\}, \{v_3, v_9\},\\
&\quad \{v_3, v_{10}\}, \{v_4, v_5\}, \{v_4, v_6\}, \{v_4, v_9\}, \{v_5, v_8\},\\
&\quad \{v_6, v_8\}, \{v_6, v_9\}, \{v_7, v_8\}, \{v_7, v_9\}, \{v_8, v_{10}\},\\
&\quad \{v_9, v_{10}\}\}
\end{aligned}
$$

$$
\begin{array}{ll}
v_1: & v_2, v_8\\
v_2: & v_1, v_3\\
v_3: & v_2, v_5, v_9, v_{10}\\
v_4: & v_5, v_6, v_9\\
v_5: & v_3, v_4, v_8\\
v_6: & v_4, v_8, v_9\\
v_7: & v_8, v_9\\
v_8: & v_1, v_5, v_6, v_7, v_9, v_{10}\\
v_9: & v_3, v_4, v_6, v_7, v_8, v_{10}\\
v_{10}: & v_3, v_8, v_9
\end{array}
$$



$$
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1\\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0\\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0\\
1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1\\
0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0
\end{pmatrix}
$$

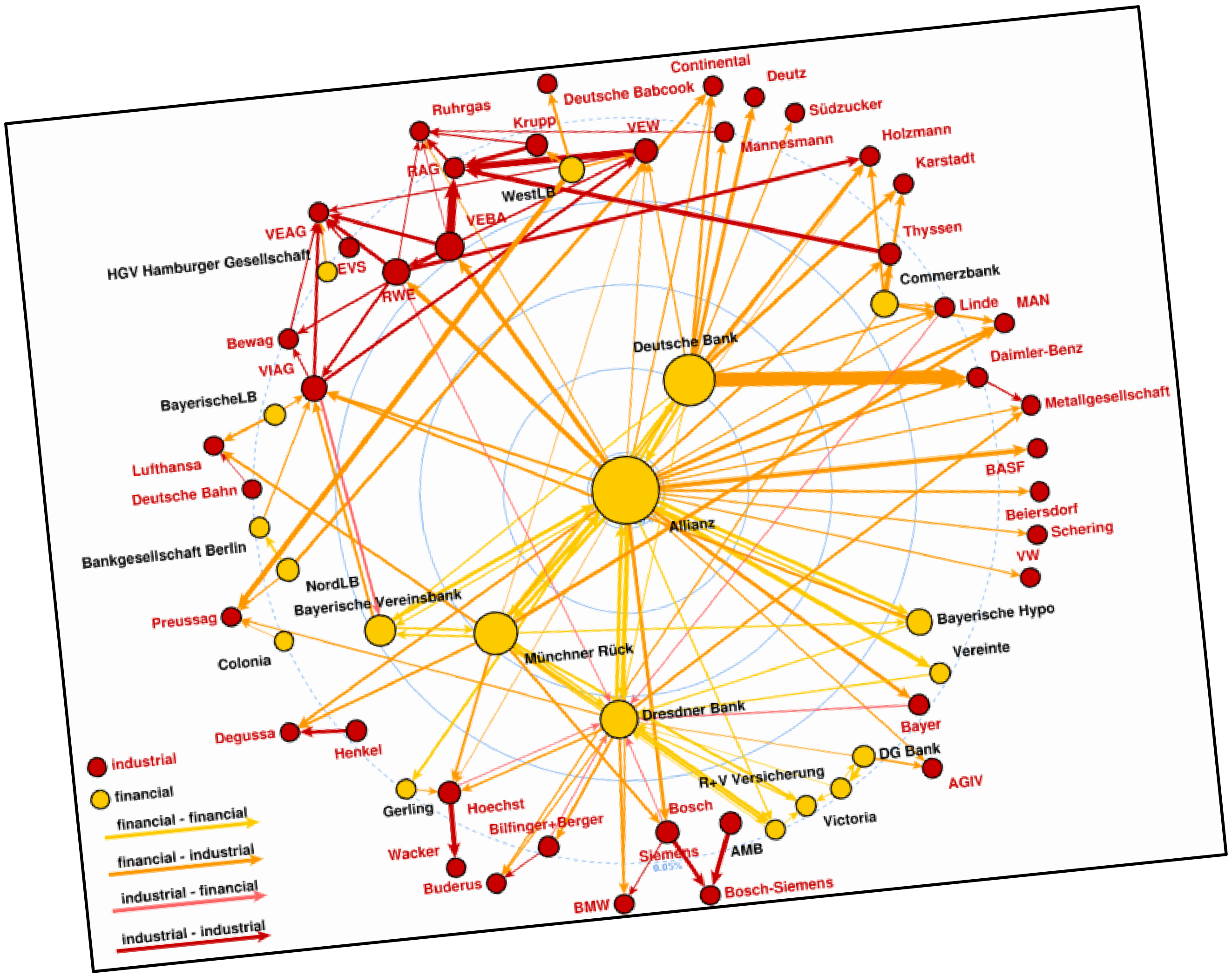


Why to draw graphs?

Graphs are mathematical models of real physical and abstract networks (social netowrks, metabolical networks, VLSI-network, UML-diagrams, citation networks, ...)

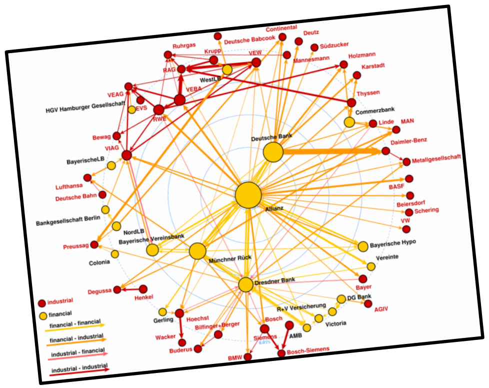
Why to draw graphs?

Graphs are mathematical models of real physical and abstract networks (social netowrks, metabolical networks, VLSI-network, UML-diagrams, citation networks, ...)



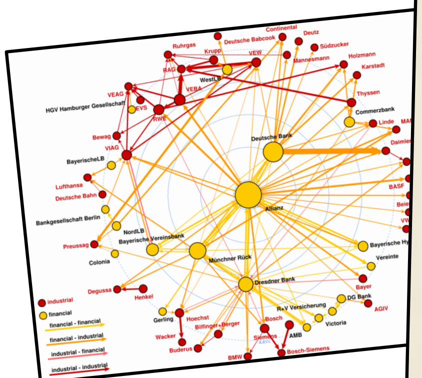
Why to draw graphs?

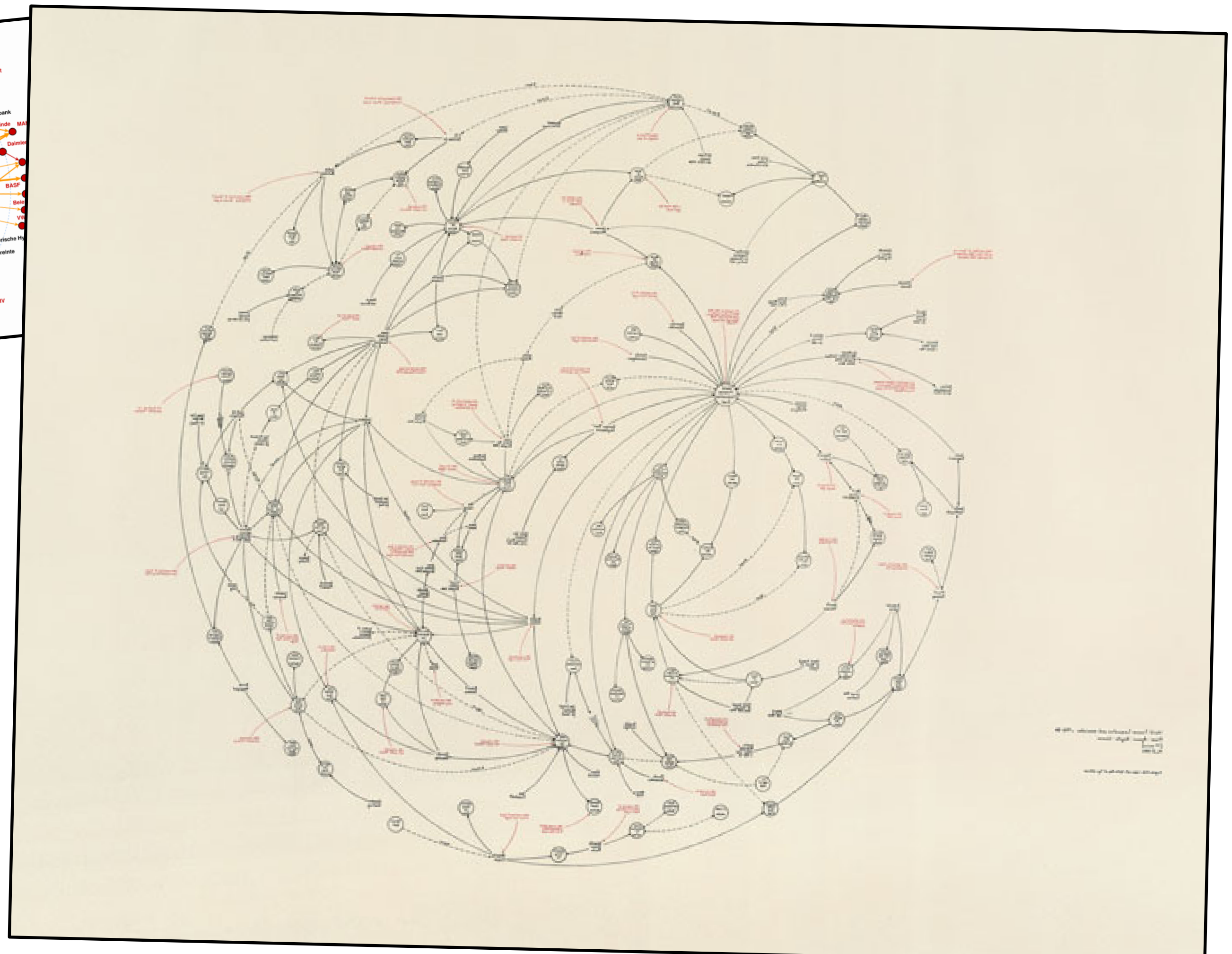
Graphs are mathematical models of real physical and abstract networks (social netowrks, metabolical networks, VLSI-network, UML-diagrams, citation networks, ...)



Why to draw graphs?

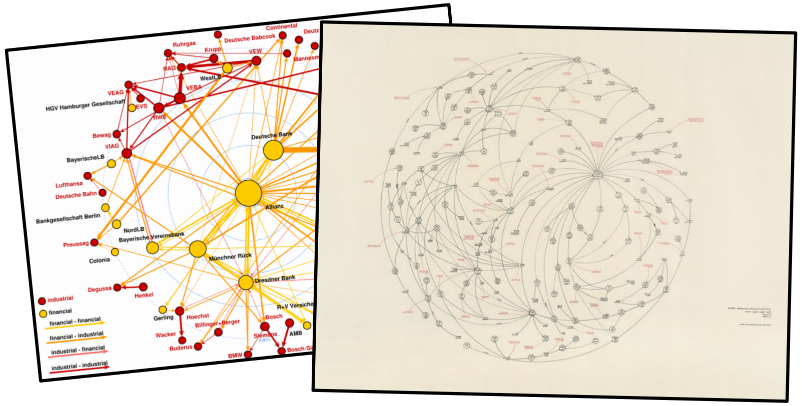
Graphs are mathematical models of real physical and abstract networks (social netowrks, metabolical networks, VLSI-network, UML-diagrams, citation networks, ...)





Why to draw graphs?

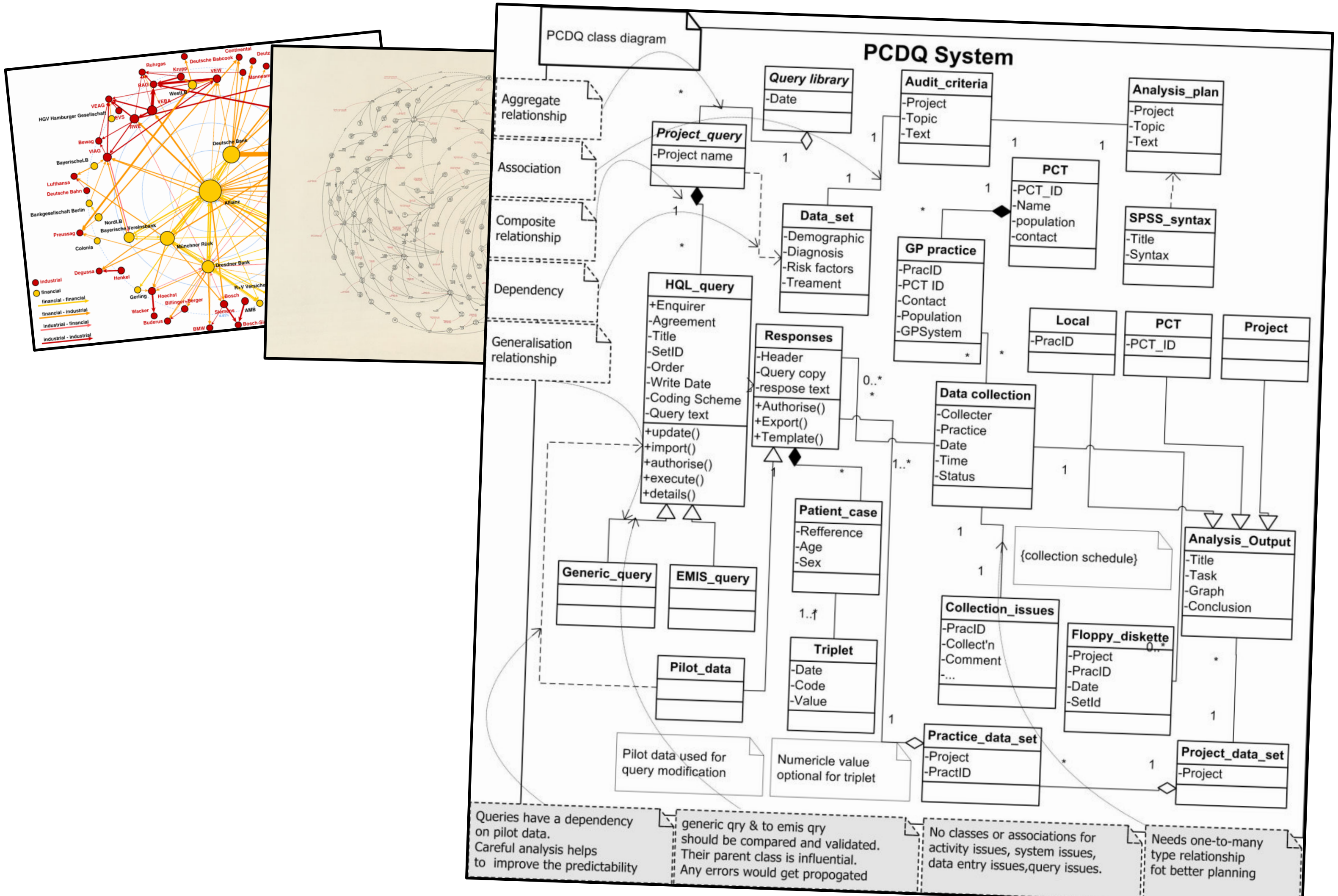
Graphs are mathematical models of real physical and abstract networks (social netowrks, metabolical networks, VLSI-network, UML-diagrams, citation networks, ...)



Karlsruhe Institute of Technology

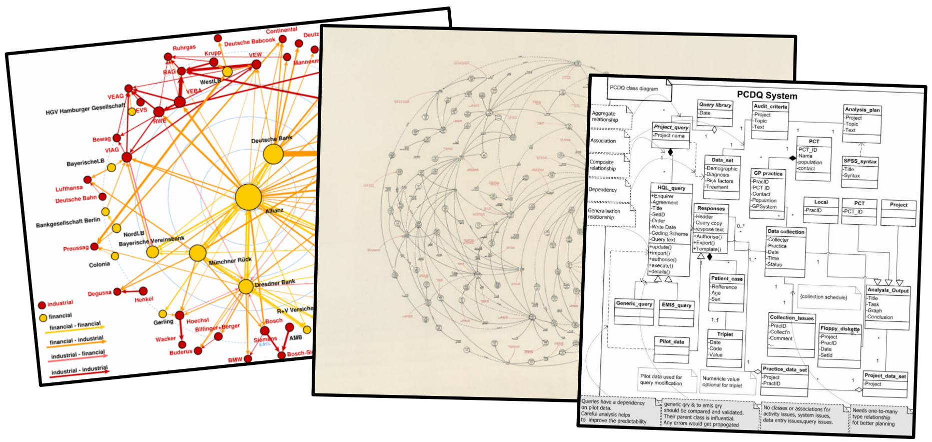
Why to draw graphs?

Graphs are mathematical models of real physical and abstract networks (social netowrks, metabolical networks, VLSI-network, UML-diagrams, citation networks, ...)



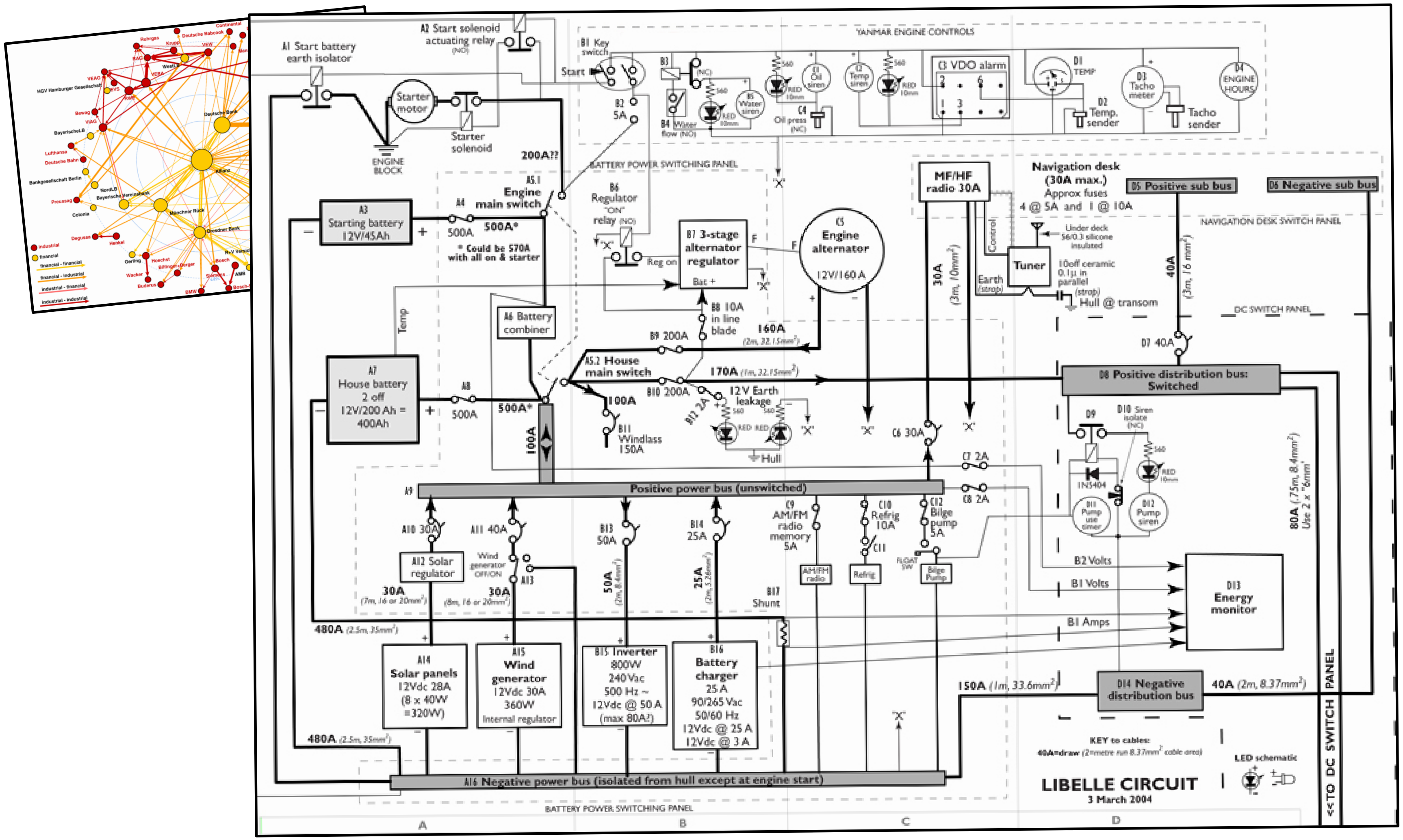
Why to draw graphs?

Graphs are mathematical models of real physical and abstract networks (social netowrks, metabolical networks, VLSI-network, UML-diagrams, citation networks, ...)



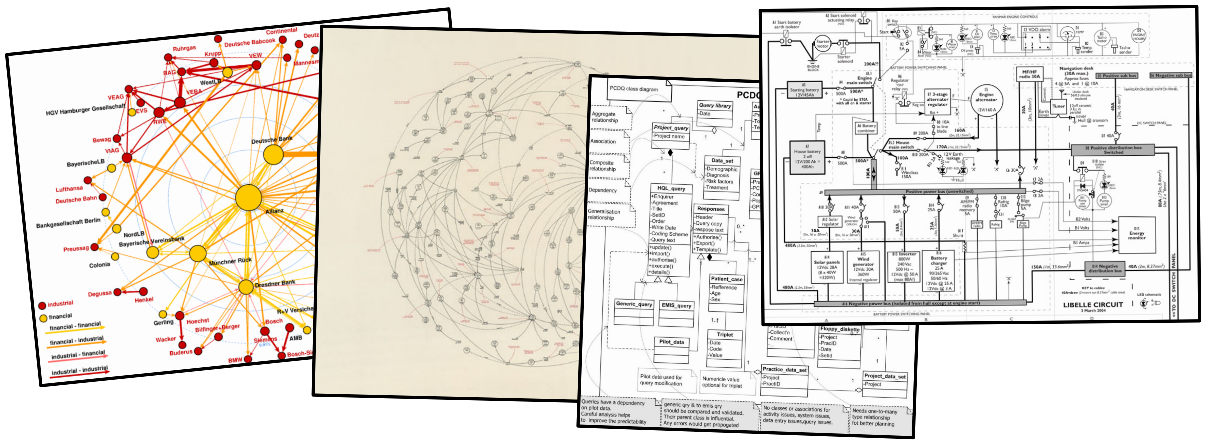
Why to draw graphs?

Graphs are mathematical models of real physical and abstract networks (social netowrks, metabolical networks, VLSI-network, UML-diagrams, citation networks, ...)



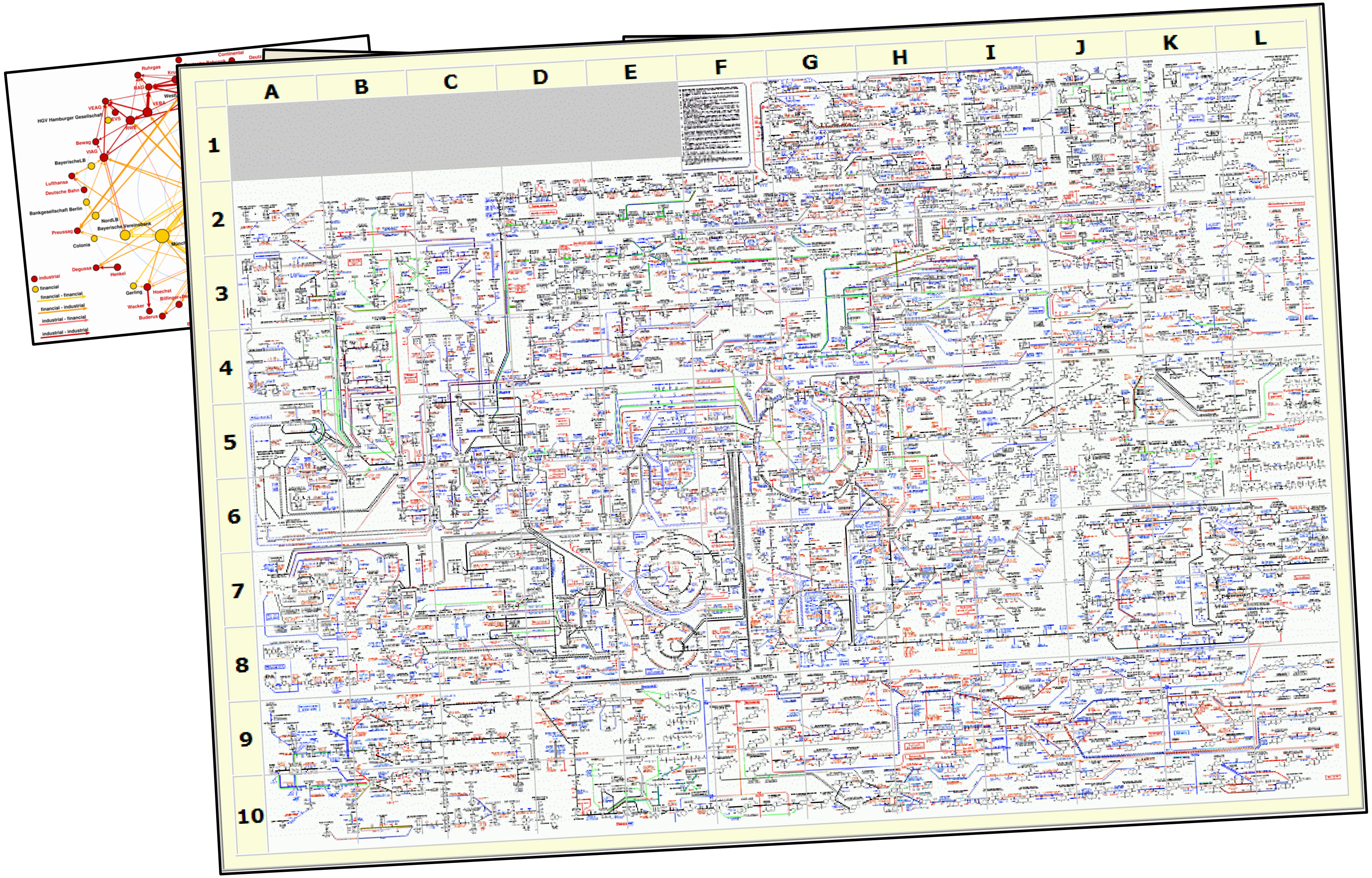
Why to draw graphs?

Graphs are mathematical models of real physical and abstract networks (social netowrks, metabolical networks, VLSI-network, UML-diagrams, citation networks, ...)



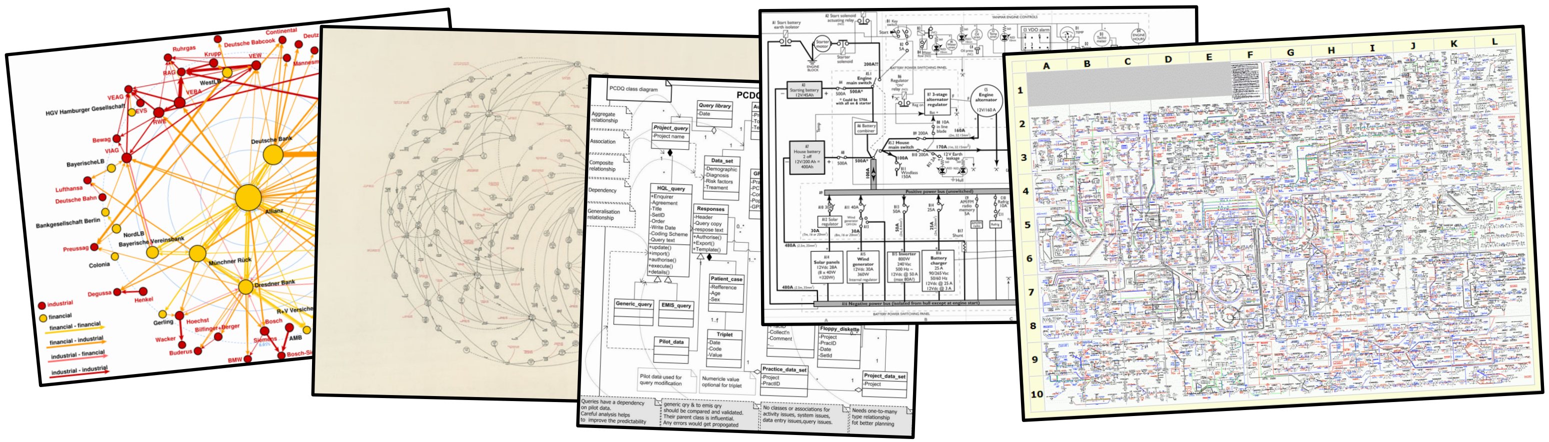
Why to draw graphs?

Graphs are mathematical models of real physical and abstract networks (social netowrks, metabolical networks, VLSI-network, UML-diagrams, citation networks, ...)



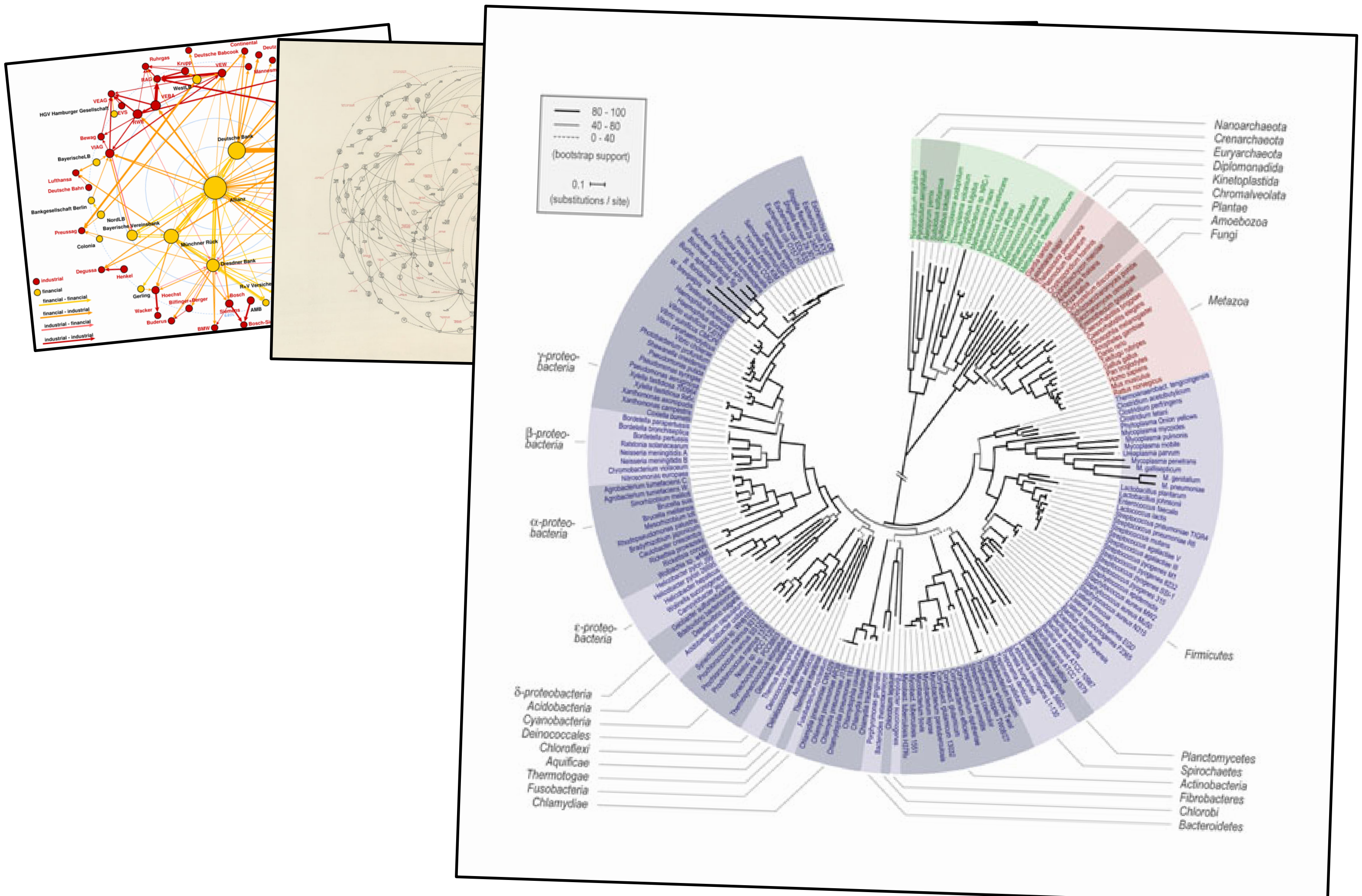
Why to draw graphs?

Graphs are mathematical models of real physical and abstract networks (social netowrks, metabolical networks, VLSI-network, UML-diagrams, citation networks, ...)



Why to draw graphs?

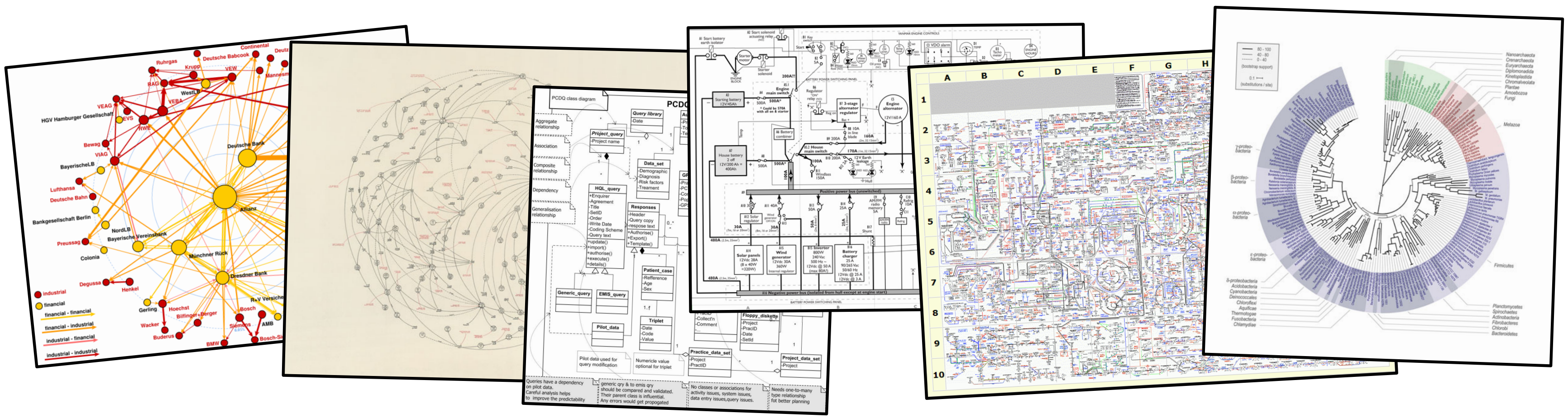
Graphs are mathematical models of real physical and abstract networks (social netowrks, metabolical networks, VLSI-network, UML-diagrams, citation networks, ...)



Why to draw graphs?

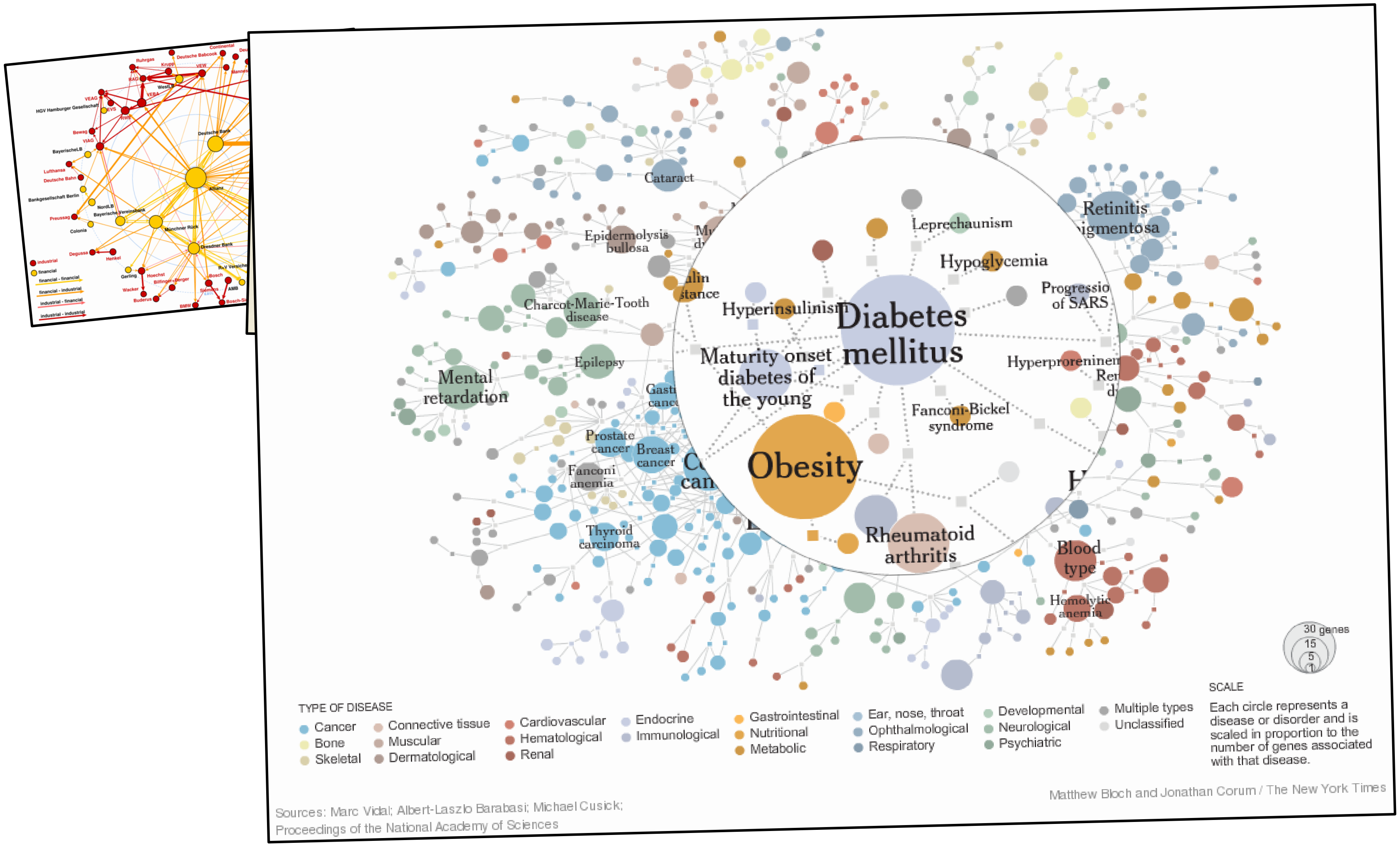
Karlsruhe Institute of Technology

Graphs are mathematical models of real physical and abstract networks (social netowrks, metabolical networks, VLSI-network, UML-diagrams, citation networks, ...)



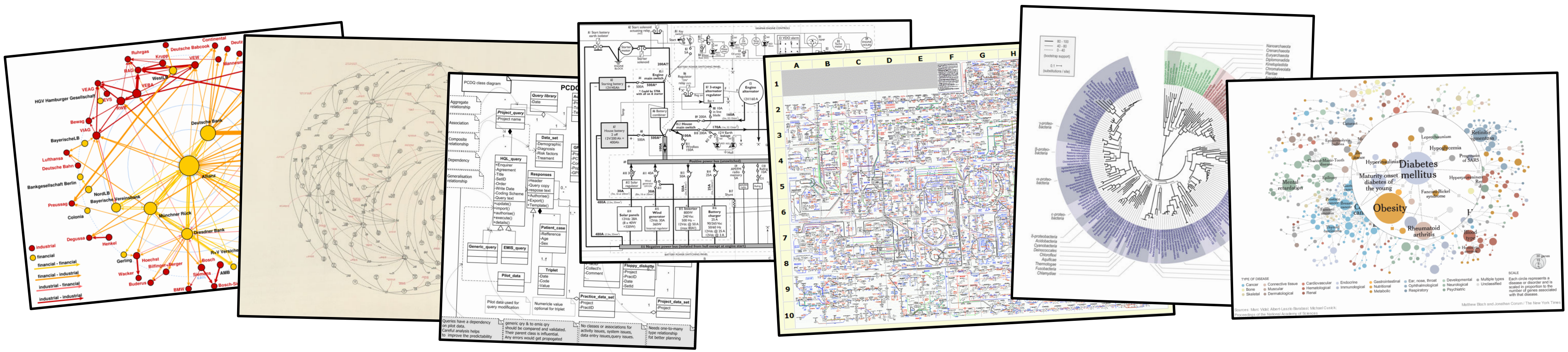
Why to draw graphs?

Graphs are mathematical models of real physical and abstract networks (social netowrks, metabolical networks, VLSI-network, UML-diagrams, citation networks, ...)



Why to draw graphs?

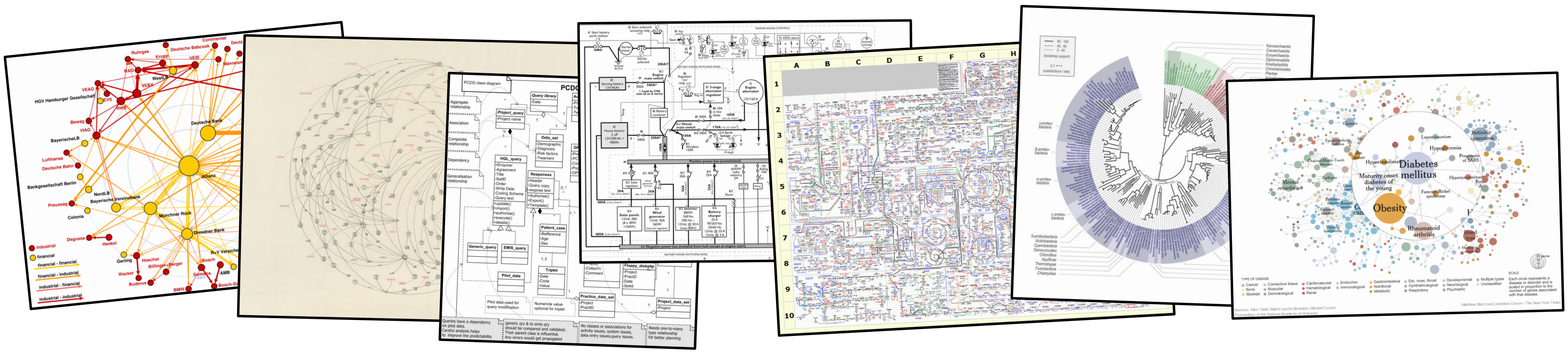
Graphs are mathematical models of real physical and abstract networks (social netowrks, metabolical networks, VLSI-network, UML-diagrams, citation networks, ...)



Why to draw graphs?

Karlsruhe Institute of Technology

Graphs are mathematical models of real physical and abstract networks (social netowrks, metabolical networks, VLSI-network, UML-diagrams, citation networks, ...)



- **People think visually** – without a good visualization, complex graphs are not understandable to us
- A visualization helps to **communicate** and **explore** the graphs/networks
- We need **algorithms** to draw graphs, and make graphs and networks accessible to people

How to draw graphs?

Guiding questions

- what makes a drawing good?
- what is a bad drawing?
- are there measures to quantify that?

How to draw graphs?

Guiding questions

- what makes a drawing good?
- what is a bad drawing?
- are there measures to quantify that?

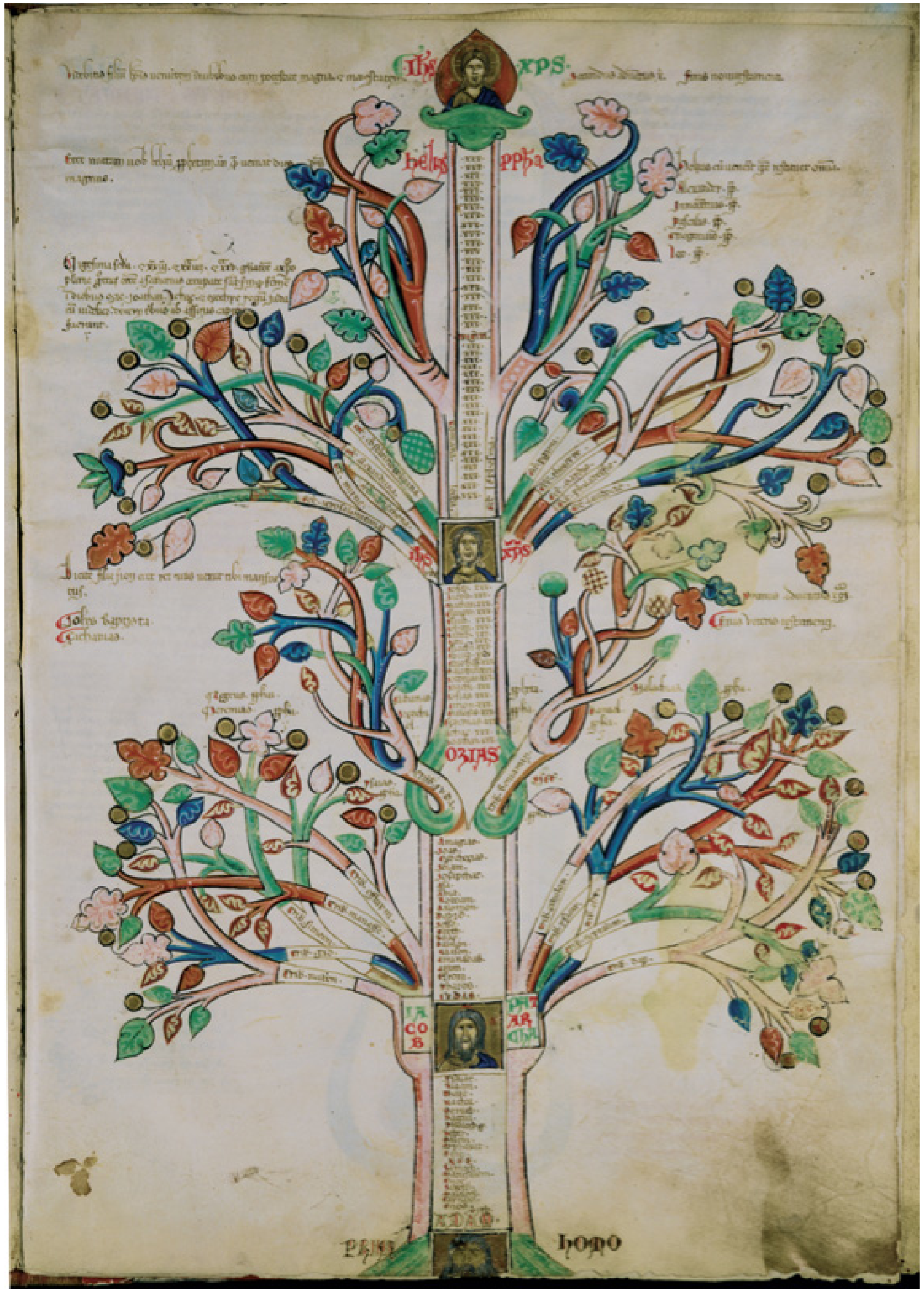
Warm-up WE WILL DRAW COUPLE OF GRAPHS

- Teams of two
- ”tasks“ in form of adjacency matrix/list
- Given time 15 minutes
- We will shortly discuss the results afterwards
- https://www.yworks.com/downloads#yEd

Network Visualizations

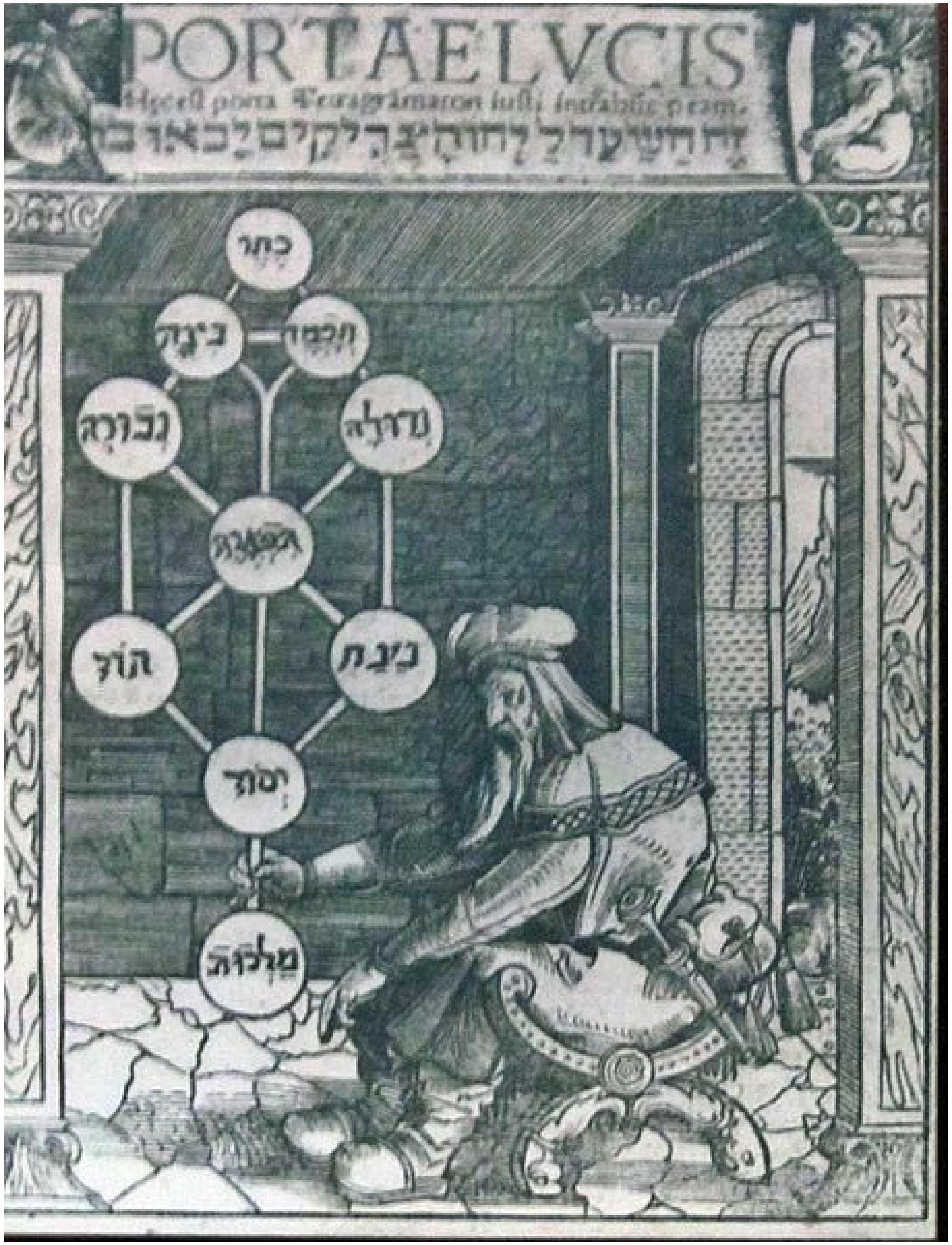
a small Film

Biblical characters and events (1202)



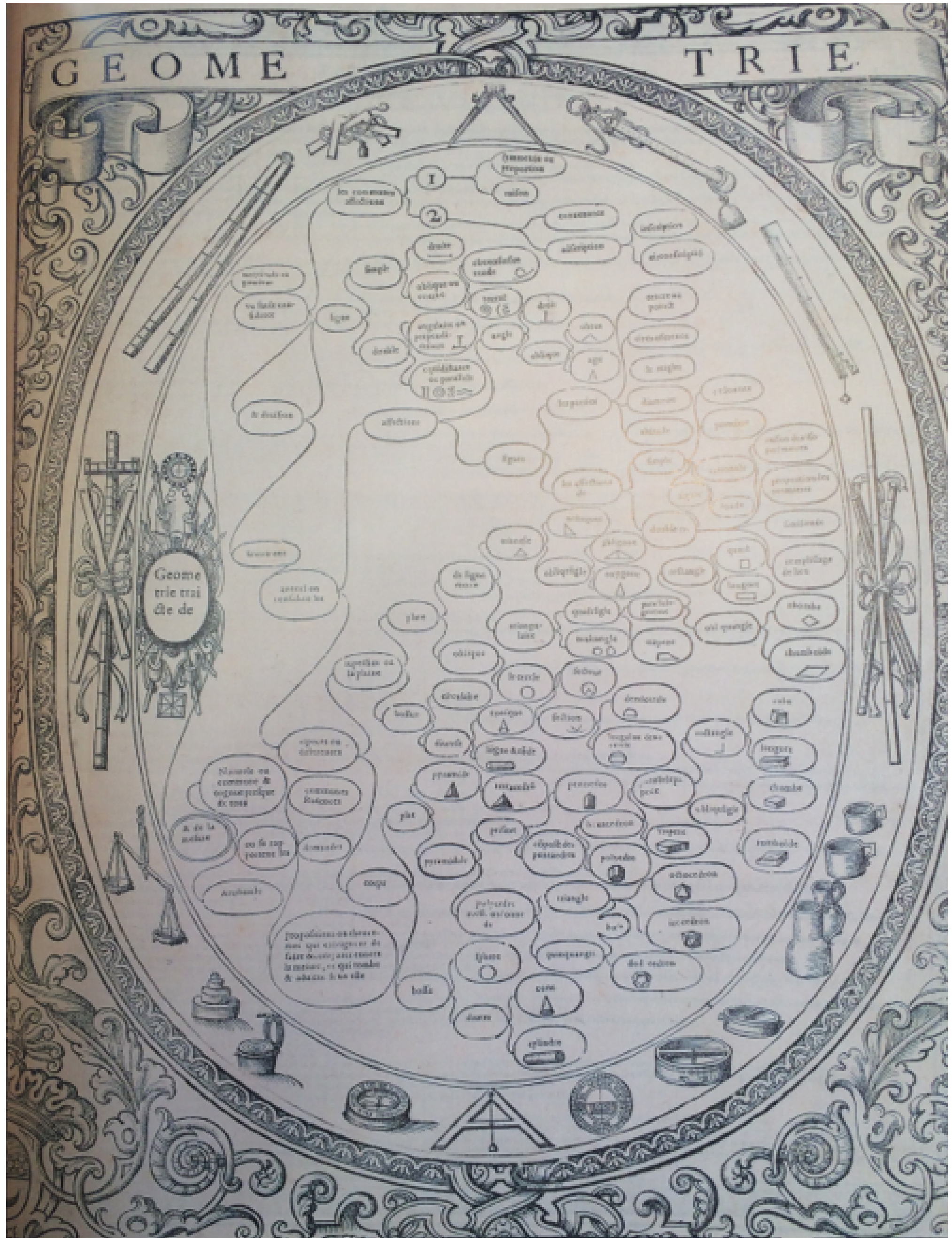
Source: Joachim de Fiore

"Tree of Life" (1516)



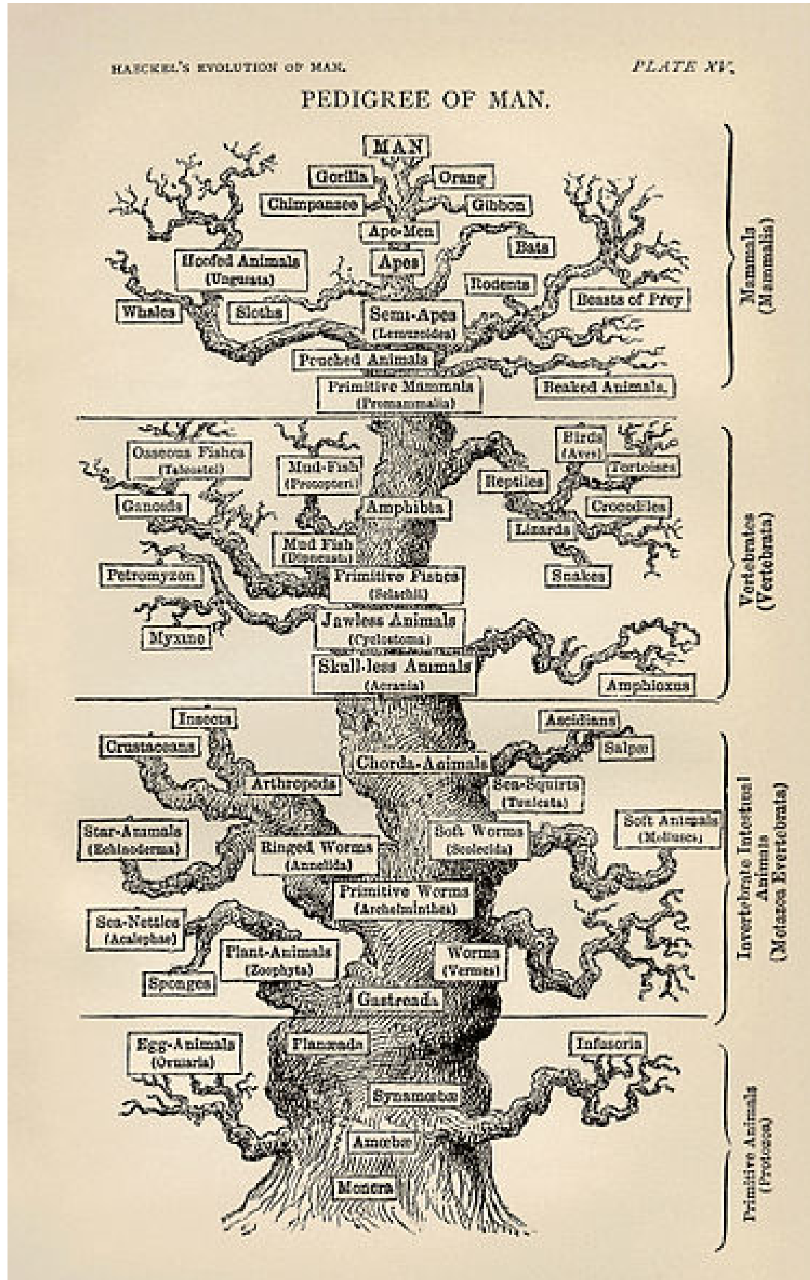
Source: Paul Riccius, Portae Lucis

Geometrical Concepts (1587)



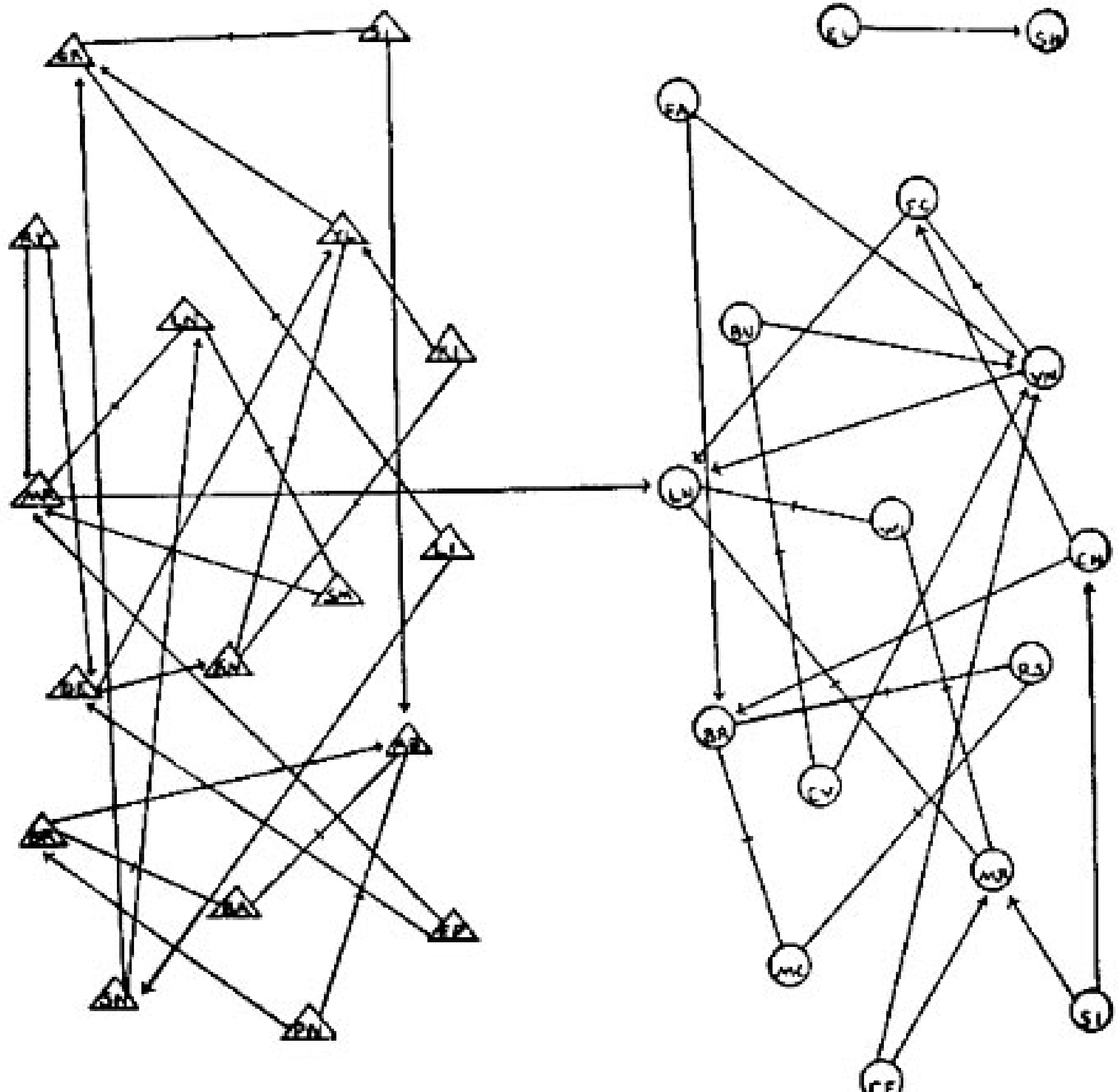
Source: Christophe de Savigny

Genealogical Tree (1879)



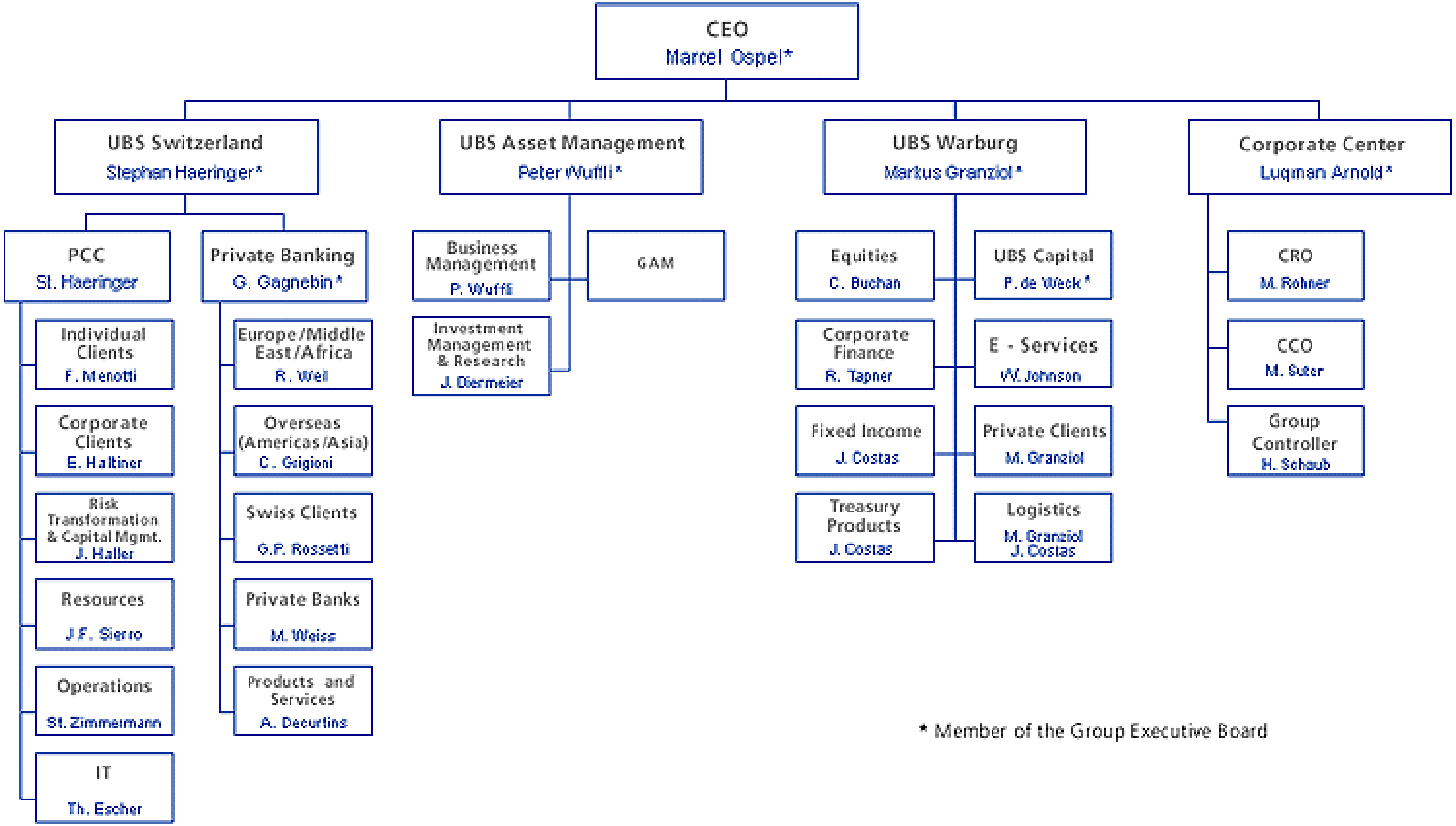
Source: Ernst Haeckel

Sociogramm (1933)



Source: Moreno, 1933

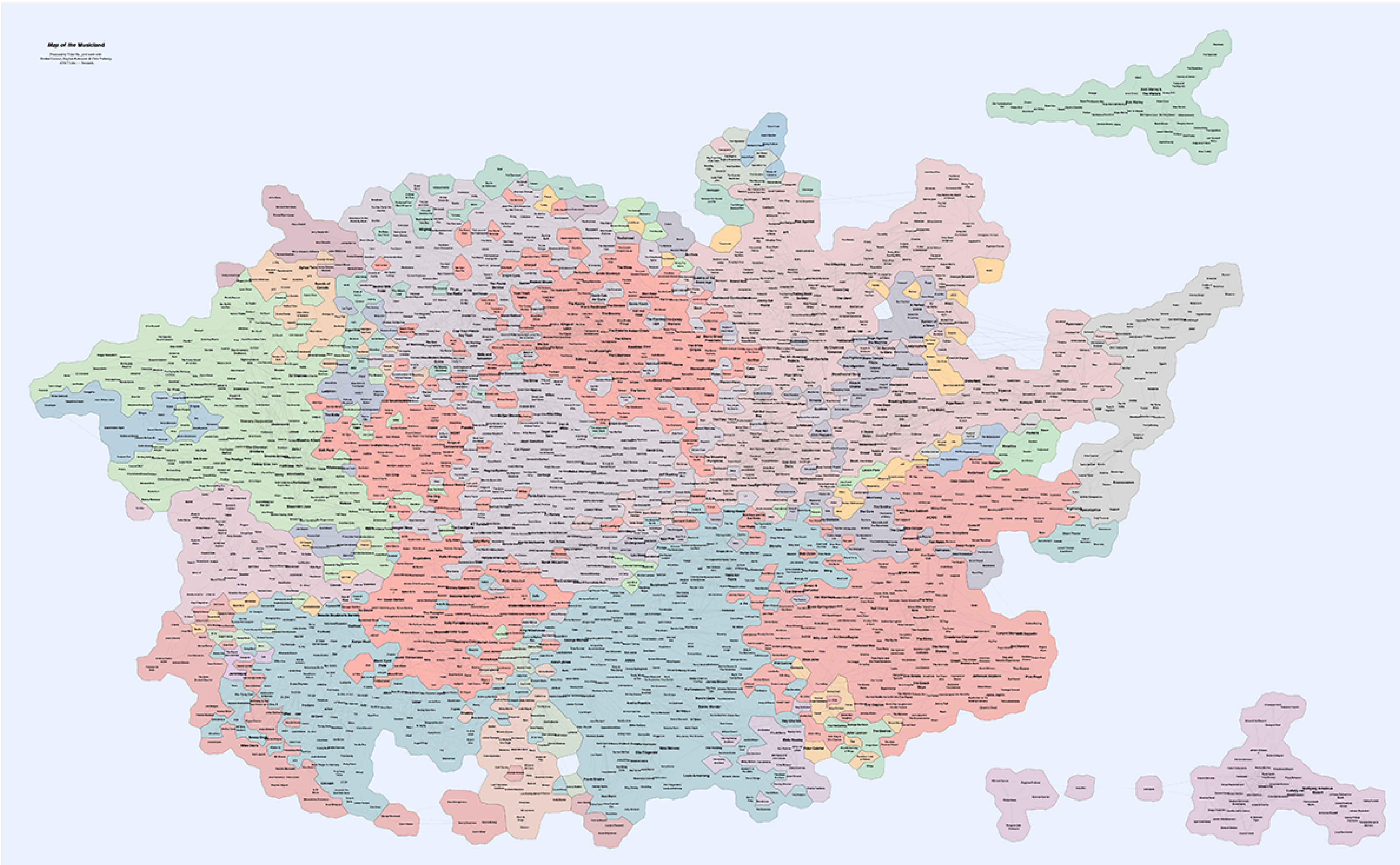
Social Network – Organization within UBS



CPAN Developer-Graph

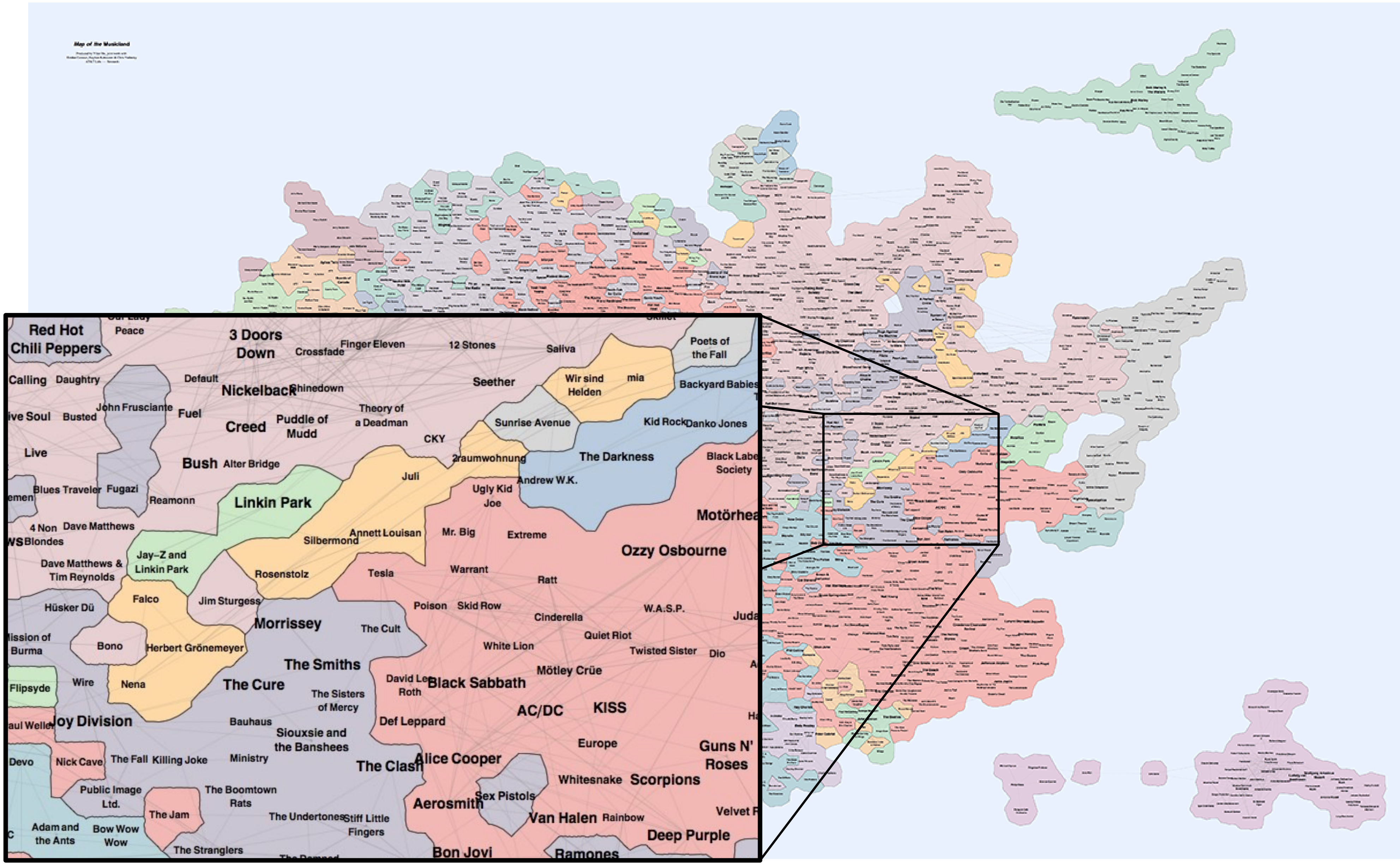
Source: cpan-explorer.org

last.fm Graph of musics as political map



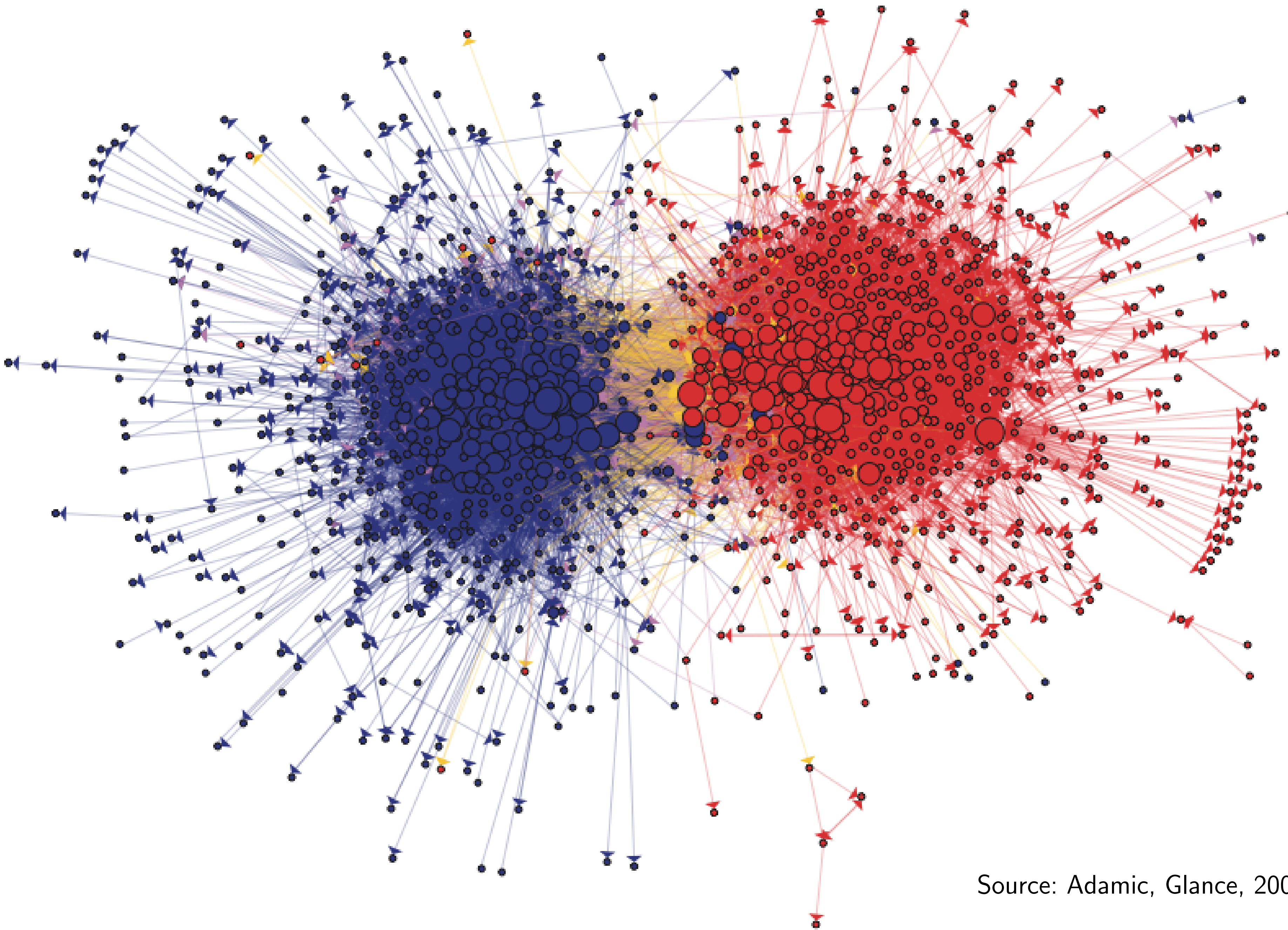
(Gansner, Hu, Kobourov: GMap, 2009)

last.fm Graph of musics as political map



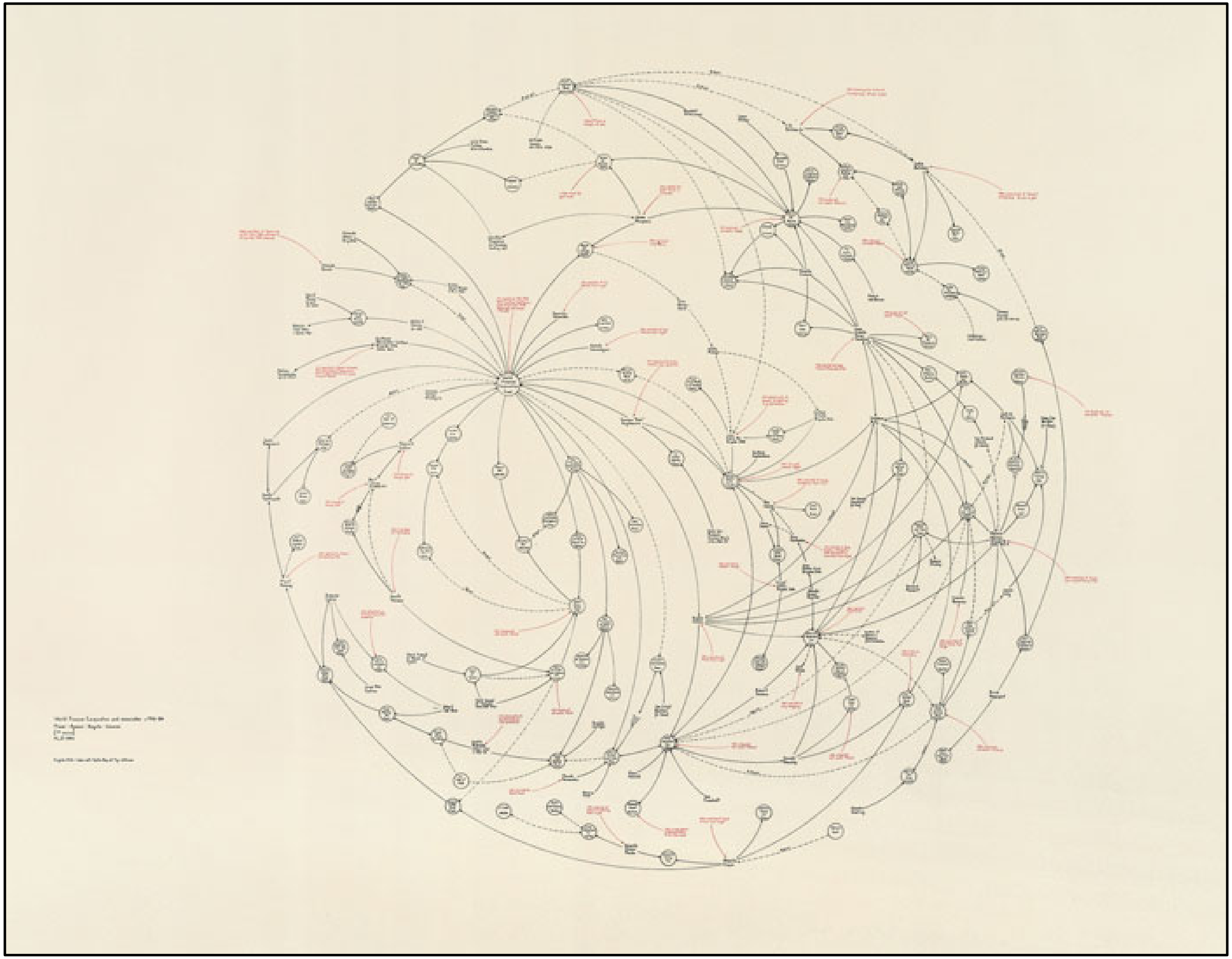
(Gansner, Hu, Kobourov: GMap, 2009)

Blogosphere 2004 Elections USA



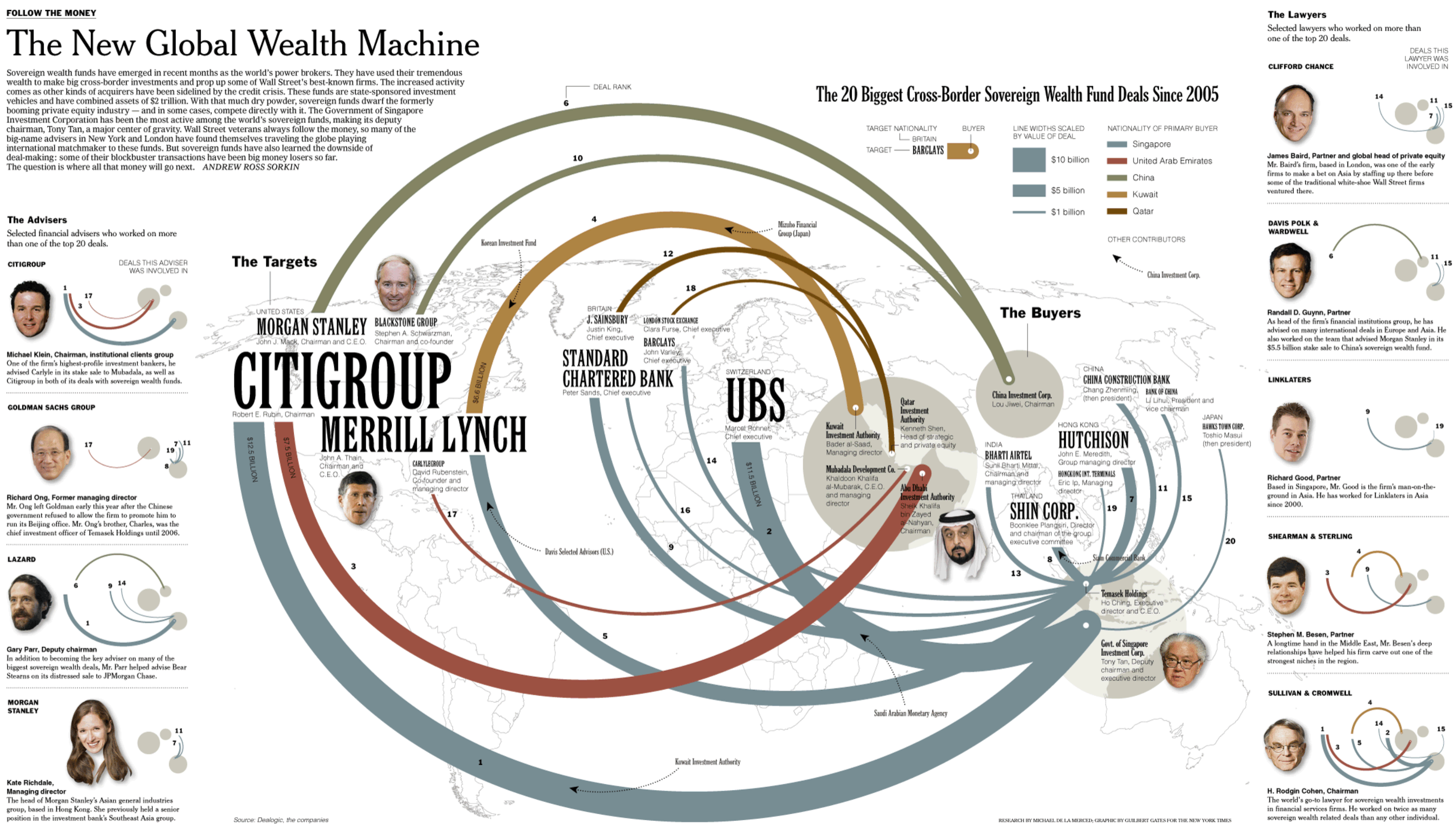
Source: Adamic, Glance, 2005

Social Network – World Finance System

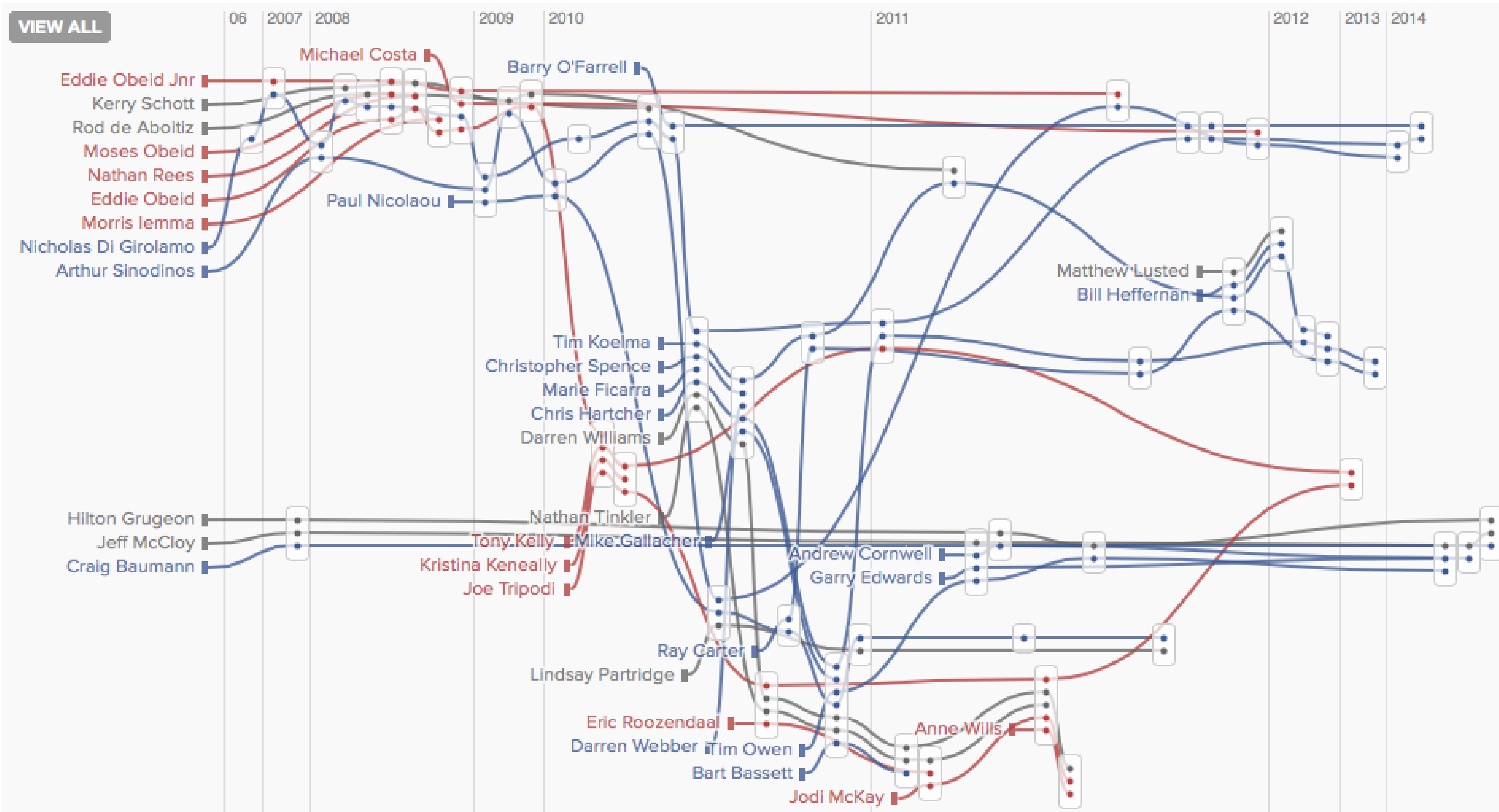


World Finance Corporation
© Mark Lombardi

Social Networks – State Funds

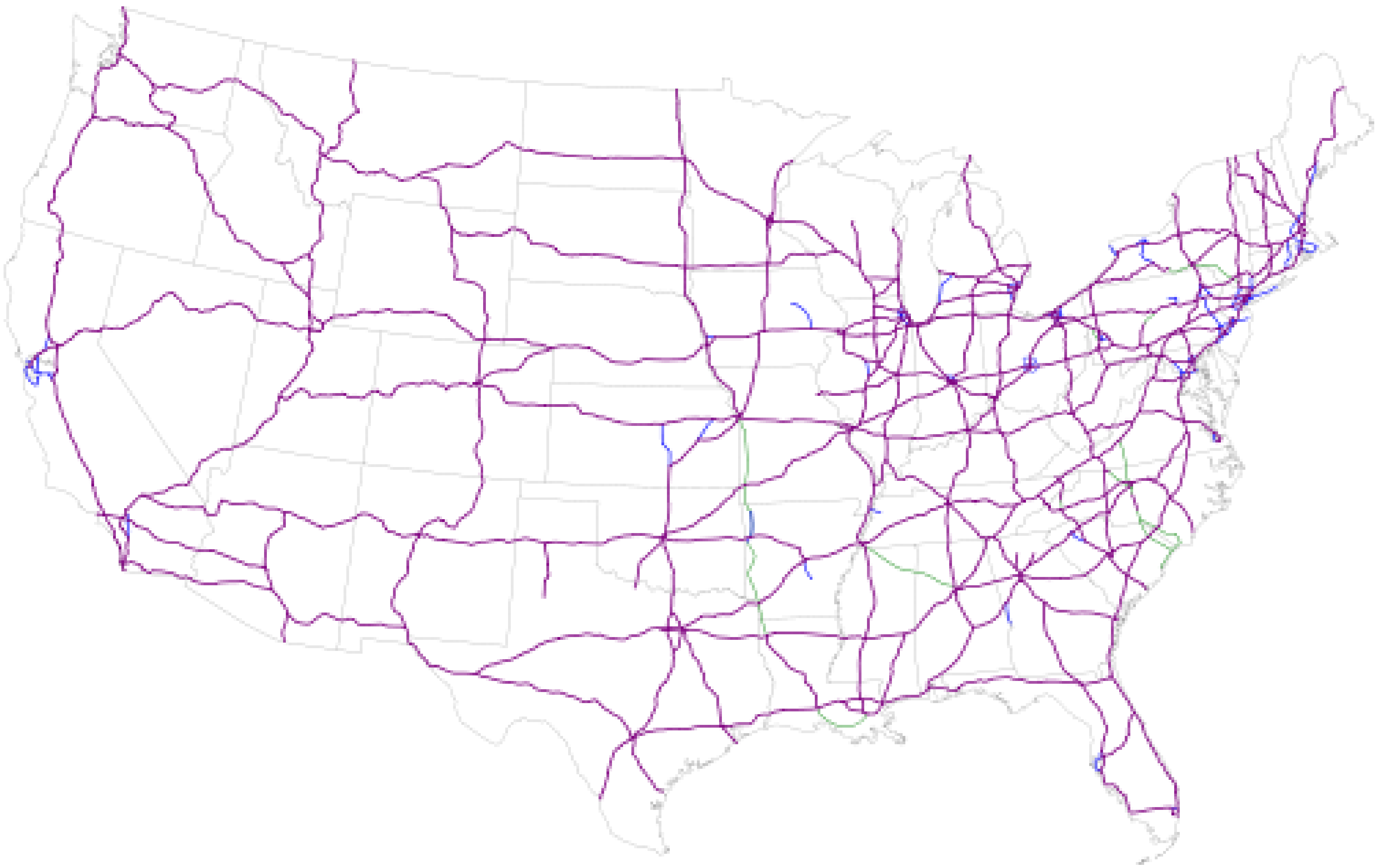


Temporal Graph Layout: Storylines

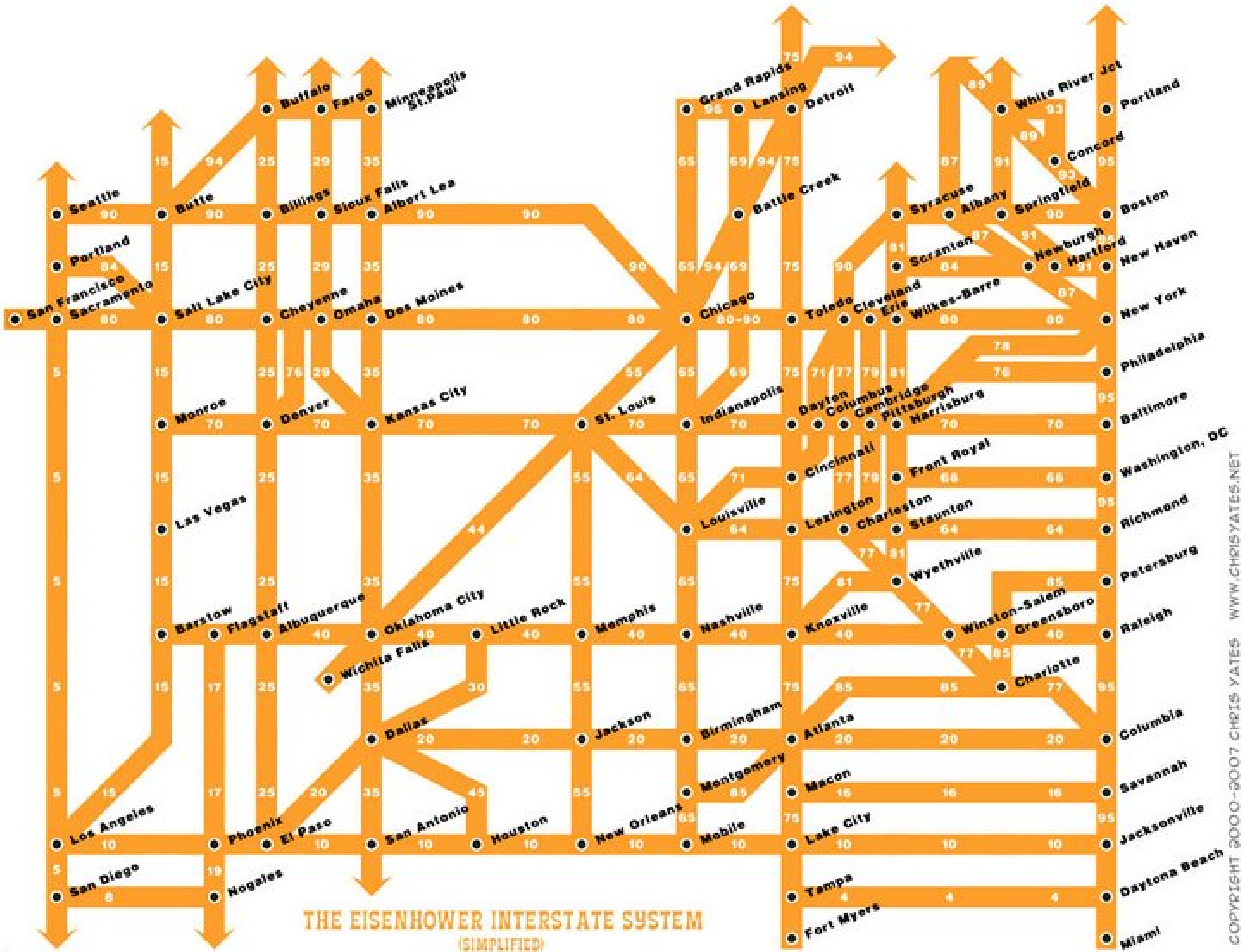


Source: ABC news, Australia

Traffic network – Highways USA



Traffic network – Highways USA

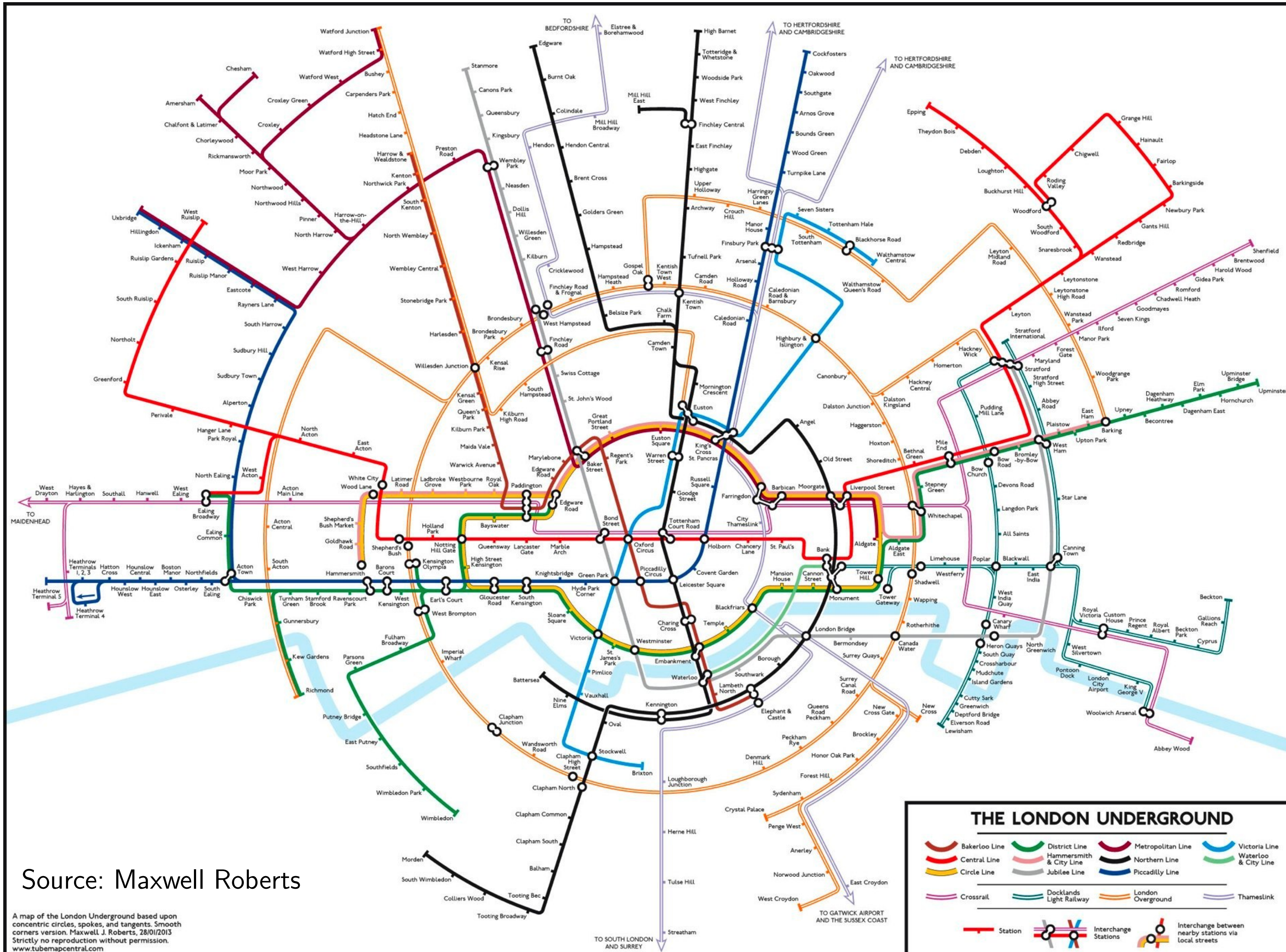


London Tube Map (1933)

Source: Henry Beck

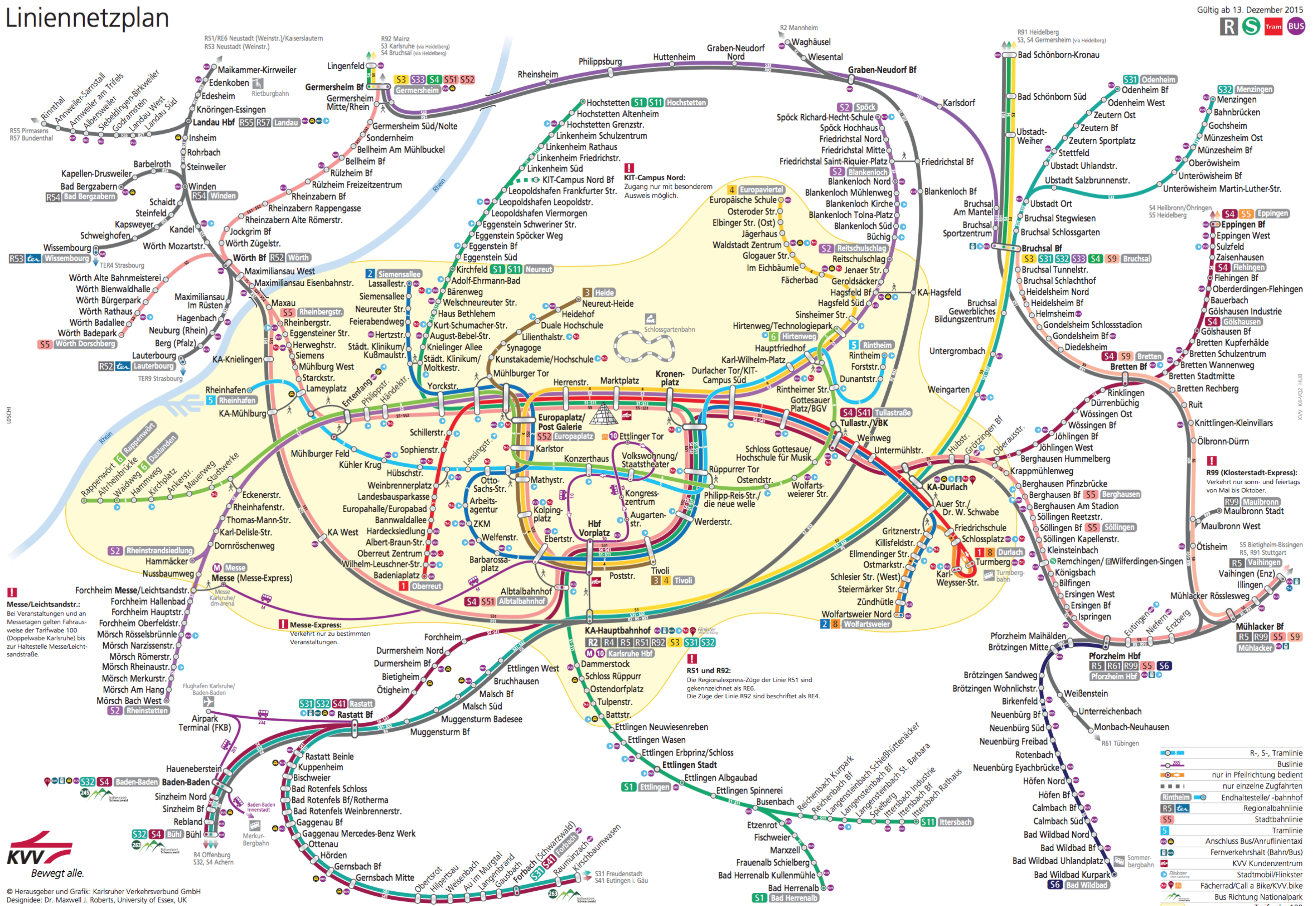
Co-centric Tube Map

Karlsruhe Institute of Technology



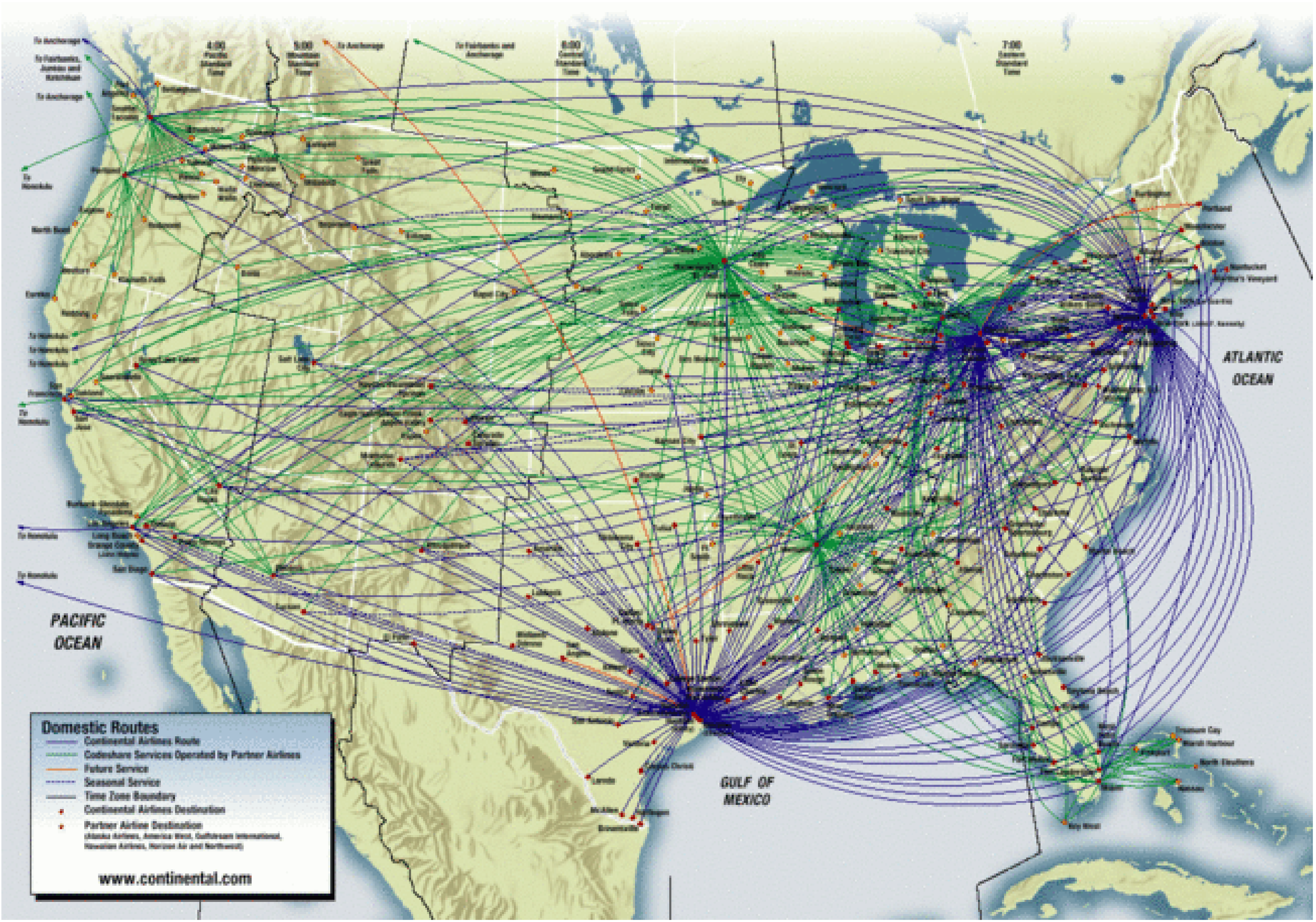
Source: Maxwell Roberts

Curvilinear S/U-bahn map

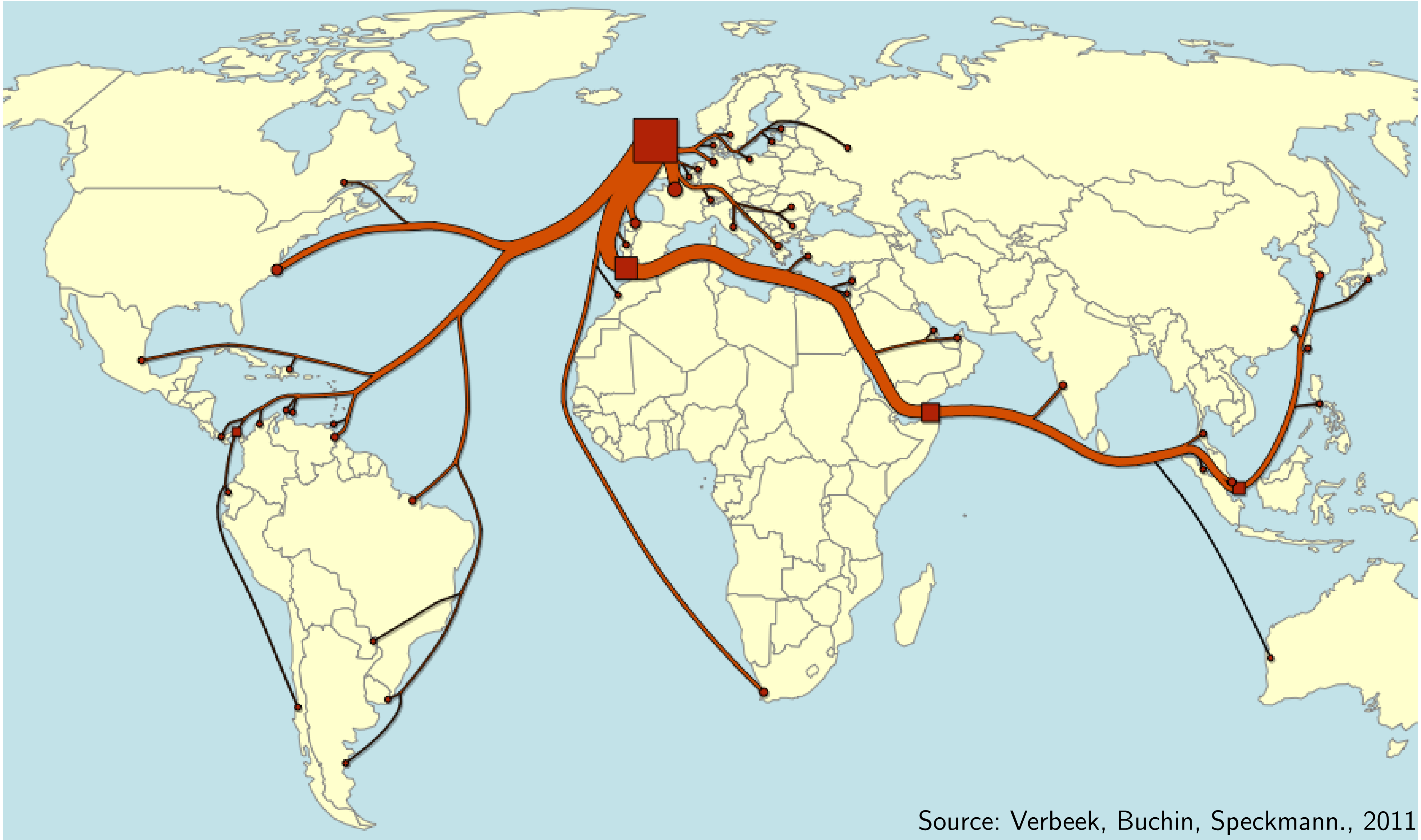


Source: KVV and Maxwell Roberts

Flight Connections

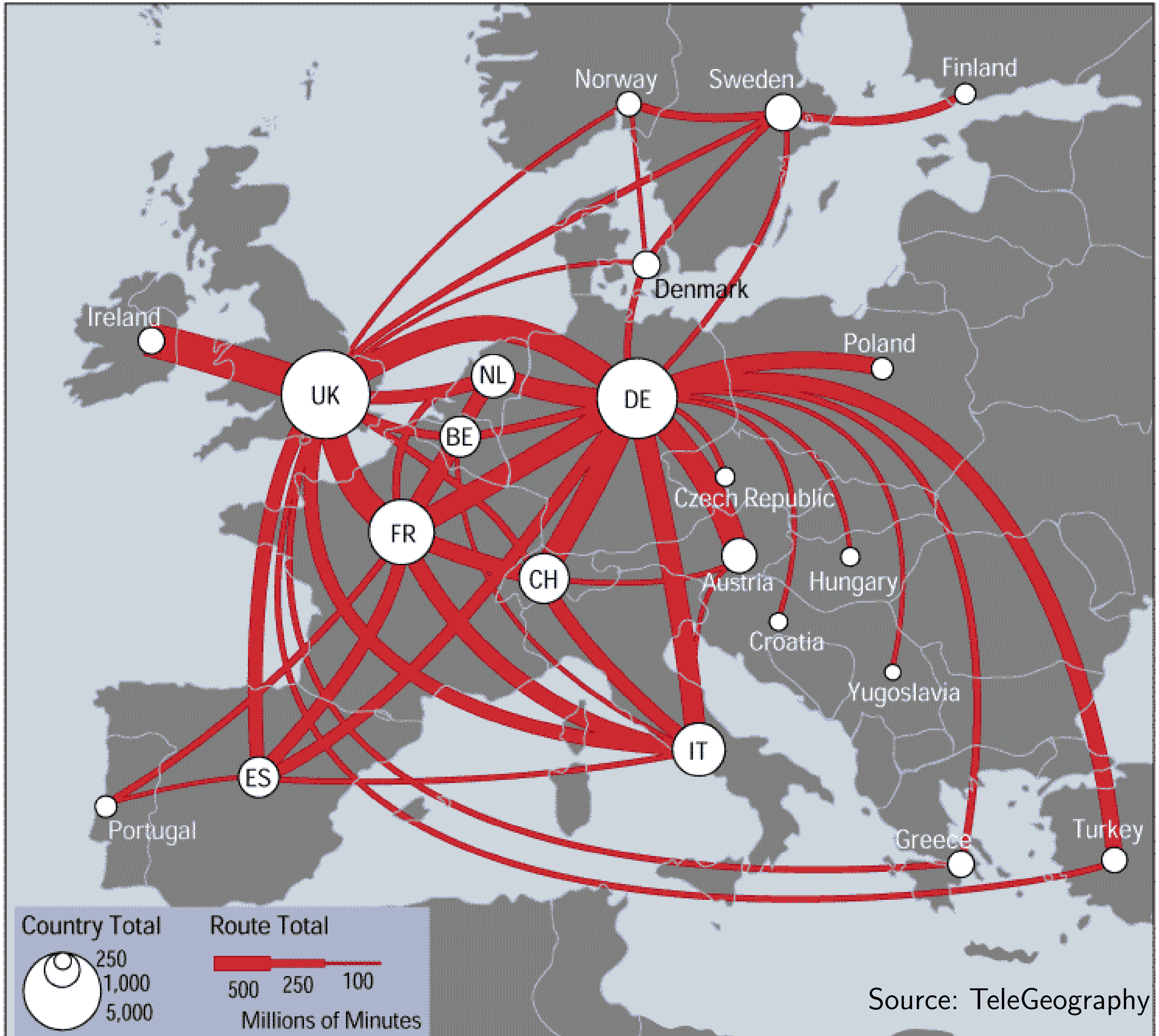


Flow-Map: Whiskey Export



Source: Verbeek, Buchin, Speckmann., 2011

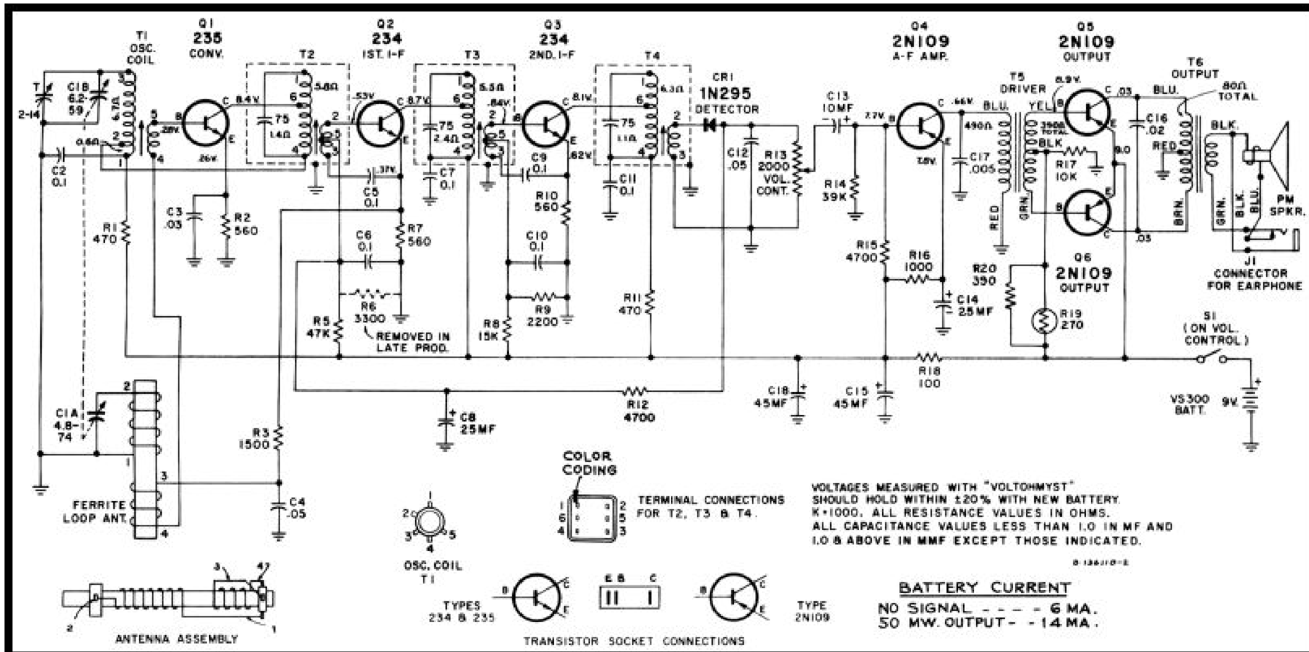
Telephony Map

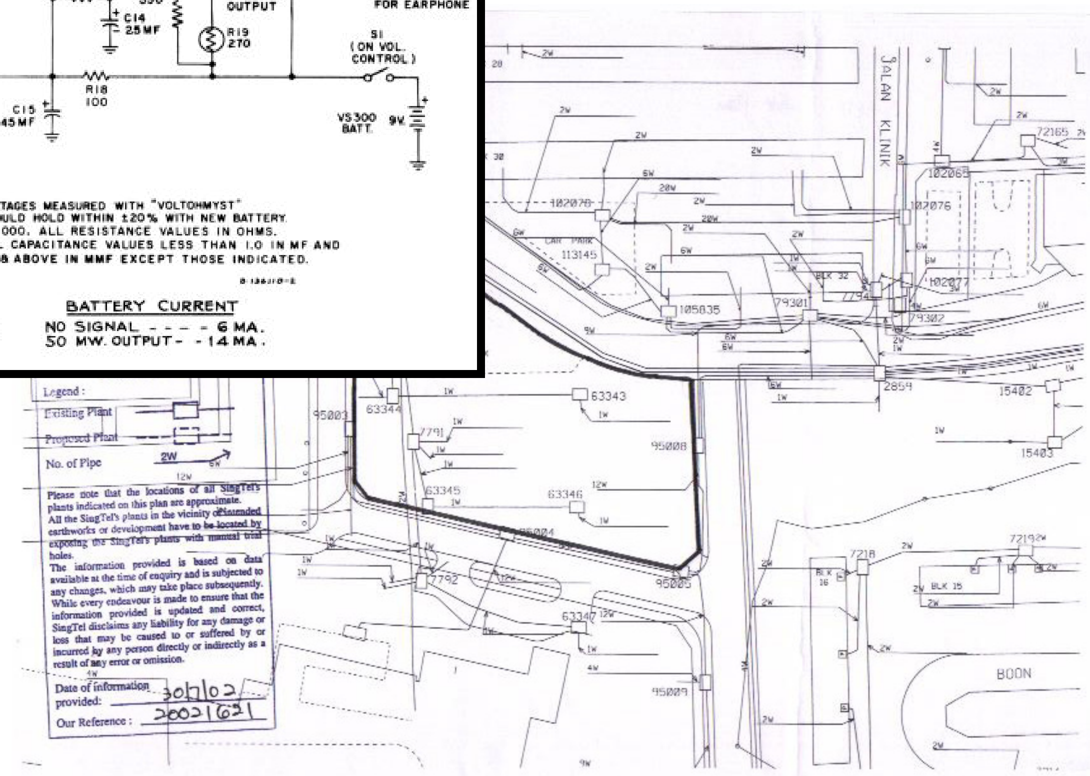


Monitoring of Energy Network

Source: Eir Grid, Irland

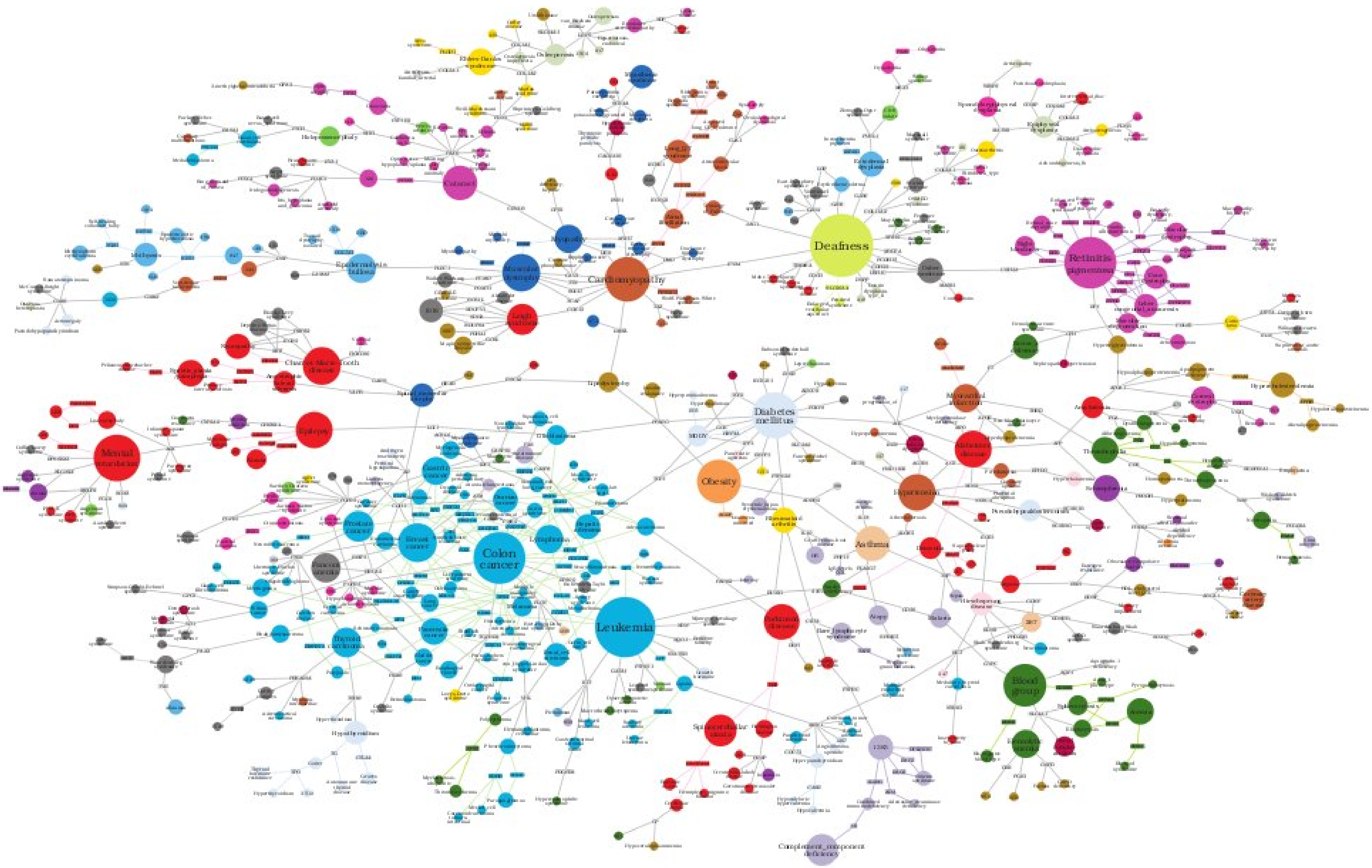
Wiring plan/ Cable plan



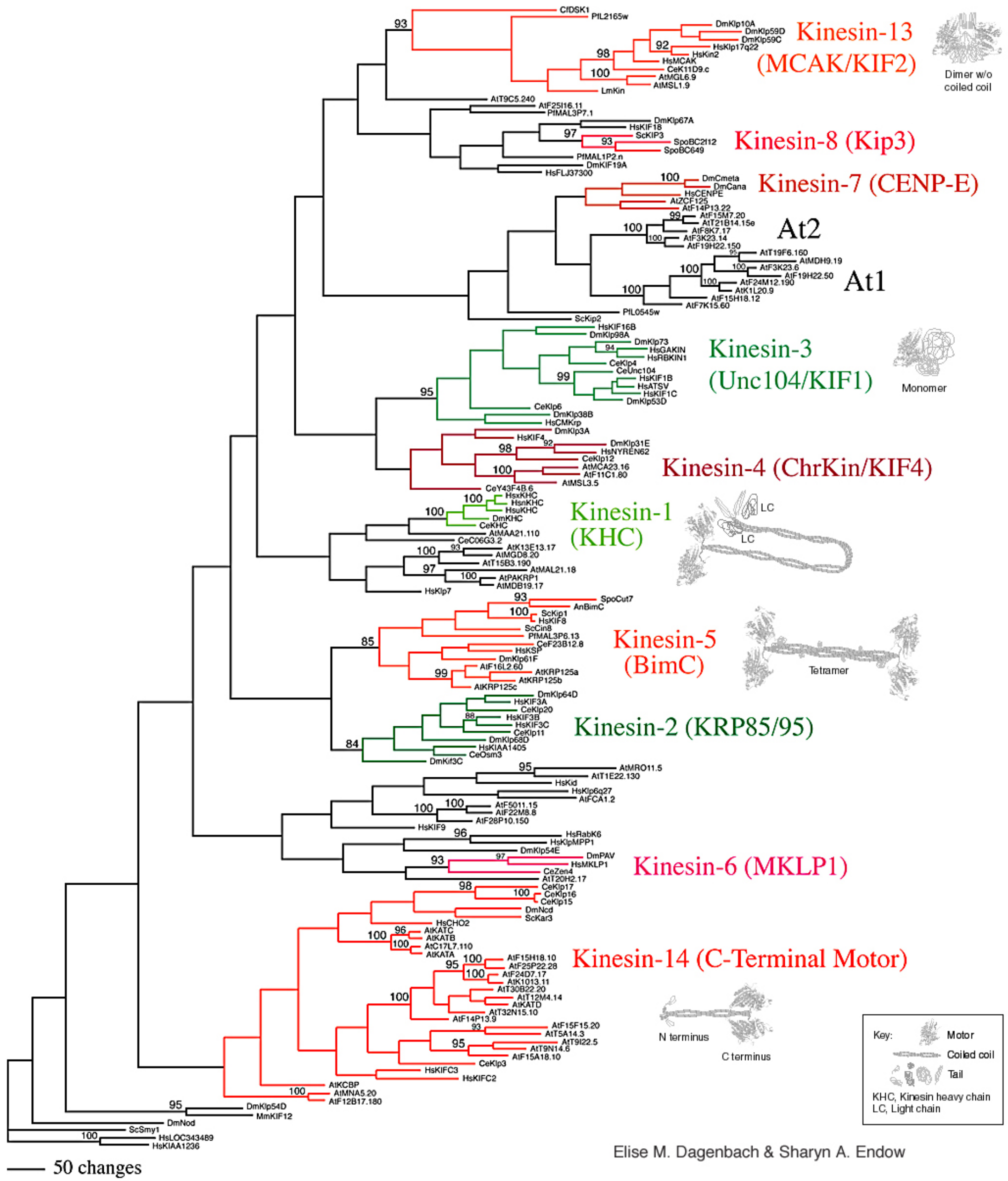


Medicine – Deseases

Karlsruhe Institute of Technology

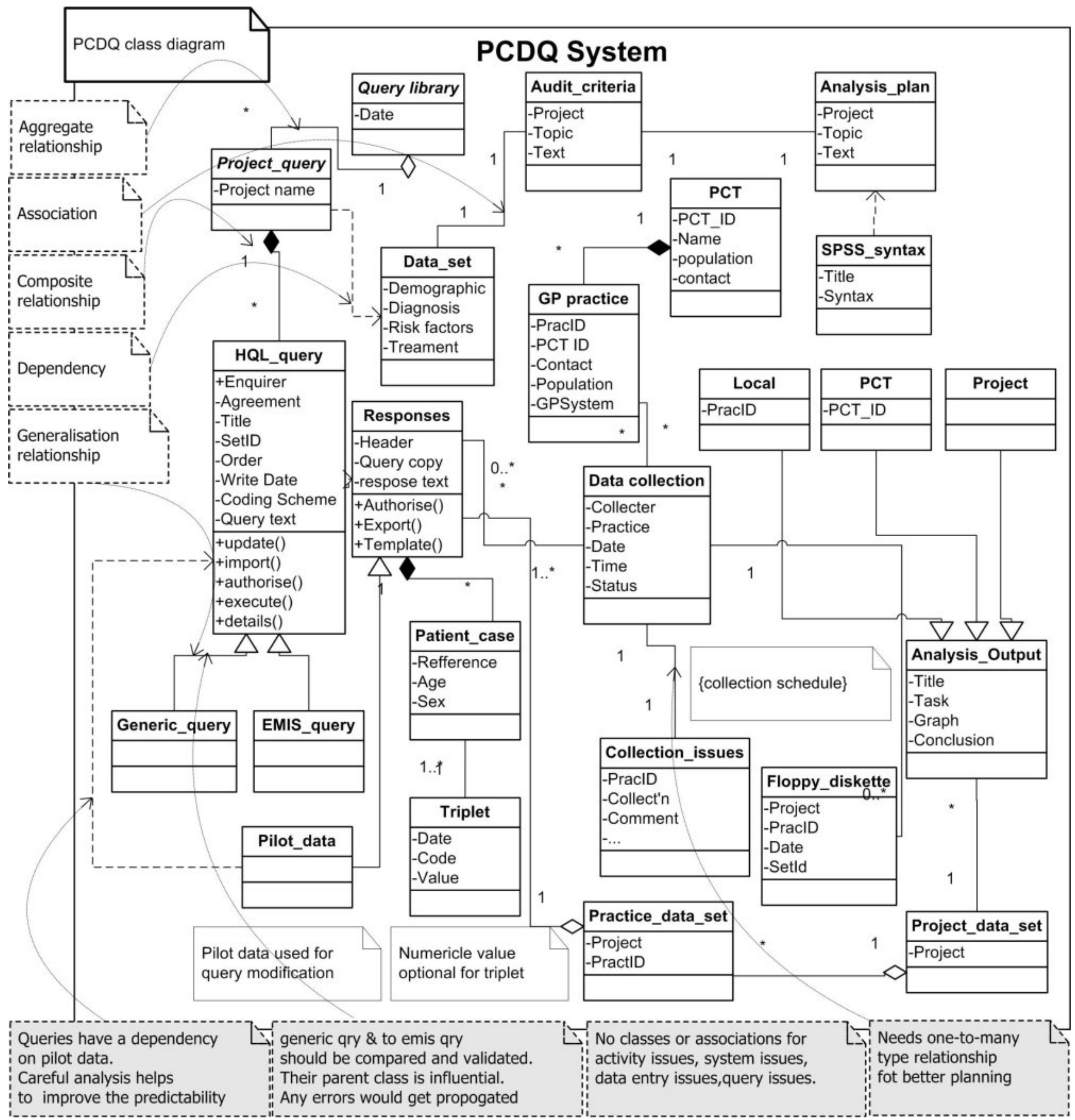


Medicine – phylogenetic Tree

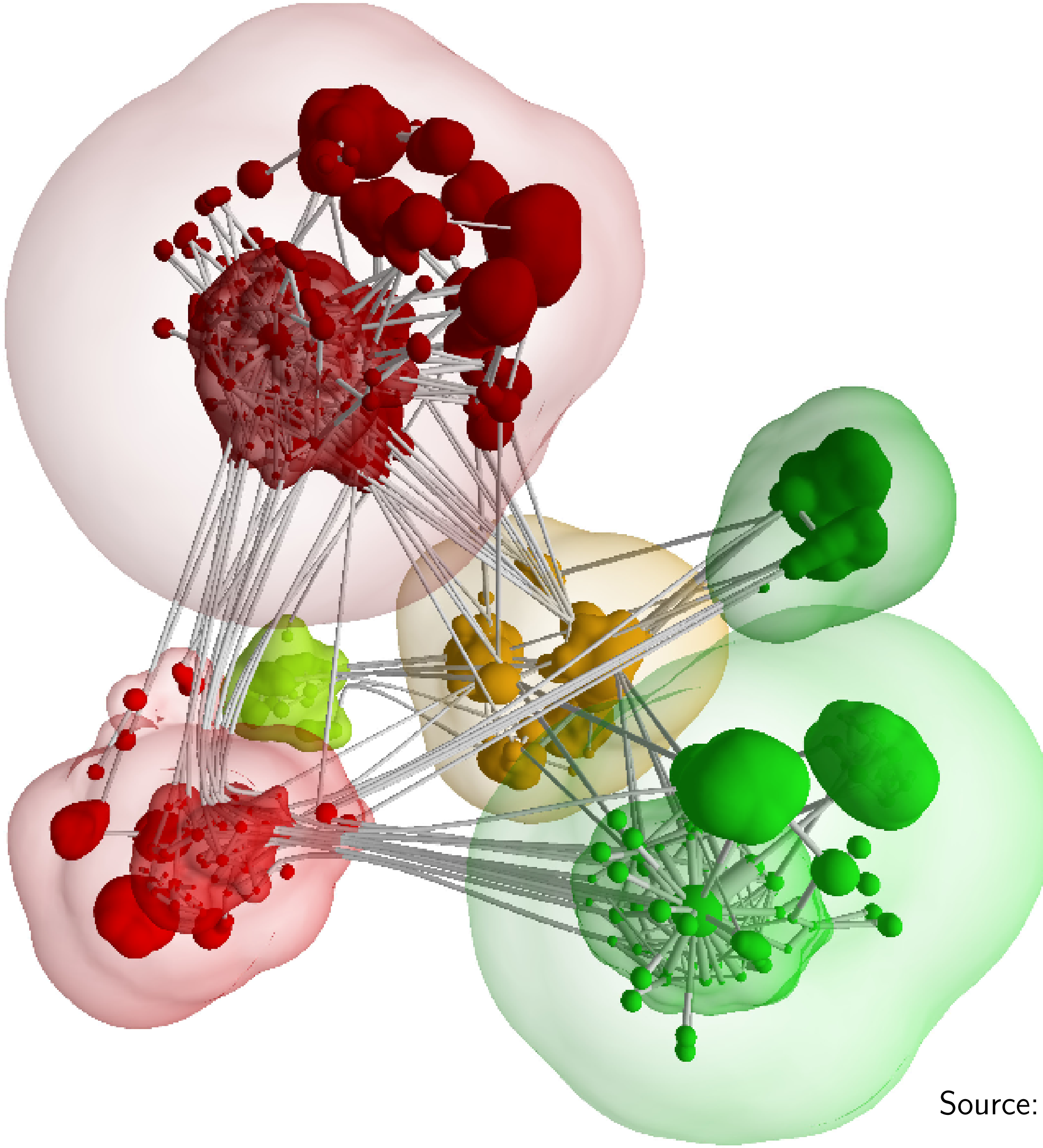


Elise M. Dagenbach & Sharyn A. Endow

Software-Network – UML Diagram

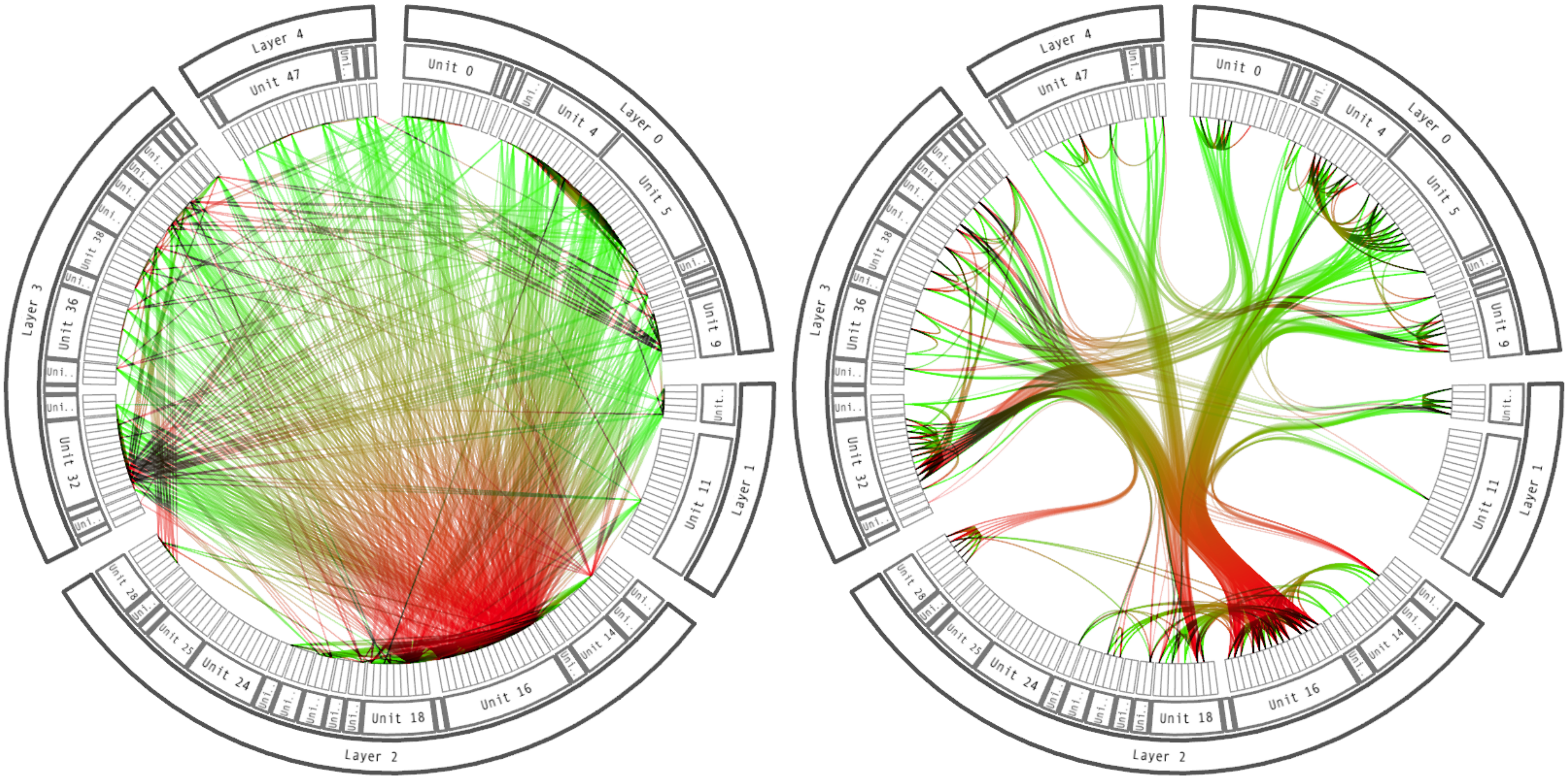


Clustered Software-Graph in 3D



Source: Balzer, Deussen, 2007

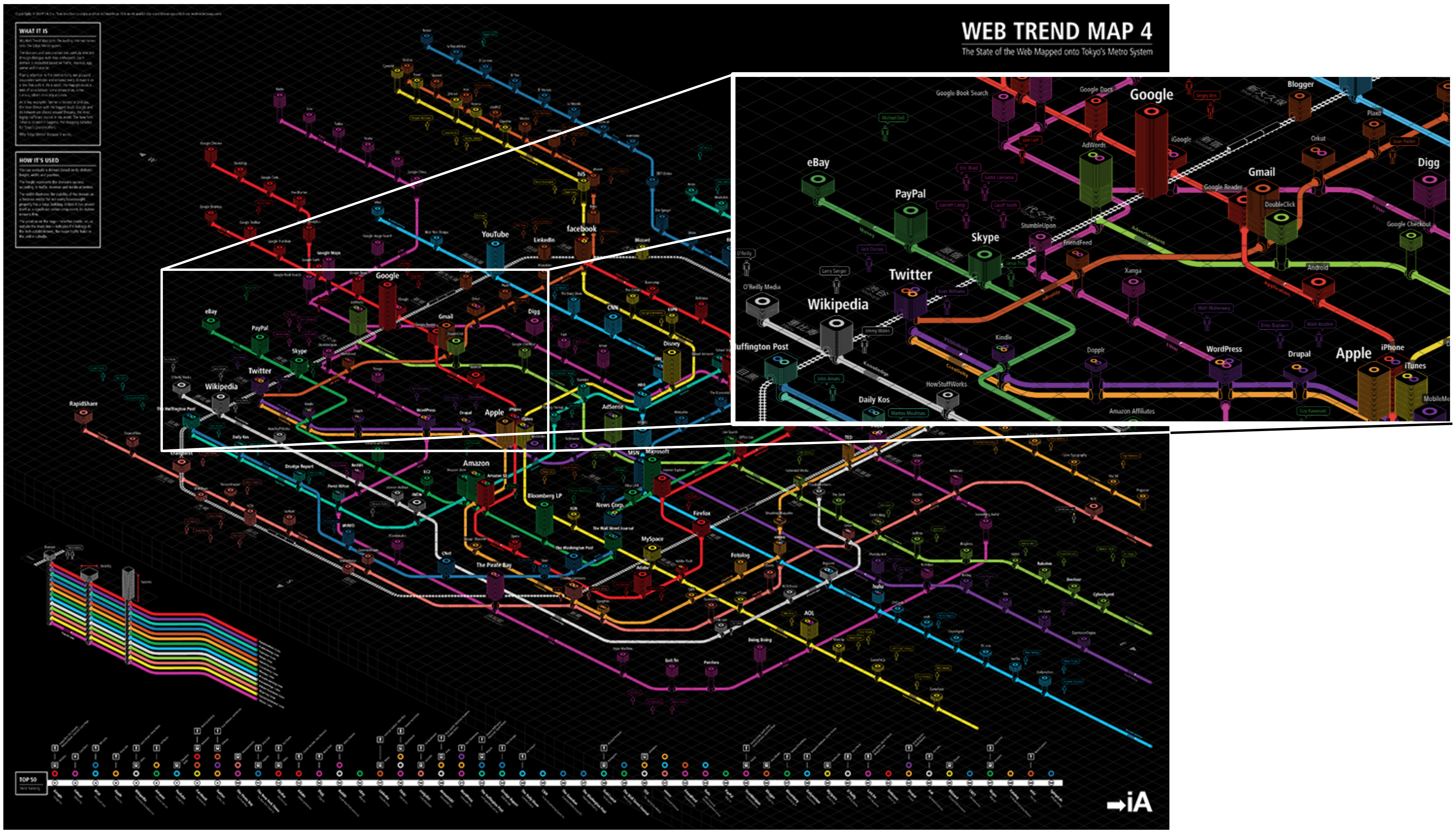
Software Call-Graph with edge-bundling



Source: Danny Holten, 2011

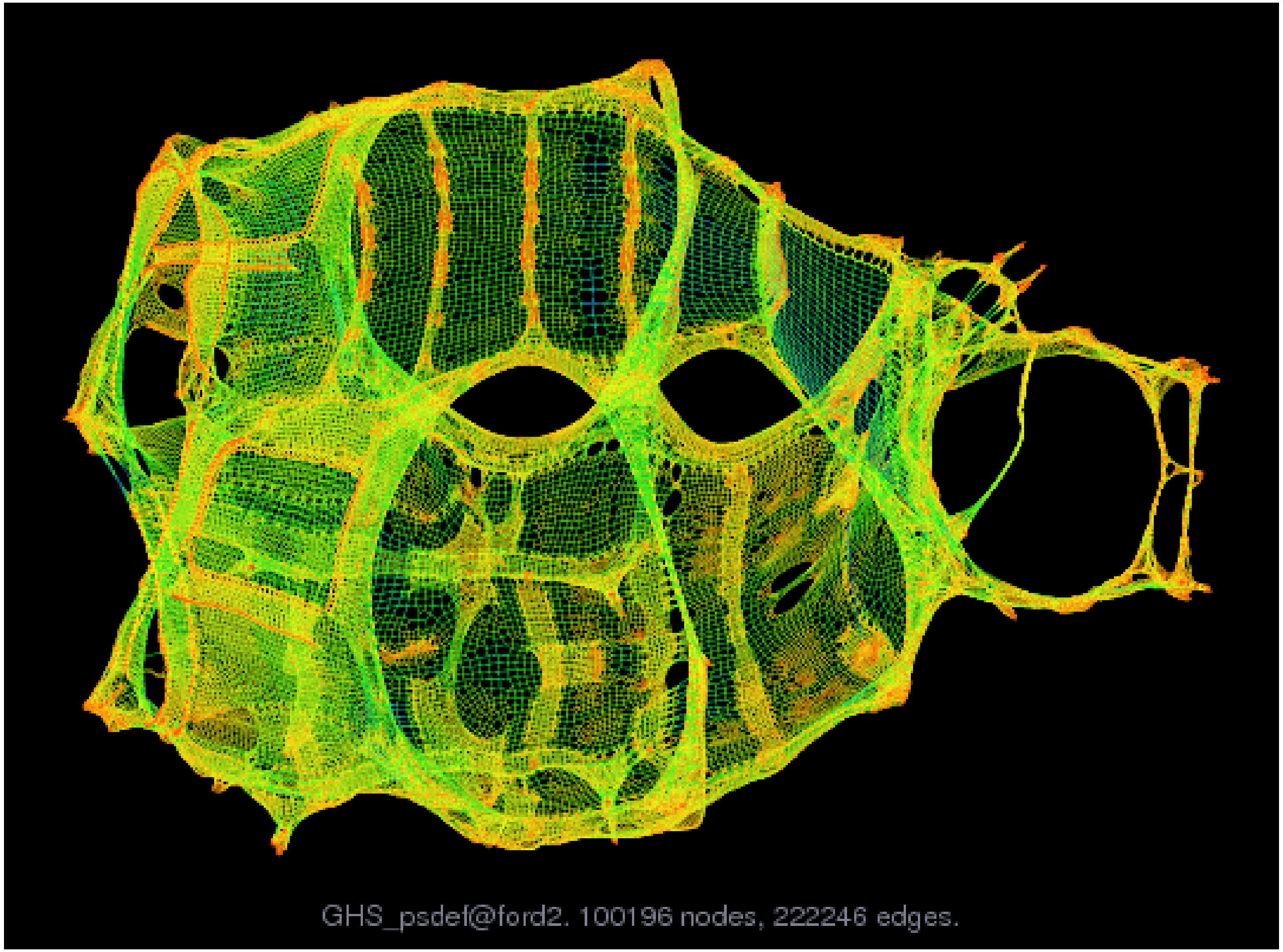
Web Trend Map

Karlsruhe Institute of Technology



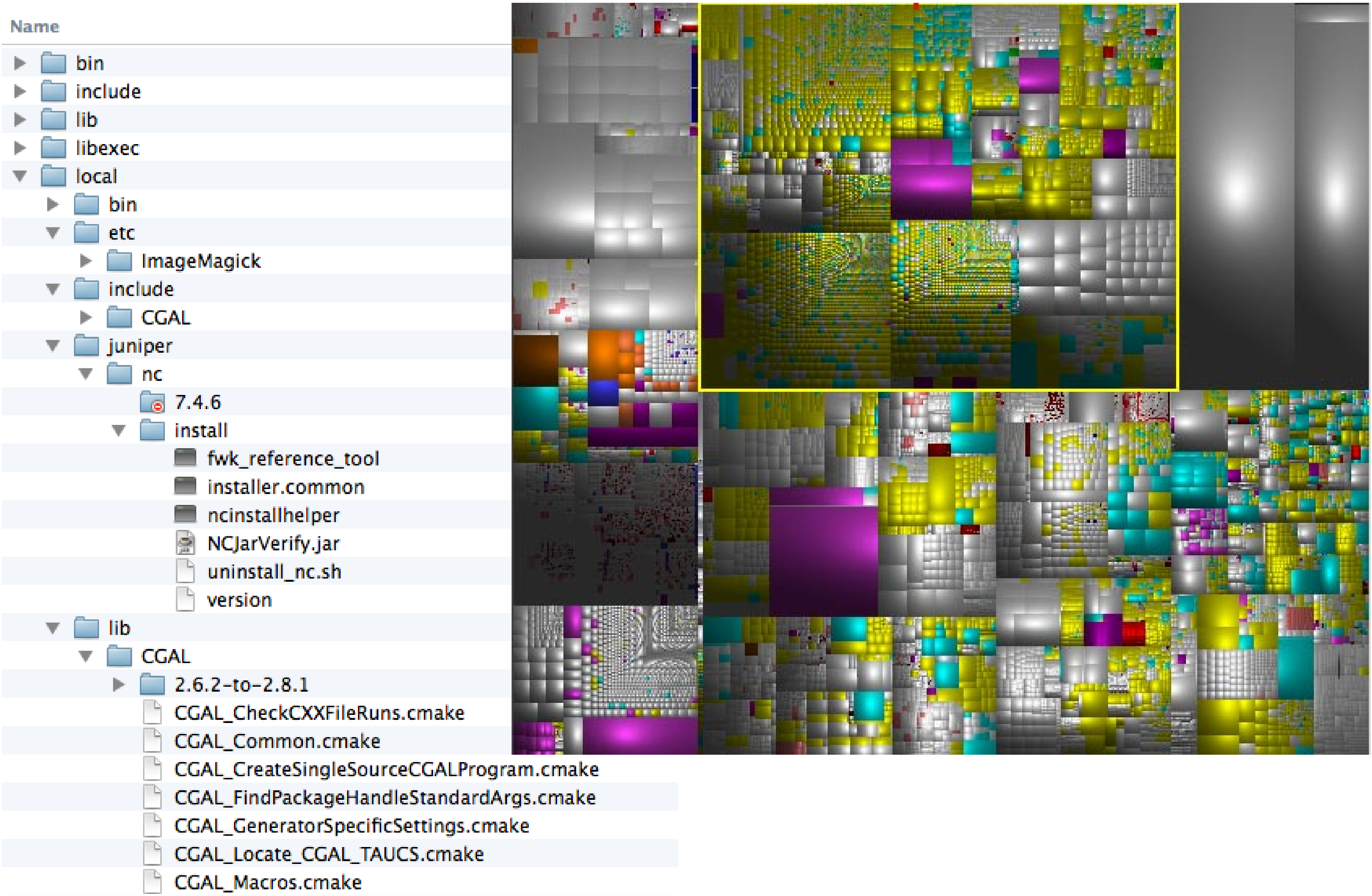
Source: information Architects, 2009

Large Graphs – Object Mesh

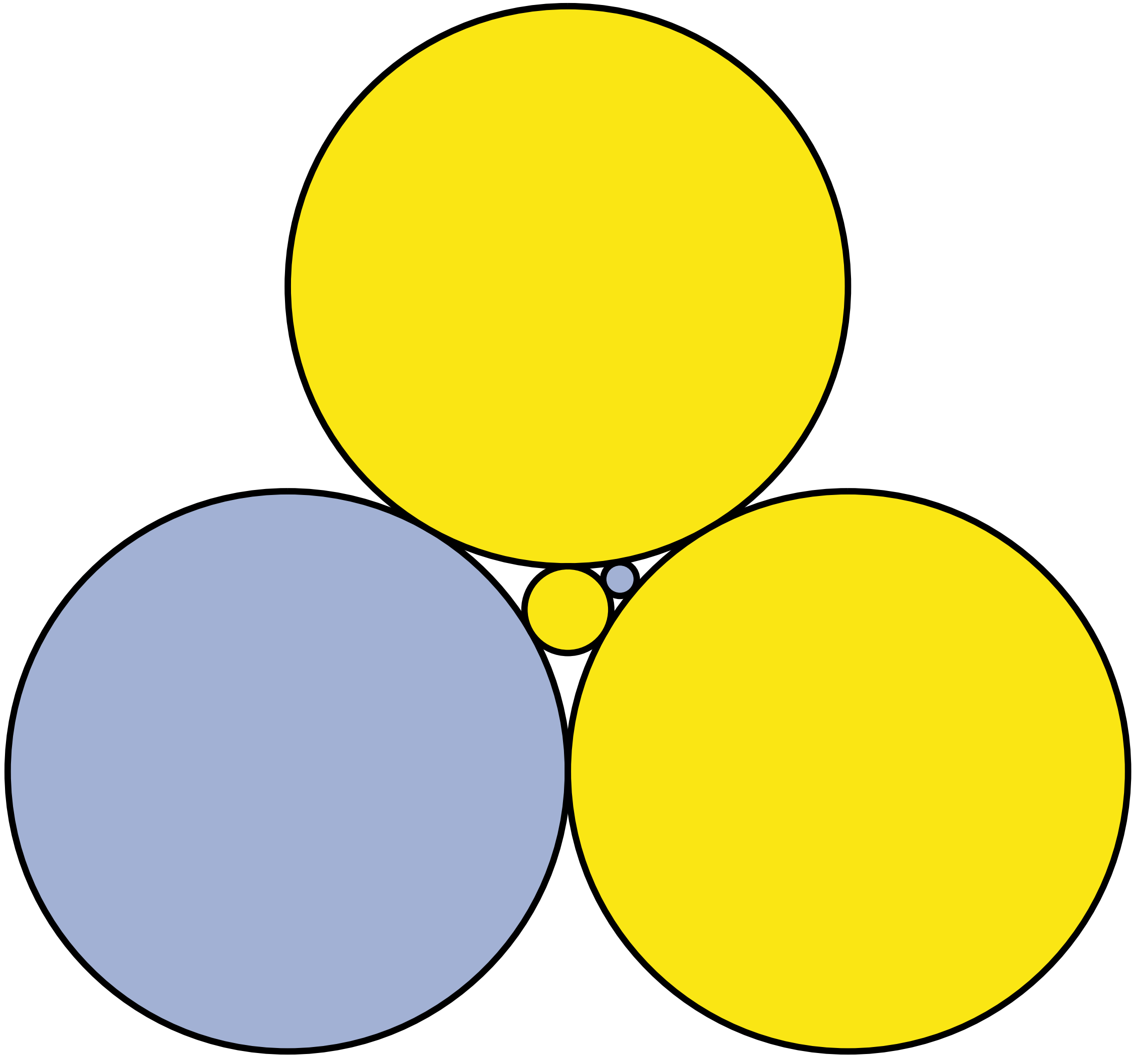


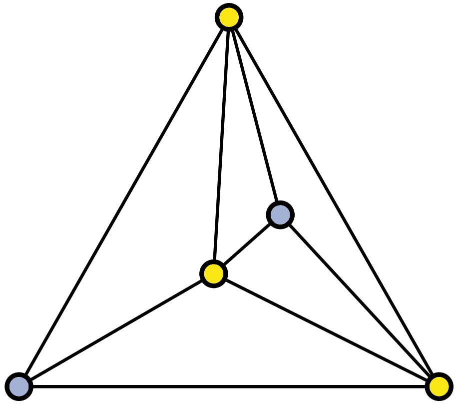
Source: Yifan Hu

Alternative Visualizations: Explorer vs Treemap



Alternative Visualizations: Contact map





Tools

Libraries for graph visualization

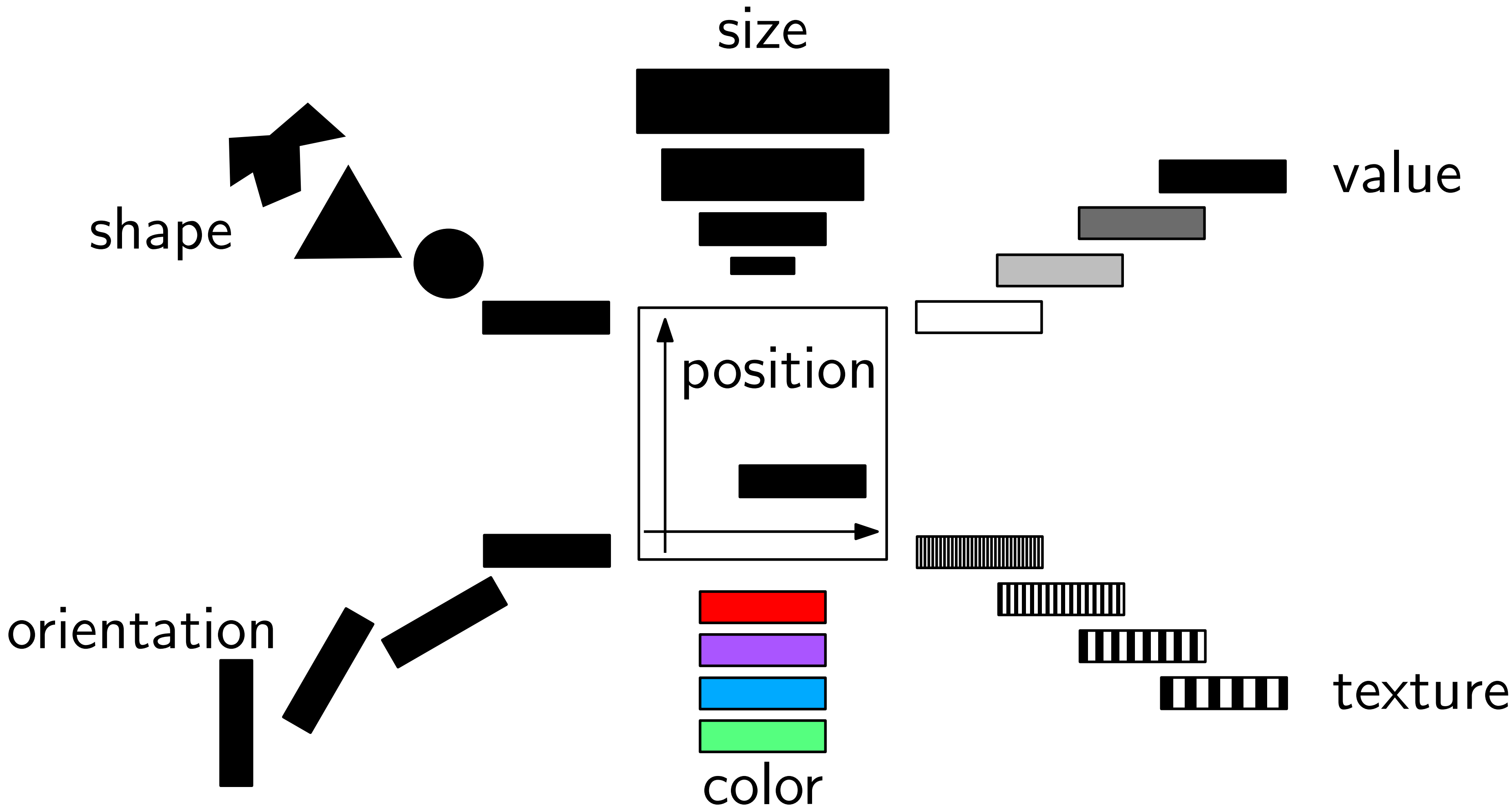
- JUNG `jung.sourceforge.net` (Java)
- OGDF `www.ogdf.net` (C++)

Visualization tools

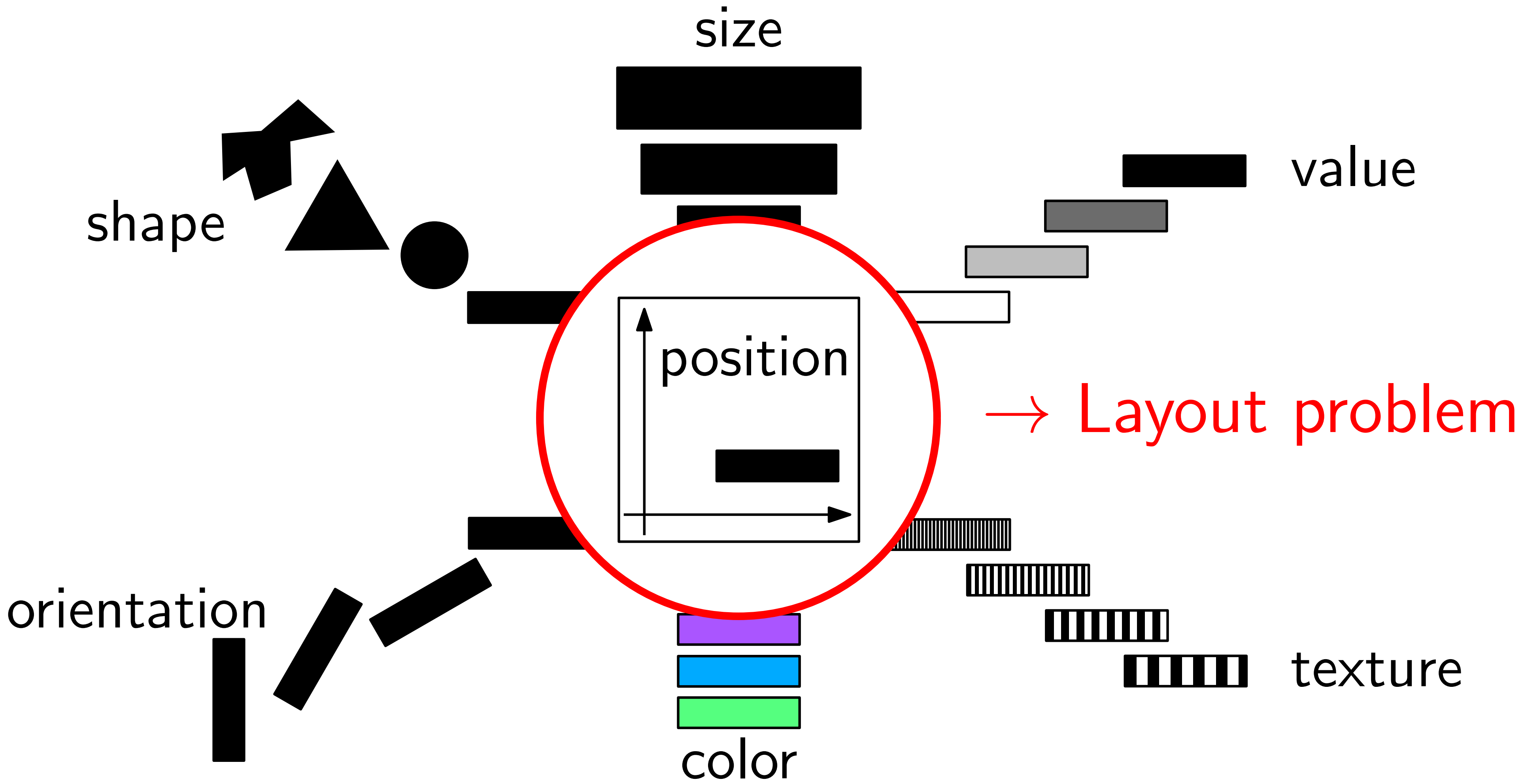
- visone `visone.info`
- graphviz `www.graphviz.org`
- yEd `www.yworks.com`
- Gephi `www.gephi.org`

Basic Definitions

Visual Variables according to Bertin (1967)



Visual Variables according to Bertin (1967)



Layout Problem

Restriction in the following: **drawing** is always meant to be in form of a node-link diagram

Graph visualization problem

given : Graph $G = (V, E)$
find: good drawing Γ of G

- $\Gamma : V \to \mathbb{R}^2$, nodes $v \mapsto$ point $\Gamma(v)$
- $\Gamma : E \to$ curves in $\mathbb{R}^2$, edge $\{u, v\} \mapsto$ simple open curve $c_{uv} : [0, 1] \to \mathbb{R}^2$ where $c_{uv}(0) = \Gamma(u)$ and $c_{uv}(1) = \Gamma(v)$

Layout Problem

Restriction in the following: **drawing** is always meant to be in form of a node-link diagram

Graph visualization problem

given : Graph $G = (V, E)$

find: good drawing Γ of G

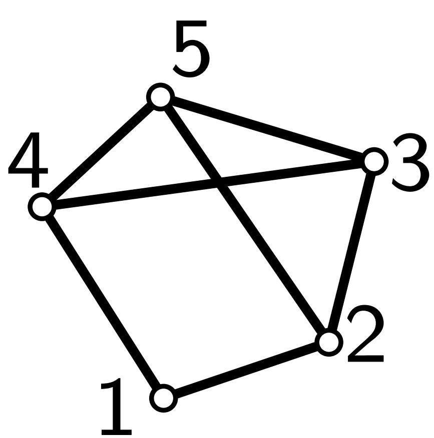
- $\Gamma : V \to \mathbb{R}^2$, nodes $v \mapsto$ point $\Gamma(v)$
- $\Gamma : E \to$ curves in $\mathbb{R}^2$, edge $\{u, v\} \mapsto$ simple open curve $c_{uv} : [0, 1] \to \mathbb{R}^2$ where $c_{uv}(0) = \Gamma(u)$ and $c_{uv}(1) = \Gamma(v)$

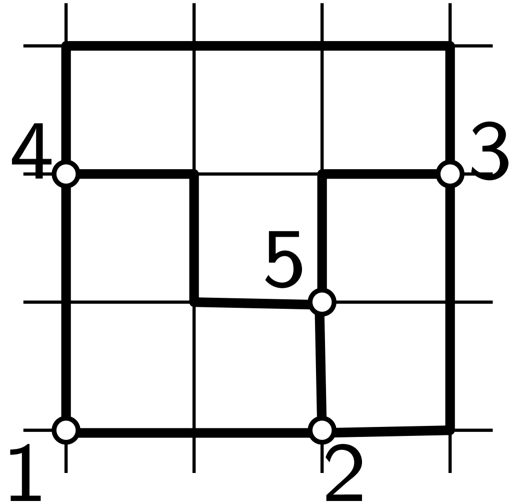
But what is a **good** Drawing?

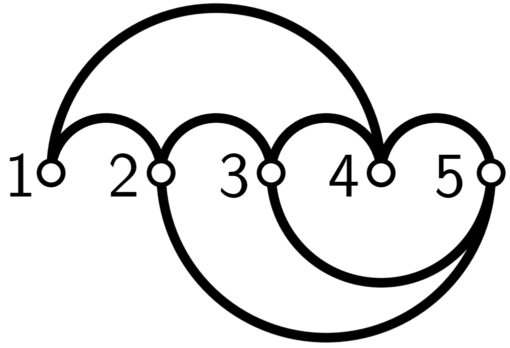
Requirement to the graph layout

1) **Drawing conventions**, required properties, for example

- straight-line edges
- orthogonal edges (with bends 90 degrees)
- Drawing on a grid
- crossing-free
- ...







Requirement to the graph layout

1) **Drawing conventions**, required properties

2) **Aesthetics** (to be optimized), for example:

- Number of crossing
- Number of bends
- Uniform edge length
- Area/length
- Angular resolution
- Symmetry
- . . .

→ often lead to NP-hard optimization problems!
→ often several competing criteria

Requirement to the graph layout

1) **Drawing conventions**, required properties

2) **Aesthetics** (to be optimized)

3) **Partial/local constraints**, for example:

- Positions of several vertices
- Relative positions of vertices
- Group of nodes drawn close to each other

Layout Problem – Second Attempt

Graph visualization problem

given: Graph $G = (V, E)$

find: good drawing Γ of G, that

- complies with drawing conventions
- optimizes aesthetics
- satisfies local/partial constraints

Layout Problem – Second Attempt

Graph visualization problem

given: Graph $G = (V, E)$
find: good drawing Γ of G, that

- complies with drawing conventions
- optimizes aesthetics
- satisfies local/partial constraints

- this definition drives to interesting algorithmic problems

Lecture topics

- **Drawings of Trees and other recursively defined graph classes**
- **straight-line drawings of planar graphs**
- incremental layouts
- orthogonal drawings
- contact representation of graphs
- hierarchical drawings of layered graphs
- force-based drawing algorithm
- ...

Lecture topics

- **Drawings of Trees and other recursively defined graph classes**
- **straight-line drawings of planar graphs**
- incremental layouts
- orthogonal drawings
- contact representation of graphs
- hierarchical drawings of layered graphs
- force-based drawing algorithm
- ...

Next Meetings

Lecture on 24.10 9:45

Execise on 26.10 9:45 $\rightarrow$ please bring your Laptops

$\rightarrow$ install the Software yEd (`www.yworks.com`)