Exercise Sheet 6

Assignment: January 30, 2017
Discussion: February 8, 2017

1 Edges with Many Bends

Define a family of embedded planar graphs with maximum degree 4 and $O(n)$ vertices, such that for each bend-minimal orthogonal drawing of the given embedding there is an edge that has $\Omega(n)$ bends. Hint: Consider spirals.

2 Extended Canonical Ordering for 4-Connected Graphs

A planar graph $G = (V, E)$ is called *proper triangular planar* (PTP, for short) if the following coditions hold:

- Every interior face of G is a triangle and the exterior face of G is a quadrangle;
- G has no separating triangles.

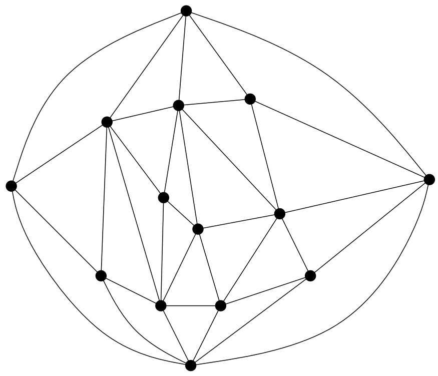
Let $G = (V, E)$ be a PTP graph with vertices a, b, c, d on the outer face. A labeling $v_1 = a, v_2 = c, v_3, \dots, v_n = d$ of the vertices of G is called an *extended canonical ordering* of G if for every $4 \leq k \leq n$:

- The subgraph G_{k-1} induced by $v_1, \dots, v_{k-1}$ is biconnected and the boundary C_{k-1} of G_{k-1} contains the edge (a, b);
- v_k is in the exterior face of G_{k-1}, and its neighbors in G_{k-1} form (at least 2-element) subinterval of the path $C_{k-1} \setminus (a, b)$. If $k \leq k-2$, v_k has at least 2 neighbors in $G \setminus G_{k-1}$.

Let $G = (V, E)$ be a PTP graph with vertices a, b, c, d on the outer face.

(a) Prove that the graph obtained from G by removal of vertices c, d and all edges incident to them is biconnected.
(b) Let $C = \{a = u_1, \dots, u_k = b, a\}$ be a simple cycle of G such that $c, d \notin C$. Let G_C denote the graph induced by the vertices of G laying inside C (including the vertices of C). Let $v_i \in C$, $2 \leq i \leq k-1$ such that no internal chord of C is incident to v_i. Show that $G \setminus \{v_i\}$ is biconnected.
(c) Let C be as above and let (v_i, v_j), $1 \leq i < j \leq k$, be an internal chord of C. Show that there exists a vertex v_l, $i < l < j$, which is adjacent to at least two vertices of $G \setminus G_C$.
(d) Using (a-c) prove that G has an extended canonical ordering such that $v_1 = a, v_2 = b, v_{n-1} = c, v_n = d$.

3 Construction of Rectangular Dual



Consider the graph G of the figure. Check whether G satisfies the necessary conditions to have a rectangular dual. In affirmative, construct a rectangular dual of G.

4 Contact Representation of Maximal Planar Graphs

Prove that a maximal planar graph admits a contact representation with T-shapes. An example of such a representation is shown in the figure.
Hint: Use canonical ordering in the way similar to the construction of a visibility representation (Exercise Sheet 1).