Exercises 6 – Duality & Well-Separated Pair Composition

Discussion: To be announced.

Duality (11.01.2016)

Exercise 1 – Duality I. In the lecture we have seen that the dual of a line segment is a double wedge, with a wedge left and right to the point that is dual to the line containing the line segment.

1. What is the dual of a triangle with vertices $p$, $q$ and $r$?
2. What is the dual of a circle through the points $p$, $q$ and $r$?

Exercise 2 – Duality II. Let $L$ be a set of $n$ lines in the plane. We want to find an axis-aligned rectangle $B(L)$ that contains all vertices of the arrangement $A(L)$. Describe an algorithm that computes $B(L)$ in $O(n \log n)$ time.

Exercise 3 – Duality III. Let $R$ be a set of $n$ red points in the plane, and let $B$ be a set of $n$ blue points in the plane. We call a line $\ell$ a separator of $R$ and $B$, if all blue points lie on one side and all red points on the other side of $\ell$.

1. Describe an algorithm that decides in $O(n \log n)$ time whether a separator of $R$ and $B$ exists.
2. Describe an randomized algorithm, that decides in $O(n)$ expected time whether a separator of $R$ and $B$ exists.

Exercise 4 – Duality IV. Let $S$ be a set of $n$ points in the plane. Describe an $O(n^2)$ algorithm that computes the line on which most points of $S$ lie.

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Well-Separated Pair Decomposition (13.01.2016)

**Exercise 5 – Foundations.** Let $s > 0$ and let $x := 2/s + 1$. Further, let $S := \{x^i \mid 0 \leq i \leq n - 1, i \in \mathbb{N}\}$ and let $\{A_j, B_j\}$ $(1 \leq j \leq m)$ be an arbitrary $s$-WSPD for $S$. Show that

$$
\sum_{j=1}^{m} (|A_j| + |B_j|) = \left(\frac{n}{2}\right) + m
$$

*Hint:* For each $j$ at least one of both sets $A_j$ and $B_j$ is a singleton.

**Exercise 6 – Neighbor I.** Let $P$ be a set of $n$ points in $\mathbb{R}^d$. Let $p \in P$ and let $q \in P$ be the next neighbor of $p$ in $P$, i.e., $|pq| = \min\{|pr| : r \in P, r \neq p\}$. Consider an arbitrary $s$-WSPD for $P$ with $s > 2$.

1. Let $\{A, B\}$ be a pair in this decomposition and assume that $p$ lies in $A$ and $q$ lies in $B$. Show that $A$ only contains $p$.
2. Show that the size of an arbitrary $s$-WSPD with $s > 2$ is at least $n/2$.

**Exercise 7 – Neighbor II.** Let $P$ be a set of $n$ points in $\mathbb{R}^d$. Further, let $p, q \in P$ be a pair of points with minimal distance to each other, i.e., $|pq| = \min\{|ab| : a \in P, b \in P\}$. Consider an arbitrary $s$-WSPD $W$ for $P$ with $s > 2$. Show that $W$ contains the pair $\{\{p\}, \{q\}\}$. 