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Algorithmen zur Visualisierung von Graphen
WS 14/15
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Exercise Sheet 6

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1 Extended Canonical Ordering for 4-Connected Graphs

A planar graph $G = (V, E)$ is called *proper triangular planar* (PTP, for short) if the following coditions hold:

- Every interior face of G is a triangle and the exterior face of G is a quadrangle;
- G has no separating triangles.

Let $G = (V, E)$ be a PTP graph with vertices a, b, c, d on the outer face. A labeling $v_1 = a, v_2 = c, v_3, \ldots, v_n = d$ of the vertices of G is called an *extended canonical ordering* of G if for every $4 \leq k \leq n$:

- The subgraph G_{k-1} induced by $v_1, \ldots, v_{k-1}$ is biconnected and the boundary C_{k-1} of G_{k-1} contains the edge (a, b);
- v_k is in the exterior face of G_{k-1}, and its neighbors in G_{k-1} form (at least 2-element) subinterval of the path $C_{k-1} \setminus (a, b)$. If $k \leq n-2$, v_k has at least 2 neighbors in $G \setminus G_{k-1}$.

Let $G = (V, E)$ be a PTP graph with vertices a, b, c, d on the outer face.

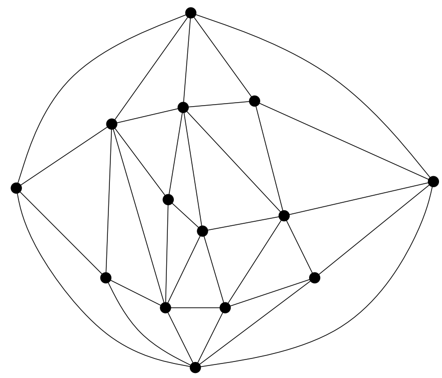
(a) Prove that the graph obtained from G by removal of vertices c, d and all edges incident to them is biconnected.

(b) Let $C = \{a = u_1, \ldots, u_k = b, a\}$ be a simple cycle of G such that $c, d \notin C$. Let G_C denote the graph induced by the vertices of G laying inside C (including the vertices of C). Let $v_i \in C$, $2 \leq i \leq k-1$ such that no internal chord of C is incident to v_i. Show that $G \setminus \{v_i\}$ is biconnected.

(c) Let C be as above and let (v_i, v_j), $1 \leq i < j \leq k$, be an internal chord of C. Show that there exists a vertex v_l, $i < l < j$, which is adjacent to at least two vertices of $G \setminus G_C$.

(d) Using (a-c) prove that G has an extended canonical ordering such that $v_1 = a, v_2 = b, v_{n-1} = c, v_n = d$.

2 Construction of Rectangular Dual



Consider the graph G of the figure. Check whether G satisfies the necessary conditions to have a rectangular dual. In affirmative, construct a rectangular dual of G.

3 *st*-Ordering and *st*-Digraphs

Let $G = (V, E)$ be a biconnected planar graph and let $f : V \to \mathbb{N}$ be the function giving an *st*-ordering of the vertices of G. In the following an undirected edge between vertices u and v is denoted by $\{u, v\}$, and an edge directed from u to v is denoted by (u, v). Let $\overrightarrow{G} = (V, \overrightarrow{E})$ be a directed graph, where $\overrightarrow{E} = \{(u, v) | \{u, v\} \in E \;\&\; f(u) < f(v)\}$. I.e. $\overrightarrow{G}$ is just an orientation of G, where each edge $\{u, v\}$ is assigned a direction from u to v if $f(u) < f(v)$ or a direction from v to u, otherwise. Prove that $\overrightarrow{G}$ is an *st*-digraph.

Hint: To achieve that prove that:

(a) $\overrightarrow{G}$ contains a single source vertex and a single sink vertex. A *source* (*sink*) of a directed graph is a vertex without incoming (outgoing) edges.

(b) $\overrightarrow{G}$ is acyclic, i.e. it does not contain any directed cycle.

4 Property of *st*-Ordering

Let $G = (V, E)$ be a biconnected planar graph with a given embedding and let $v_1, \ldots, v_n$ be an *st*-ordering of G such that v_1, v_n belong to the outer face of G. Let G_i denote the plane subgraph of G induced by the vertices $v_1, \ldots, v_i$. Prove that v_{i+1} belongs to the outer face of G_i.

5 Ear decomposition.

Let $G = (V, E)$ such that for each edge $\{s, t\} \in E$, G has an open ear decomposition that starts with $\{s, t\}$. Show that G is 2-connected. (Recall that the reverse statement was proven in the lecture.)