Computational Geometry · Lecture

Projects & Polygon Triangulation

INSTITUTE FOR THEORETICAL INFORMATICS · FACULTY OF INFORMATICS

Tamara Mchedlidze · Darren Strash

02.11.2015

Projects

Project Specifications

Groups:

- 2 or 3 students, assigned to a supervisor

Tools:

- Use any programming language, should run with little effort on Linux

Due date:

- 08.02.2016

Visualization:

- Need to visualize output; ipe, svg, video? This is also helpful for debugging output
- Do not need to use graphics APIs, unless you really want to

Group Presentations:

- Each groups gives a 20-minute presentation (last 2 weeks of class)

Project—Next Steps

Project proposals: due in 2 weeks (16.11.2015):

- No more than 1 page
- Formalize your problem
- Describe geometric primitives
- State any simplifying assumptions
- Describe the algorithms / data structures you will use
- **Avoid brute force algorithms**

Supervisor:

- Assigned supervisor (Tamara, Darren, or Benjamin)
- Meet with supervisor in two weeks to discuss proposal.

Project—Next Steps

Project proposals: due in 2 weeks (16.11.2015):

- No more than 1 page
- Formalize your problem
- Describe geometric primitives
- State any simplifying assumptions
- Describe the algorithms / data structures you will use
- **Avoid brute force algorithms**

Supervisor:

- Assigned supervisor (Tamara, Darren, or Benjamin)
- Meet with supervisor in two weeks to discuss proposal.

Now, form groups and sit by each other!

Project—Next Steps

Project proposals: due in 2 weeks (16.11.2015):

- No more than 1 page
- Formalize your problem
- Describe geometric primitives
- State any simplifying assumptions
- Describe the algorithms / data structures you will use
- **Avoid brute force algorithms**

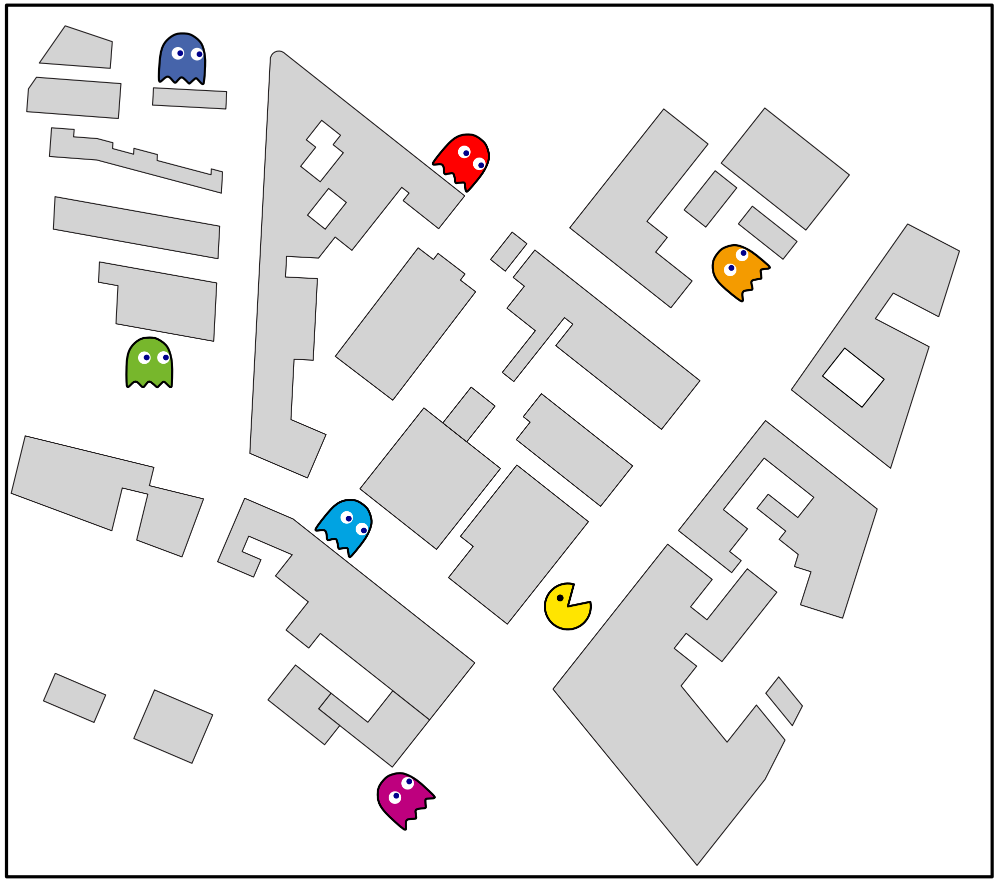
Supervisor:

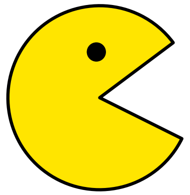
- Assigned supervisor (Tamara, Darren, or Benjamin)
- Meet with supervisor in two weeks to discuss proposal.

Now, form groups and sit by each other!

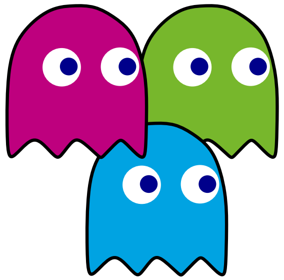
Time to present the projects...

Scenario: Development of the game Pacman.





Pacman

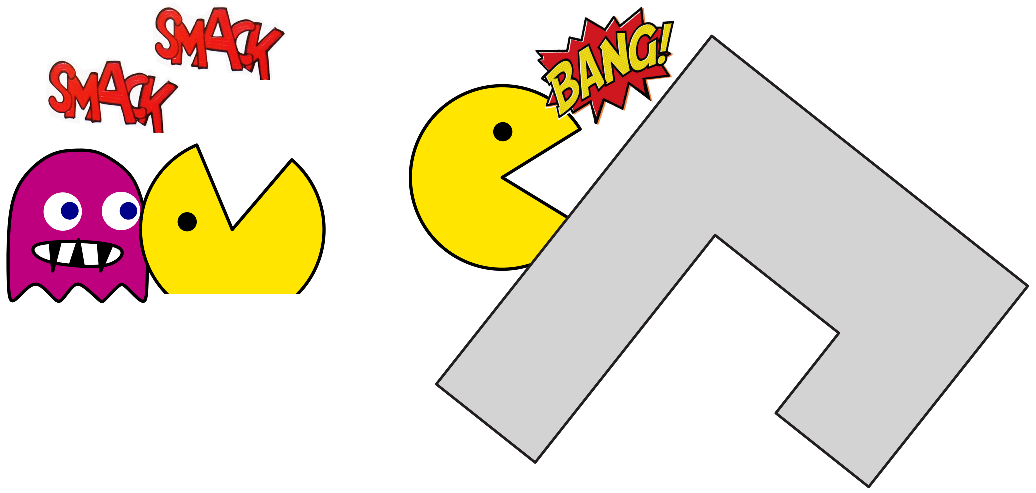


THE Ghosts.

Story:

Pacman is strolling through a large city with many buildings, gathering items. However, incredibly many ghosts want to hinder Pacman by eating him.

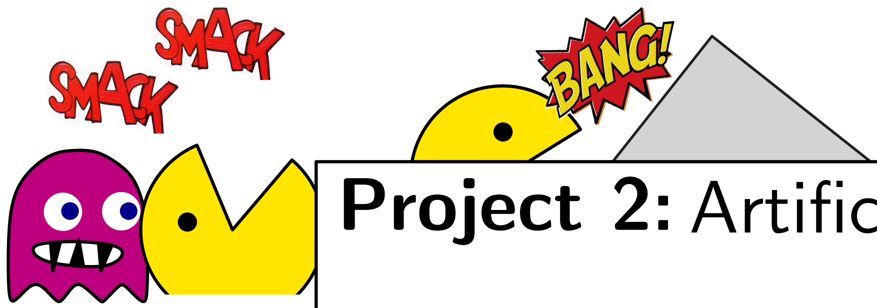
Project 1: Collision Detection



moving objects $\leftrightarrow$ moving objects

moving objects $\leftrightarrow$ buildings

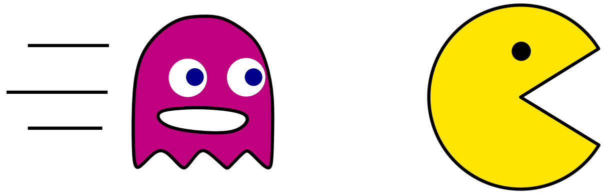
Project 1: Collision Detection



moving obje

moving obje

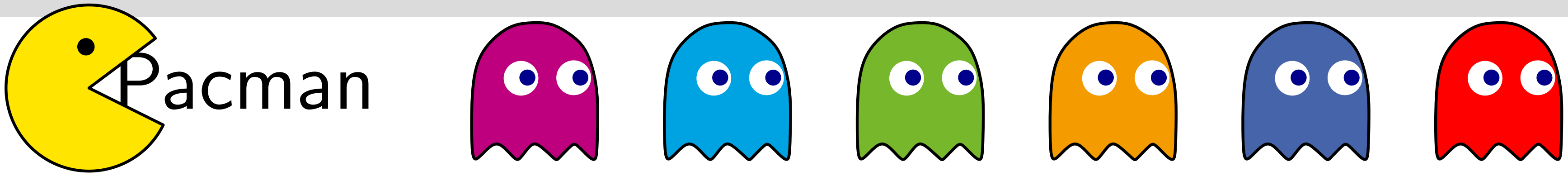
Project 2: Artifical Intelligence I



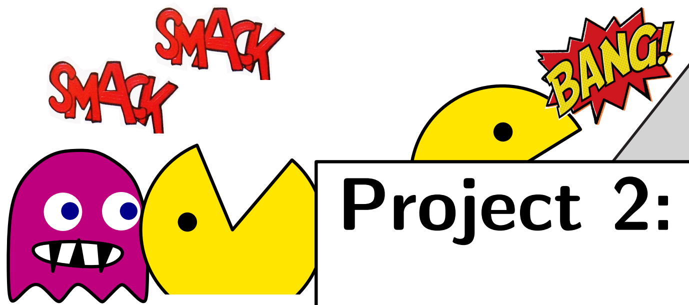
Ghosts seeing Pacman follow him until they lose eye contact.

Fast visibility check:

Is Pacman visible from point p?

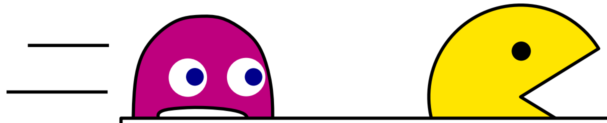


Project 1: Collision Detection



moving obj
moving obj

Project 2: Artifical Intelligence I

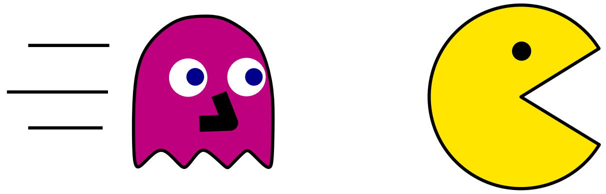


Ghosts se
until they

Fast visi

Is Pacm

Project 3: Artifical Intelligence II



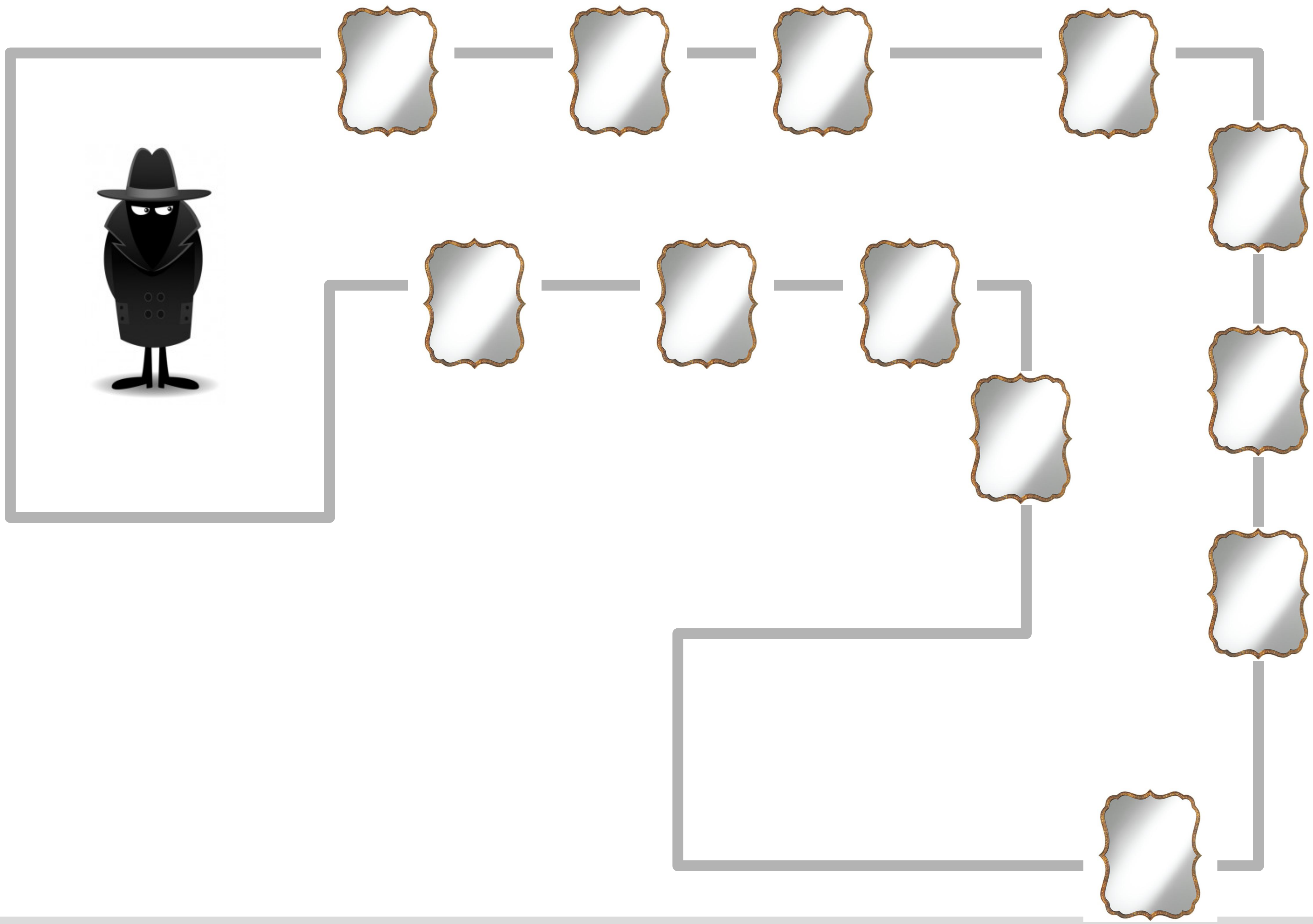
Ghosts smelling Pacman follow him until they lose the track.

Fast smell check:

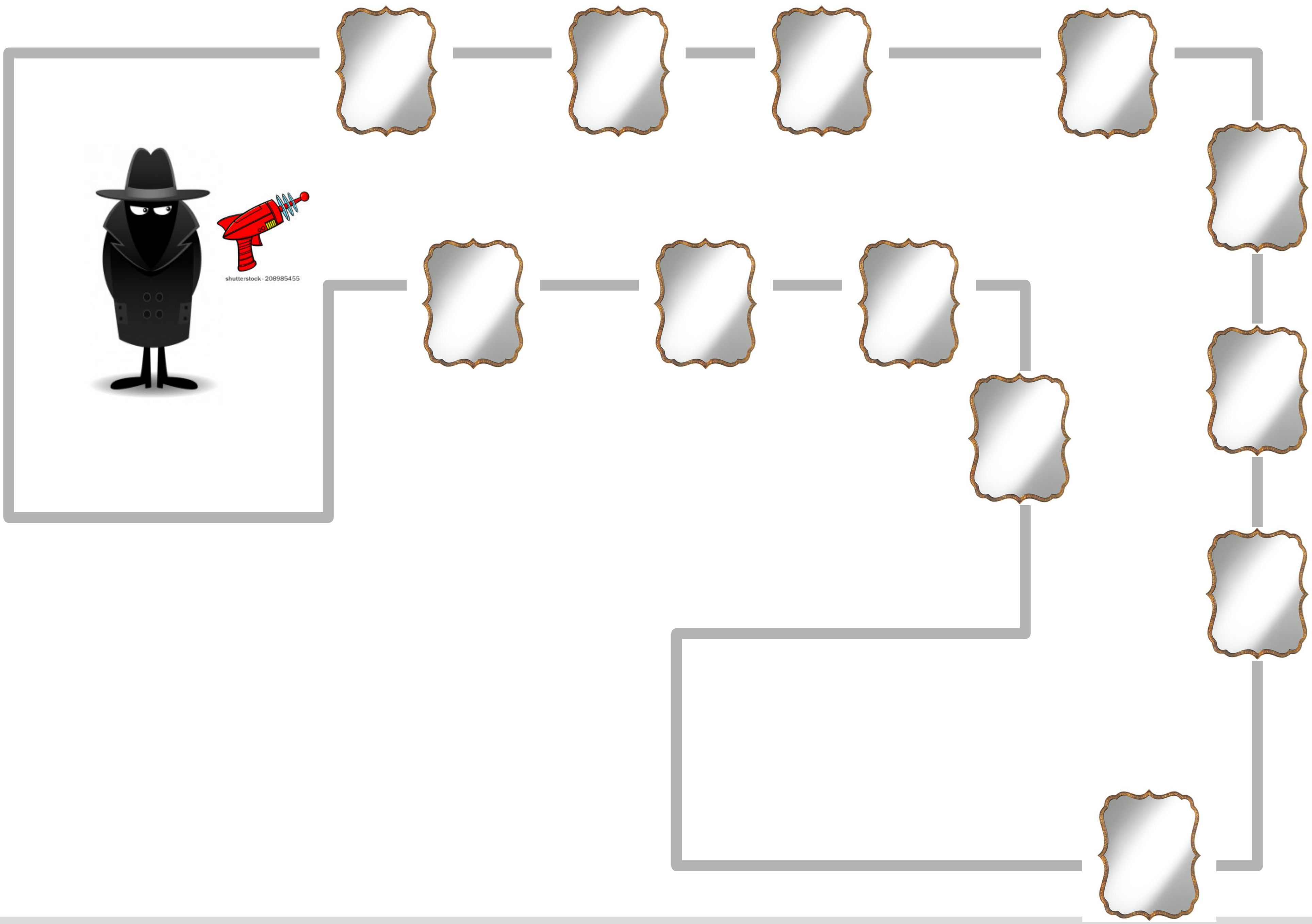
Can Packman be smelled from point p?

Secret Agent

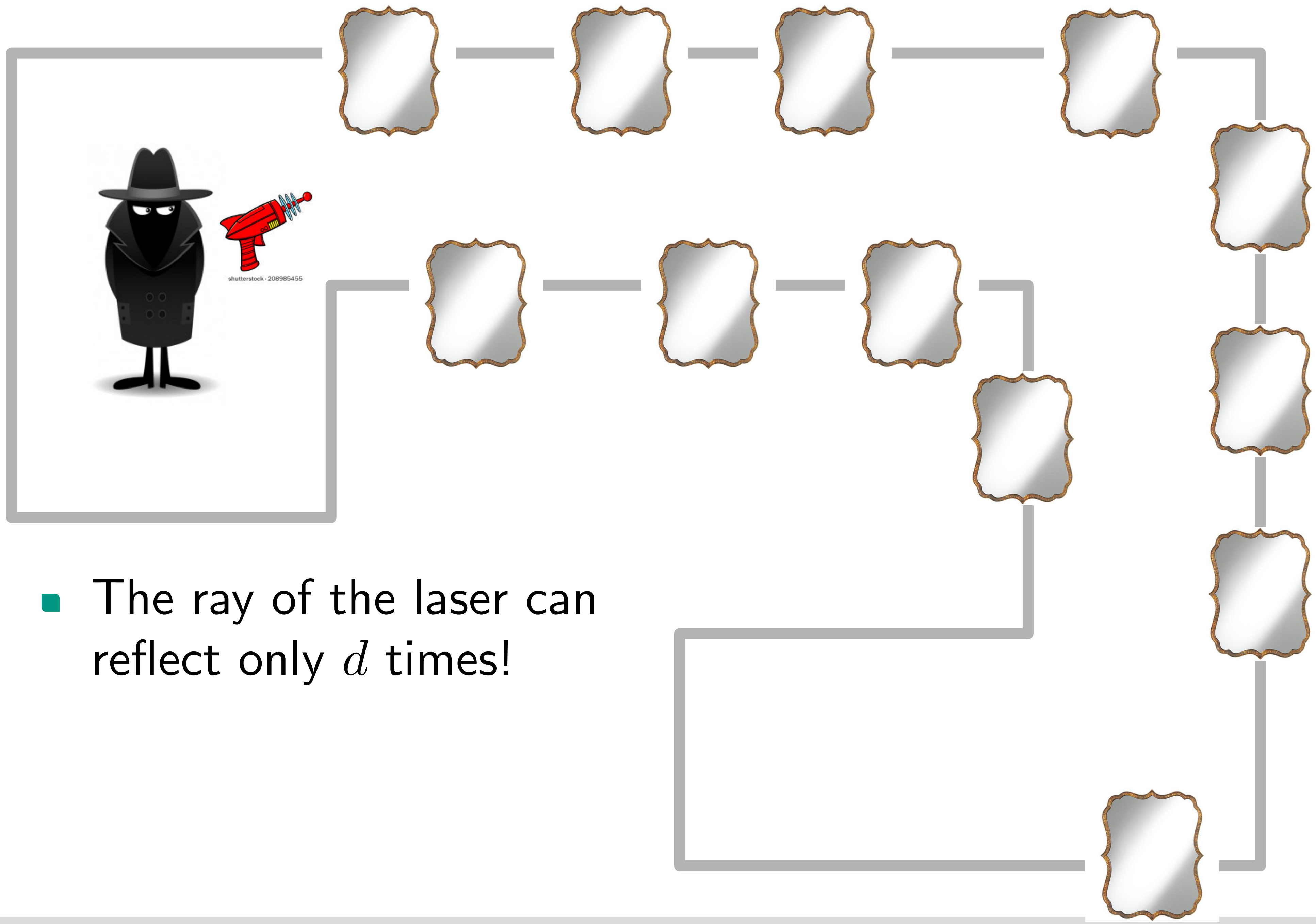
Shooting Secret Agent



Shooting Secret Agent

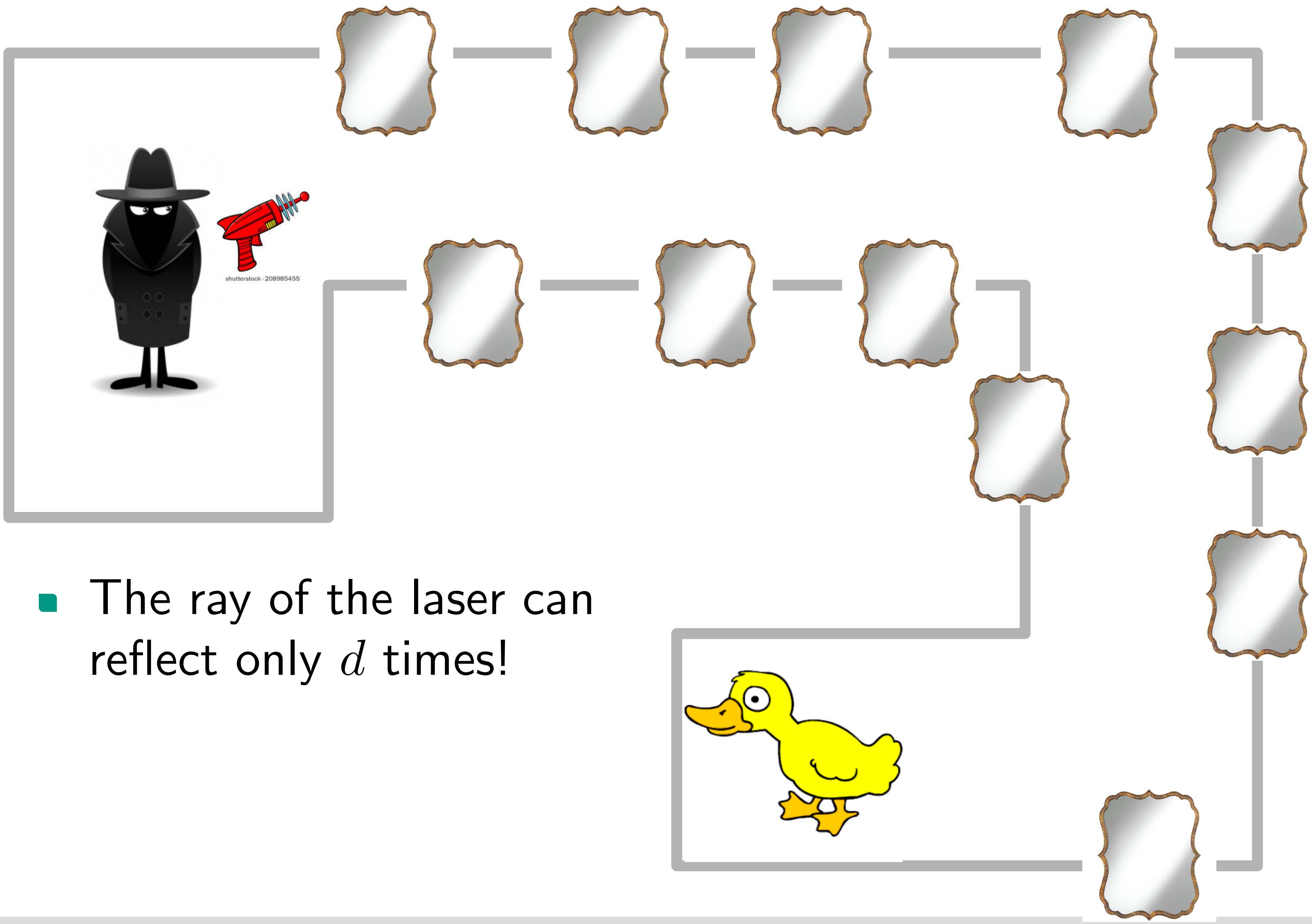


Shooting Secret Agent



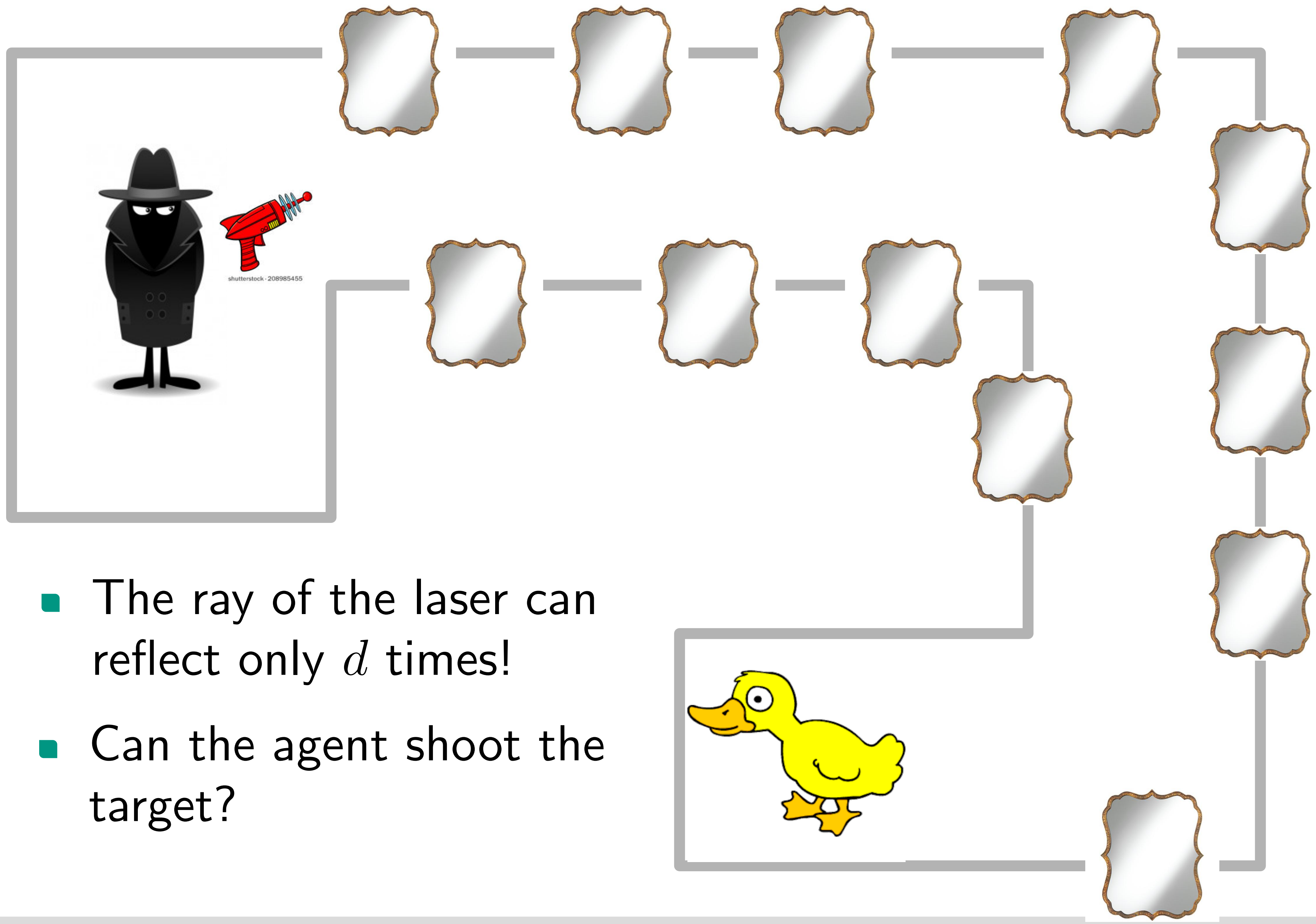
- The ray of the laser can reflect only d times!

Shooting Secret Agent



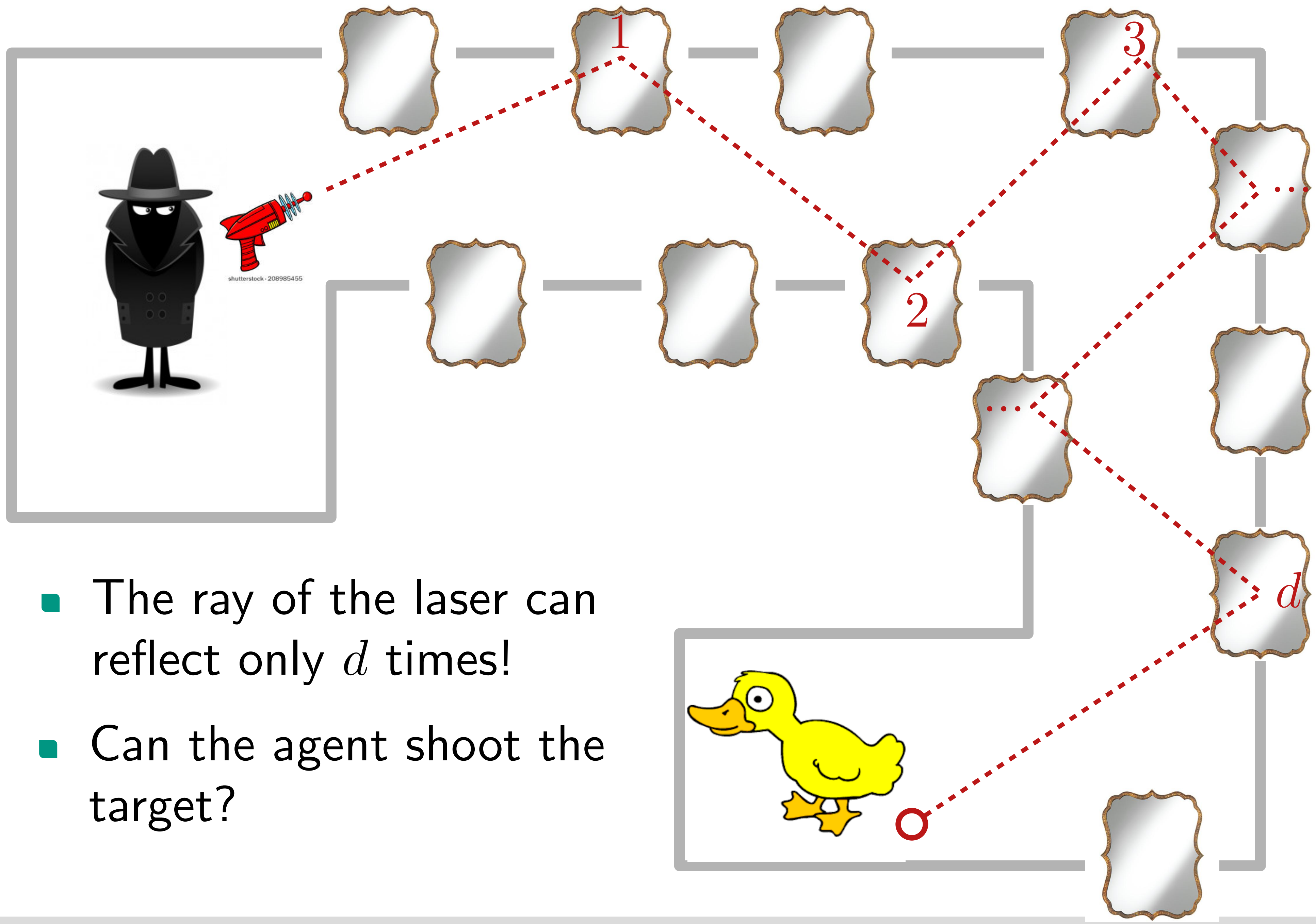
- The ray of the laser can reflect only d times!

Shooting Secret Agent



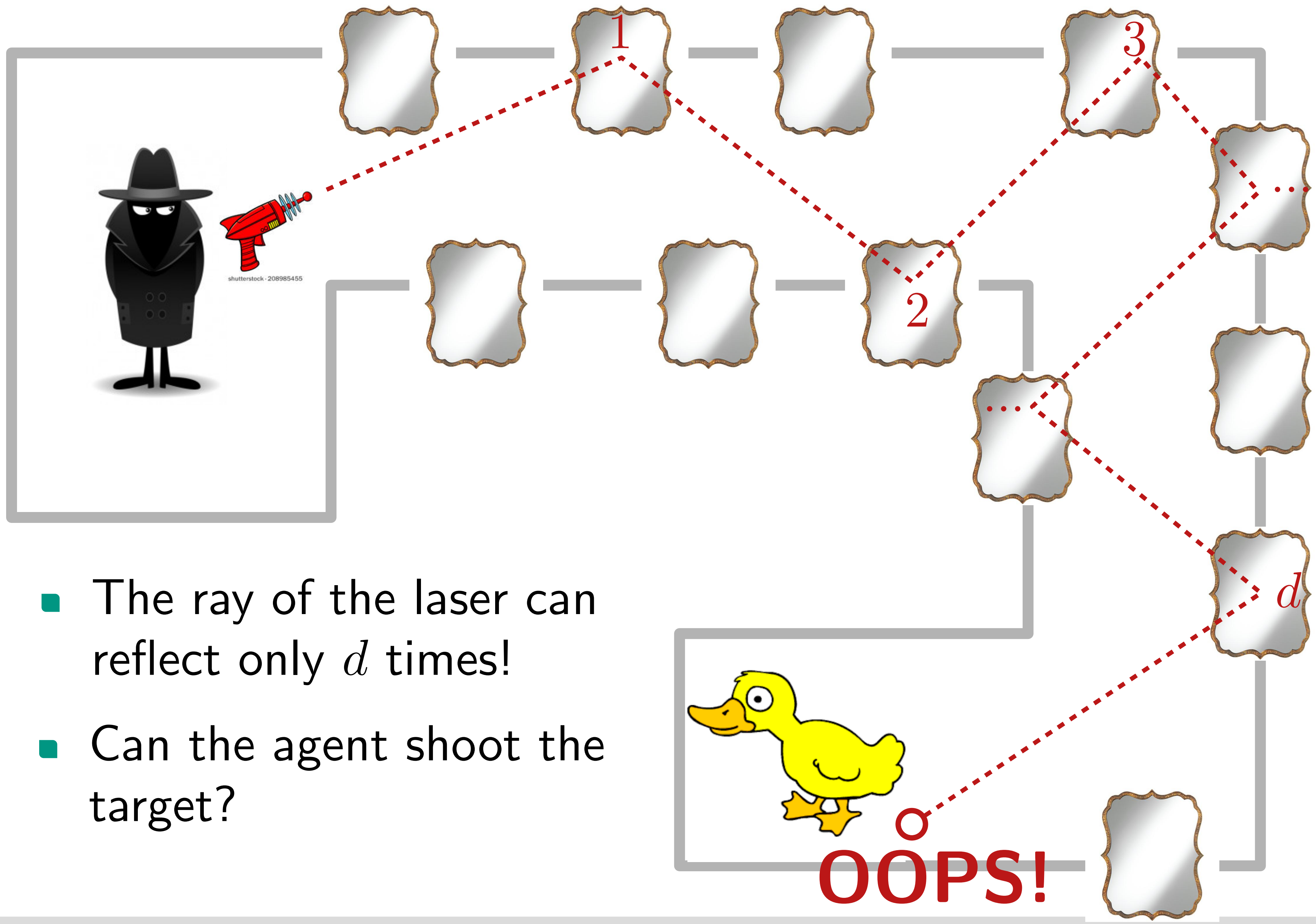
- The ray of the laser can reflect only d times!
- Can the agent shoot the target?

Shooting Secret Agent



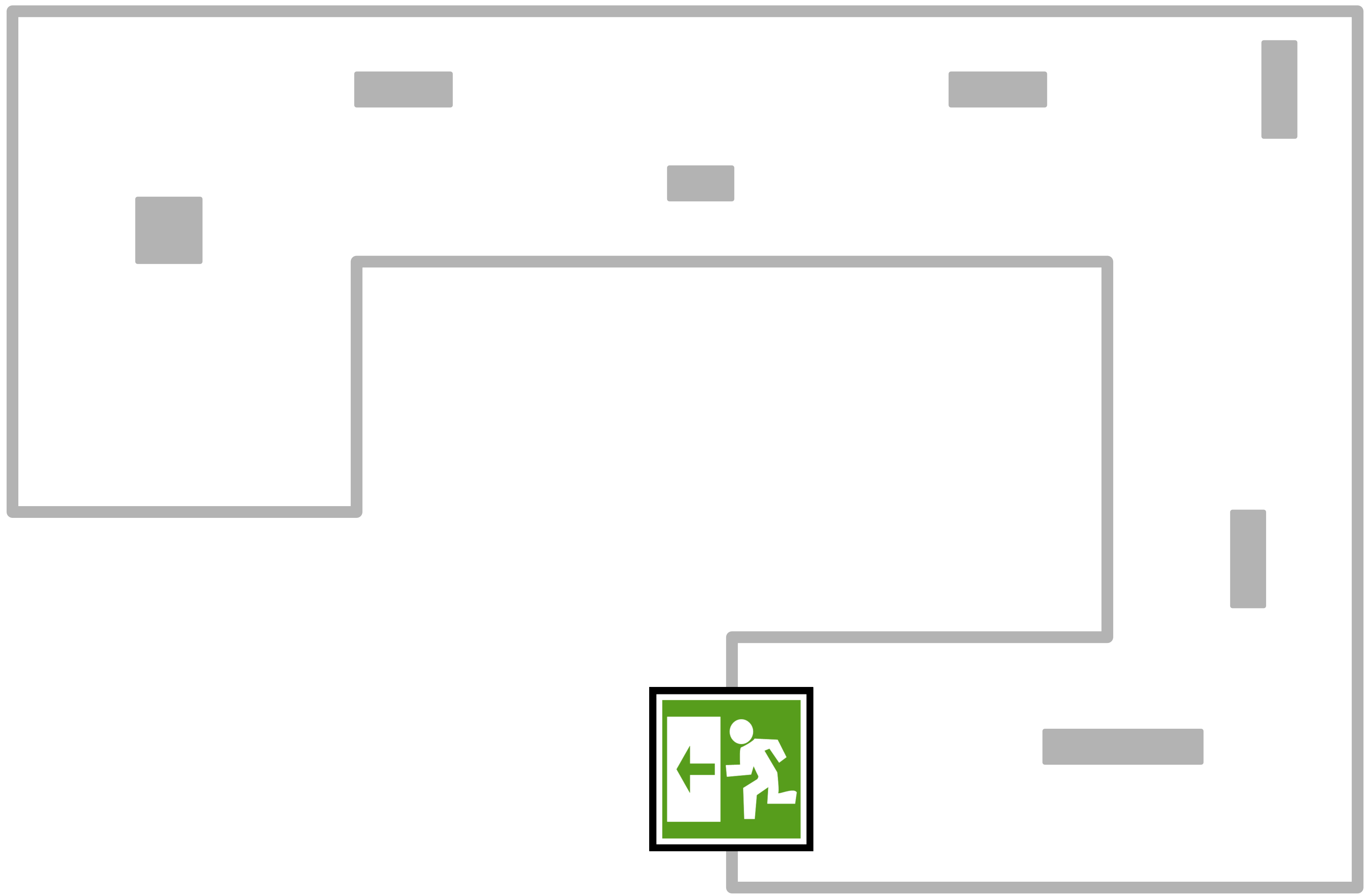
- The ray of the laser can reflect only d times!
- Can the agent shoot the target?

Shooting Secret Agent

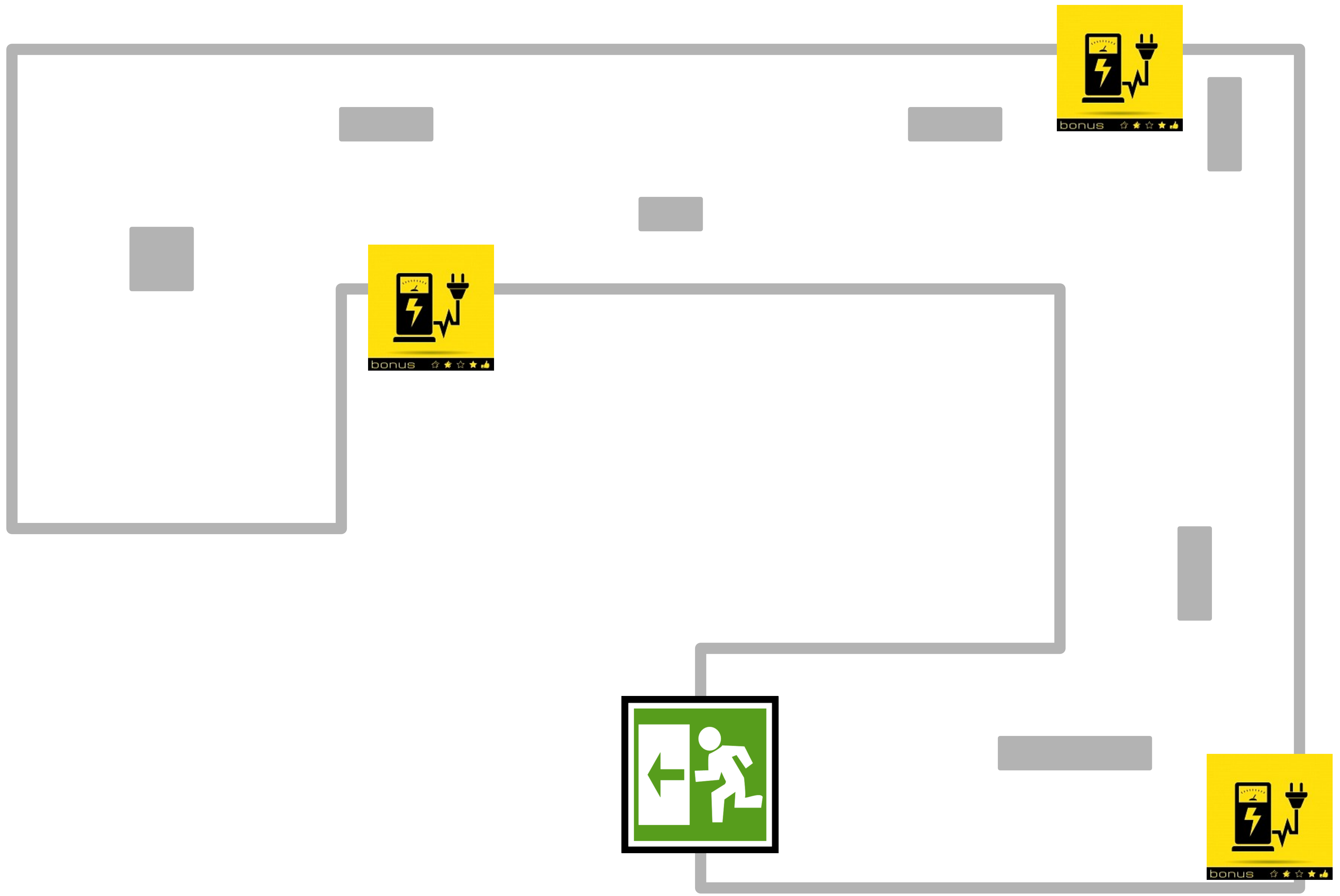


- The ray of the laser can reflect only d times!
- Can the agent shoot the target?

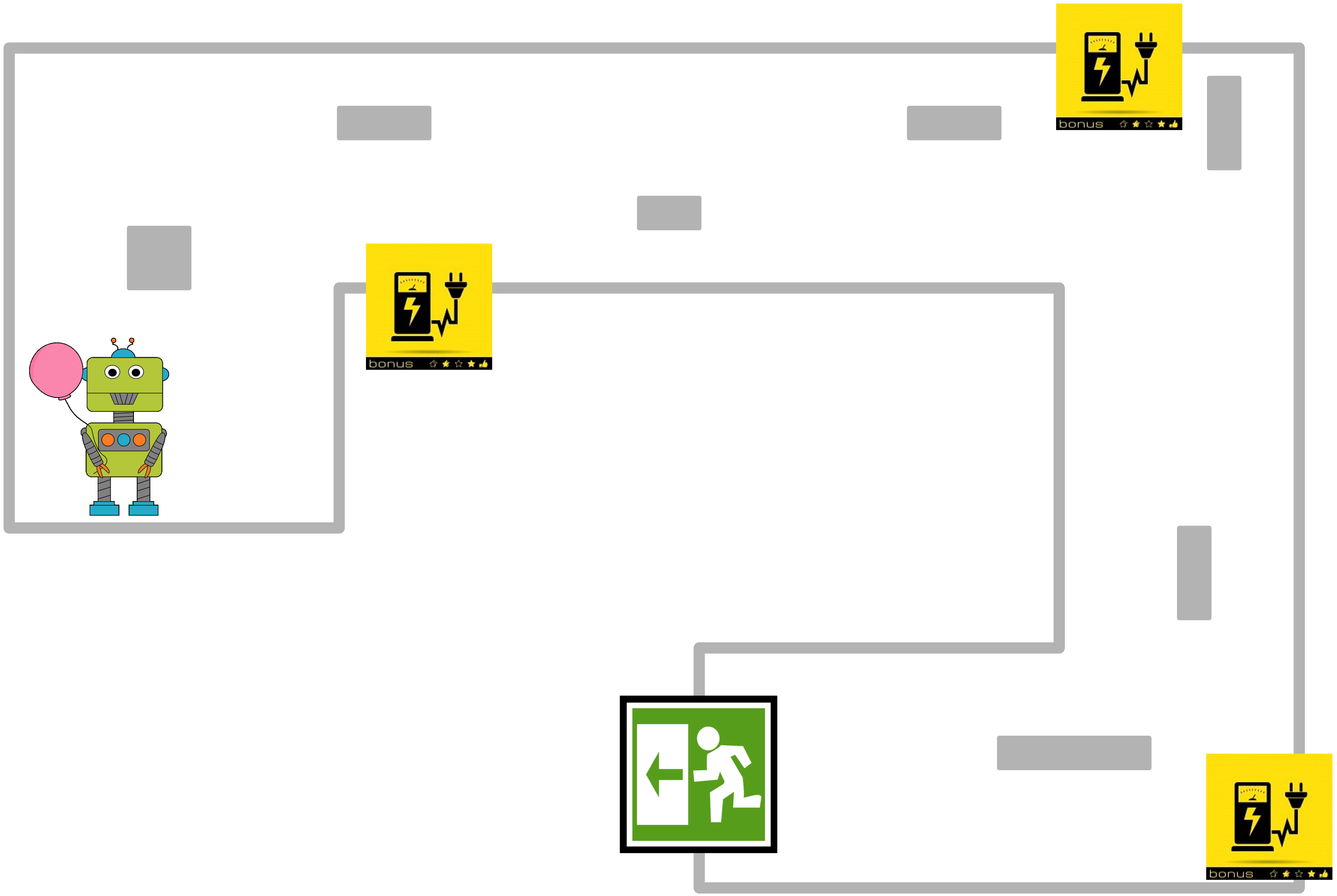
Secret Agent's Robot



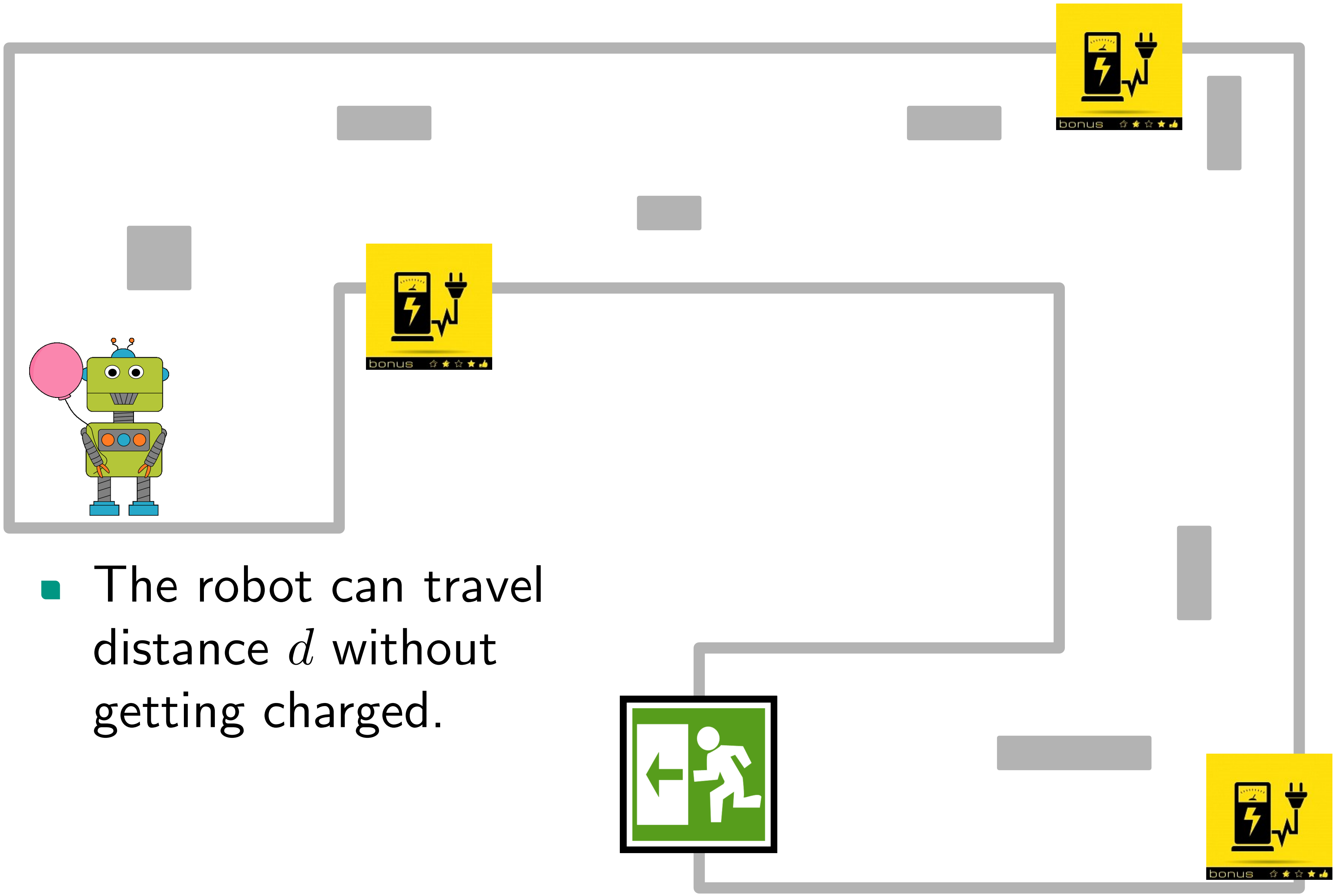
Secret Agent’s Robot



Secret Agent's Robot

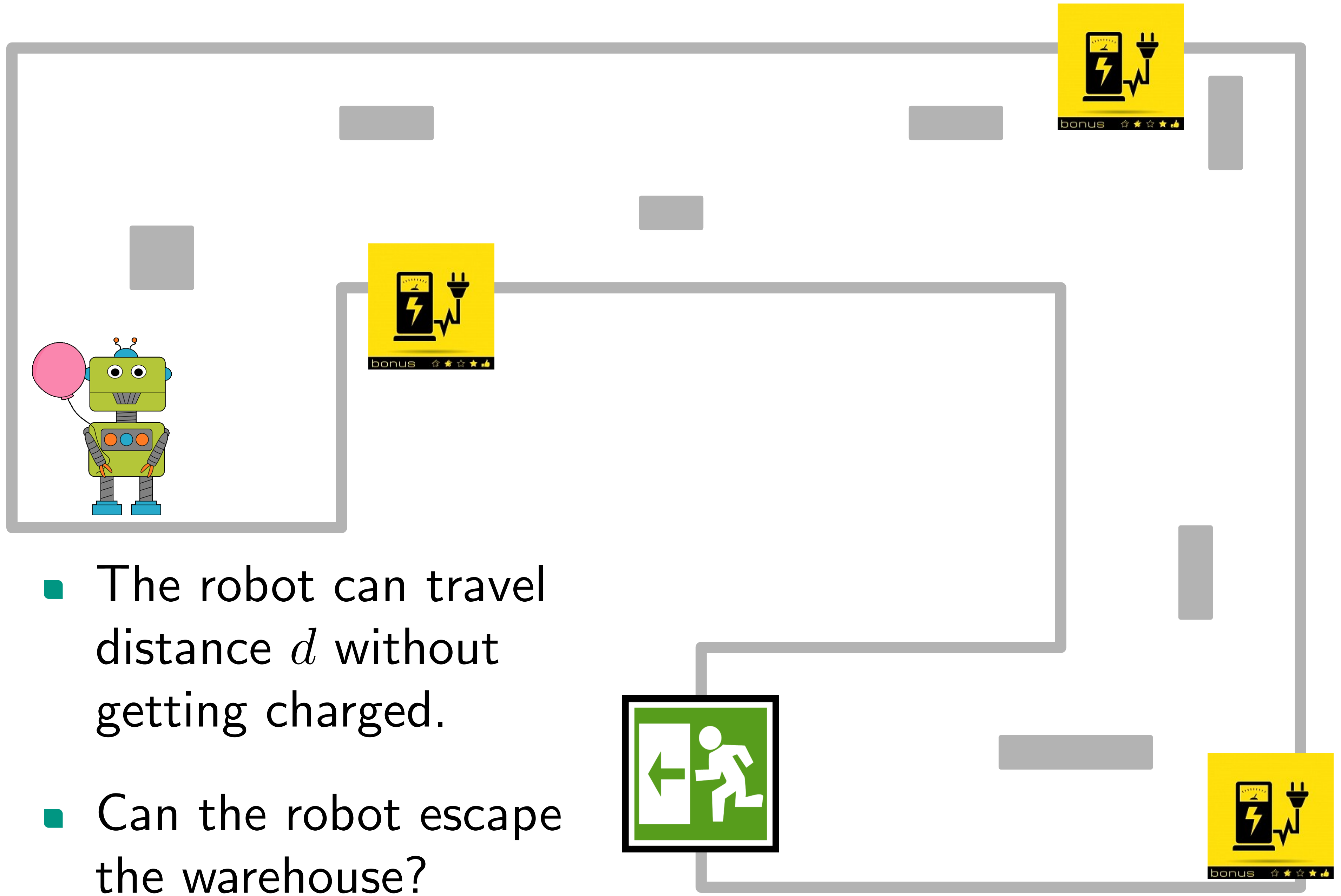


Secret Agent's Robot



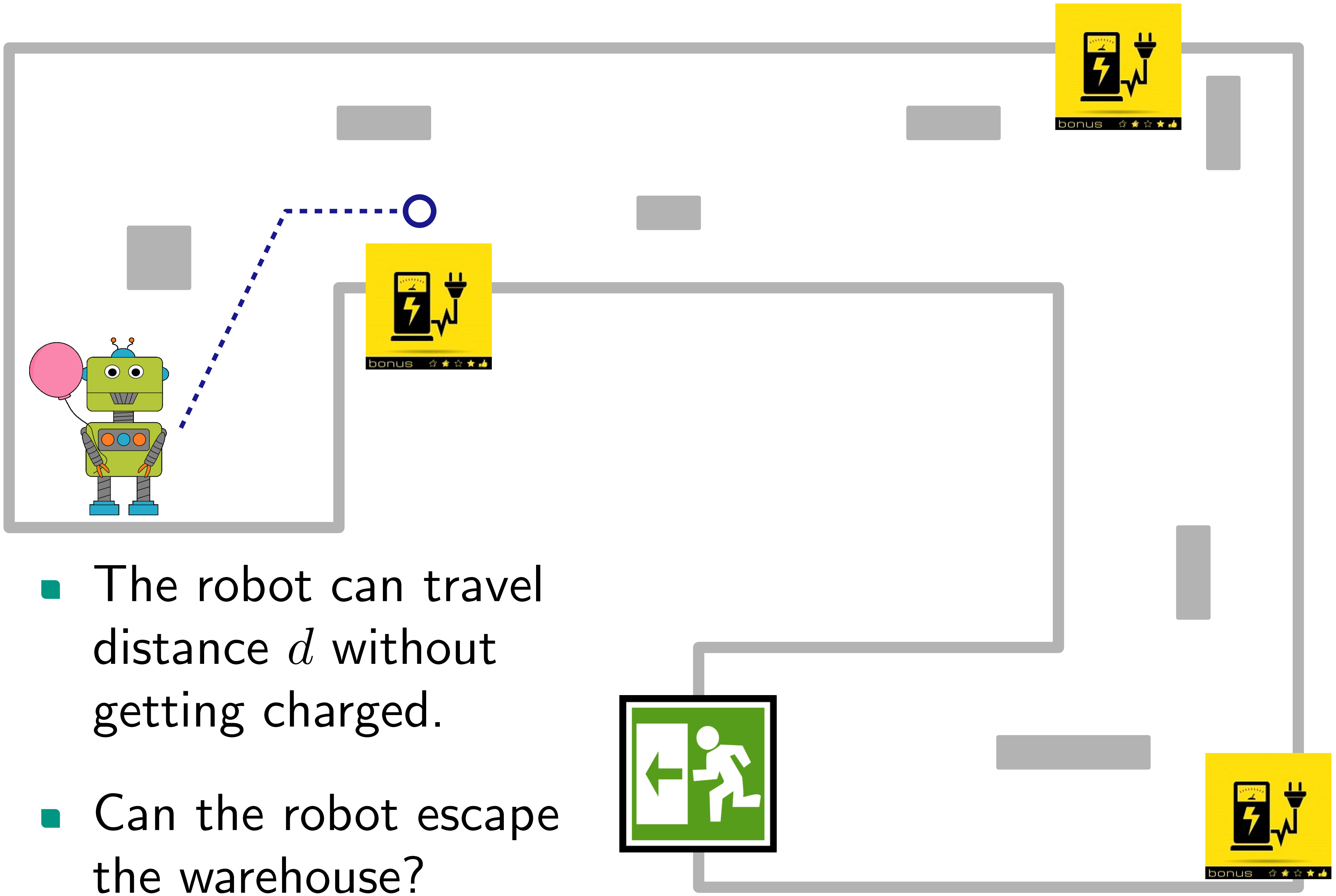
- The robot can travel distance d without getting charged.

Secret Agent's Robot



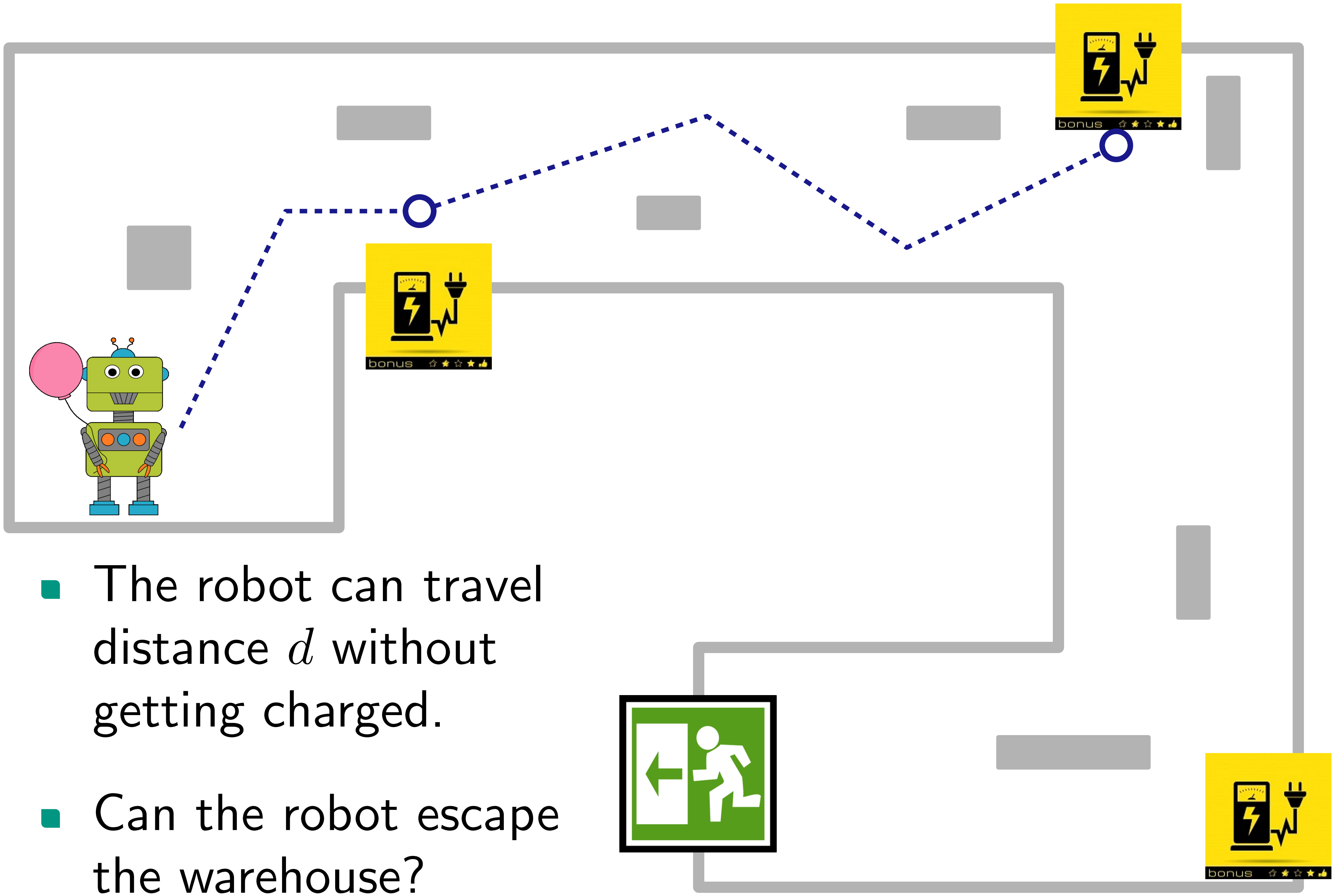
- The robot can travel distance d without getting charged.
- Can the robot escape the warehouse?

Secret Agent's Robot



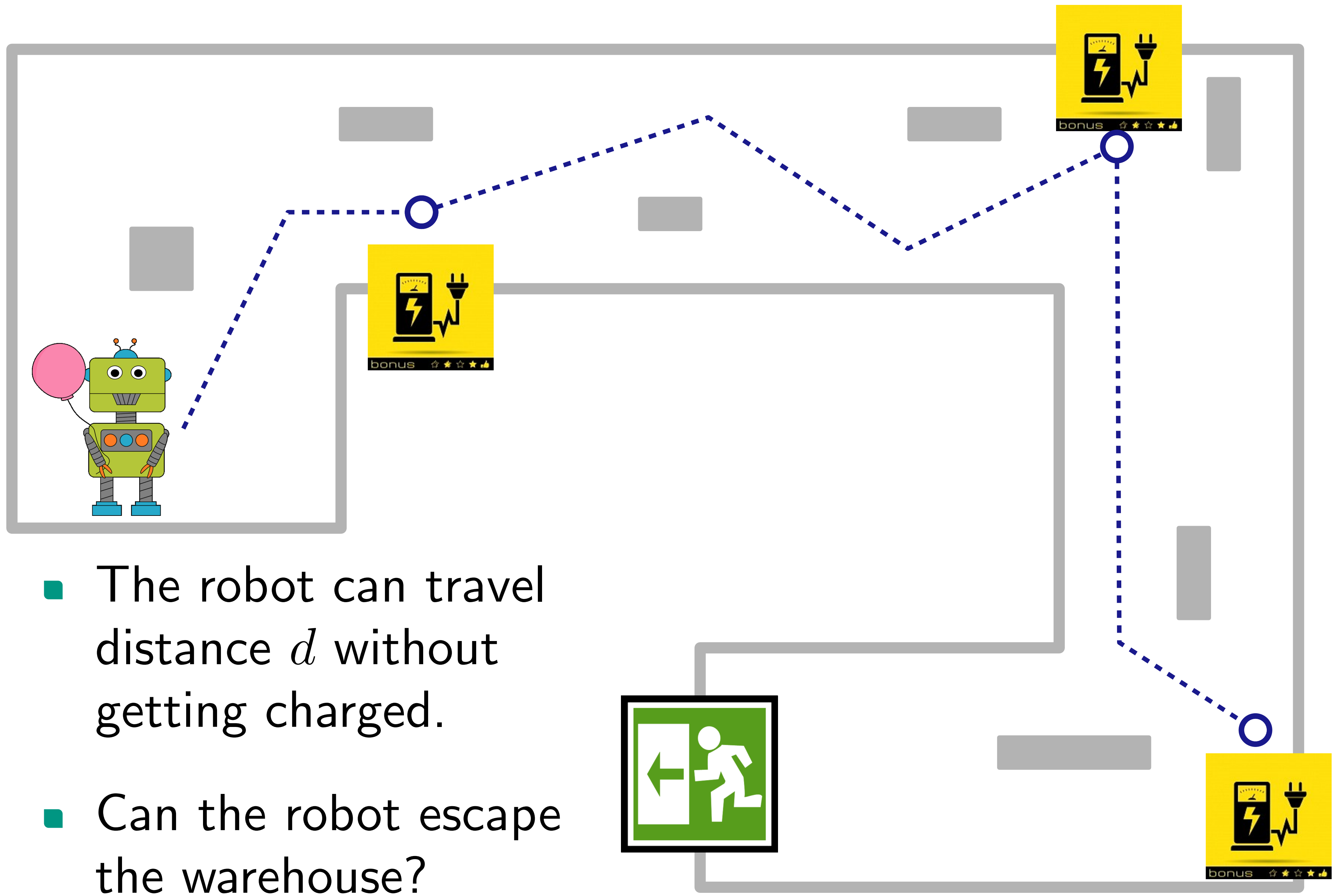
- The robot can travel distance d without getting charged.
- Can the robot escape the warehouse?

Secret Agent's Robot



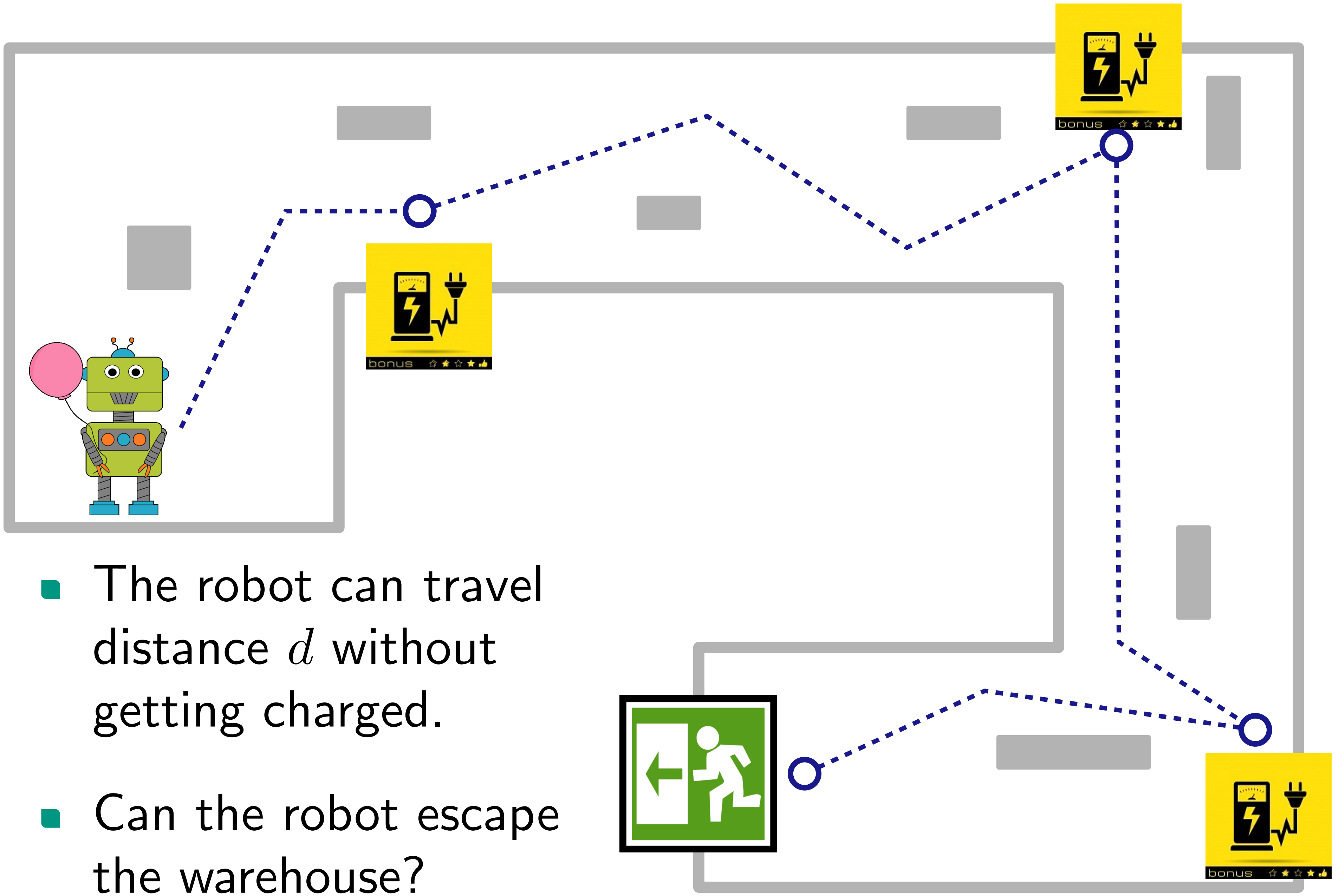
- The robot can travel distance d without getting charged.
- Can the robot escape the warehouse?

Secret Agent's Robot



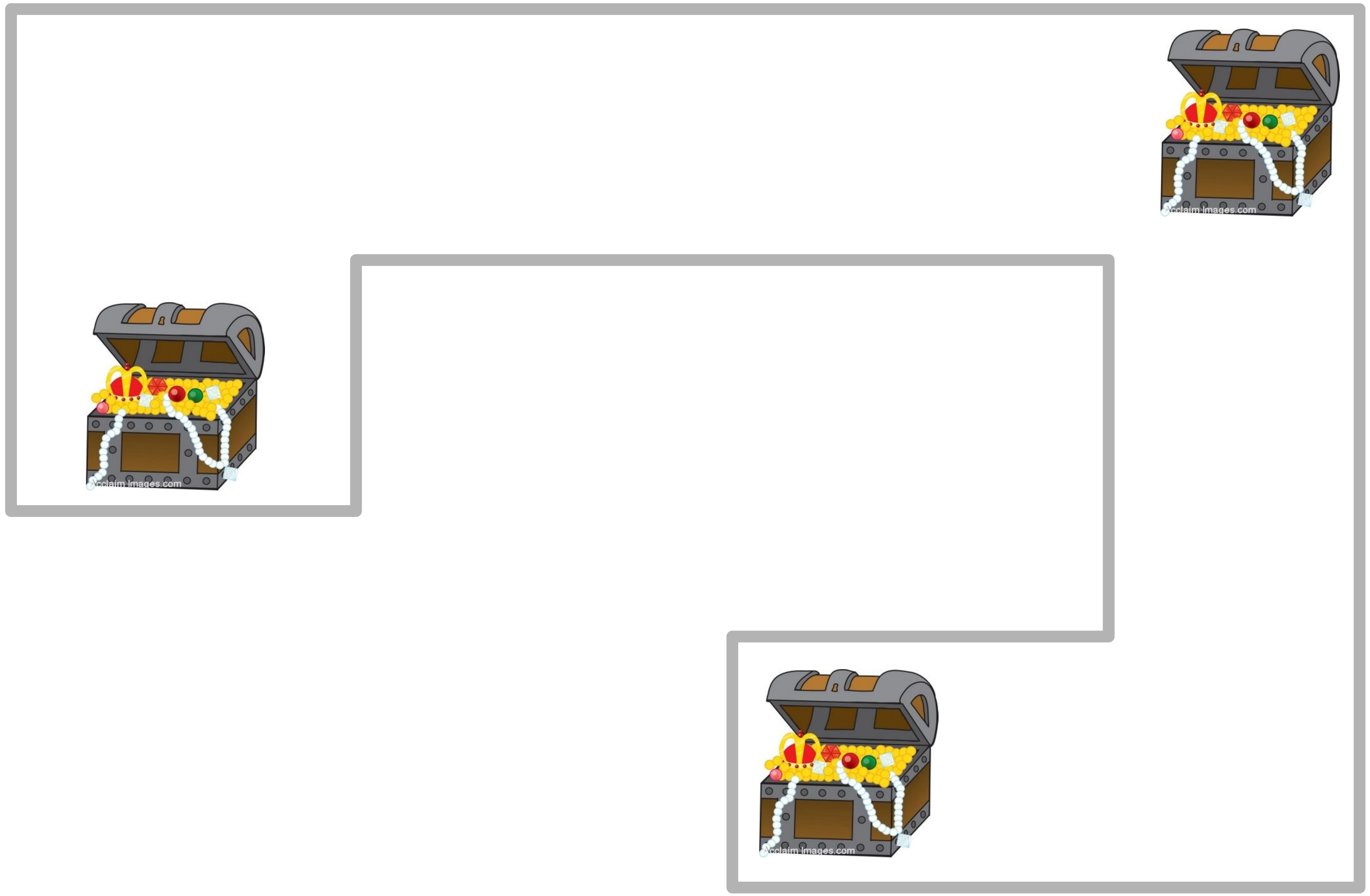
- The robot can travel distance d without getting charged.
- Can the robot escape the warehouse?

Secret Agent's Robot

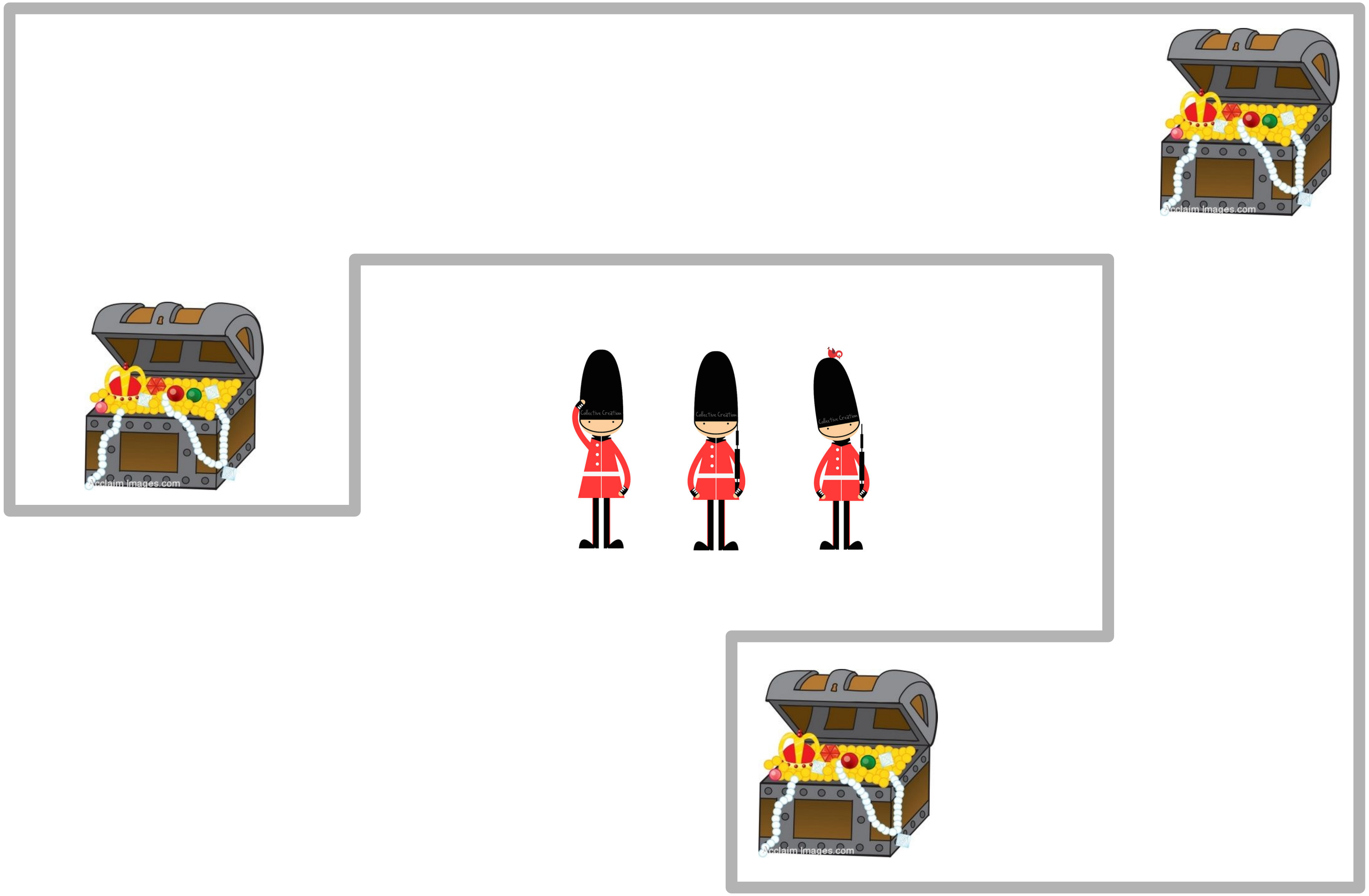


- The robot can travel distance d without getting charged.
- Can the robot escape the warehouse?

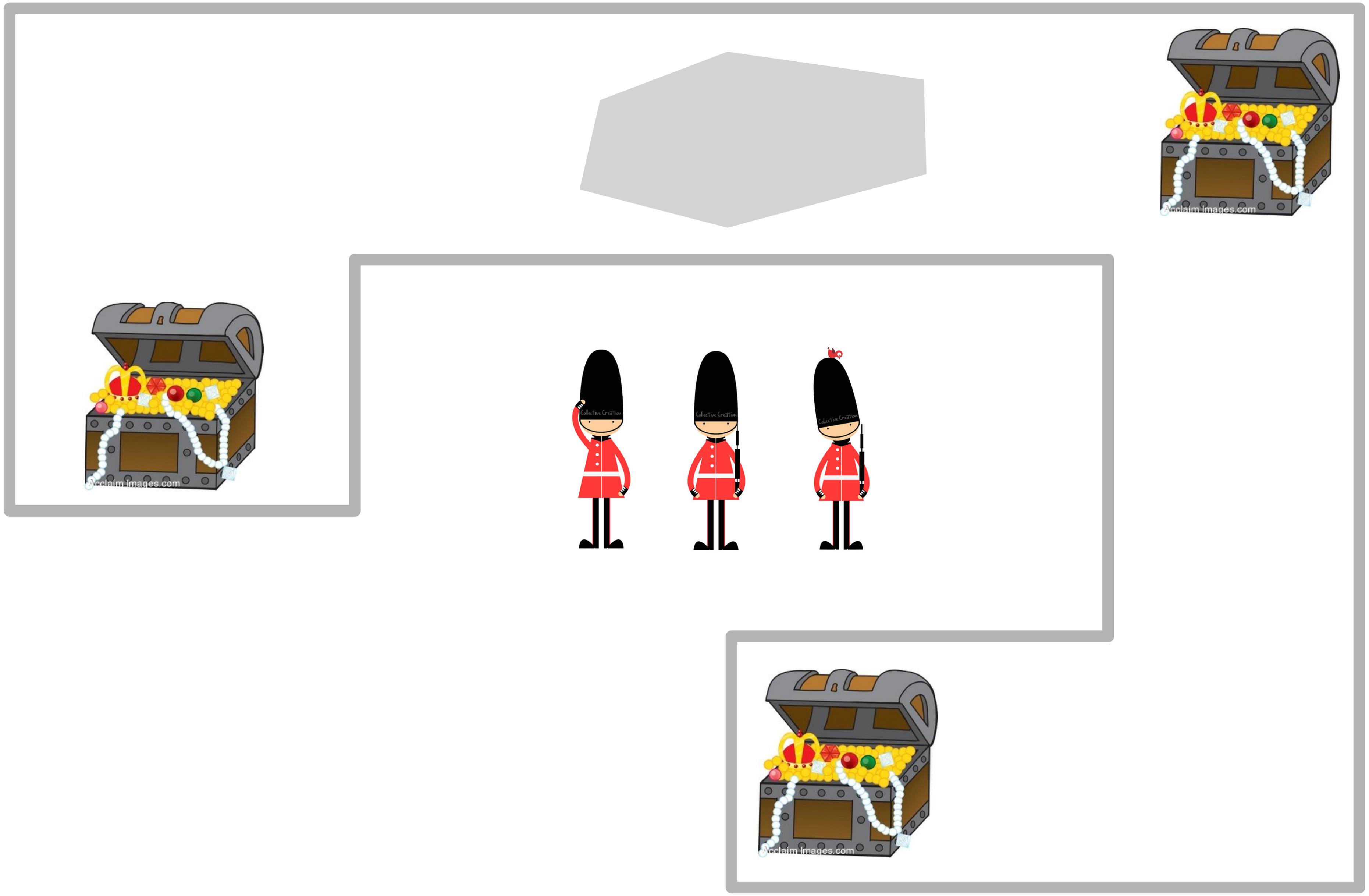
Secret Agent Protects Jewels



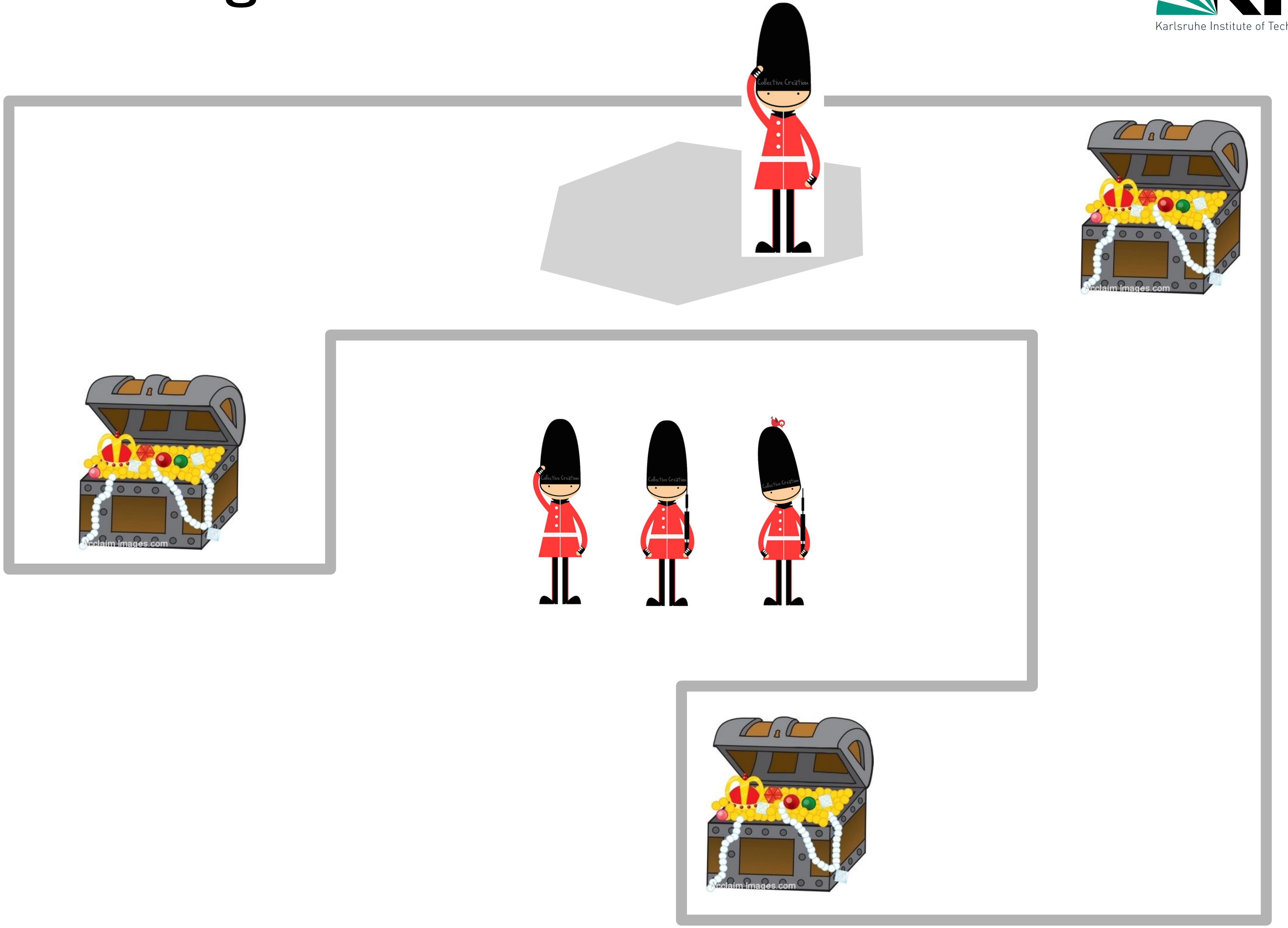
Secret Agent Protects Jewels



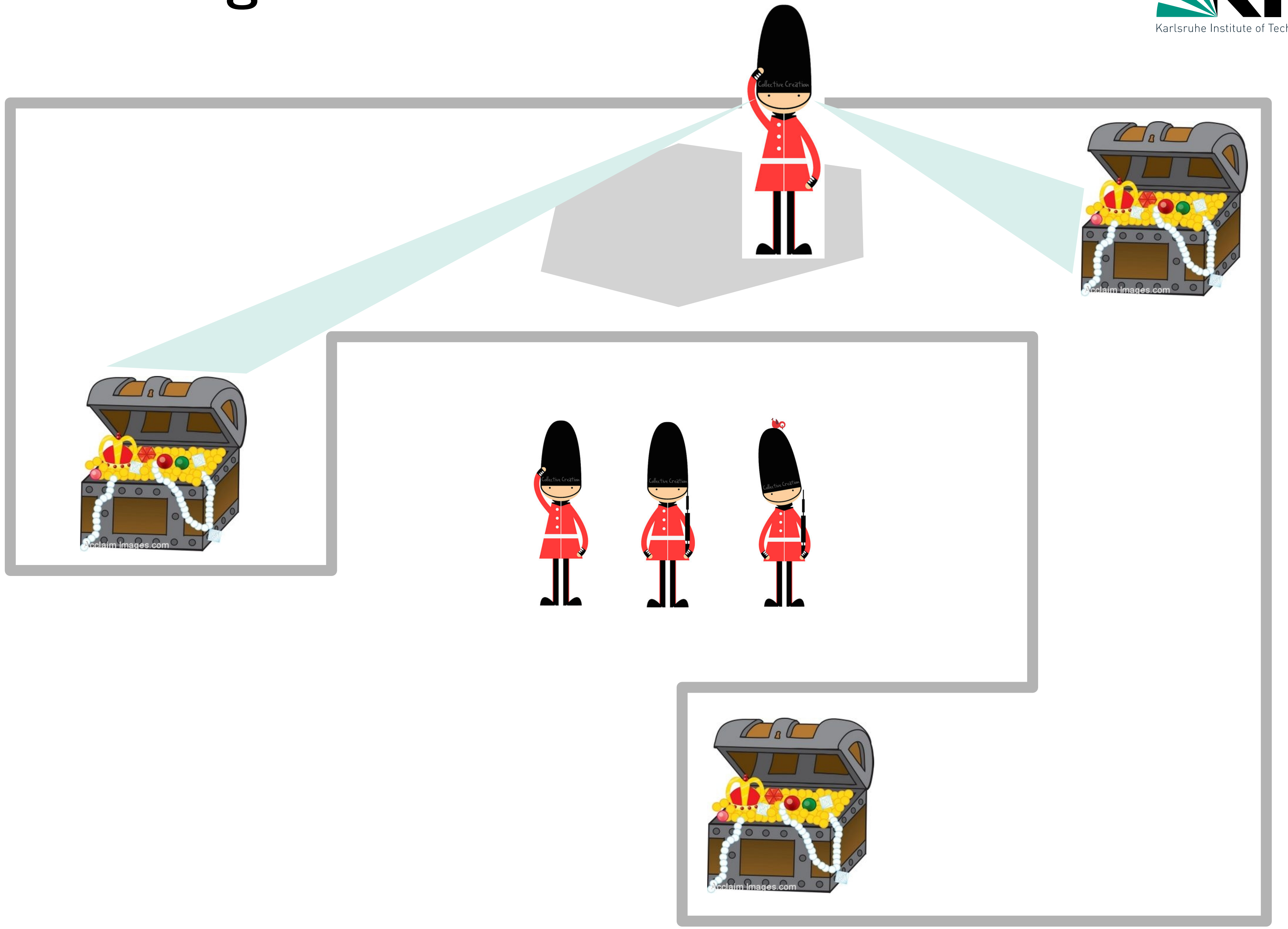
Secret Agent Protects Jewels



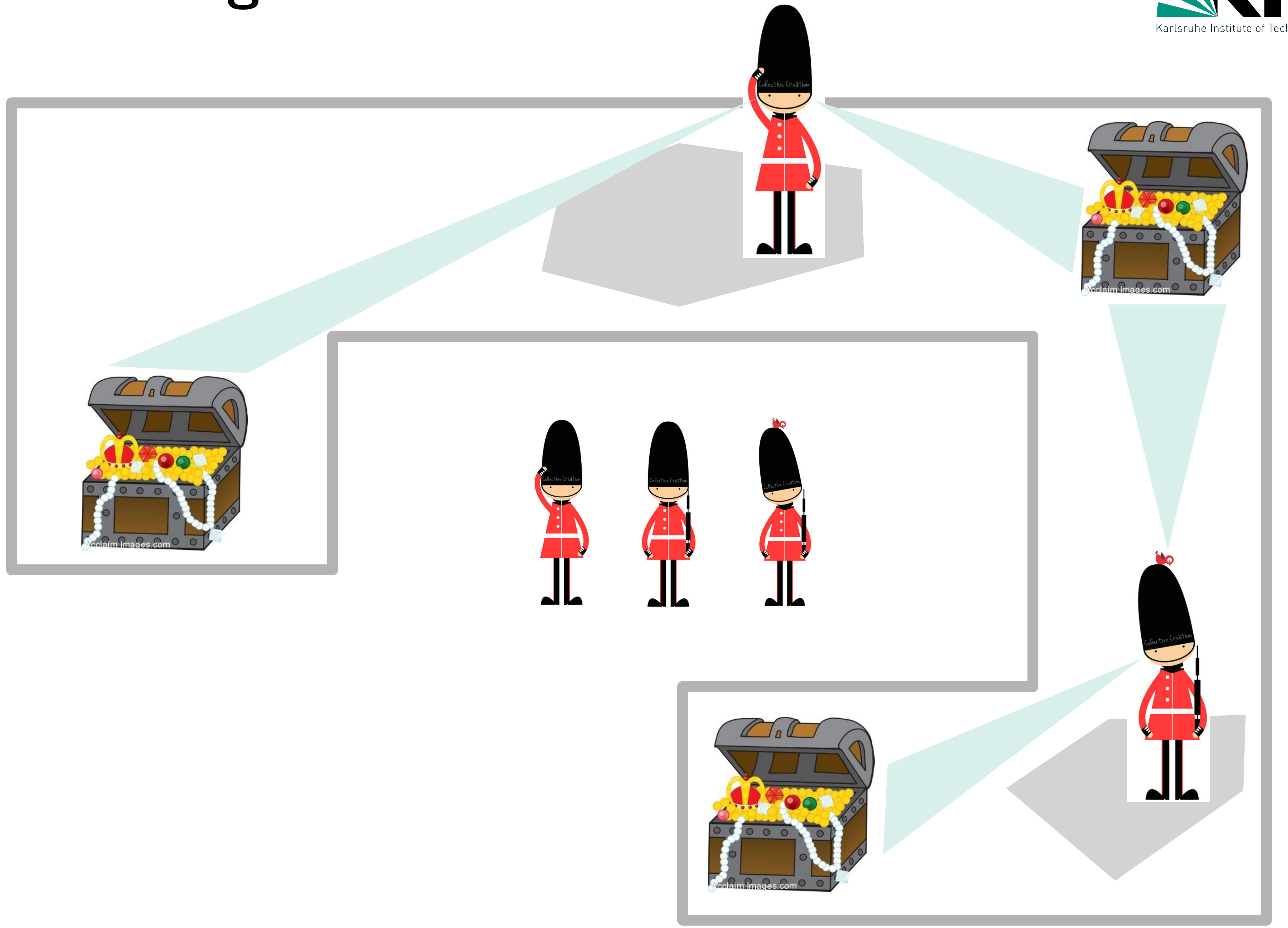
Secret Agent Protects Jewels



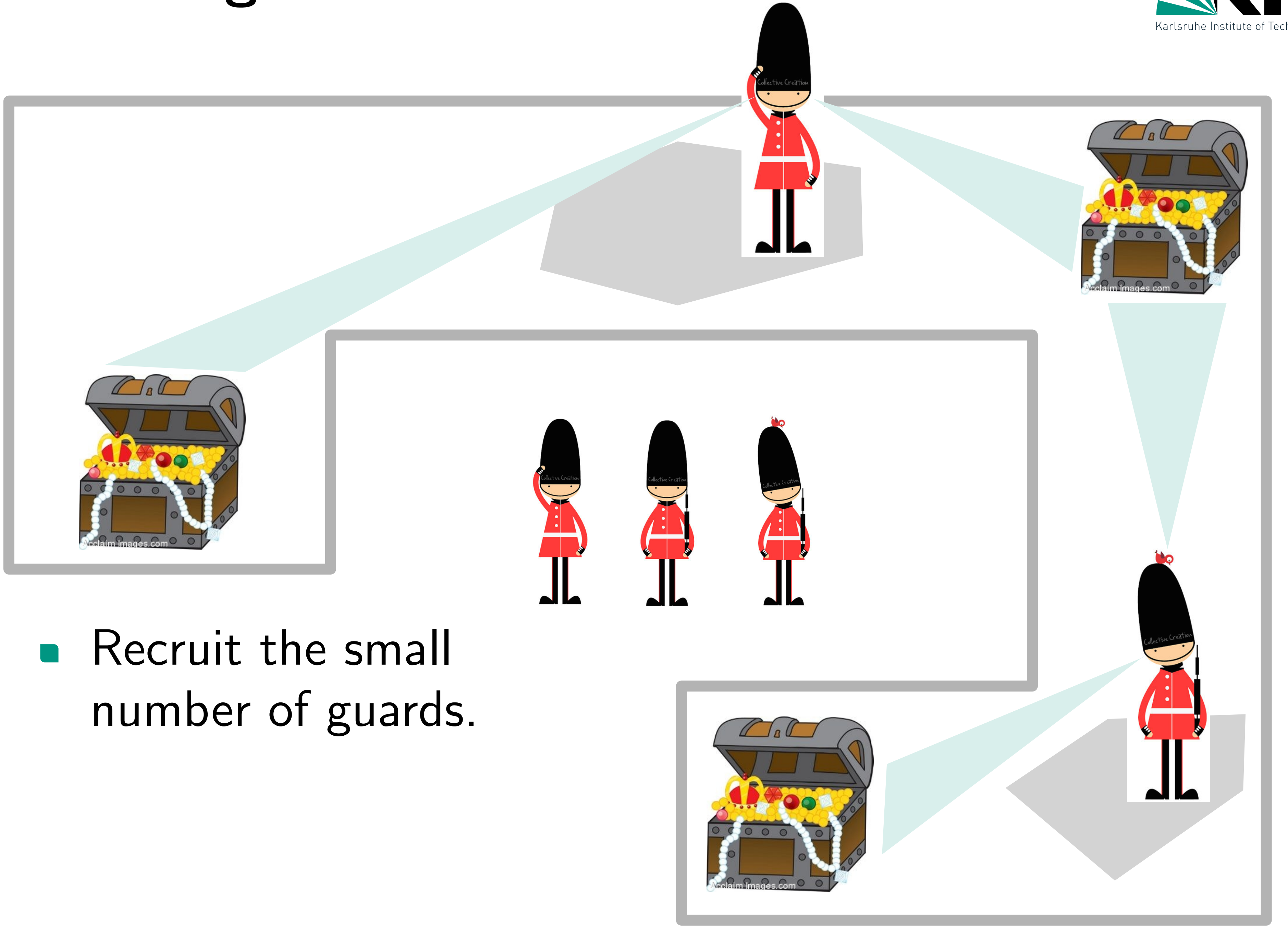
Secret Agent Protects Jewels



Secret Agent Protects Jewels

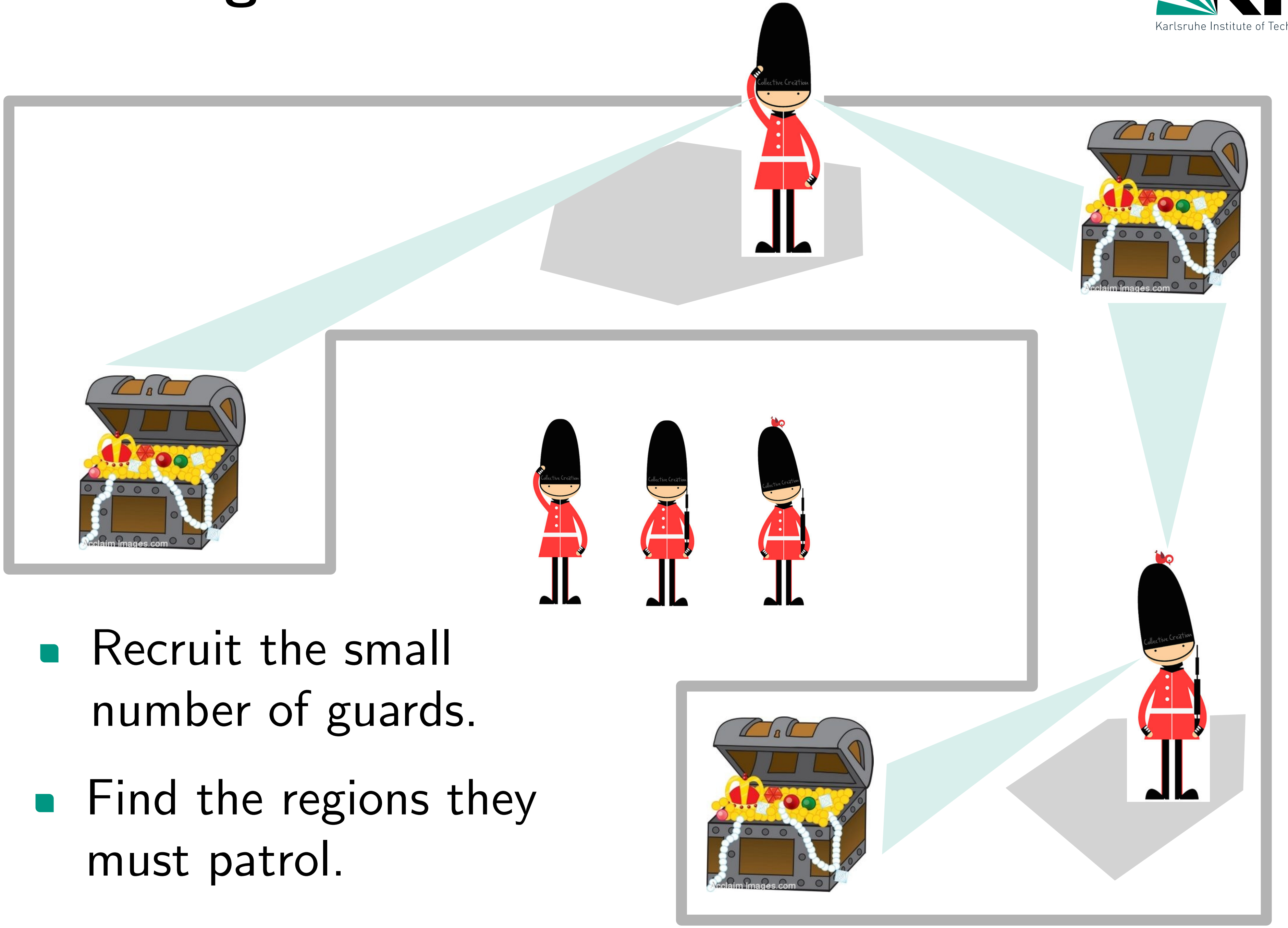


Secret Agent Protects Jewels



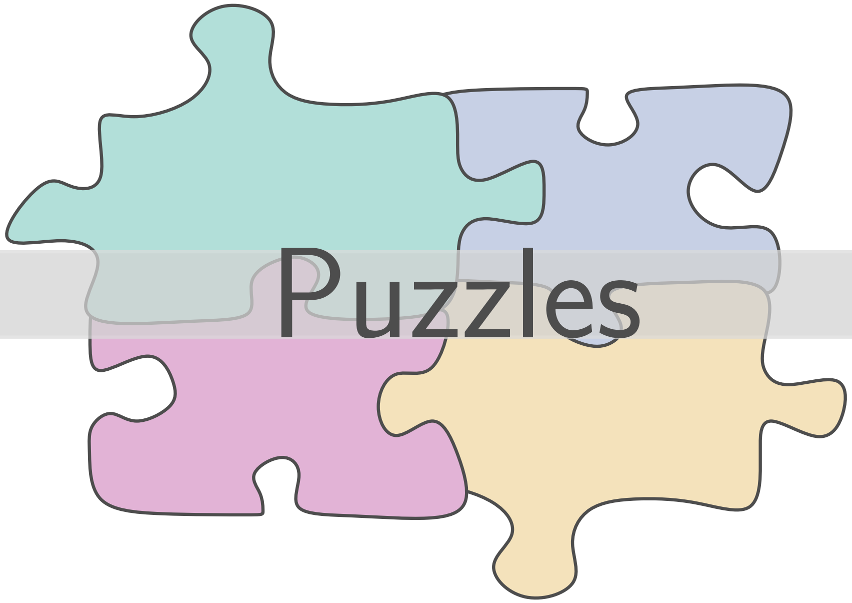
- Recruit the small number of guards.

Secret Agent Protects Jewels



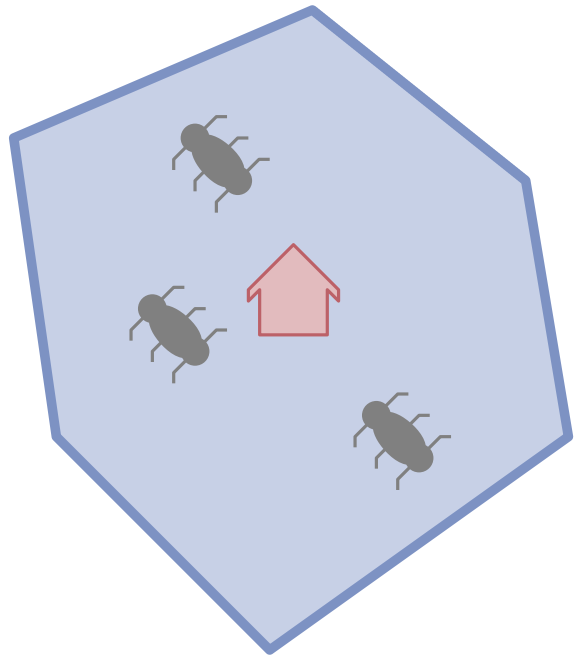
- Recruit the small number of guards.
- Find the regions they must patrol.

Puzzles



Puzzles

Puzzle # 1: Food fit



Given:

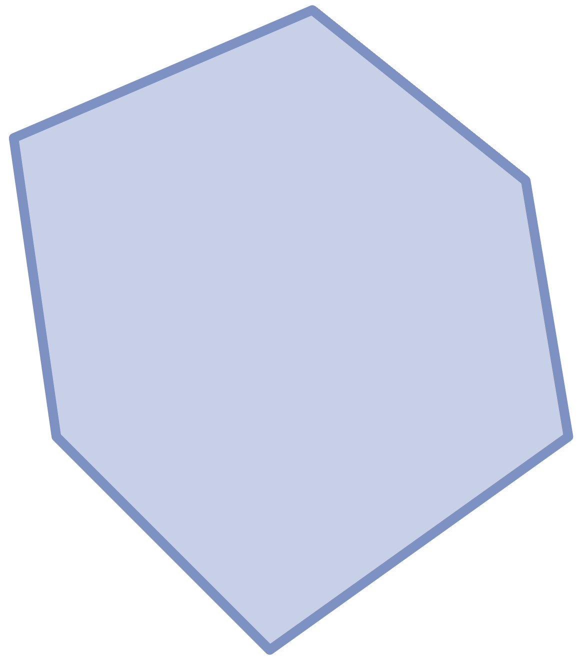
- A region representing an ant colony's home
- A "picnic" containing food items

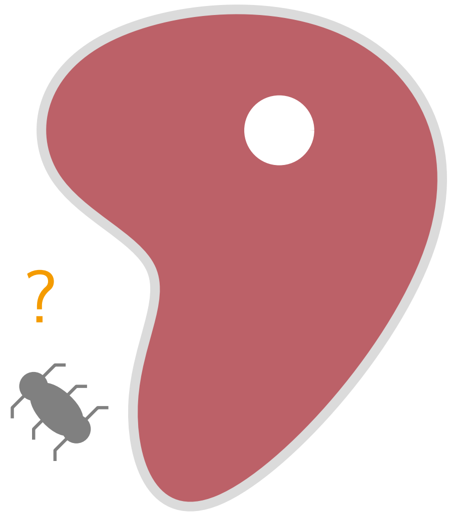
Output:

- Which food items can the ants fit in their home?

Puzzles

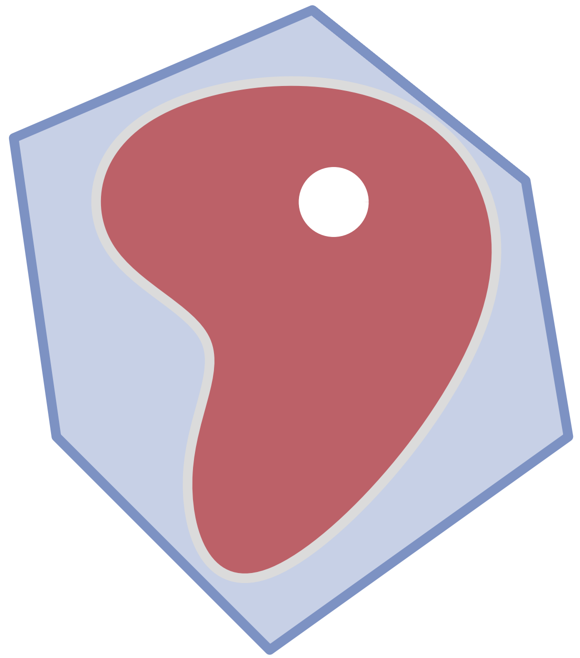
Puzzle # 1: Food fit





Puzzles

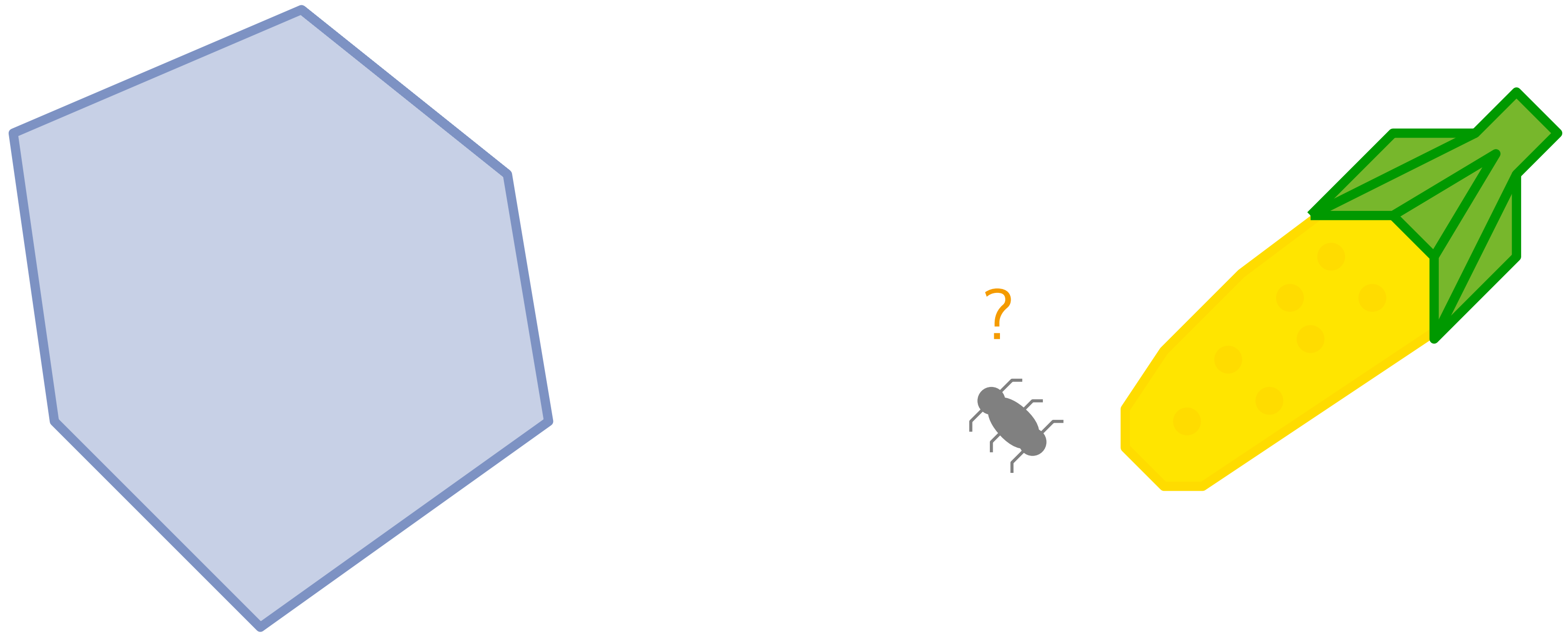
Puzzle # 1: Food fit



Fits!

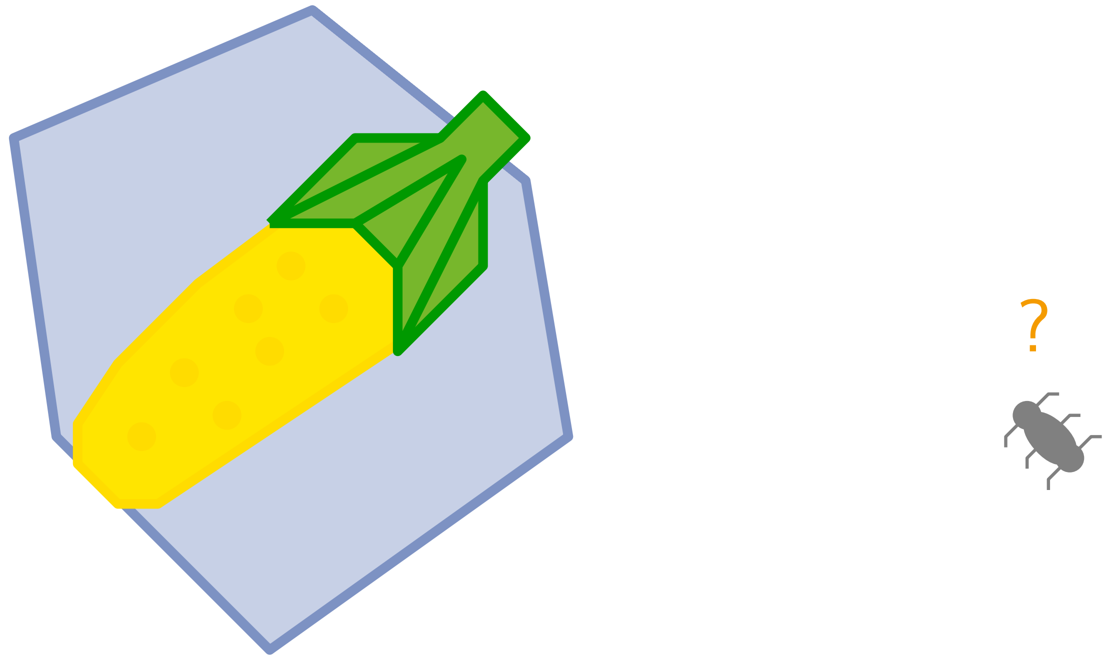
Puzzles

Puzzle # 1: Food fit



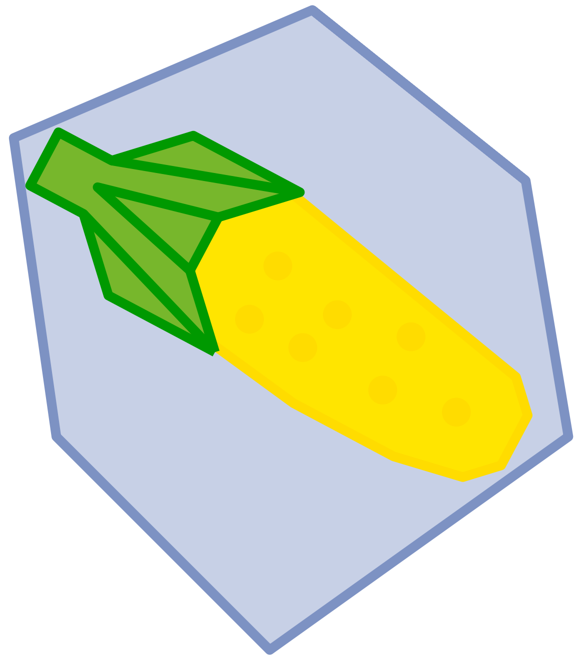
Puzzles

Puzzle # 1: Food fit



Puzzles

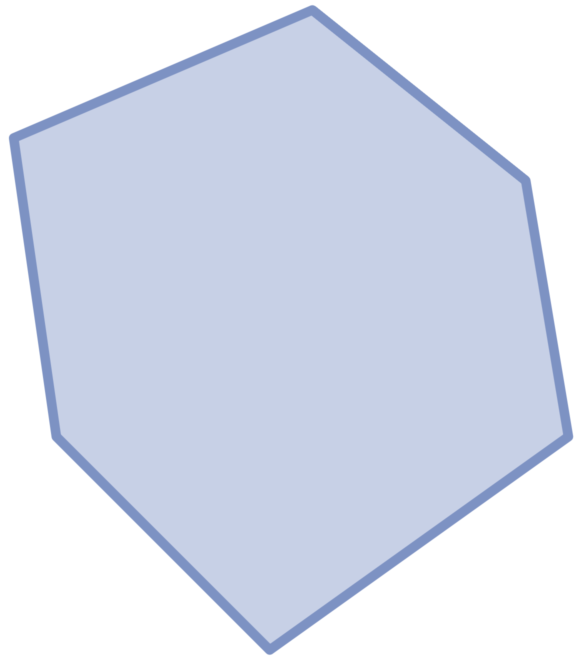
Puzzle # 1: Food fit

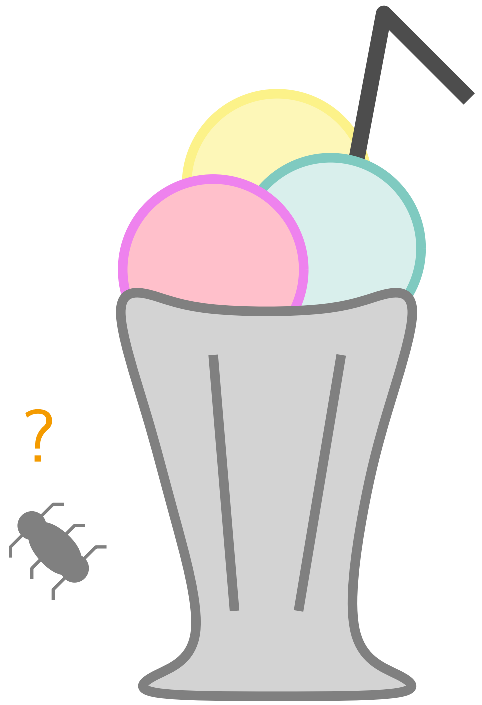


Fits!

Puzzles

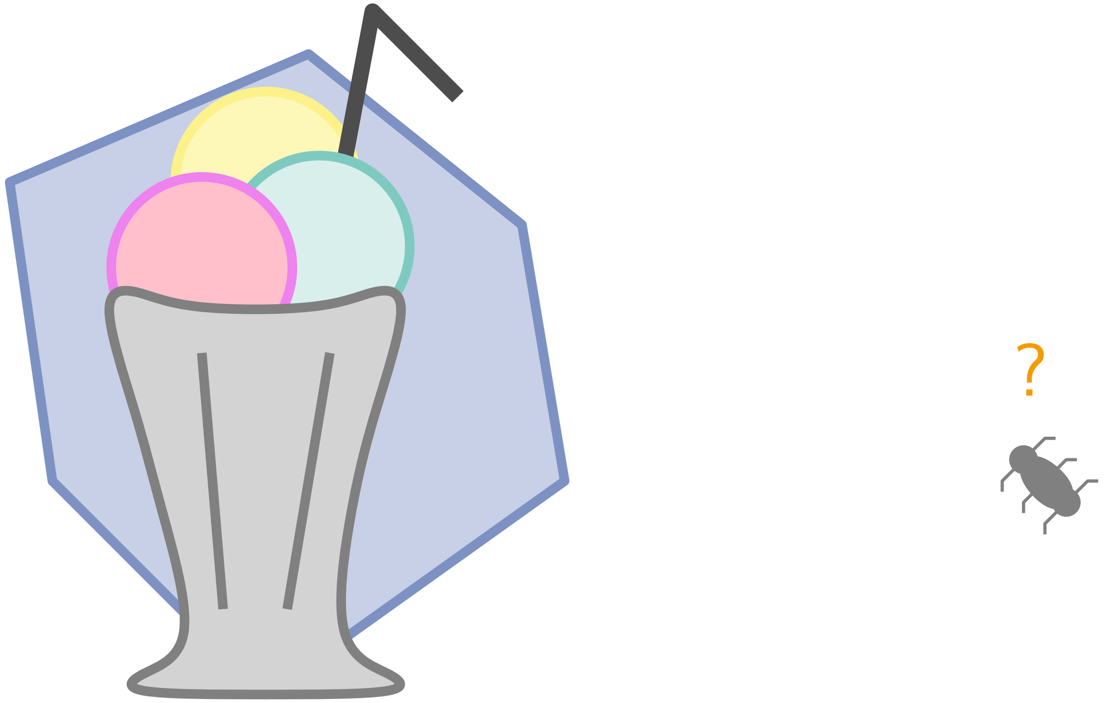
Puzzle # 1: Food fit





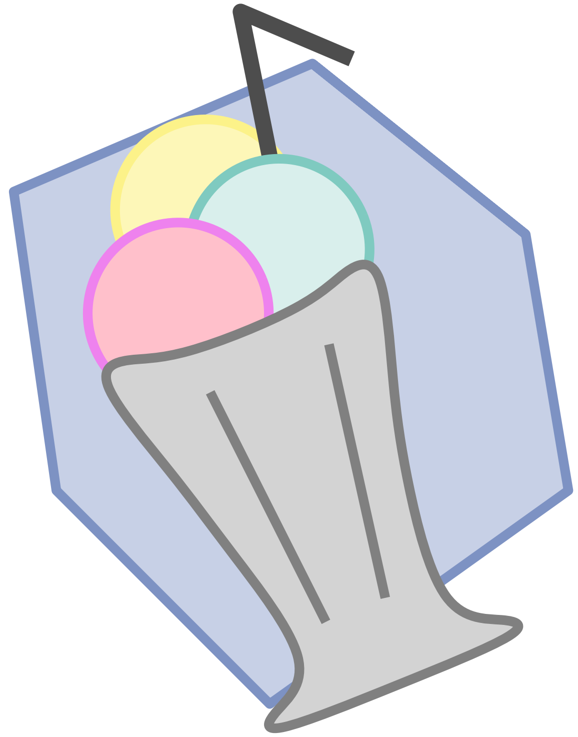
Puzzles

Puzzle # 1: Food fit



Puzzles

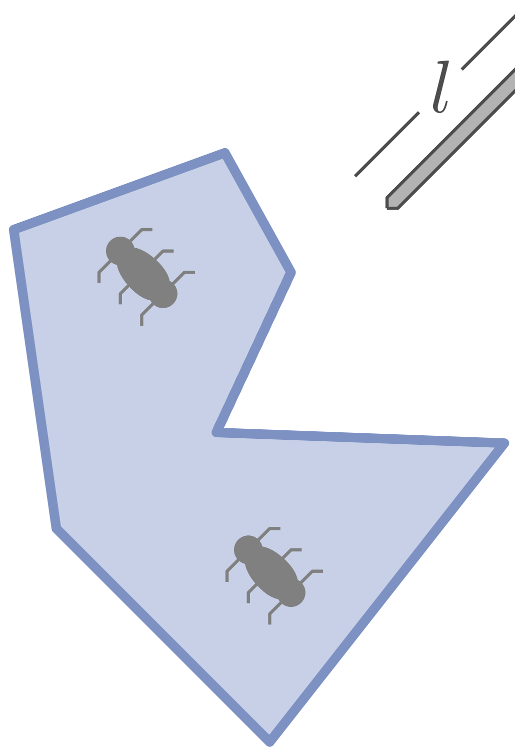
Puzzle # 1: Food fit



Does not fit!

Puzzles

Puzzle # 2: Protect the colony



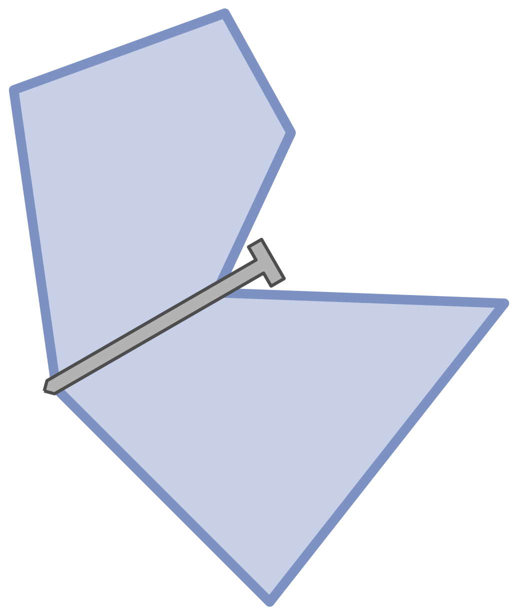
Given:

- A region representing an ant colony's home
- A nail of length l, to be hammered into the home

Output:

- Will the nail break the home apart?
- How can we make "small" changes to protect the home?

Puzzle # 2: Protect the colony



Given:

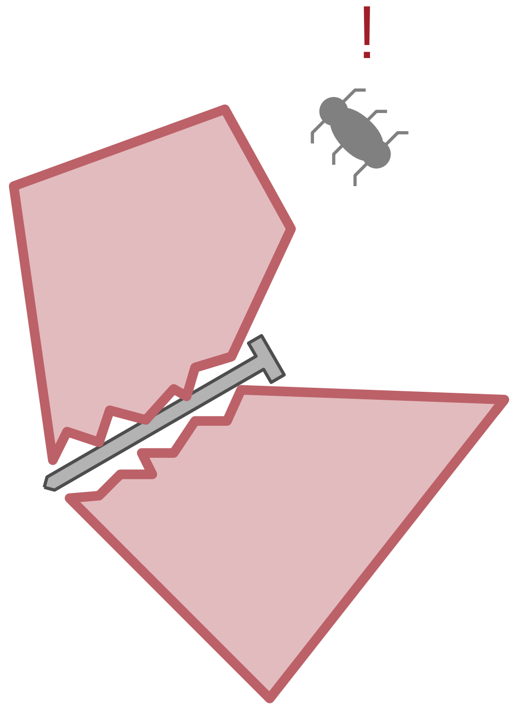
- A region representing an ant colony's home
- A nail of length l, to be hammered into the home

Output:

- Will the nail break the home apart?
- How can we make "small" changes to protect the home?

Puzzles

Puzzle # 2: Protect the colony



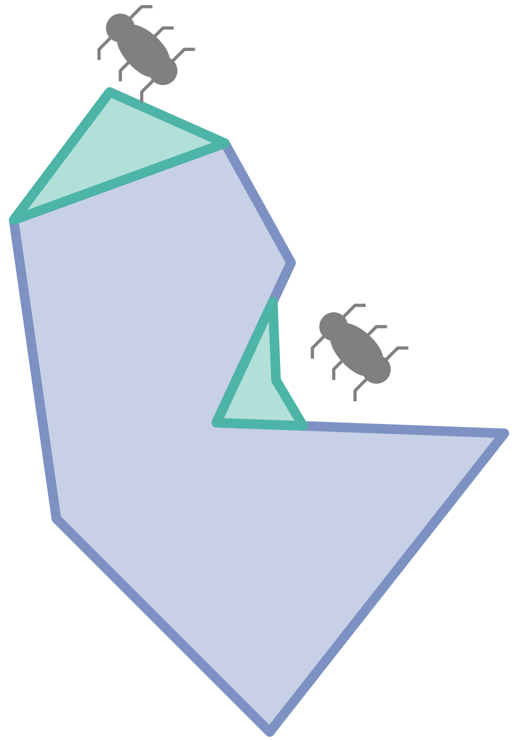
Given:

- A region representing an ant colony's home
- A nail of length l, to be hammered into the home

Output:

- Will the nail break the home apart?
- How can we make "small" changes to protect the home?

Puzzle # 2: Protect the colony



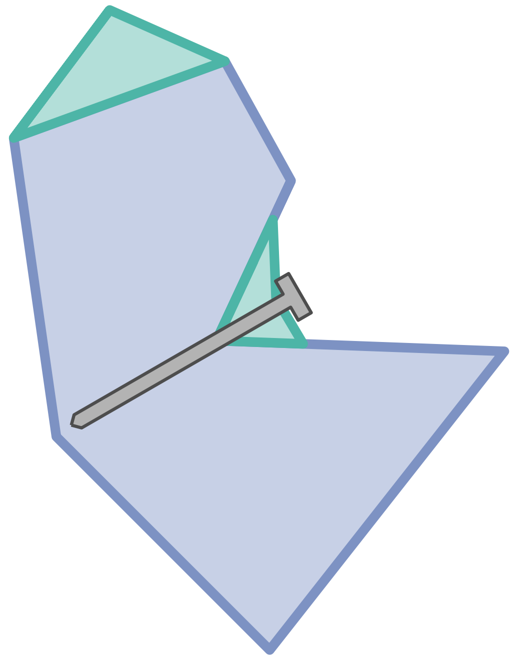
Given:

- A region representing an ant colony's home
- A nail of length l, to be hammered into the home

Output:

- Will the nail break the home apart?
- How can we make "small" changes to protect the home?

Puzzle # 2: Protect the colony



Given:

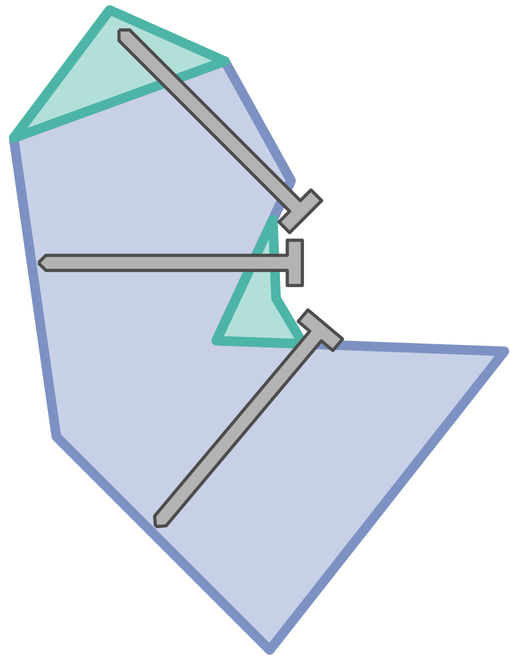
- A region representing an ant colony's home
- A nail of length l, to be hammered into the home

Output:

- Will the nail break the home apart?
- How can we make "small" changes to protect the home?

Puzzles

Puzzle # 2: Protect the colony



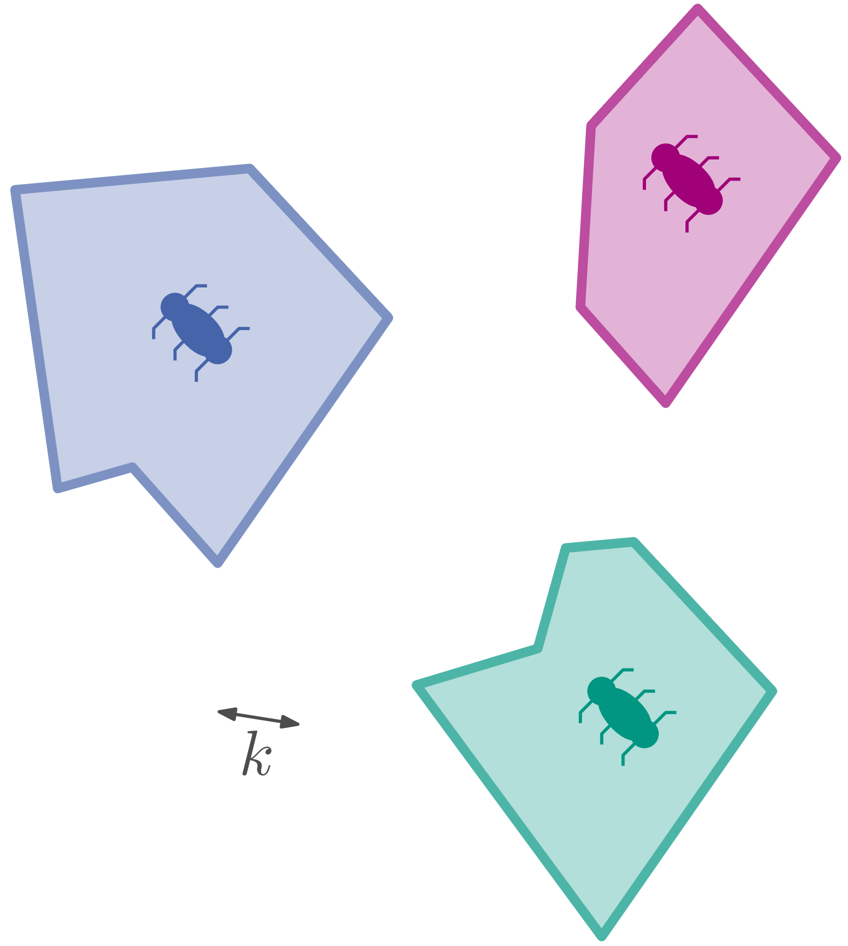
Given:

- A region representing an ant colony's home
- A nail of length l, to be hammered into the home

Output:

- Will the nail break the home apart?
- How can we make "small" changes to protect the home?

Puzzle # 3: Keep away



Given:

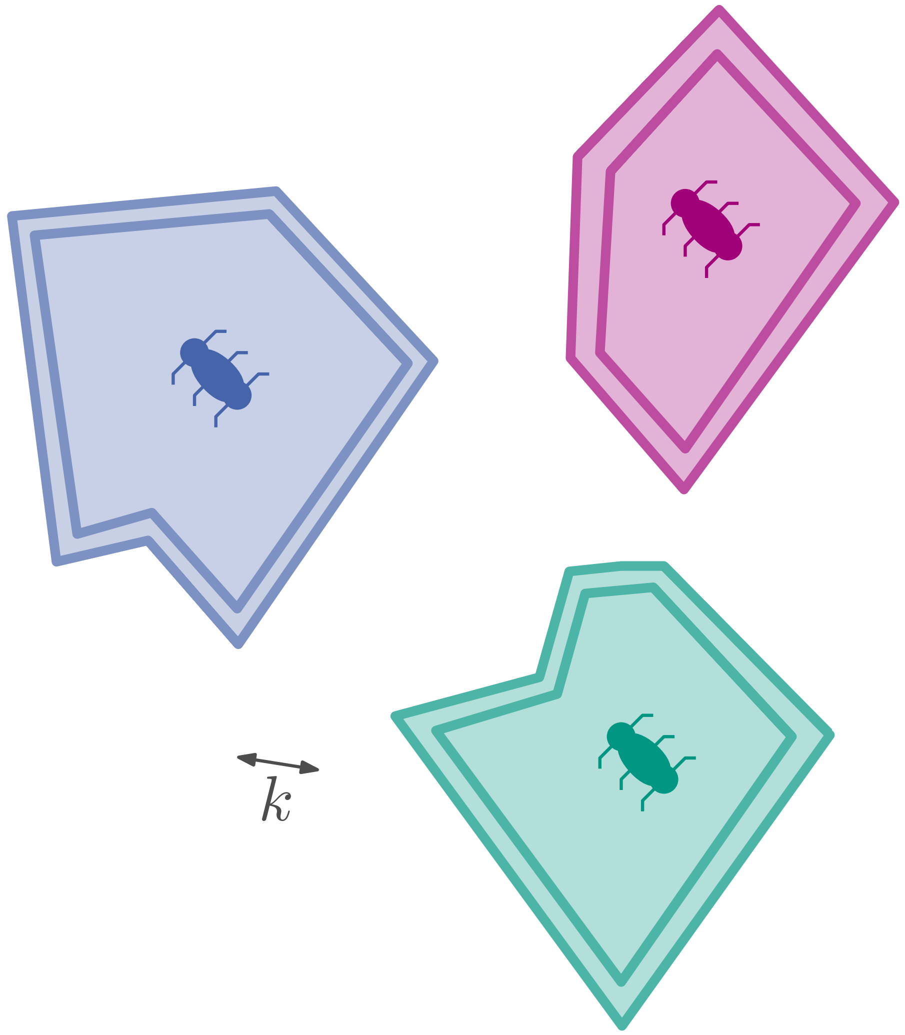
- A collection of ant colonies that grow over time
- A keep-away distance k

Output:

- The state of the colonies after t time steps

Puzzles

Puzzle # 3: Keep away



Given:

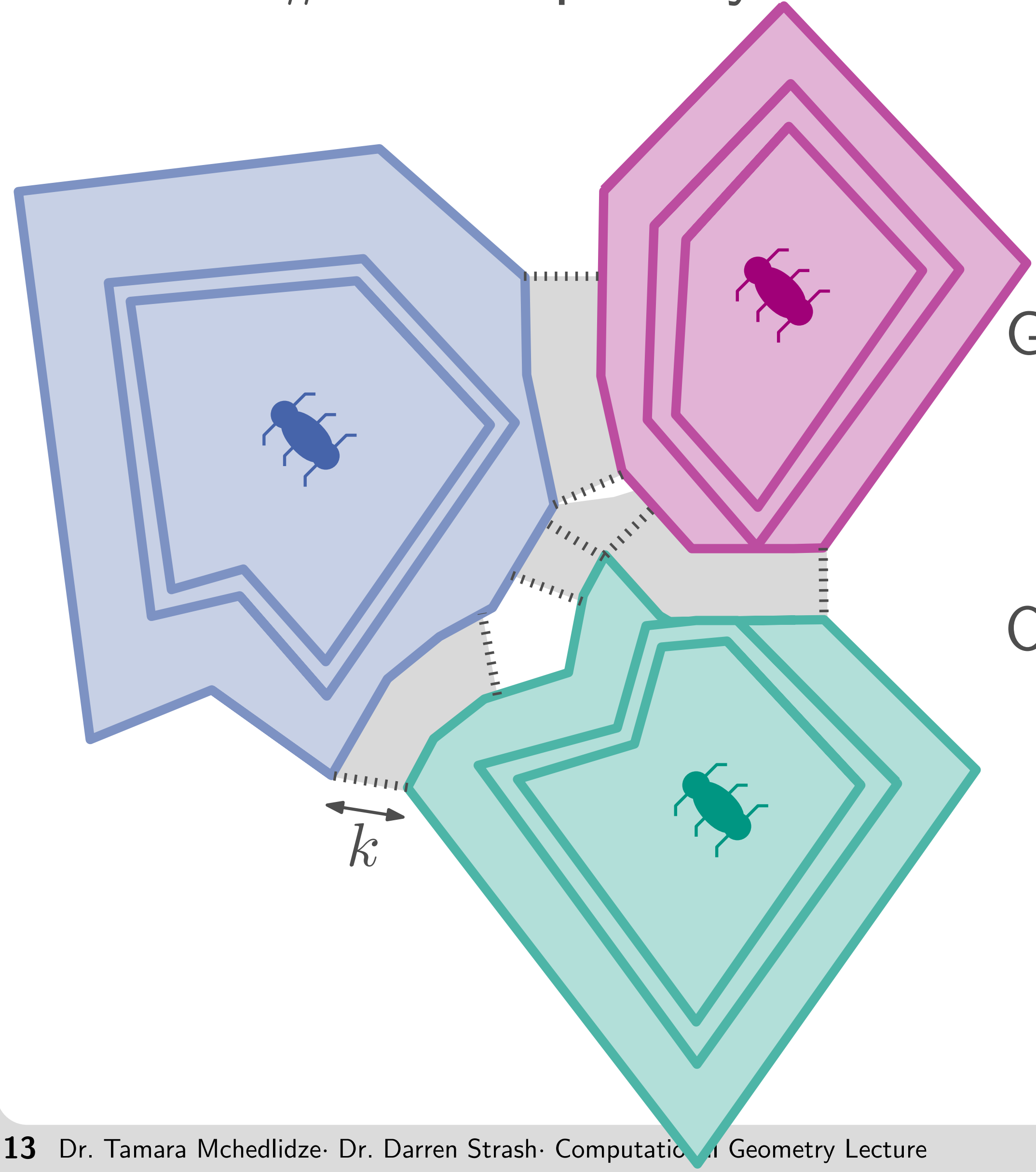
- A collection of ant colonies that grow over time
- A keep-away distance k

Output:

- The state of the colonies after t time steps

Puzzles

Puzzle # 3: Keep away



Given:

- A collection of ant colonies that grow over time
- A keep-away distance k

Output:

- The state of the colonies after t time steps

Project Selection

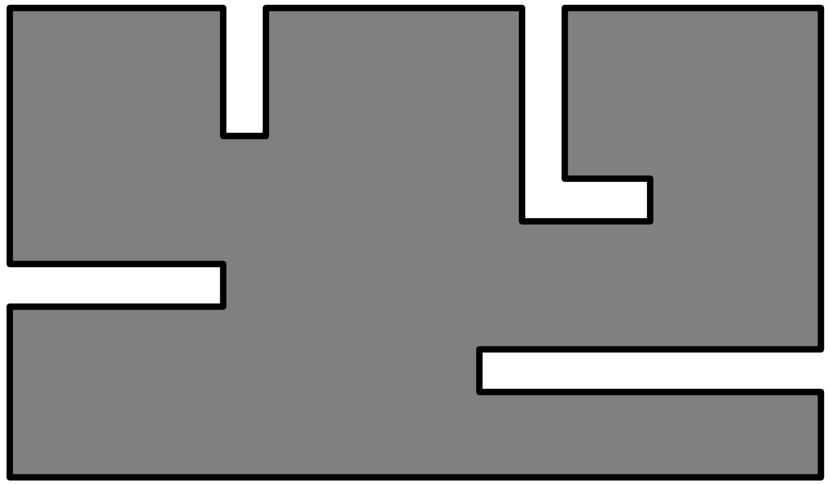
Projects

1. **Pacman:** Collision detection
2. **Pacman:** Ghosts activate on sight
3. **Pacman:** Ghosts activate on smell
4. **Secret Agent:** Shoot the laser
5. **Secret Agent:** Save the robot
6. **Secret Agent:** Guard the jewels
7. **Puzzle:** Food fit
8. **Puzzle:** Protect the colony
9. **Puzzle:** Keep away

Polygon Triangulation

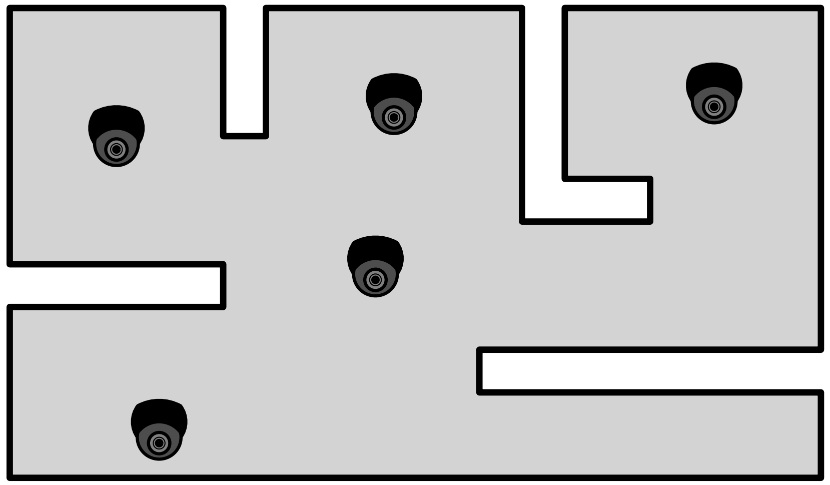
The Art-Gallery-Problem

Task: Install a number of cameras in an art gallery so that every part of the galery is visible to at least one of them.



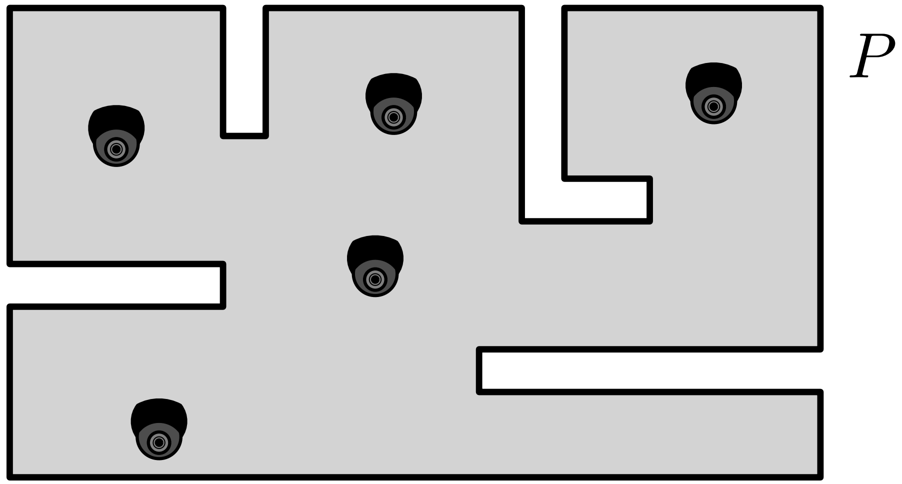
The Art-Gallery-Problem

Task: Install a number of cameras in an art gallery so that every part of the galery is visible to at least one of them.



The Art-Gallery-Problem

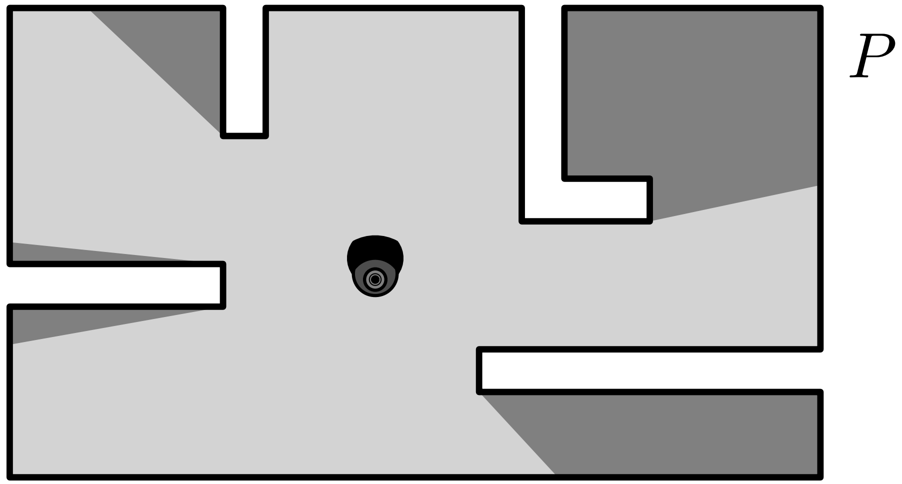
Task: Install a number of cameras in an art gallery so that every part of the galery is visible to at least one of them.



Assumption: Art gallery is a *simple* polygon P with n corners (no self-intersections, no holes)

The Art-Gallery-Problem

Task: Install a number of cameras in an art gallery so that every part of the galery is visible to at least one of them.

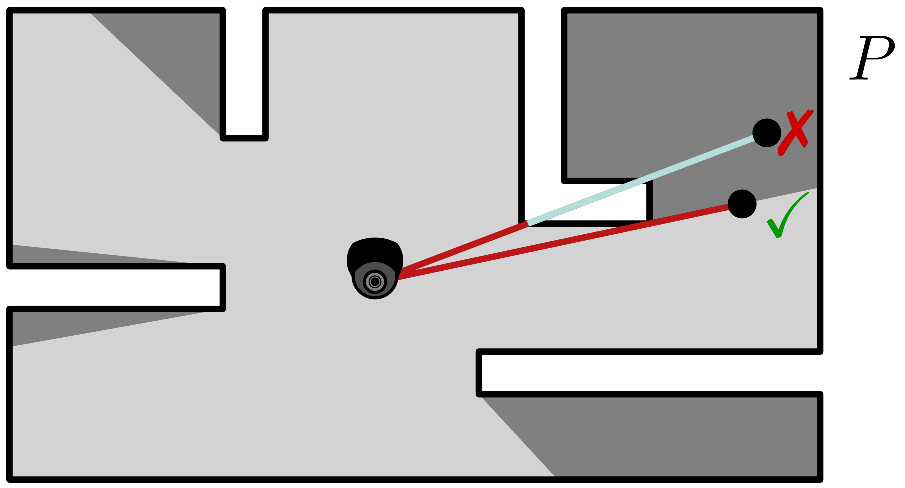


Assumption: Art gallery is a *simple* polygon P with n corners (no self-intersections, no holes)

Observation: each camera observes a star-shaped region

The Art-Gallery-Problem

Task: Install a number of cameras in an art gallery so that every part of the galery is visible to at least one of them.



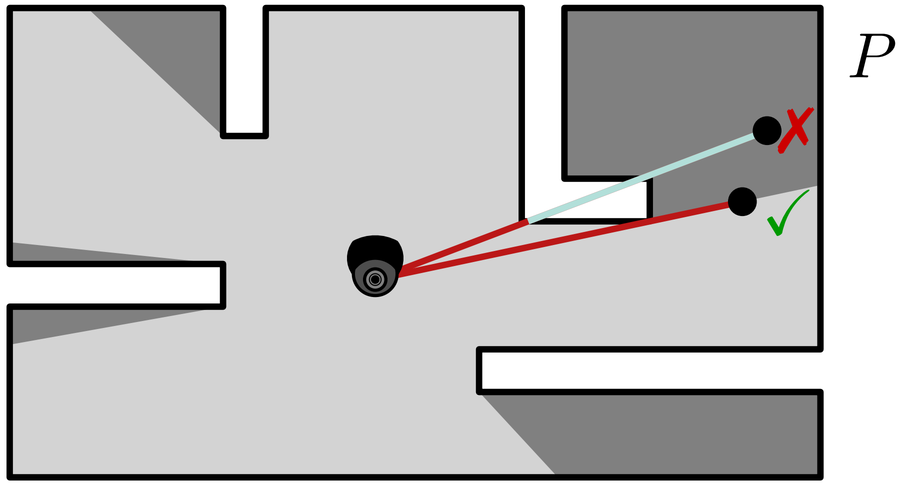
Assumption: Art gallery is a *simple* polygon P with n corners (no self-intersections, no holes)

Observation: each camera observes a star-shaped region

Definition: Point $p \in P$ is *visible* from $c \in P$ if $\overline{cp} \in P$

The Art-Gallery-Problem

Task: Install a number of cameras in an art gallery so that every part of the galery is visible to at least one of them.



Assumption: Art gallery is a *simple* polygon P with n corners (no self-intersections, no holes)

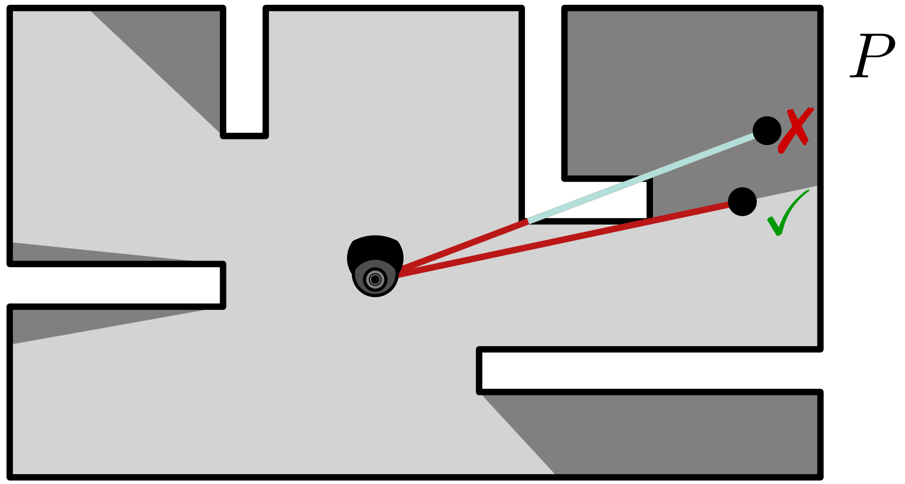
Observation: each camera observes a star-shaped region

Definition: Point $p \in P$ is *visible* from $c \in P$ if $\overline{cp} \in P$

Goal: Use as few cameras as possible!

The Art-Gallery-Problem

Task: Install a number of cameras in an art gallery so that every part of the galery is visible to at least one of them.



Assumption: Art gallery is a *simple* polygon P with n corners (no self-intersections, no holes)

Observation: each camera observes a star-shaped region

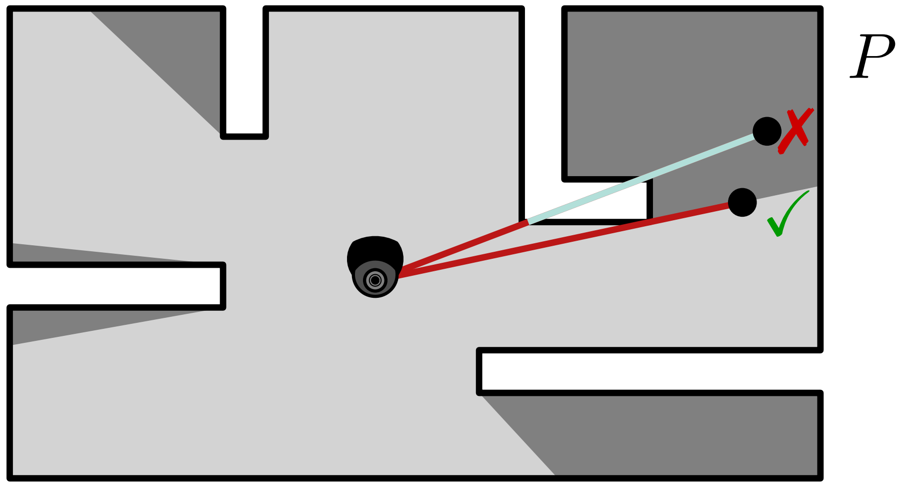
Definition: Point $p \in P$ is *visible* from $c \in P$ if $\overline{cp} \in P$

Goal: Use as few cameras as possible!

$\rightarrow$ The number depends on the number of corners n and on the shape of P

The Art-Gallery-Problem

Task: Install a number of cameras in an art gallery so that every part of the galery is visible to at least one of them.



Assumption: Art gallery is a *simple* polygon P with n corners (no self-intersections, no holes)

Observation: each camera observes a star-shaped region

Definition: Point $p \in P$ is *visible* from $c \in P$ if $\overline{cp} \in P$

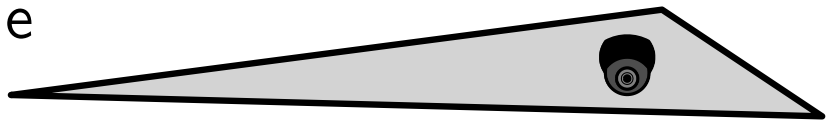
Goal: Use as few cameras as possible!

NP-hard!

$\rightarrow$ The number depends on the number of corners n and on the shape of P

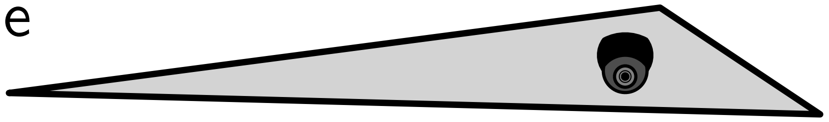
Problem Simplification

Observation: It is easy to guard a triangle

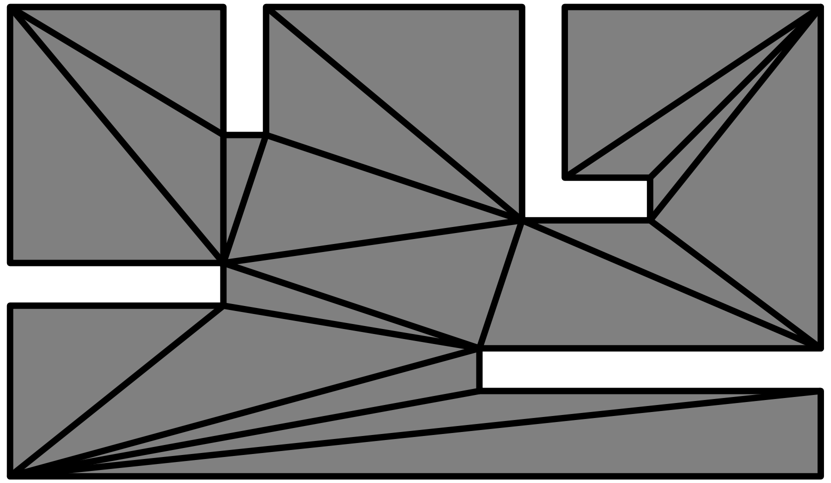


Problem Simplification

Observation: It is easy to guard a triangle

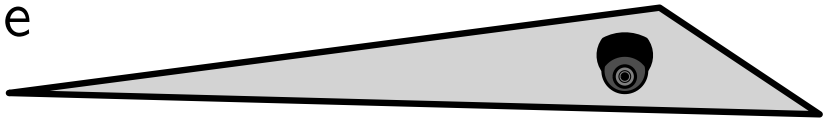


Idea: Decompose P into triangles and guard each of them

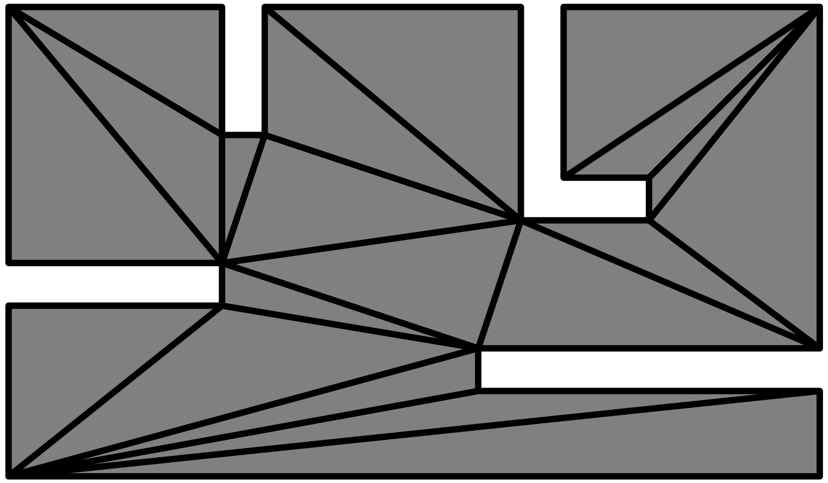


Problem Simplification

Observation: It is easy to guard a triangle



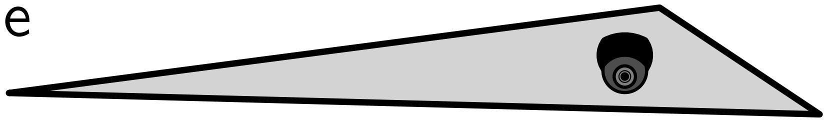
Idea: Decompose P into triangles and guard each of them



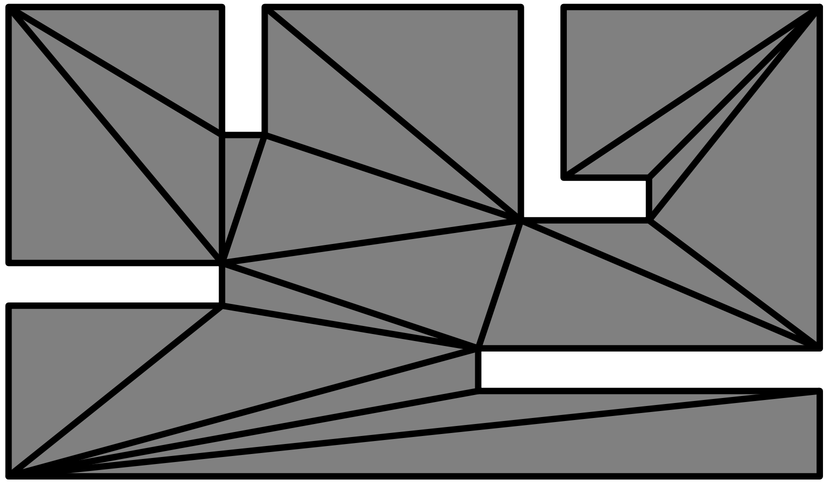
Theorem 1: Each simple polygon with n corners admits a triangulation; any such triangulation contains exactly $n-2$ triangles.

Problem Simplification

Observation: It is easy to guard a triangle



Idea: Decompose P into triangles and guard each of them

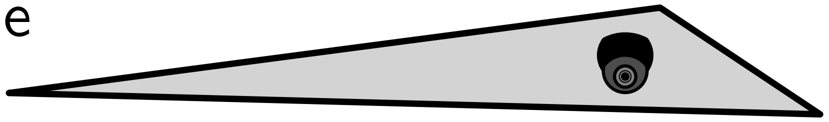


Theorem 1: Each simple polygon with n corners admits a triangulation; any such triangulation contains exactly $n - 2$ triangles.

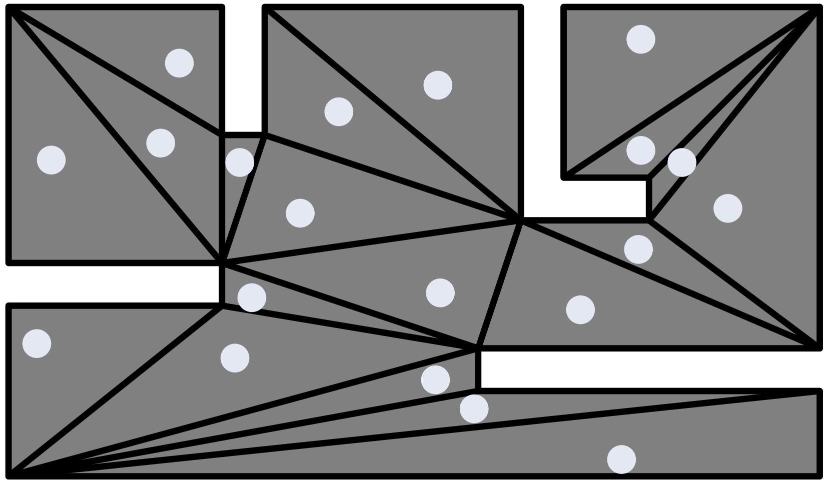
The proof implies a recursive $O(n^2)$-Algorithm!

Problem Simplification

Observation: It is easy to guard a triangle



Idea: Decompose P into triangles and guard each of them

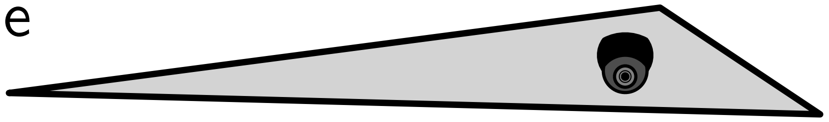


Theorem 1: Each simple polygon with n corners admits a triangulation; any such triangulation contains exactly $n-2$ triangles.

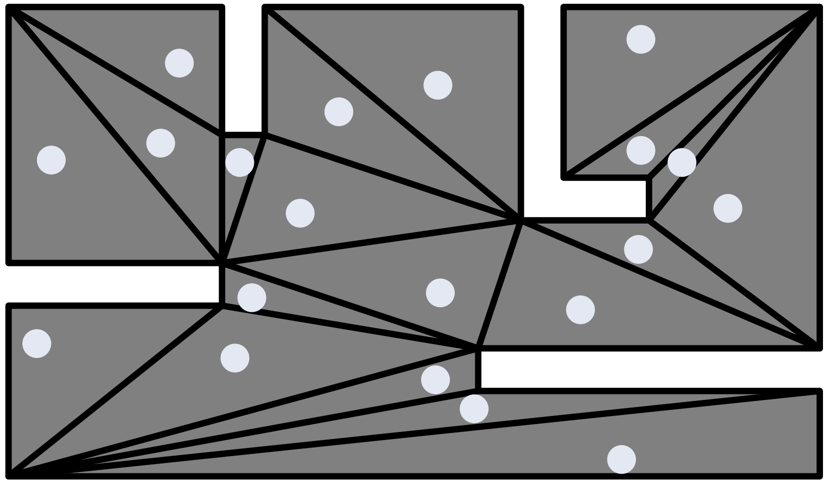
- P could be guarded by $n-2$ cameras placed in the triangles

Problem Simplification

Observation: It is easy to guard a triangle



Idea: Decompose P into triangles and guard each of them



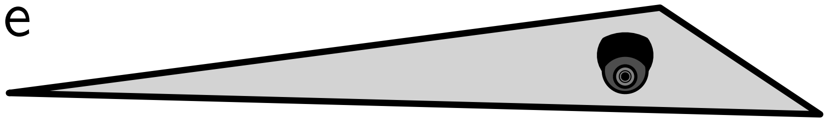
Theorem 1: Each simple polygon with n corners admits a triangulation; any such triangulation contains exactly $n-2$ triangles.

- P could be guarded by $n-2$ cameras placed in the triangles

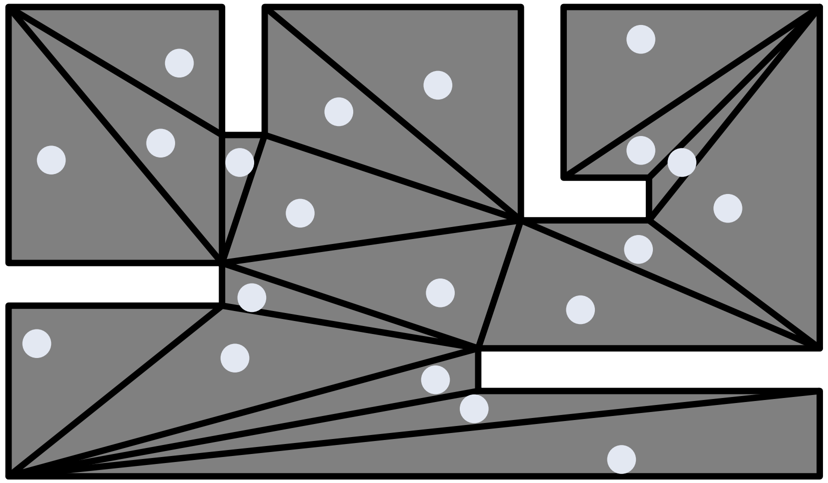
Can we do better?

Problem Simplification

Observation: It is easy to guard a triangle



Idea: Decompose P into triangles and guard each of them

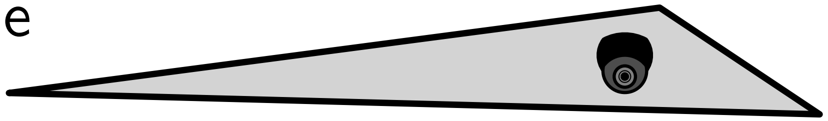


Theorem 1: Each simple polygon with n corners admits a triangulation; any such triangulation contains exactly $n-2$ triangles.

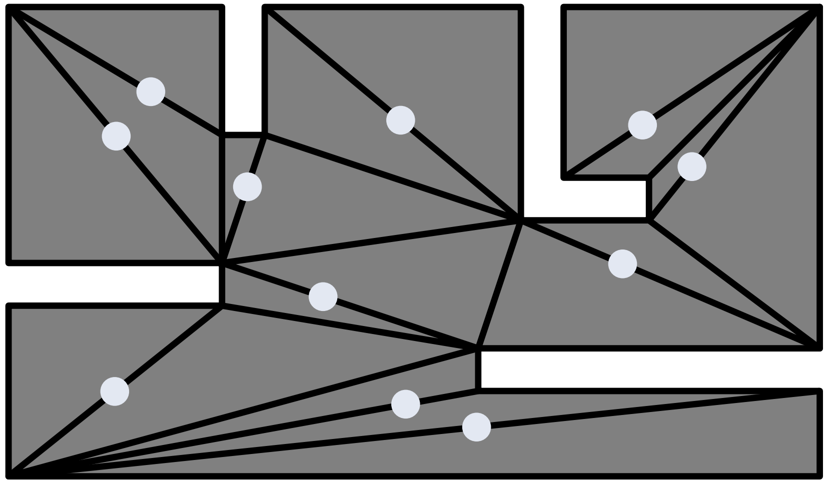
- P could be guarded by $n-2$ cameras placed in the triangles
- P can be guarded by $\approx n/2$ cameras placed on the diagonals

Problem Simplification

Observation: It is easy to guard a triangle



Idea: Decompose P into triangles and guard each of them

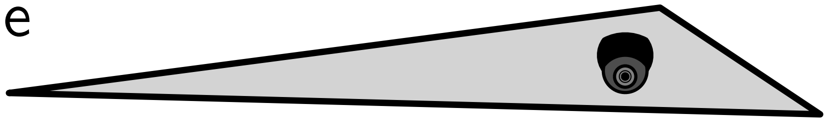


Theorem 1: Each simple polygon with n corners admits a triangulation; any such triangulation contains exactly $n-2$ triangles.

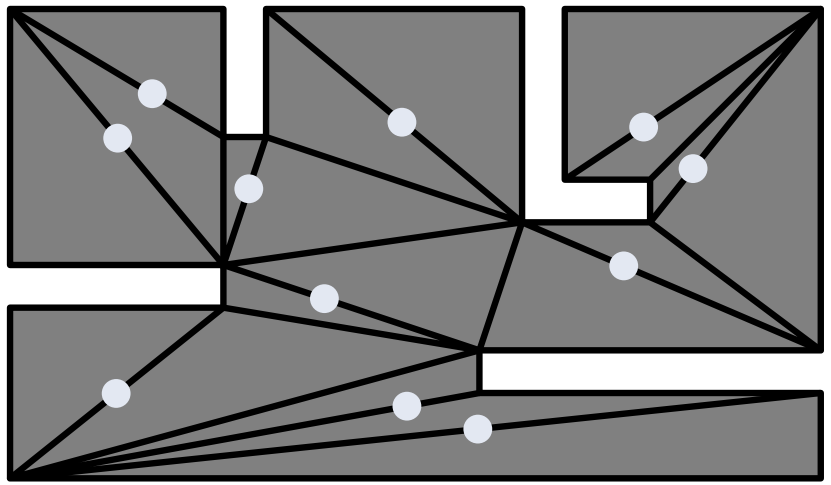
- P could be guarded by $n-2$ cameras placed in the triangles
- P can be guarded by $\approx n/2$ cameras placed on the diagonals

Problem Simplification

Observation: It is easy to guard a triangle



Idea: Decompose P into triangles and guard each of them

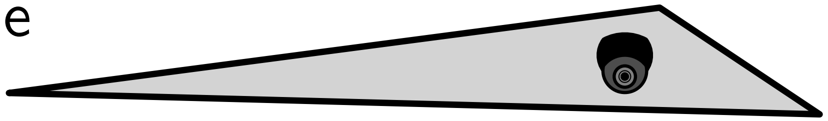


Theorem 1: Each simple polygon with n corners admits a triangulation; any such triangulation contains exactly $n-2$ triangles.

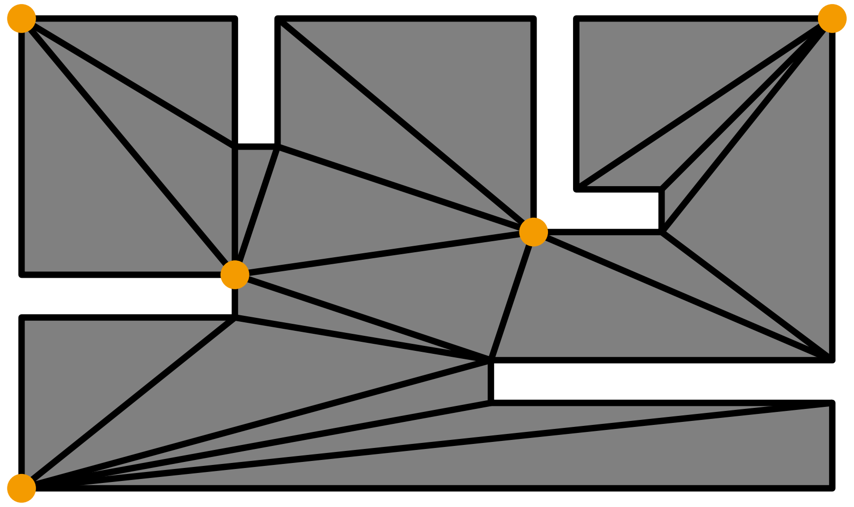
- P could be guarded by $n-2$ cameras placed in the triangles
- P can be guarded by $\approx n/2$ cameras placed on the diagonals
- P can be observed by even smaller number of cameras placed on the corners

Problem Simplification

Observation: It is easy to guard a triangle



Idea: Decompose P into triangles and guard each of them



Theorem 1: Each simple polygon with n corners admits a triangulation; any such triangulation contains exactly $n-2$ triangles.

- P could be guarded by $n-2$ cameras placed in the triangles
- P can be guarded by $\approx n/2$ cameras placed on the diagonals
- P can be observed by even smaller number of cameras placed on the corners

The Art-Gallery-Theorem [Chvátal '75]

Theorem 2: For a simple polygon with n vertices, $\lfloor n/3 \rfloor$ cameras are sometimes necessary and always sufficient to guard it.

The Art-Gallery-Theorem [Chvátal '75]

Theorem 2: For a simple polygon with n vertices, $\lfloor n/3 \rfloor$ cameras are sometimes necessary and always sufficient to guard it.

Proof:

- Find a simple polygon with n corners that requires $\approx n/3$ cameras!

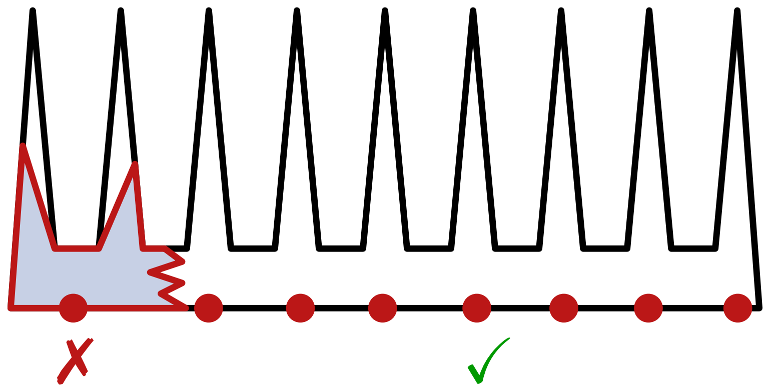
Discuss it with your neighbour for 2 minutes

The Art-Gallery-Theorem [Chvátal '75]

Theorem 2: For a simple polygon with n vertices, $\lfloor n/3 \rfloor$ cameras are sometimes necessary and always sufficient to guard it.

Proof:

- Find a simple polygon with n corners that requires $\approx n/3$ cameras!

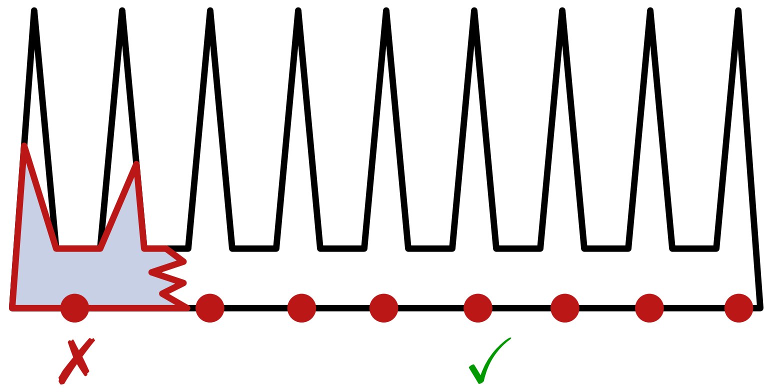


The Art-Gallery-Theorem [Chvátal '75]

Theorem 2: For a simple polygon with n vertices, $\lfloor n/3 \rfloor$ cameras are sometimes necessary and always sufficient to guard it.

Proof:

- Find a simple polygon with n corners that requires $\approx n/3$ cameras!



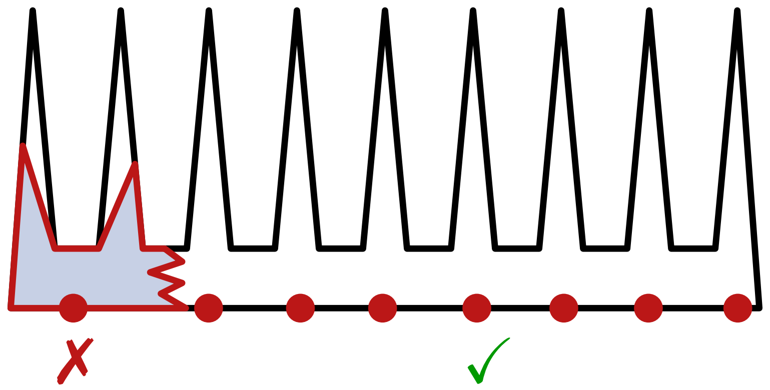
- Sufficiency on the board

The Art-Gallery-Theorem [Chvátal '75]

Theorem 2: For a simple polygon with n vertices, $\lfloor n/3 \rfloor$ cameras are sometimes necessary and always sufficient to guard it.

Proof:

- Find a simple polygon with n corners that requires $\approx n/3$ cameras!



- Sufficiency on the board

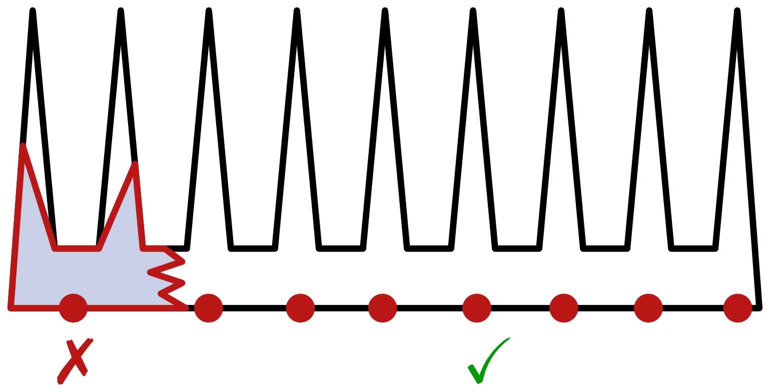
Conclusion: Given a triangulation, the $\lfloor n/3 \rfloor$ cameras that guard the polygon can be placed in $O(n)$ time.

The Art-Gallery-Theorem [Chvátal '75]

Theorem 2: For a simple polygon with n vertices, $\lfloor n/3 \rfloor$ cameras are sometimes necessary and always sufficient to guard it.

Proof:

- Find a simple polygon with n corners that requires $\approx n/3$ cameras!



- Sufficiency on the board

Conclusion: Given a triangulation, the $\lfloor n/3 \rfloor$ cameras that guard the polygon can be placed in $O(n)$ time.

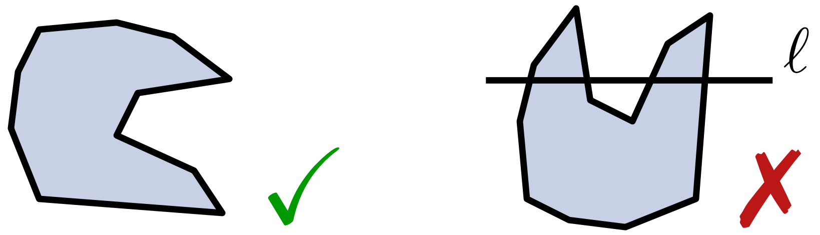
Can we do better than $O(n^2)$ described before?

Proof of Art-Gallery-Theorem: Overview

3-step process:

- Step 1: Decompose P into y-monotone polygons

Definition: A polygon is y-monotone, if for any horizontal line ℓ, the interection $\ell \cap P$ is connected.



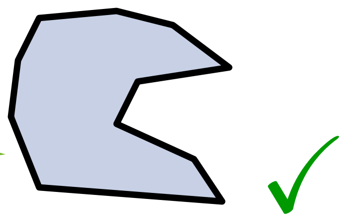
Proof of Art-Gallery-Theorem: Overview

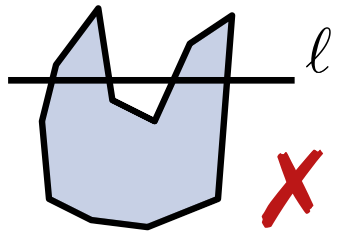
3-step process:

- Step 1: Decompose P into y-monotone polygons

Definition: A polygon is y-monotone, if for any horizontal line ℓ, the interection $\ell \cap P$ is connected.

The two paths from the topmost to the bottommost point bounding the polygon, never go upward


✓



✗

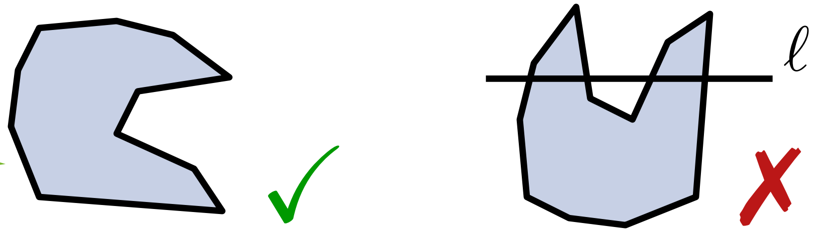
Proof of Art-Gallery-Theorem: Overview

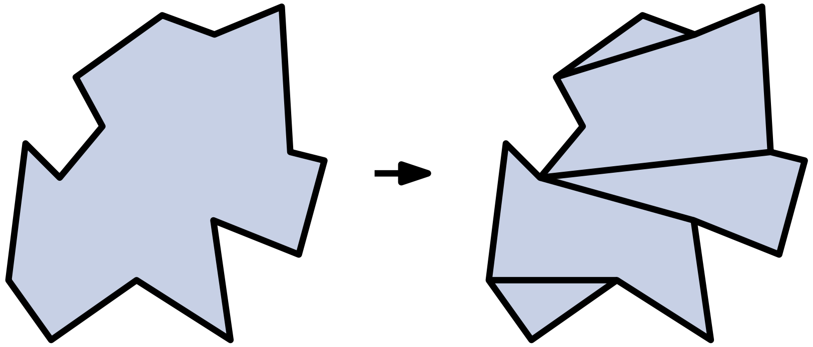
3-step process:

- Step 1: Decompose P into y-monotone polygons

Definition: A polygon is y-monotone, if for any horizontal line ℓ, the interection $\ell \cap P$ is connected.

The two paths from the topmost to the bottommost point bounding the polygon, never go upward



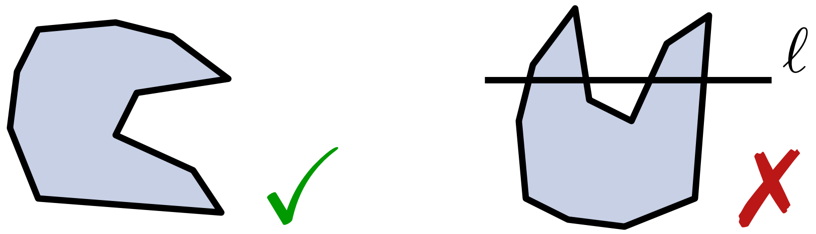


Proof of Art-Gallery-Theorem: Overview

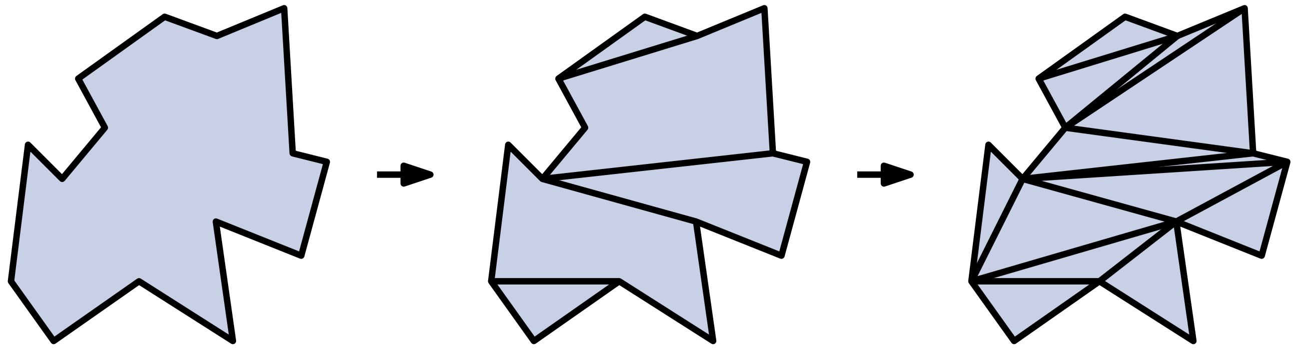
3-step process:

- Step 1: Decompose P into y-monotone polygons

  Definition: A polygon is y-monotone, if for any horizontal line ℓ, the interection $\ell \cap P$ is connected.



- Step 2: Triangulate the resulting y-monotone polygons

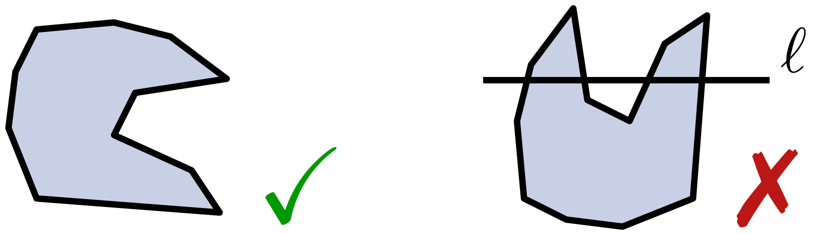


Proof of Art-Gallery-Theorem: Overview

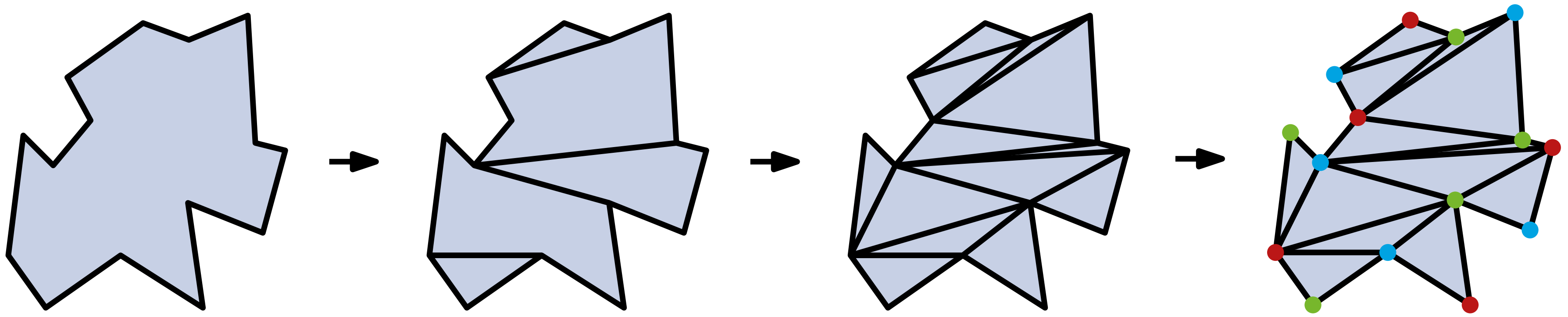
3-step process:

- Step 1: Decompose P into y-monotone polygons

  Definition: A polygon is y-monotone, if for any horizontal line ℓ, the interection $\ell \cap P$ is connected.

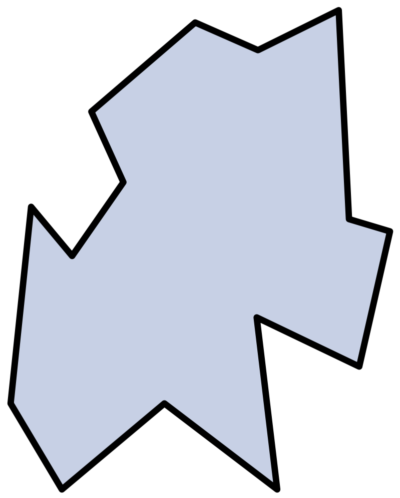


- Step 2: Triangulate the resulting y-monotone polygons
- Step 3: use DFS to color the vertices of the polygon



Partition into y-monotone Polygons

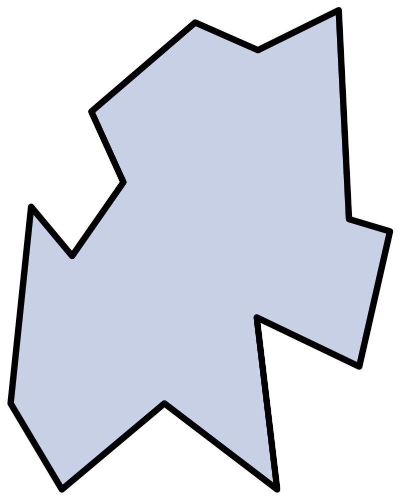
Idea: Five different types of vertices



Partition into y-monotone Polygons

Idea: Five different types of vertices

– *Turn vertices*:

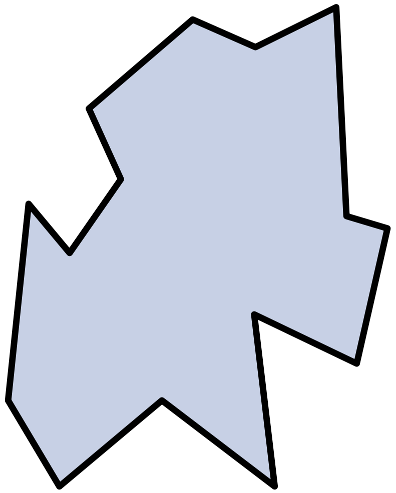


– *regular* vertices

Partition into y-monotone Polygons

Idea: Five different types of vertices

– *Turn vertices*:
vertical change in direction

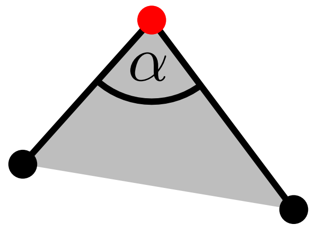


– *regular* vertices

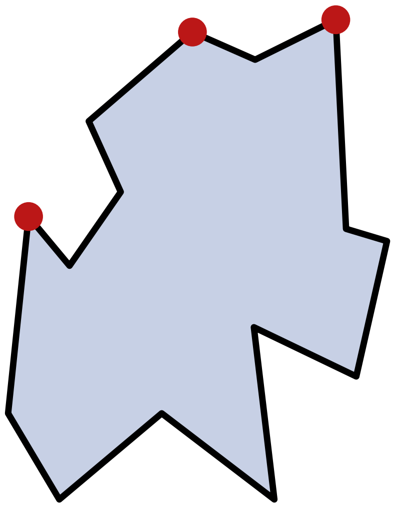
Partition into y-monotone Polygons

Idea: Five different types of vertices

- *Turn vertices*:
  vertical change in direction
  - *start* vertices



if $\alpha < 180°$

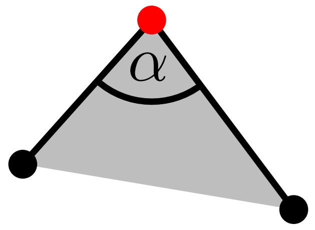


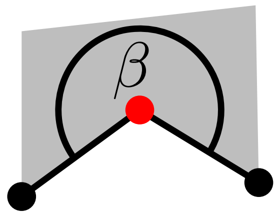
- *regular* vertices

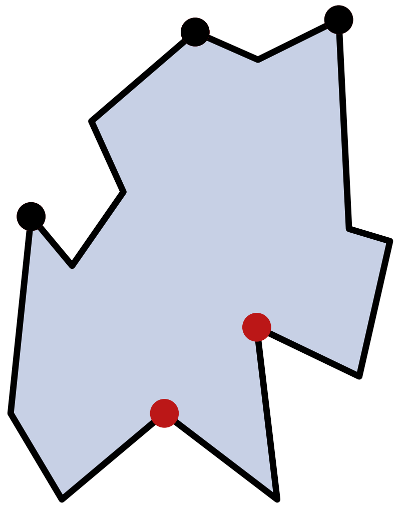
Partition into y-monotone Polygons

Idea: Five different types of vertices

– *Turn vertices*:
vertical change in direction

- *start* vertices  if $\alpha < 180°$

- *split* vertices  if $\beta > 180°$



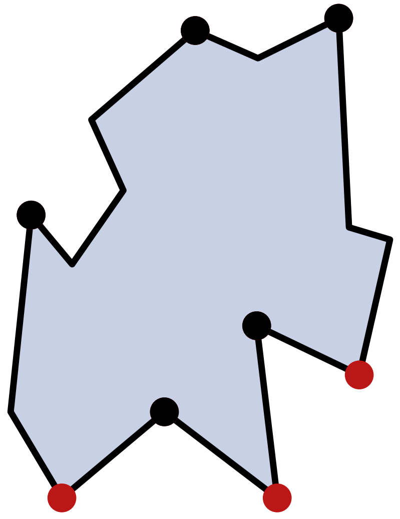
– *regular* vertices

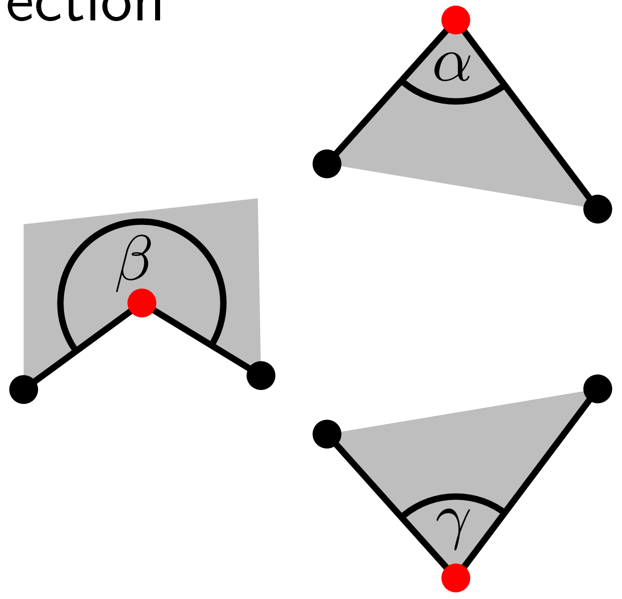
Partition into y-monotone Polygons

Idea: Five different types of vertices

– *Turn vertices*:
vertical change in direction

- *start* vertices
- *split* vertices
- *end* vertices





if $\alpha < 180°$

if $\beta > 180°$

if $\gamma < 180°$

– *regular* vertices

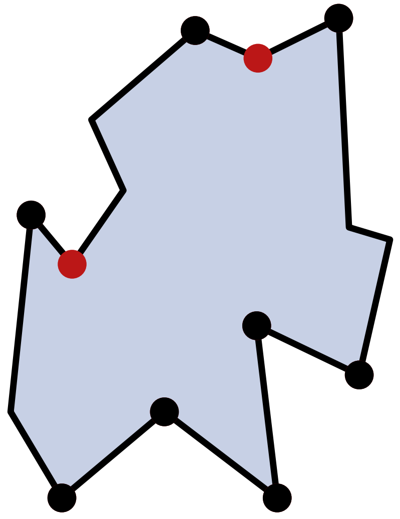
Partition into y-monotone Polygons

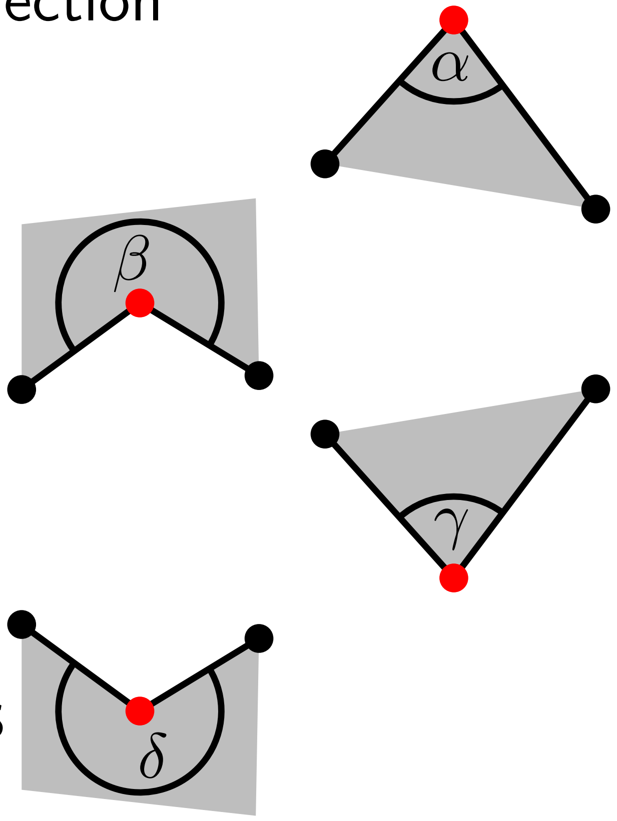
Idea: Five different types of vertices

– *Turn vertices*:
vertical change in direction

- *start* vertices
- *split* vertices
- *end* vertices
- *merge* vertices

– *regular* vertices





if $\alpha < 180°$

if $\beta > 180°$

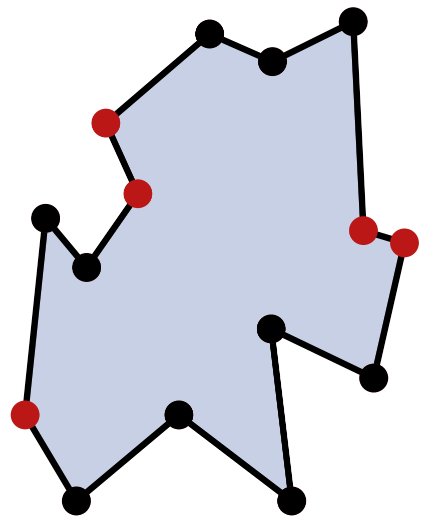
if $\gamma < 180°$

if $\delta > 180°$

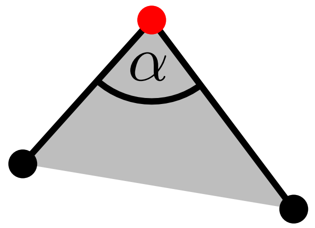
Partition into y-monotone Polygons

Idea: Five different types of vertices

– *Turn vertices*:
  vertical change in direction

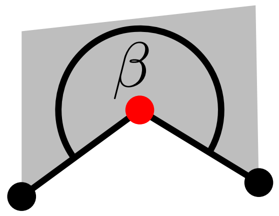


- *start* vertices



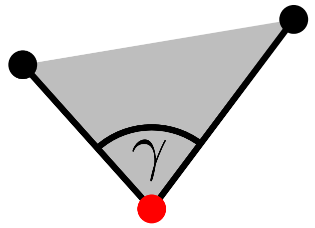
if $\alpha < 180°$

- *split* vertices



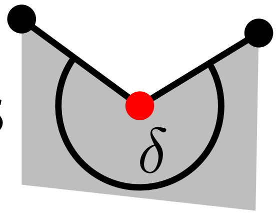
if $\beta > 180°$

- *end* vertices



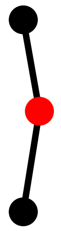
if $\gamma < 180°$

- *merge* vertices



if $\delta > 180°$

– *regular* vertices



Characterization

Lemma 1: A polygon is y-monotone if it has no split vertices or merge vertices.

Characterization

Lemma 1: A polygon is y-monotone if it has no split vertices or merge vertices.

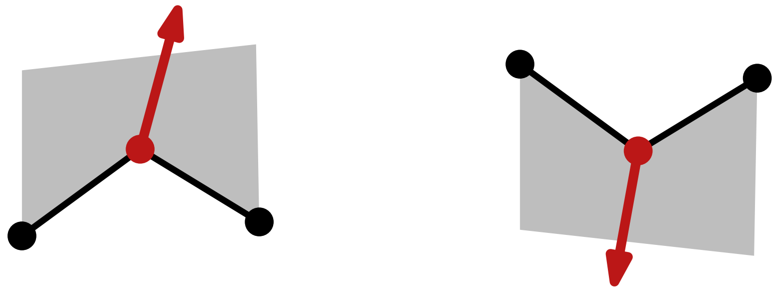
Proof: On the blackboard

Characterization

Lemma 1: A polygon is y-monotone if it has no split vertices or merge vertices.

Proof: On the blackboard

$\Rightarrow$ We need to eliminate all split and merge vertices by using diagonals

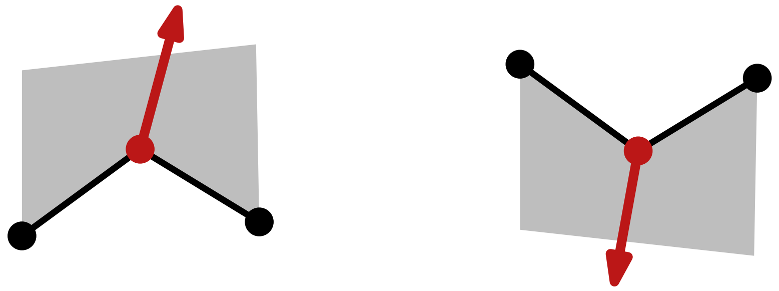


Characterization

Lemma 1: A polygon is y-monotone if it has no split vertices or merge vertices.

Proof: On the blackboard

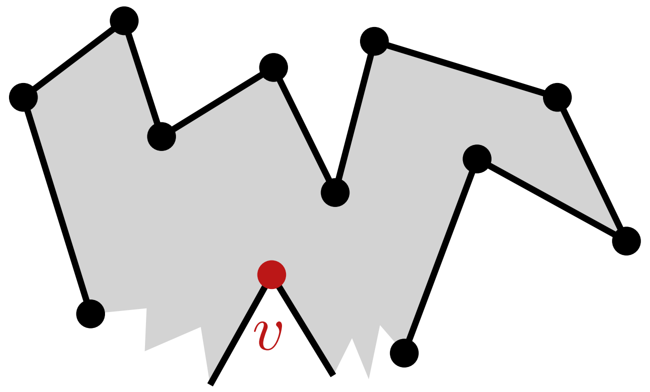
$\Rightarrow$ We need to eliminate all split and merge vertices by using diagonals



Observation: The diagonals should neither cross the edges of P nor the other diagonals

Ideas for Sweep-Line-Algorithm

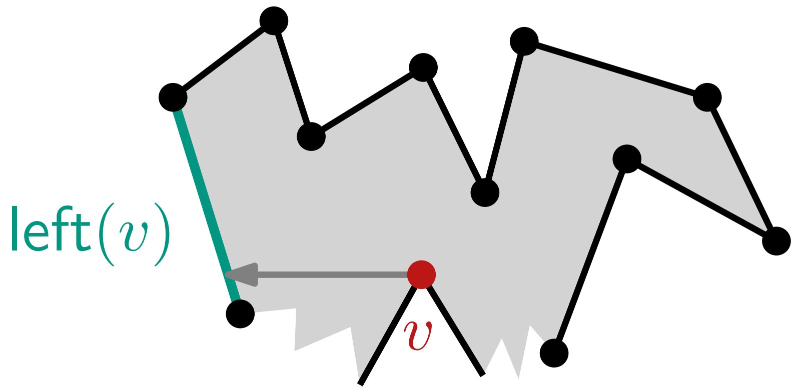
1) Diagonals for the split vertices



Ideas for Sweep-Line-Algorithm

1) Diagonals for the split vertices

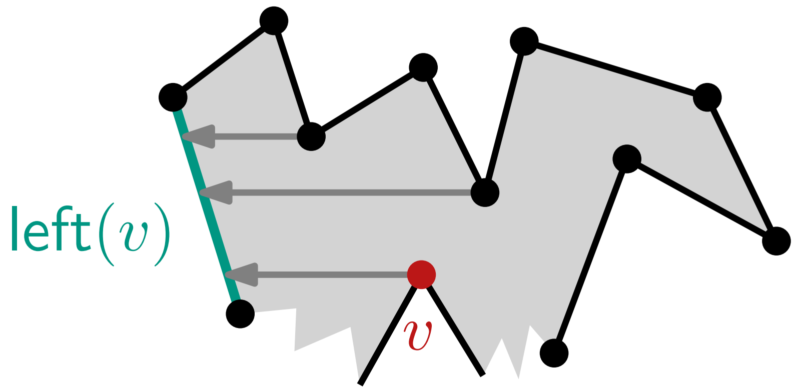
- compute for each vertex v its left adjacent edge left(v) with respect to the horizontal sweep line ℓ



Ideas for Sweep-Line-Algorithm

1) Diagonals for the split vertices

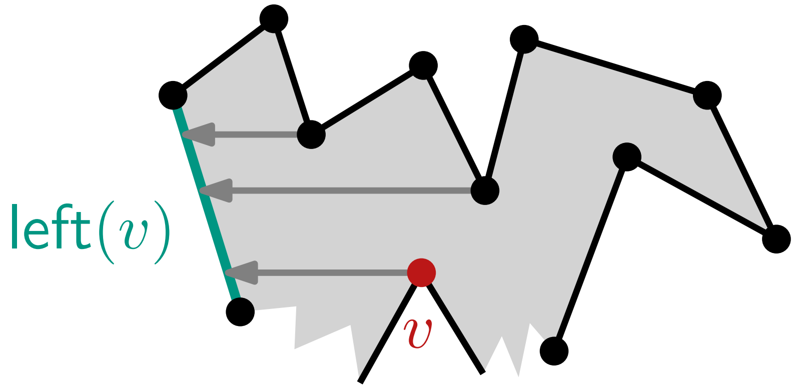
- compute for each vertex v its left adjacent edge left(v) with respect to the horizontal sweep line ℓ



Ideas for Sweep-Line-Algorithm

1) Diagonals for the split vertices

- compute for each vertex v its left adjacent edge left(v) with respect to the horizontal sweep line ℓ

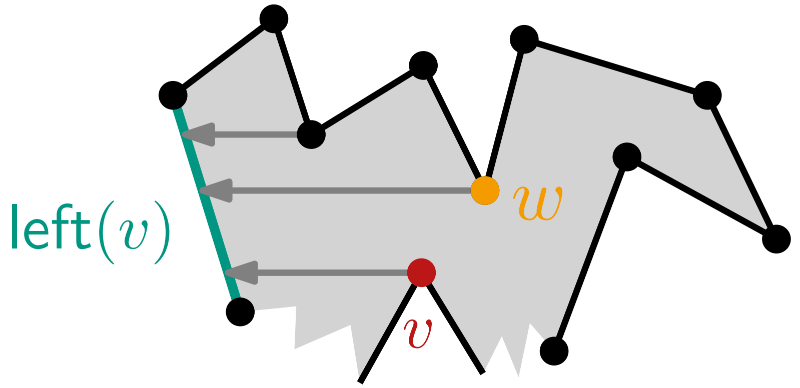


- connect split vertex v to the nearest vertex w above v, such that left$(w) =$ left(v)

Ideas for Sweep-Line-Algorithm

1) Diagonals for the split vertices

- compute for each vertex v its left adjacent edge left(v) with respect to the horizontal sweep line ℓ

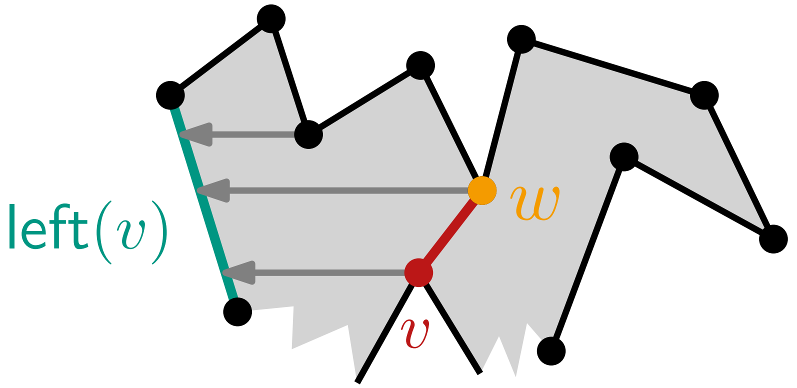


- connect split vertex v to the nearest vertex w above v, such that left$(w) =$ left(v)

Ideas for Sweep-Line-Algorithm

1) Diagonals for the split vertices

- compute for each vertex v its left adjacent edge left(v) with respect to the horizontal sweep line ℓ

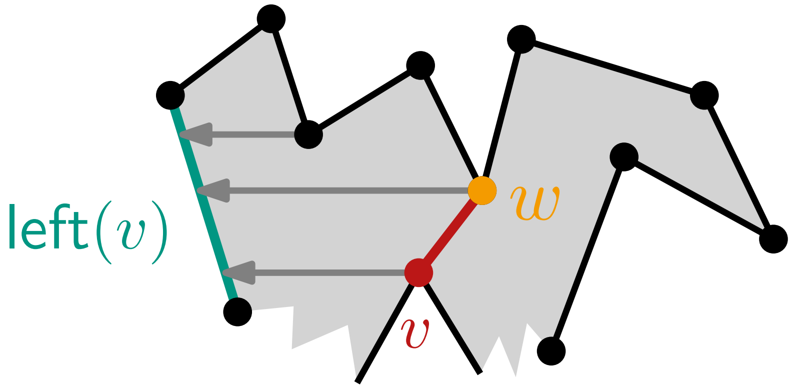


- connect split vertex v to the nearest vertex w above v, such that left$(w) =$ left(v)

Ideas for Sweep-Line-Algorithm

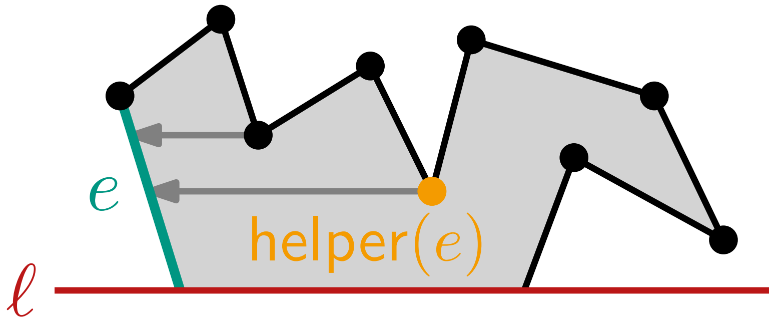
1) Diagonals for the split vertices

- compute for each vertex v its left adjacent edge left(v) with respect to the horizontal sweep line ℓ



- connect split vertex v to the nearest vertex w above v, such that left(w) = left(v)

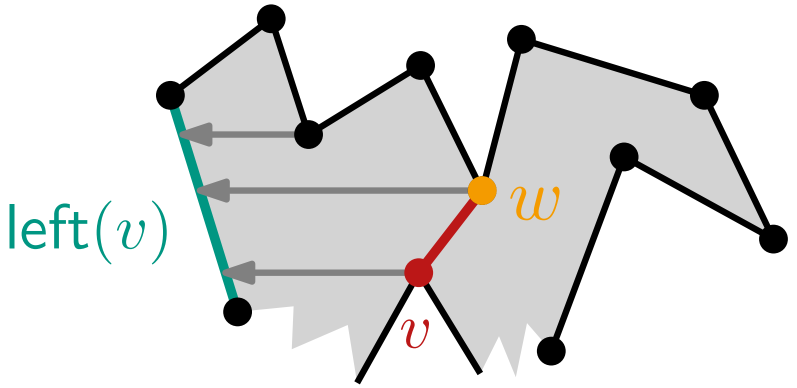
- for each edge e save the botommost vertex w such that left$(w) = e$; notation helper(e) := w



Ideas for Sweep-Line-Algorithm

1) Diagonals for the split vertices

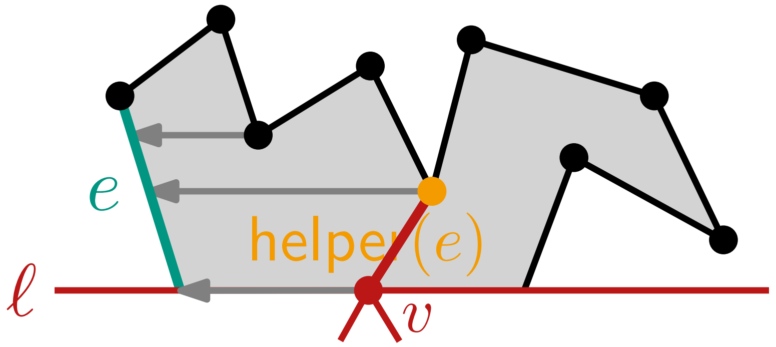
- compute for each vertex v its left adjacent edge left(v) with respect to the horizontal sweep line ℓ



- connect split vertex v to the nearest vertex w above v, such that left(w) = left(v)

- for each edge e save the botommost vertex w such that left$(w) = e$; notation helper(e) := w

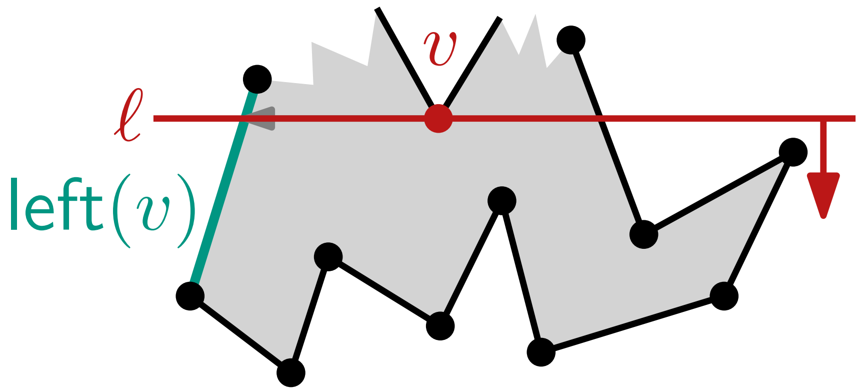
- when ℓ passes through a split vertex v, we connect v with helper(left(v))



Ideas for Sweep-Line-Algorithm

2) Diagonals for merge vertices

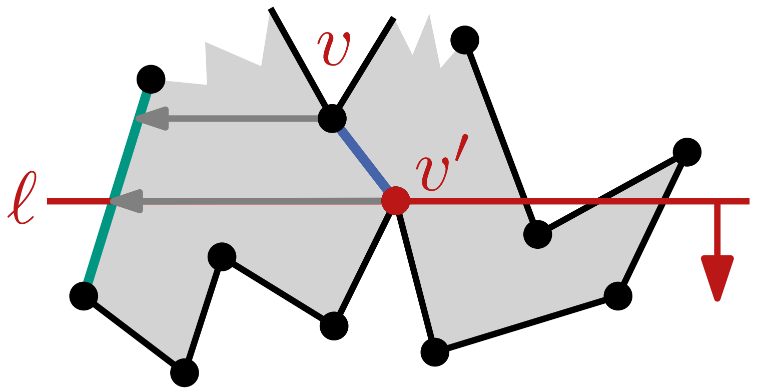
- when the vertex v is reached, we set helper(left(v)) $= v$



Ideas for Sweep-Line-Algorithm

2) Diagonals for merge vertices

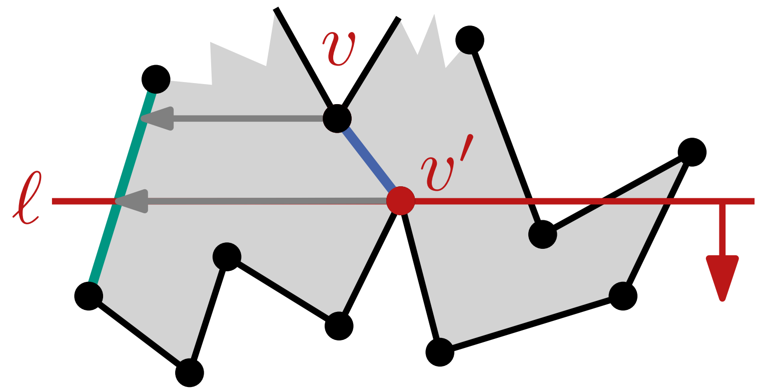
- when the vertex v is reached, we set helper(left(v)) $= v$
- when we reach a split vertex v' such that left(v') $=$ left(v) the diagonal (v, v') is introduced

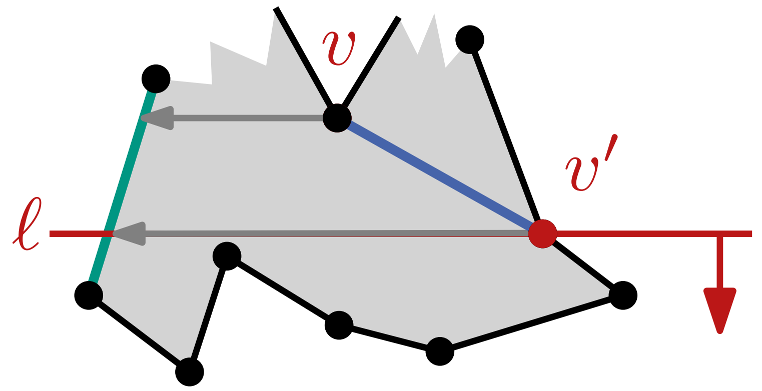


Ideas for Sweep-Line-Algorithm

2) Diagonals for merge vertices

- when the vertex v is reached, we set helper(left$(v)) = v$
- when we reach a split vertex v' such that left(v') = left(v) the diagonal (v, v') is introduced
- in case we reach a regular vertex v' such that helper(left(v')) is v the diagonal (v, v') is introduced

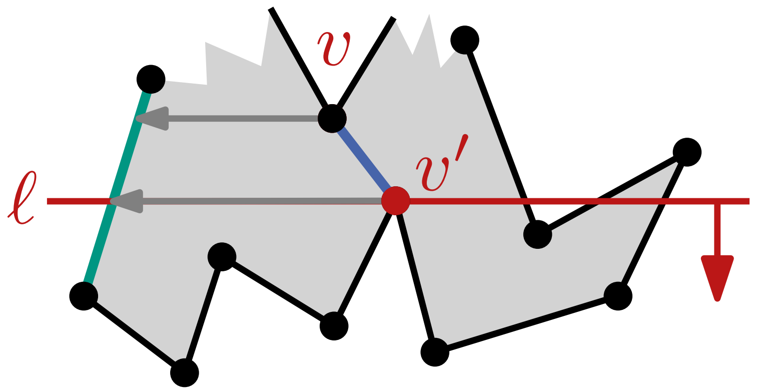


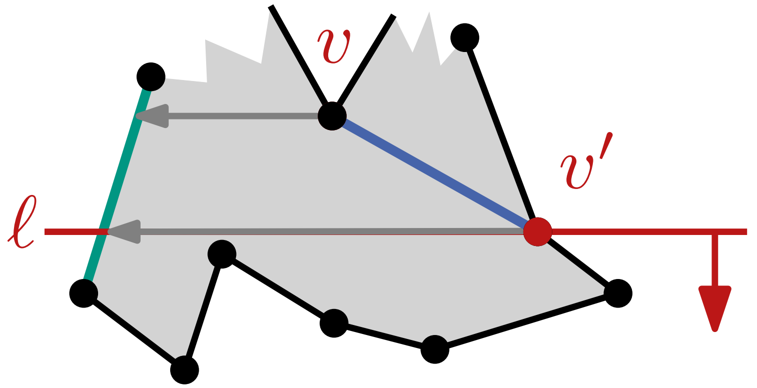


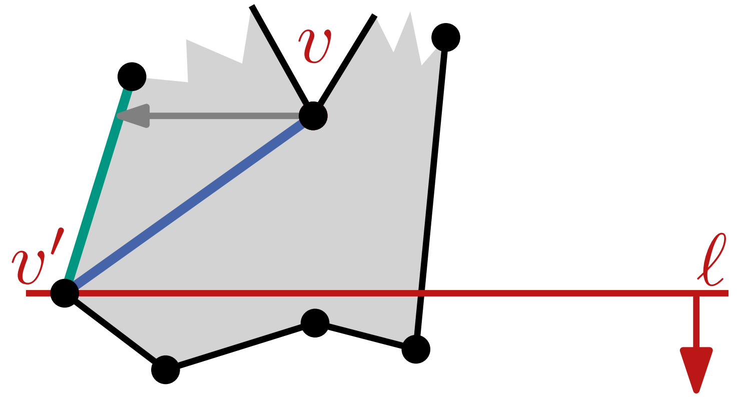
Ideas for Sweep-Line-Algorithm

2) Diagonals for merge vertices

- when the vertex v is reached, we set helper(left$(v)) = v$
- when we reach a split vertex v' such that left(v') = left(v) the diagonal (v, v') is introduced
- in case we reach a regular vertex v' such that helper(left(v')) is v the diagonal (v, v') is introduced
- if the end of v' of left(v) is reached, then the diagonal (v, v') is introduced







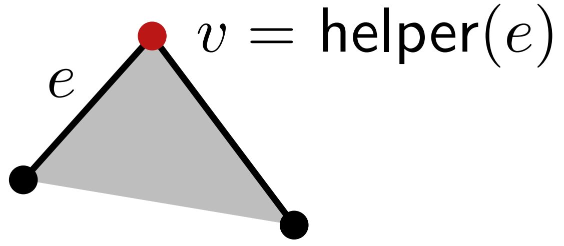
Algorithm MakeMonotone(P)

```
MakeMonotone(Polygon $P$)
  $\mathcal{D} \leftarrow$ doubly-connected edge list for $(V(P), E(P))$
  $\mathcal{Q} \leftarrow$ priority queue for $V(P)$ sorted lexicographically; $\mathcal{T} \leftarrow \emptyset$ (binary
  search tree for sweep-line status)
  while $\mathcal{Q} \neq \emptyset$ do
      $v \leftarrow \mathcal{Q}$.nextVertex()
      $\mathcal{Q}$.deleteVertex($v$)
      handleVertex($v$)
  return $\mathcal{D}$
```

Algorithm MakeMonotone(P)

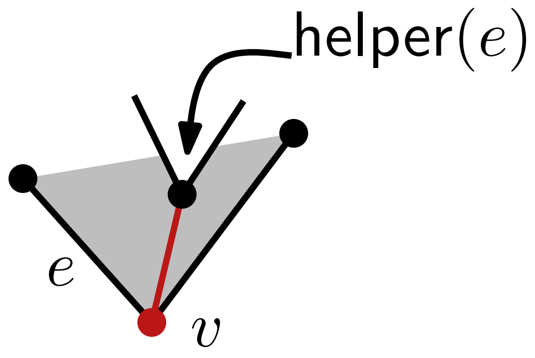
```
**MakeMonotone**(Polygon $P$)
    $\mathcal{D} \leftarrow$ doubly-connected edge list for $(V(P), E(P))$
    $\mathcal{Q} \leftarrow$ priority queue for $V(P)$ sorted lexicographically; $\mathcal{T} \leftarrow \emptyset$ (binary search tree for sweep-line status)
    **while** $\mathcal{Q} \neq \emptyset$ **do**
        $v \leftarrow \mathcal{Q}$.nextVertex()
        $\mathcal{Q}$.deleteVertex($v$)
        handleVertex($v$)
    **return** $\mathcal{D}$
```

```
**handleStartVertex**(vertex $v$)
    $\mathcal{T} \leftarrow$ add the left edge $e$
    helper($e$) $\leftarrow v$
```

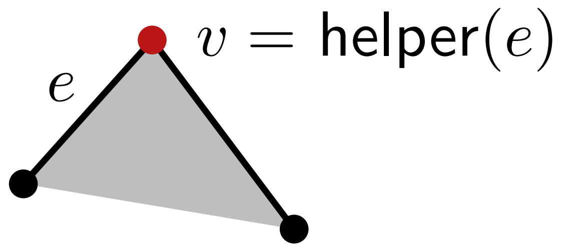


Algorithm MakeMonotone(P)

```
MakeMonotone(Polygon $P$)
    $\mathcal{D} \leftarrow$ doubly-connected edge list for $(V(P), E(P))$
    $\mathcal{Q} \leftarrow$ priority queue for $V(P)$ sorted lexicographically; $\mathcal{T} \leftarrow \emptyset$ (binary
    search tree for sweep-line status)
    while $\mathcal{Q} \neq \emptyset$ do
        $v \leftarrow \mathcal{Q}$.nextVertex()
        $\mathcal{Q}$.deleteVertex($v$)
        handleVertex($v$)
    return $\mathcal{D}$
```



```
handleStartVertex(vertex $v$)
    $\mathcal{T} \leftarrow$ add the left edge $e$
    helper($e$) $\leftarrow v$
```

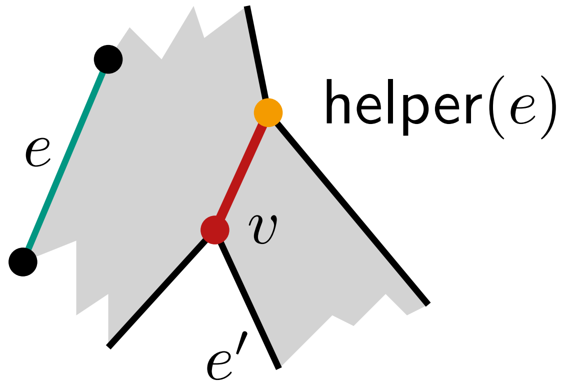


```
handleEndVertex(vertex $v$)
    $e \leftarrow$ left edge
    if isMergeVertex(helper($e$)) then
        $\mathcal{D} \leftarrow$ add edge (helper($e$), $v$)
    remove $e$ from $\mathcal{T}$
```

Algorithm MakeMonotone(P)

```
**MakeMonotone**(Polygon $P$)
  $\mathcal{D} \leftarrow$ doubly-connected edge list for $(V(P), E(P))$
  $\mathcal{Q} \leftarrow$ priority queue for $V(P)$ sorted lexicographically; $\mathcal{T} \leftarrow \emptyset$ (binary
  search tree for sweep-line status)
  while $\mathcal{Q} \neq \emptyset$ do
      $v \leftarrow \mathcal{Q}$.nextVertex()
      $\mathcal{Q}$.deleteVertex($v$)
      handleVertex($v$)
  return $\mathcal{D}$
```

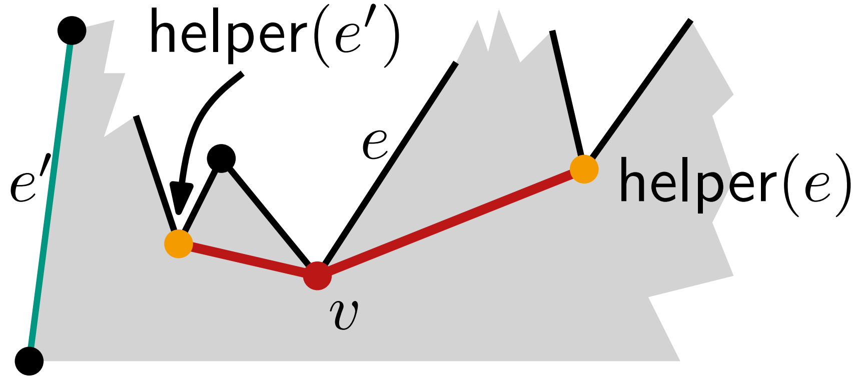
```
**handleSplitVertex**(vertex $v$)
  $e \leftarrow$ Edge to the left of $v$ in $\mathcal{T}$
  $\mathcal{D} \leftarrow$ add edge (helper($e$), $v$)
  helper($e$) $\leftarrow v$
  $\mathcal{T} \leftarrow$ add the right edge $e'$ of $v$
  helper($e'$) $\leftarrow v$
```



Algorithm MakeMonotone(P)

```
**MakeMonotone**(Polygon $P$)
    $\mathcal{D} \leftarrow$ doubly-connected edge list for $(V(P), E(P))$
    $\mathcal{Q} \leftarrow$ priority queue for $V(P)$ sorted lexicographically; $\mathcal{T} \leftarrow \emptyset$ (binary search tree for sweep-line status)
    while $\mathcal{Q} \neq \emptyset$ do
        $v \leftarrow \mathcal{Q}$.nextVertex()
        $\mathcal{Q}$.deleteVertex($v$)
        handleVertex($v$)
    return $\mathcal{D}$
```

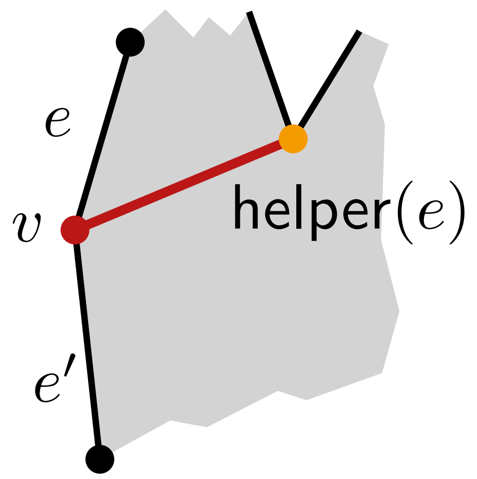
```
**handleMergeVertex**(vertex $v$)
    $e \leftarrow$ right edge
    if isMergeVertex(helper($e$)) then
        $\mathcal{D} \leftarrow$ add edge (helper($e$), $v$)
    remove $e$ from $\mathcal{T}$
    $e' \leftarrow$ edge to the left of $v$ in $\mathcal{T}$
    if isMergeVertex(helper($e'$)) then
        $\mathcal{D} \leftarrow$ add edge (helper($e'$), $v$)
    helper($e'$) $\leftarrow v$
```

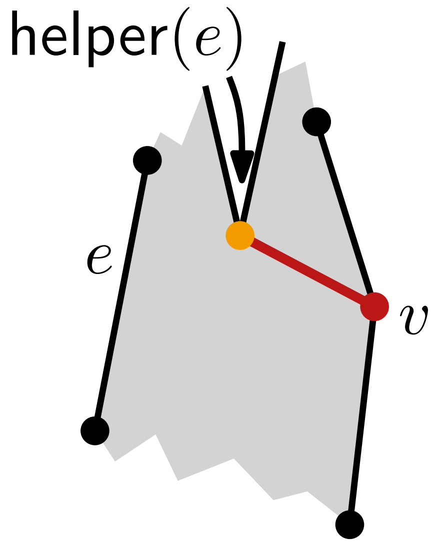


Algorithm MakeMonotone(P)

```
MakeMonotone(Polygon $P$)
    $\mathcal{D} \leftarrow$ doubly-connected edge list for $(V(P), E(P))$
    $\mathcal{Q} \leftarrow$ priority queue for $V(P)$ sorted lexicographically; $\mathcal{T} \leftarrow \emptyset$ (binary
    search tree for sweep-line status)
    while $\mathcal{Q} \neq \emptyset$ do
        $v \leftarrow \mathcal{Q}$.nextVertex()
        $\mathcal{Q}$.deleteVertex($v$)
        handleVertex($v$)
    return $\mathcal{D}$
```

```
handleRegularVertex(vertex $v$)
    if $P$ lies locally to the left of $v$ then
        $e, e' \leftarrow$ above, below edge
        if isMergeVertex(helper($e$)) then
            $\mathcal{D} \leftarrow$ add edge (helper($e$), $v$)
        remove $e$ from $\mathcal{T}$
        $\mathcal{T} \leftarrow$ add $e'$; helper($e'$) $\leftarrow v$
    else
        $e \leftarrow$ edge to the left of $v$
        add $e$ to $\mathcal{T}$
        if isMergeVertex(helper($e$)) then
            $\mathcal{D} \leftarrow$ add (helper($e$), $v$)
        helper($e$) $\leftarrow v$
```





Analysis

Lemma 2: The algorithm MakeMonotone computes a set of crossing-free diagonals of P, which partitions P into y-monotone polygons.

Analysis

Lemma 2: The algorithm MakeMonotone computes a set of crossing-free diagonals of P, which partitions P into y-monotone polygons.

Theorem 3: A simple polygon with n vertices can be partitioned into y-monotone polygons in $O(n \log n)$ time and $O(n)$ space.

Analysis

Lemma 2: The algorithm MakeMonotone computes a set of crossing-free diagonals of P, which partitions P into y-monotone polygons.

Theorem 3: A simple polygon with n vertices can be partitioned into y-monotone polygons in $O(n \log n)$ time and $O(n)$ space.

- Construct priority queue $\mathcal{Q}$: $O(n)$
- Initialize sweep-line status $\mathcal{T}$: $O(1)$
- Handle a single event: $O(\log n)$
    - $\mathcal{Q}$.deleteMax: $O(\log n)$
    - Find, remove, add element in $\mathcal{T}$: $O(\log n)$
    - Add diagonals to $\mathcal{D}$: $O(1)$

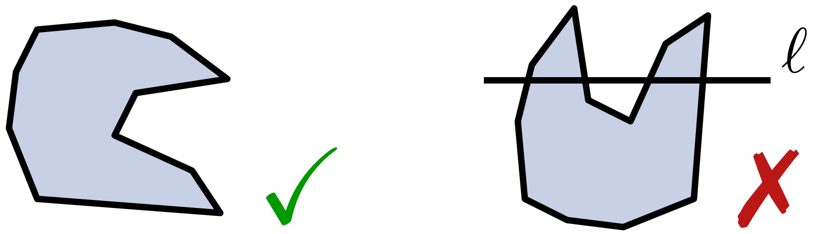
- Space: obviously $O(n)$

Proof of Art-Gallery-Theorem: Overview

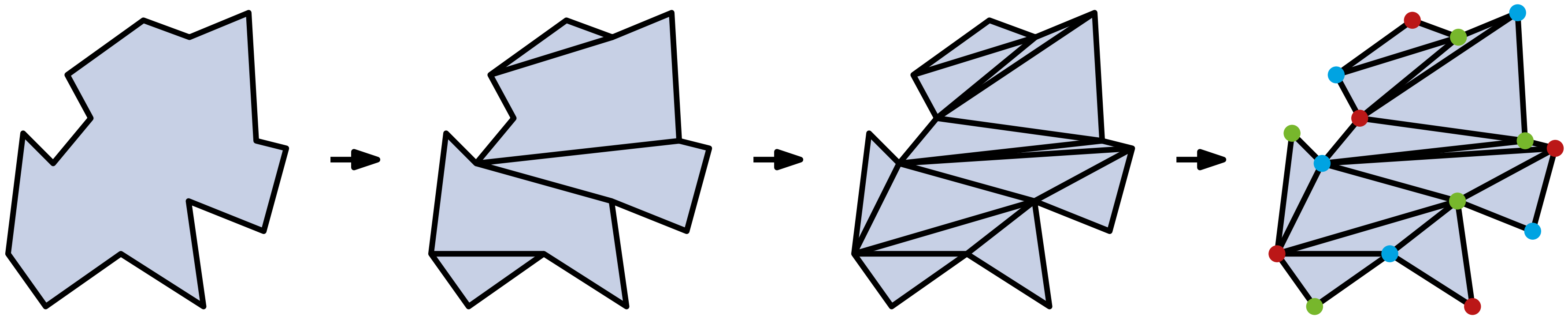
Three-step procedure:

- Step 1: Decompose P in y-monotone polygons ✓

Definition: A polygon P is y-monotone, if for each horizontal line ℓ the intersection $\ell \cap P$ is connected.



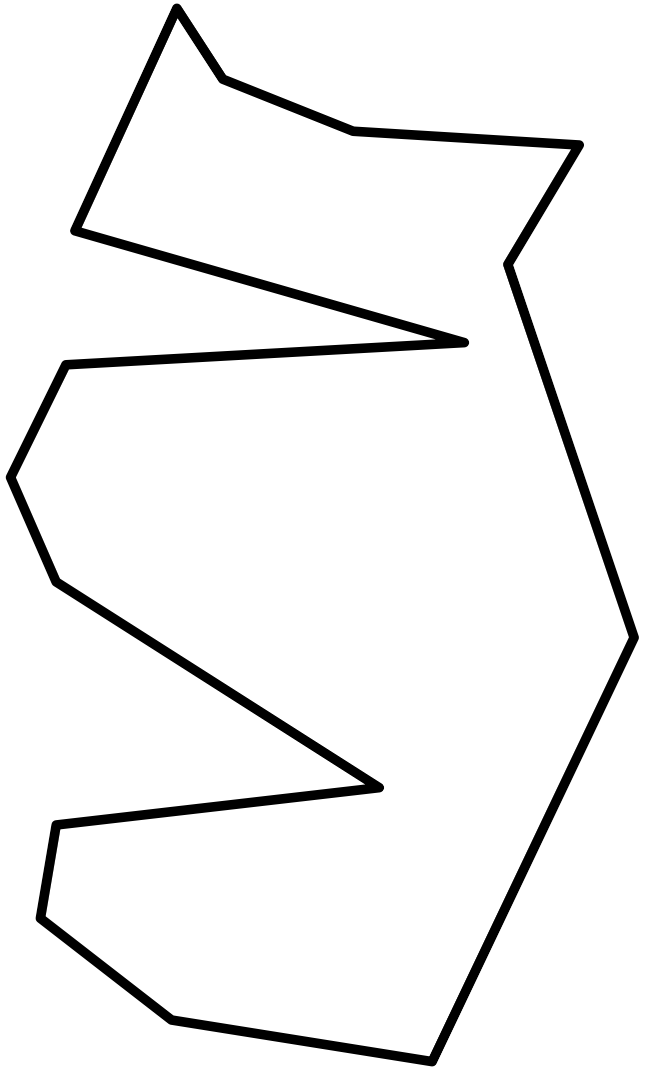
- Step 2: Triangulate y-monotone polygons **ToDo!**
- Step 3: use DFS to color the triangulated polygon ✓



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

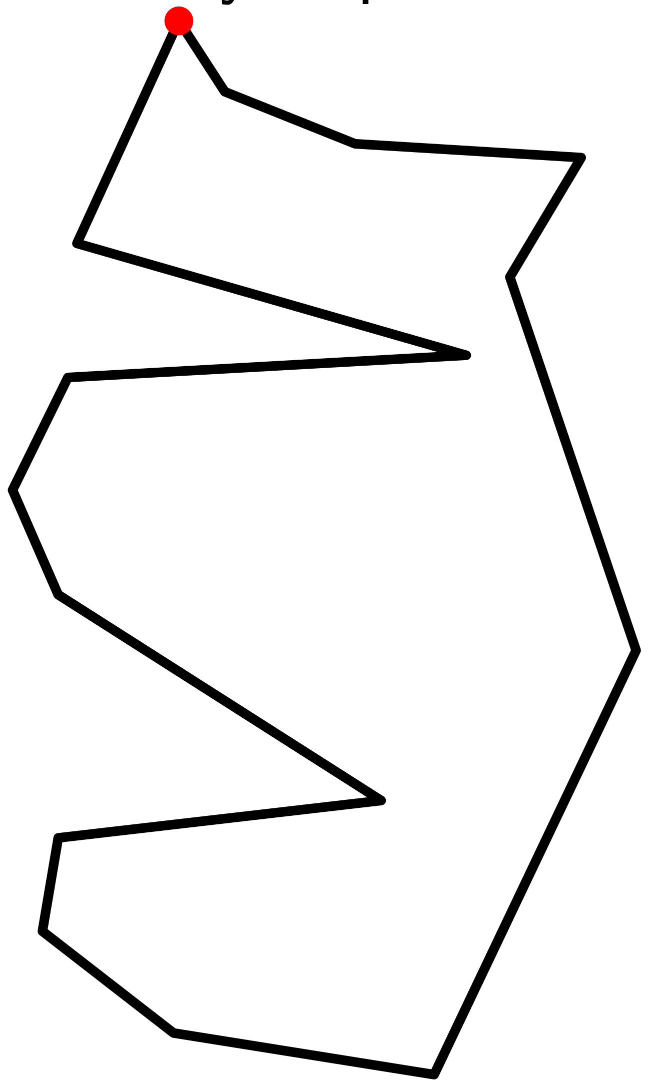
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

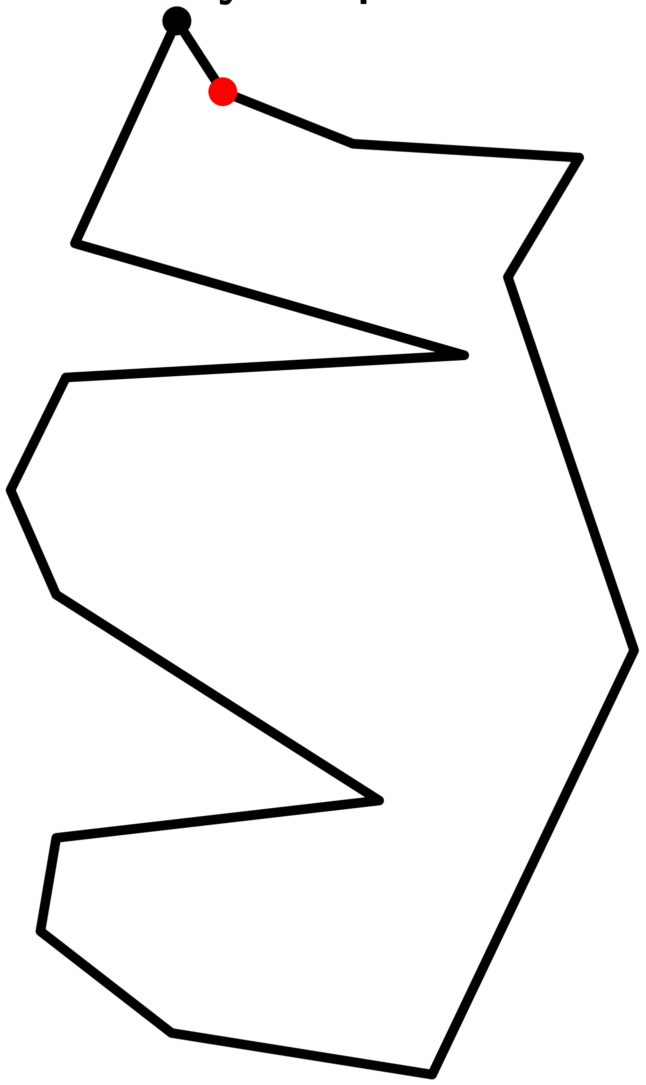
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

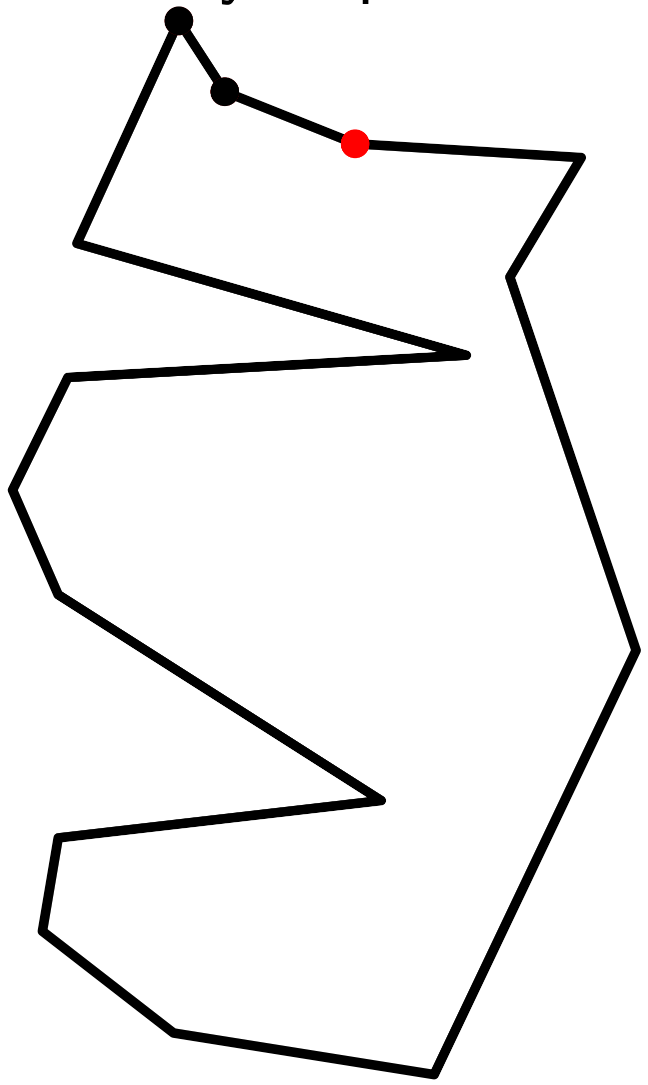
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

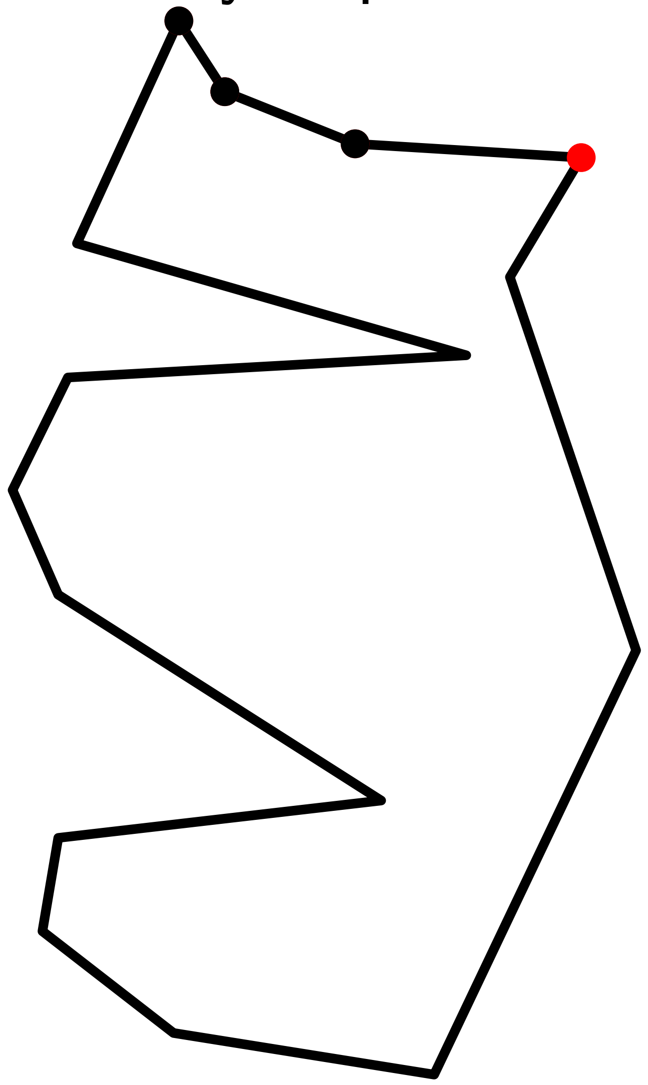
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

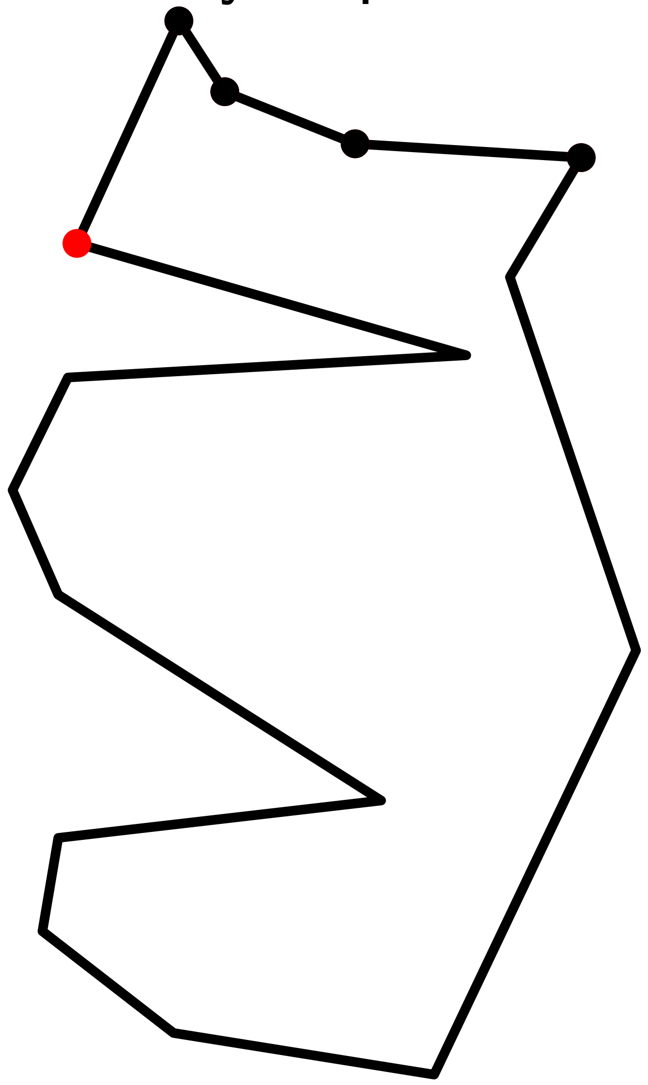
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

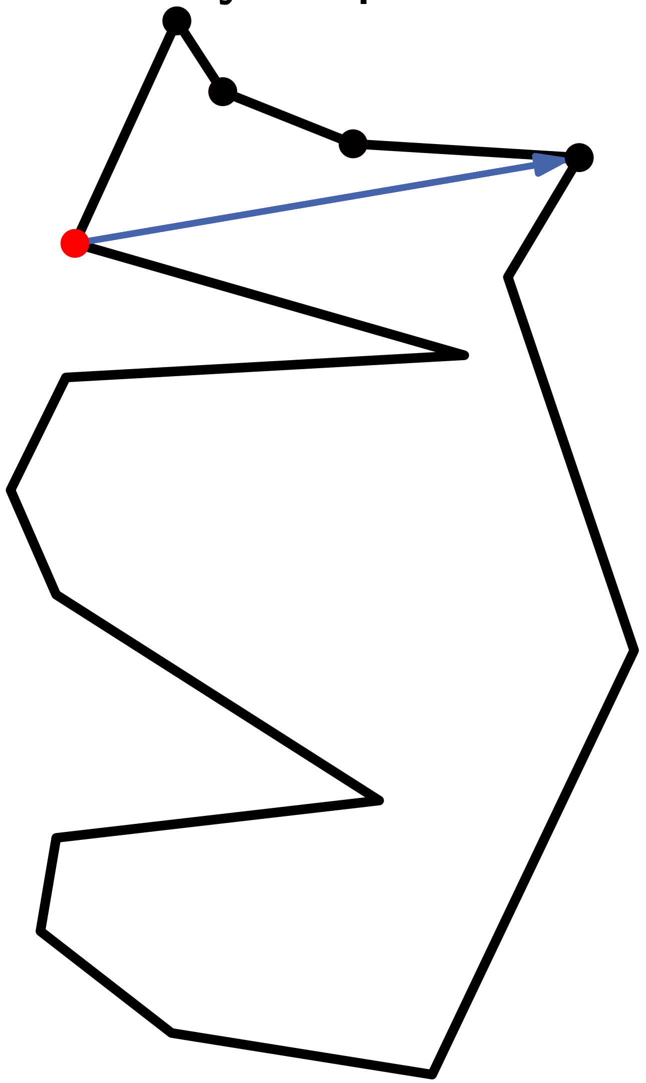
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

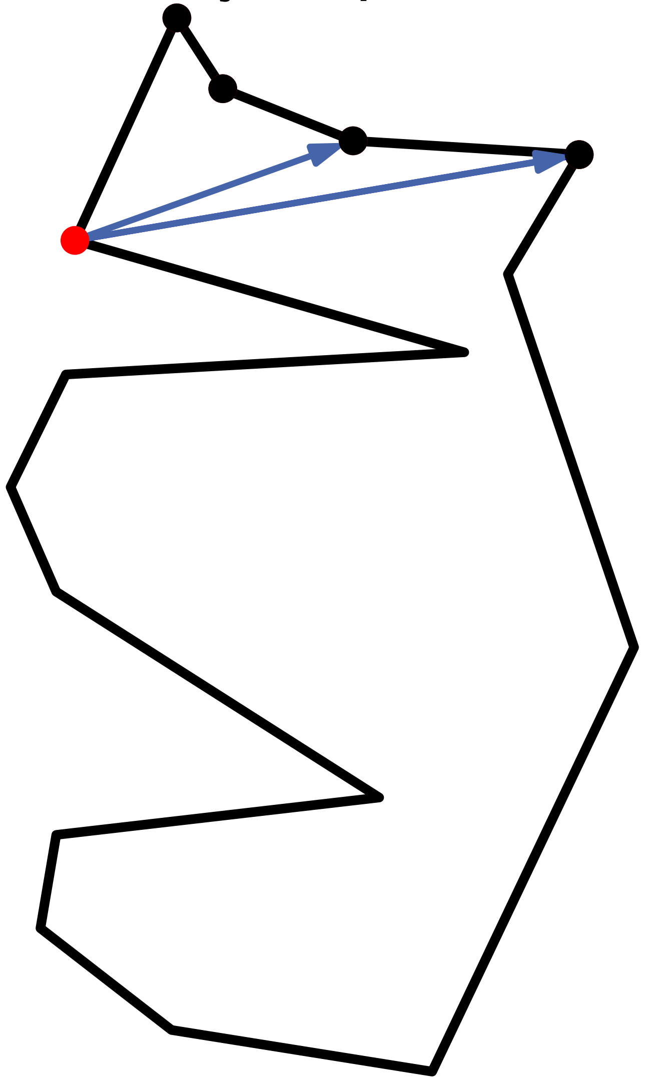
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

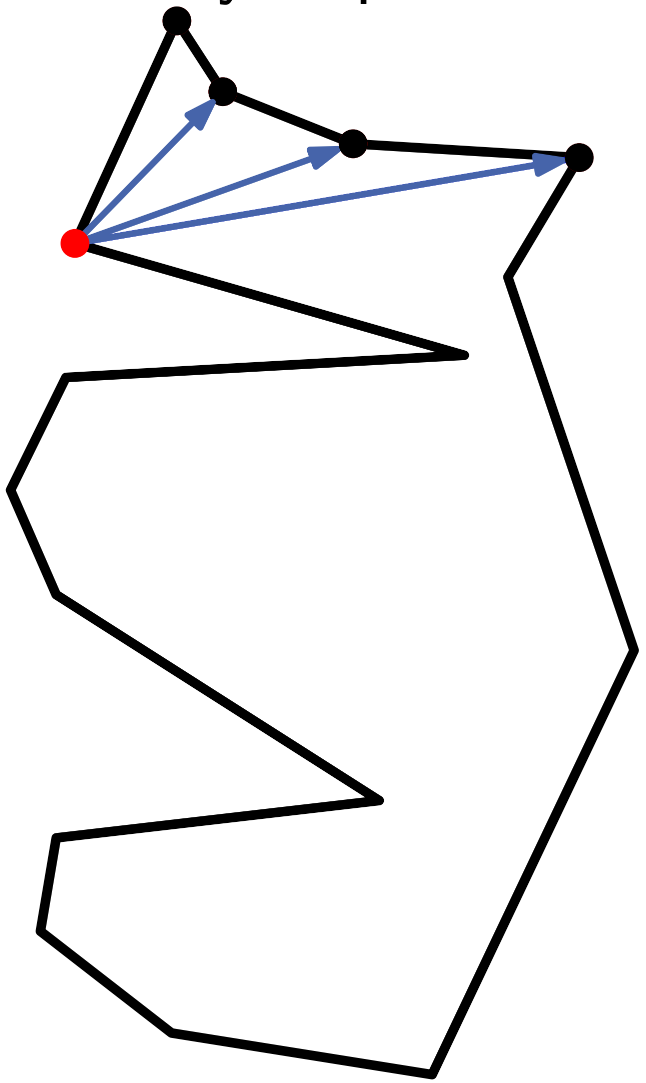
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

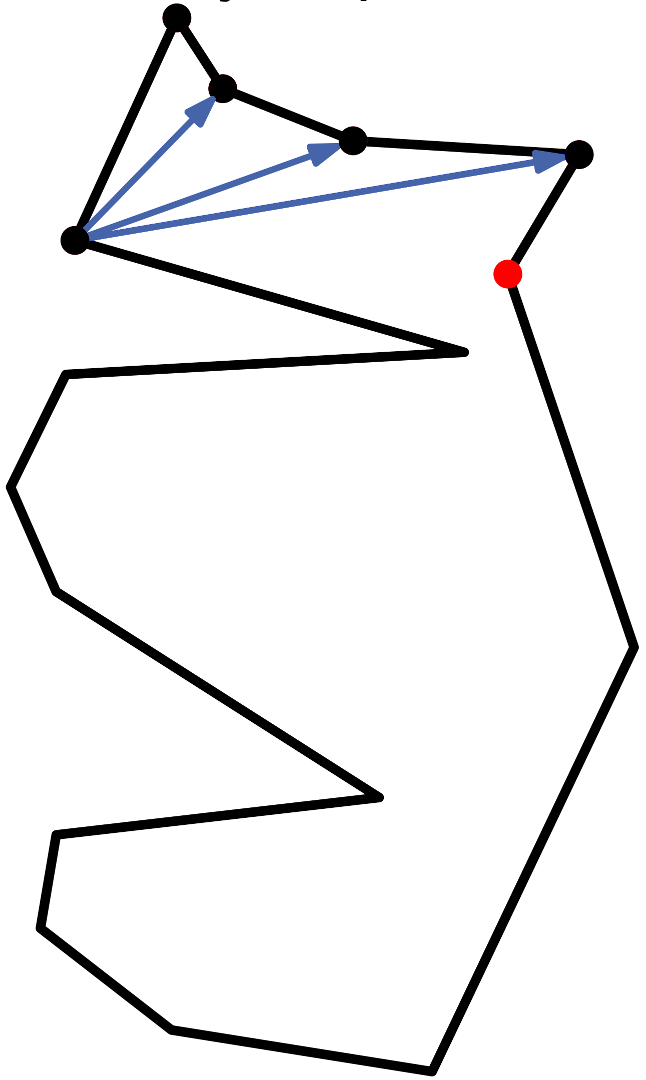
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

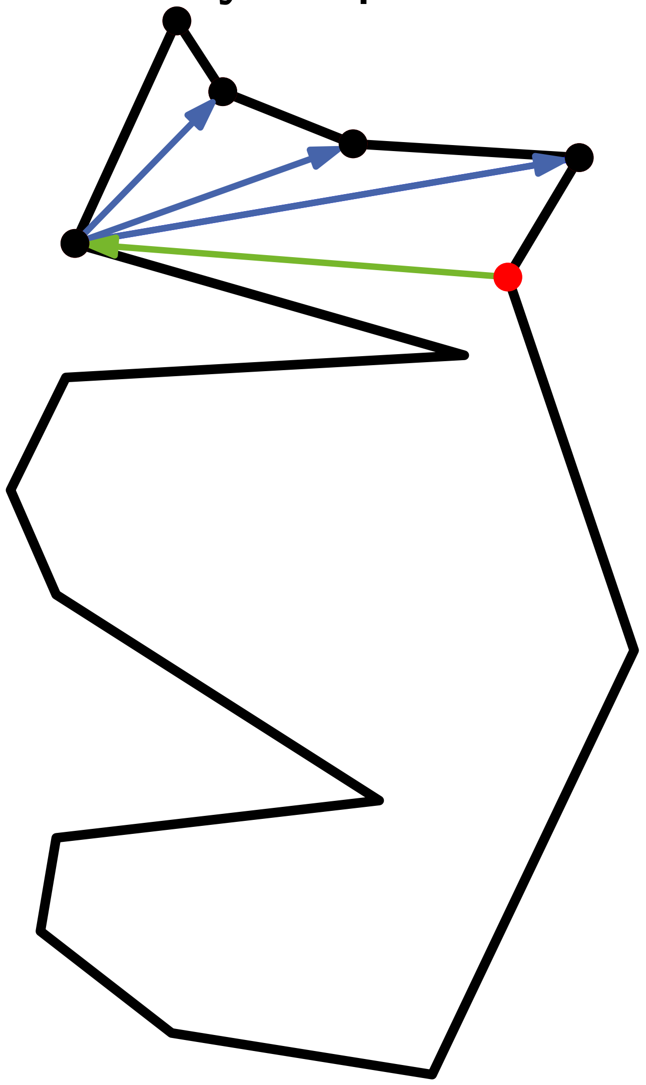
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

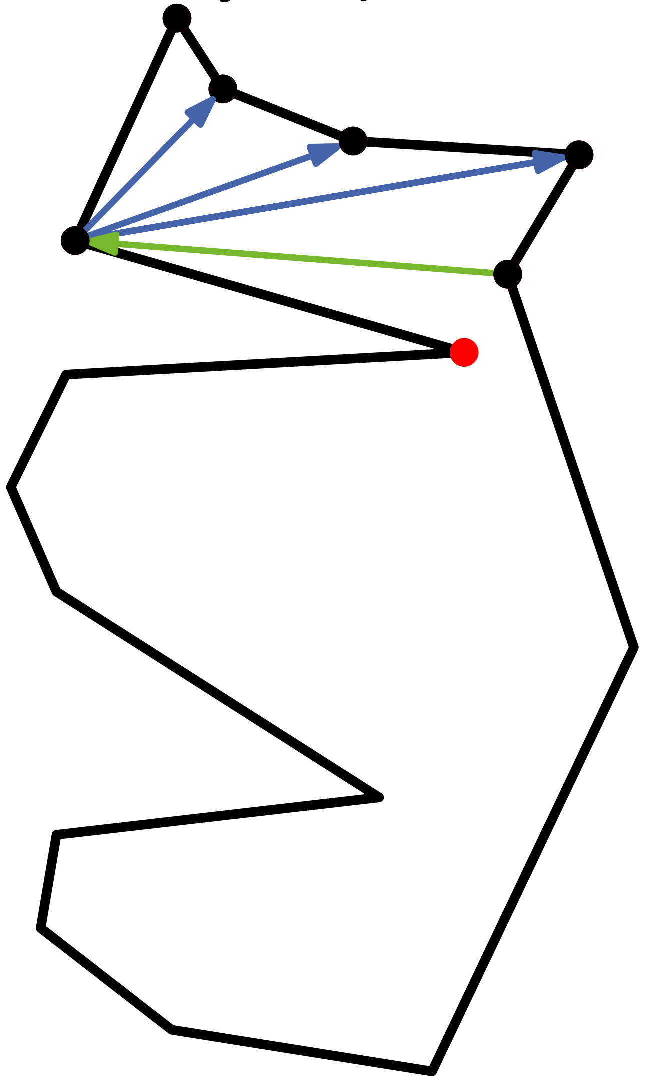
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

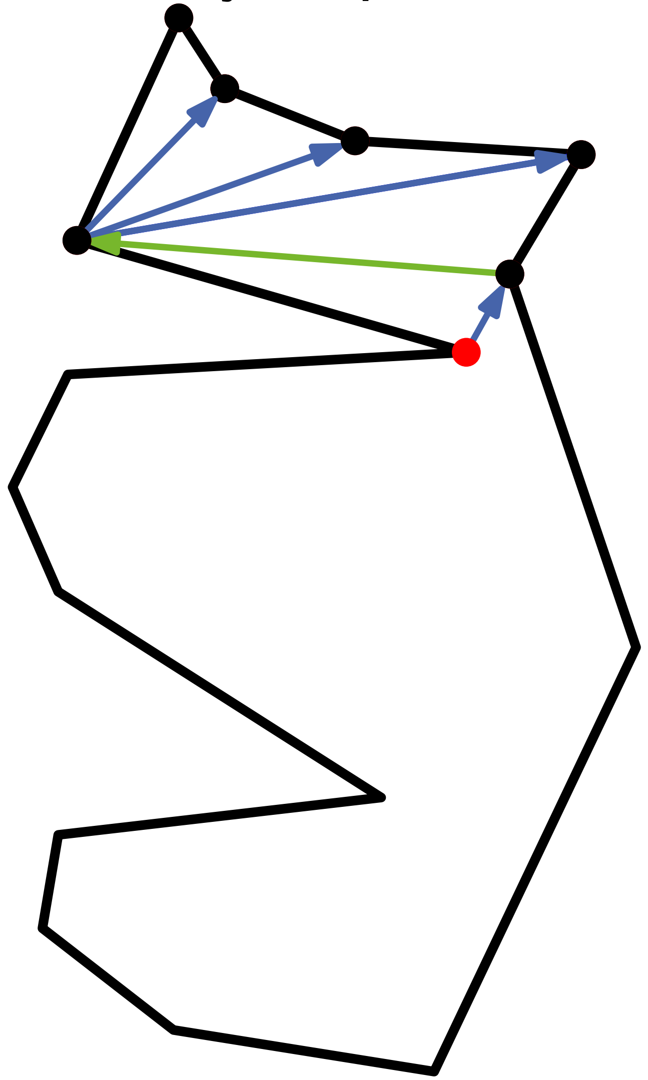
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

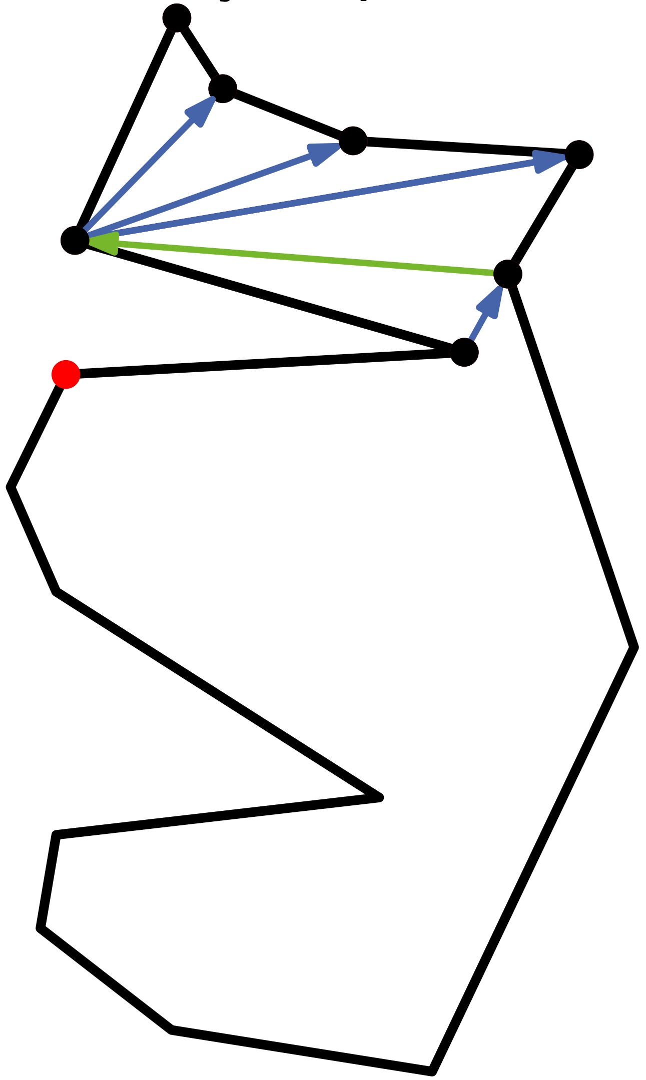
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

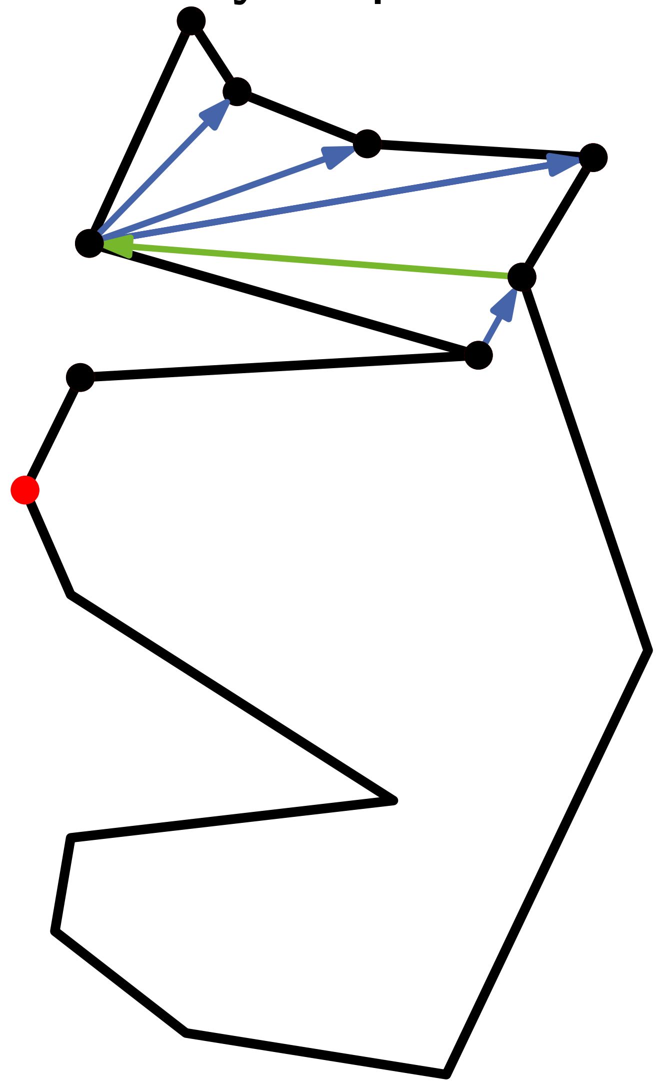
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

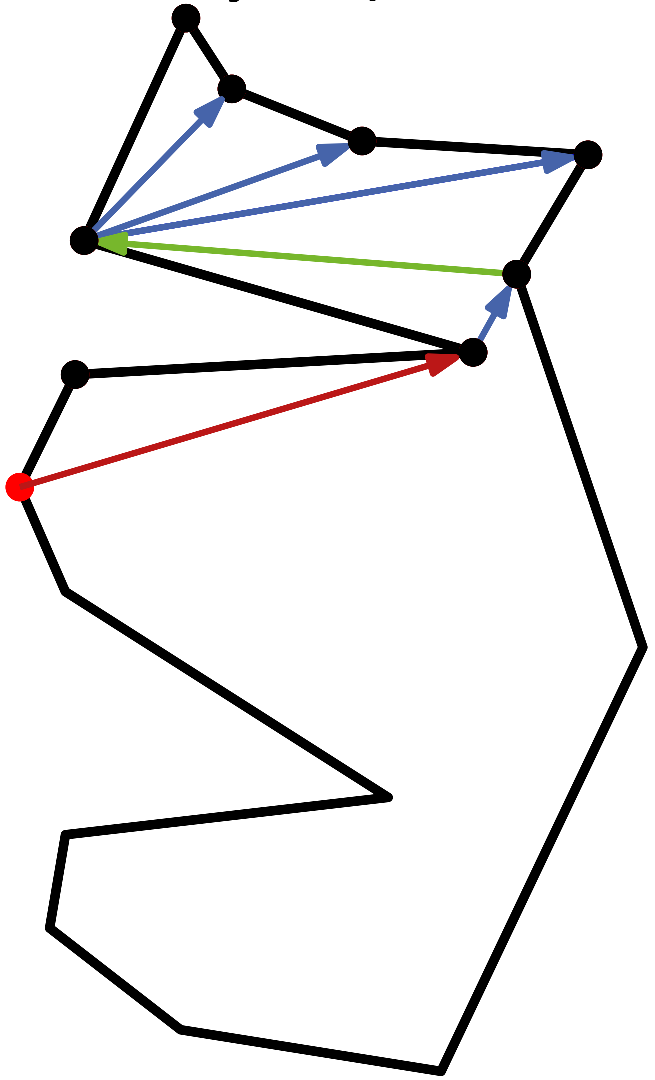
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

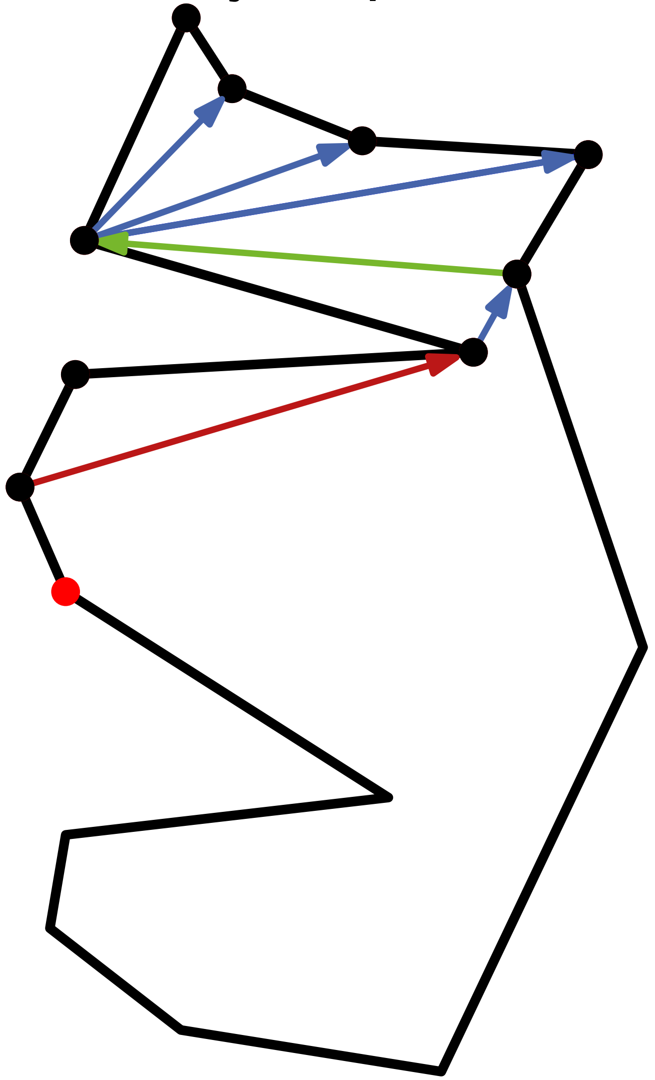
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

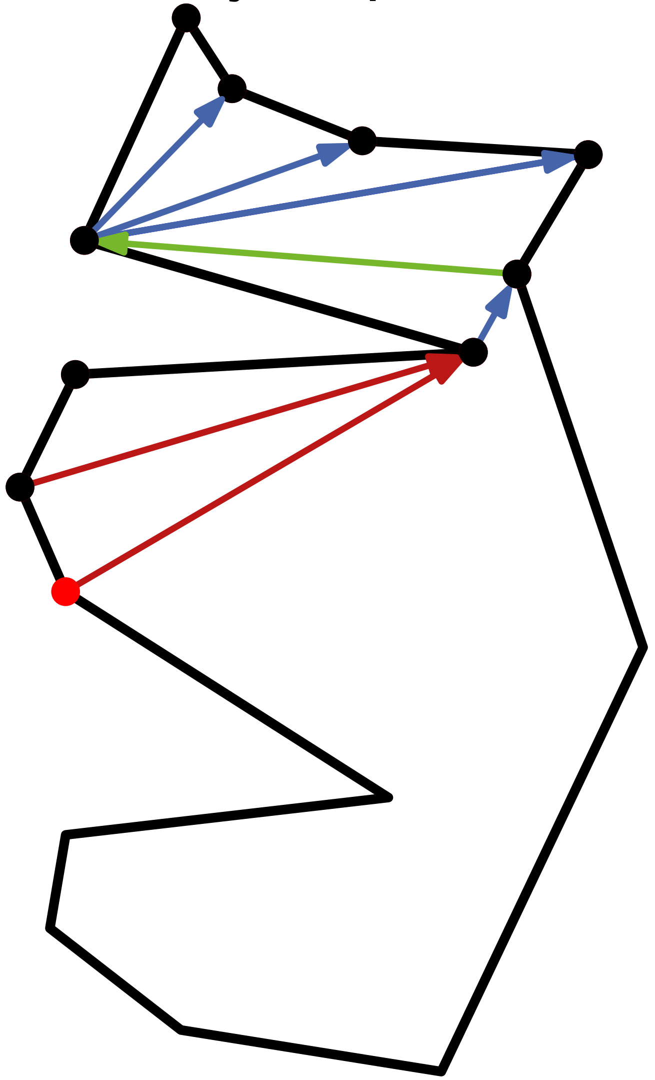
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

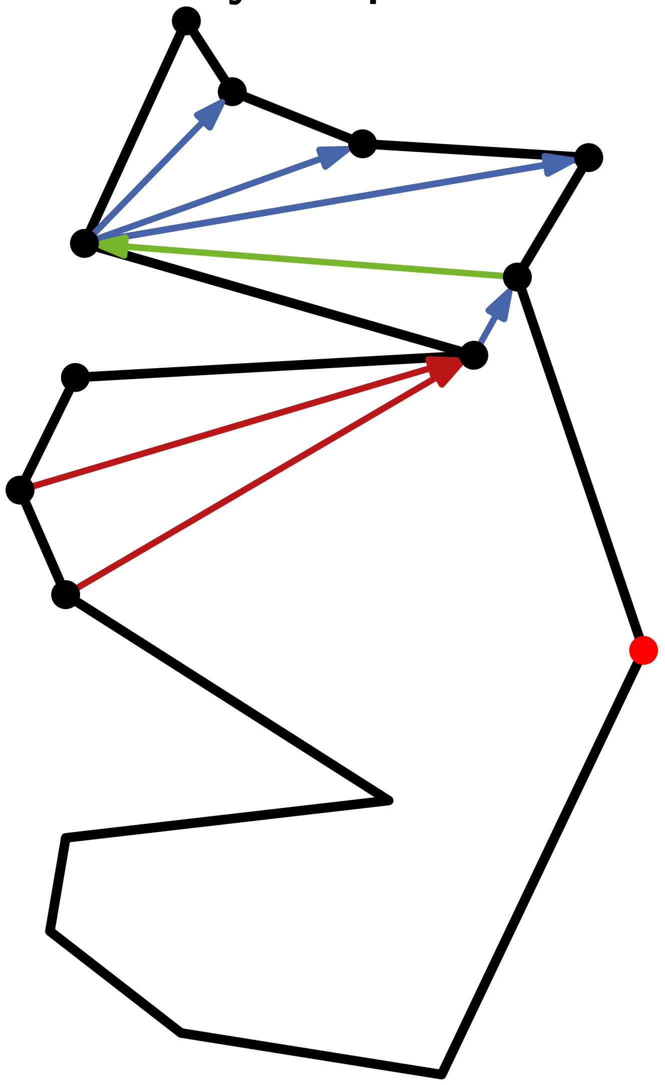
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

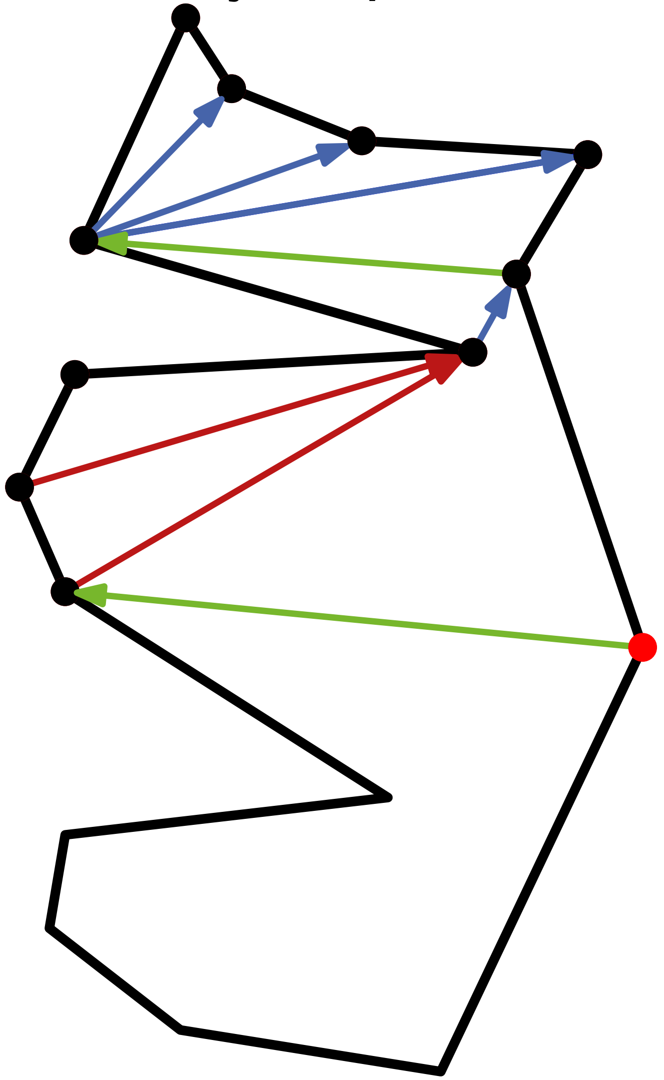
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

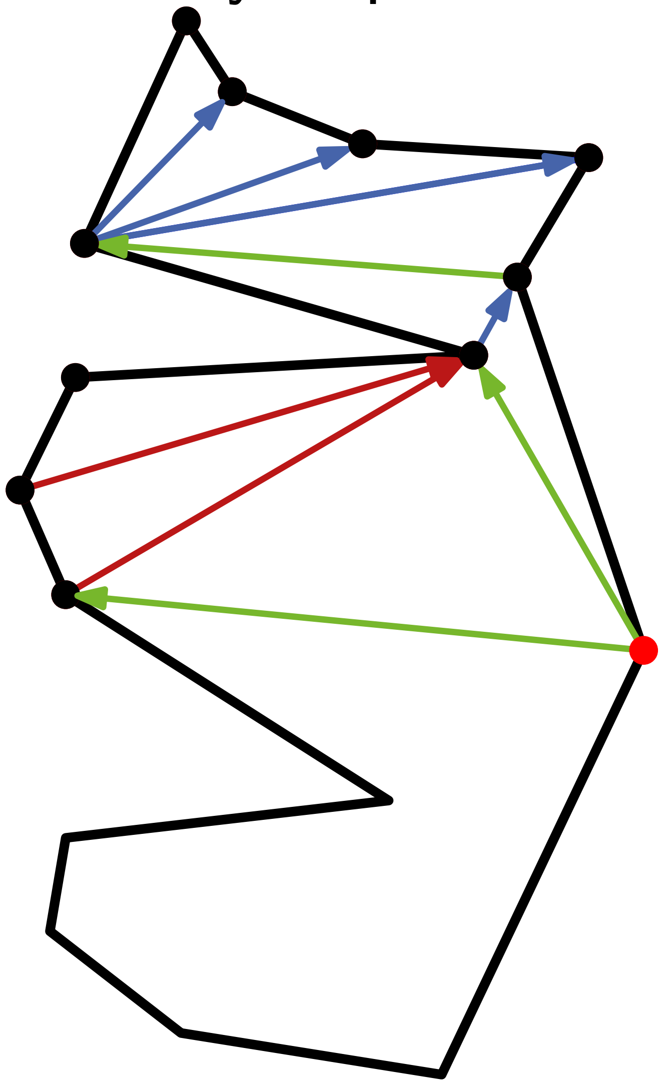
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

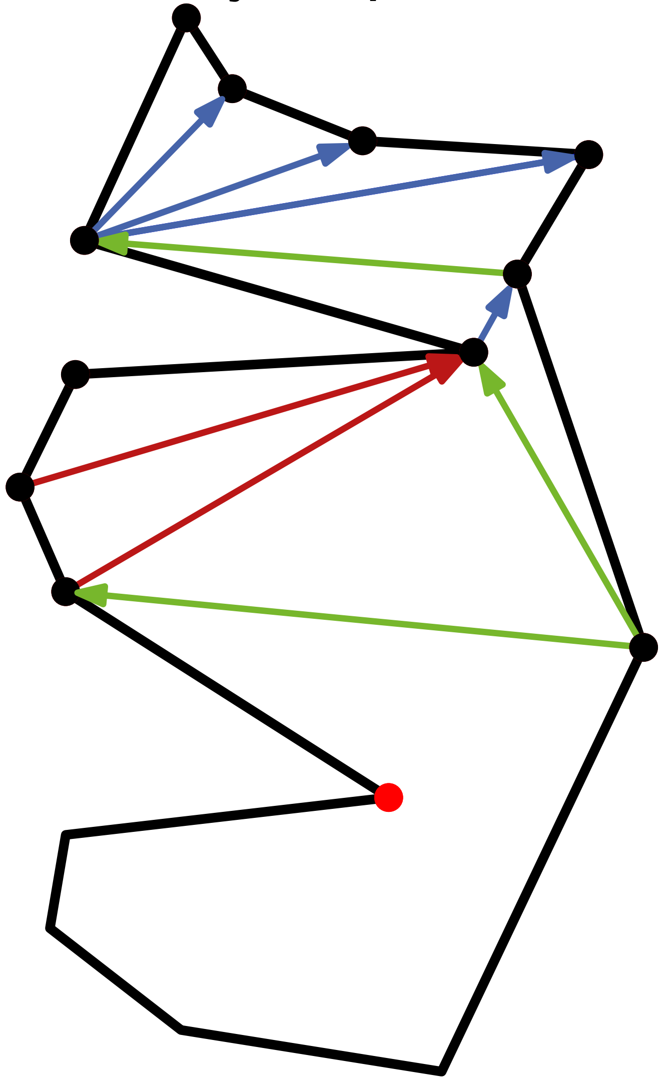
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

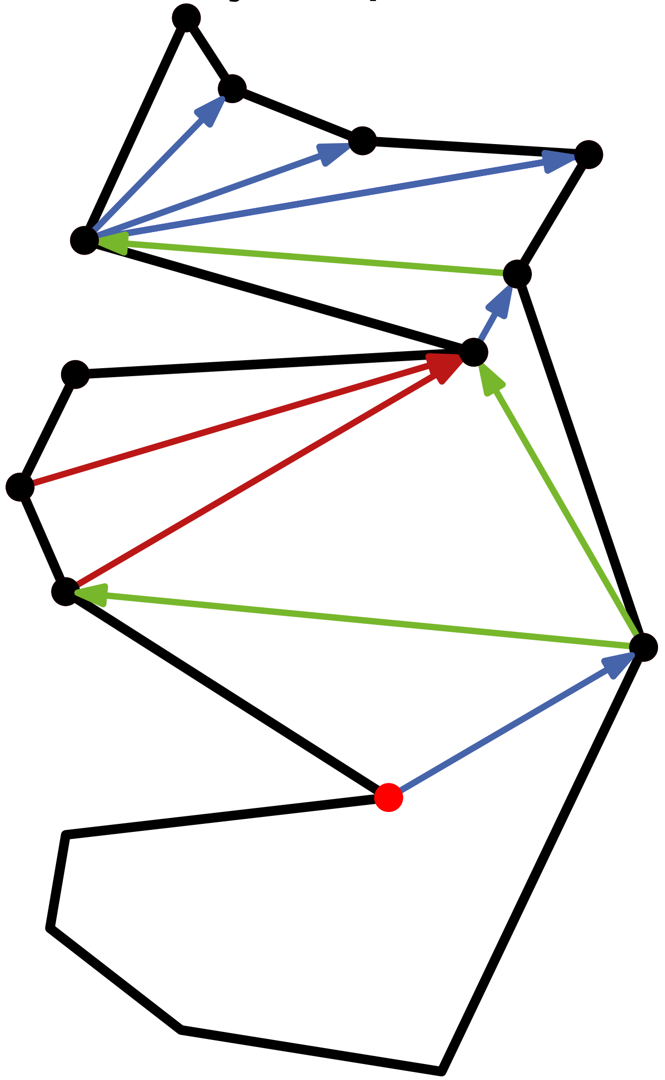
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

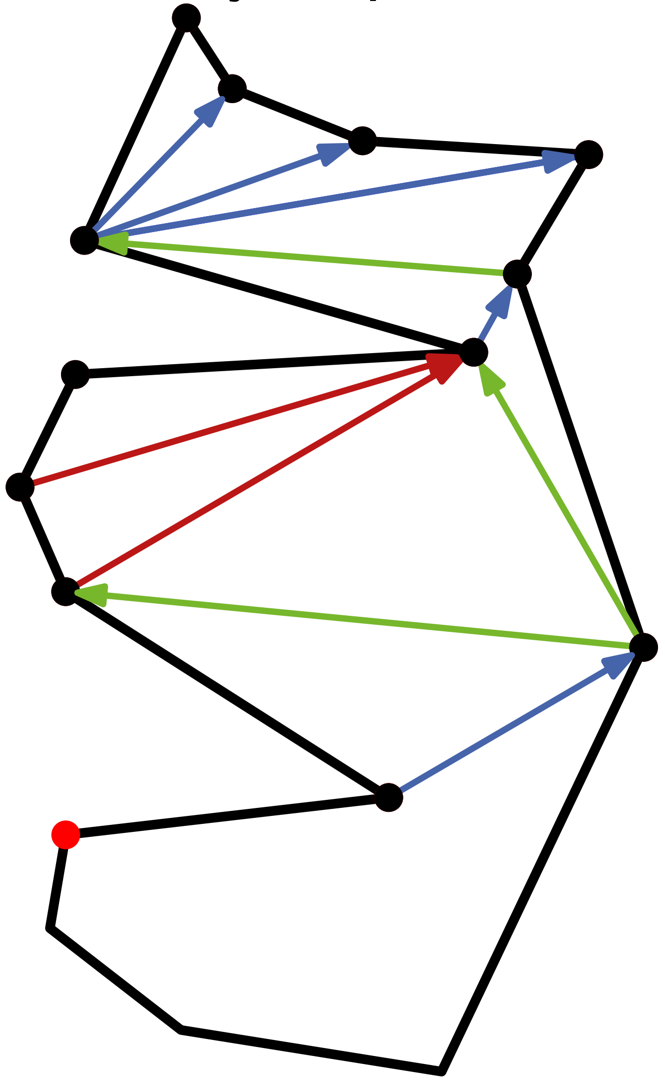
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

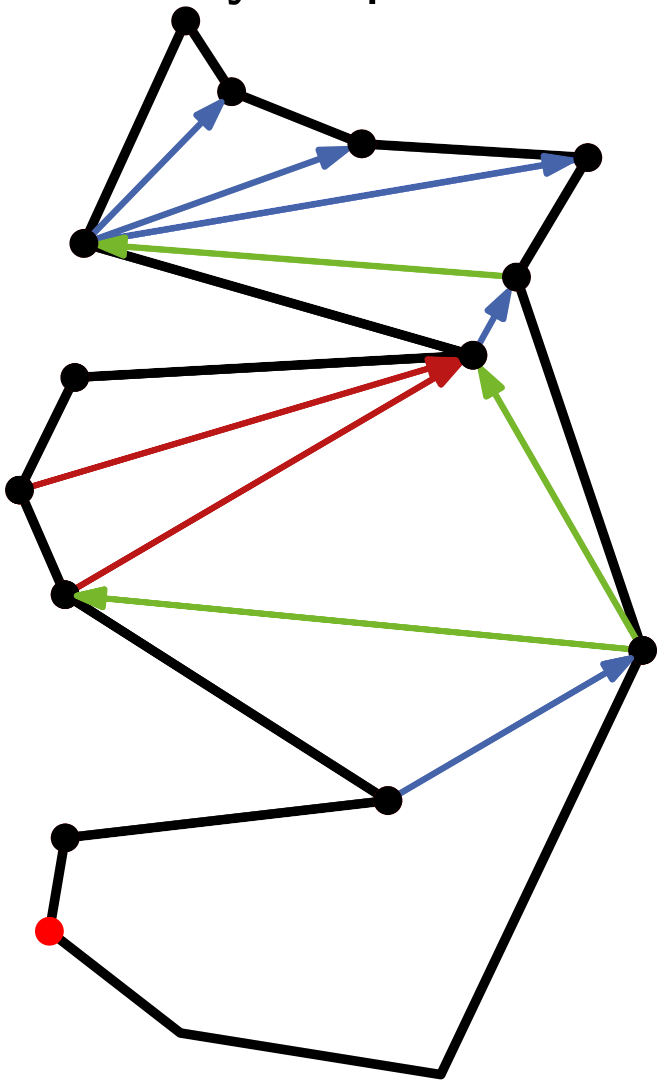
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

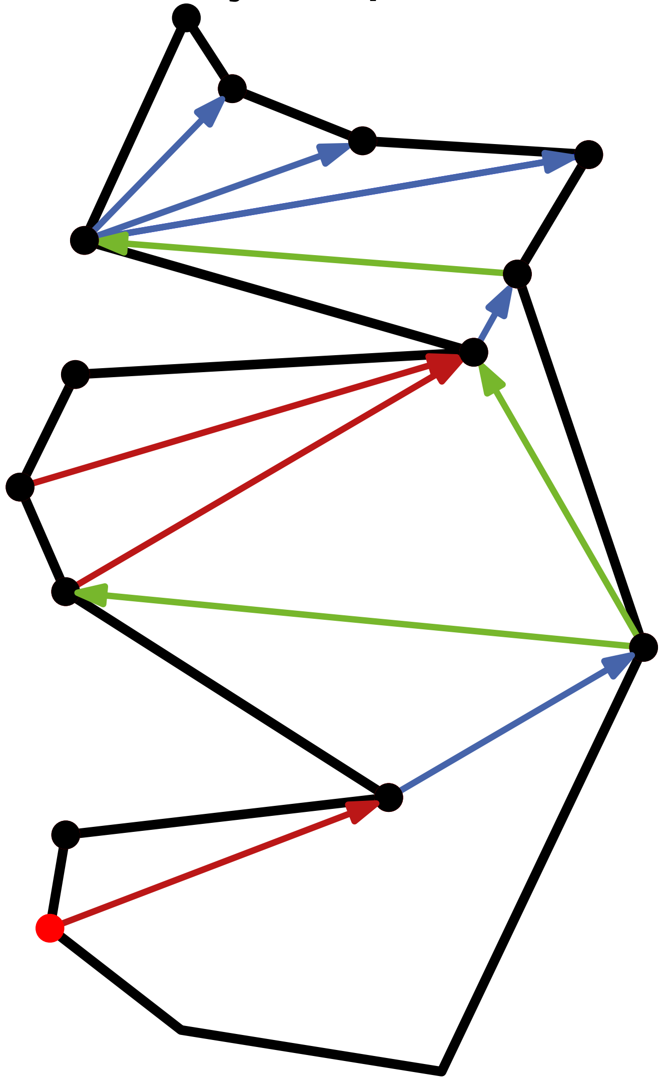
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

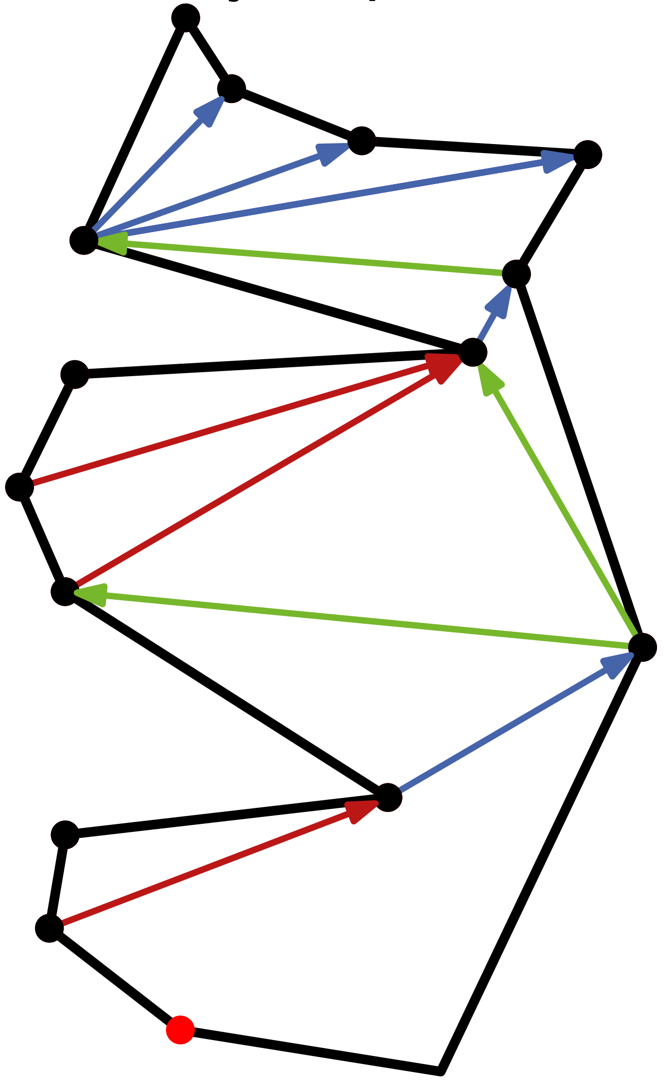
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

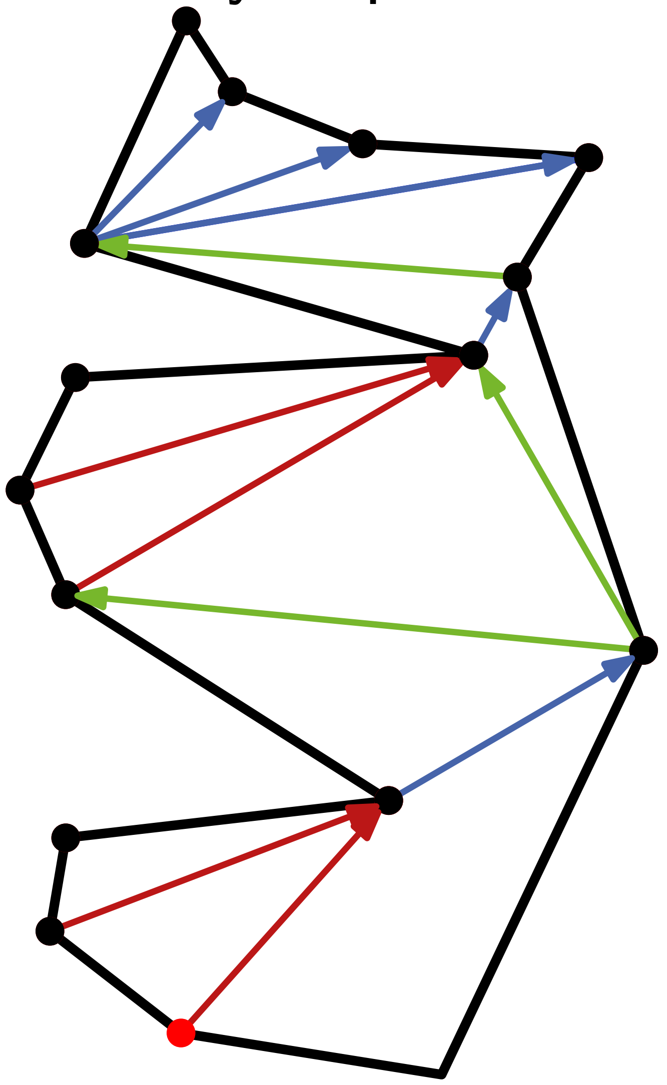
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

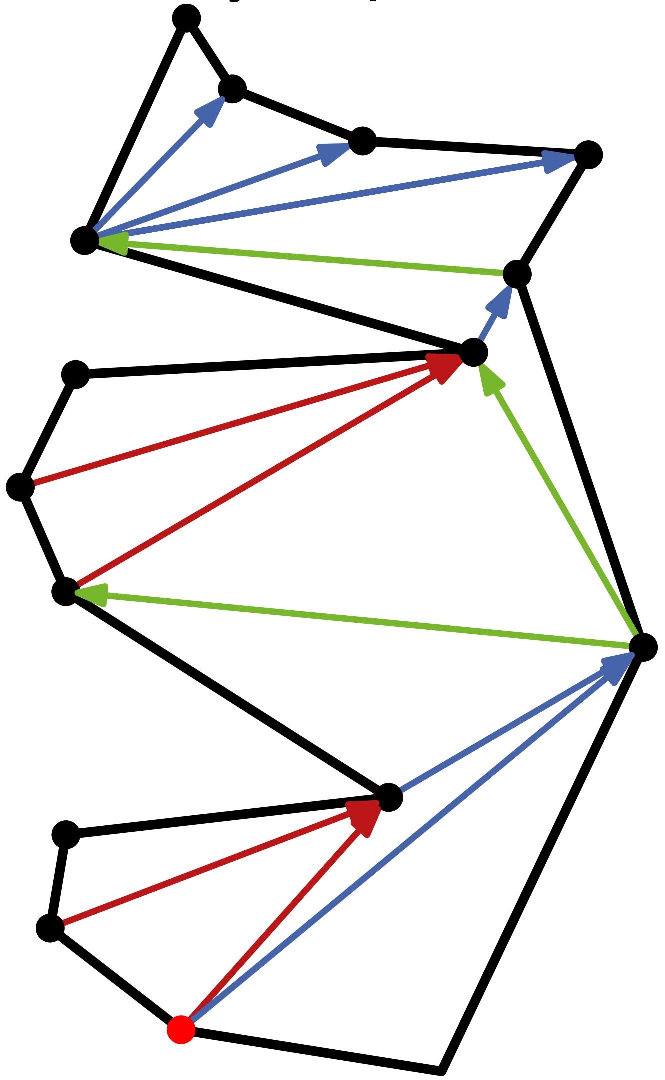
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

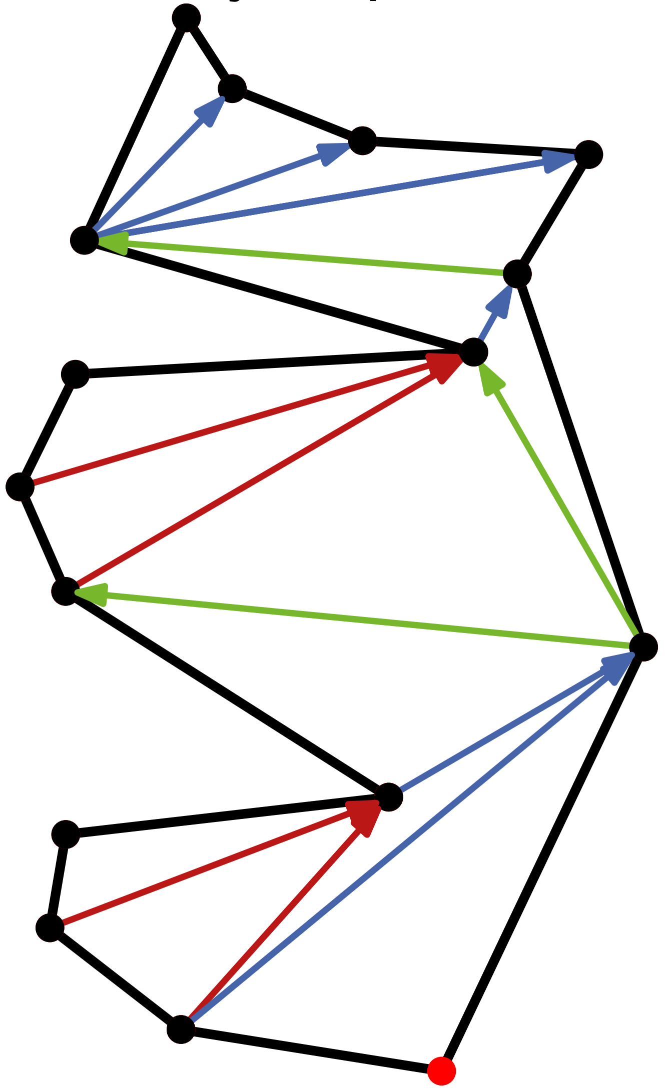
Approach: Greedy, top down traversal of both sides



Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

Approach: Greedy, top down traversal of both sides

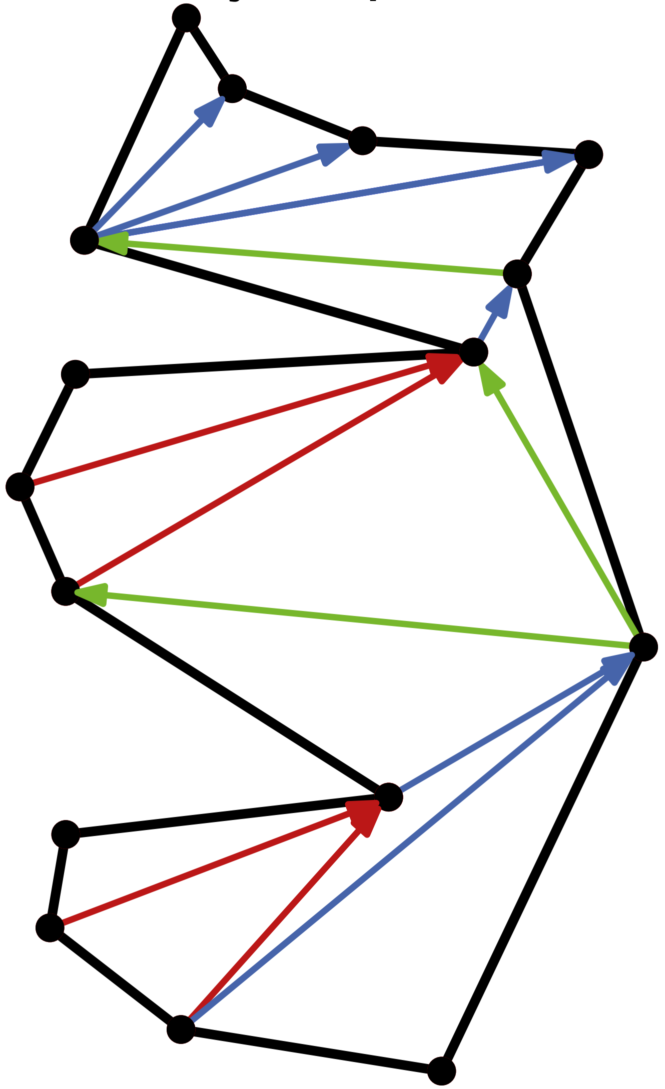


Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

Approach: Greedy, top down traversal of both sides

Invariant?

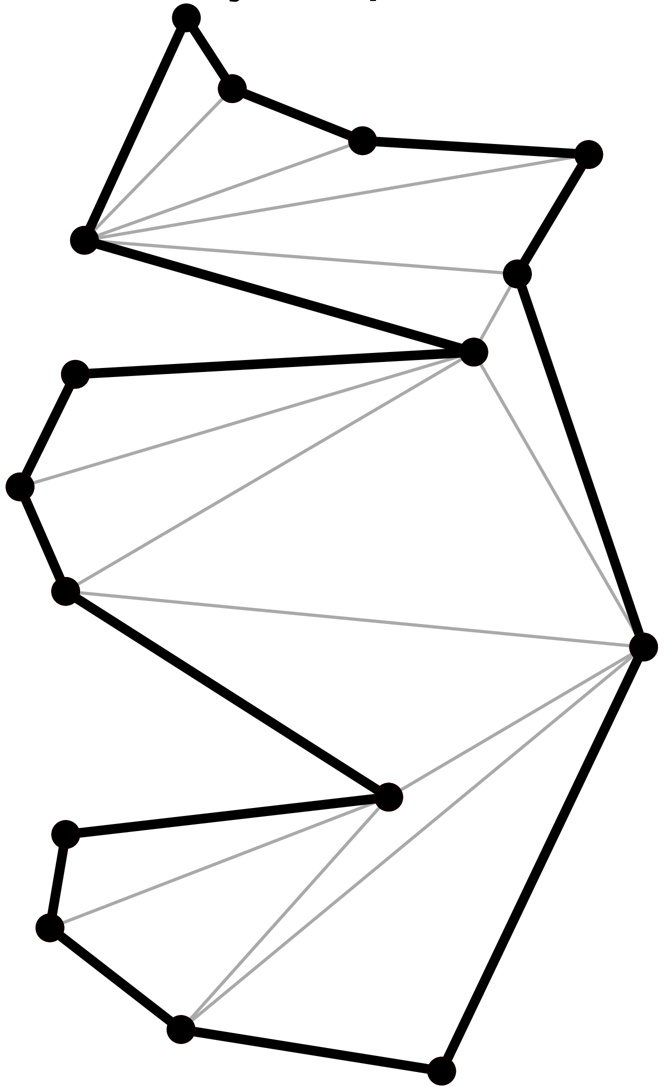


Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

Approach: Greedy, top down traversal of both sides

Invariant?

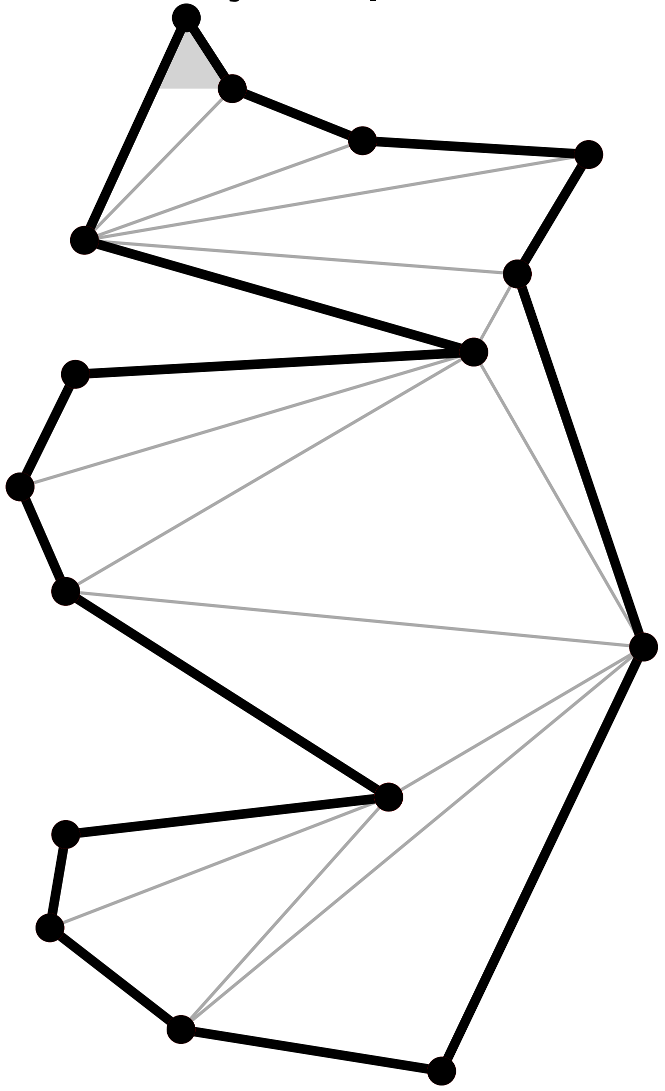


Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

Approach: Greedy, top down traversal of both sides

Invariant?

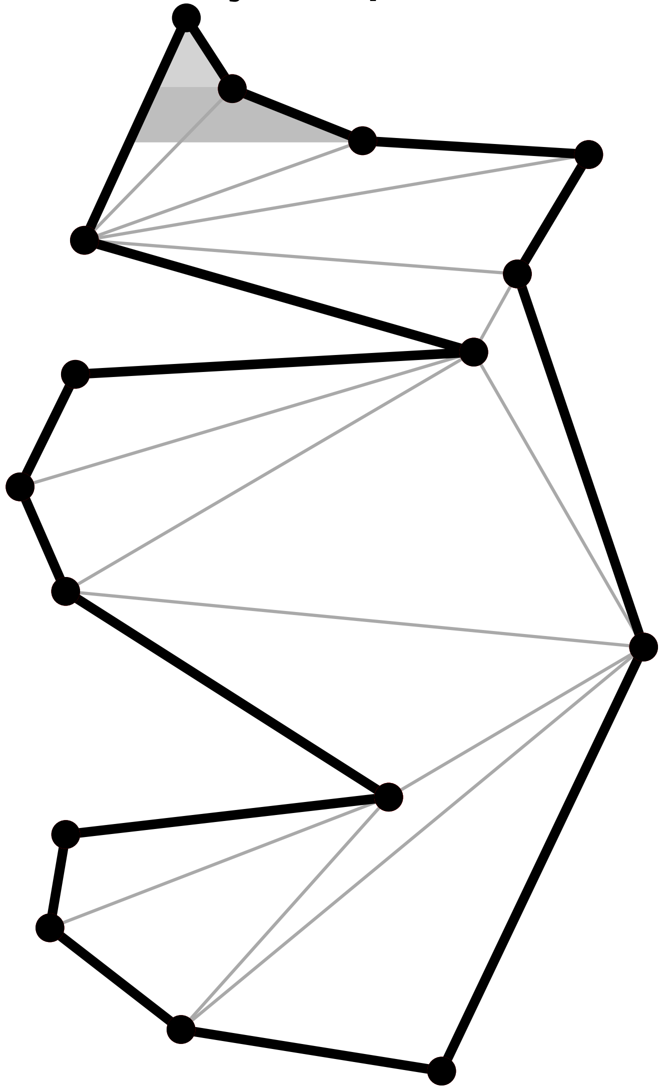


Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

Approach: Greedy, top down traversal of both sides

Invariant?

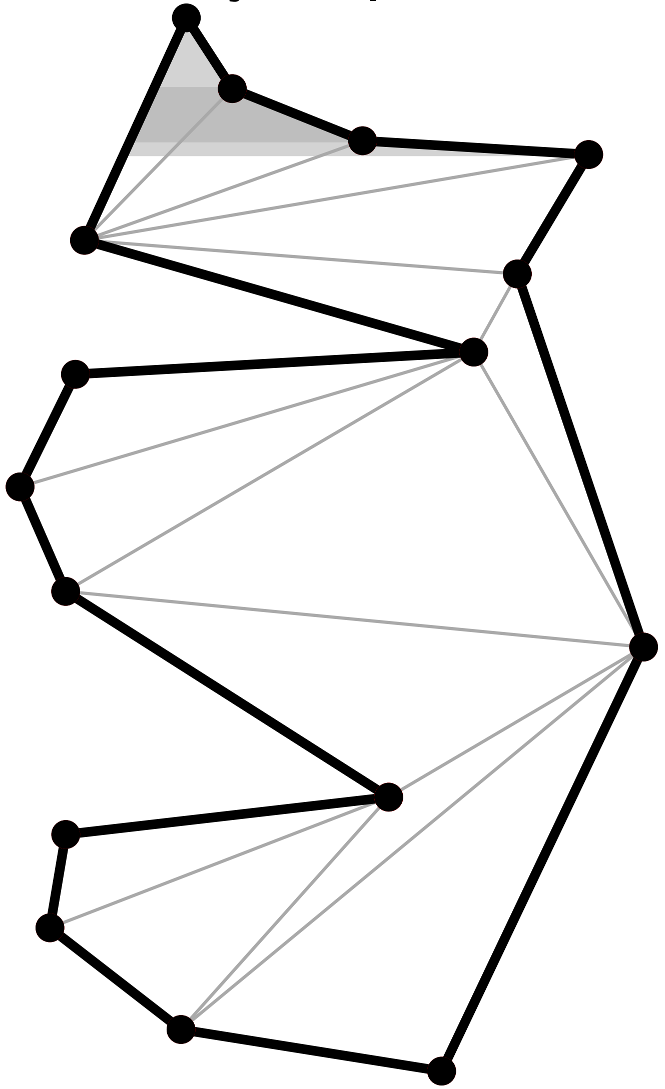


Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

Approach: Greedy, top down traversal of both sides

Invariant?

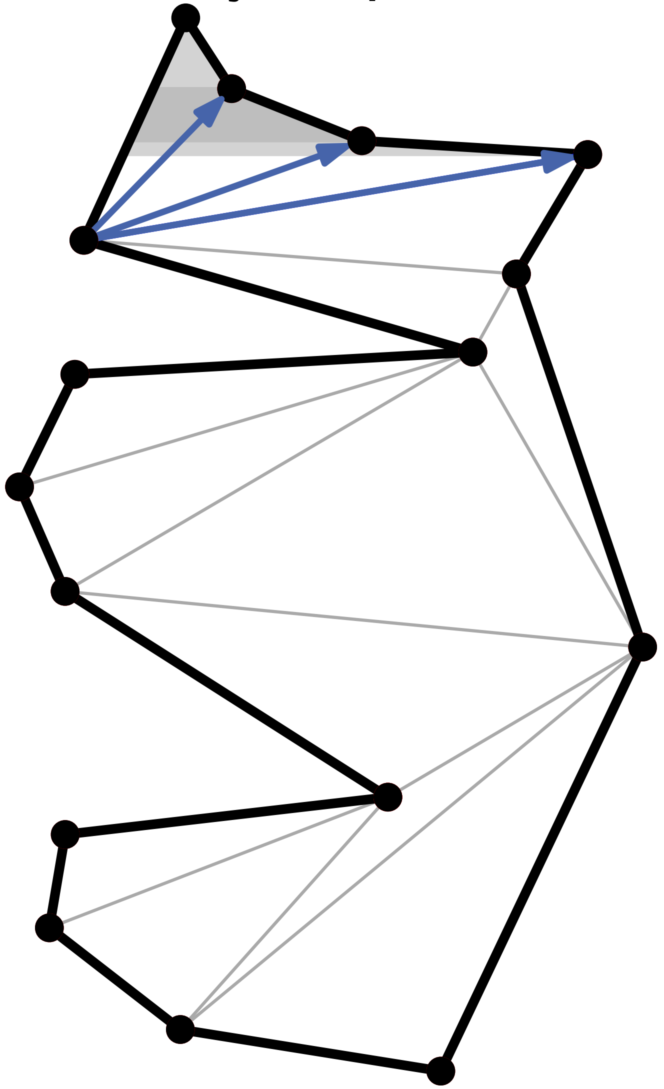


Triangulate y-monotone Polygon

Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

Approach: Greedy, top down traversal of both sides

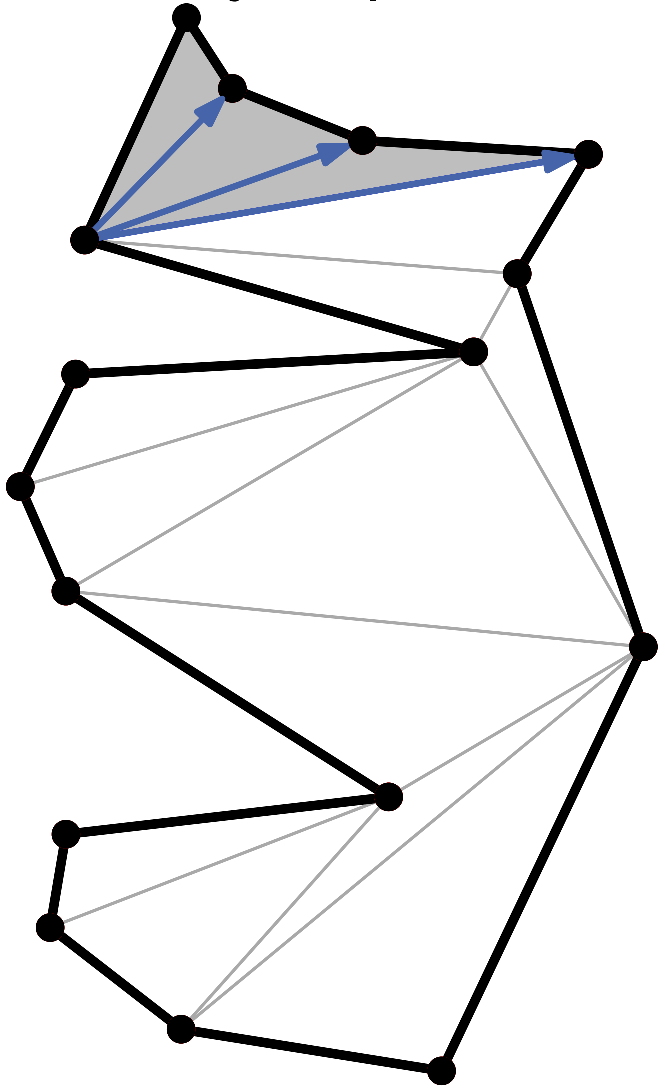
Invariant?



Triangulate y-monotone Polygon

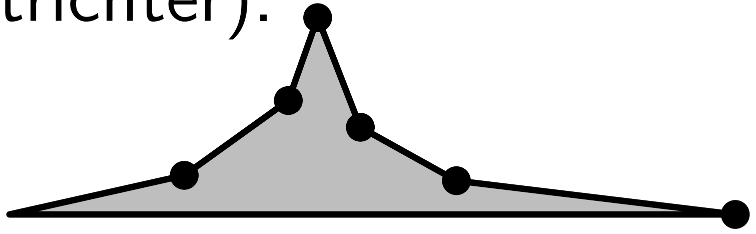
Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

Approach: Greedy, top down traversal of both sides



Invariant?

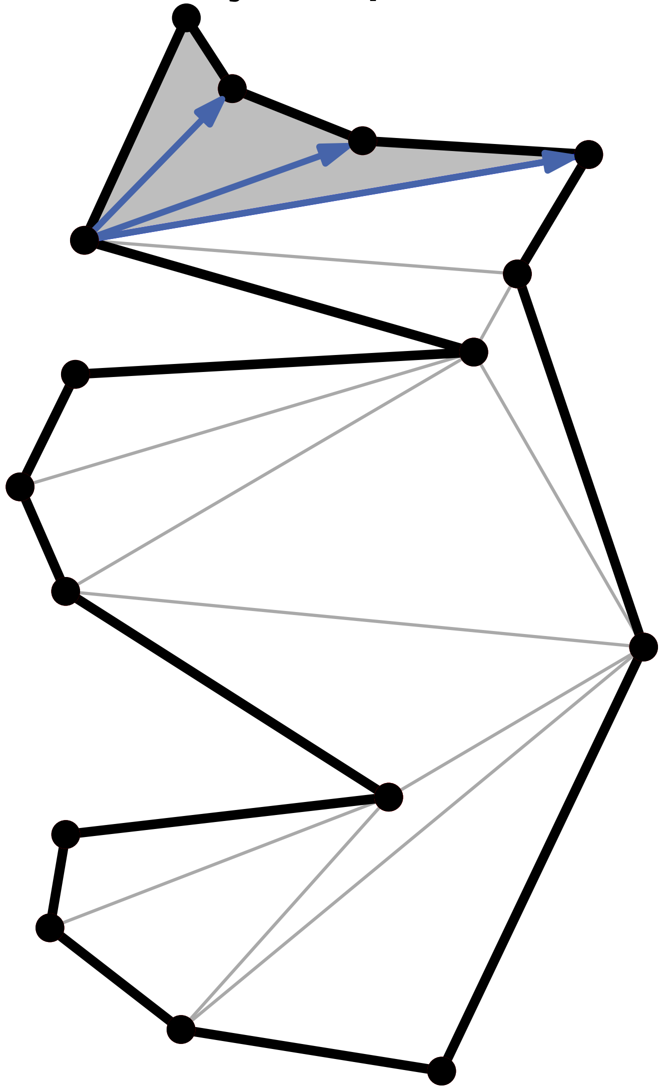
The already visited but not triangulated polygon has the shape of a *funnel* (trichter).



Triangulate y-monotone Polygon

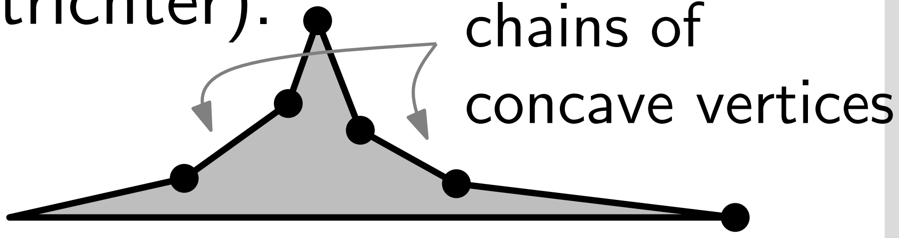
Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

Approach: Greedy, top down traversal of both sides



Invariant?

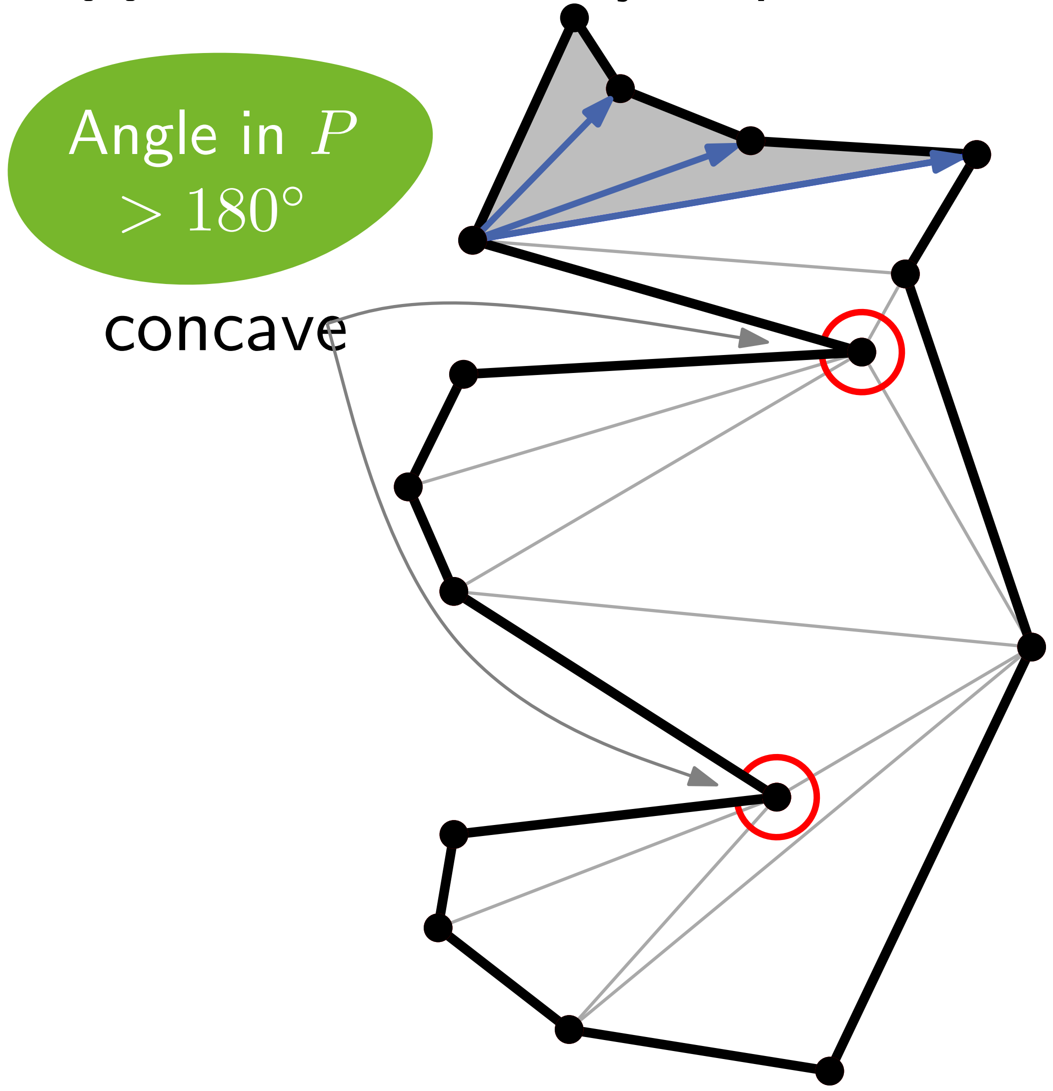
The already visited but not triangulated polygon has the shape of a *funnel* (trichter).



Triangulate y-monotone Polygon

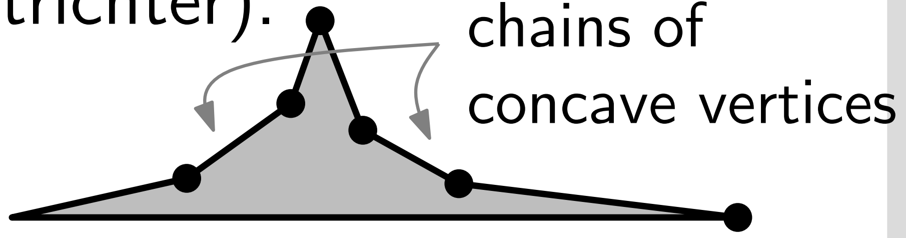
Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

Approach: Greedy, top down traversal of both sides



Invariant?

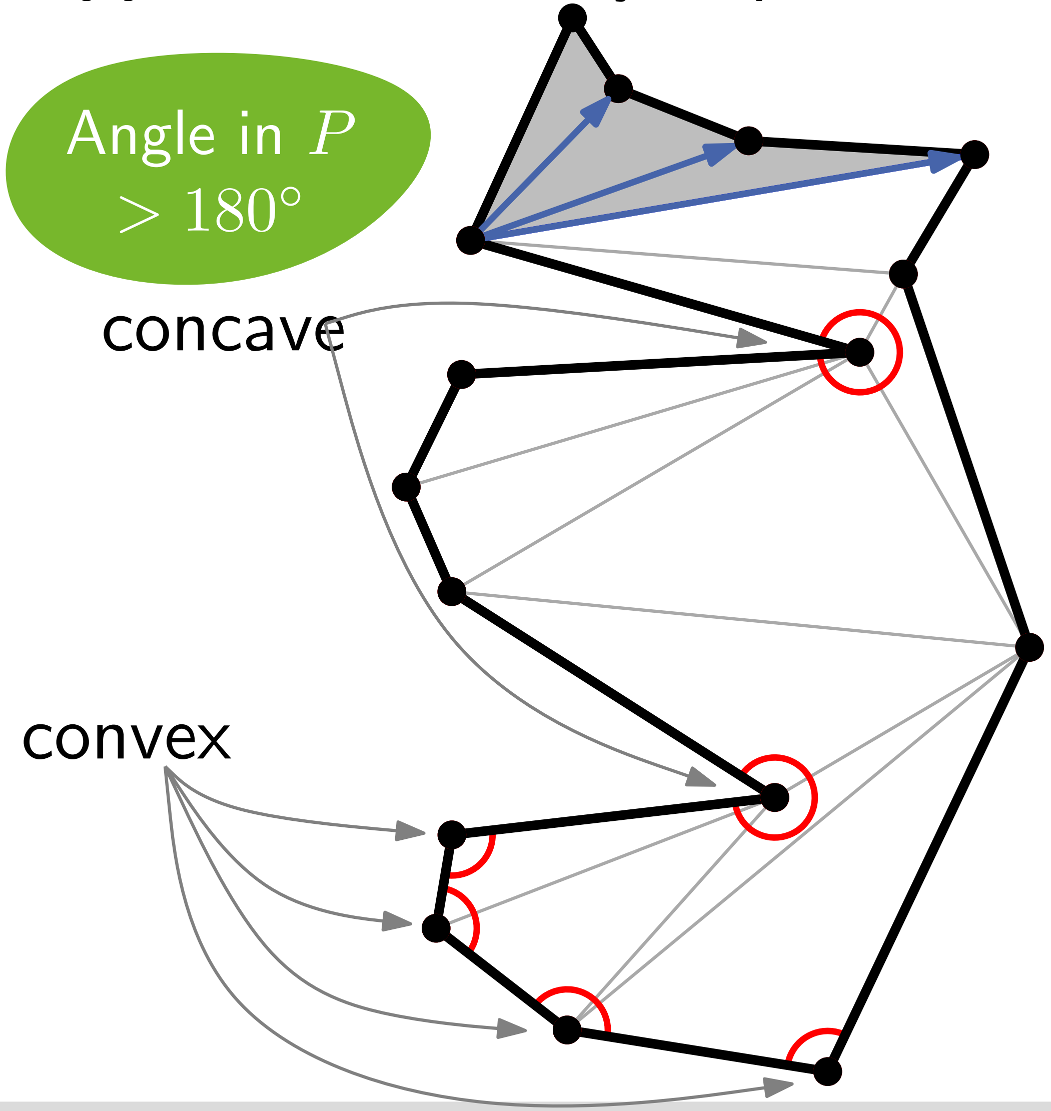
The already visited but not triangulated polygon has the shape of a *funnel* (trichter).



Triangulate y-monotone Polygon

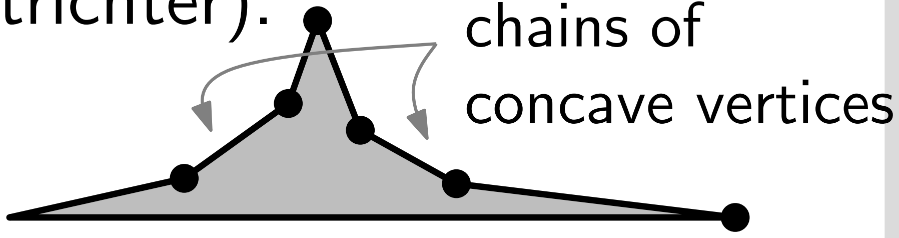
Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

Approach: Greedy, top down traversal of both sides



Invariant?

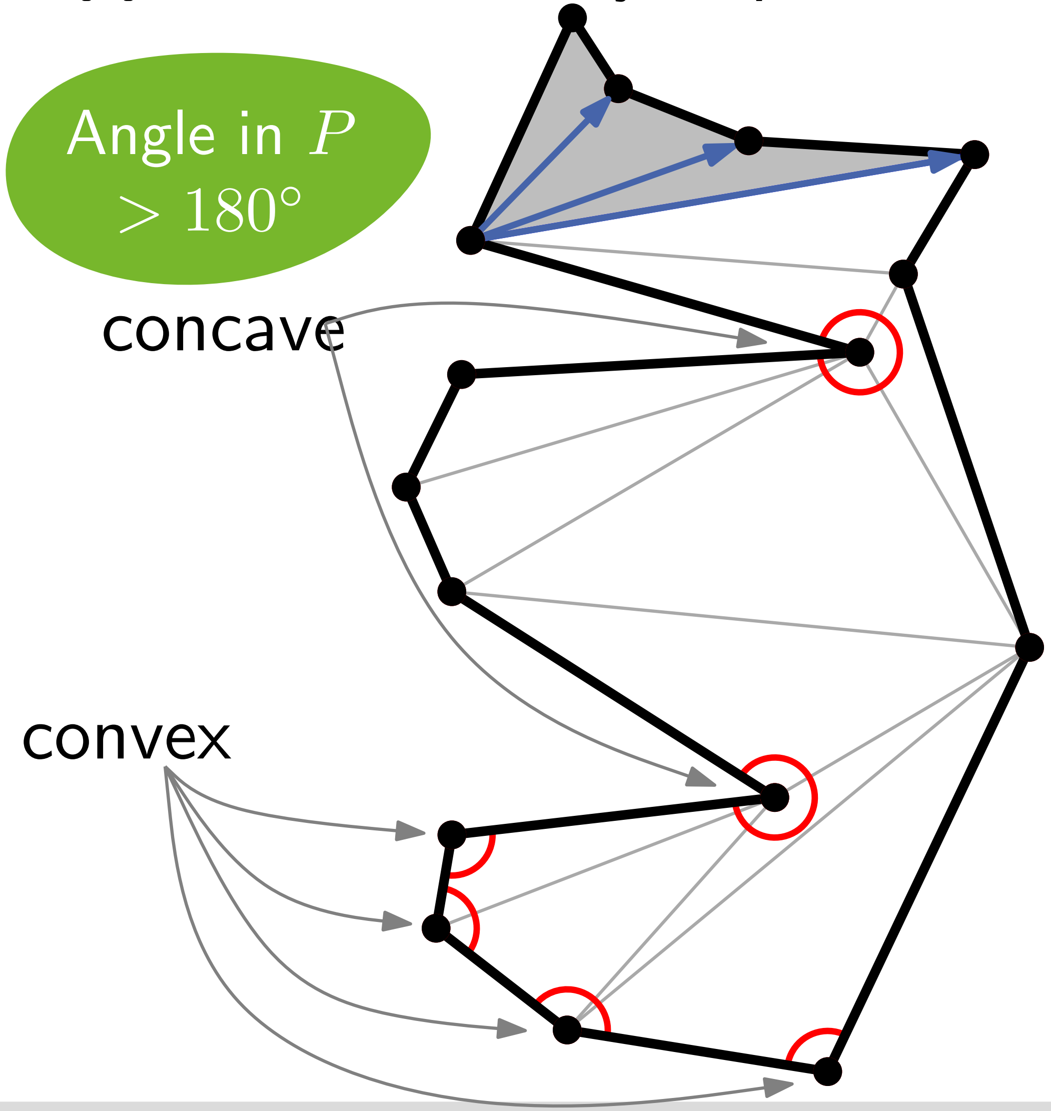
The already visited but not triangulated polygon has the shape of a *funnel* (trichter).



Triangulate y-monotone Polygon

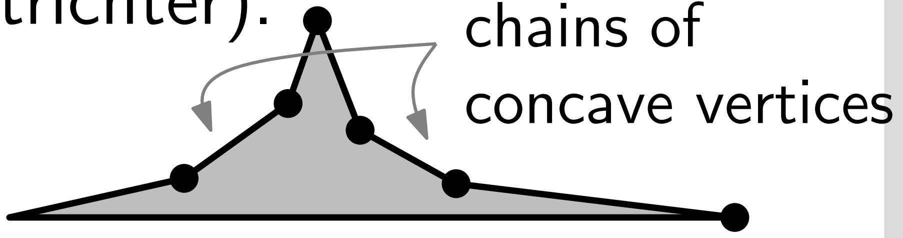
Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

Approach: Greedy, top down traversal of both sides

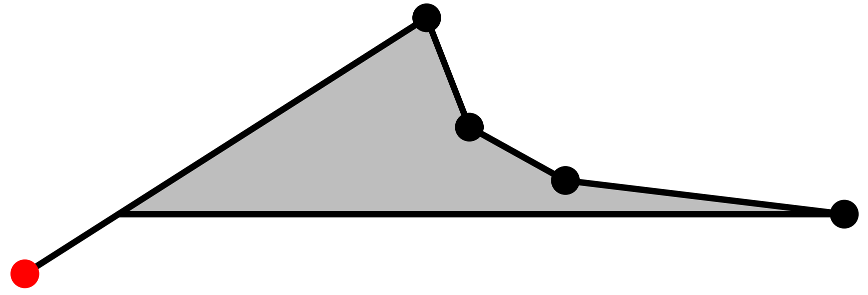


Invariant?

The already visited but not triangulated polygon has the shape of a *funnel* (trichter).



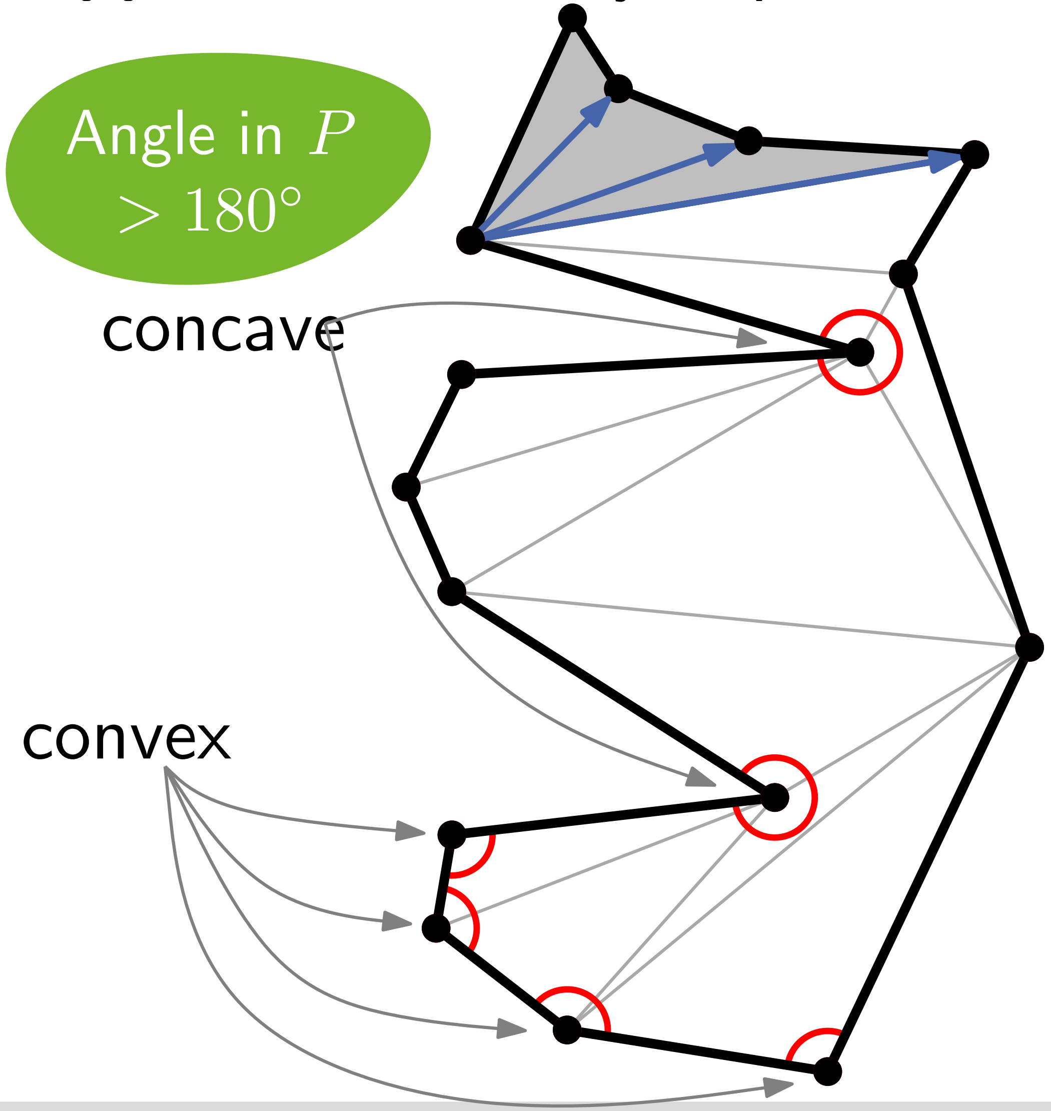
In our case:



Triangulate y-monotone Polygon

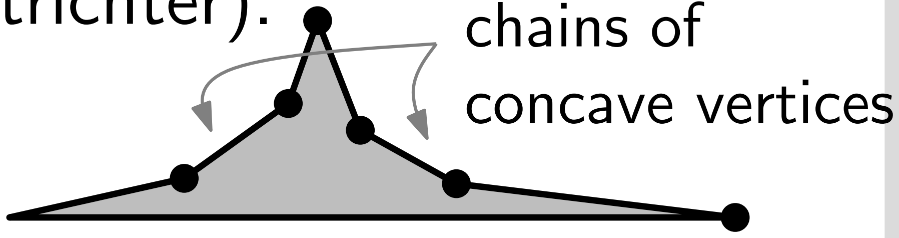
Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

Approach: Greedy, top down traversal of both sides

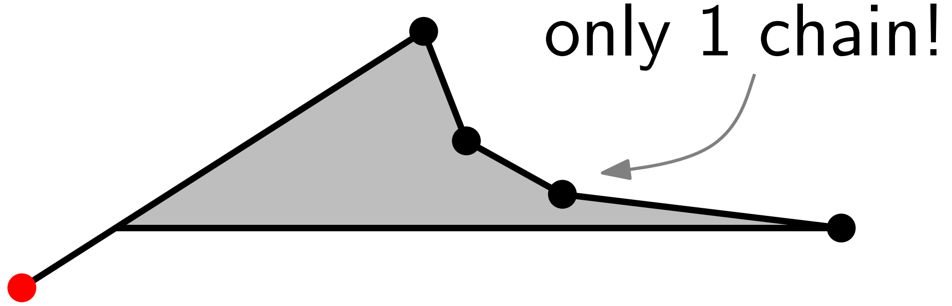


Invariant?

The already visited but not triangulated polygon has the shape of a *funnel* (trichter).



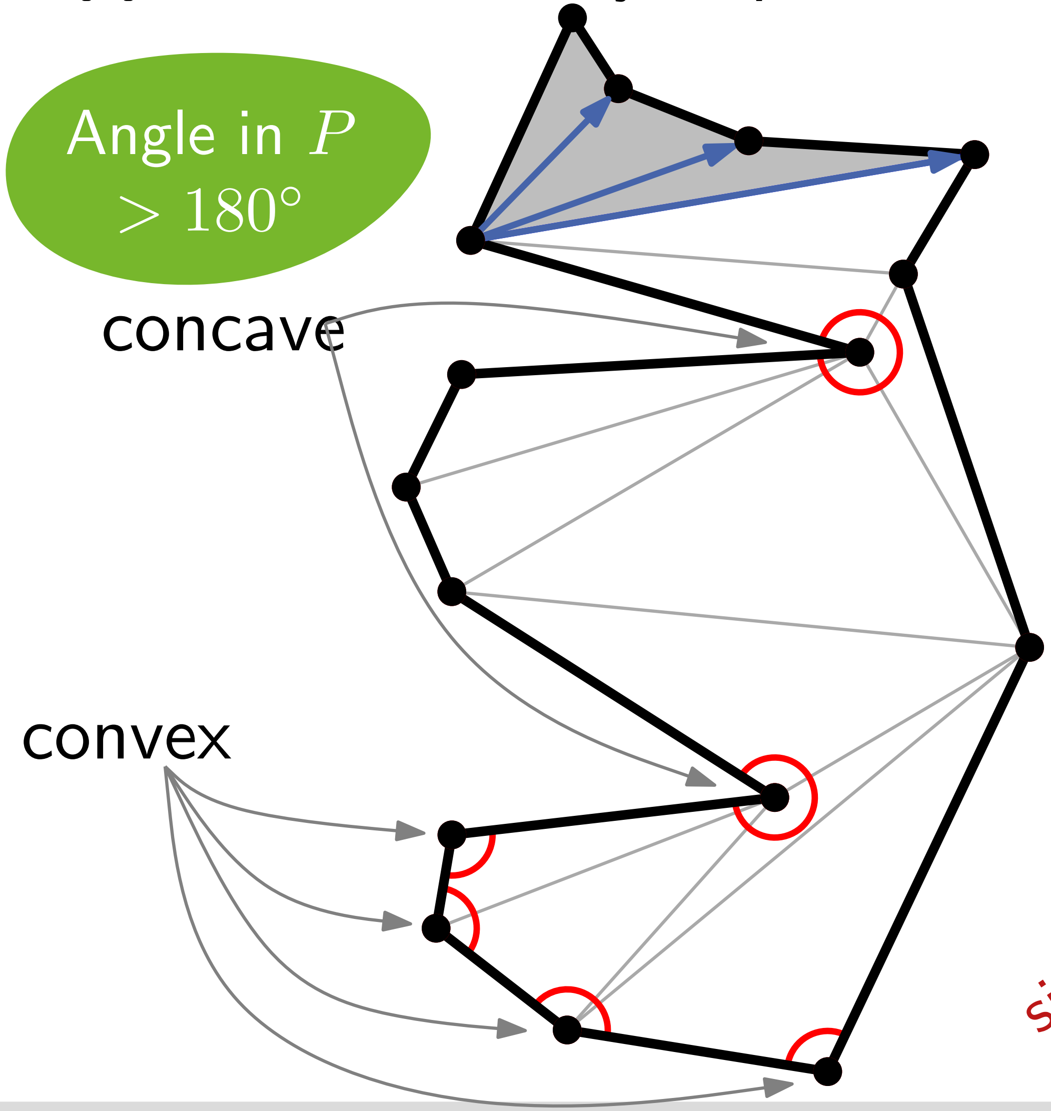
In our case:



Triangulate y-monotone Polygon

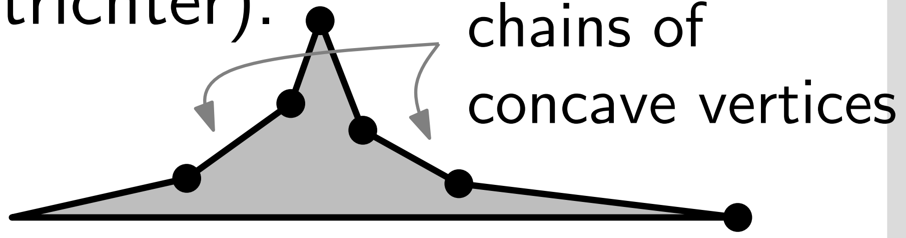
Reminder: The left and the right boundary of the polygon have decreasing y-coordinates

Approach: Greedy, top down traversal of both sides

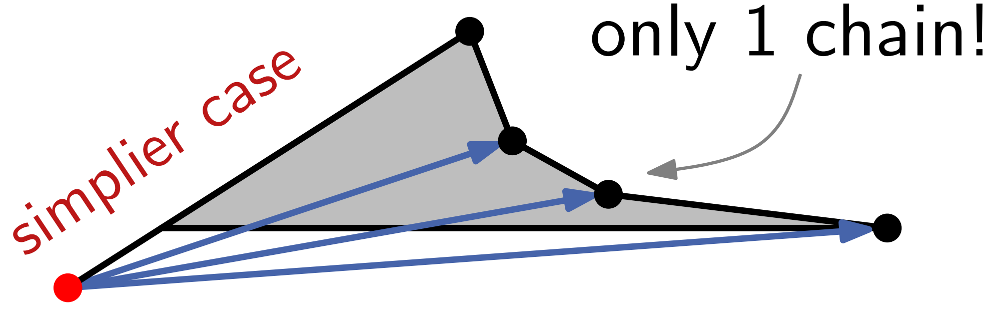


Invariant?

The already visited but not triangulated polygon has the shape of a *funnel* (trichter).



In our case:



Algorithm TriangulateMonotonePolygon

```
TriangulateMonotonePolygon(Polygon $P$ as doubly-connected list of
edges)
    Merge vertices on left and right chains into desc. seq. $\to u_1, \ldots, u_n$
```

Algorithm TriangulateMonotonePolygon

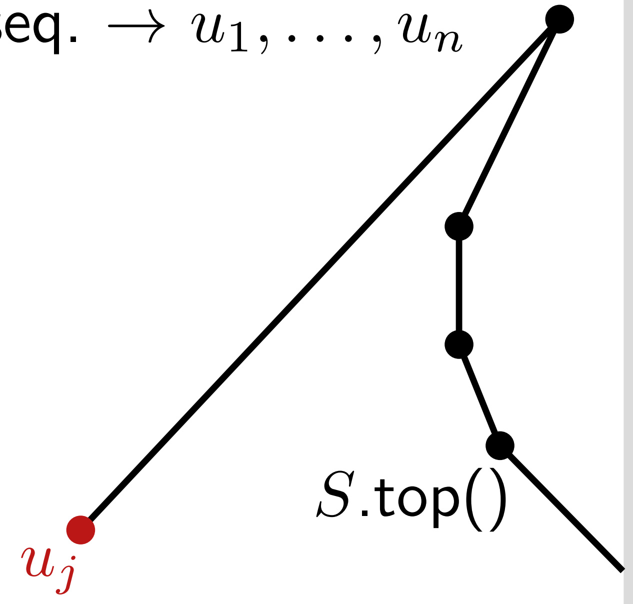
TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

```
Merge vertices on left and right chains into desc. seq. $\rightarrow u_1, \ldots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push($u_1$); $S$.push($u_2$)
for $j \leftarrow 3$ to $n - 1$ do
    if $u_j$ and $S$.top() from different paths then
        while not $S$.empty() do
            $v \leftarrow S$.pop()
            if not $S$.empty() then draw $(u_j, v)$
        $S$.push($u_{j-1}$); $S$.push($u_j$)
```

Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

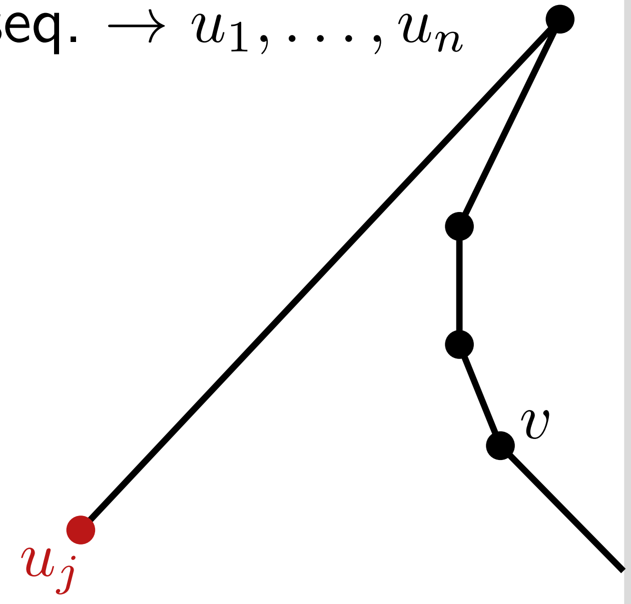
```
Merge vertices on left and right chains into desc. seq. $\rightarrow u_1, \ldots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push($u_1$); $S$.push($u_2$)
for $j \leftarrow 3$ to $n-1$ do
    if $u_j$ and $S$.top() from different paths then
        while not $S$.empty() do
            $v \leftarrow S$.pop()
            if not $S$.empty() then draw $(u_j, v)$
        $S$.push($u_{j-1}$); $S$.push($u_j$)
```



Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

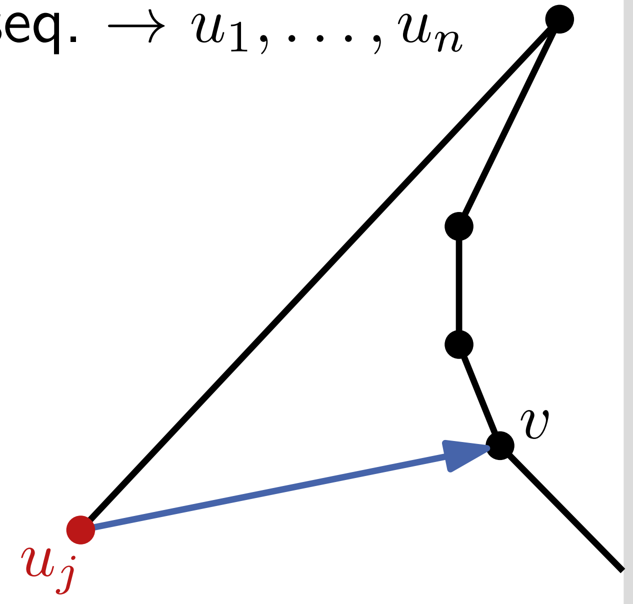
```
Merge vertices on left and right chains into desc. seq. $\rightarrow u_1, \ldots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push($u_1$); $S$.push($u_2$)
for $j \leftarrow 3$ to $n-1$ do
    if $u_j$ and $S$.top() from different paths then
        while not $S$.empty() do
            $v \leftarrow S$.pop()
            if not $S$.empty() then draw $(u_j, v)$
        $S$.push($u_{j-1}$); $S$.push($u_j$)
```



Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

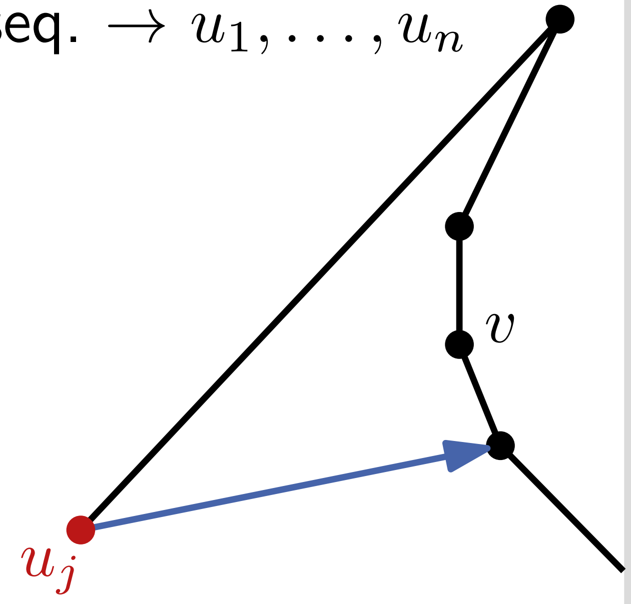
```
Merge vertices on left and right chains into desc. seq. $\rightarrow u_1, \ldots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push($u_1$); $S$.push($u_2$)
for $j \leftarrow 3$ to $n-1$ do
    if $u_j$ and $S$.top() from different paths then
        while not $S$.empty() do
            $v \leftarrow S$.pop()
            if not $S$.empty() then draw $(u_j, v)$
        $S$.push($u_{j-1}$); $S$.push($u_j$)
```



Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

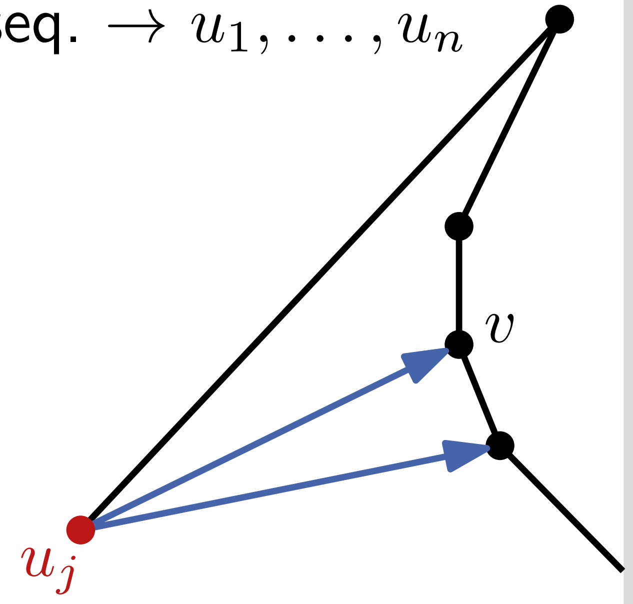
```
Merge vertices on left and right chains into desc. seq. $\rightarrow u_1, \dots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push($u_1$); $S$.push($u_2$)
**for** $j \leftarrow 3$ **to** $n-1$ **do**
    **if** $u_j$ and $S$.top() from different paths **then**
        **while not** $S$.empty() **do**
            $v \leftarrow S$.pop()
            **if not** $S$.empty() **then** draw $(u_j, v)$
        $S$.push($u_{j-1}$); $S$.push($u_j$)
```



Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

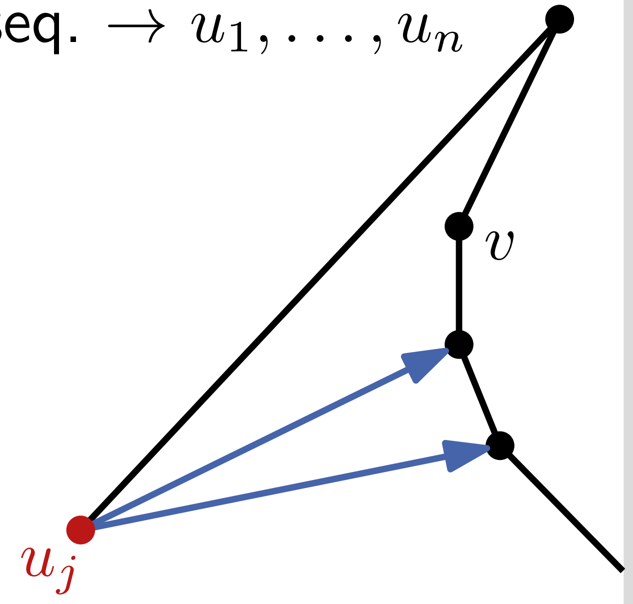
```
Merge vertices on left and right chains into desc. seq. $\rightarrow u_1, \ldots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push($u_1$); $S$.push($u_2$)
for $j \leftarrow 3$ to $n-1$ do
    if $u_j$ and $S$.top() from different paths then
        while not $S$.empty() do
            $v \leftarrow S$.pop()
            if not $S$.empty() then draw $(u_j, v)$
        $S$.push($u_{j-1}$); $S$.push($u_j$)
```



Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

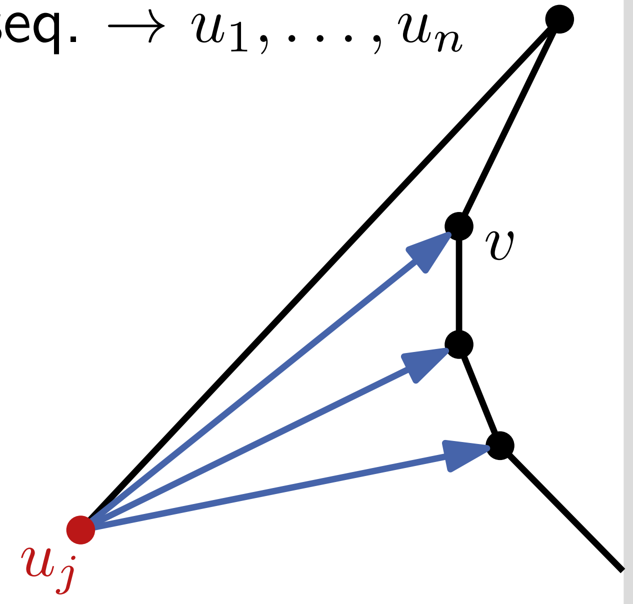
```
Merge vertices on left and right chains into desc. seq. $\rightarrow u_1, \ldots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push$(u_1)$; $S$.push$(u_2)$
**for** $j \leftarrow 3$ **to** $n-1$ **do**
    **if** $u_j$ and $S$.top() from different paths **then**
        **while not** $S$.empty() **do**
            $v \leftarrow S$.pop()
            **if not** $S$.empty() **then** draw $(u_j, v)$
        $S$.push$(u_{j-1})$; $S$.push$(u_j)$
```



Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

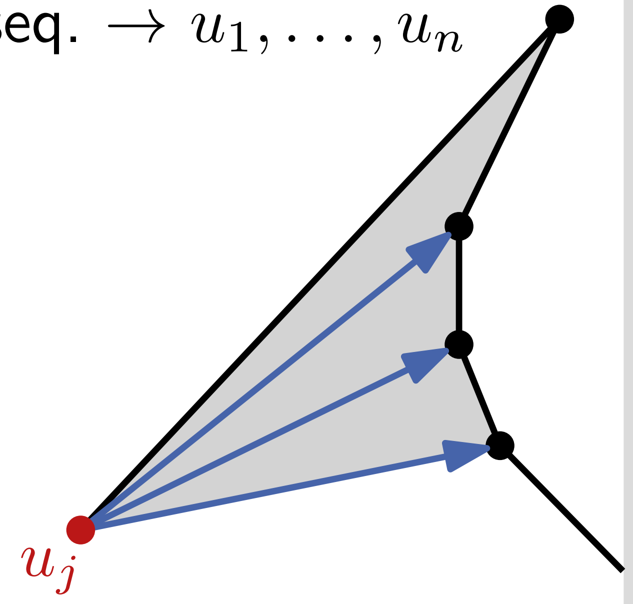
```
Merge vertices on left and right chains into desc. seq. $\rightarrow u_1, \dots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push($u_1$); $S$.push($u_2$)
for $j \leftarrow 3$ to $n-1$ do
    if $u_j$ and $S$.top() from different paths then
        while not $S$.empty() do
            $v \leftarrow S$.pop()
            if not $S$.empty() then draw $(u_j, v)$
        $S$.push($u_{j-1}$); $S$.push($u_j$)
```



Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

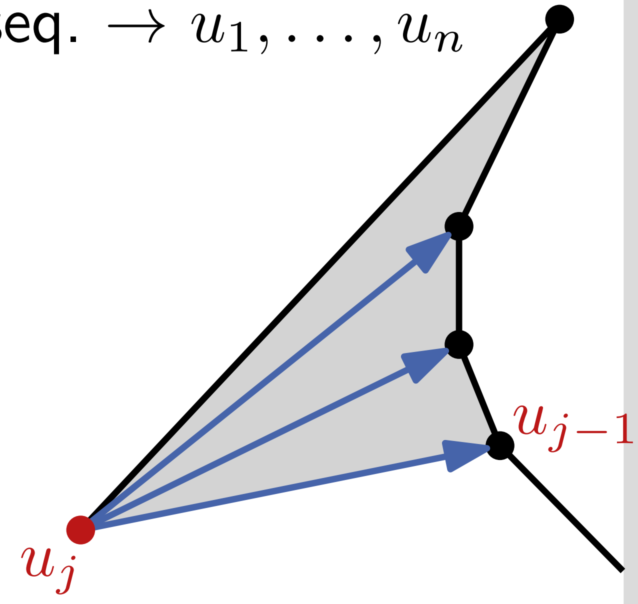
```
Merge vertices on left and right chains into desc. seq. $\rightarrow u_1, \dots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push($u_1$); $S$.push($u_2$)
for $j \leftarrow 3$ to $n-1$ do
    if $u_j$ and $S$.top() from different paths then
        while not $S$.empty() do
            $v \leftarrow S$.pop()
            if not $S$.empty() then draw $(u_j, v)$
        $S$.push($u_{j-1}$); $S$.push($u_j$)
```



Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

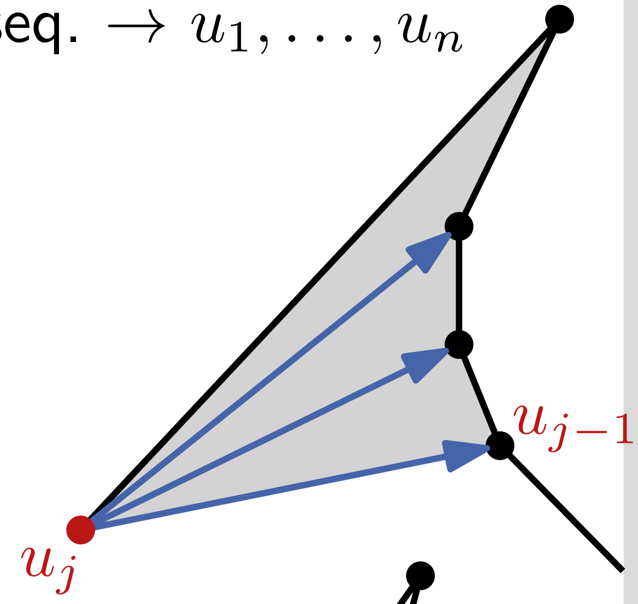
```
Merge vertices on left and right chains into desc. seq. $\rightarrow u_1, \dots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push$(u_1)$; $S$.push$(u_2)$
for $j \leftarrow 3$ to $n-1$ do
    if $u_j$ and $S$.top() from different paths then
        while not $S$.empty() do
            $v \leftarrow S$.pop()
            if not $S$.empty() then draw $(u_j, v)$
        $S$.push$(u_{j-1})$; $S$.push$(u_j)$
```

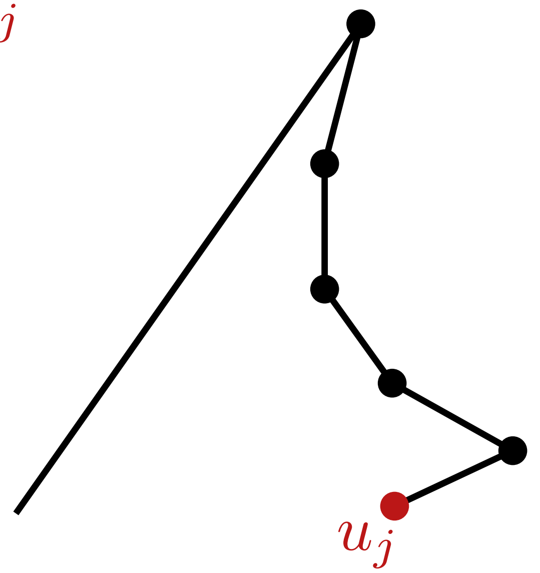


Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

```
Merge vertices on left and right chains into desc. seq. $\to u_1, \ldots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push($u_1$); $S$.push($u_2$)
**for** $j \leftarrow 3$ **to** $n-1$ **do**
    **if** $u_j$ and $S$.top() from different paths **then**
        **while not** $S$.empty() **do**
            $v \leftarrow S$.pop()
            **if not** $S$.empty() **then** draw $(u_j, v)$
        $S$.push($u_{j-1}$); $S$.push($u_j$)
    **else**
        $v \leftarrow S$.pop()
        **while not** $S$.empty() **and** $u_j$ sees $S$.top() **do**
            $v \leftarrow S$.pop()
            draw diagonal $(u_j, v)$
        $S$.push($v$); $S$.push($u_j$)
```

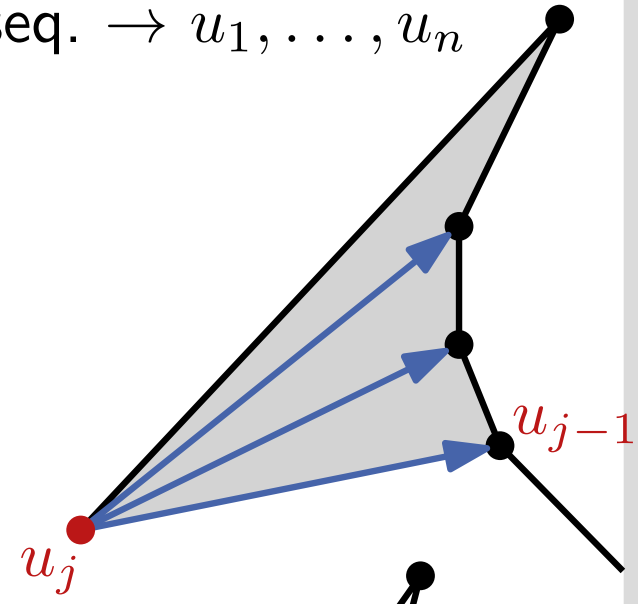


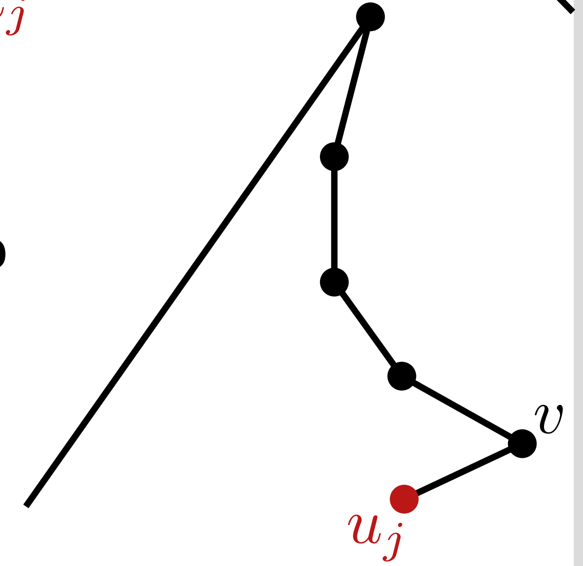


Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

```
Merge vertices on left and right chains into desc. seq. $\rightarrow u_1, \ldots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push($u_1$); $S$.push($u_2$)
for $j \leftarrow 3$ to $n-1$ do
    if $u_j$ and $S$.top() from different paths then
        while not $S$.empty() do
            $v \leftarrow S$.pop()
            if not $S$.empty() then draw $(u_j, v)$
        $S$.push($u_{j-1}$); $S$.push($u_j$)
    else
        $v \leftarrow S$.pop()
        while not $S$.empty() and $u_j$ sees $S$.top() do
            $v \leftarrow S$.pop()
            draw diagonal $(u_j, v)$
        $S$.push($v$); $S$.push($u_j$)
```

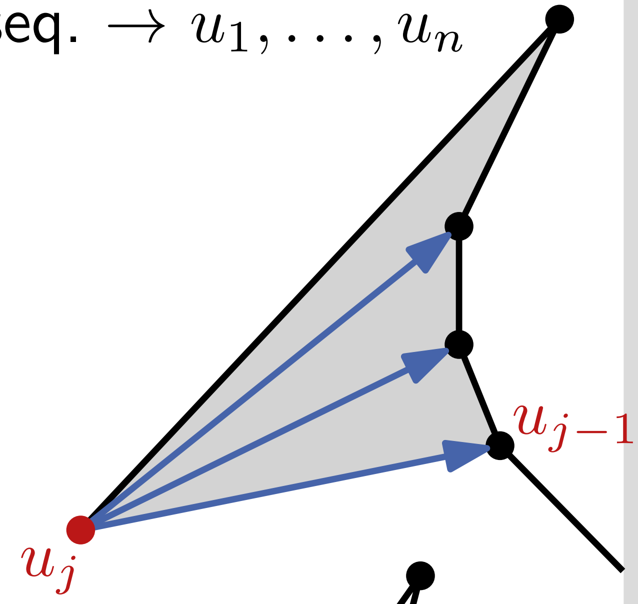


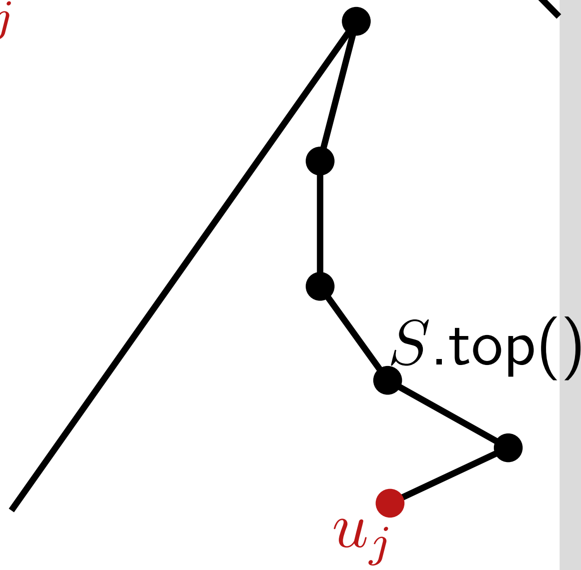


Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

```
Merge vertices on left and right chains into desc. seq. $\to u_1, \ldots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push($u_1$); $S$.push($u_2$)
**for** $j \leftarrow 3$ **to** $n-1$ **do**
    **if** $u_j$ and $S$.top() from different paths **then**
        **while not** $S$.empty() **do**
            $v \leftarrow S$.pop()
            **if not** $S$.empty() **then** draw $(u_j, v)$
        $S$.push($u_{j-1}$); $S$.push($u_j$)
    **else**
        $v \leftarrow S$.pop()
        **while not** $S$.empty() **and** $u_j$ sees $S$.top() **do**
            $v \leftarrow S$.pop()
            draw diagonal $(u_j, v)$
        $S$.push($v$); $S$.push($u_j$)
```

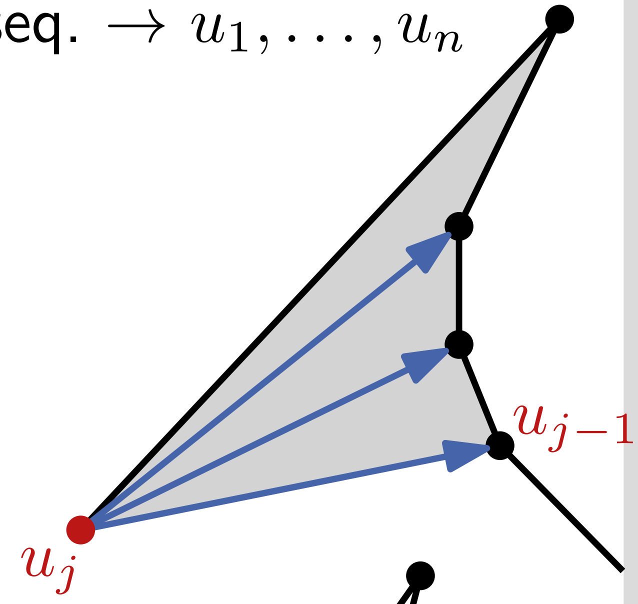


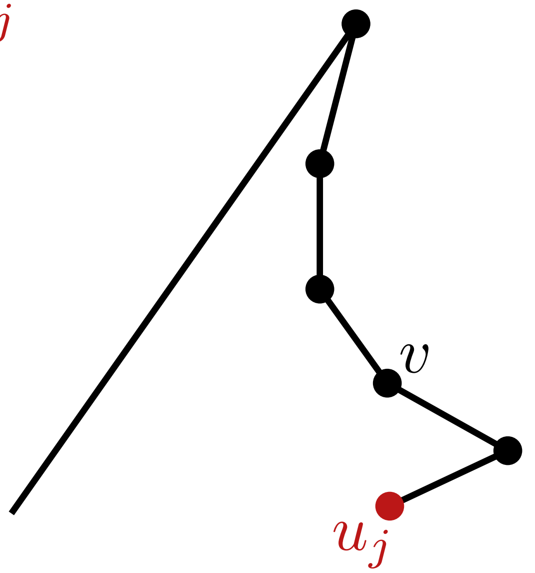


Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

```
Merge vertices on left and right chains into desc. seq. $\to u_1, \dots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push($u_1$); $S$.push($u_2$)
**for** $j \leftarrow 3$ **to** $n-1$ **do**
    **if** $u_j$ and $S$.top() from different paths **then**
        **while not** $S$.empty() **do**
            $v \leftarrow S$.pop()
            **if not** $S$.empty() **then** draw $(u_j, v)$
        $S$.push($u_{j-1}$); $S$.push($u_j$)
    **else**
        $v \leftarrow S$.pop()
        **while not** $S$.empty() **and** $u_j$ sees $S$.top() **do**
            $v \leftarrow S$.pop()
            draw diagonal $(u_j, v)$
        $S$.push($v$); $S$.push($u_j$)
```

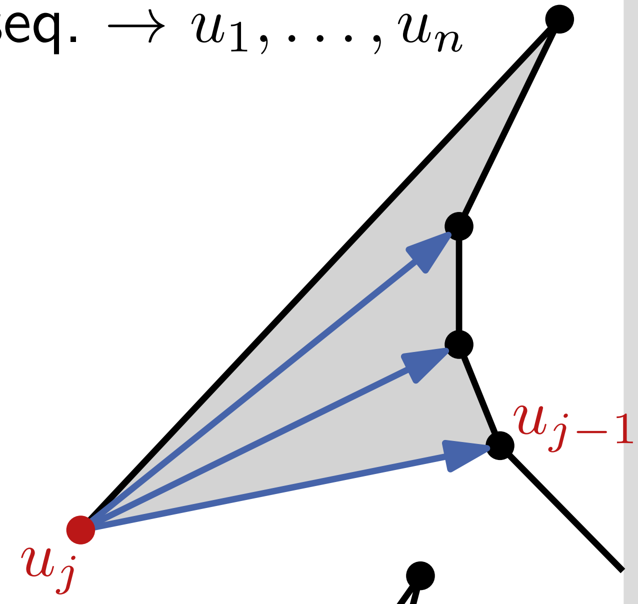


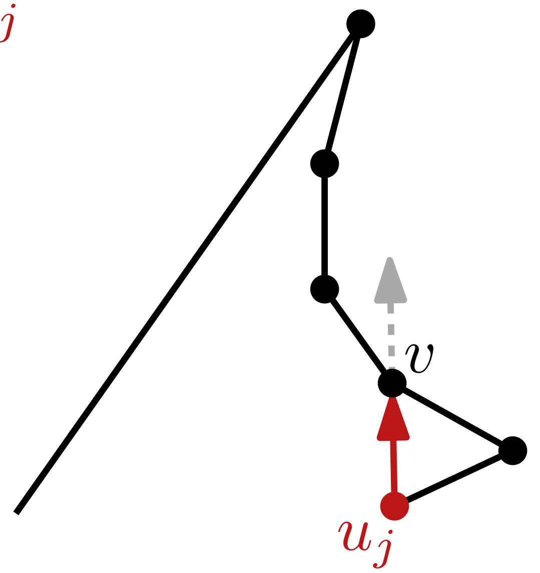


Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

```
Merge vertices on left and right chains into desc. seq. $\rightarrow u_1, \dots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push($u_1$); $S$.push($u_2$)
for $j \leftarrow 3$ to $n-1$ do
    if $u_j$ and $S$.top() from different paths then
        while not $S$.empty() do
            $v \leftarrow S$.pop()
            if not $S$.empty() then draw $(u_j, v)$
        $S$.push($u_{j-1}$); $S$.push($u_j$)
    else
        $v \leftarrow S$.pop()
        while not $S$.empty() and $u_j$ sees $S$.top() do
            $v \leftarrow S$.pop()
            draw diagonal $(u_j, v)$
        $S$.push($v$); $S$.push($u_j$)
```

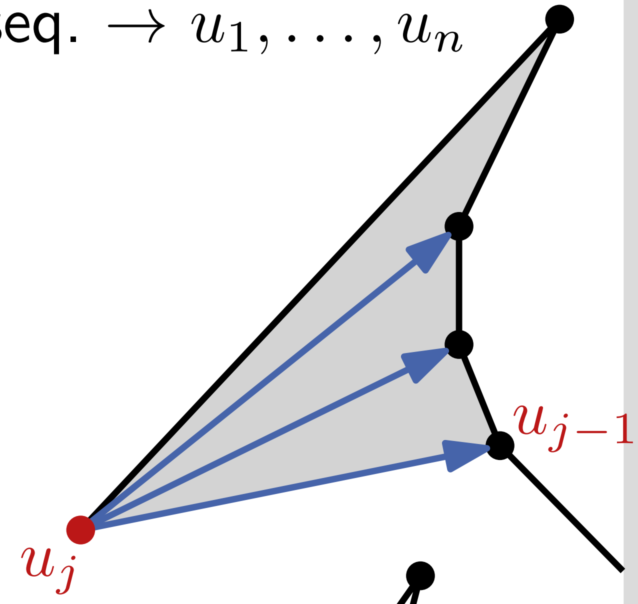


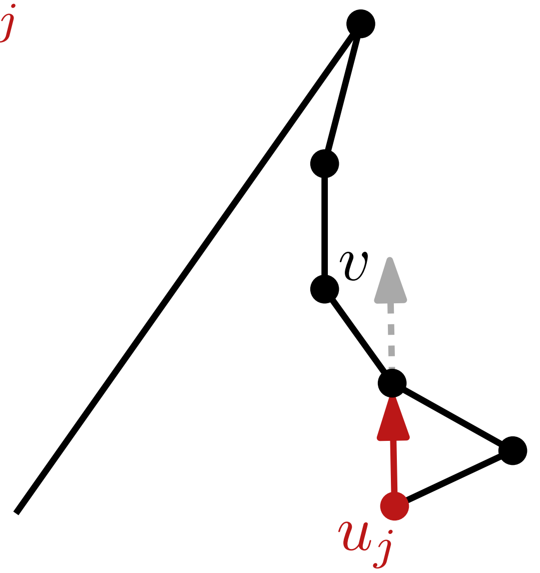


Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

```
Merge vertices on left and right chains into desc. seq. $\to u_1, \dots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push($u_1$); $S$.push($u_2$)
**for** $j \leftarrow 3$ **to** $n-1$ **do**
    **if** $u_j$ and $S$.top() from different paths **then**
        **while not** $S$.empty() **do**
            $v \leftarrow S$.pop()
            **if not** $S$.empty() **then** draw $(u_j, v)$
        $S$.push($u_{j-1}$); $S$.push($u_j$)
    **else**
        $v \leftarrow S$.pop()
        **while not** $S$.empty() **and** $u_j$ sees $S$.top() **do**
            $v \leftarrow S$.pop()
            draw diagonal $(u_j, v)$
        $S$.push($v$); $S$.push($u_j$)
```

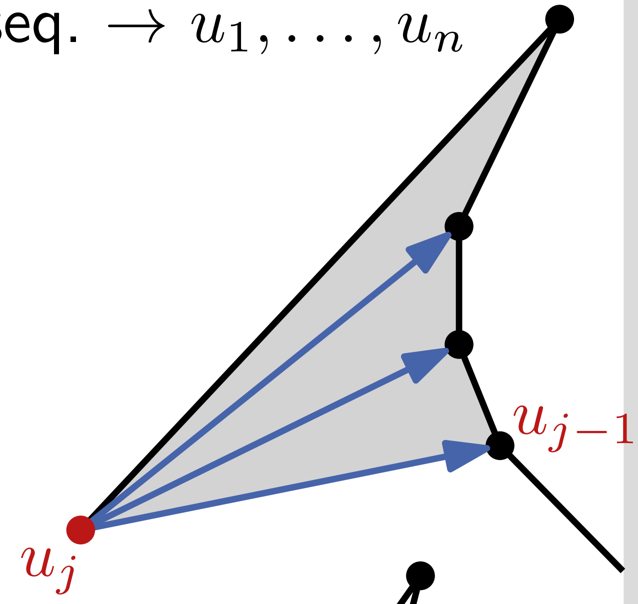


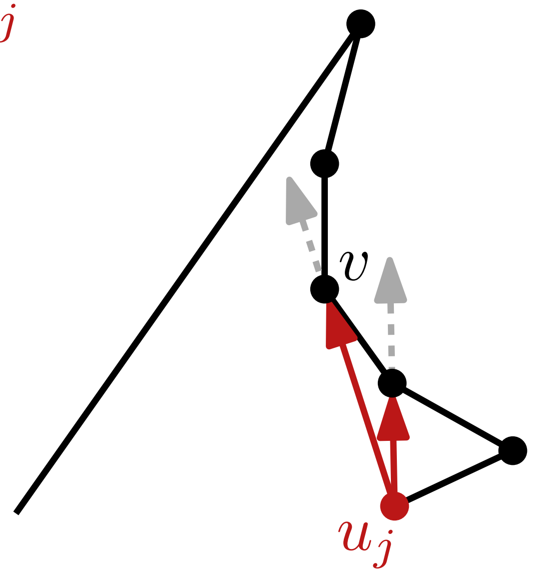


Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

```
Merge vertices on left and right chains into desc. seq. $\rightarrow u_1, \ldots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push($u_1$); $S$.push($u_2$)
for $j \leftarrow 3$ to $n-1$ do
    if $u_j$ and $S$.top() from different paths then
        while not $S$.empty() do
            $v \leftarrow S$.pop()
            if not $S$.empty() then draw $(u_j, v)$
        $S$.push($u_{j-1}$); $S$.push($u_j$)
    else
        $v \leftarrow S$.pop()
        while not $S$.empty() and $u_j$ sees $S$.top() do
            $v \leftarrow S$.pop()
            draw diagonal $(u_j, v)$
        $S$.push($v$); $S$.push($u_j$)
```

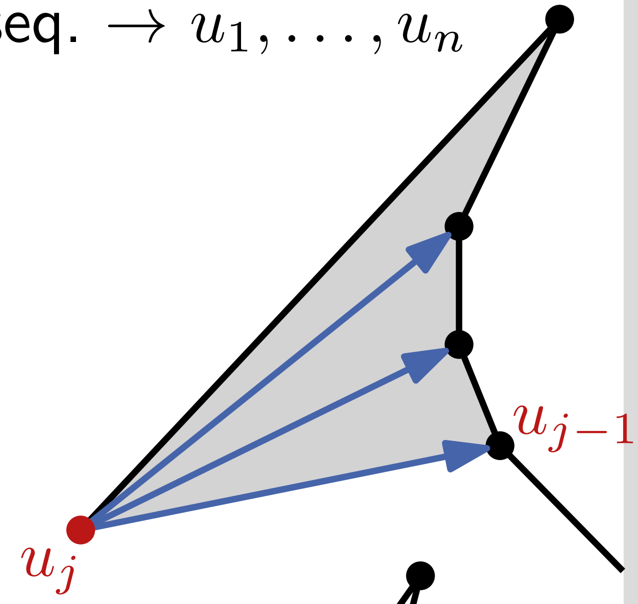


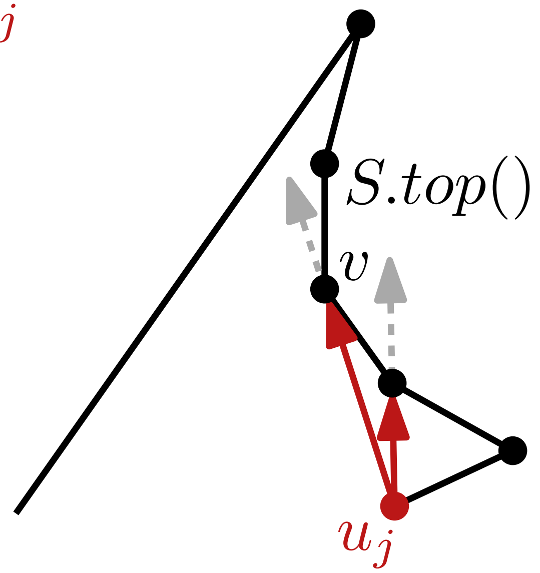


Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

```
Merge vertices on left and right chains into desc. seq. $\to u_1, \dots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push($u_1$); $S$.push($u_2$)
for $j \leftarrow 3$ to $n-1$ do
    if $u_j$ and $S$.top() from different paths then
        while not $S$.empty() do
            $v \leftarrow S$.pop()
            if not $S$.empty() then draw $(u_j, v)$
        $S$.push($u_{j-1}$); $S$.push($u_j$)
    else
        $v \leftarrow S$.pop()
        while not $S$.empty() and $u_j$ sees $S$.top() do
            $v \leftarrow S$.pop()
            draw diagonal $(u_j, v)$
        $S$.push($v$); $S$.push($u_j$)
```

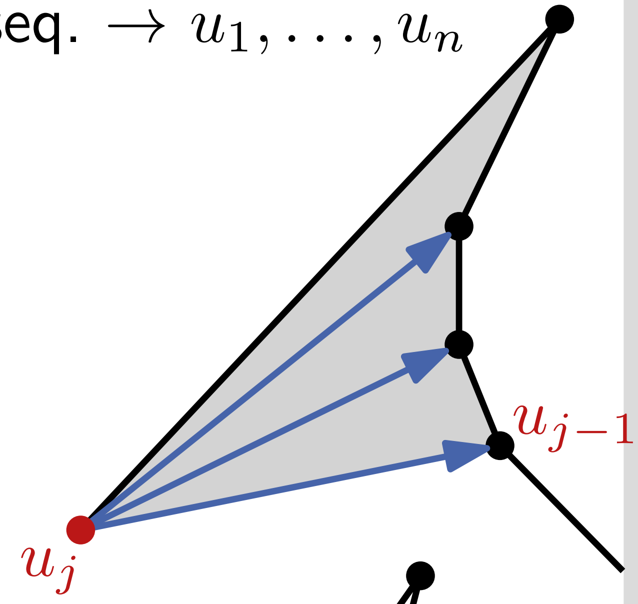


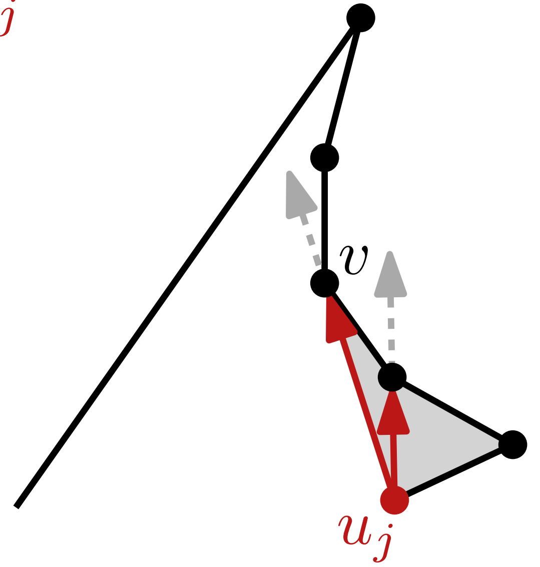


Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

```
Merge vertices on left and right chains into desc. seq. $\rightarrow u_1, \ldots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push($u_1$); $S$.push($u_2$)
for $j \leftarrow 3$ to $n-1$ do
    if $u_j$ and $S$.top() from different paths then
        while not $S$.empty() do
            $v \leftarrow S$.pop()
            if not $S$.empty() then draw $(u_j, v)$
        $S$.push($u_{j-1}$); $S$.push($u_j$)
    else
        $v \leftarrow S$.pop()
        while not $S$.empty() and $u_j$ sees $S$.top() do
            $v \leftarrow S$.pop()
            draw diagonal $(u_j, v)$
        $S$.push($v$); $S$.push($u_j$)
```

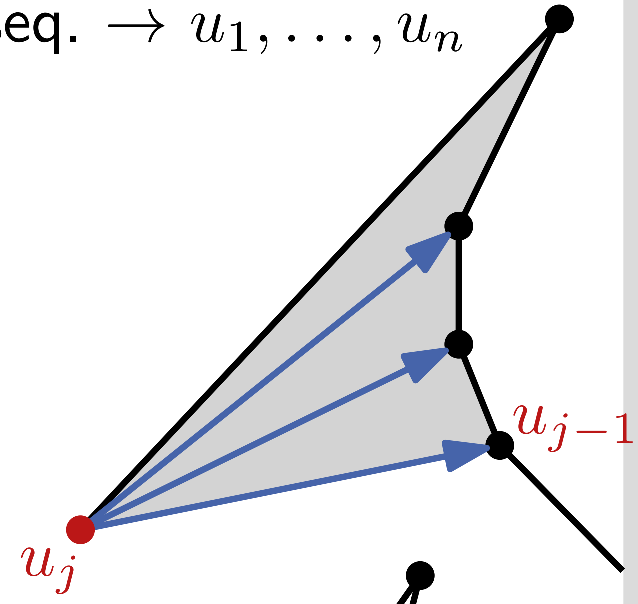


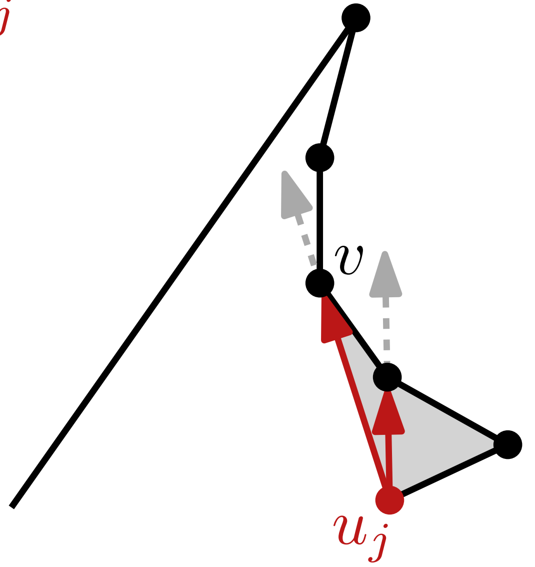


Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

```
Merge vertices on left and right chains into desc. seq. $\to u_1, \dots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push($u_1$); $S$.push($u_2$)
**for** $j \leftarrow 3$ **to** $n-1$ **do**
    **if** $u_j$ and $S$.top() from different paths **then**
        **while not** $S$.empty() **do**
            $v \leftarrow S$.pop()
            **if not** $S$.empty() **then** draw $(u_j, v)$
        $S$.push($u_{j-1}$); $S$.push($u_j$)
    **else**
        $v \leftarrow S$.pop()
        **while not** $S$.empty() **and** $u_j$ sees $S$.top() **do**
            $v \leftarrow S$.pop()
            draw diagonal $(u_j, v)$
        $S$.push($v$); $S$.push($u_j$)
Connect $u_n$ to all the vertices in $S$ (except for the first and the last)
```





Algorithm TriangulateMonotonePolygon

TriangulateMonotonePolygon(Polygon P as doubly-connected list of edges)

```
Merge vertices on left and right chains into desc. seq. $\to u_1, \dots, u_n$
Stack $S \leftarrow \emptyset$; $S$.push$(u_1)$; $S$.push$(u_2)$
**for** $j \leftarrow 3$ **to** $n-1$ **do**
    **if** $u_j$ and $S$.top() from different paths **then**
        **while not** $S$.empty() **do**
            $v \leftarrow S$.pop()
            **if not** $S$.empty() **then** draw $(u_j, v)$
        $S$.push$(u_{j-1})$; $S$.push$(u_j)$
    **else**
        $v \leftarrow S$.pop()
        **while not** $S$.empty() **and** $u_j$ sees $S$.top() **do**
            $v \leftarrow S$.pop()
            draw diagonal $(u_j, v)$
        $S$.push$(v)$; $S$.push$(u_j)$
Connect $u_n$ to all the vertices in $S$ (except for the first and the last)
```

Task:
What is the running time?

Summary

Theorem 4: A y-monotone polygon with n vertices can be triangulated in $O(n)$ time.

Summary

Theorem 4: A y-monotone polygon with n vertices can be triangulated in $O(n)$ time.

Theorem 3: A simple polygon with n vertices can be partitioned into y-monotone polygons in $O(n \log n)$ time and $O(n)$ space.

recall

Summary

Theorem 4: A y-monotone polygon with n vertices can be triangulated in $O(n)$ time.

Theorem 3: A simple polygon with n vertices can be partitioned into y-monotone polygons in $O(n \log n)$ time and $O(n)$ space.

recall

$\Downarrow$

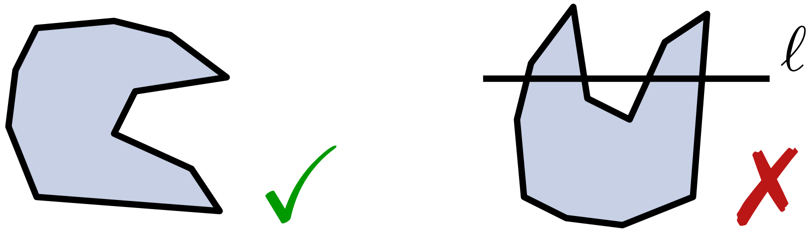
Theorem 5: A simple polygon with n vertices can be triangulated in $O(n \log n)$ time and $O(n)$ space.

Proof of Art-Gallery-Theorem: Overview

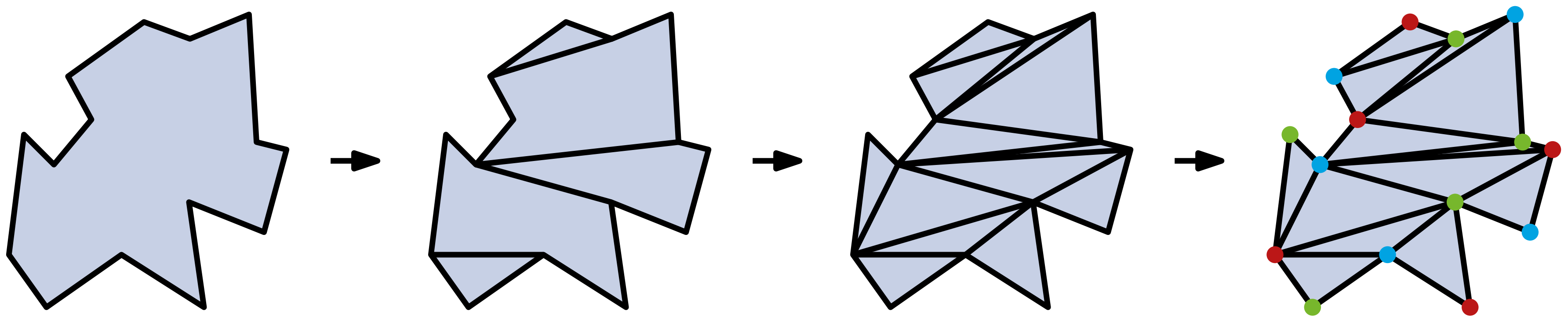
Three-step procedure:

- Step 1: Decompose P in y-monotone polygons ✓

  Definition: A polygon P is y-monotone, if for each horizontal line ℓ the intersection $\ell \cap P$ is connected.



- Step 2: Triangulate y-monotone polygons ✓
- Step 3: use DFS to color the triangulated polygon ✓



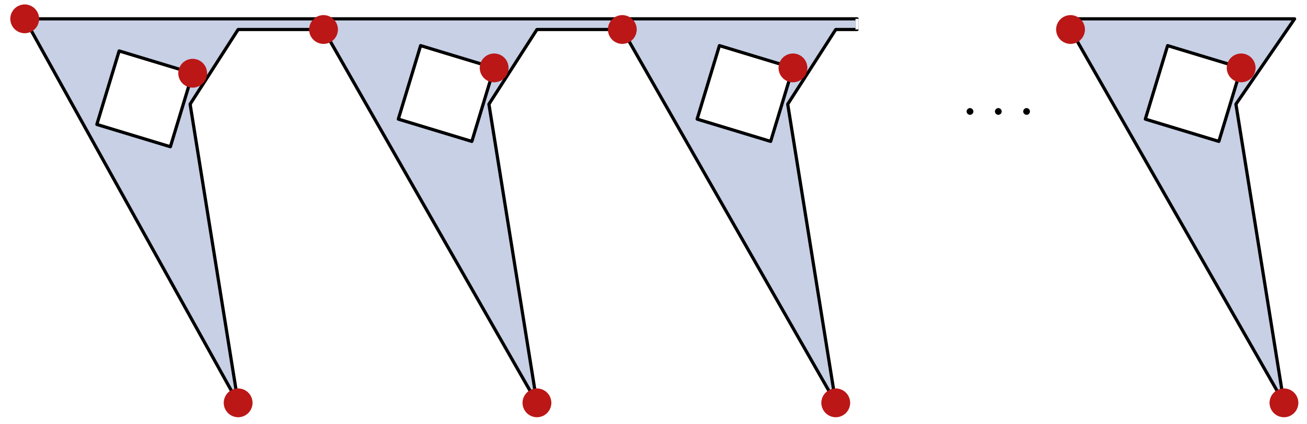
Discussion

Can the triangulation algorithm be expanded to work with polygons with holes?

Discussion

Can the triangulation algorithm be expanded to work with polygons with holes?

- Triangulation: yes
- But are $\lfloor n/3 \rfloor$ cameras still sufficient to guard it?
  No, a generalization of Art-Gallery-Theorems says that $\lfloor (n+h)/3 \rfloor$ cameras are sometimes necessary, and always sufficient, where h is the number of holes. [Hoffmann et al., 1991]



Discussion

Can the triangulation algorithm be expanded to work with polygons with holes?

- Triangulation: yes
- But are $\lfloor n/3 \rfloor$ cameras still sufficient to guard it?
  No, a generalization of Art-Gallery-Theorems says that $\lfloor (n+h)/3 \rfloor$ cameras are sometimes necessary, and always sufficient, where h is the number of holes. [Hoffmann et al., 1991]

Can we solve the triangulation problem faster for simple polygons?

Discussion

Can the triangulation algorithm be expanded to work with polygons with holes?

- Triangulation: yes
- But are $\lfloor n/3 \rfloor$ cameras still sufficient to guard it?
  No, a generalization of Art-Gallery-Theorems says that $\lfloor (n+h)/3 \rfloor$ cameras are sometimes necessary, and always sufficient, where h is the number of holes. [Hoffmann et al., 1991]

Can we solve the triangulation problem faster for simple polygons?

Yes. The question whether it is possible was open for more than a decade. In the end of 80's a faster randomized algorithm was given, and in 1990 Chazelle presented a deterministic linear-time algorithm (complicated).