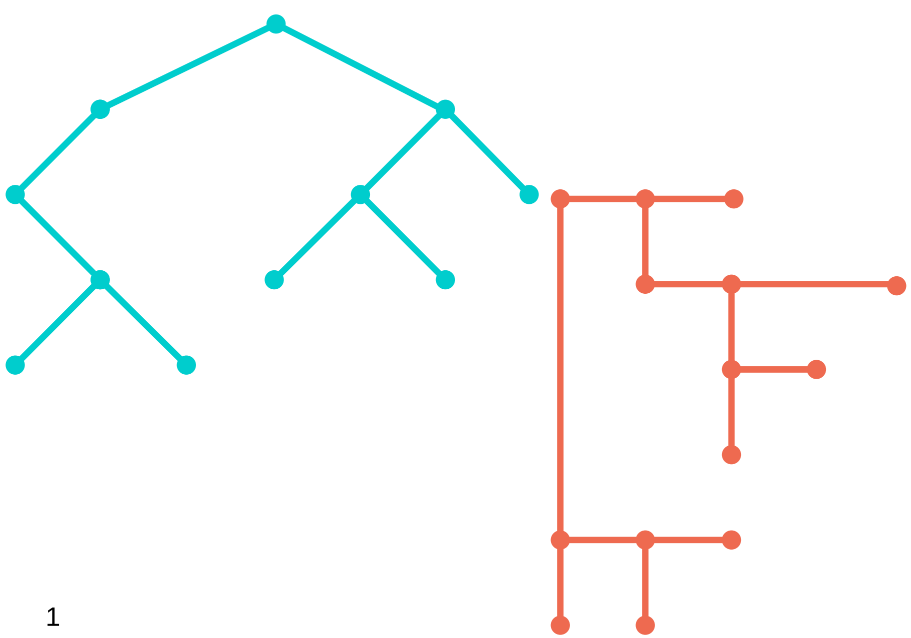
Karlsruhe Institute of Technology

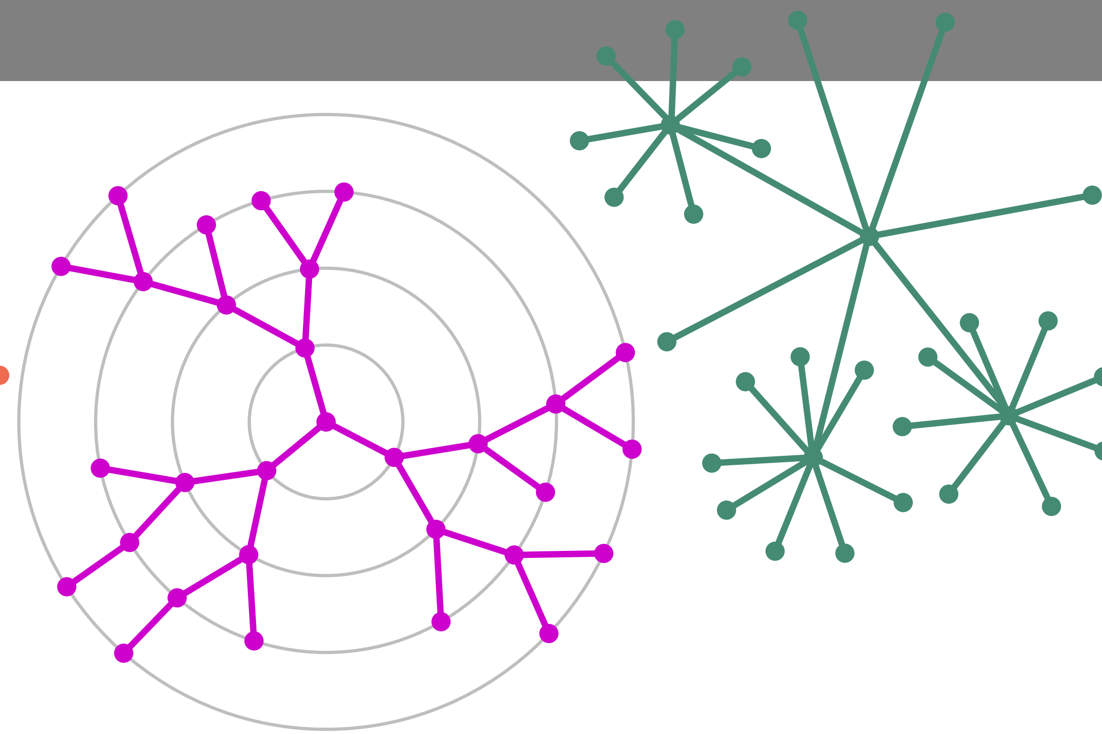
Algorithms for graph visualization

Divide and Conquer - Tree Layouts

WINTER SEMESTER 2016/2017

Tamara Mchedlidze

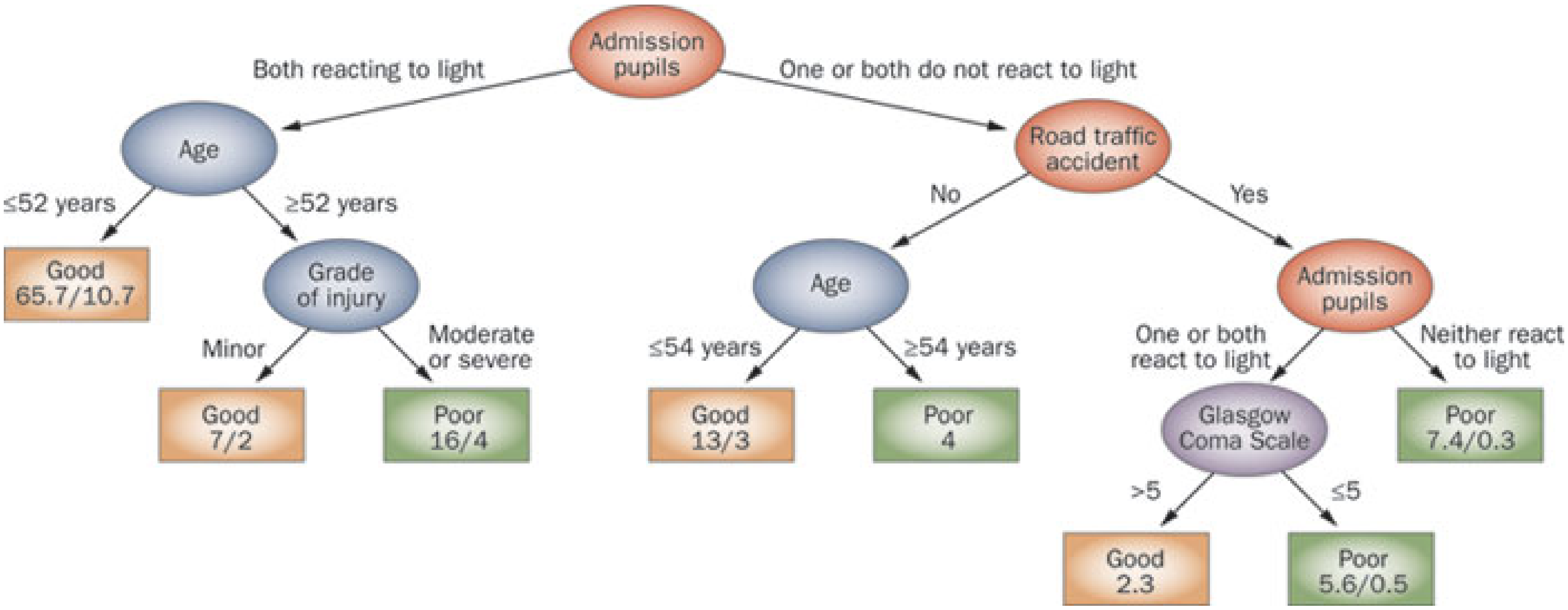




KIT – Universität des Landes Baden-Württemberg und
nationales Forschungszentrum in der Helmholtz-Gemeinschaft

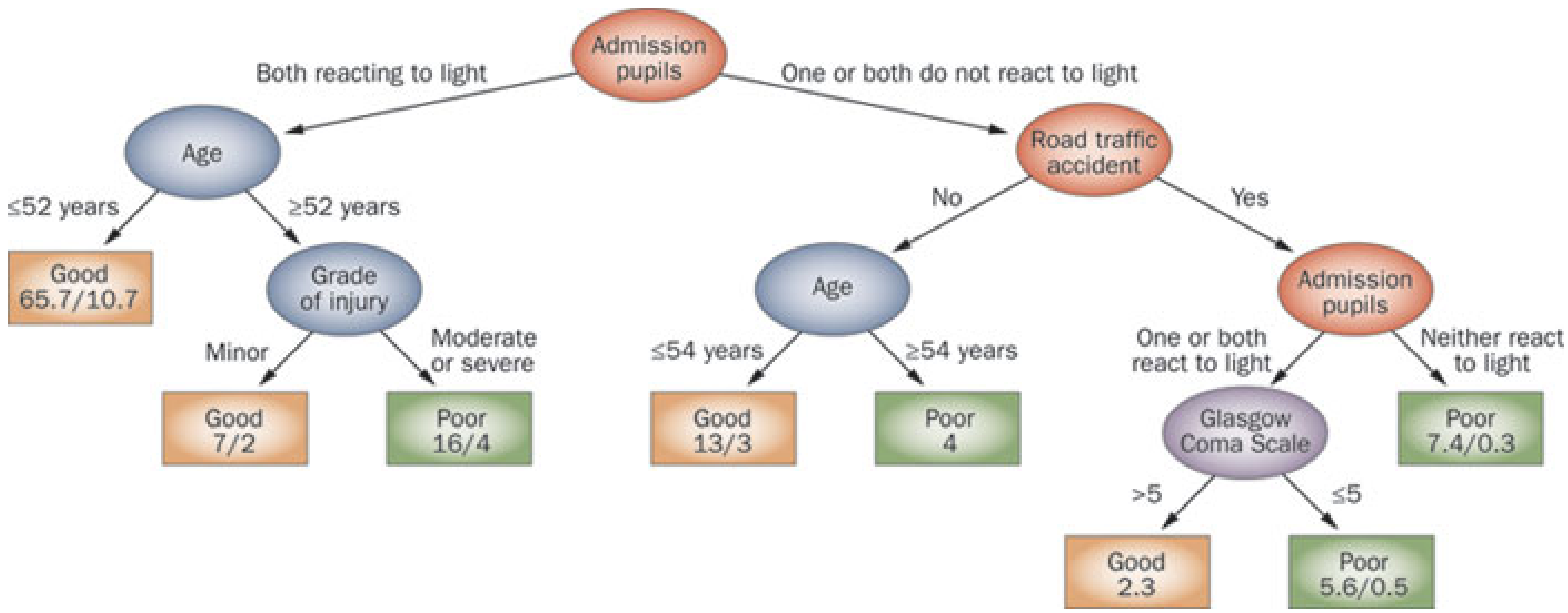
www.kit.edu

Applications



Decision tree analysis for prediction of outcome after traumatic brain injury
Nature Reviews Neurology

Applications



Level-based layout

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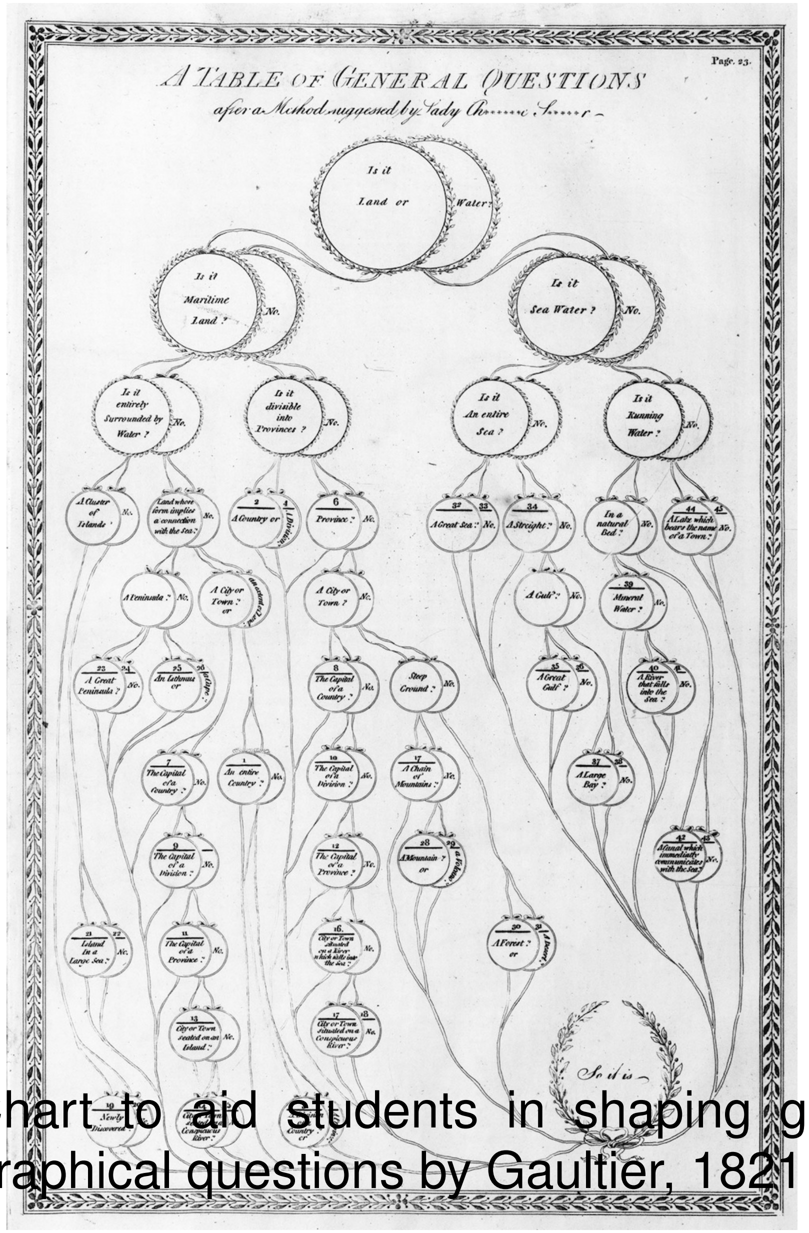
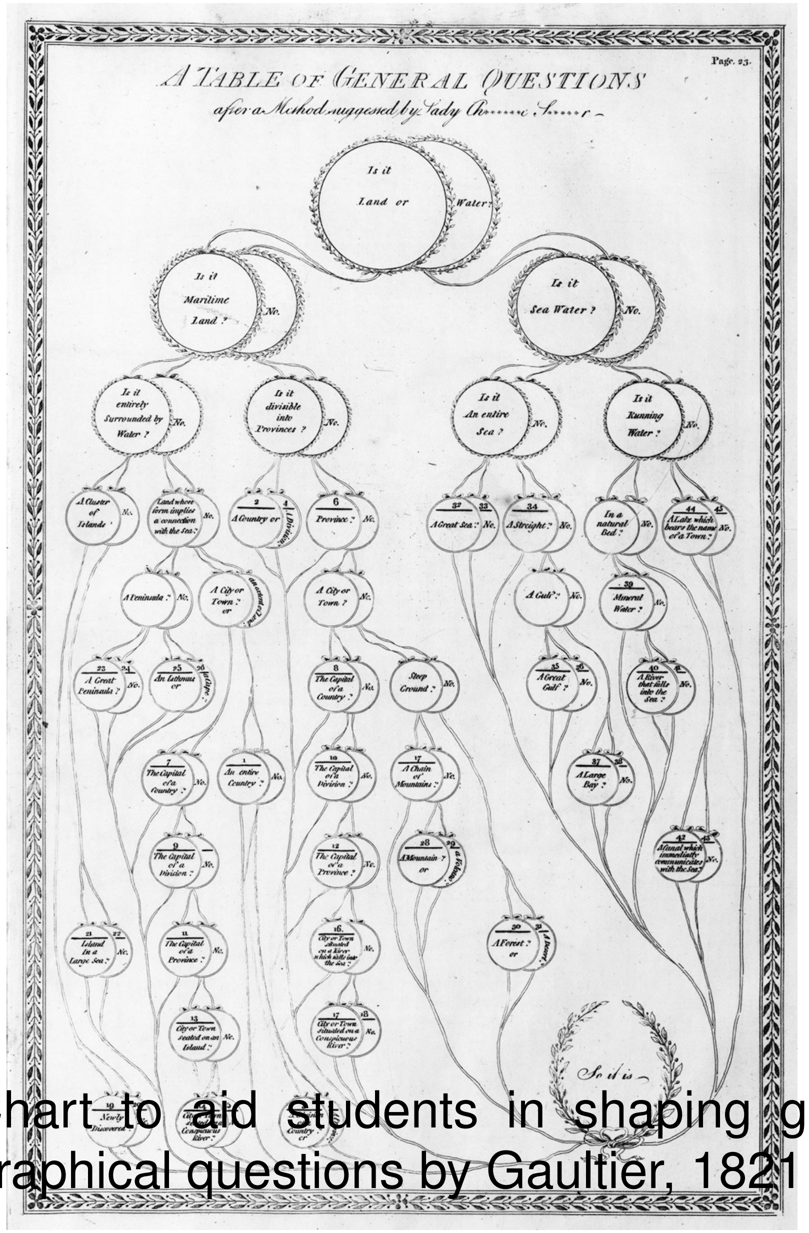


Chart to aid students in shaping geographical questions by Gaultier, 1821

Applications



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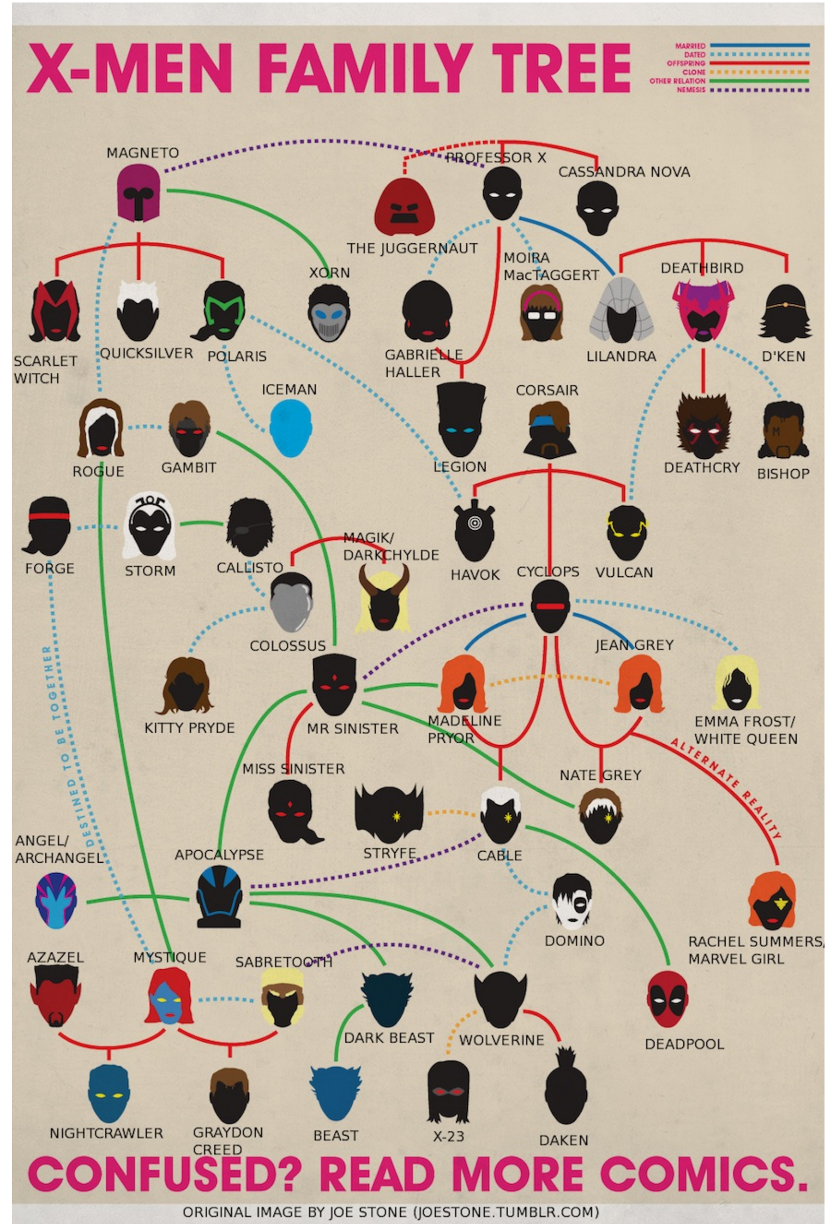

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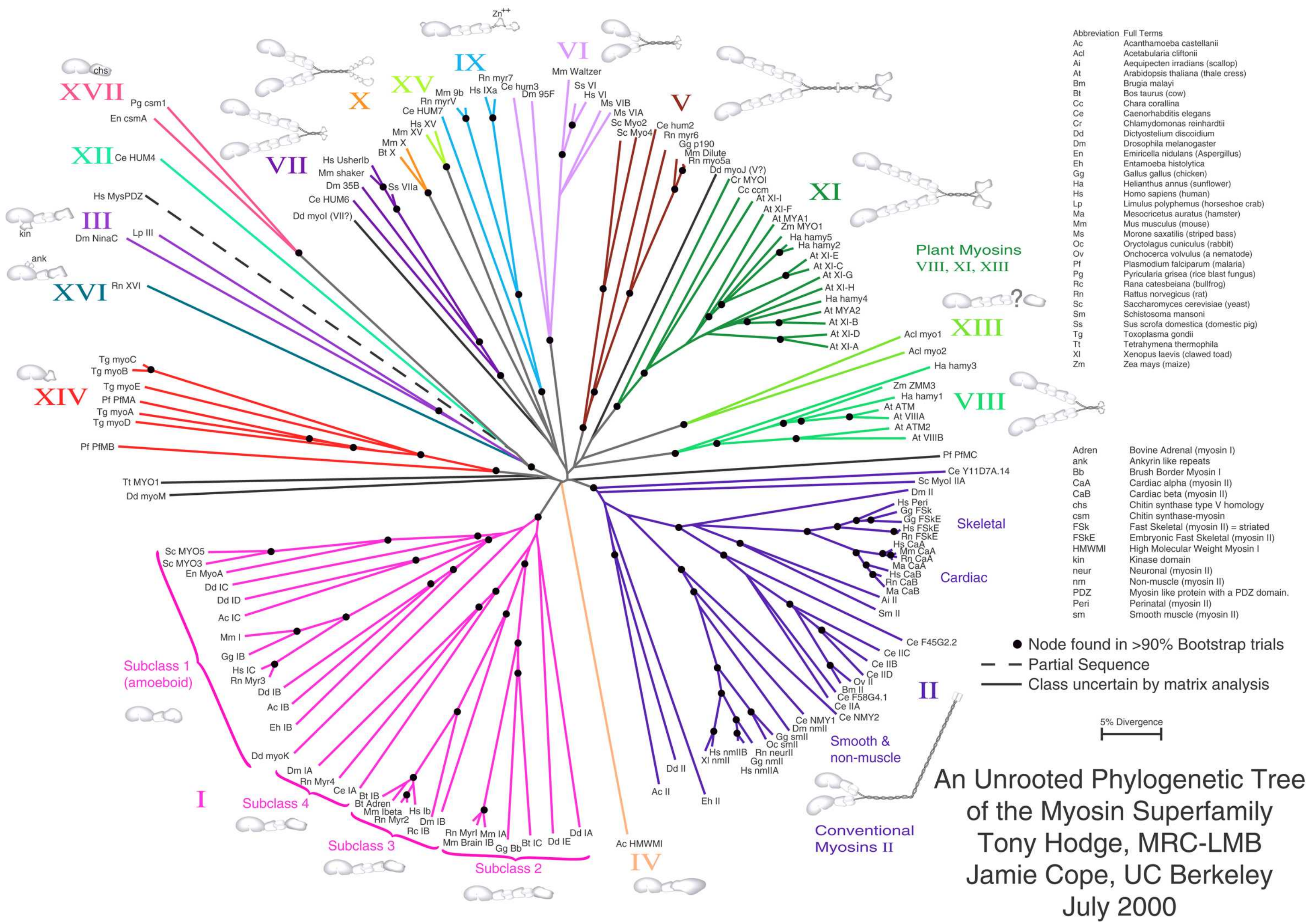
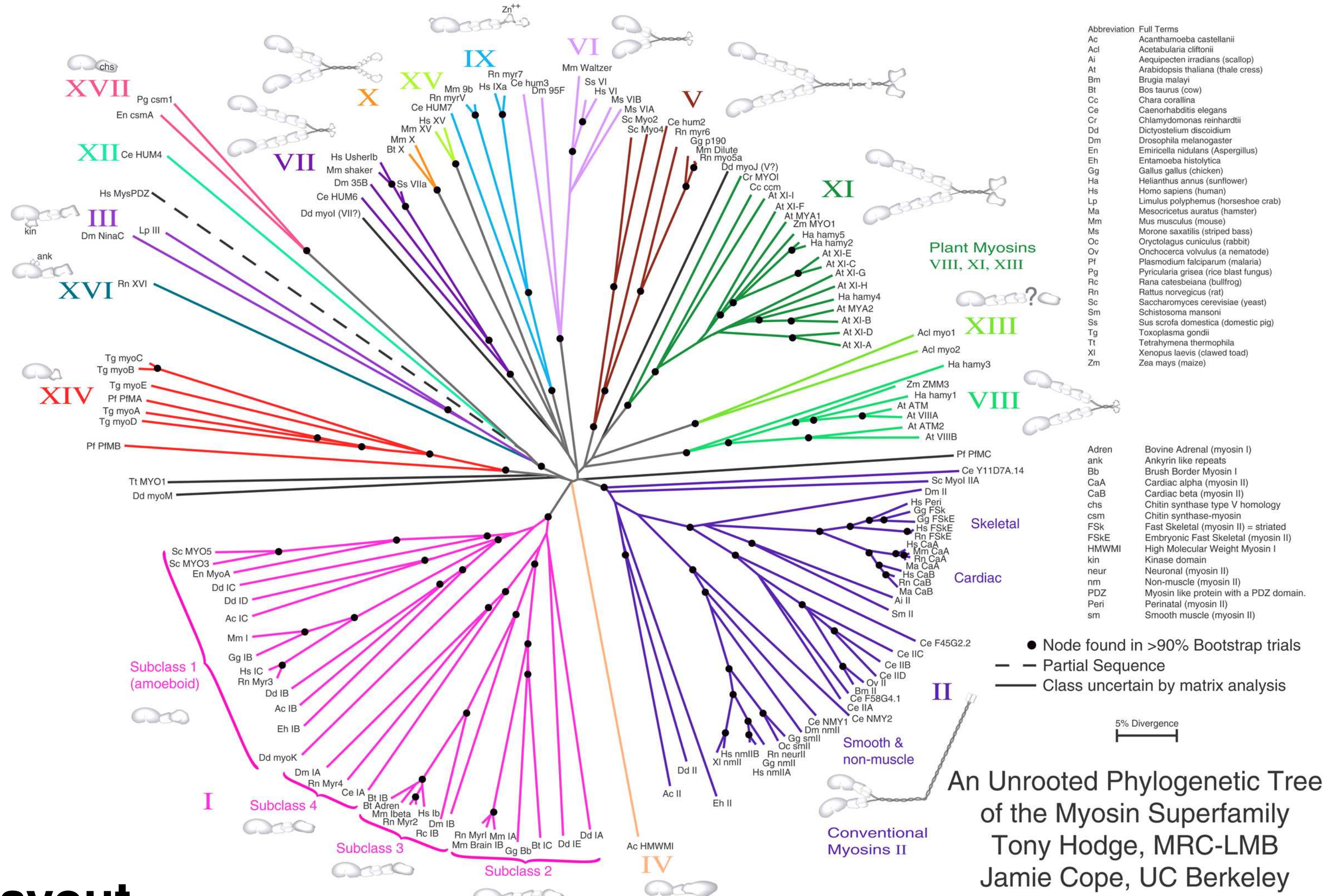


An unrooted phylogenetic tree for myosin, a superfamily of proteins.
”A myosin family tree“ *Journal of Cell Science*

Applications



Radial layout

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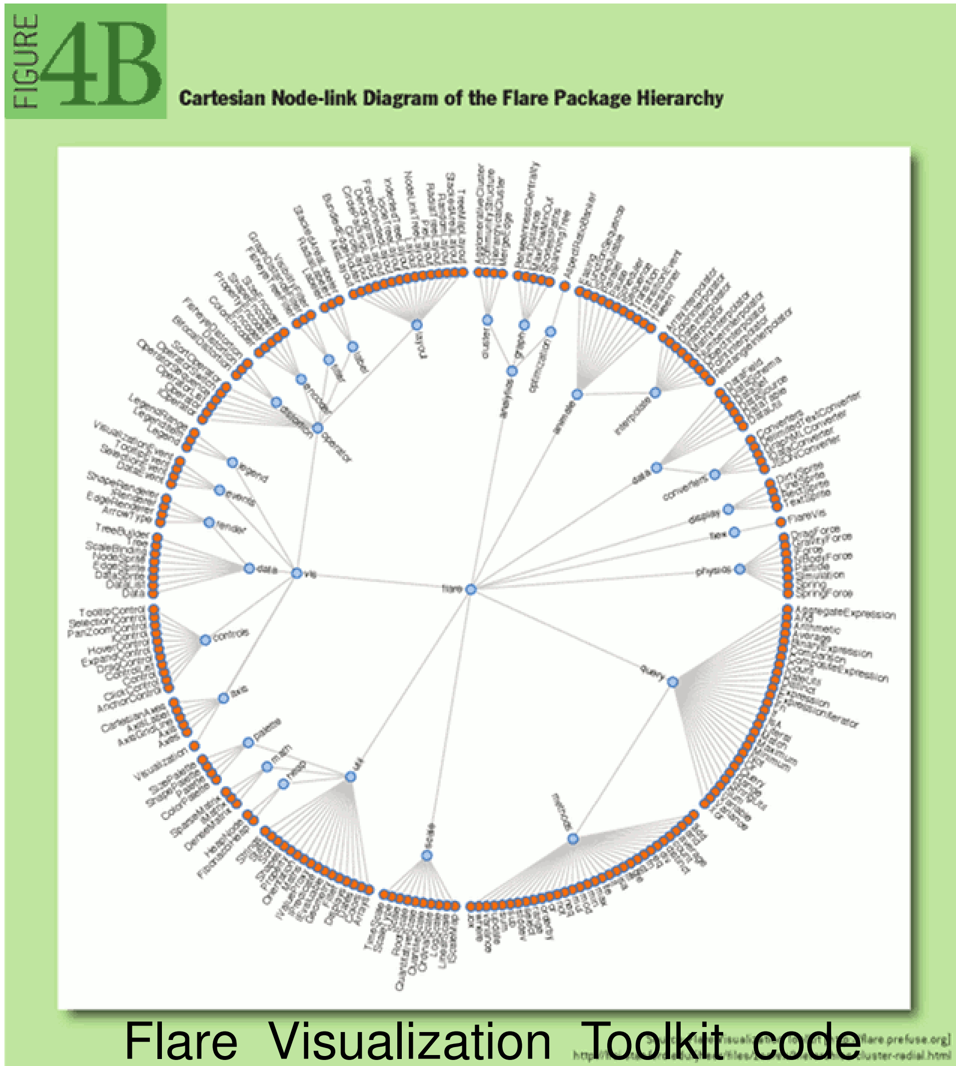
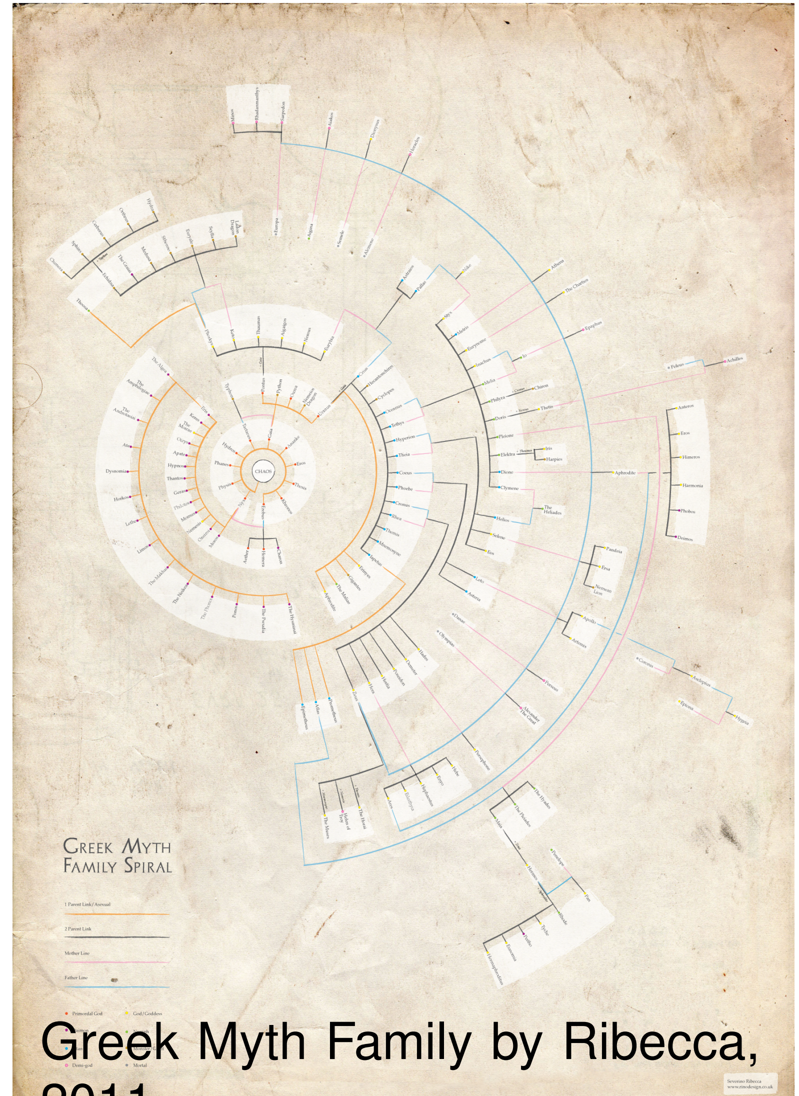


Flare Visualization Toolkit code structure by Heer, Bostock and Ogievetsky, 2010



Greek Myth Family by Ribecca, 2011

Applications

Cons cell diagram in LISP.
Cons(constructs) are memory objects which hold two values or pointers to values.

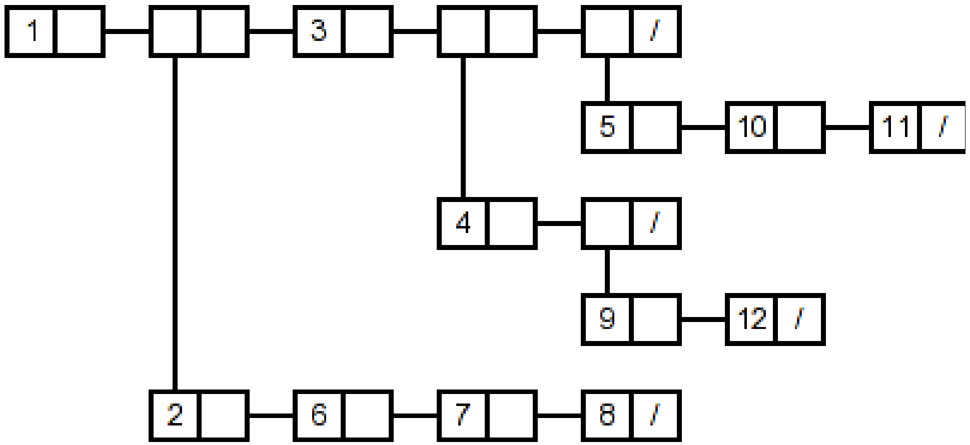


Figure 3: Diagram of cons cells of the simple tree.

`http://gajon.org/`

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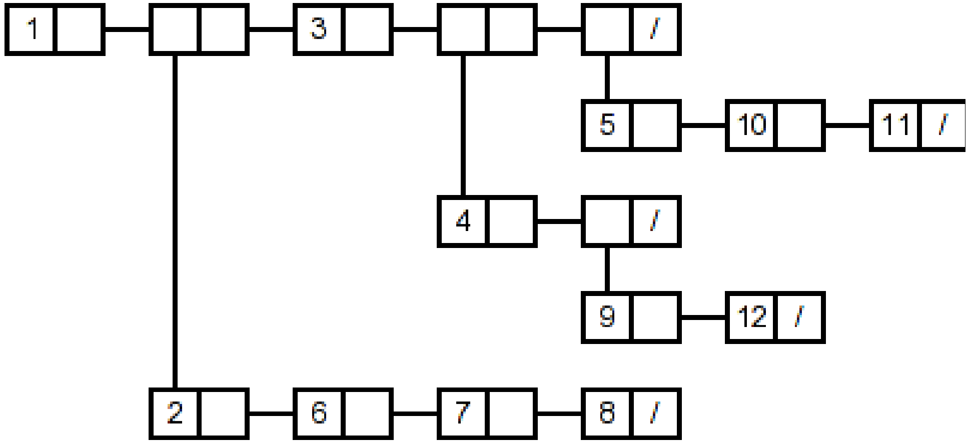

Figure 3: Diagram of cons cells of the simple tree.

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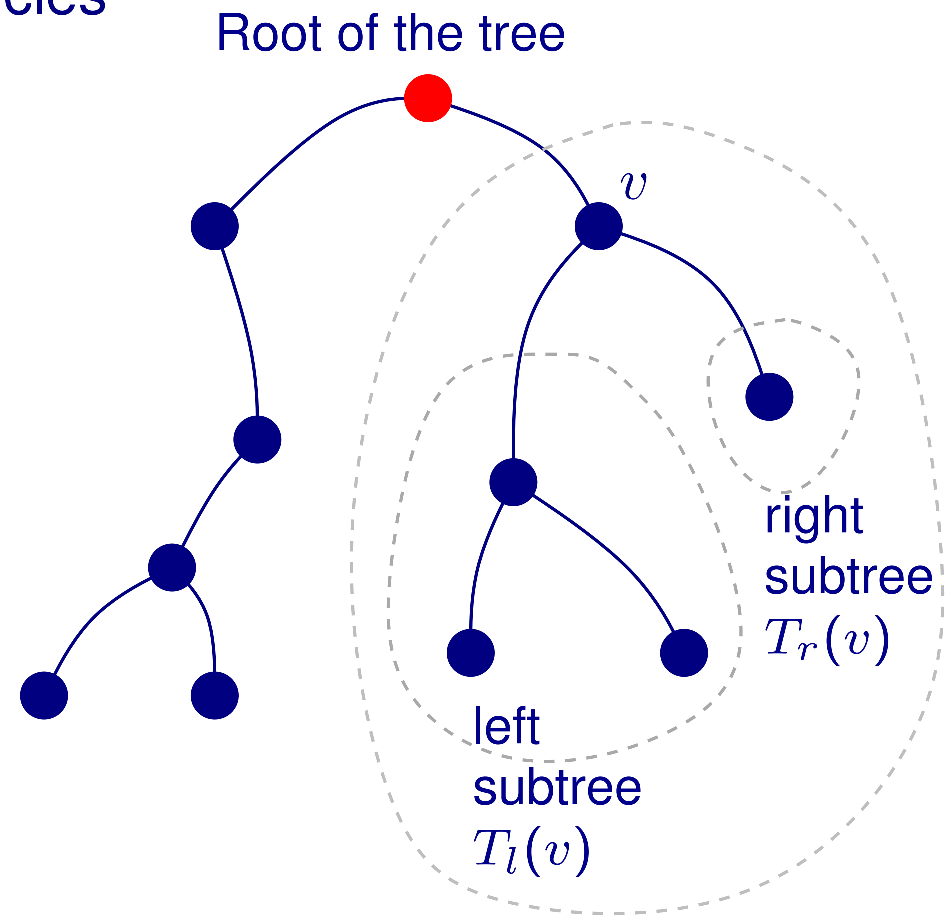
HV-layout (Horizontal-Vertical)

Overview

- Applications with tree visualization
- Layered tree drawing algorithm
- H(horizontal) V(vertical) tree drawing algorithm
- Radial tree drawing algorithm
- Other visualization styles

Basic Definitions

- Tree - connected graph without cycles
- Binary tree



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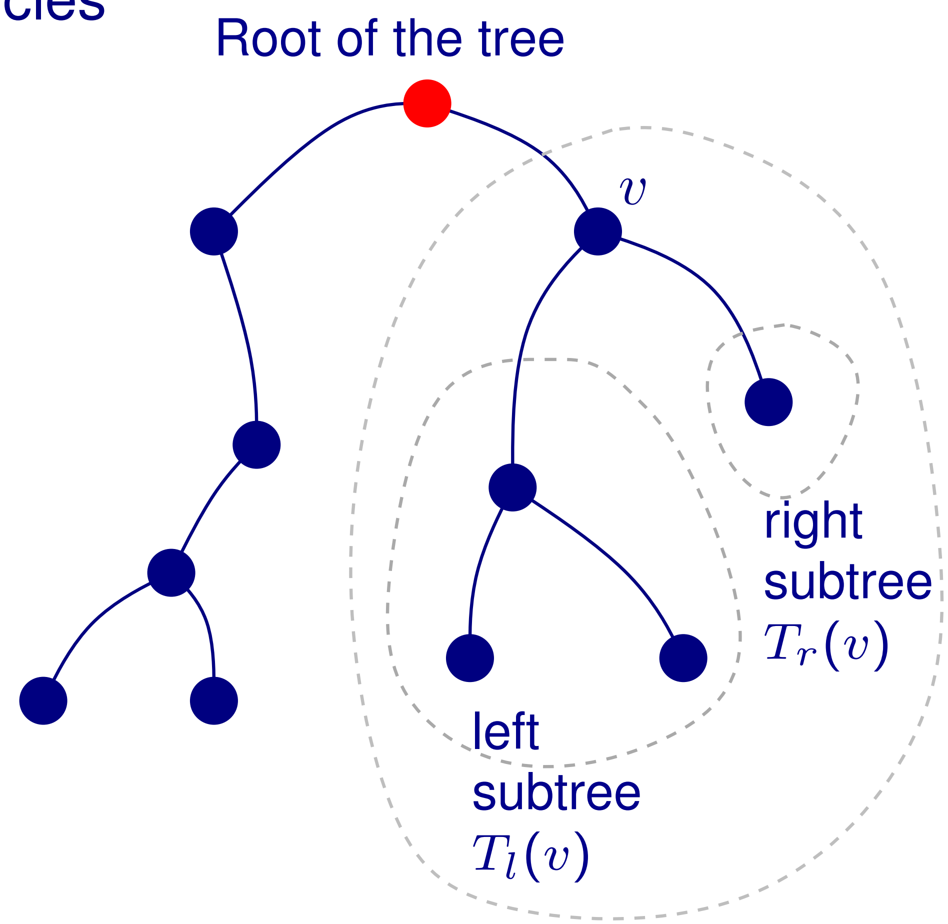
Tree traversals



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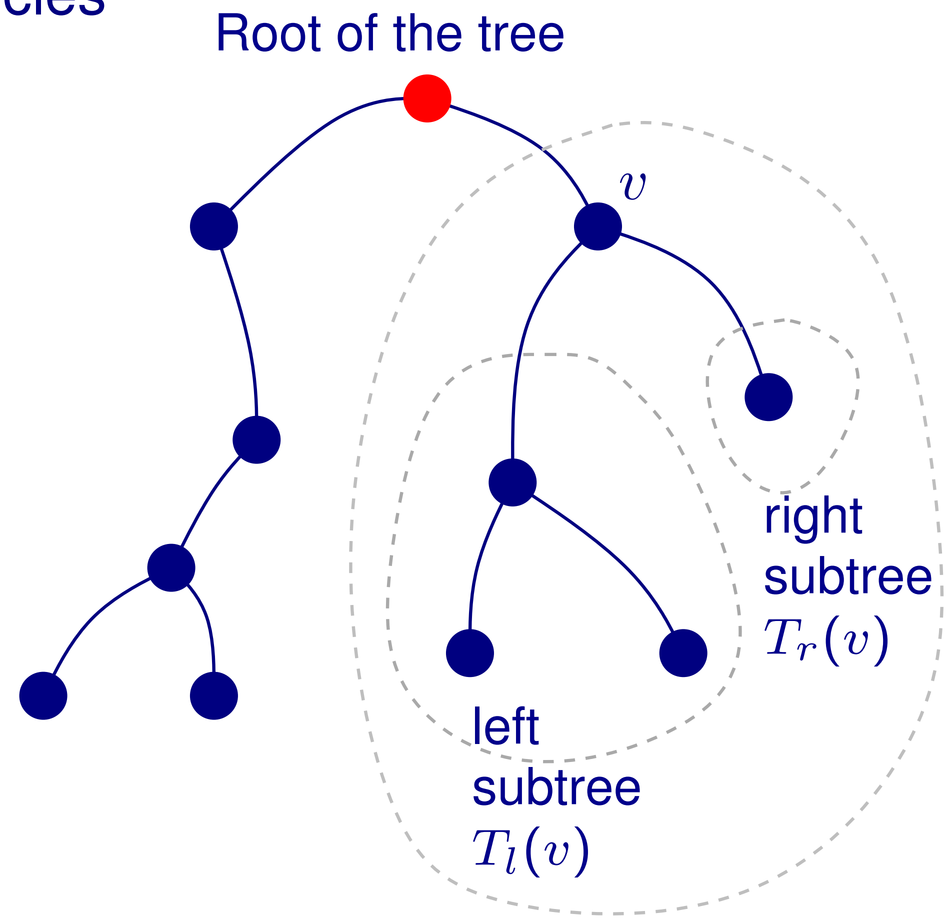
Depth-first search



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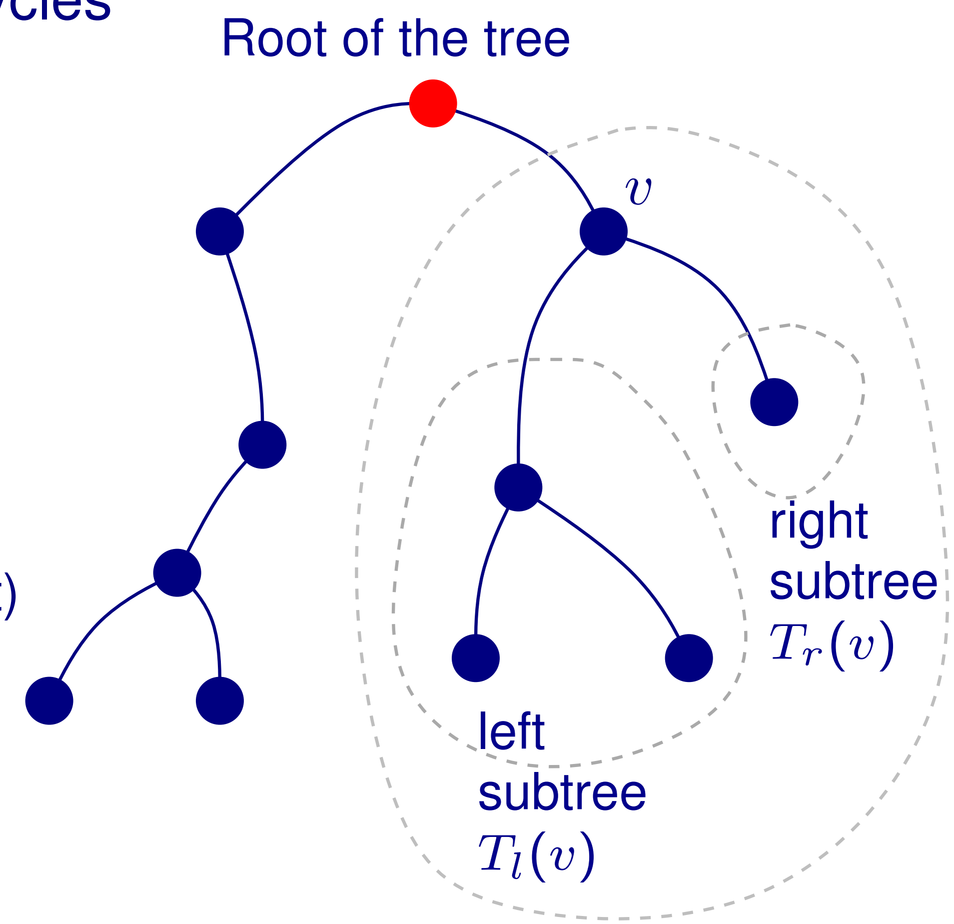
- Pre-order (First parent, then subtrees)
- In-order (Left child, parent, right child)
- Post-order (First subtrees, then parent)



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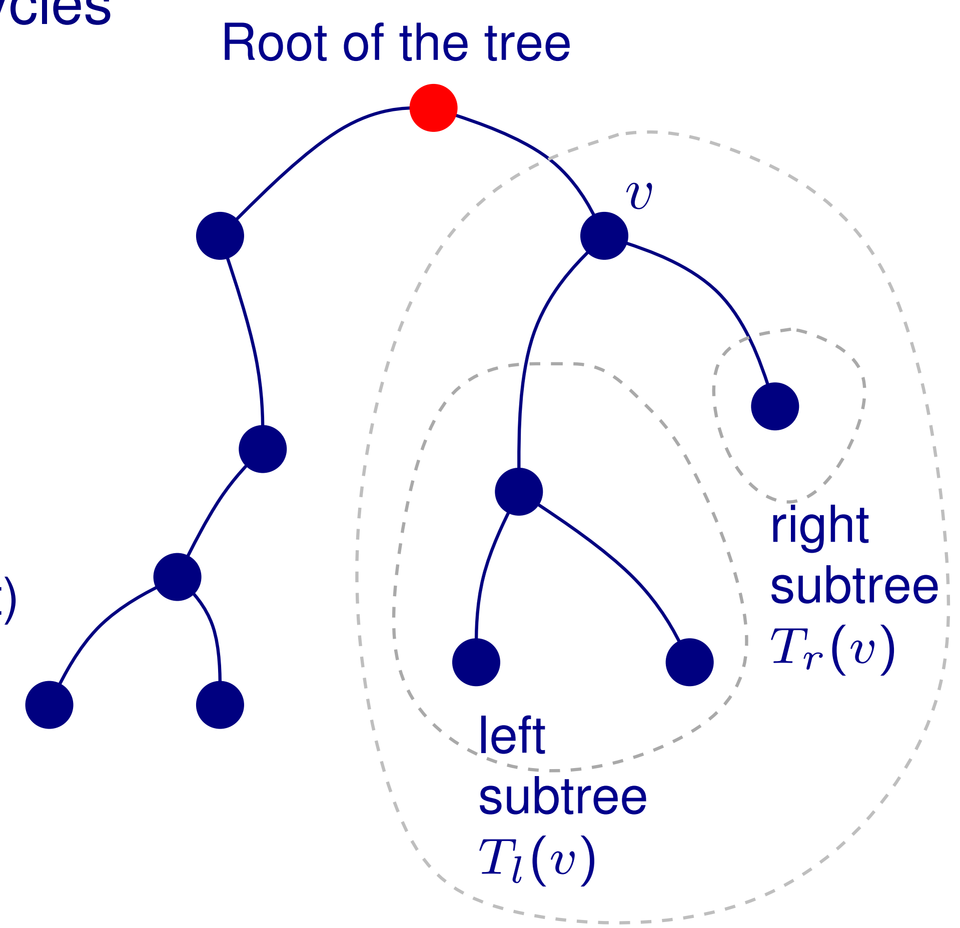
Breadth-first search



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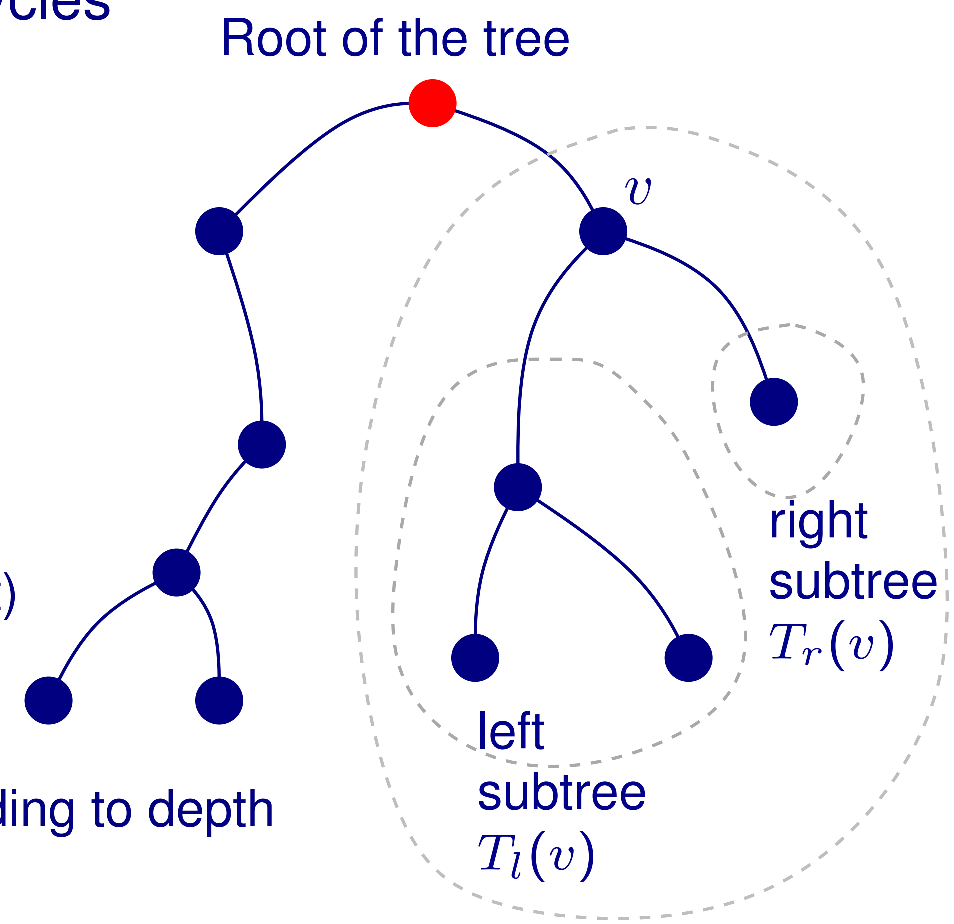
Breadth-first search

- Assignes vertices to levels corresponding to depth



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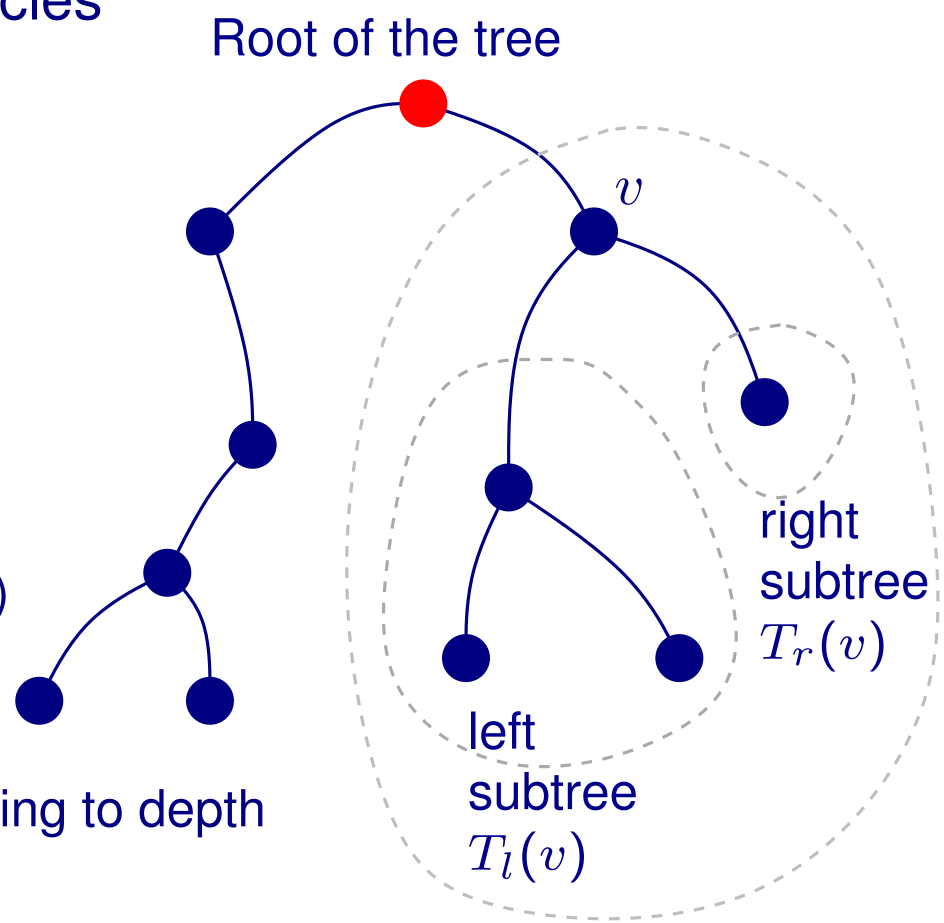

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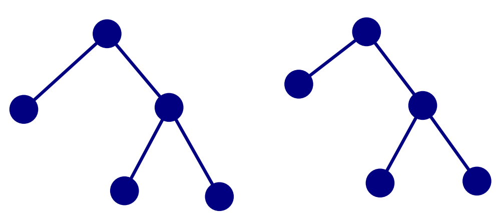
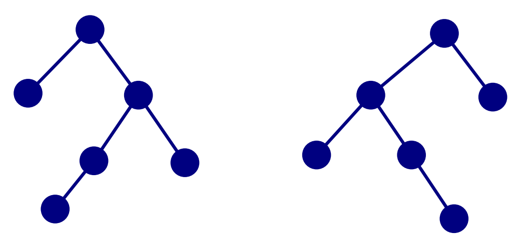
Isomorphism (of ordered trees)

Simple



Axial



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Given: A rooted binary tree

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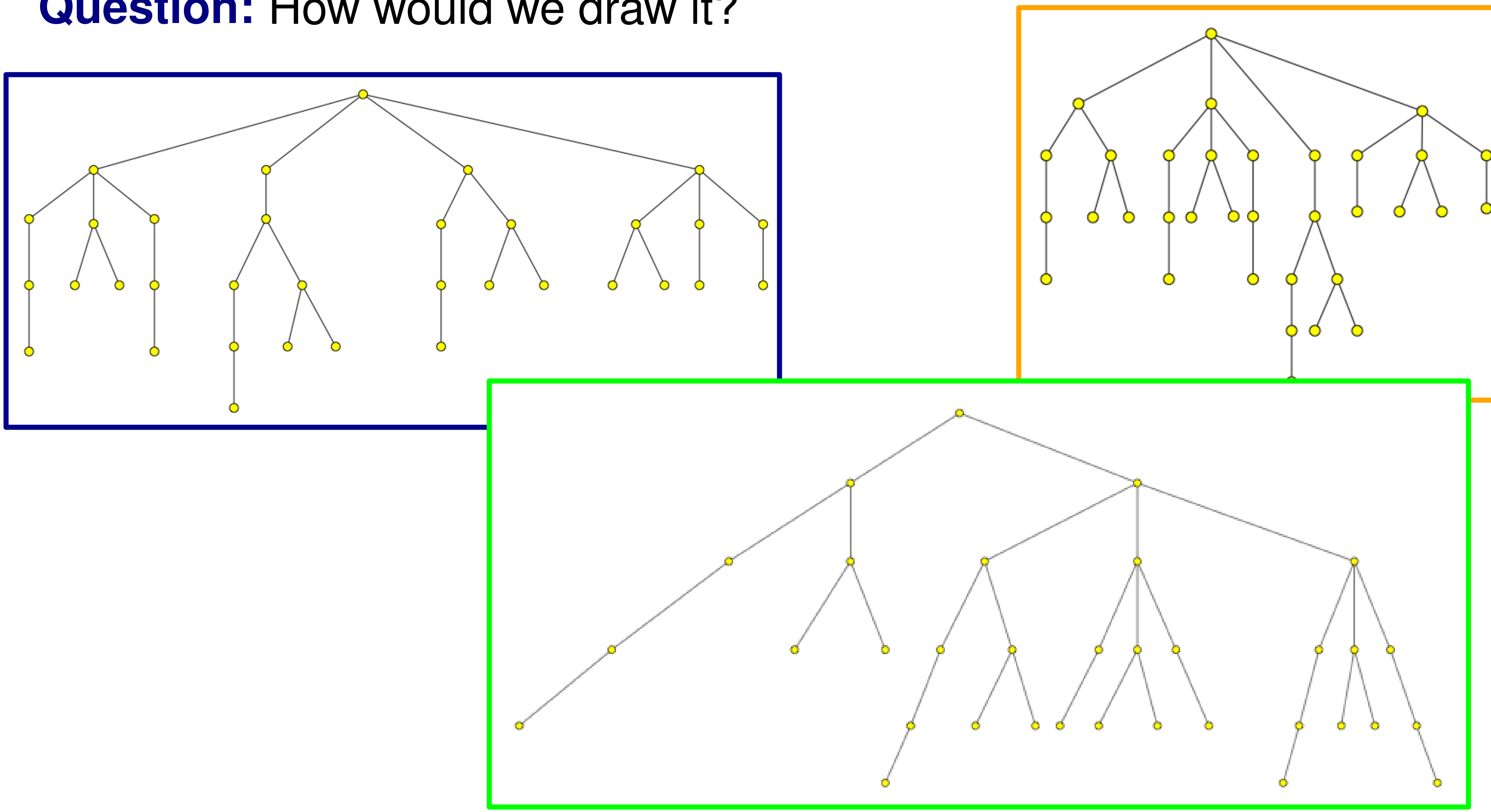

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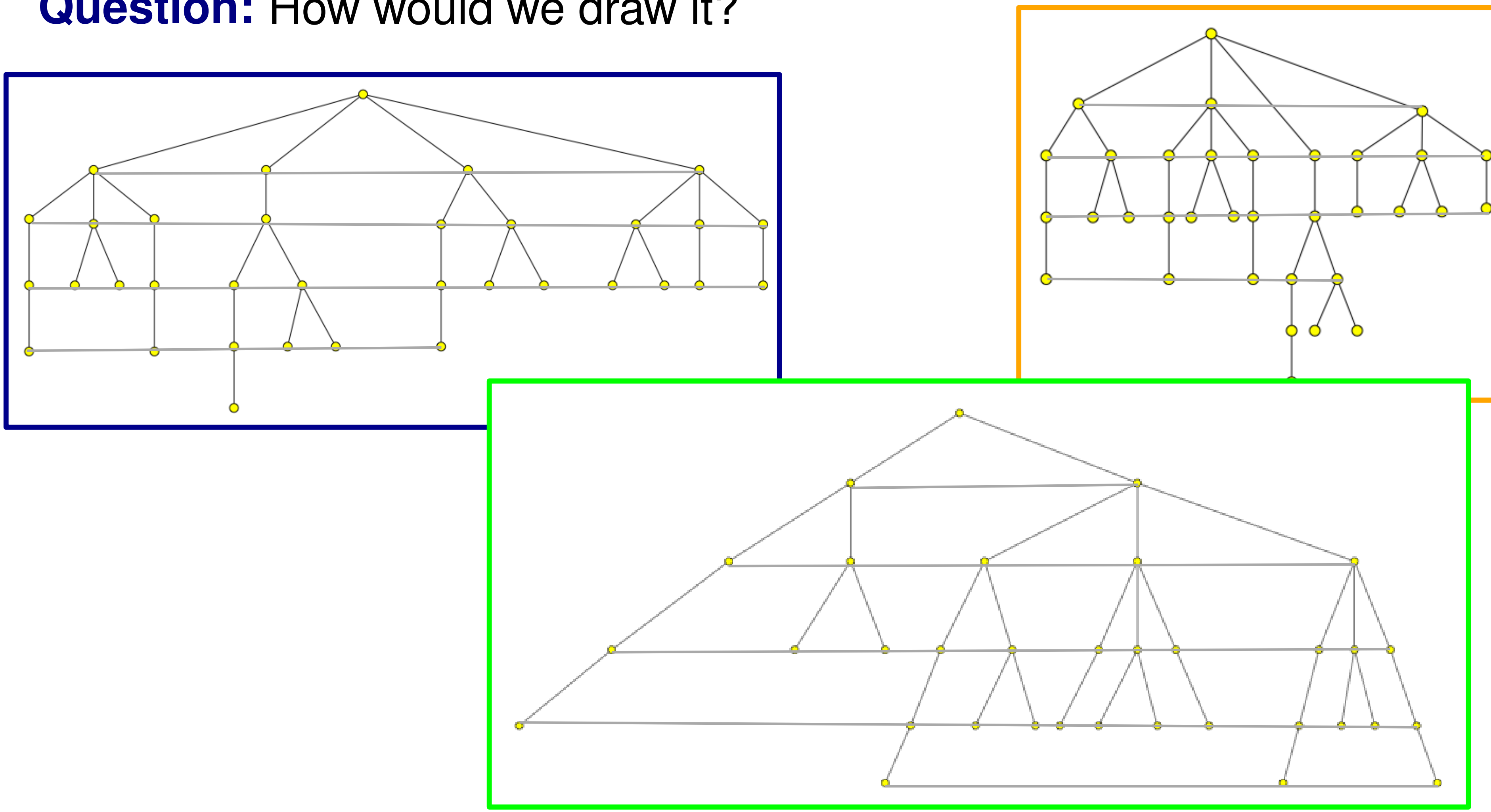

Institut für Theoretische Informatik
Lehrstuhl Algorithmik I

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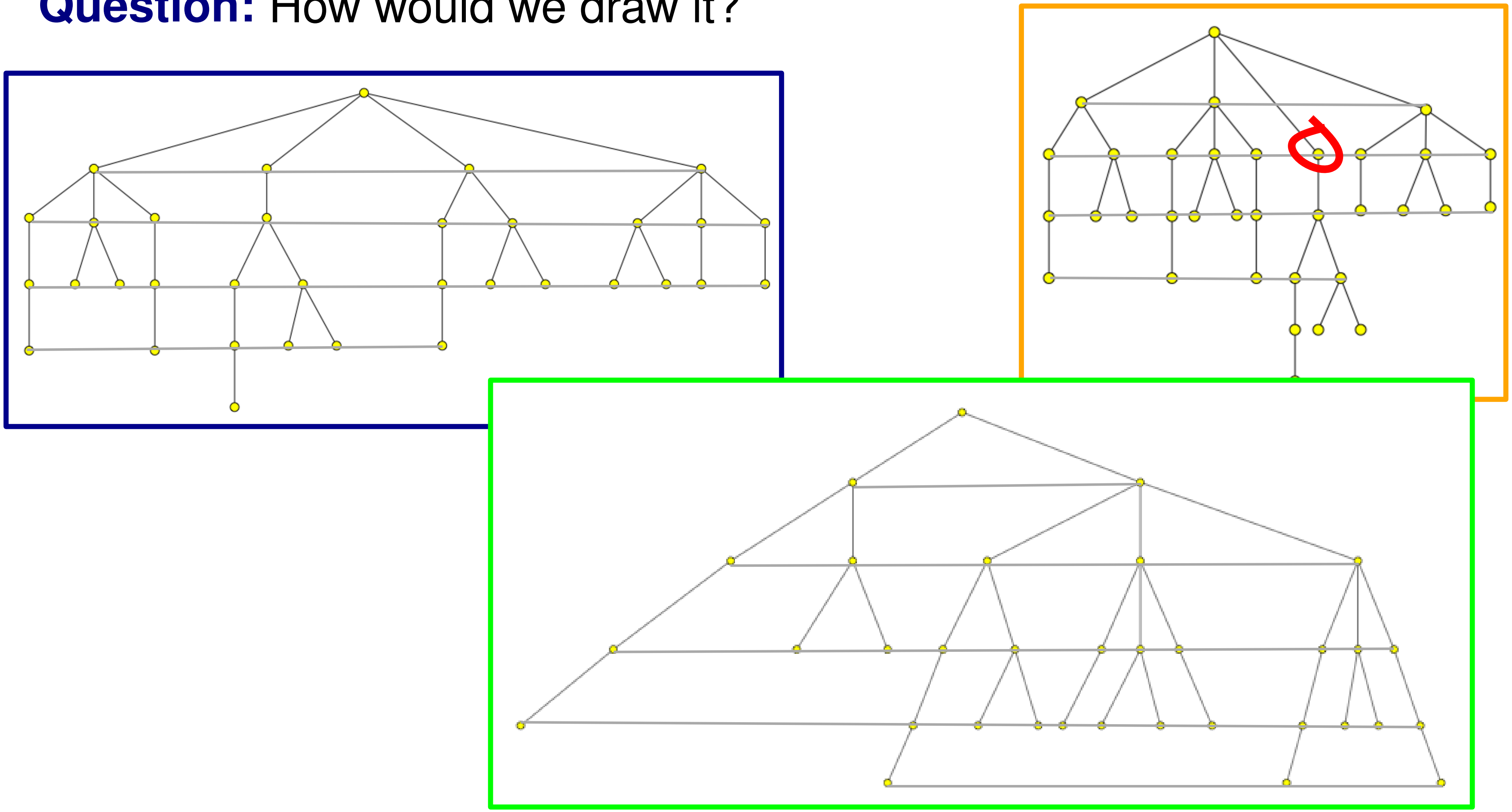

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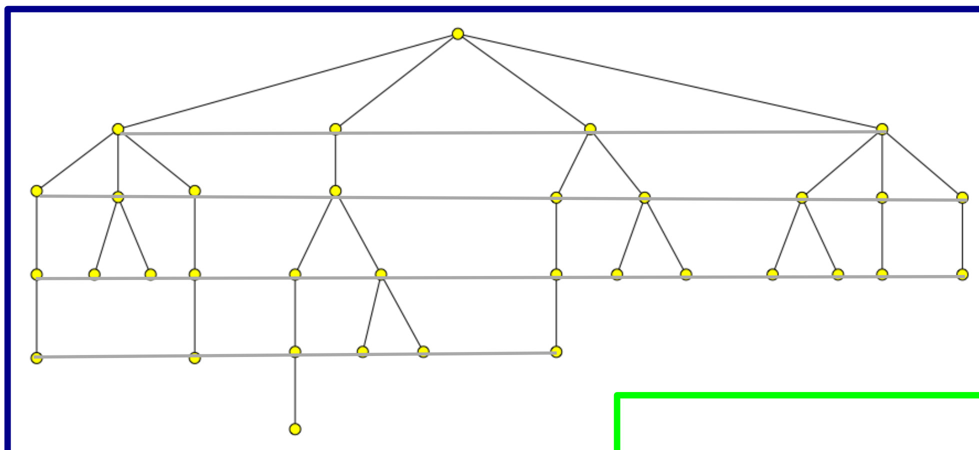

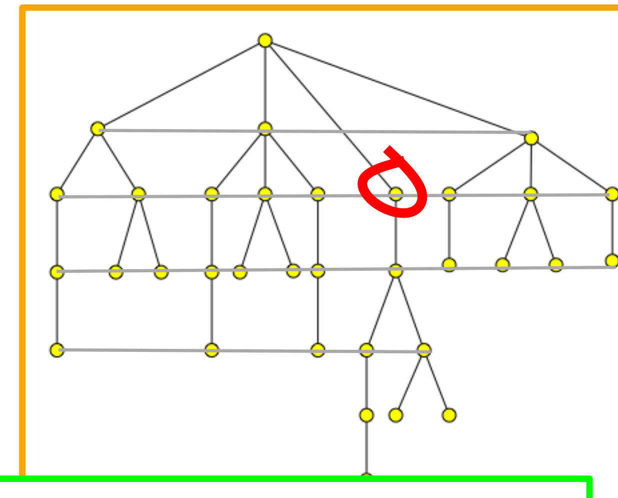

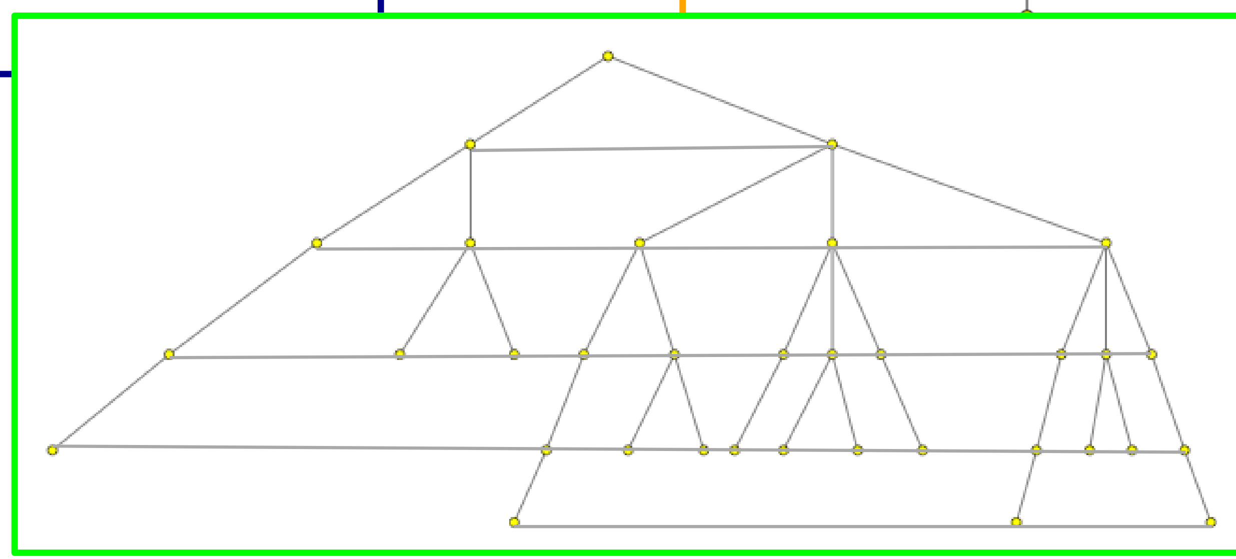

- Vertices are mapped to levels

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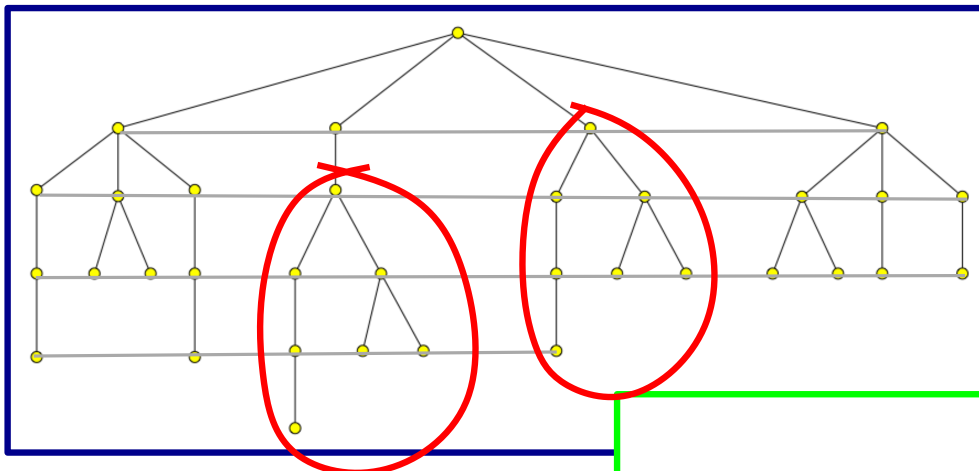

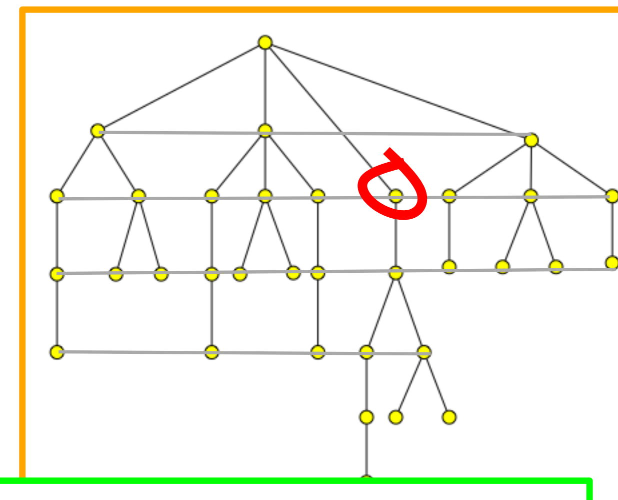

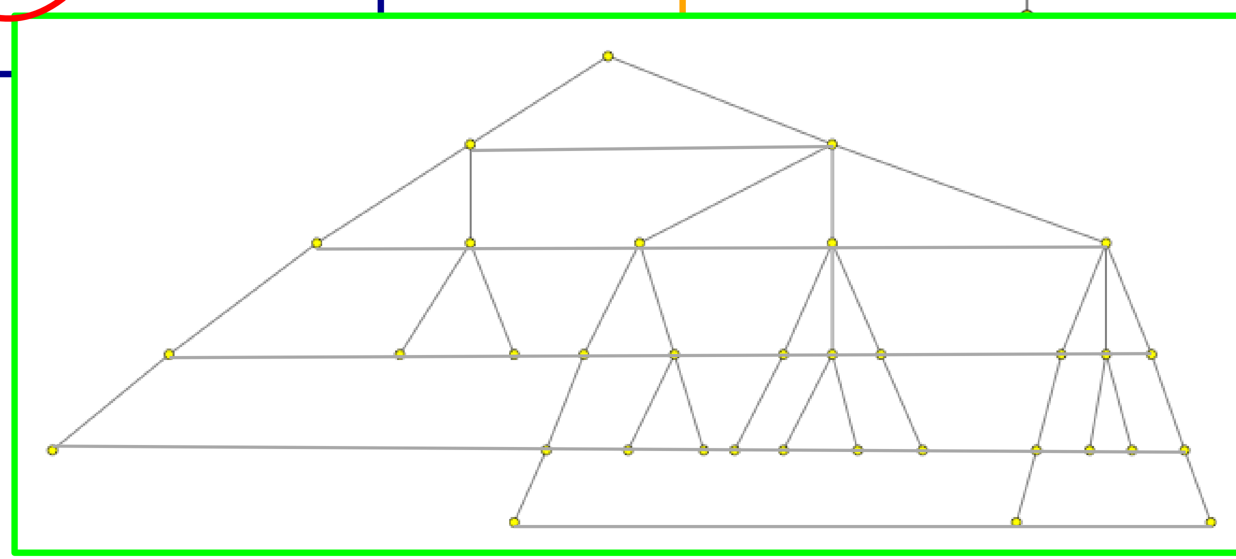

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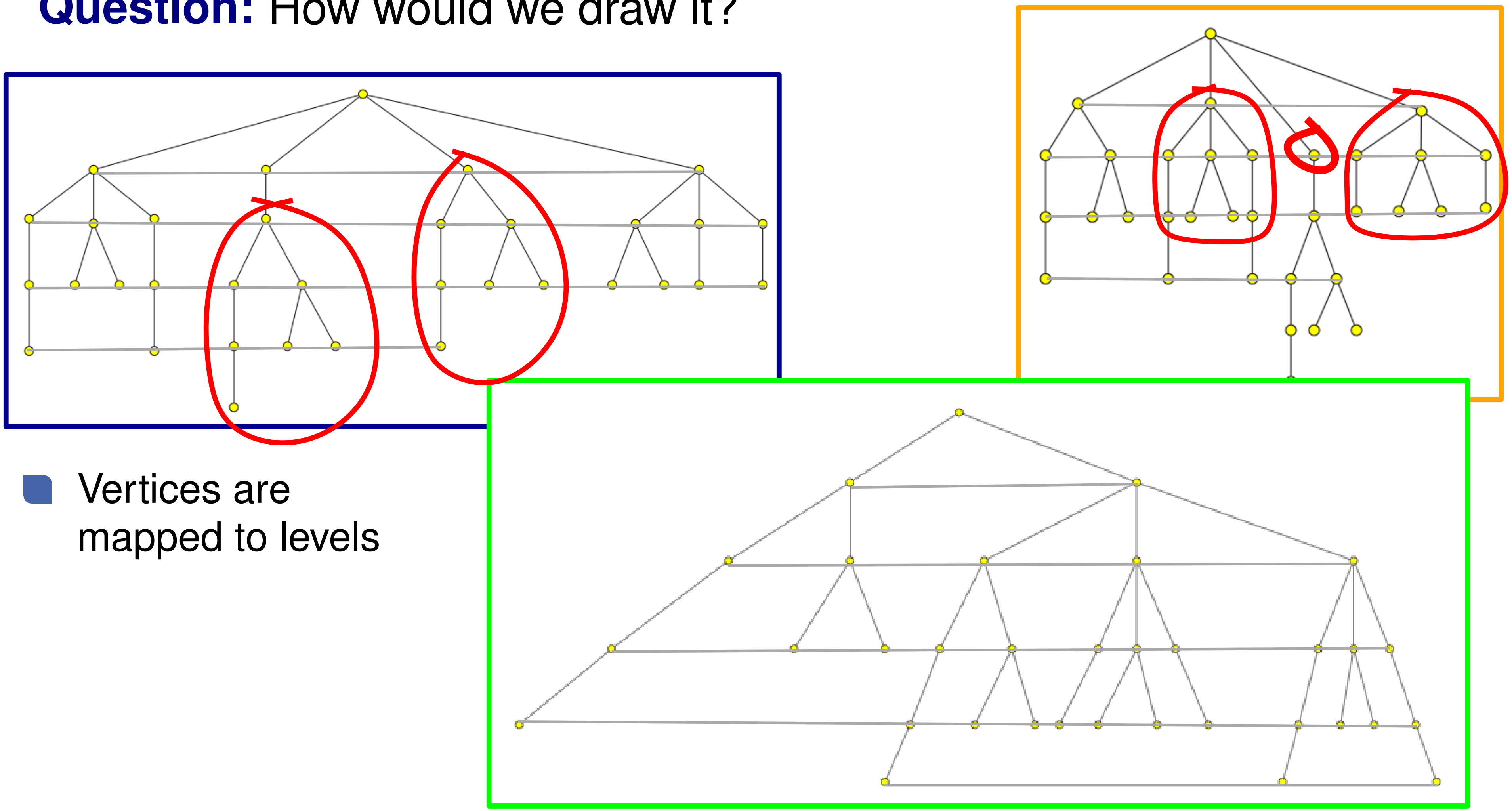

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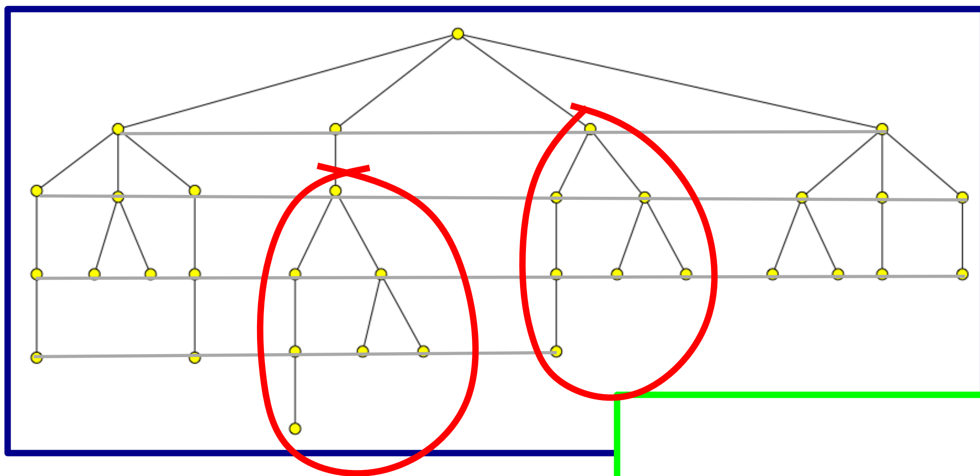

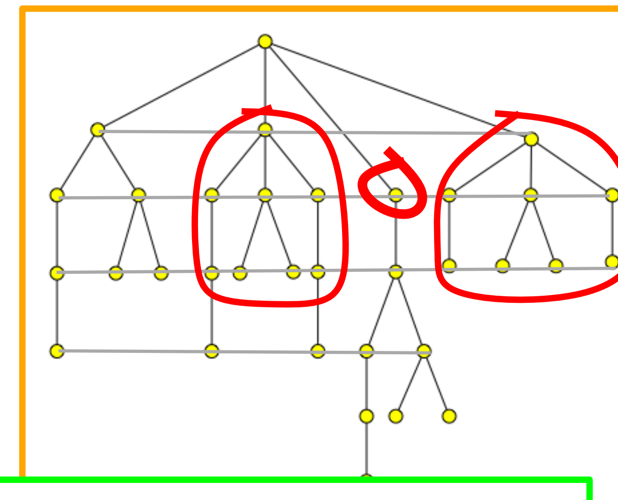

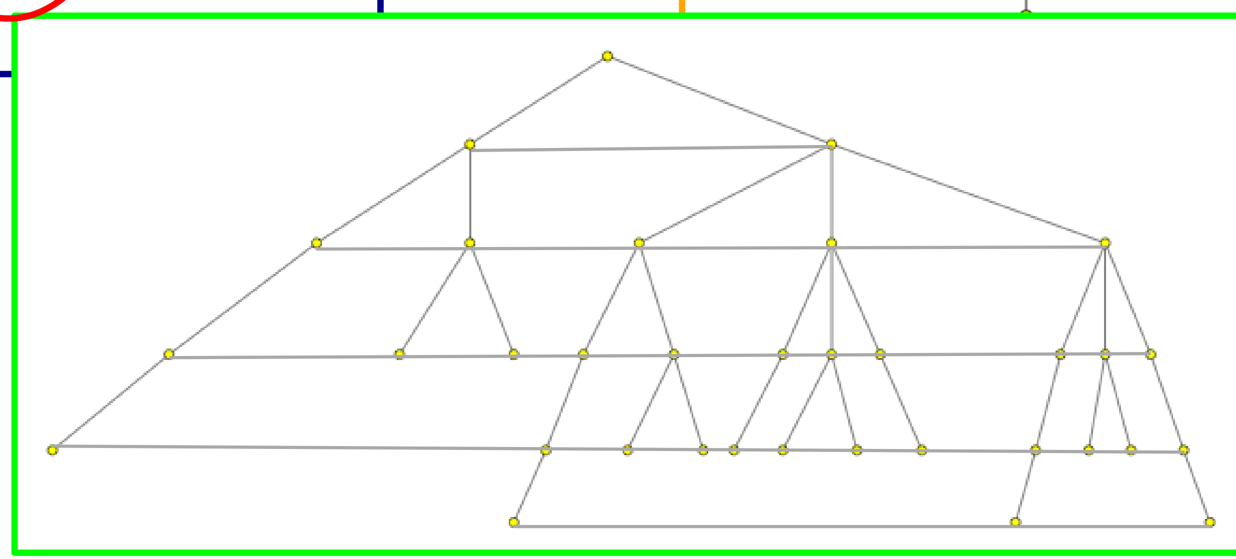

- Vertices are mapped to levels
- Isomorphic trees are drawn similarly

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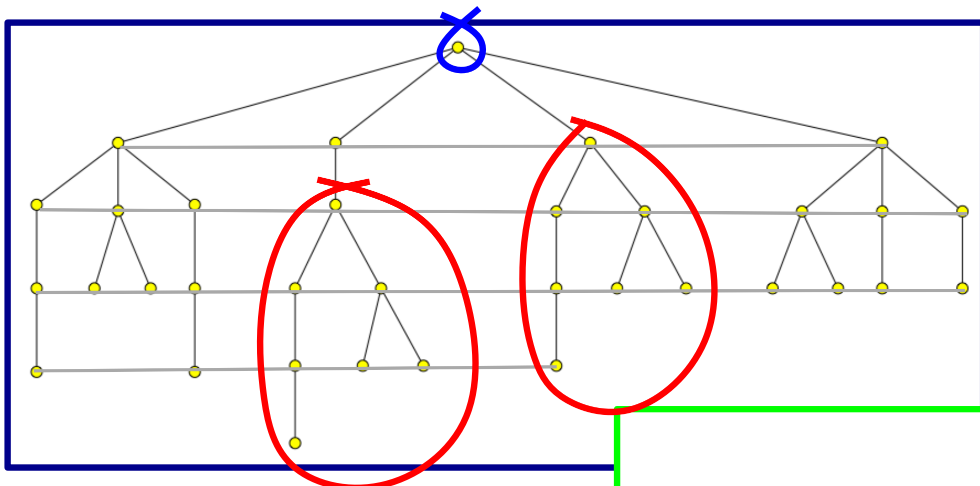

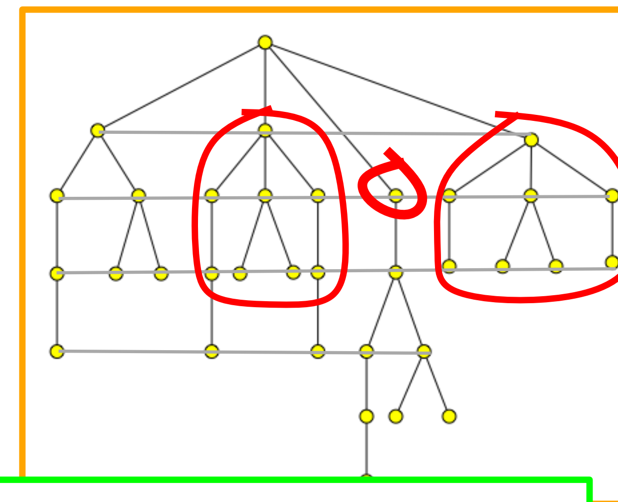

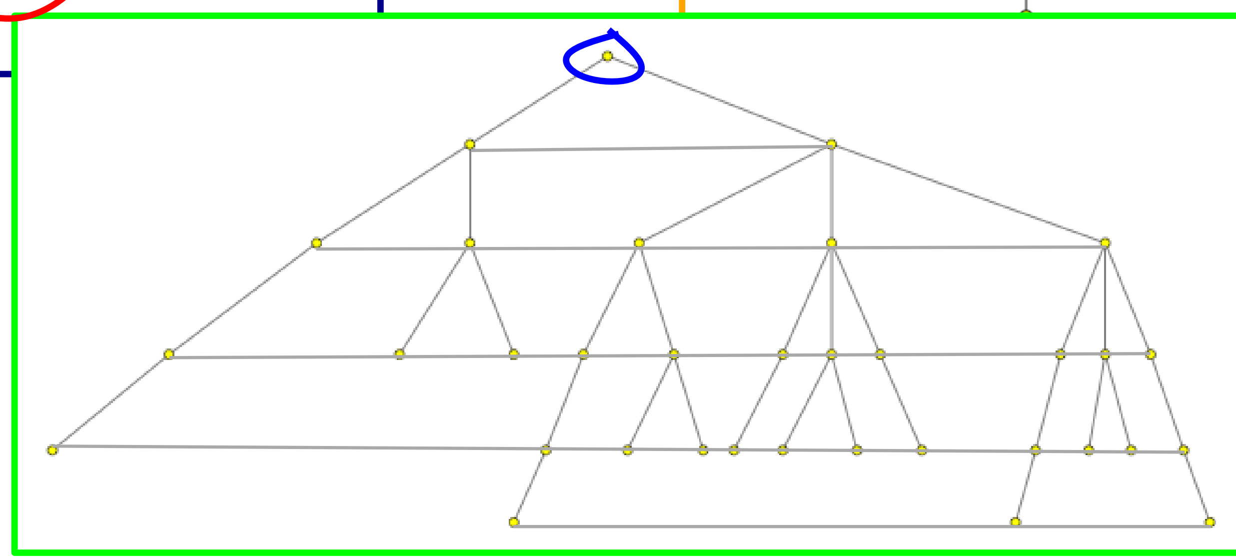

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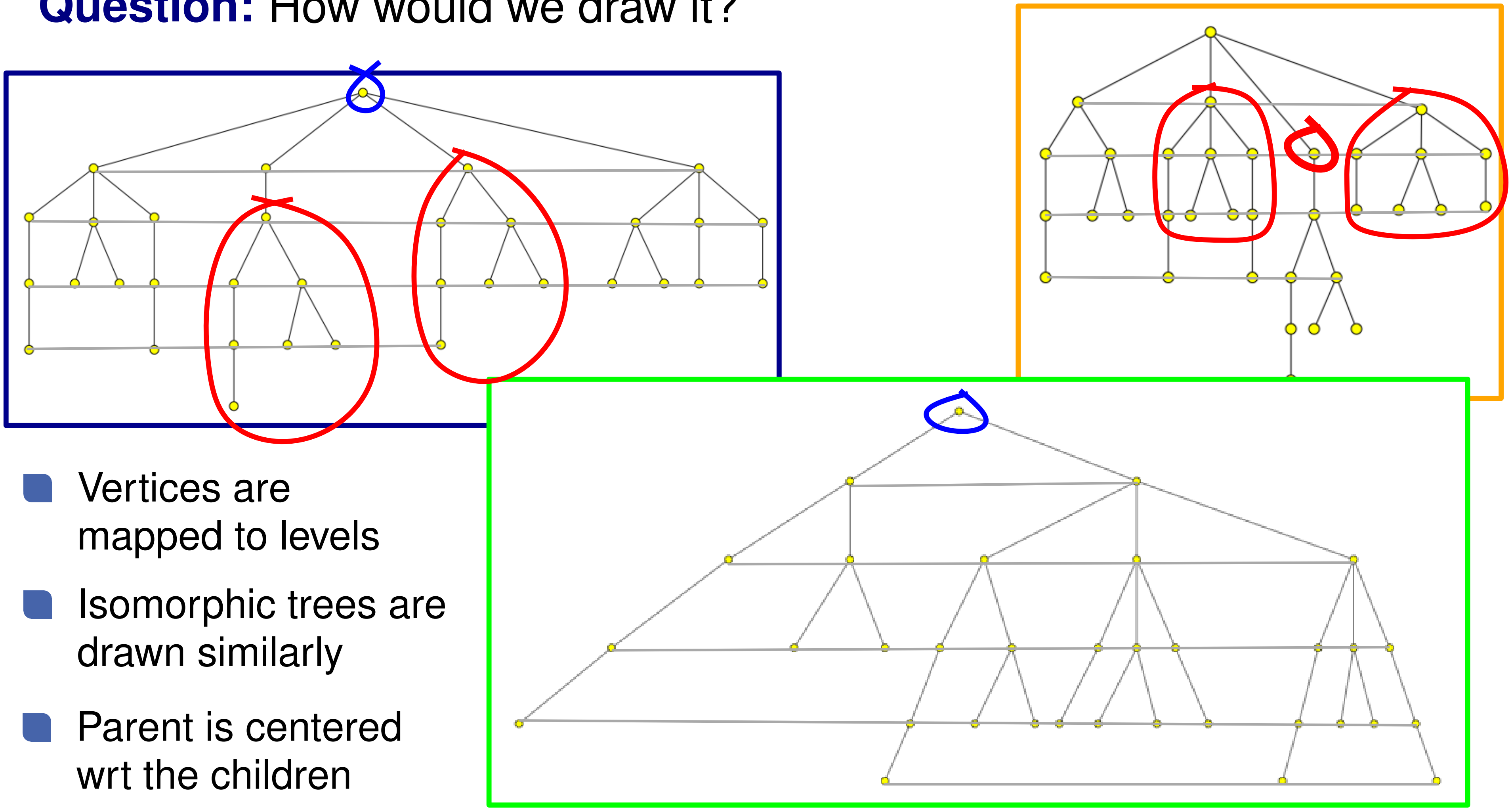


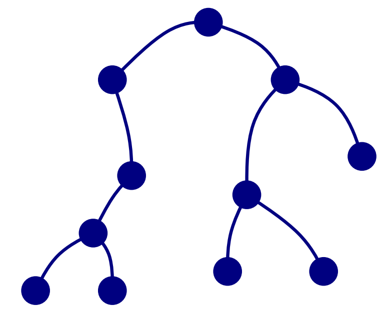
- Vertices are mapped to levels
- Isomorphic trees are drawn similarly
- Parent is centered wrt the children

Level-based Layout

Algorithm Outline:

Input: A binary tree

Output: A leveled drawing of T



Base case: A single vertex

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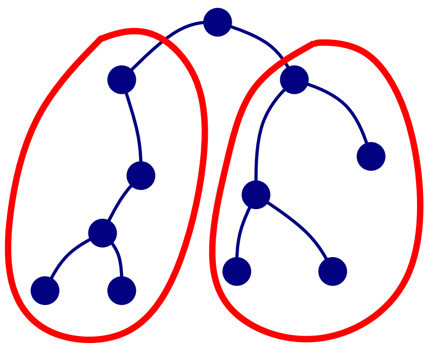

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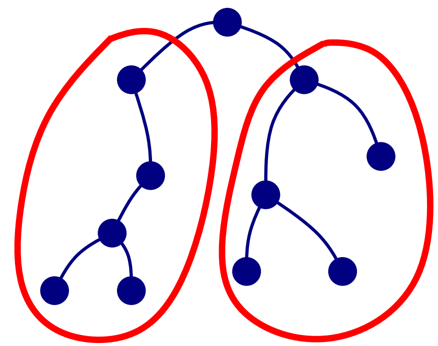

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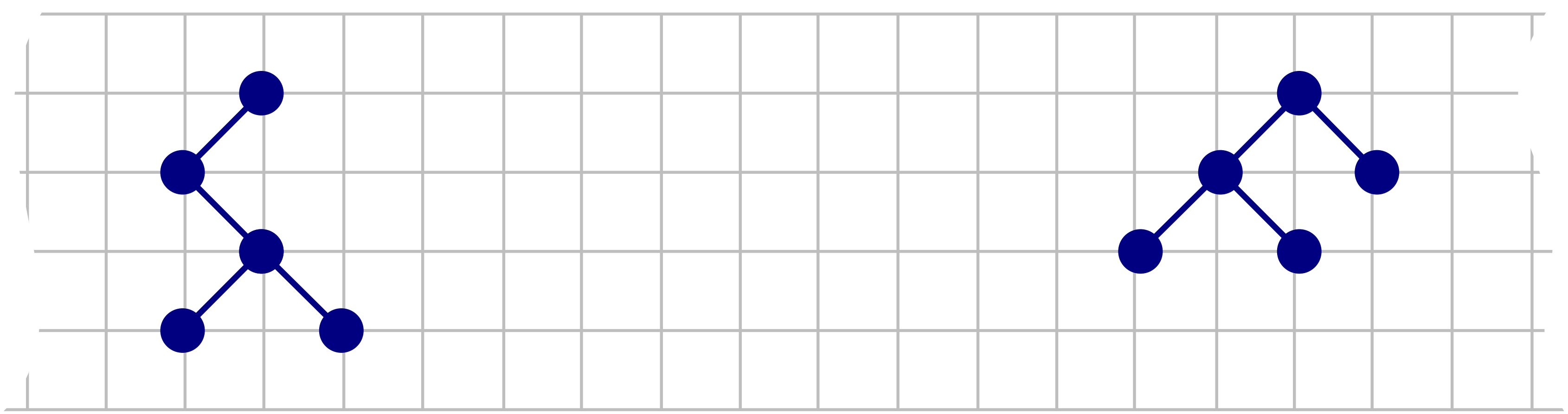

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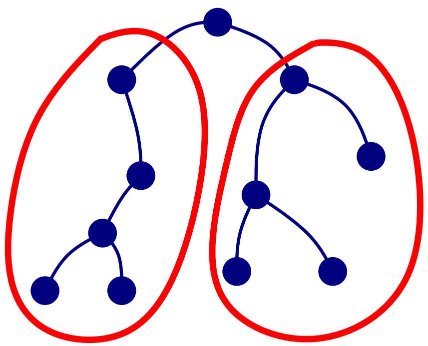

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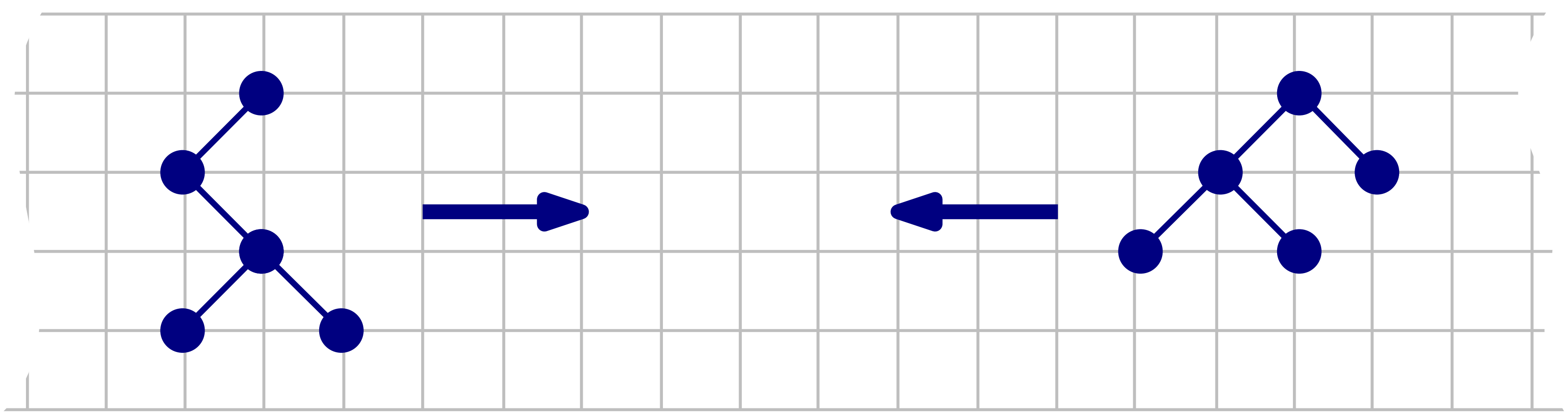

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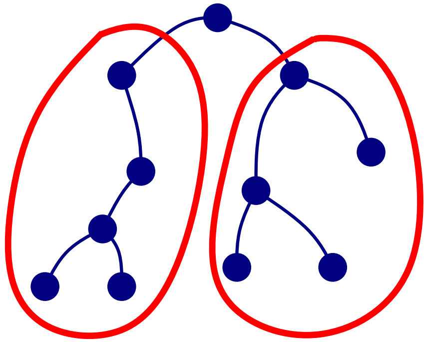

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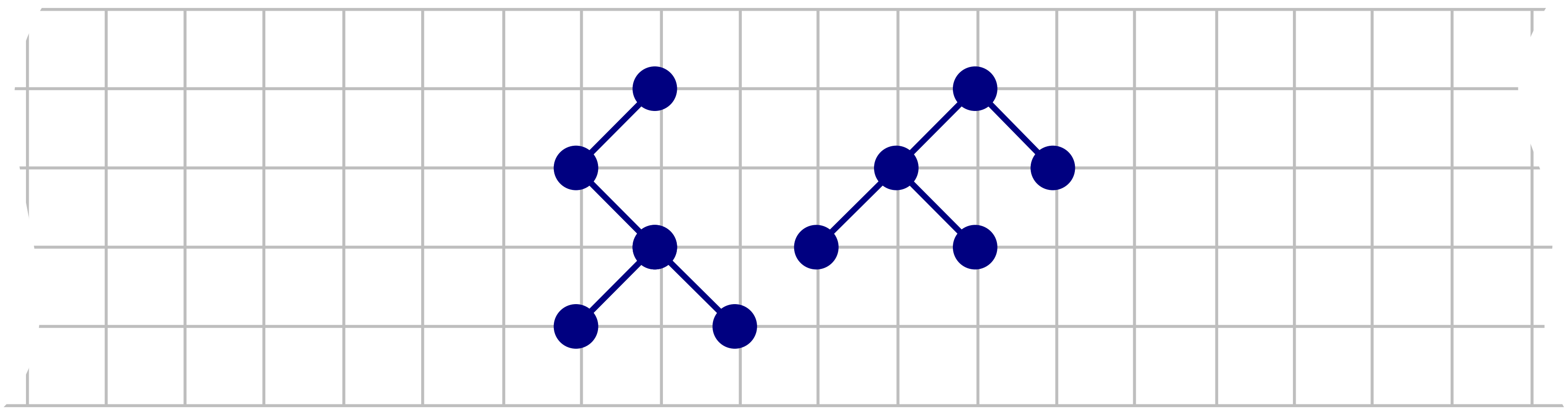

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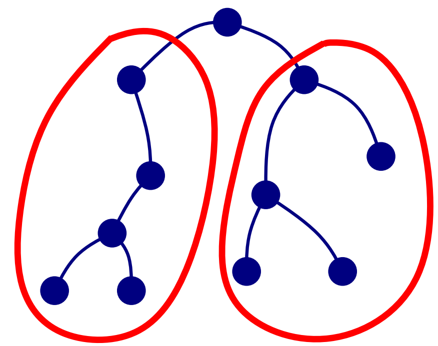

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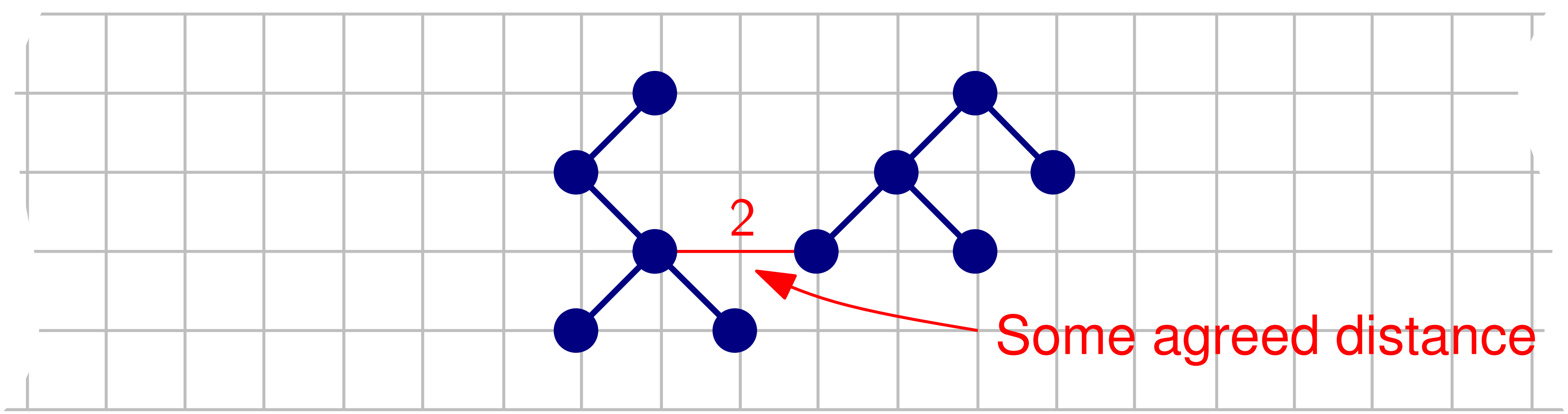

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Karlsruhe Institute of Technology

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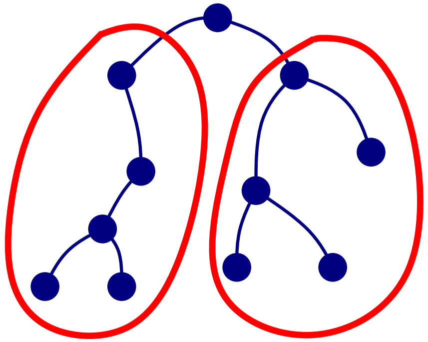

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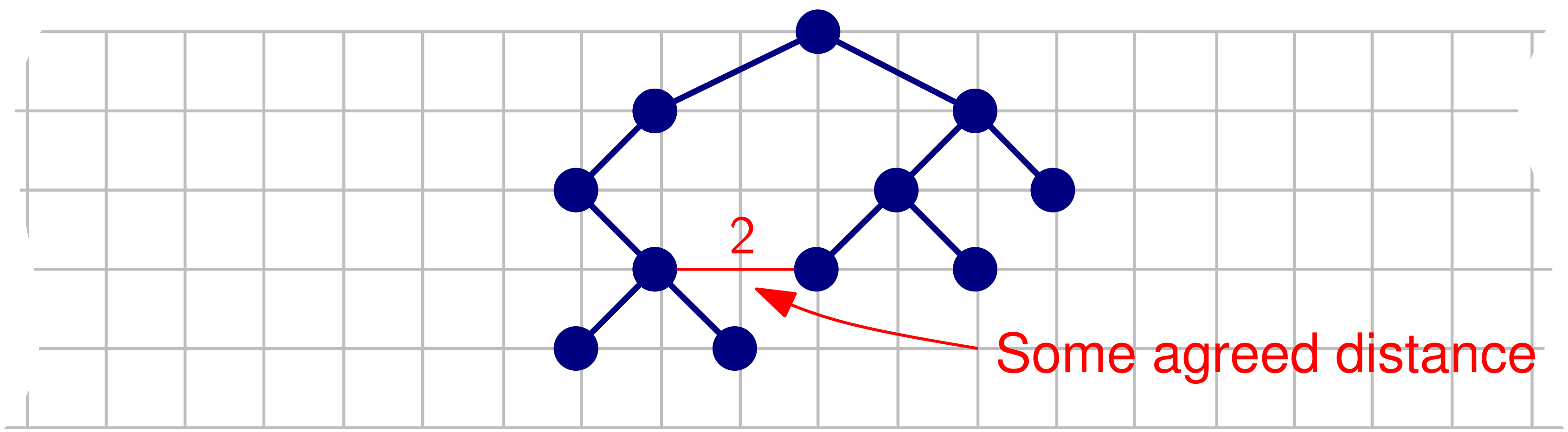

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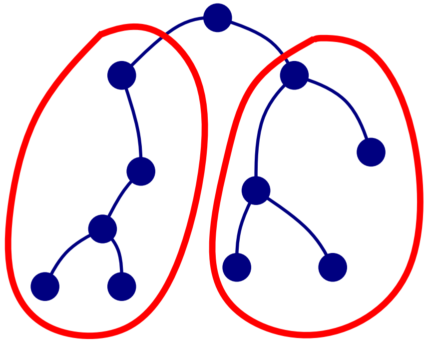

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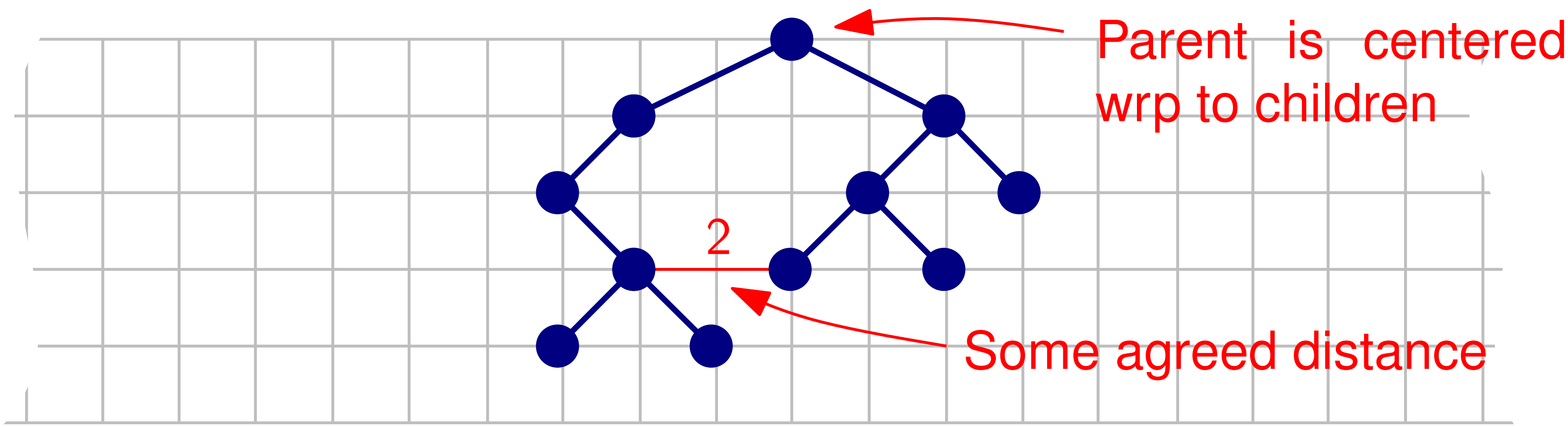

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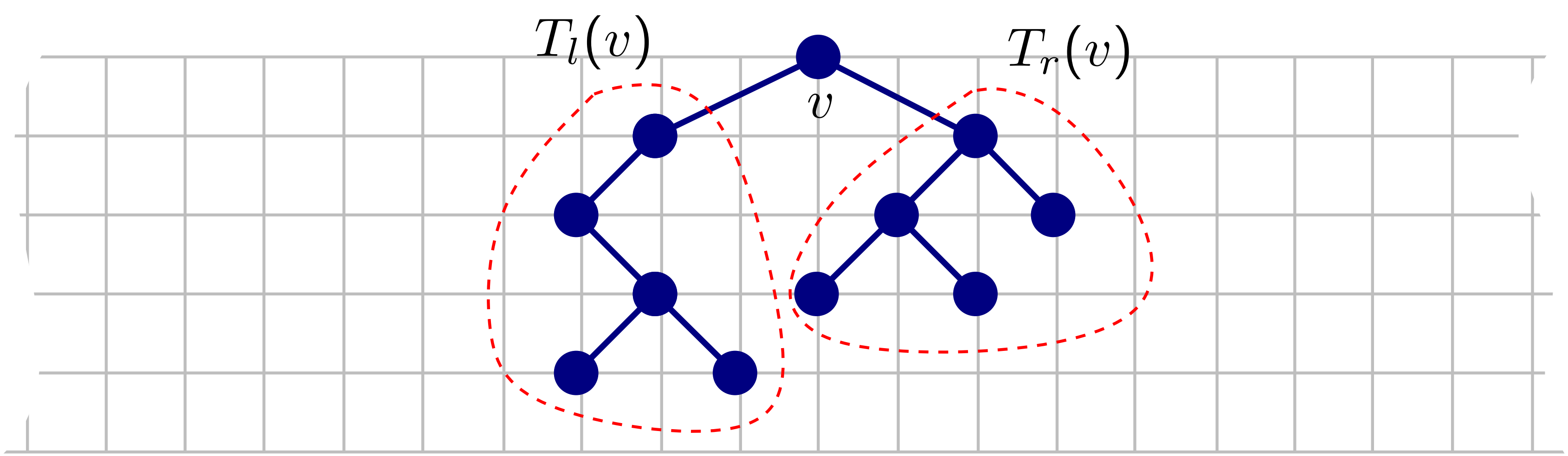
Implementation Details (postorder and preorder traversals)

Postorder traversal: For each vertex v compute horizontal displacement of the left and the right child



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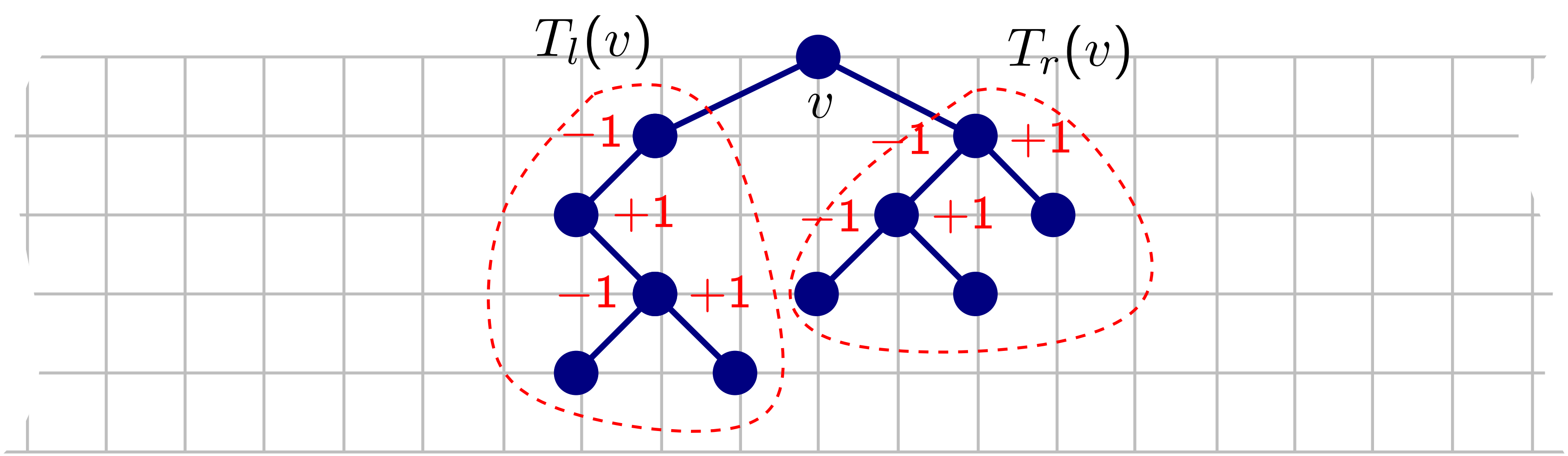

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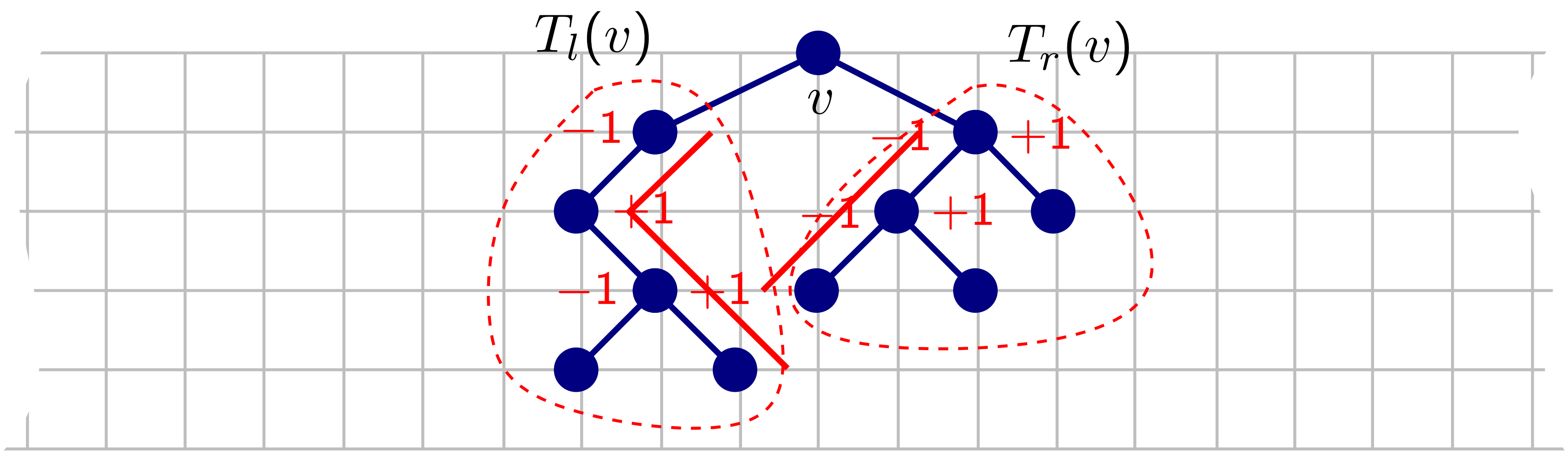
Postorder traversal: For each vertex v compute horizontal displacement of the left and the right child

- Assume at each vertex u (below v) we have stored the left and the right boundary of the subtree $T(u)$



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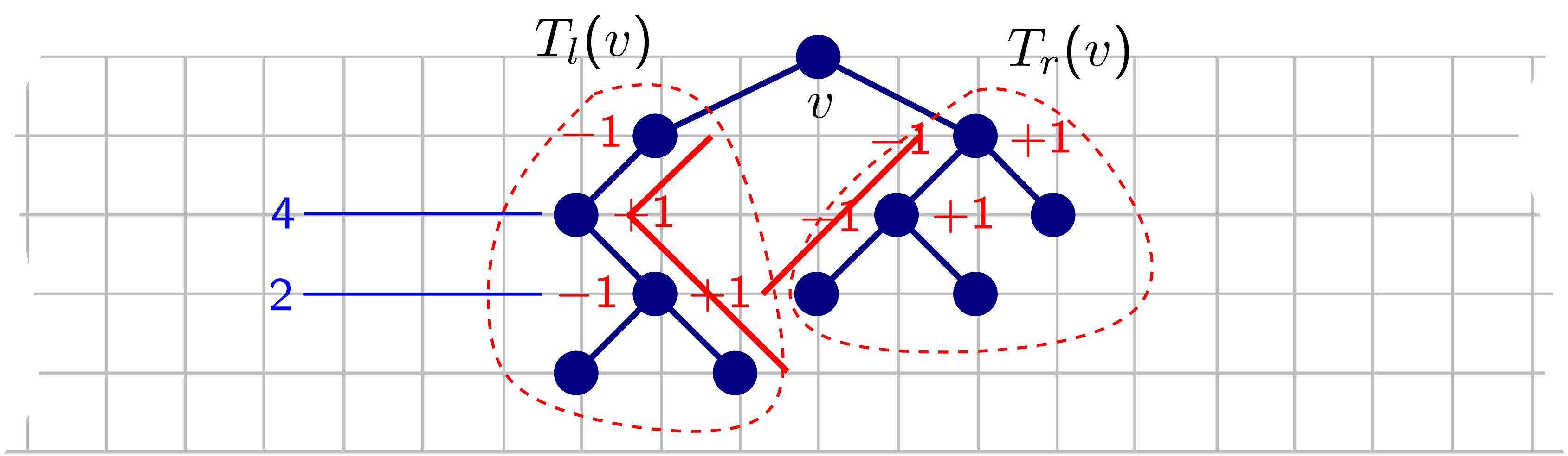

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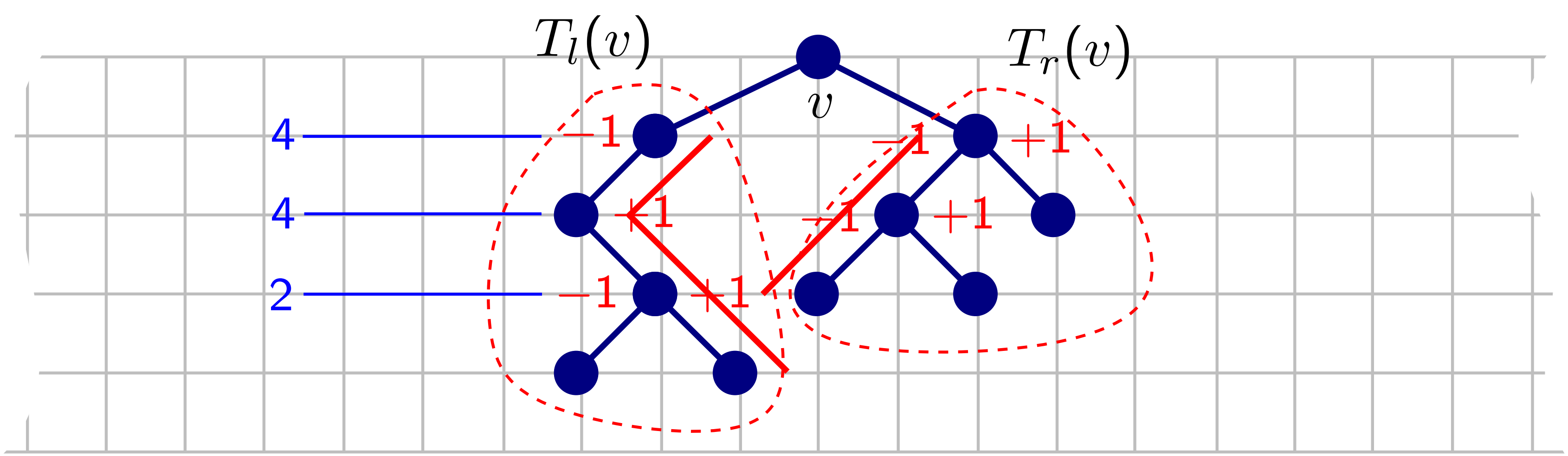

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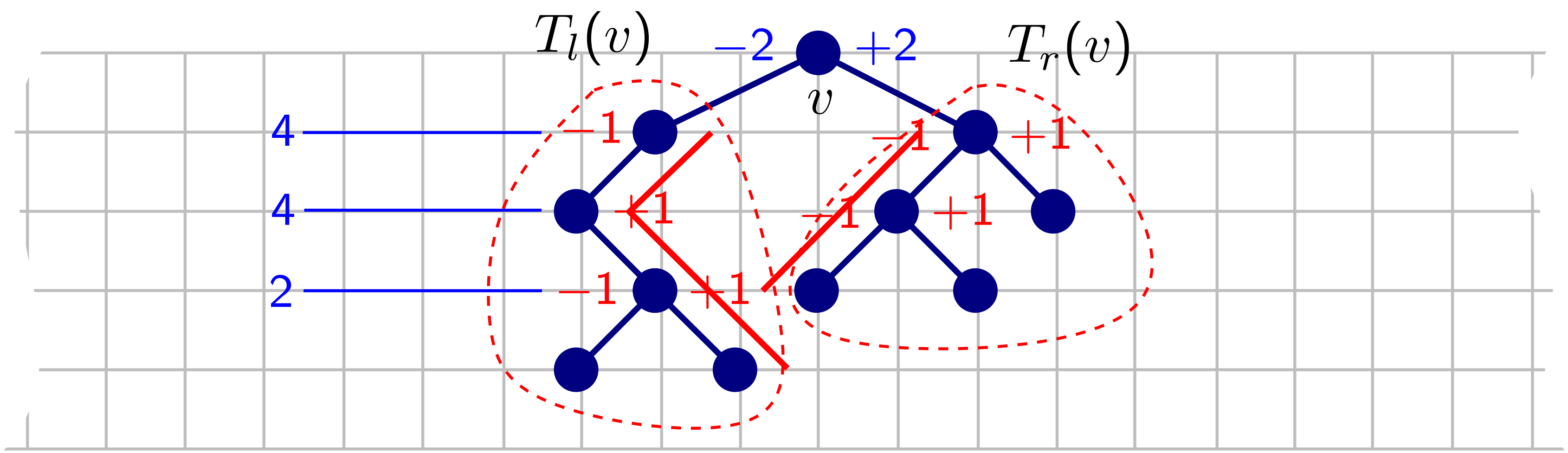

Level-based Layout

Implementation Details (postorder and preorder traversals)

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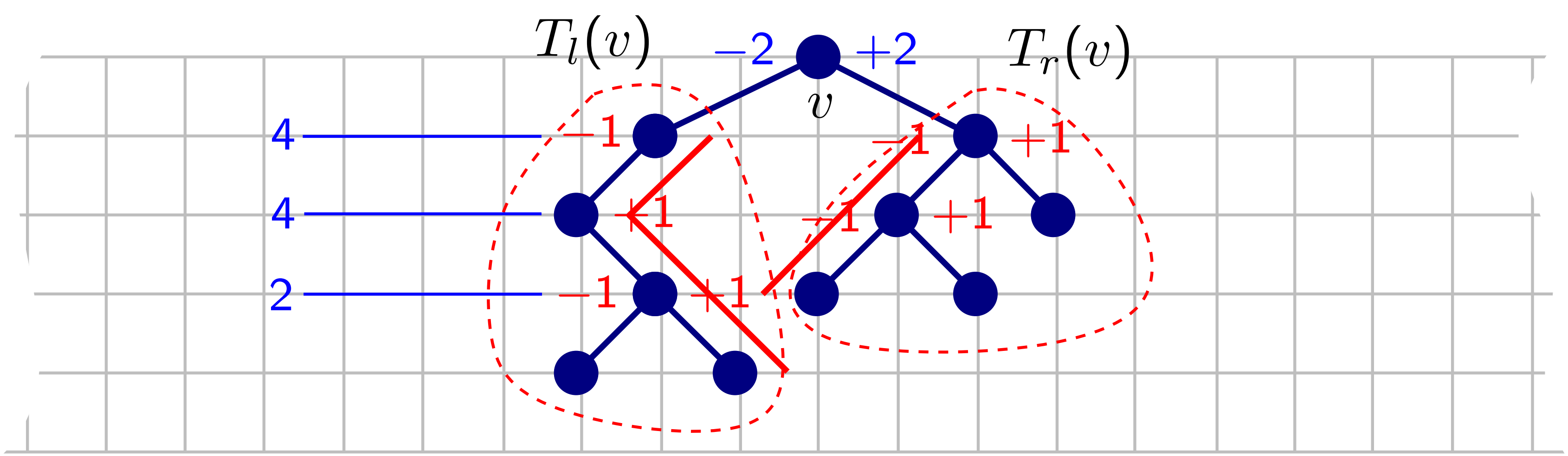

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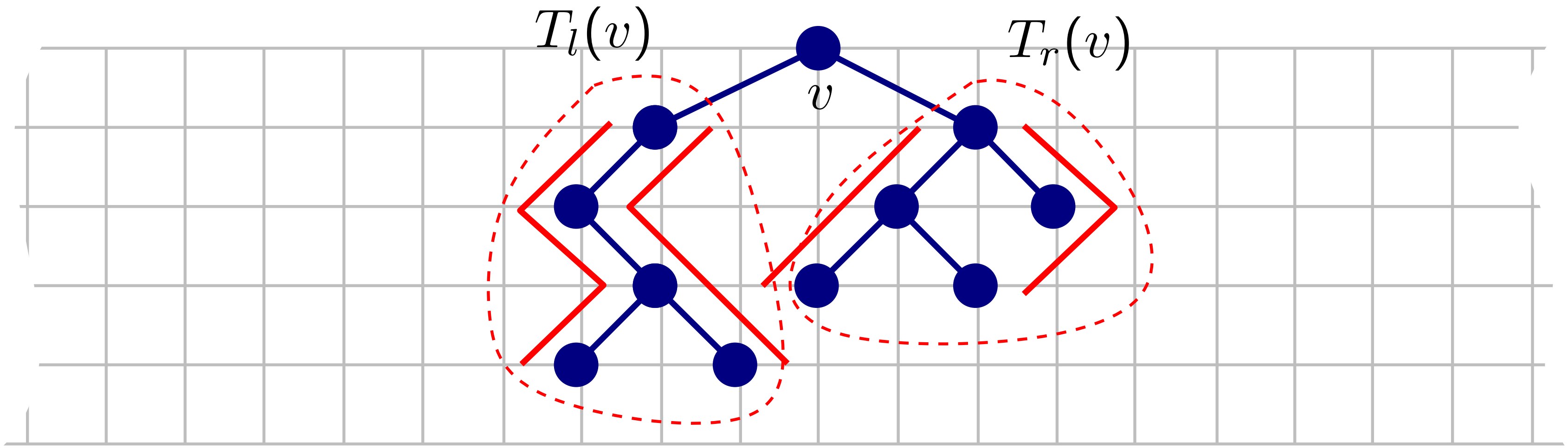

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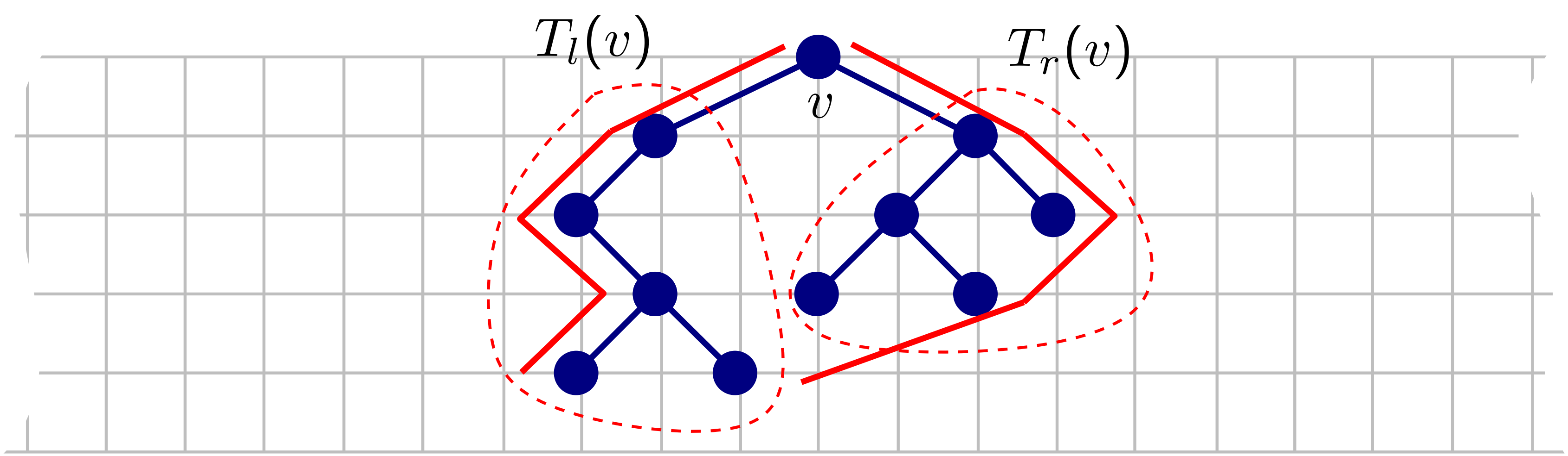

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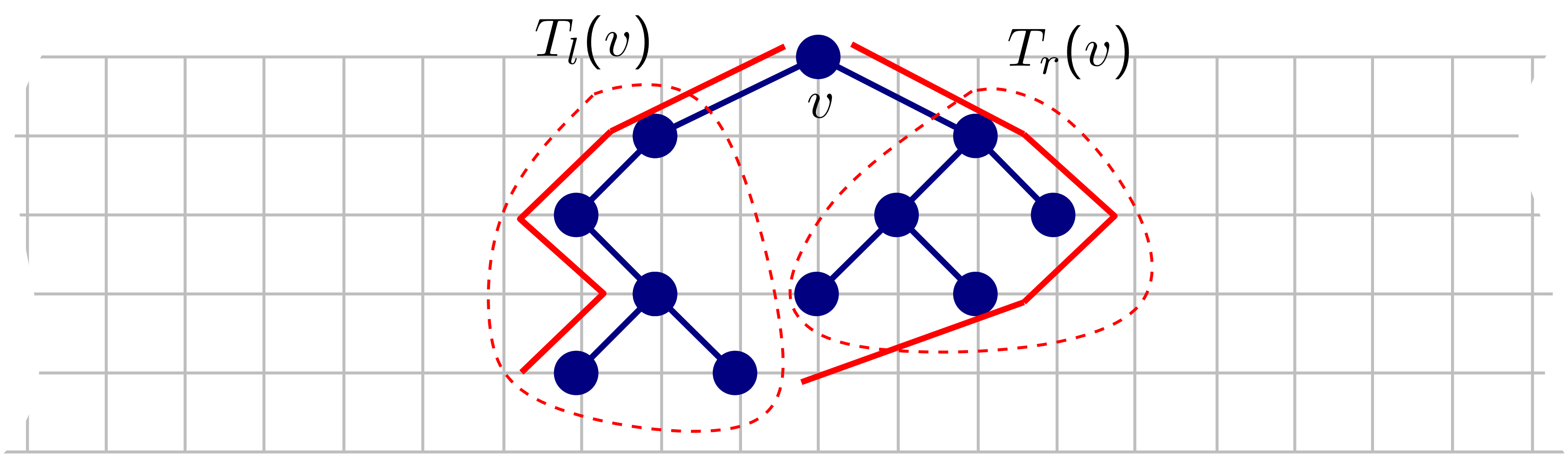

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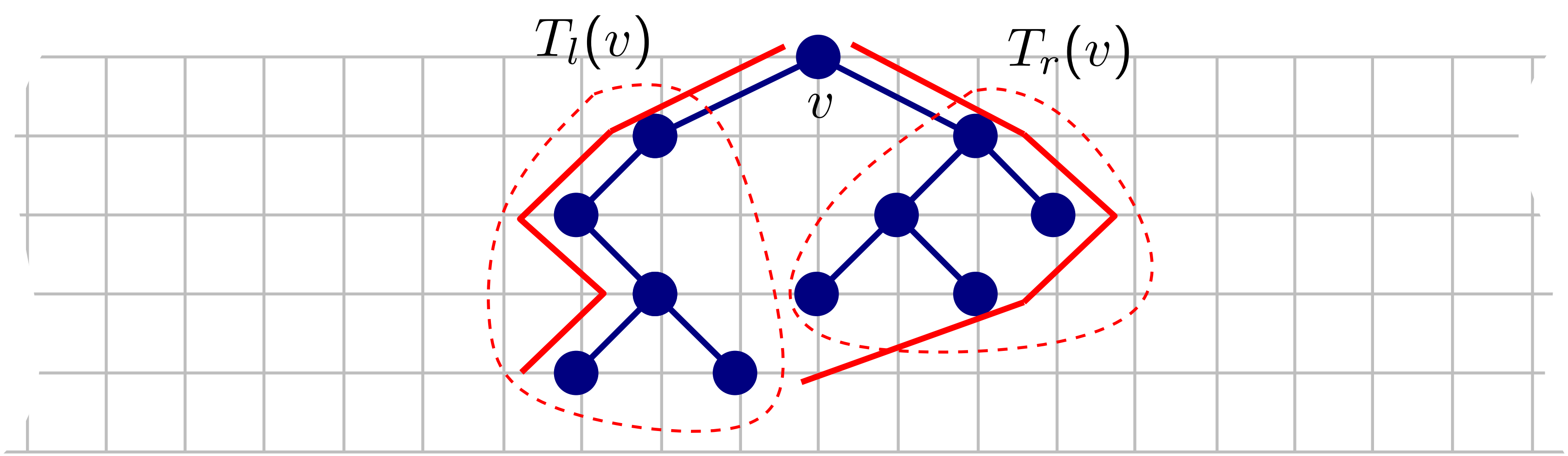

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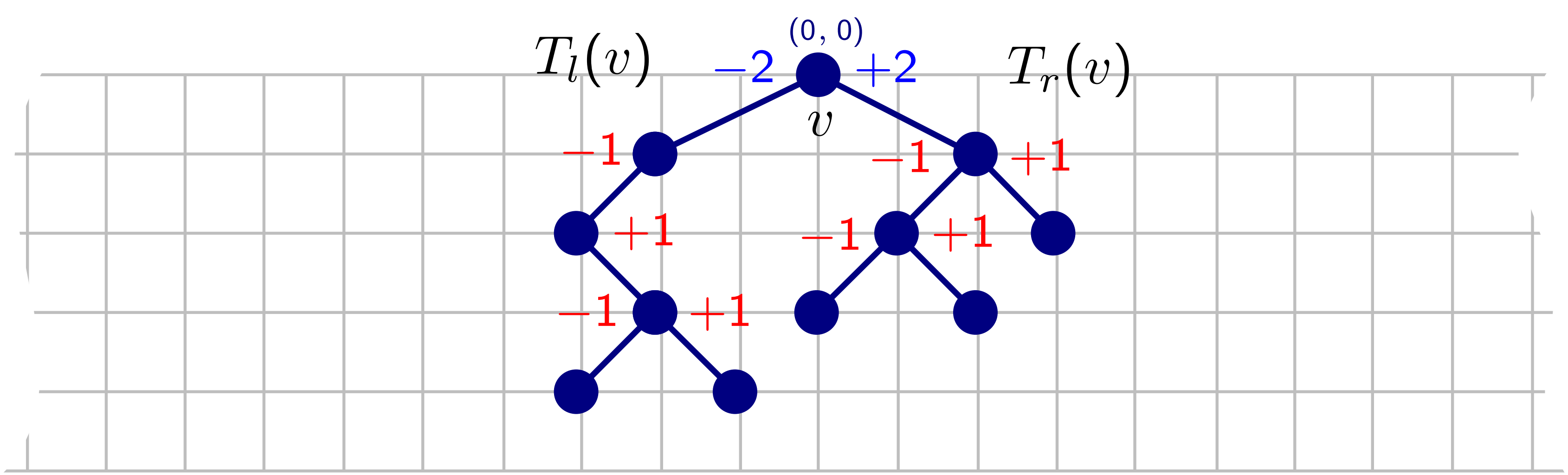

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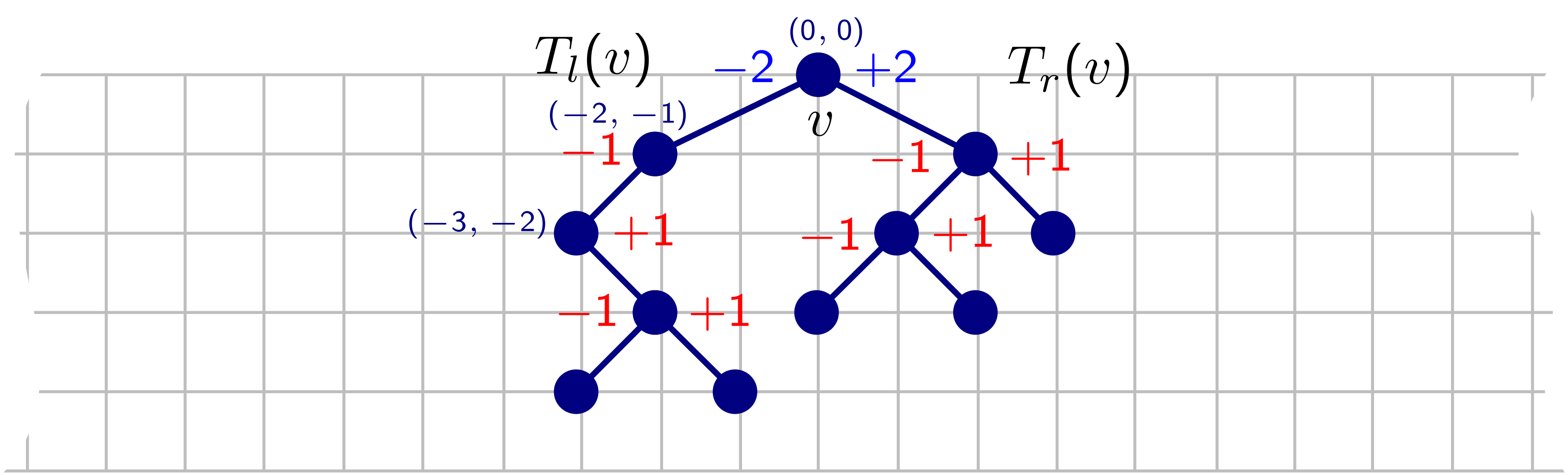

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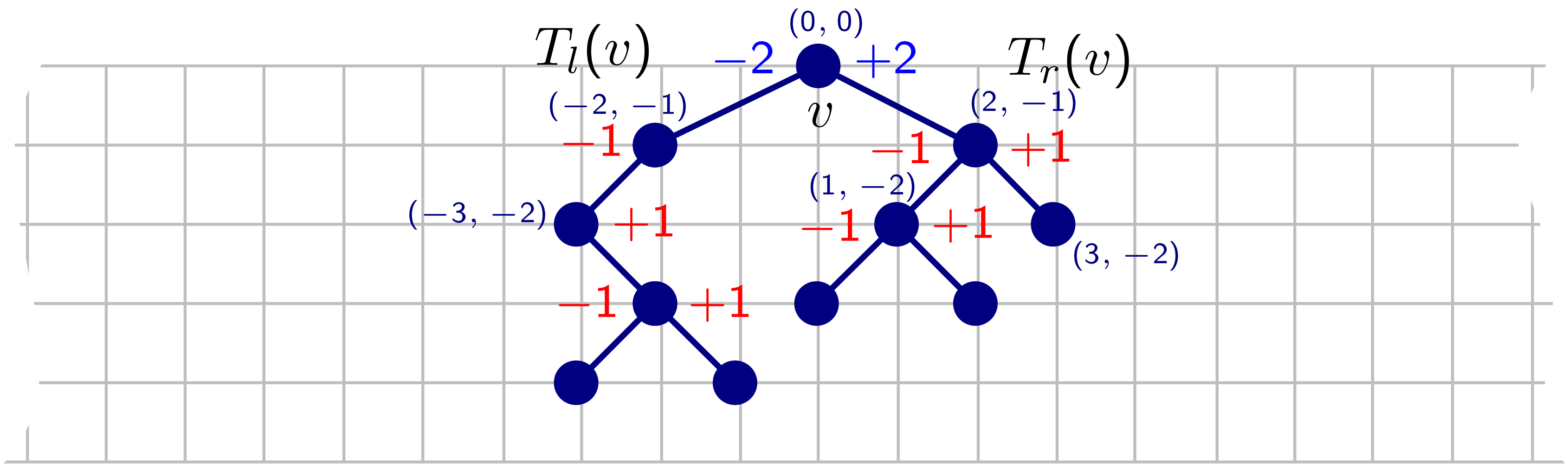

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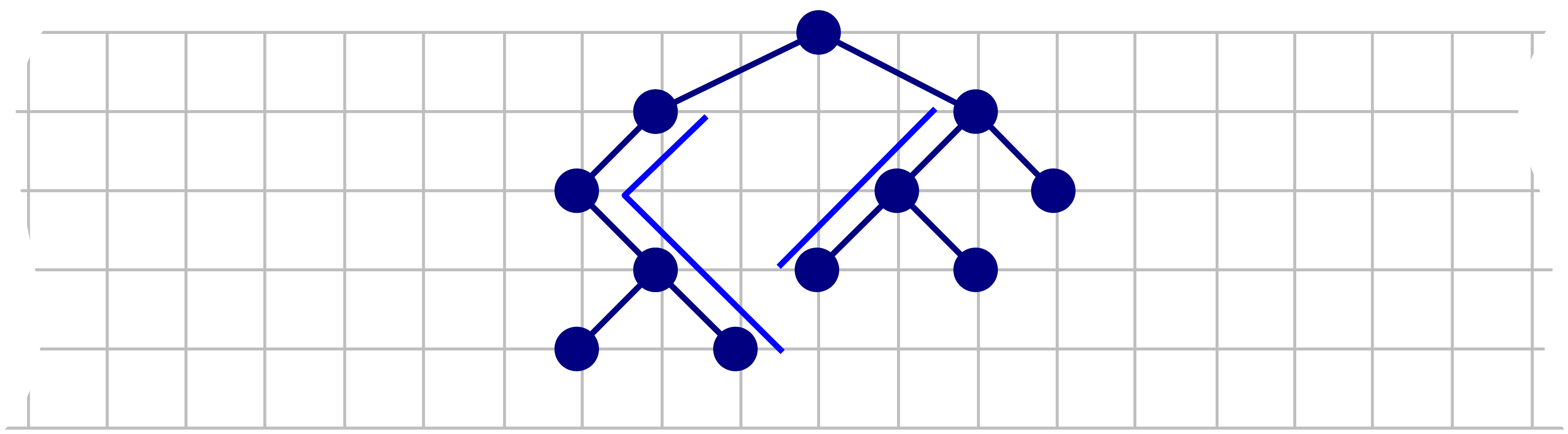

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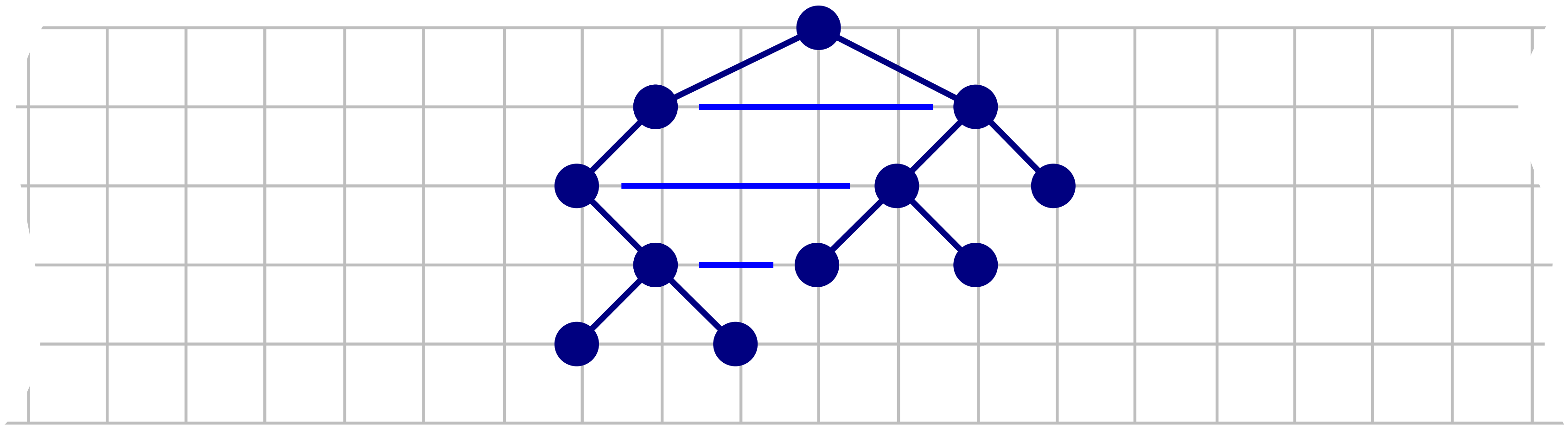

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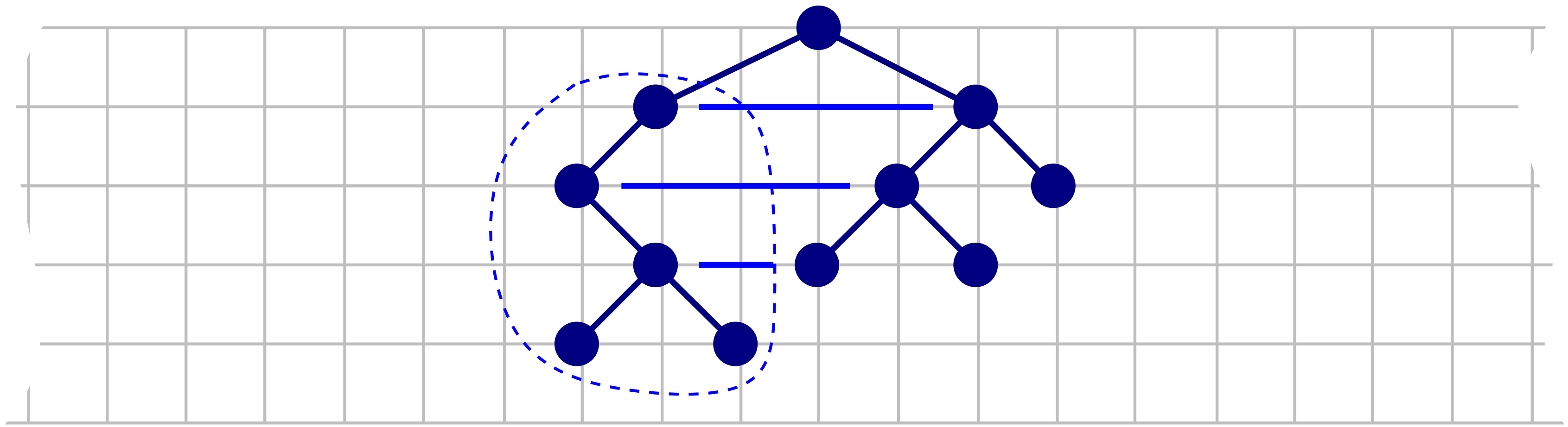

Level-based Layout

Karlsruhe Institute of Technology

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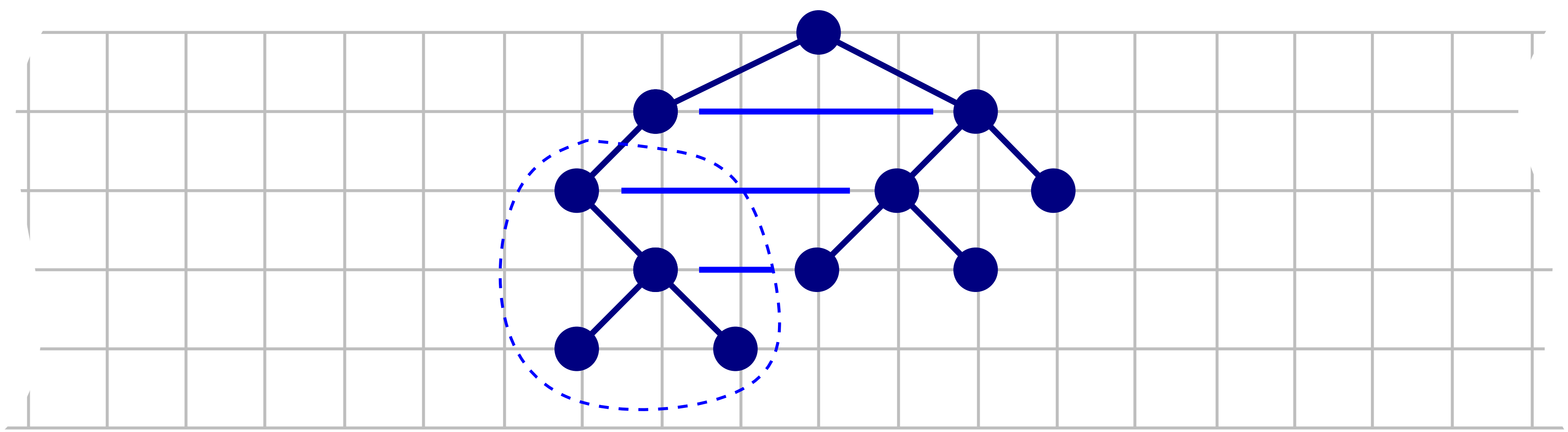

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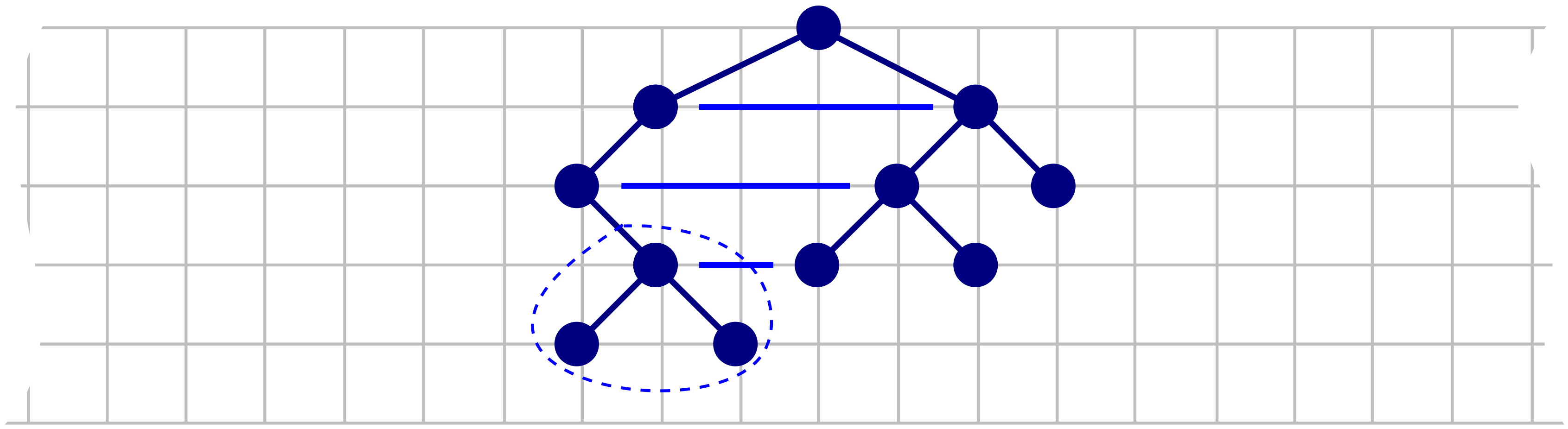

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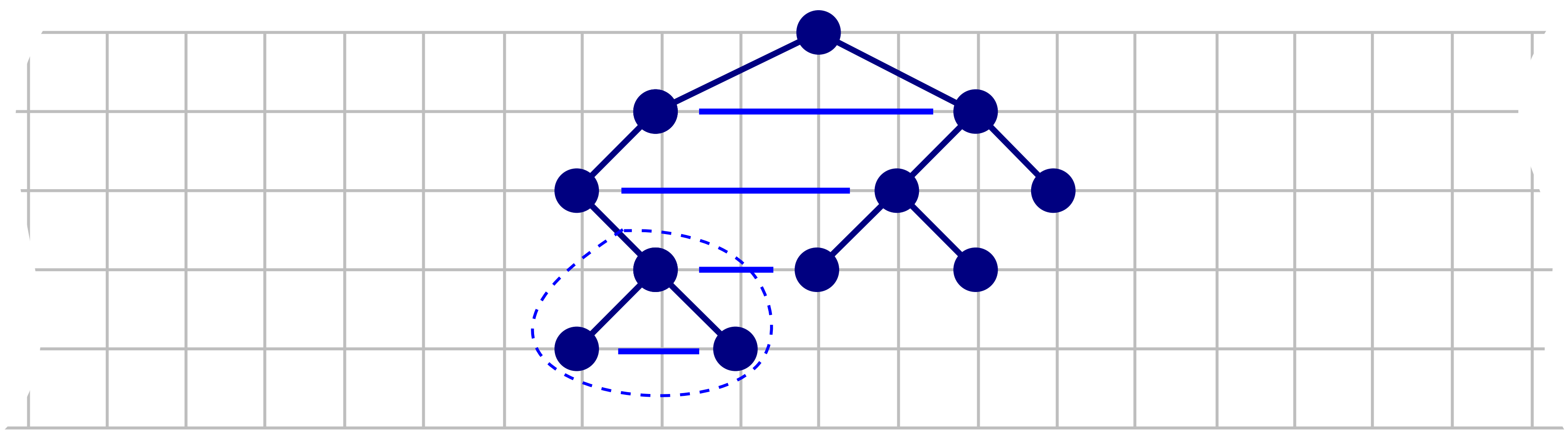

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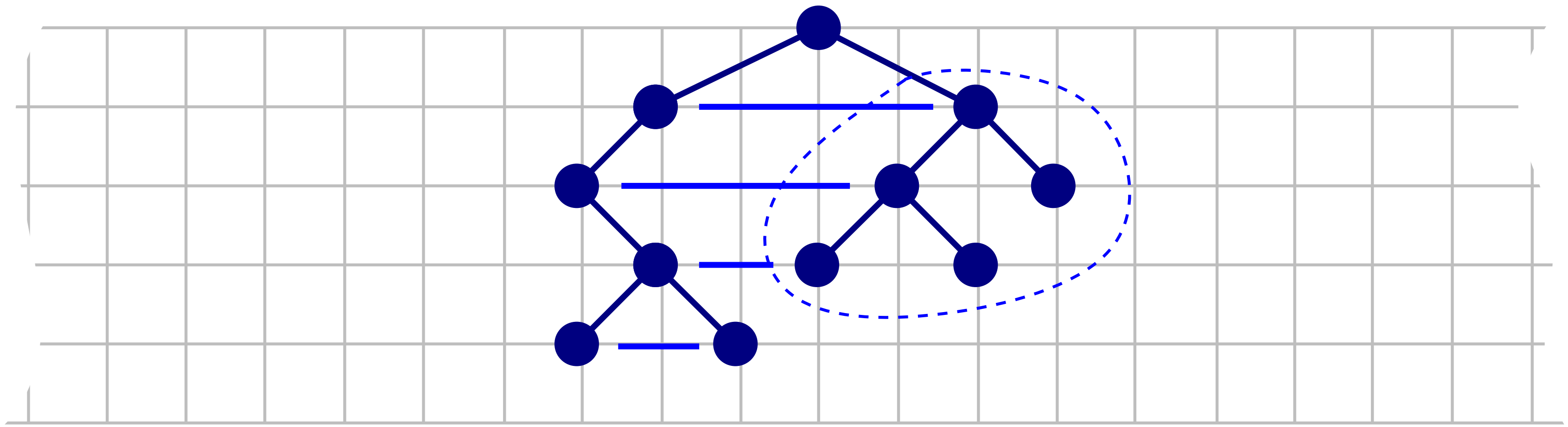

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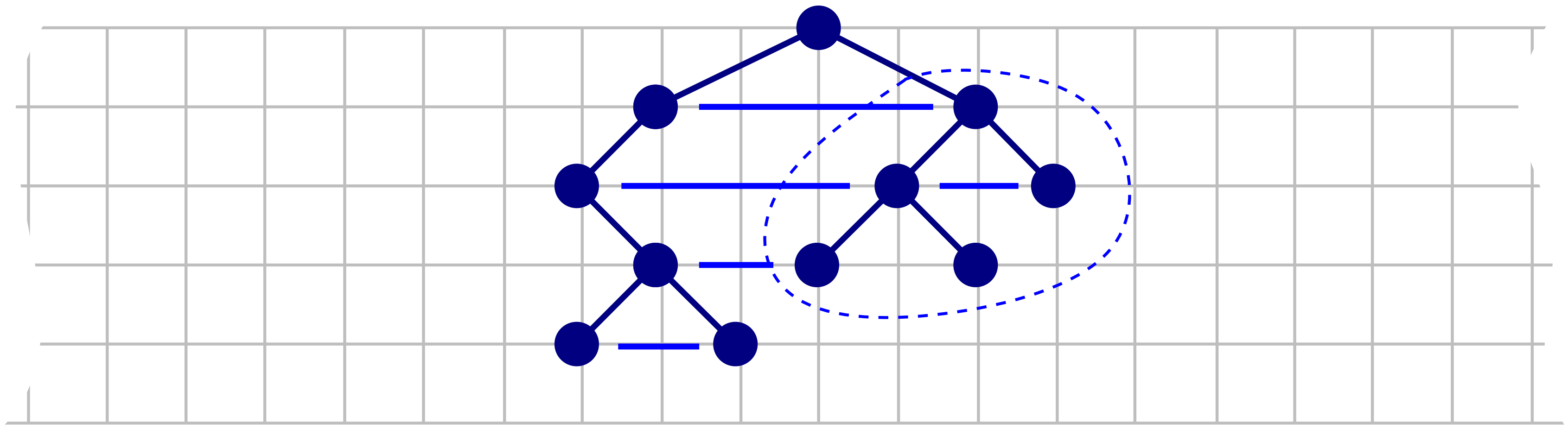

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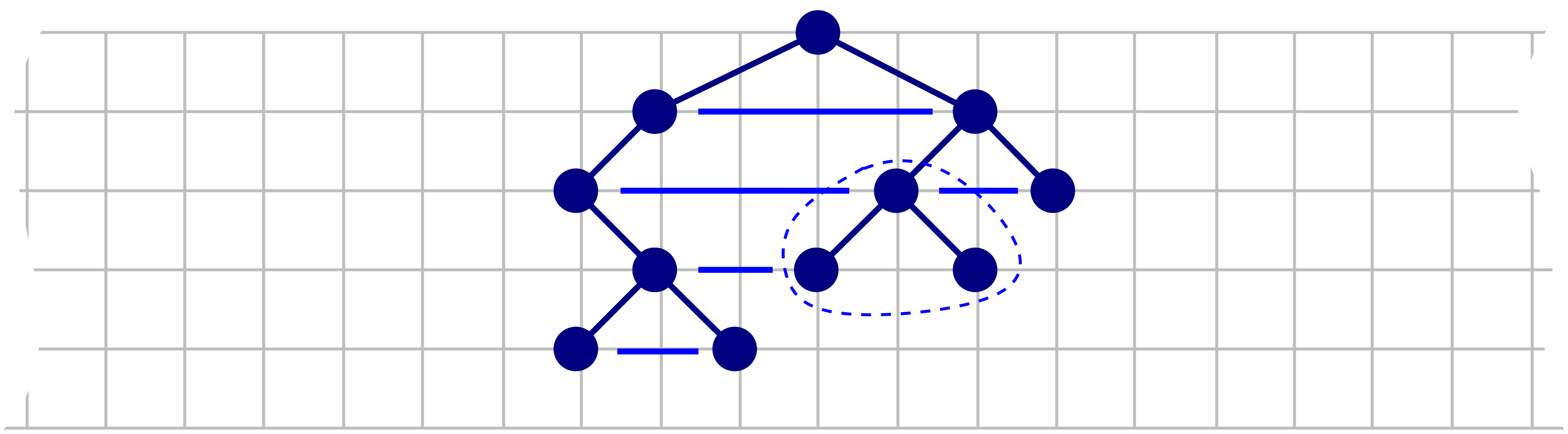

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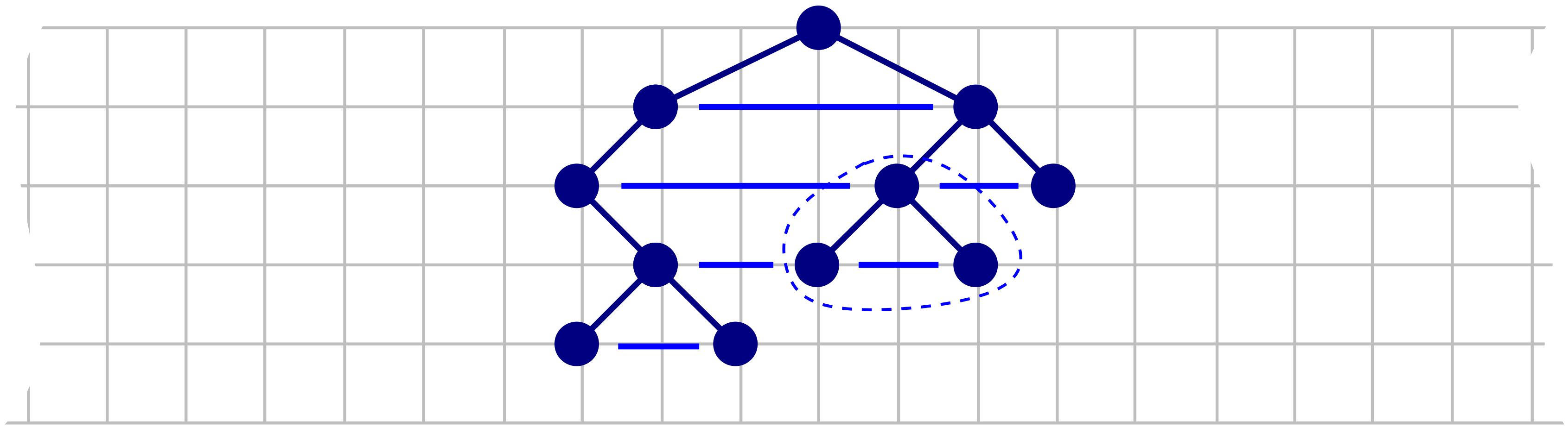

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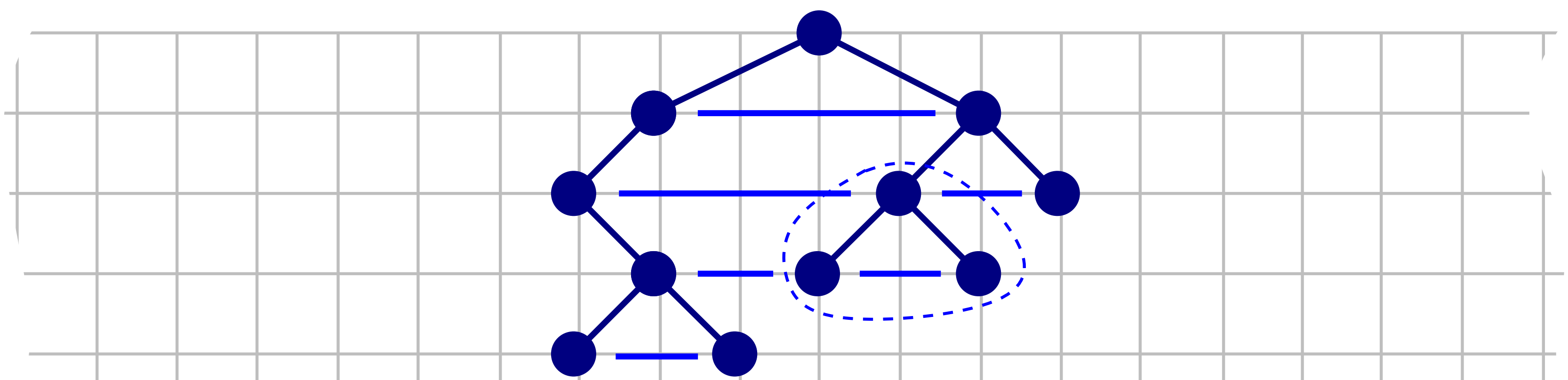

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$O(n)$

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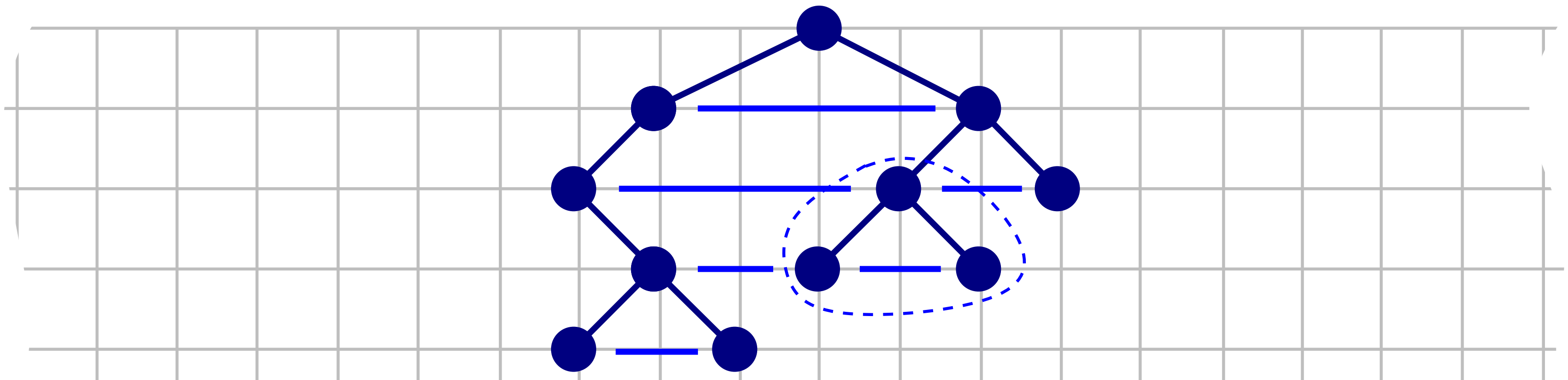

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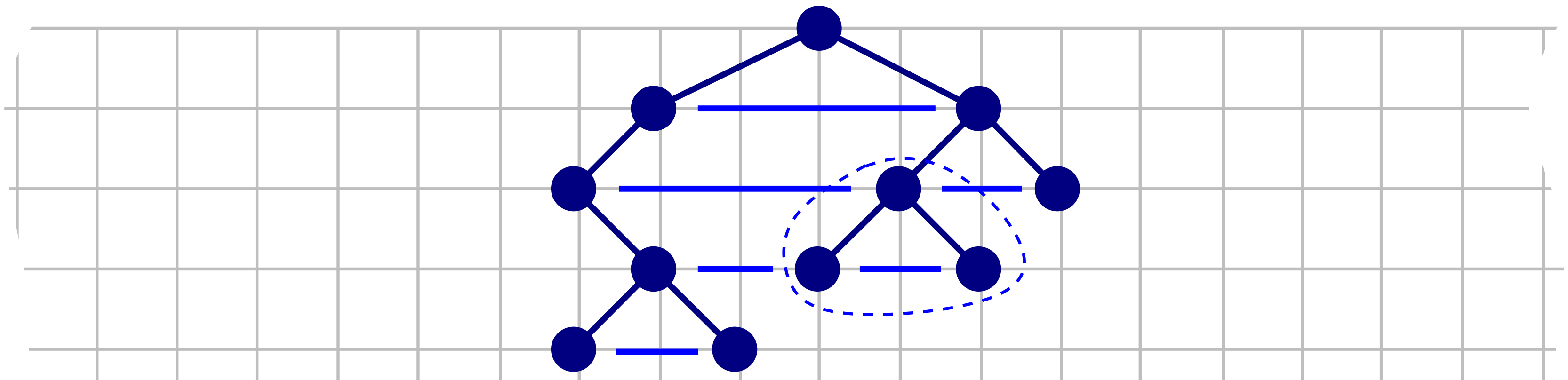

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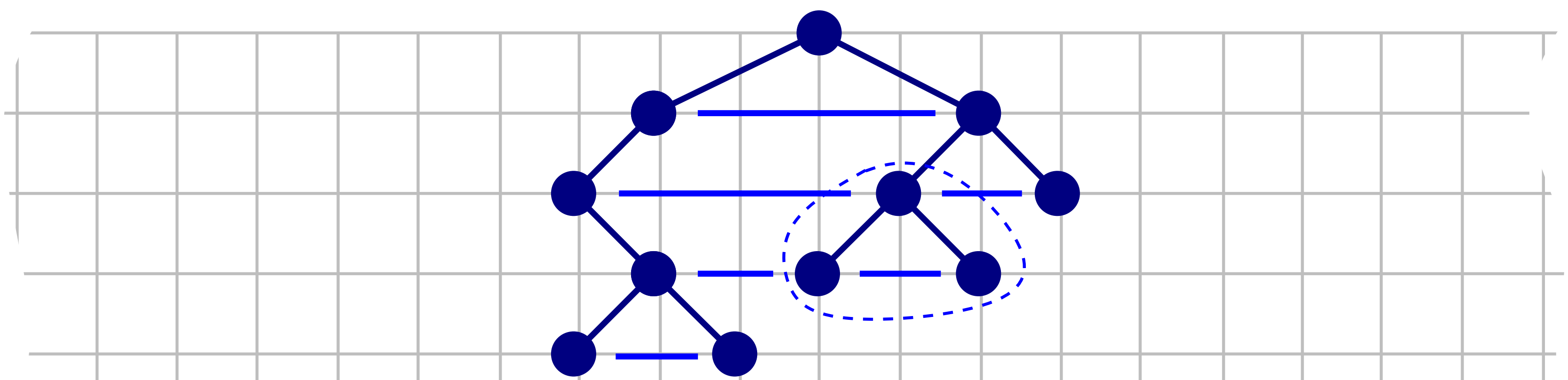

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Level-based Layout

Karlsruhe Institute of Technology

Theorem (Reingold & Tilford)

Let T be a binary tree with n vertices. Algorithm (R & T) constructs a drawing Γ of T in $O(n)$ time, such that:

- Γ is planar and straight-line
- $\forall v \in T$ y-coordinate of v is $-depth(v)$
- Vertical and horizontal distance is at least 1
- Area of Γ is

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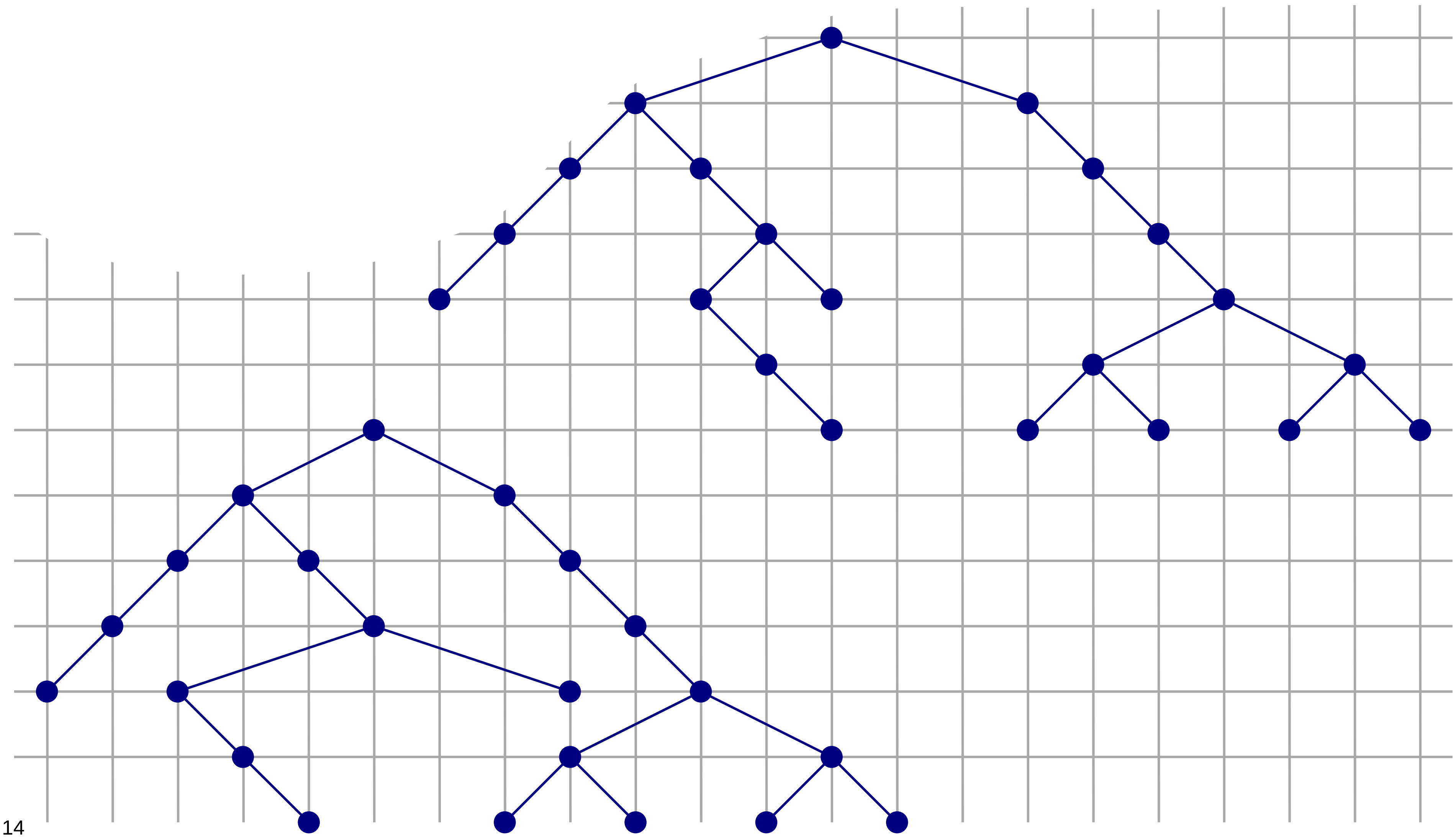
- Each vertex is centered with respect to its children
- Simply isomorphic subtrees have congruent (coincident) drawing, up to translation
- Axially isomorphic trees have congruent drawing, up to translation and reflection around y-axis

Level-based Layout

- The presented algorithm tries to minimize width



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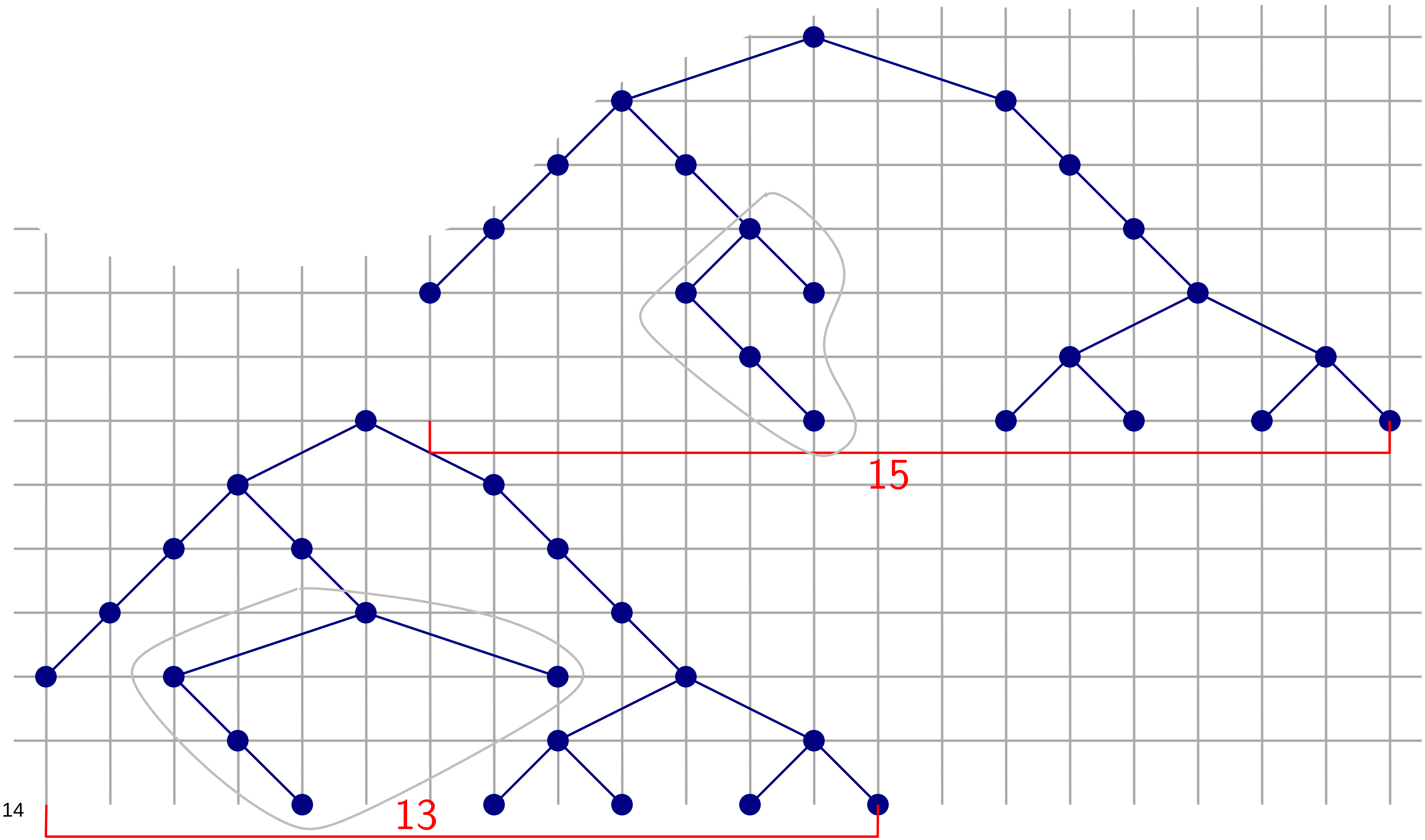

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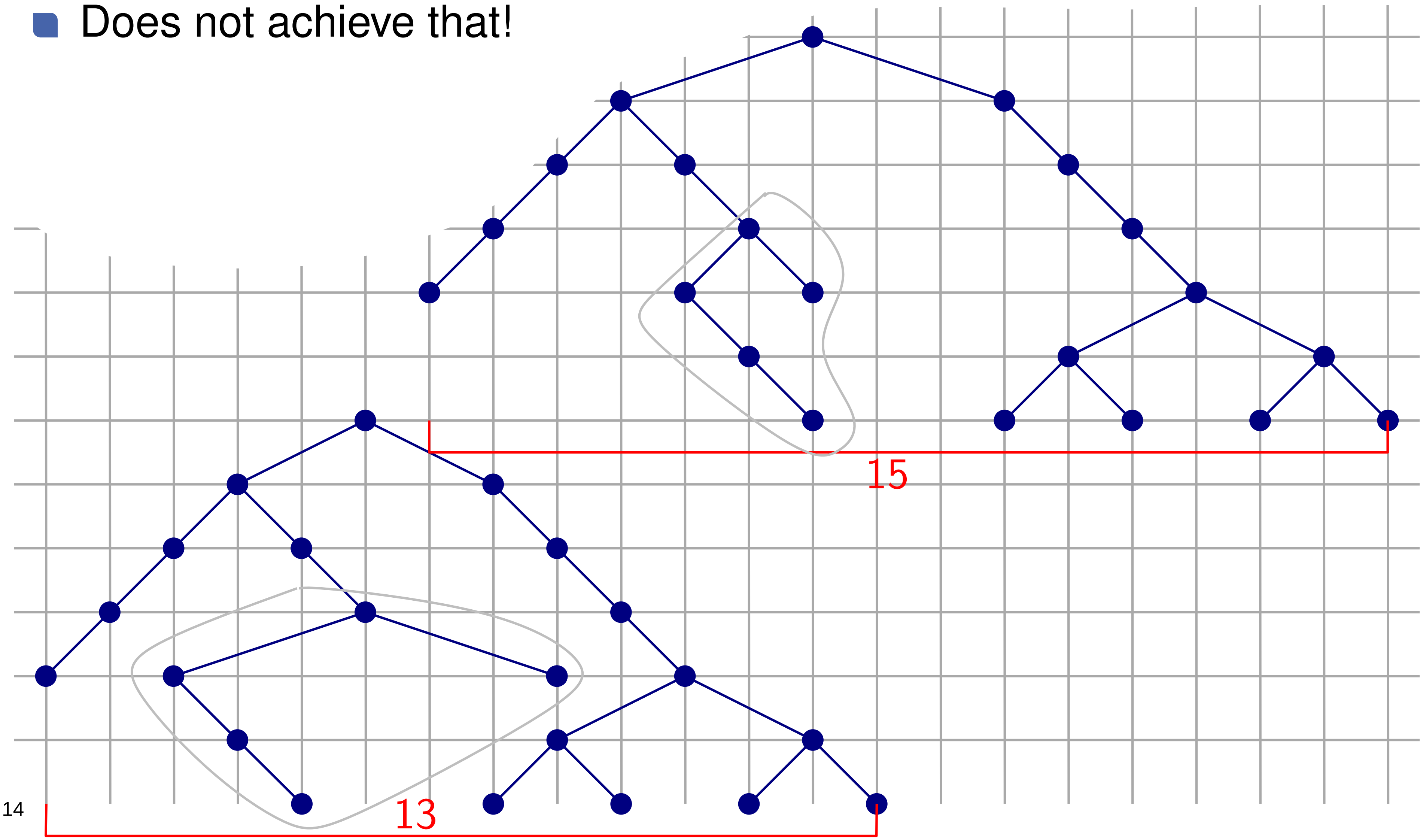

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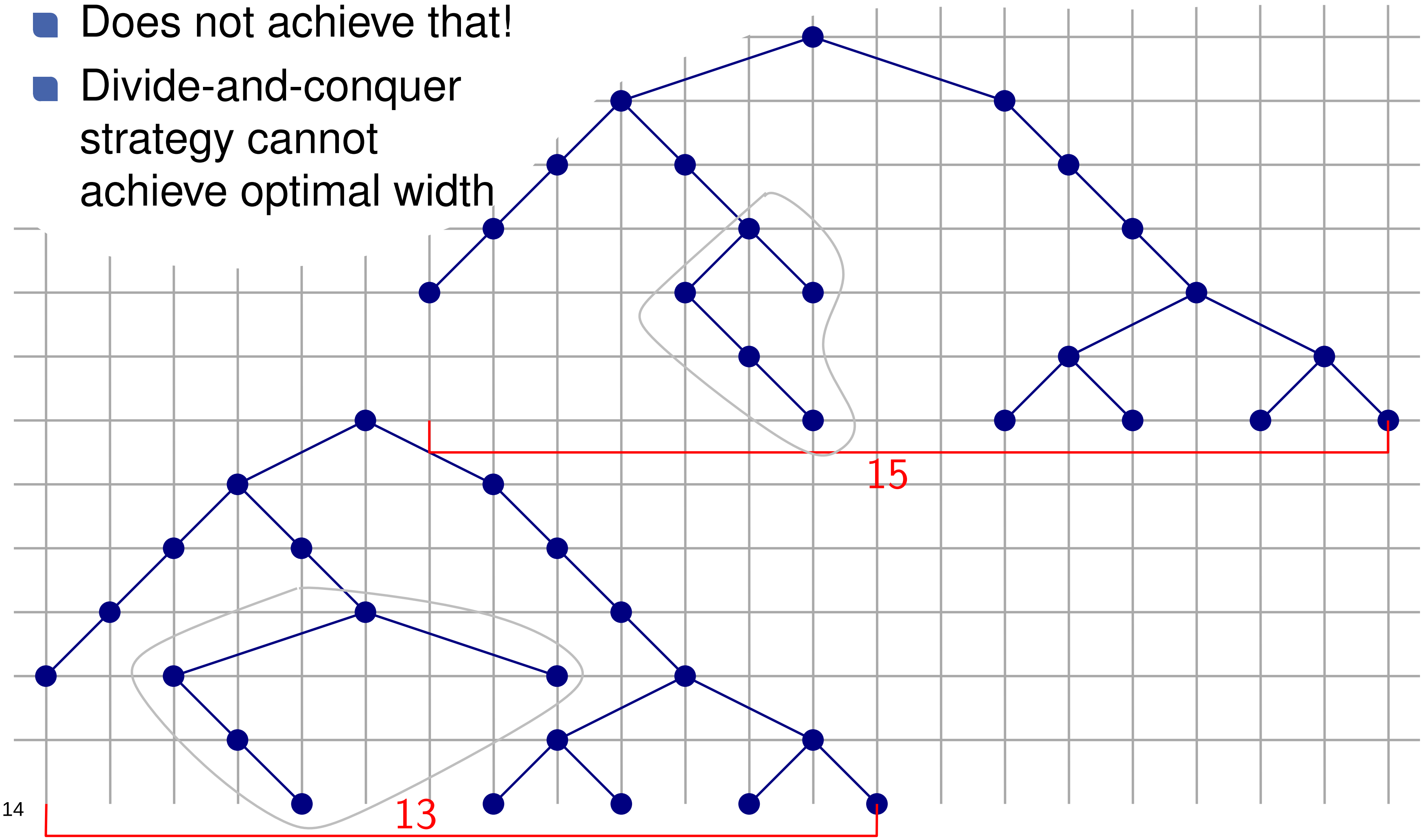

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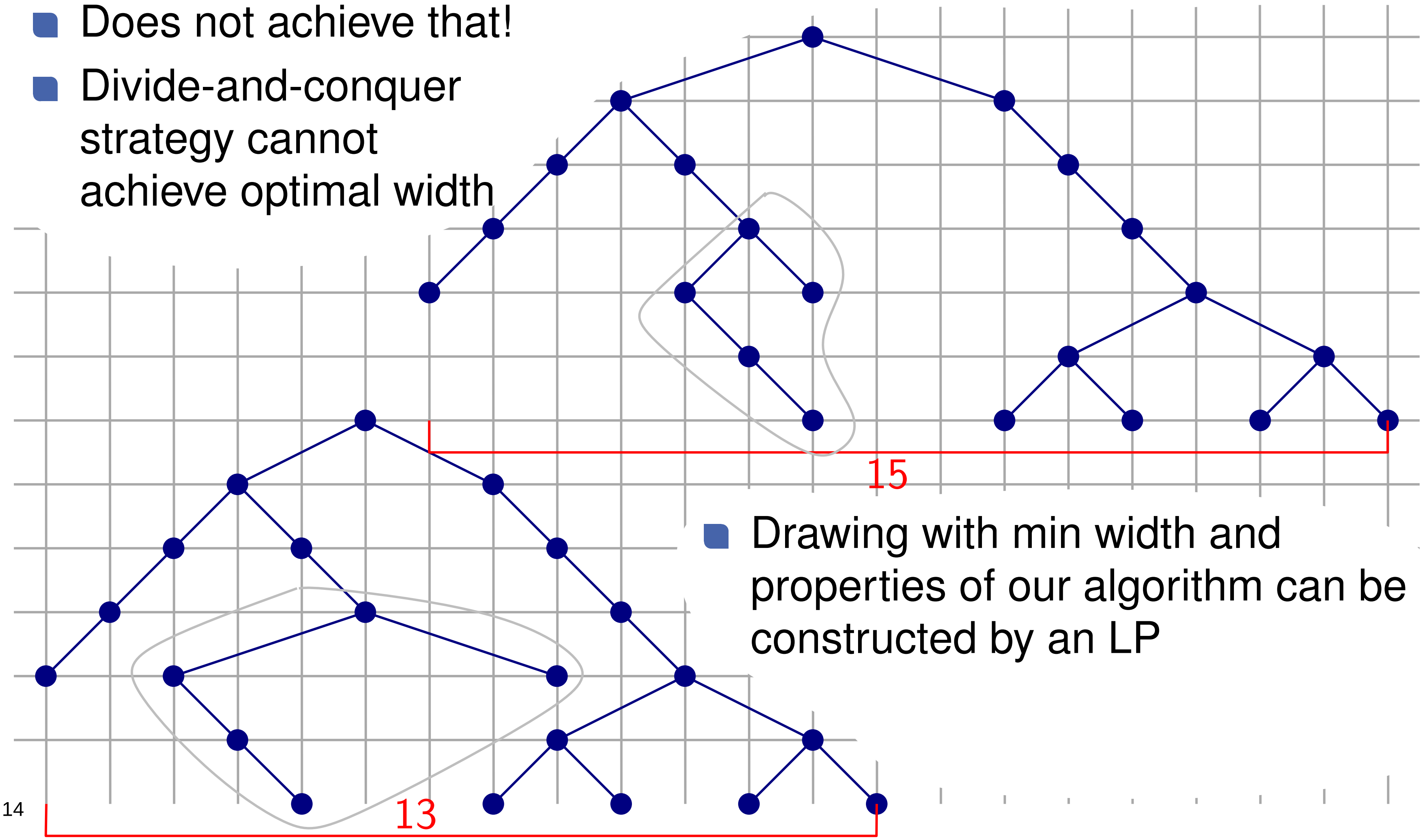

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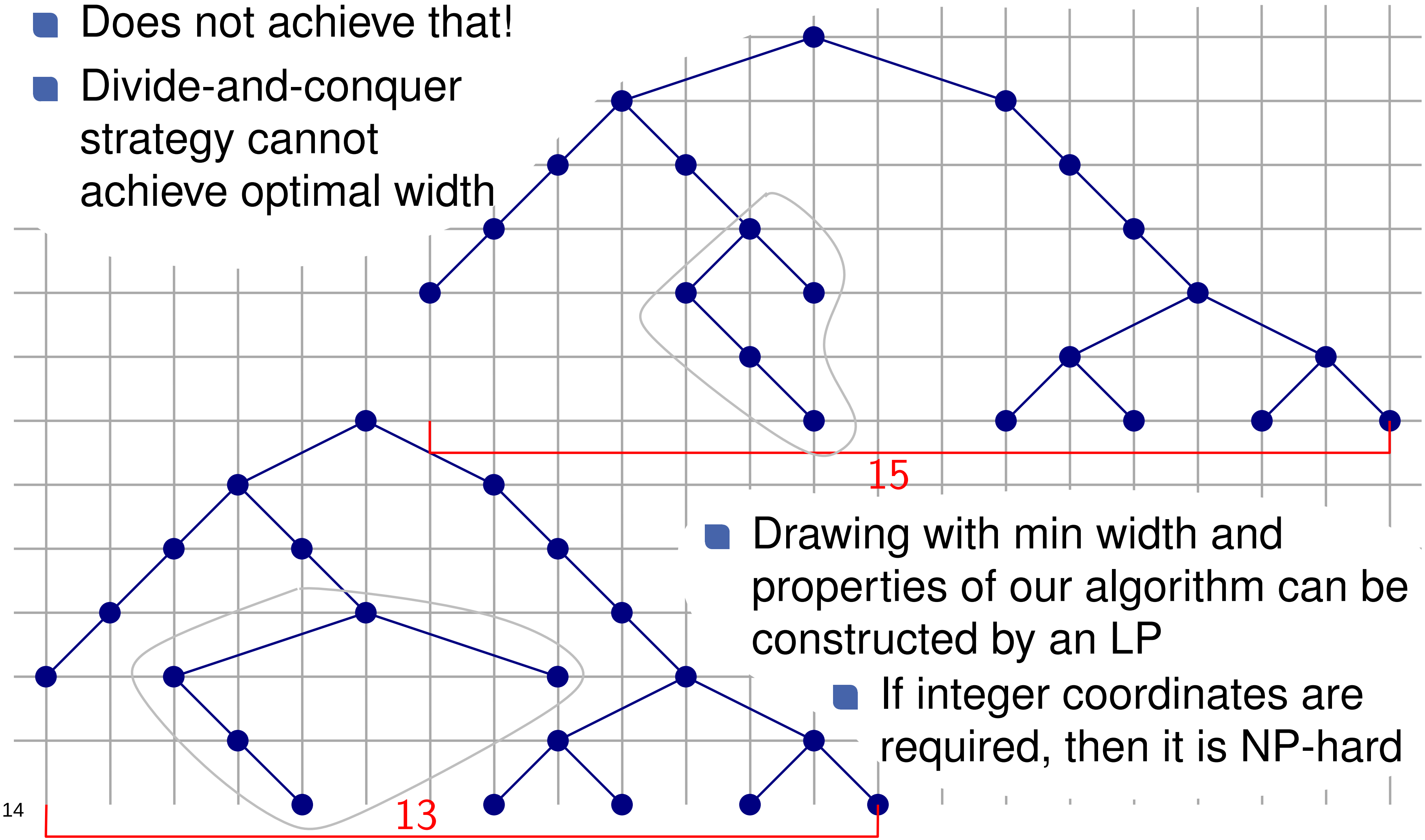

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  - If integer coordinates are required, then it is NP-hard

Level-based Layout - General trees

Algorithm Outline:

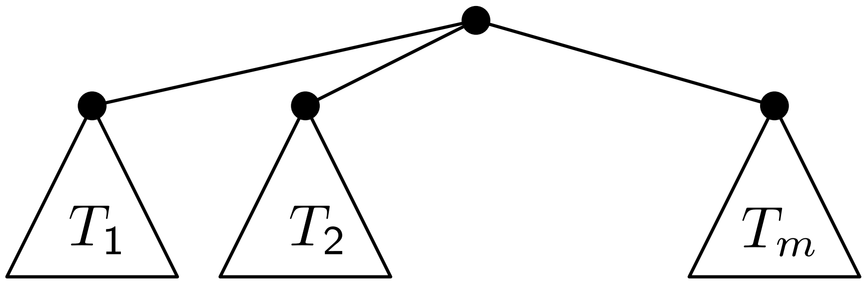
Input: A rooted tree

Output: A level-based drawing of T

Base case: A single vertex

Divide: Assume that T has subtrees $T_1, \ldots T_m$. Draw each T_i recursively.

Conquer:



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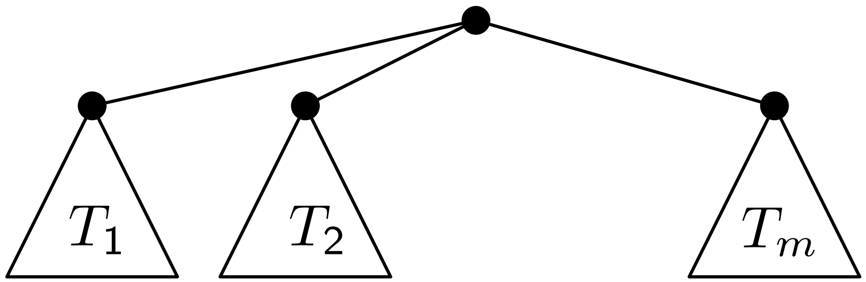

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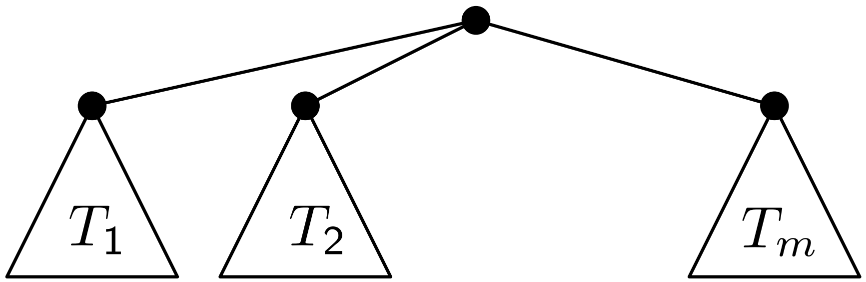


Questions?

Applications

Cons cell diagram in LISP.
Cons(constructs) are memory objects which hold two values or pointers to values.

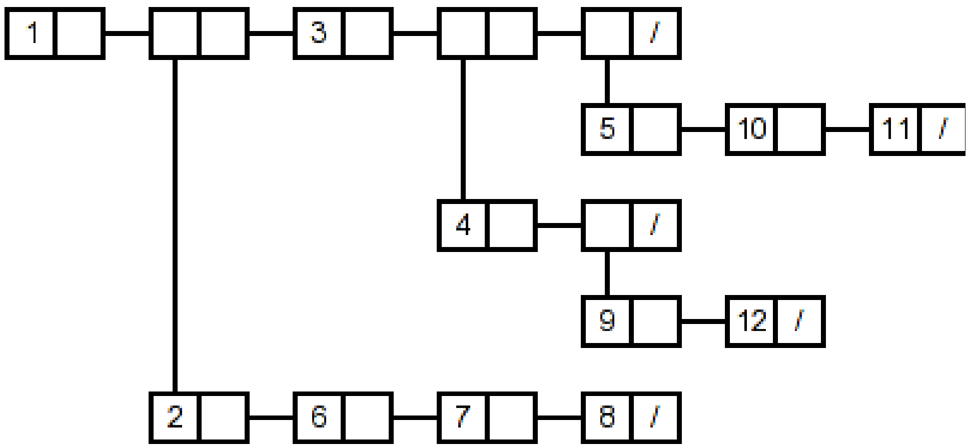


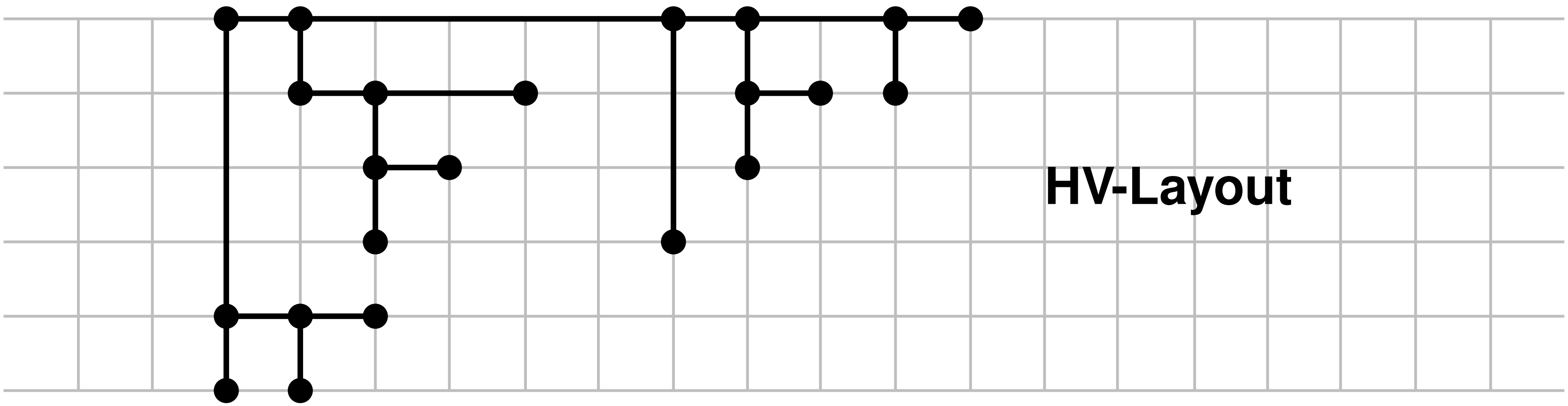
Figure 3: Diagram of cons cells of the simple tree.

`http://gajon.org/`

HV-layout (Horizontal-Vertical)

HV-Layout

Divide & Conquer Approach:



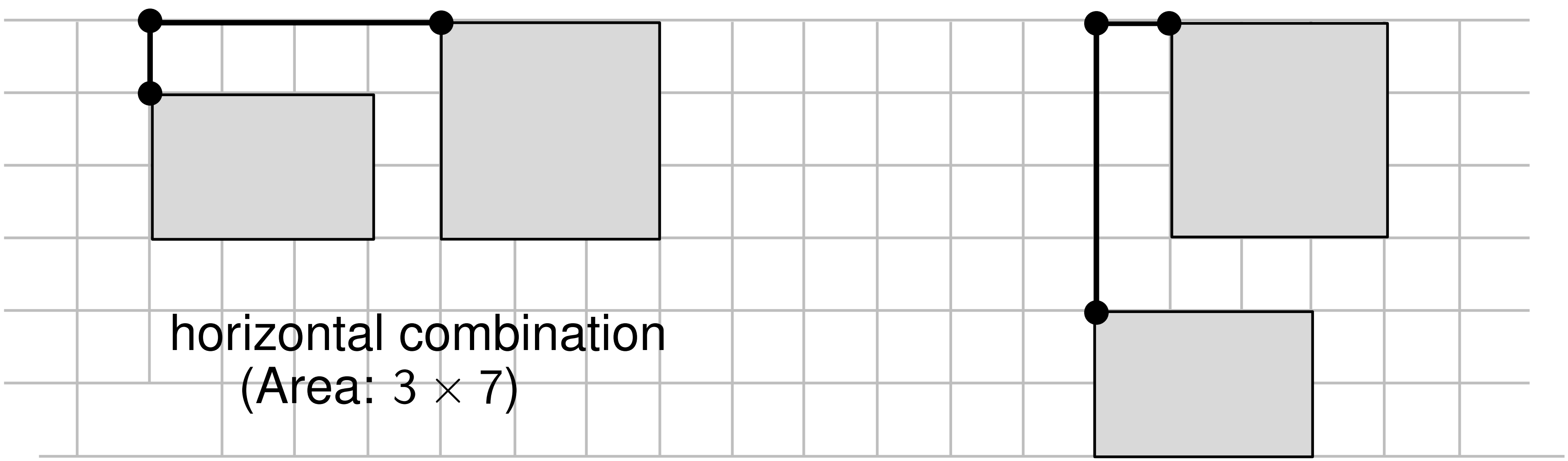
HV-Layout

Idea for binary trees:

- Children are vertically and horizontally aligned with the root
- The bounding boxes of the children do not intersect

Induction base: 

Induction step: combine layouts



vertical combination
(Area: 6 × 4)

HV-Layout

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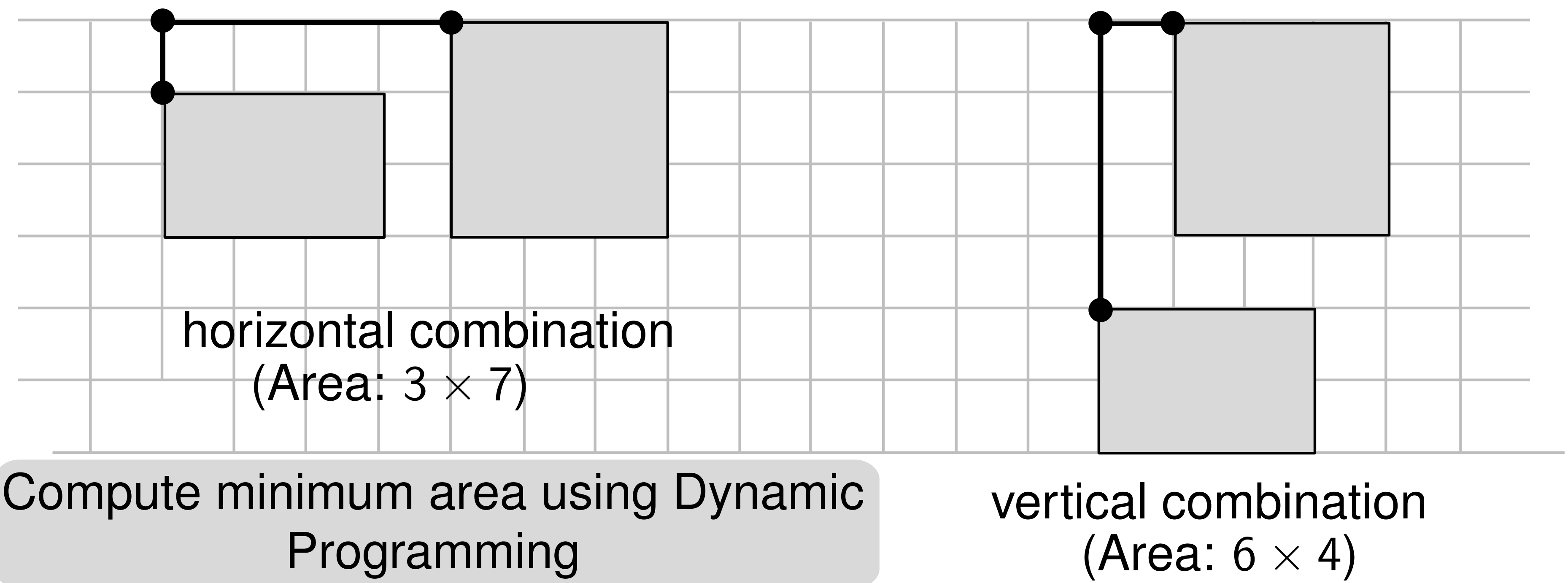

Compute minimum area using Dynamic Programming

Right-Heavy HV-Layout

Right-Heavy approach:

- At every induction step apply horizontal combination
- Place the larger subtree to the right

Right-Heavy HV-Layout

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Lemma

Let T be a binary tree. The hight of the drawing constructed by Right-Heavy approach is at most $\log n$.

Right-Heavy HV-Layout

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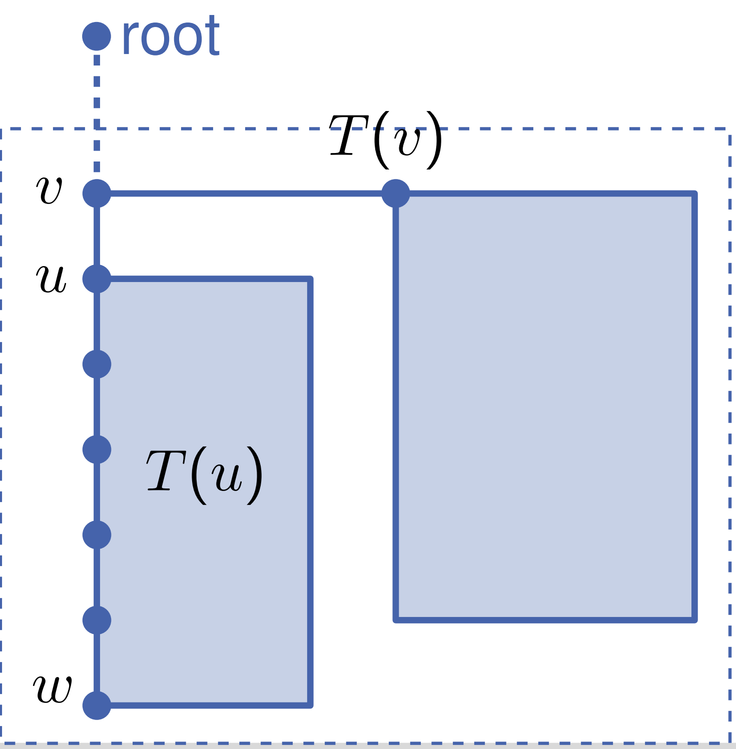

Proof:

- Each vertical edge has length 1
- Let w be the lowest node in the drawing
- Let P be a path from w to the root of T
- For every edge (u, v) in P: $|T(v)| > 2|T(u)|$
- $\Rightarrow P$ contains at most $\log n$ edges

Right-Heavy HV-Layout

Theorem

Let T be a binary tree with n vertices. The Right-Heavy algorithm constructs in $O(n)$ time a drawing Γ of T such that:

Right-Heavy HV-Layout

Theorem

Let T be a binary tree with n vertices. The Right-Heavy algorithm constructs in $O(n)$ time a drawing Γ of T such that:

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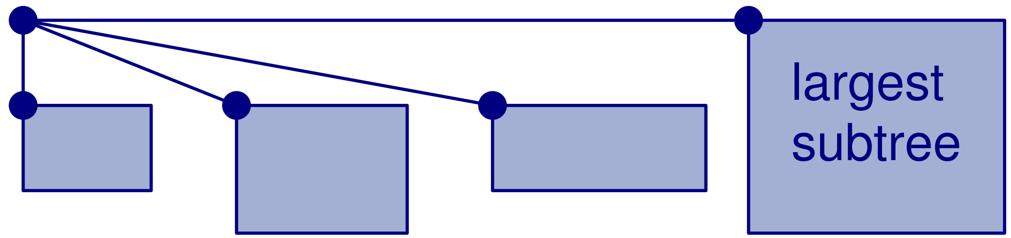

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Karlsruhe Institute of Technology

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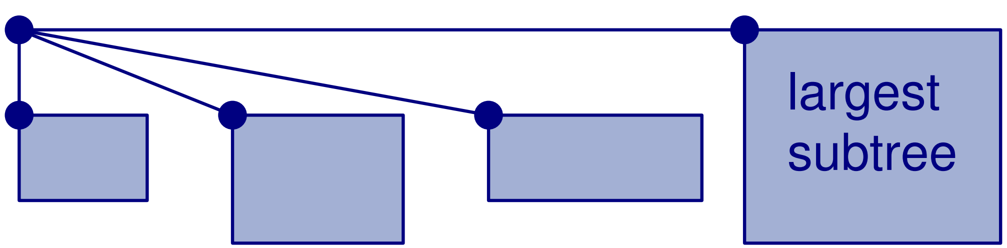
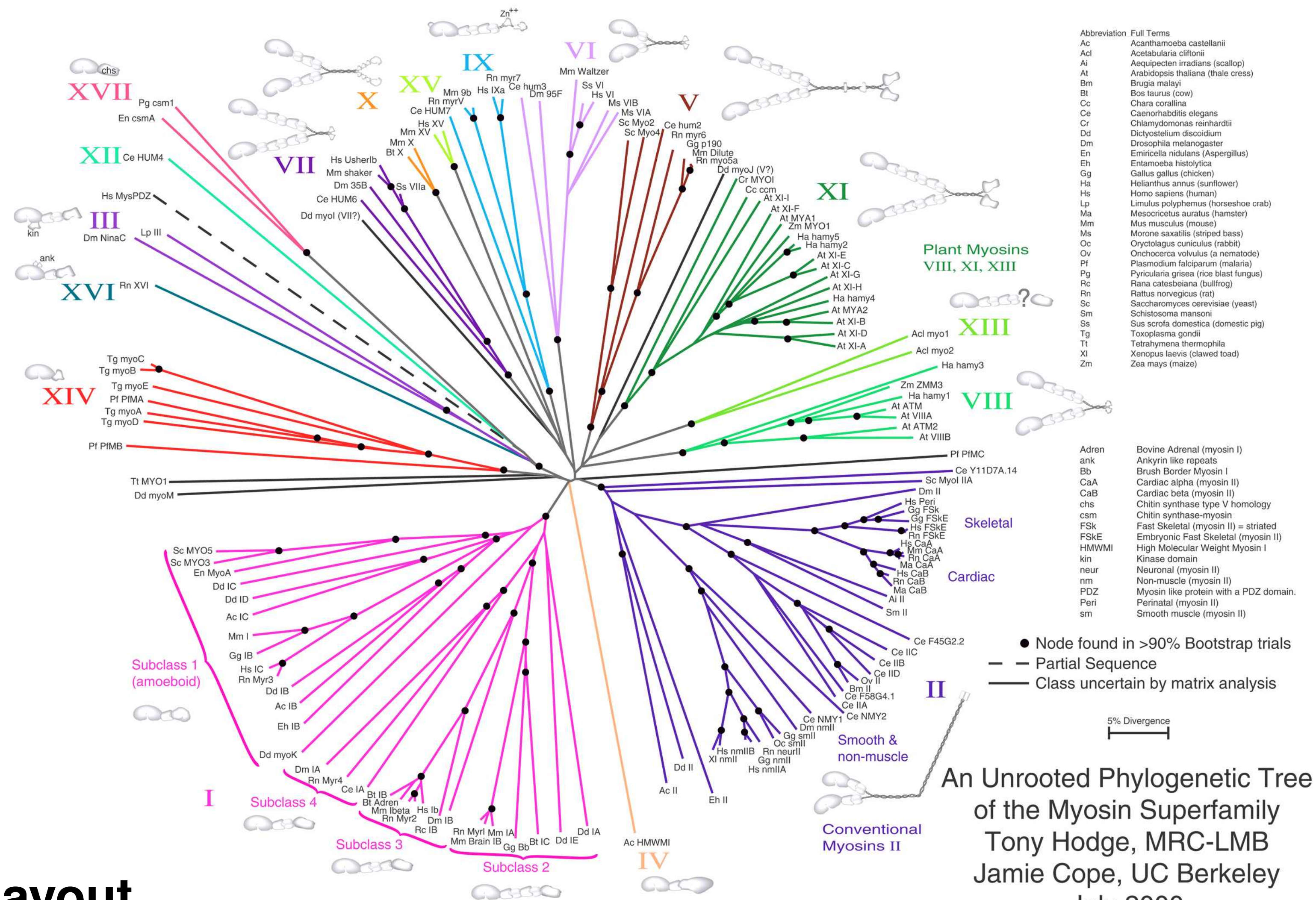


Questions?

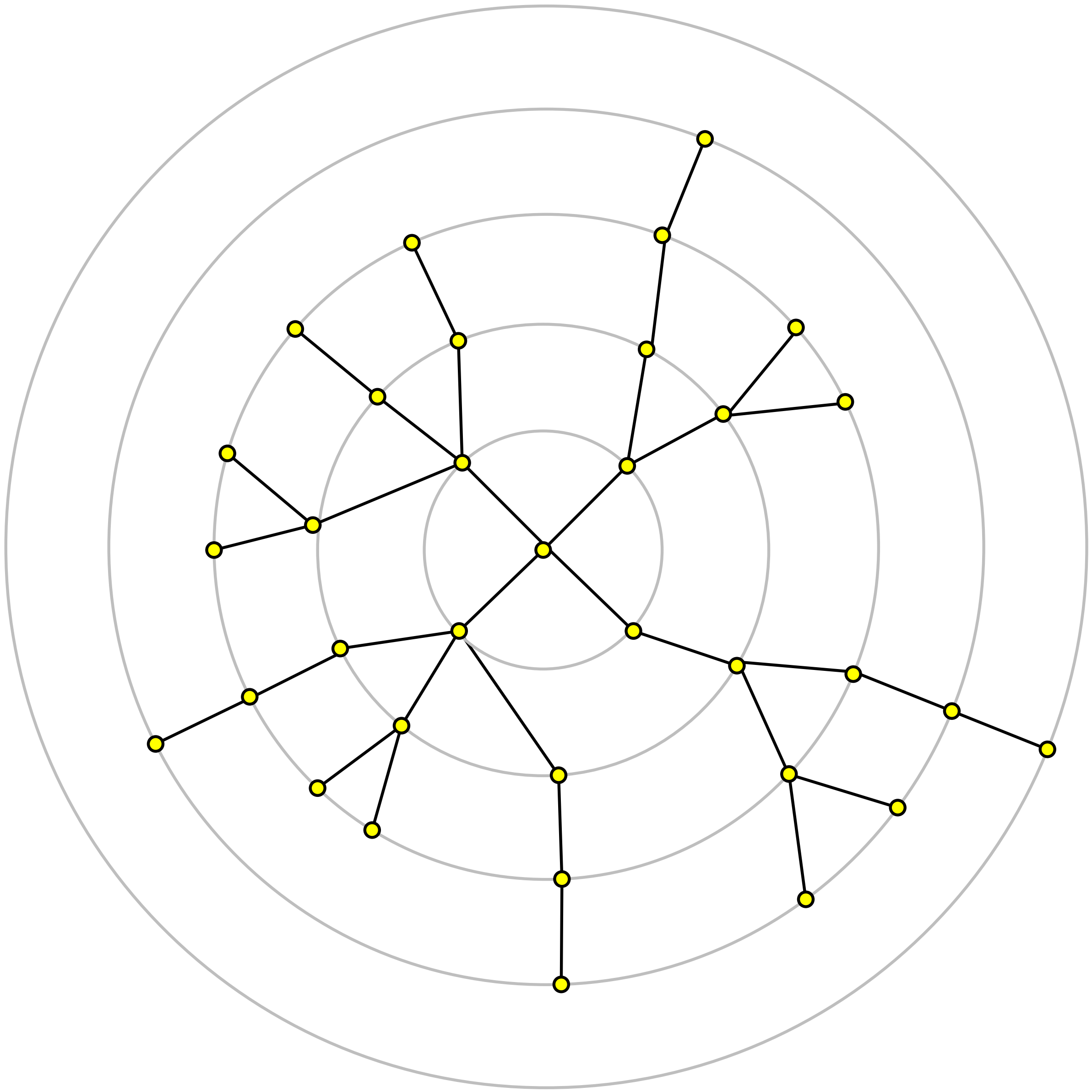
Applications



Radial layout

An unrooted phylogenetic tree for myosin, a superfamily of proteins.
"A myosin family tree" *Journal of Cell Science*

Radial Layout



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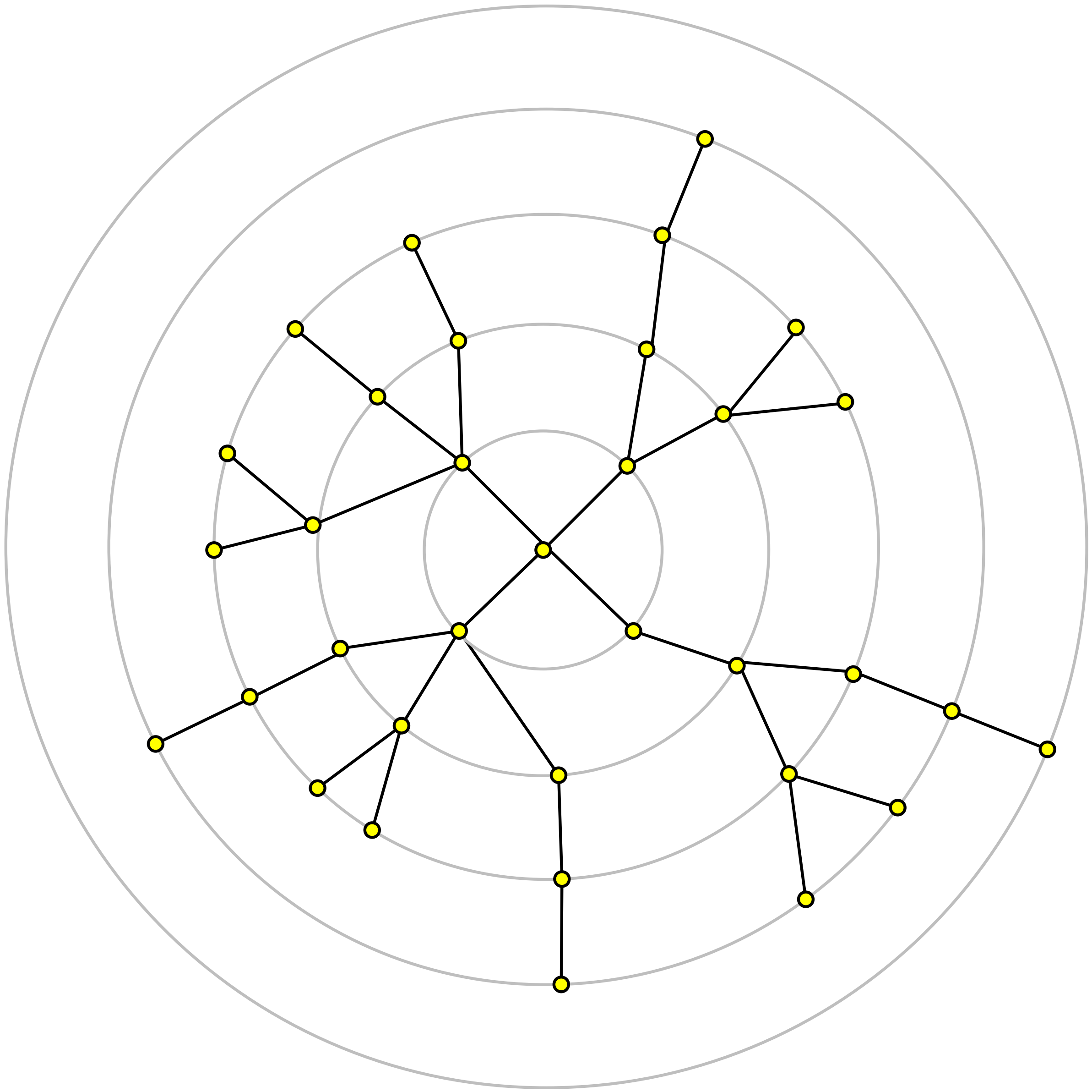

Radial Layout

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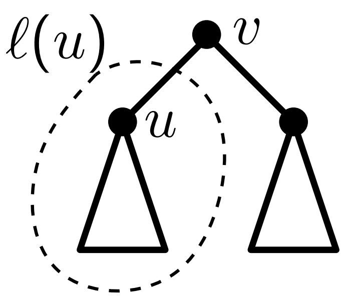
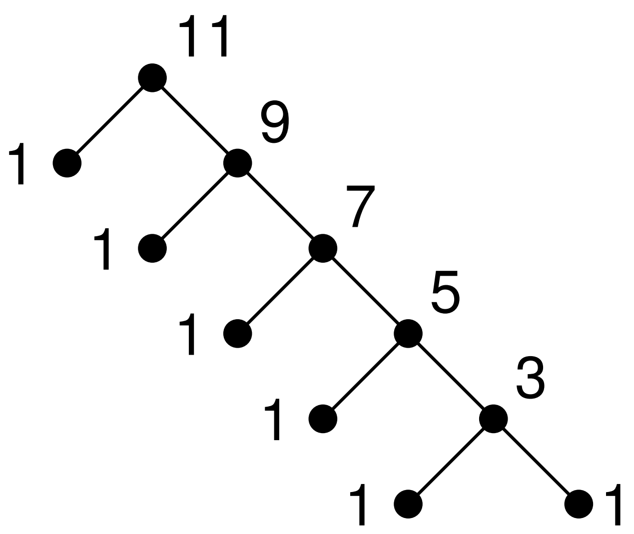
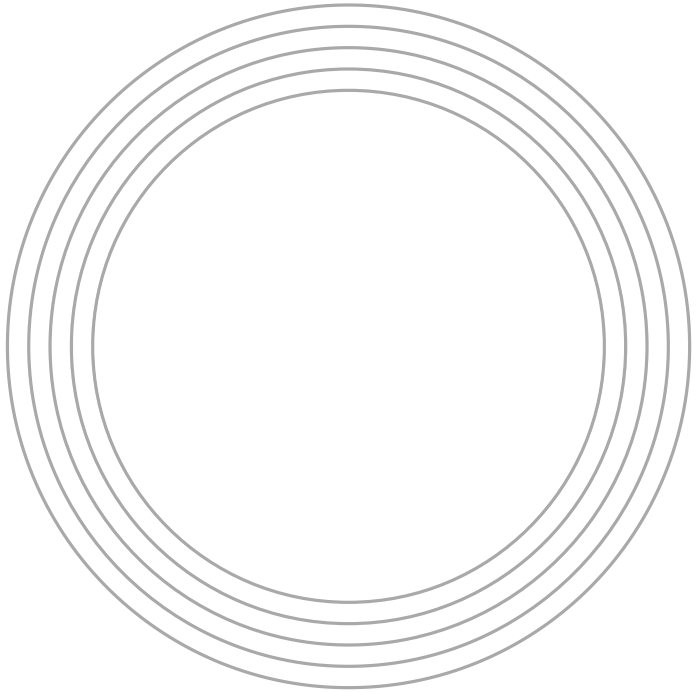
- Angle corresponding to the subtree rooted at u: $\tau_u = \frac{\ell(u)}{\ell(v)-1}$







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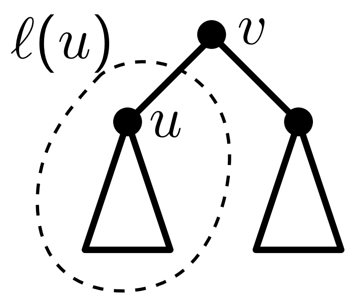

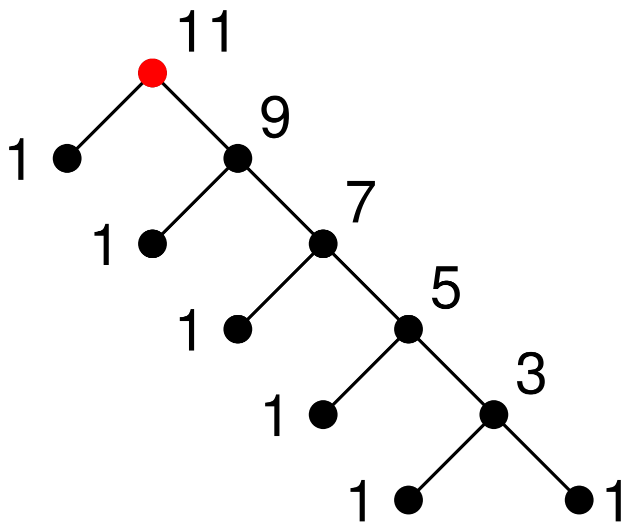

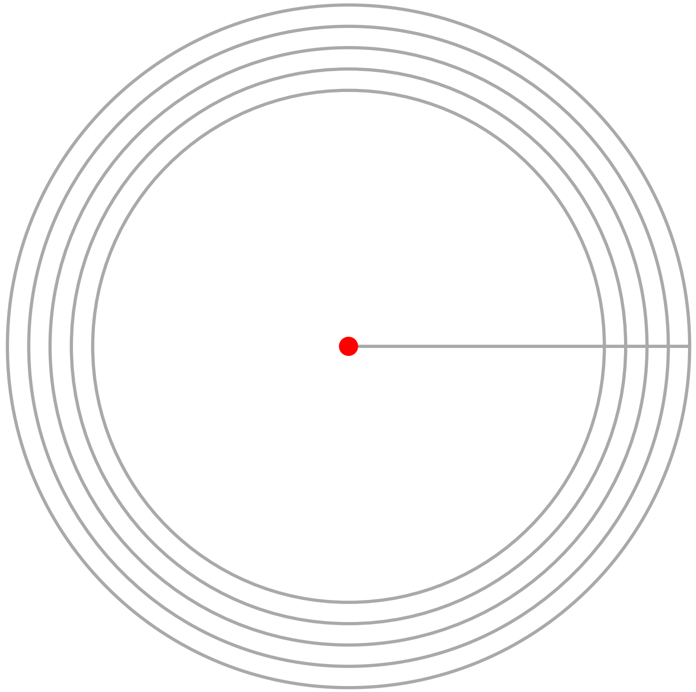

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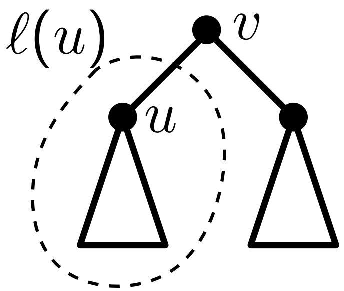

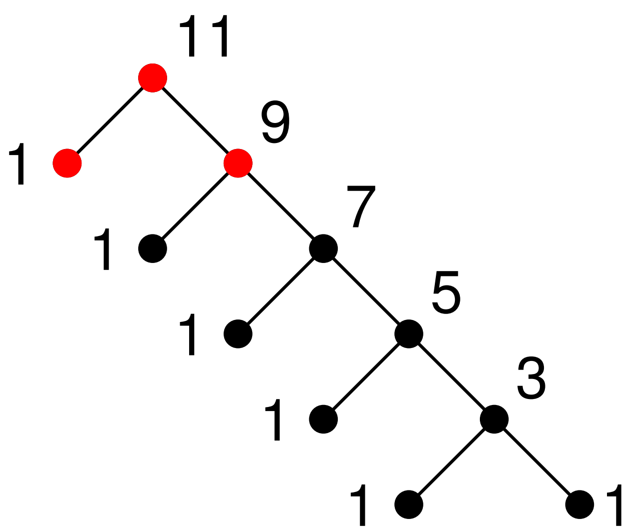

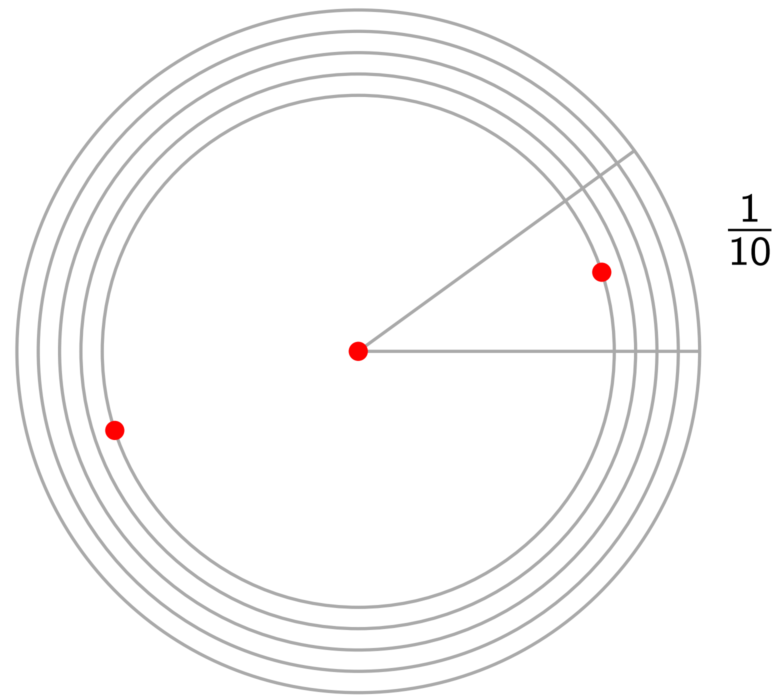

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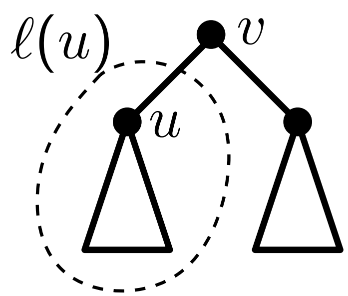

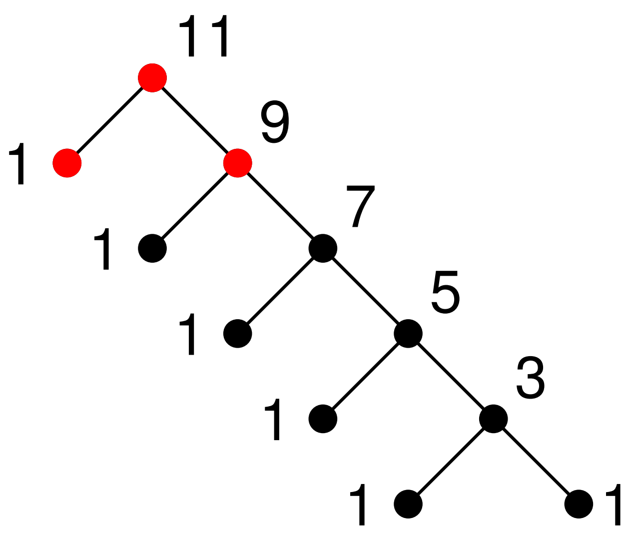

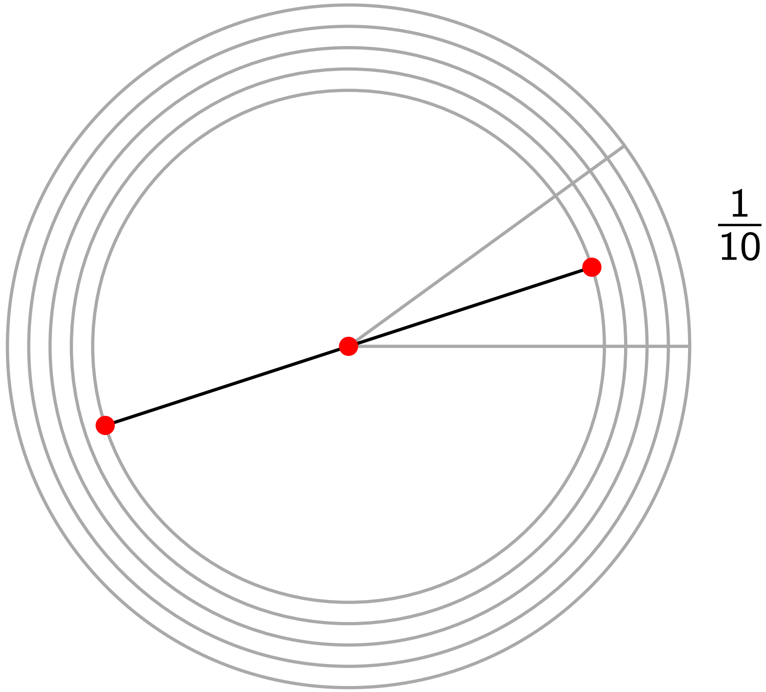

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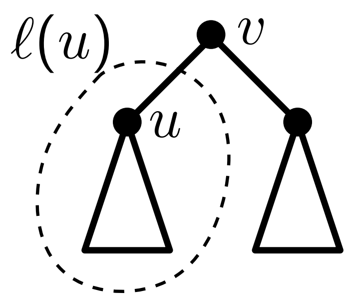

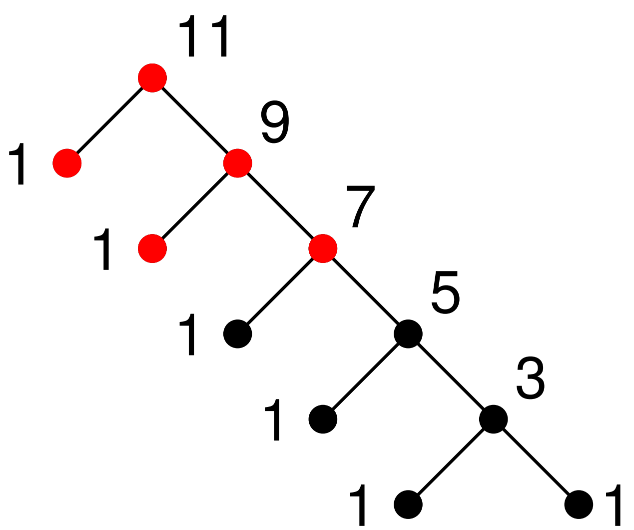

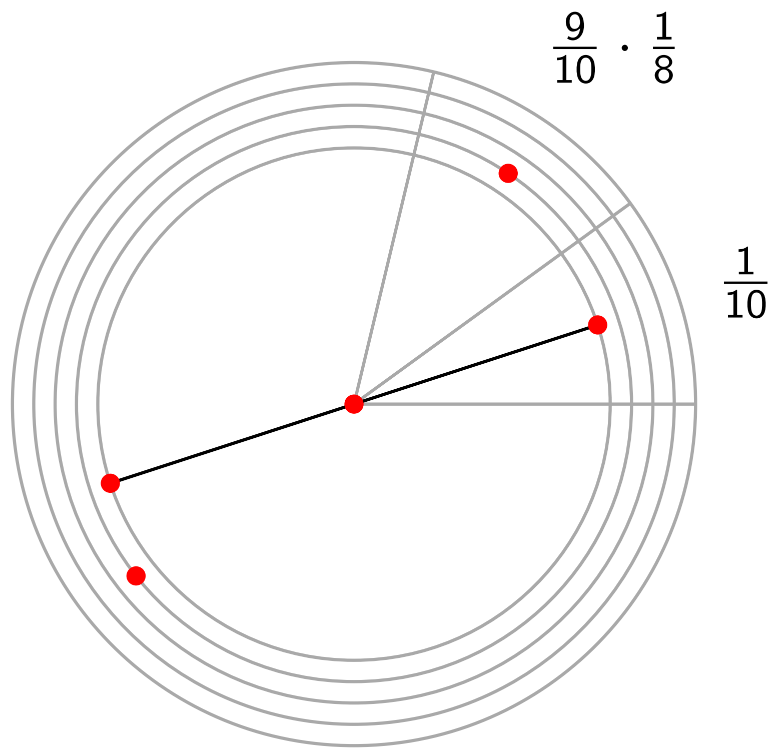

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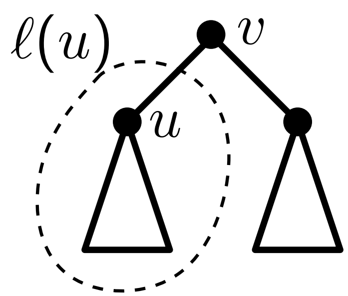

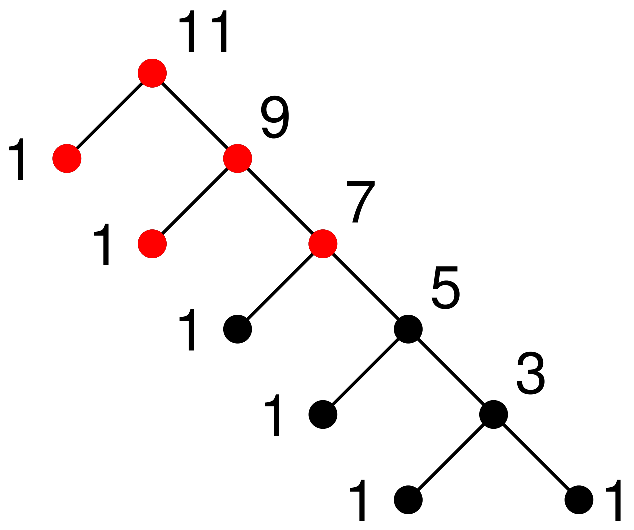

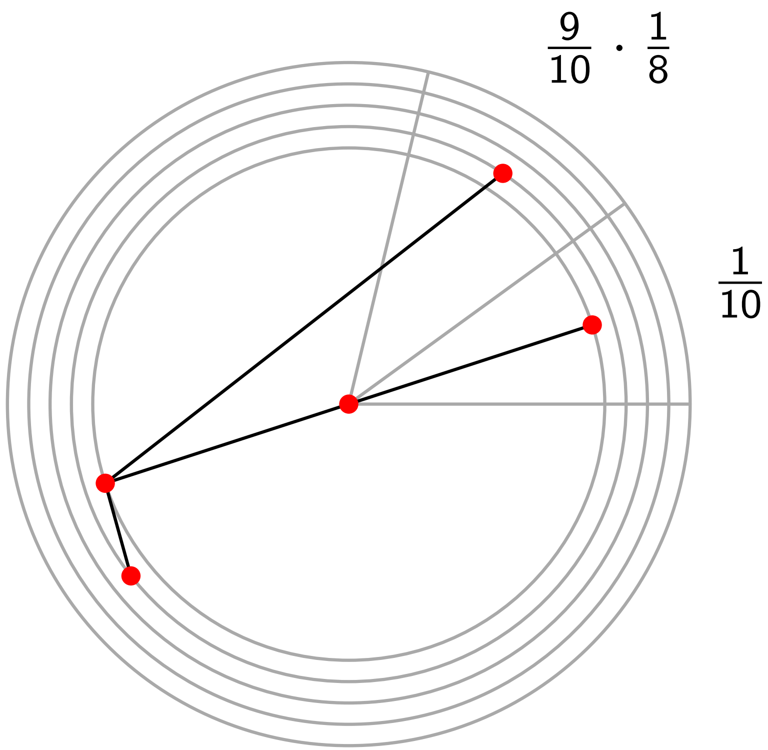

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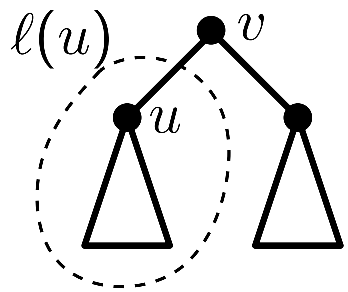

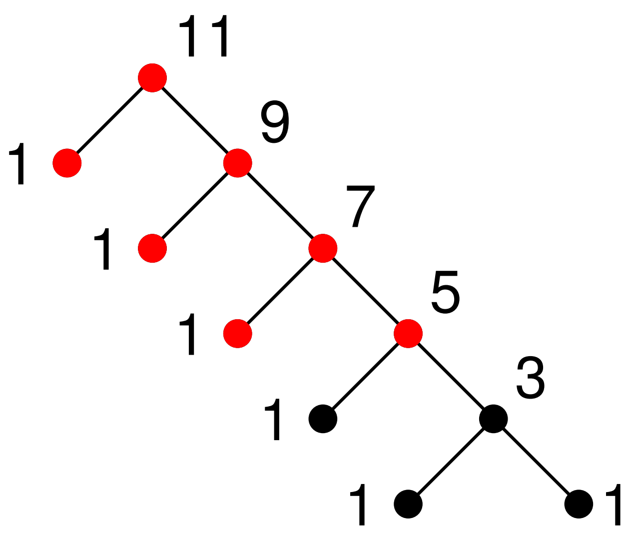

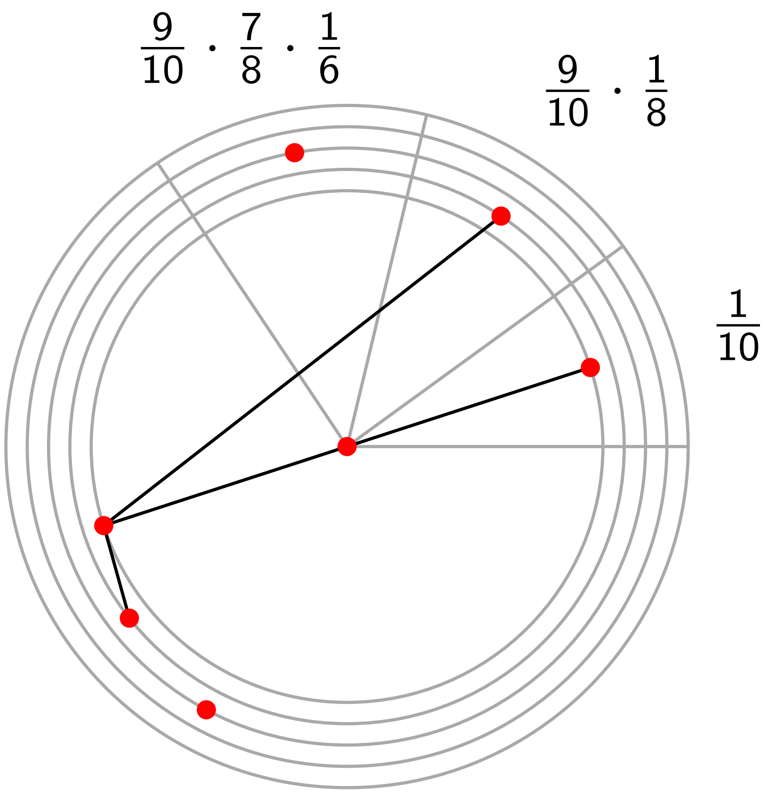

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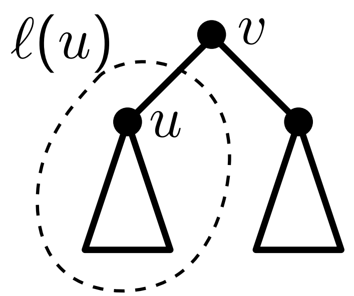
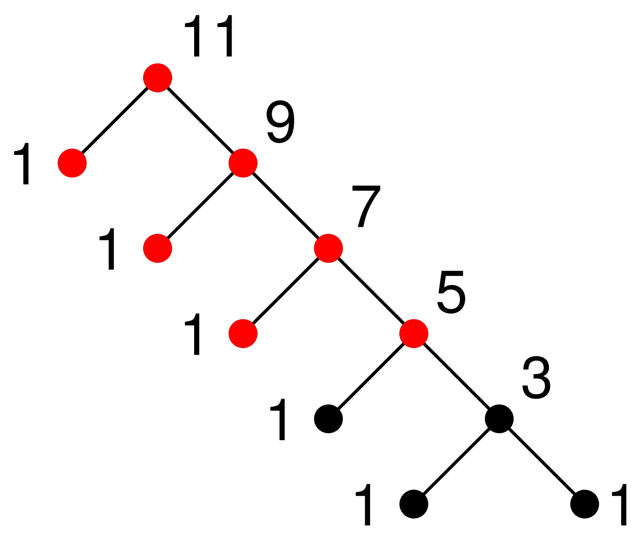
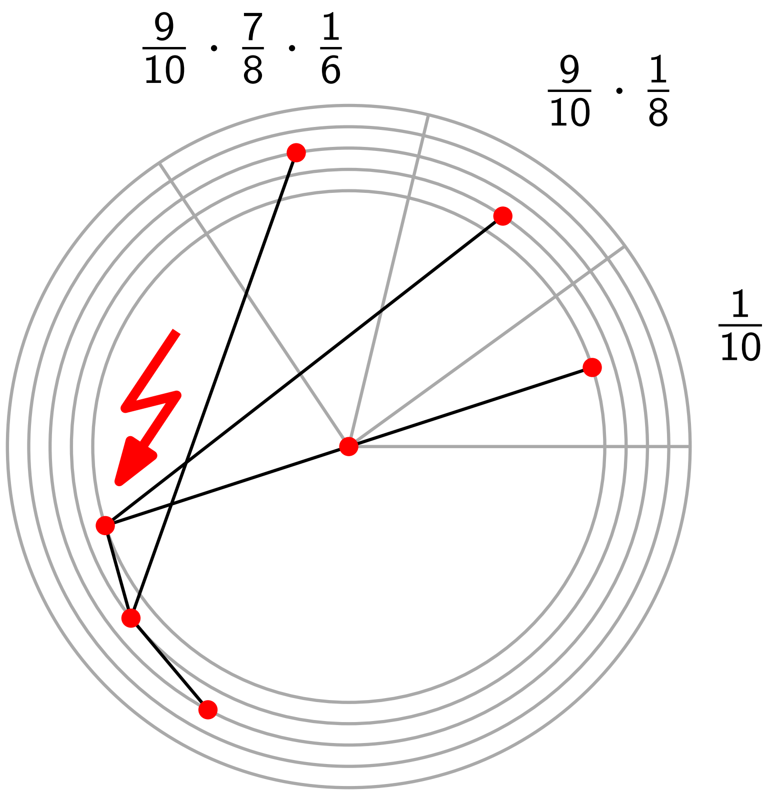
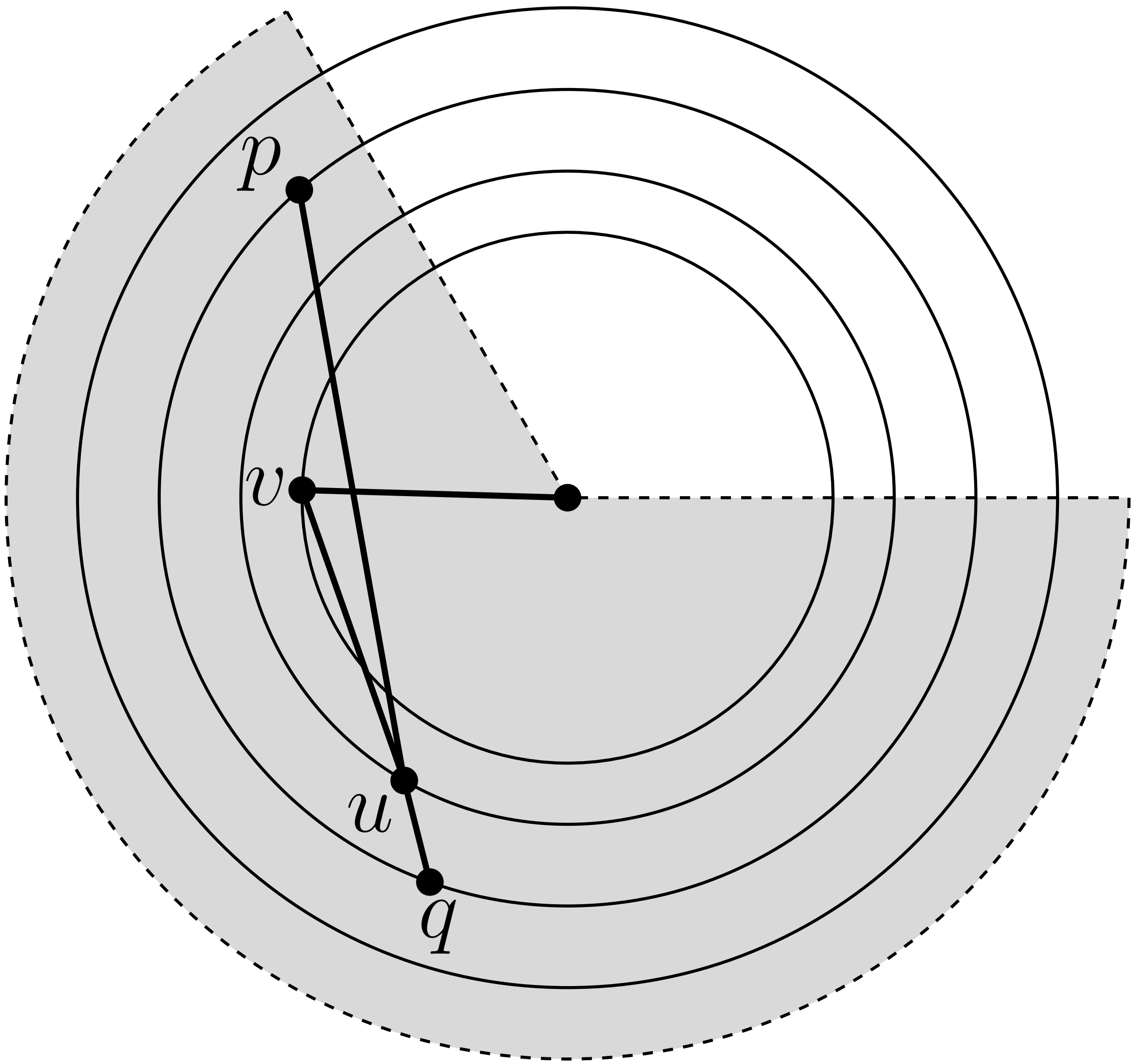






Radial Layout

How to avoid crossings:



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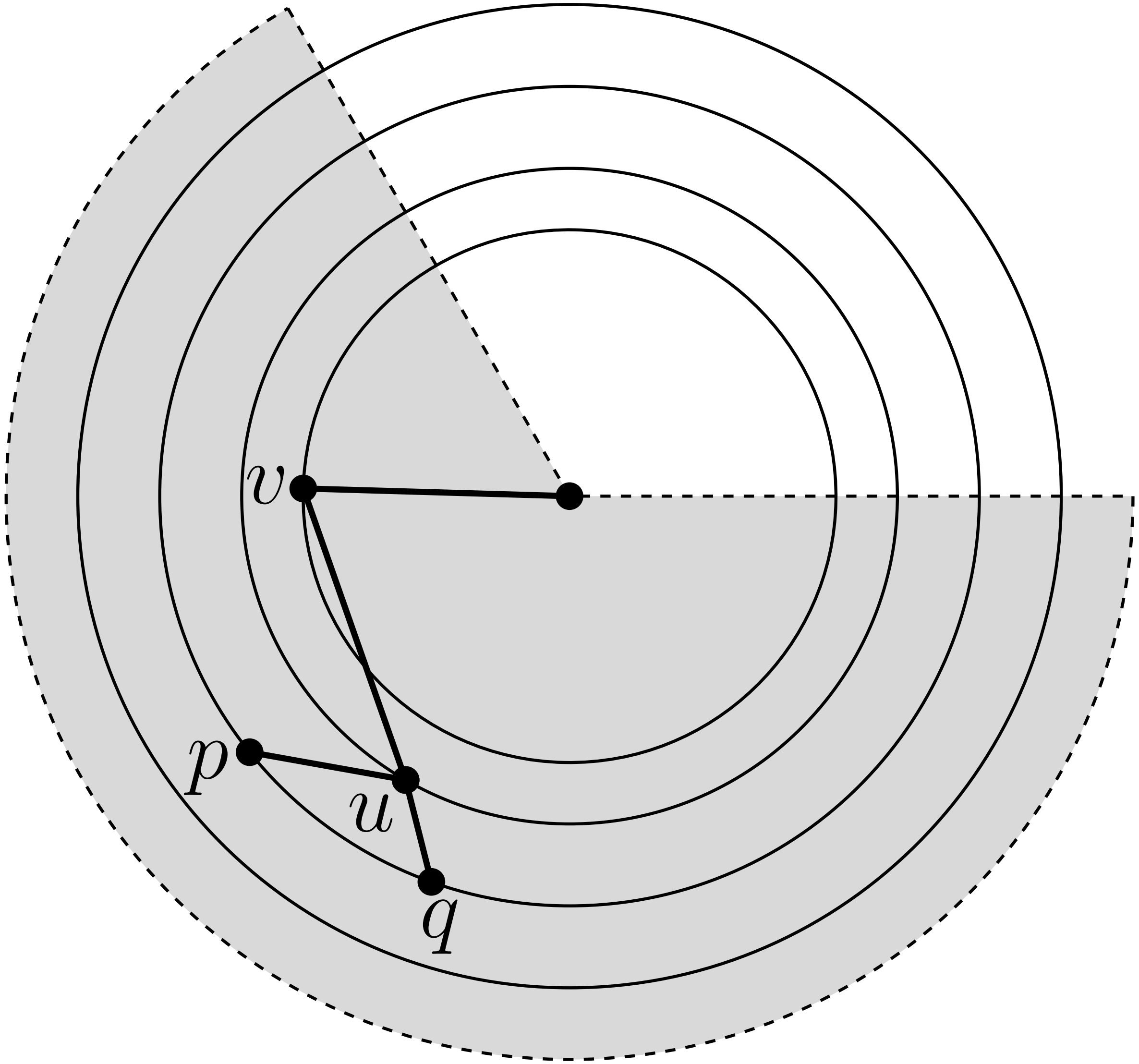

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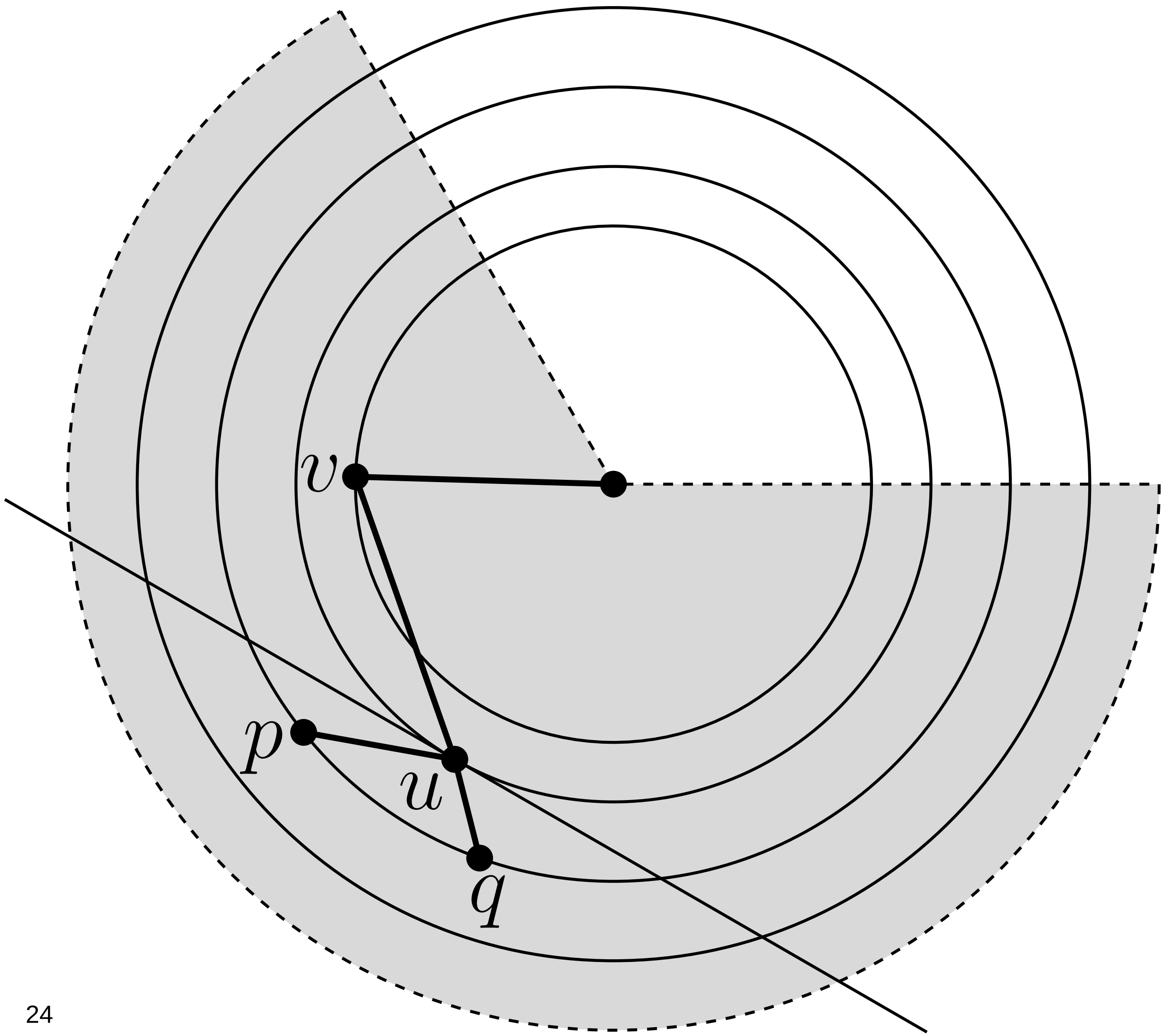

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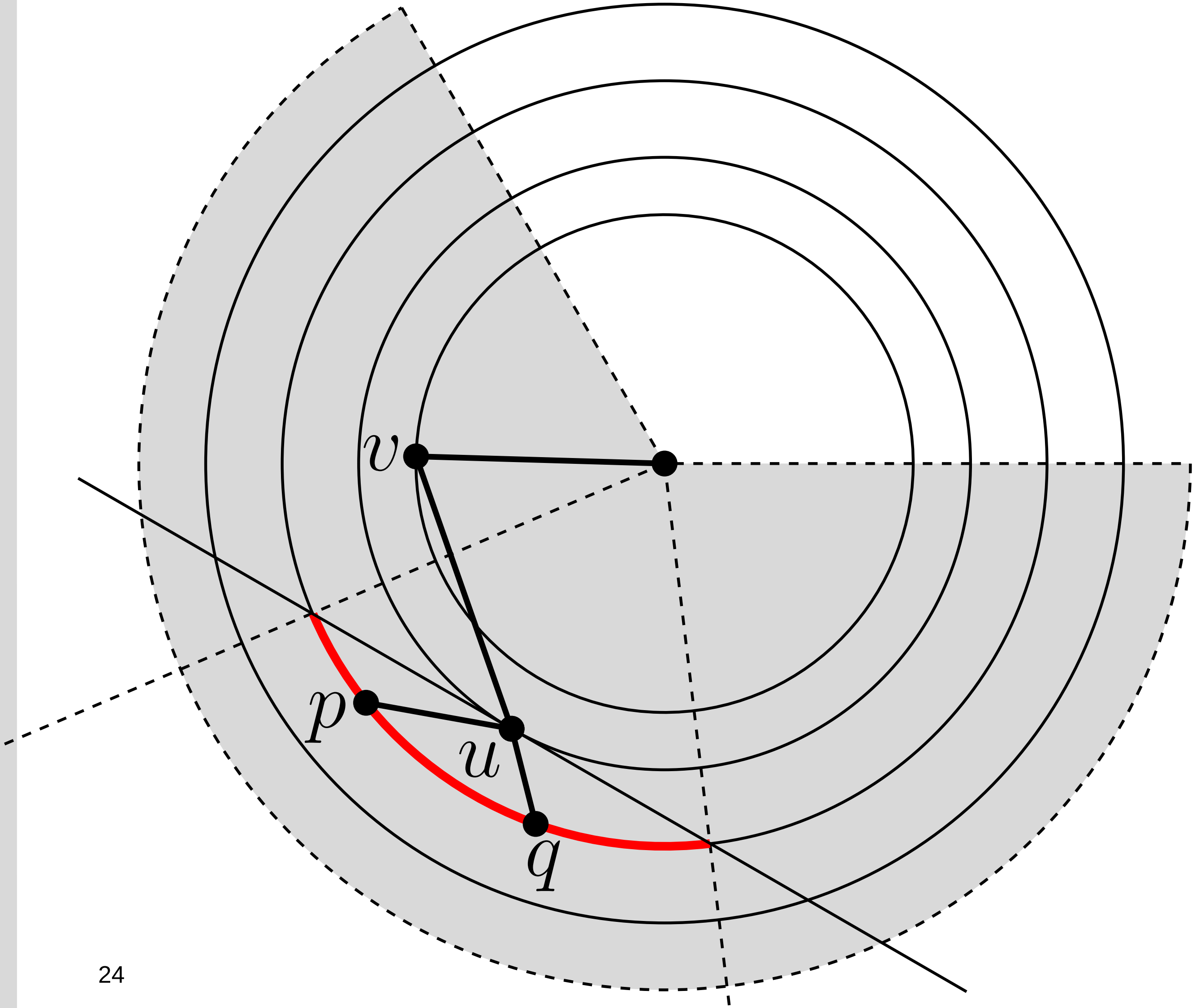

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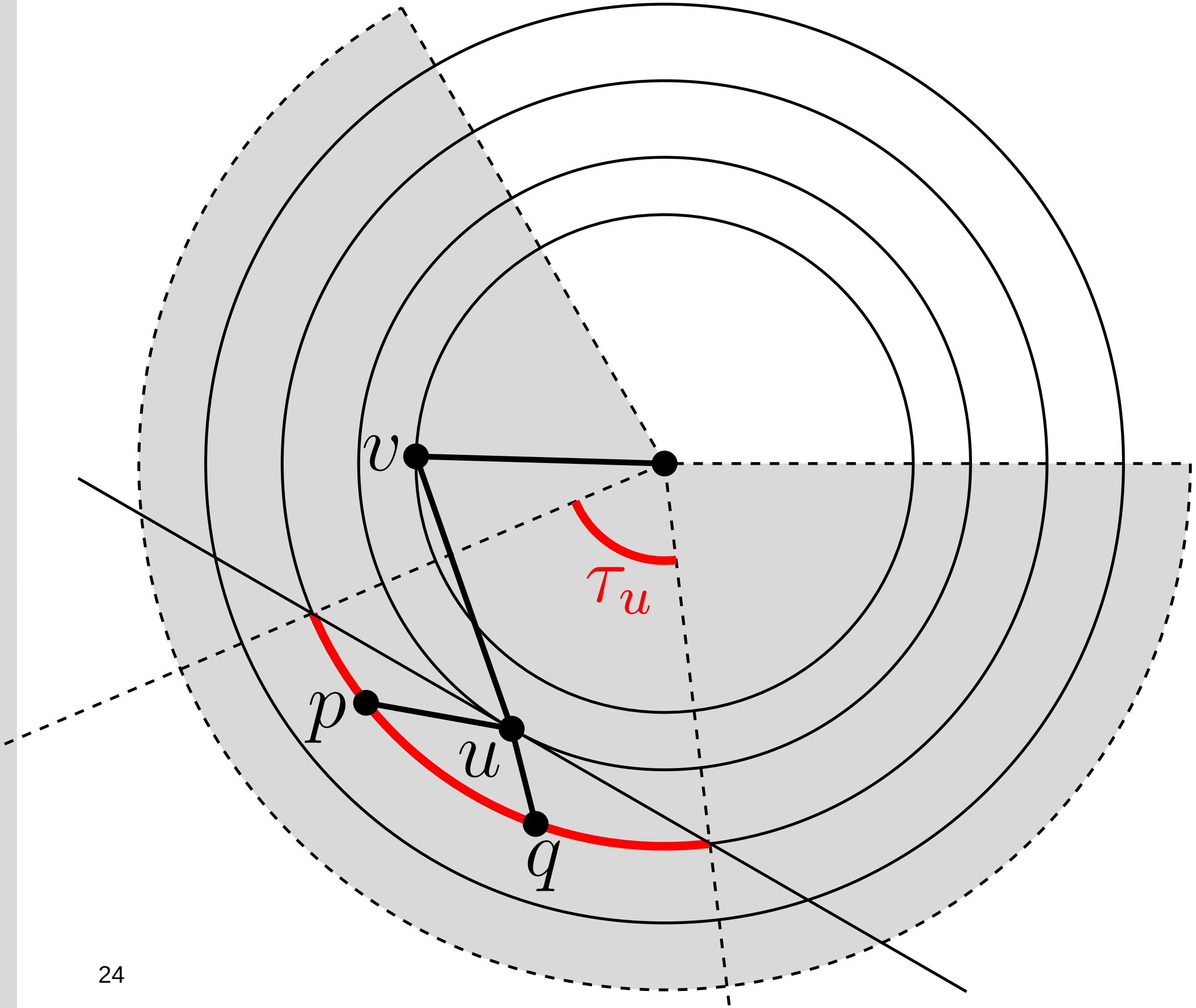

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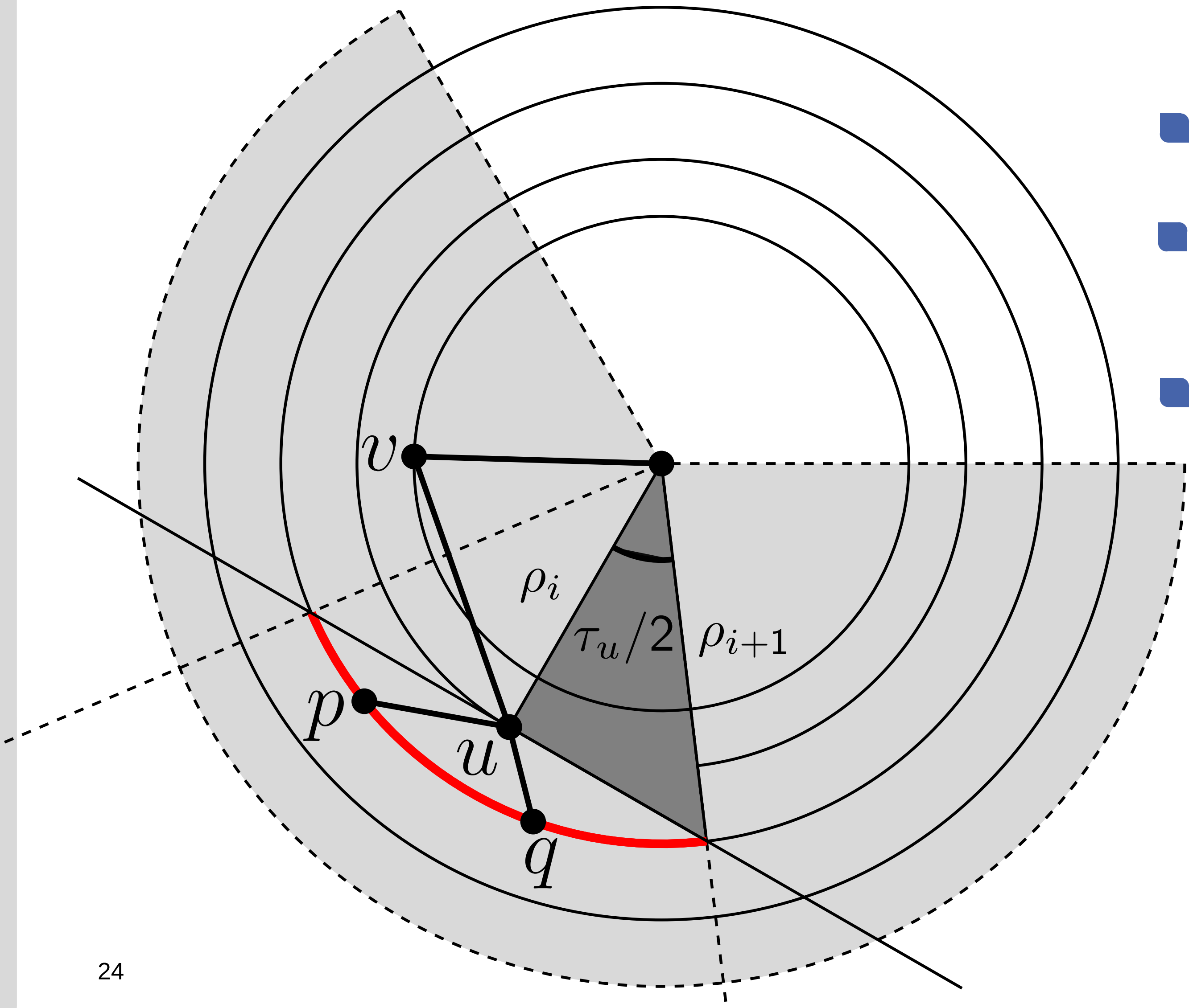

- τ_u - angle of the wedge corresponding to vertex u
- ρ_i - raduis of layer i
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Radial Layout

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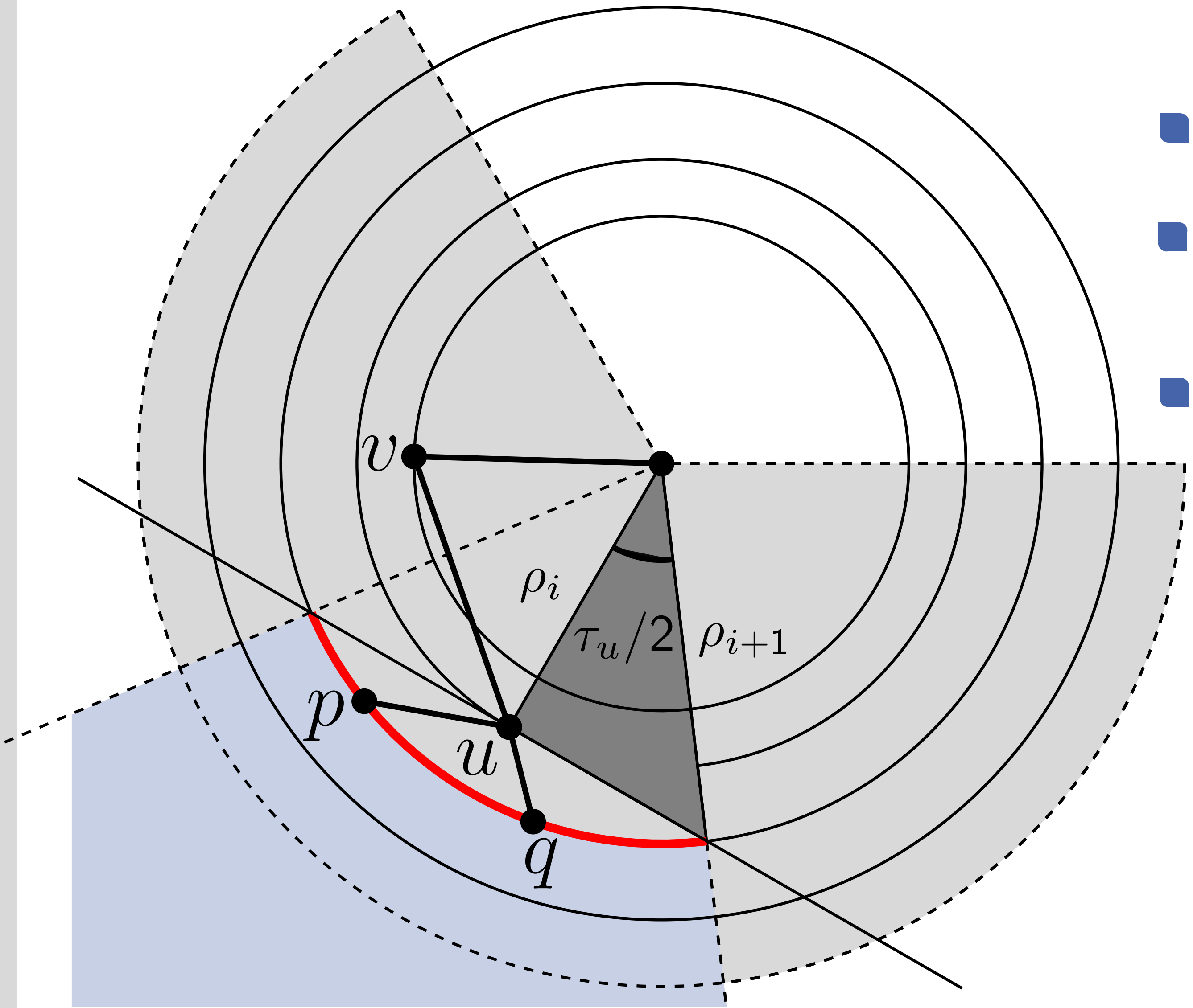

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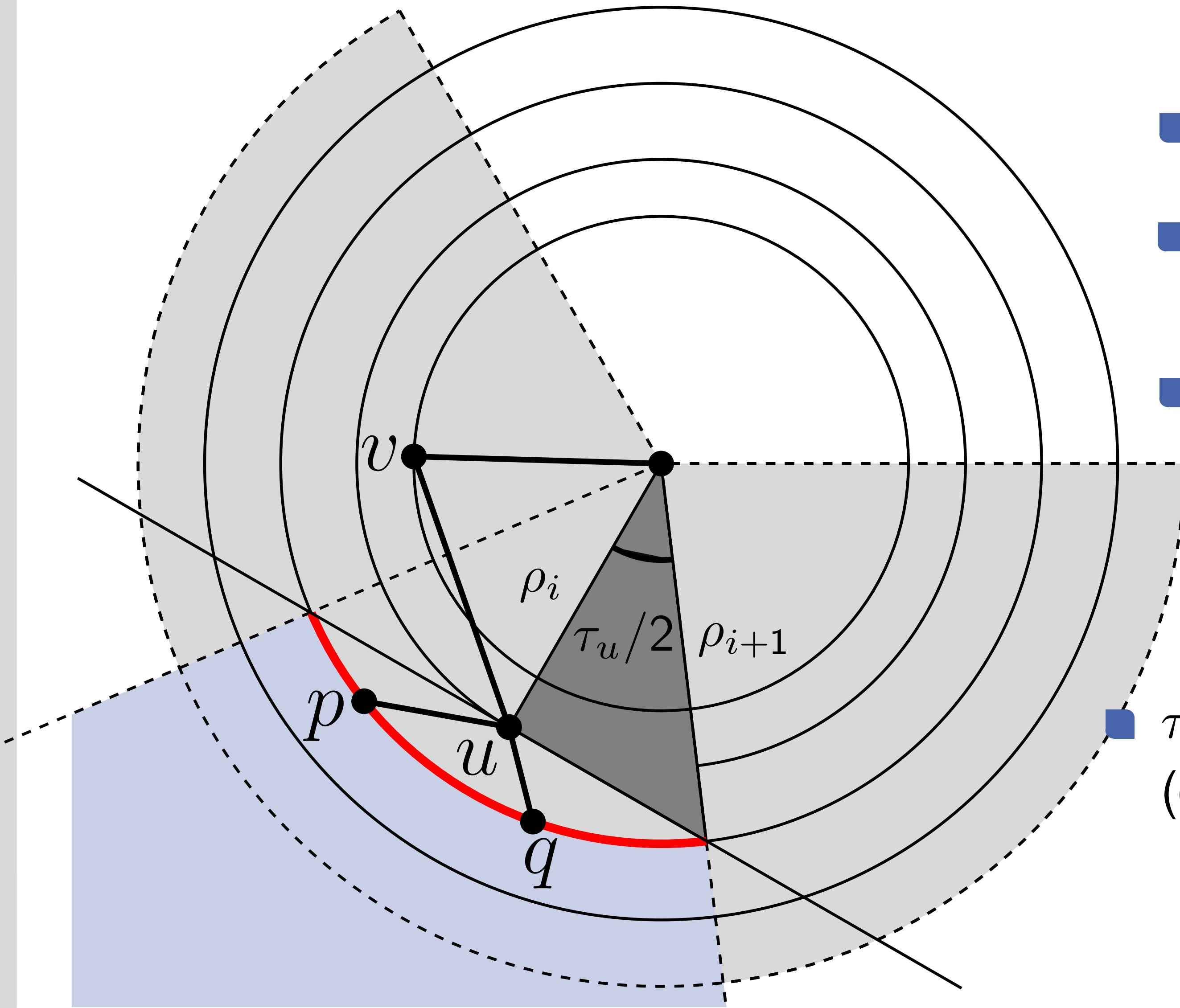

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Radial Layout

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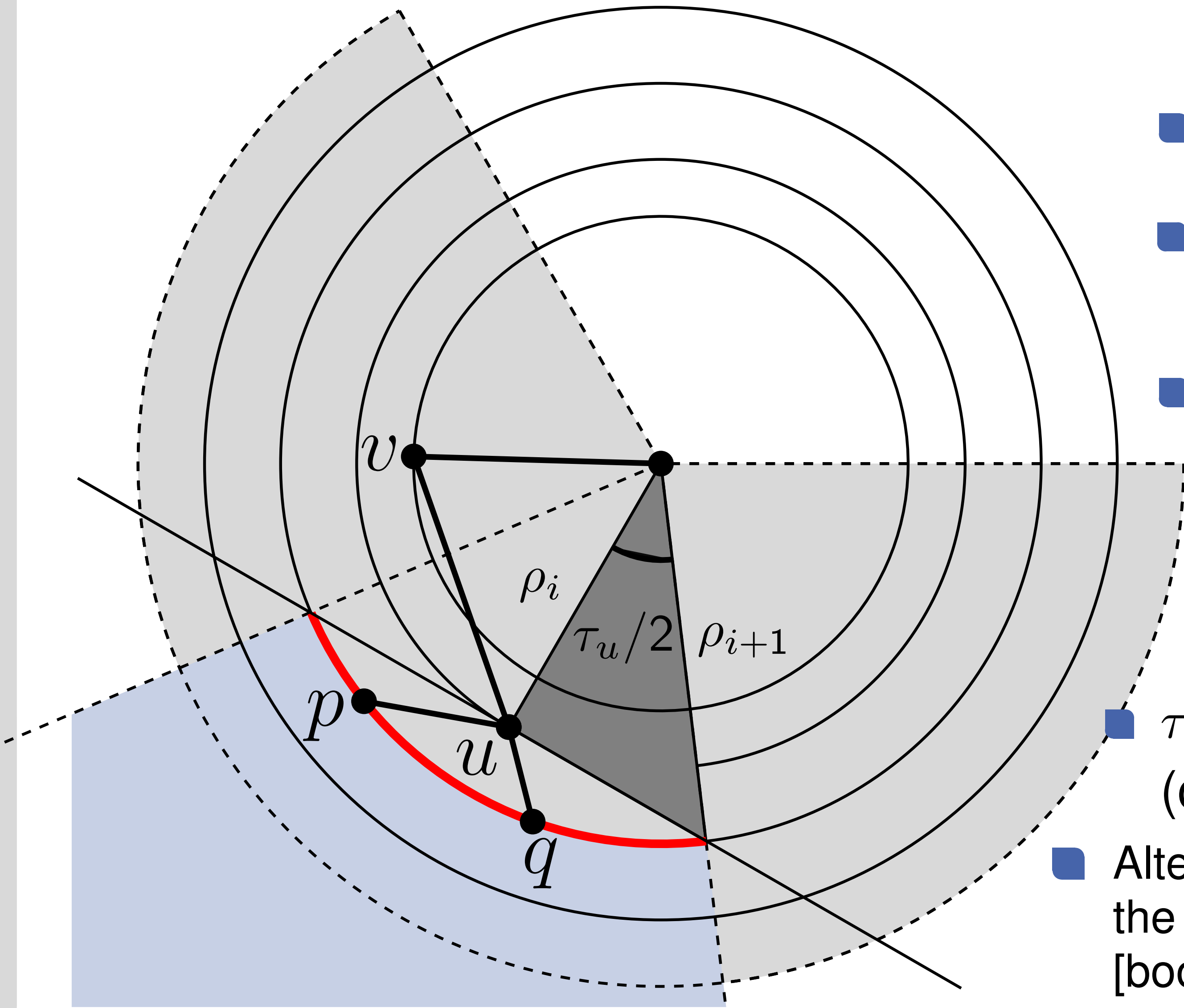

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Radial Layout

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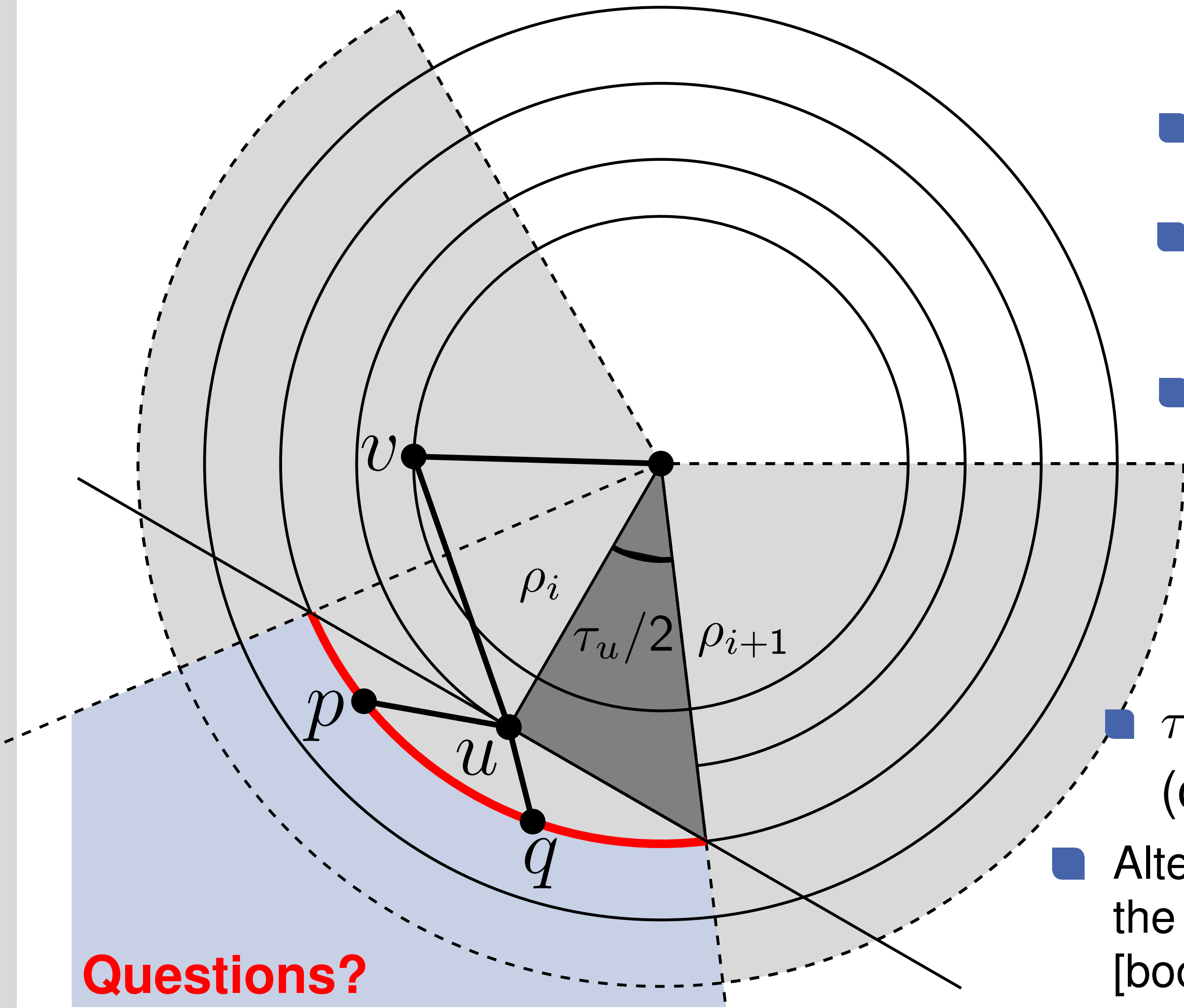

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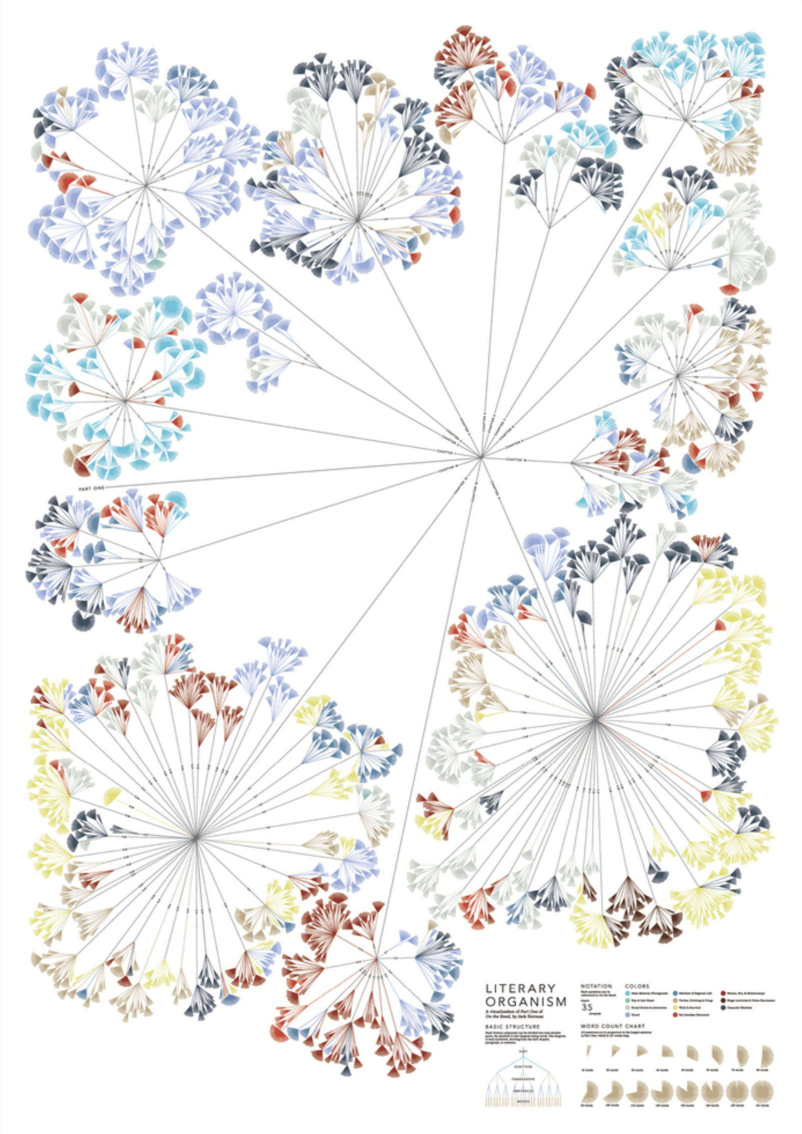
Questions?

Other Visualization Styles

Writing Without Words: the project explores methods of visually-representing text and visualises the differences in writing styles when comparing different authors.



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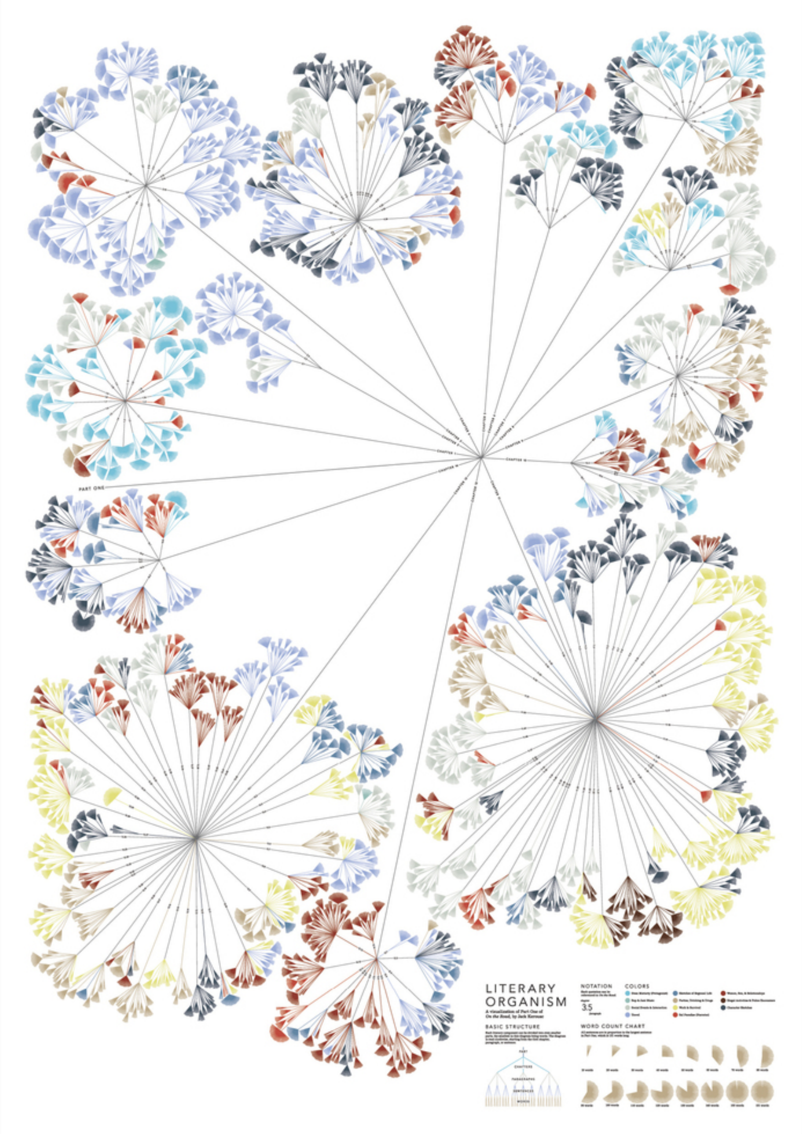
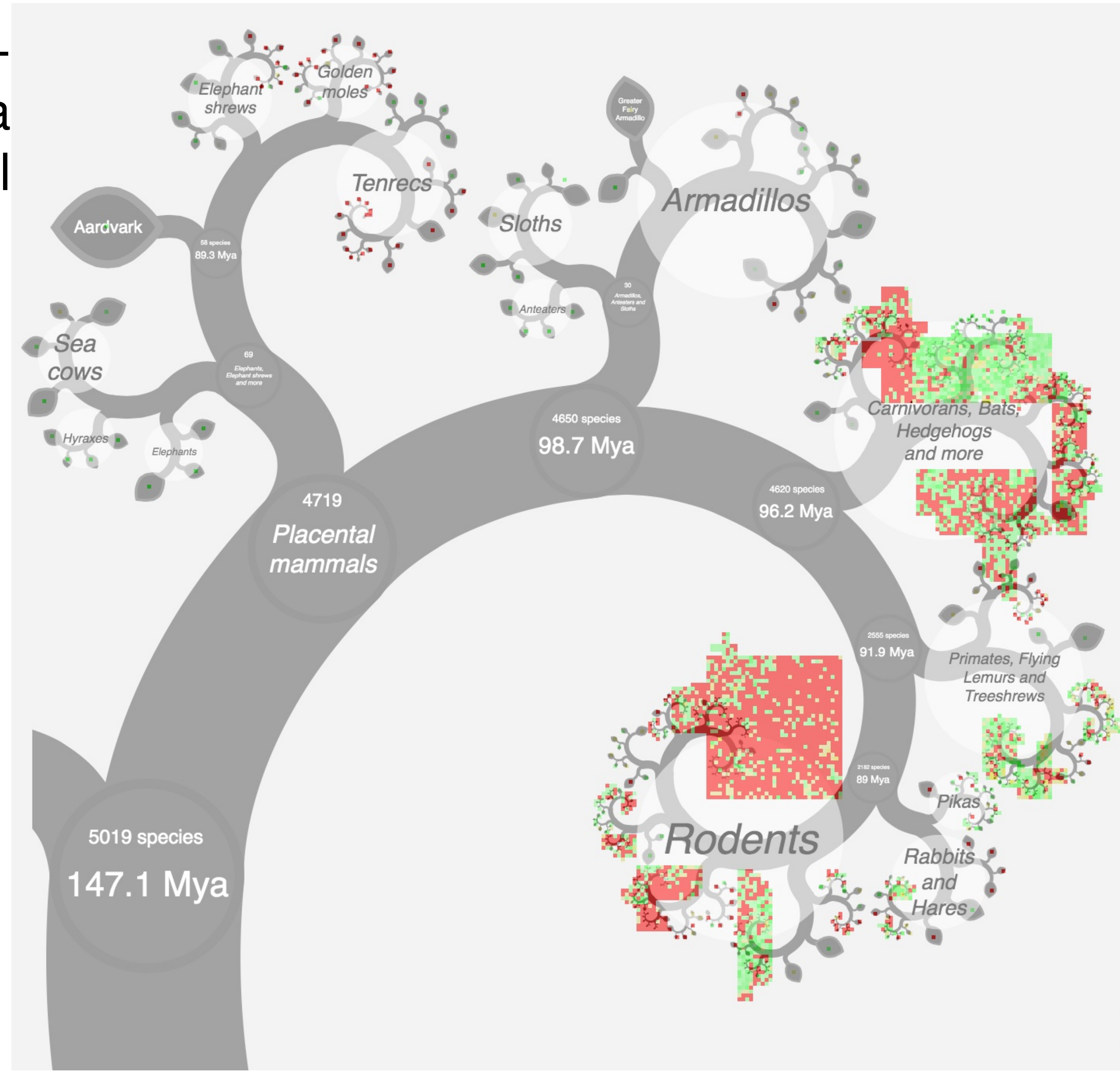


similar to Ballon layout

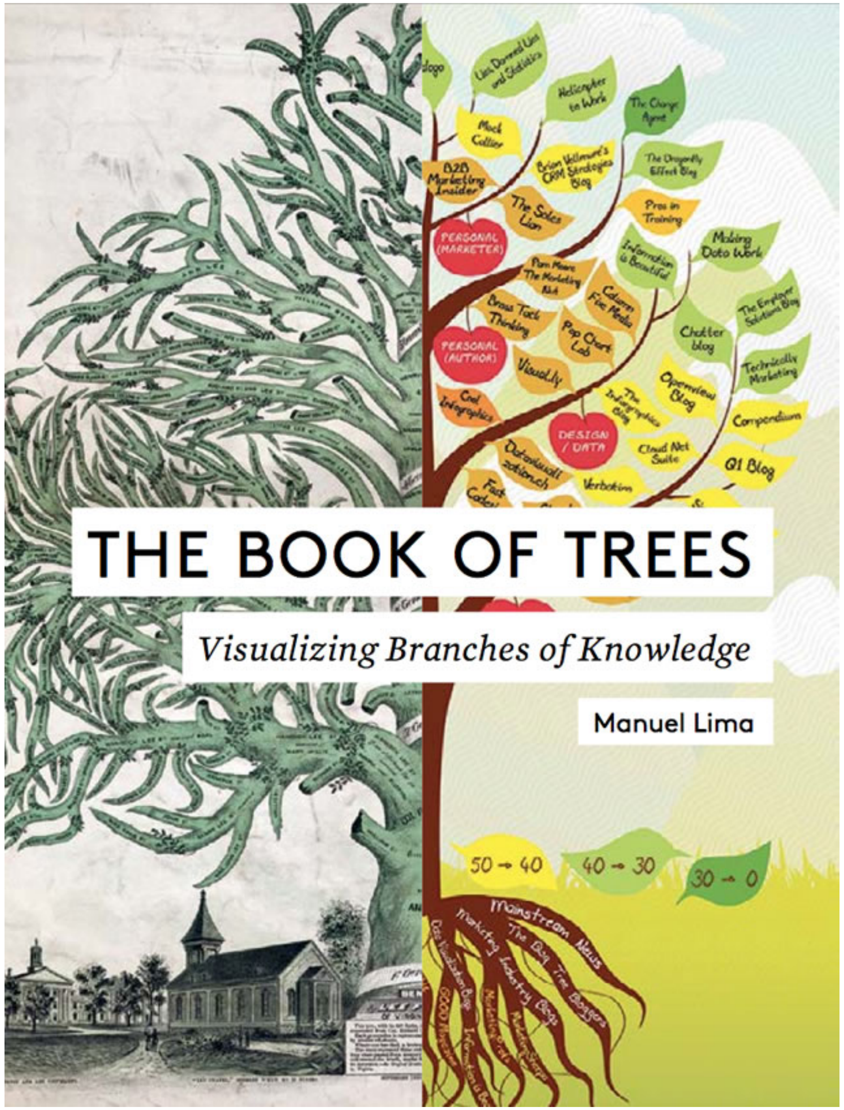
Other Visualization Styles

A phylogenetically organised display of data for all placental mammal species.



Fractal tree layout

for more applications and layouts...



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