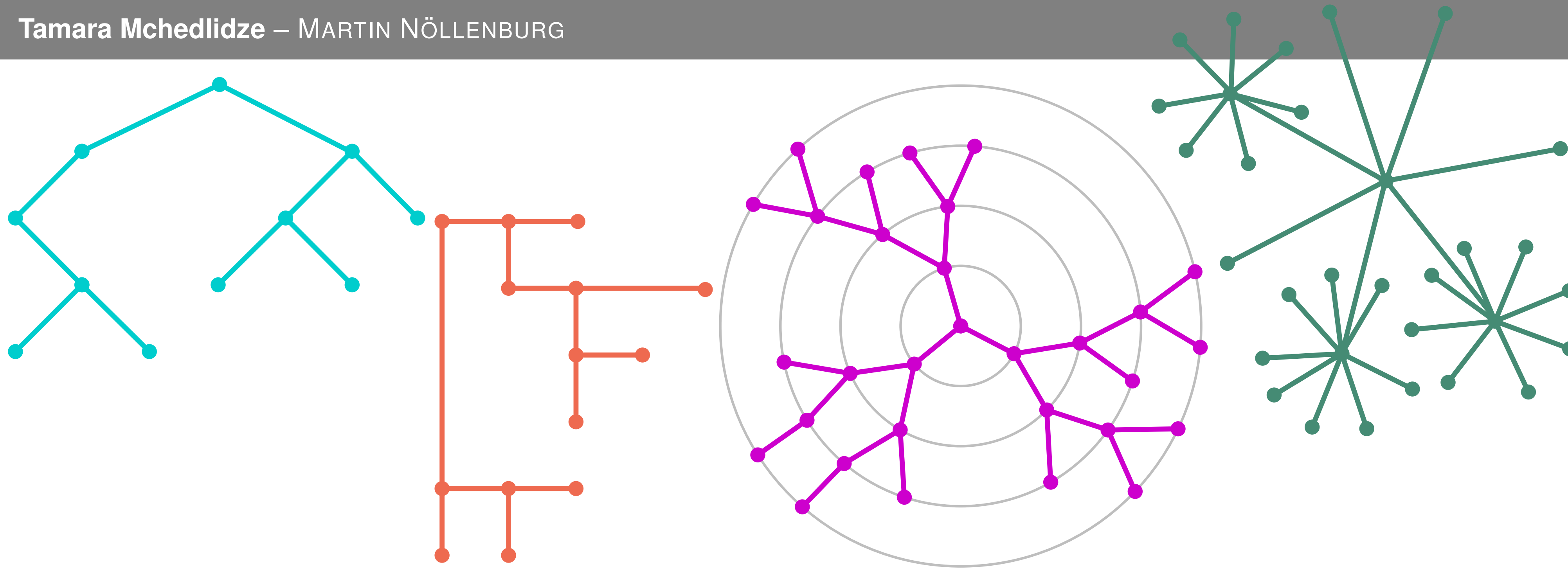
Karlsruhe Institute of Technology

Algorithms for graph visualization

Divide and Conquer - Tree Layouts

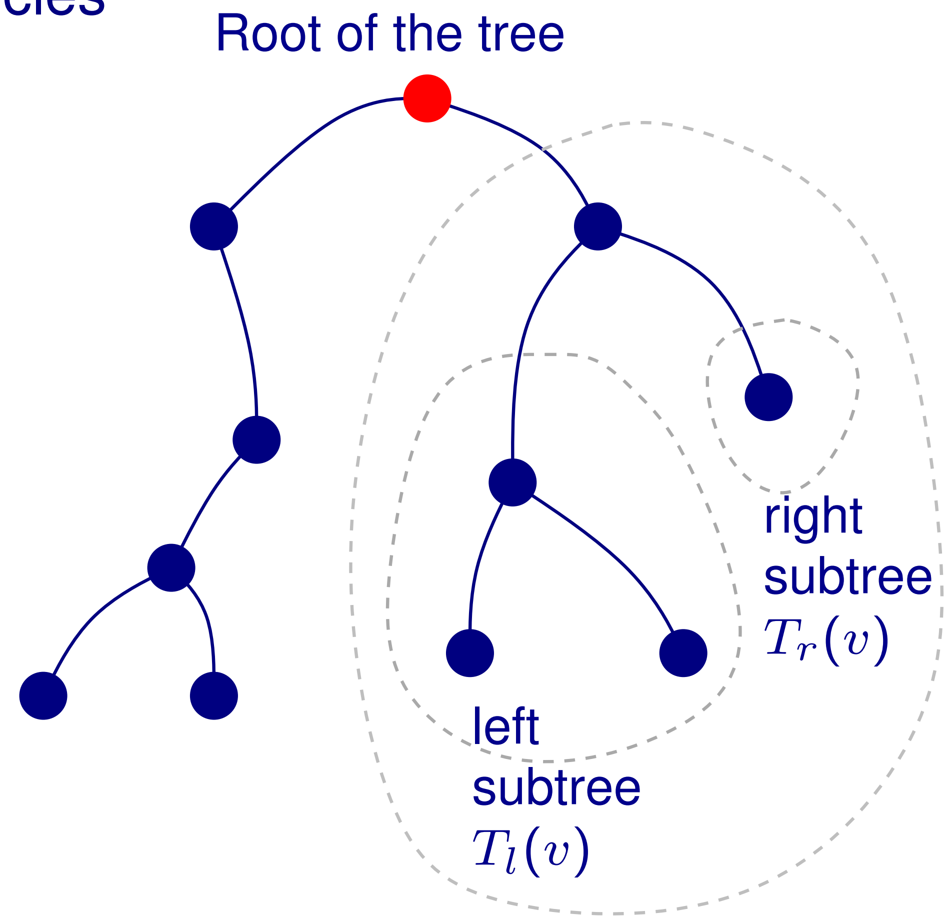
Winter semester 2014/2015

Tamara Mchedlidze – Martin Nöllenburg



Basic Definitions

- Tree - connected graph without cycles
- Binary tree



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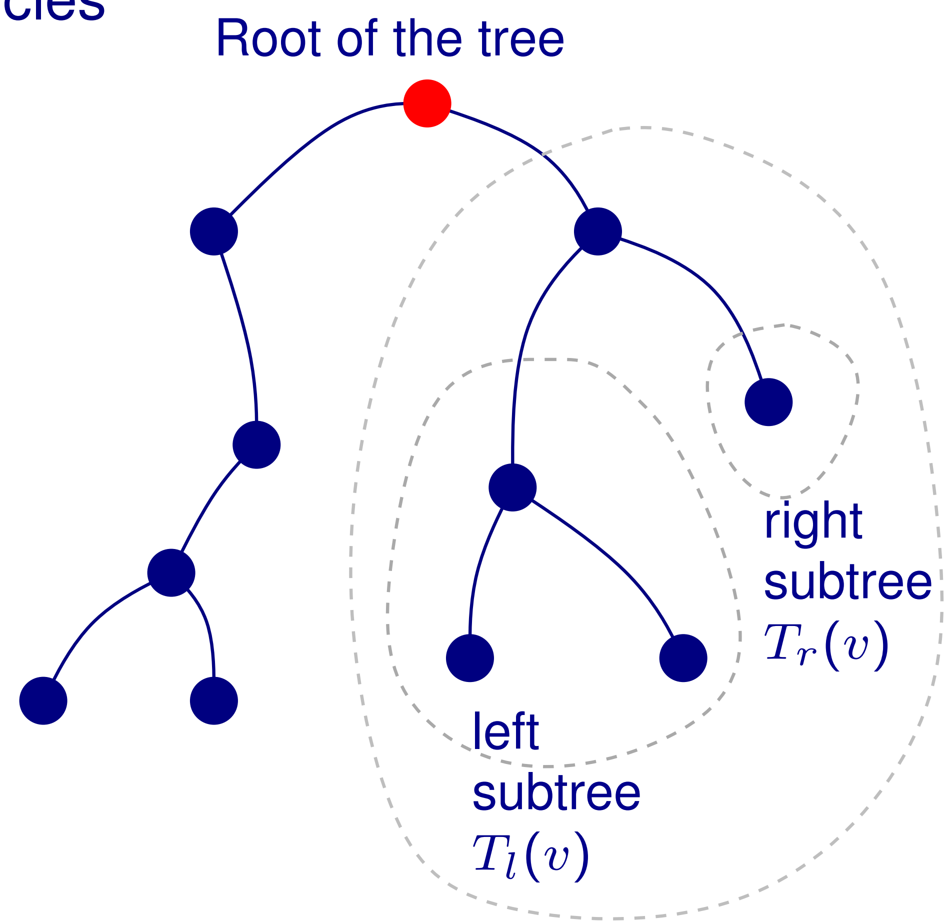

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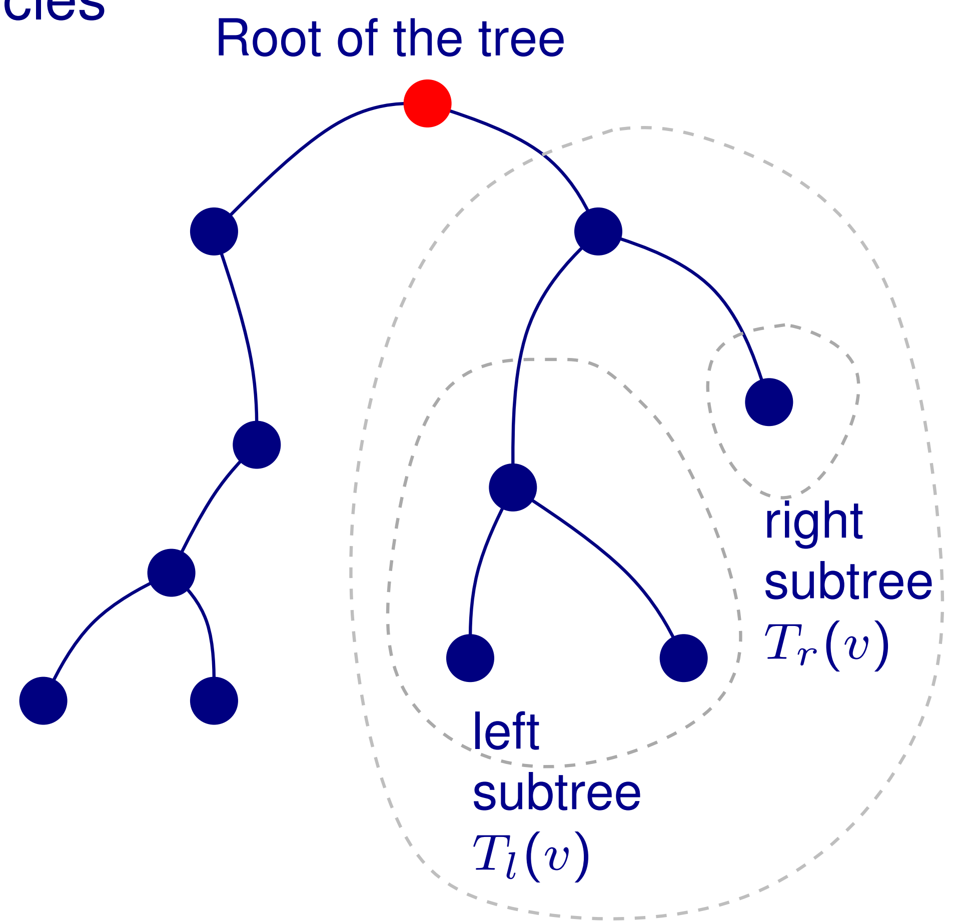
Depth-first search



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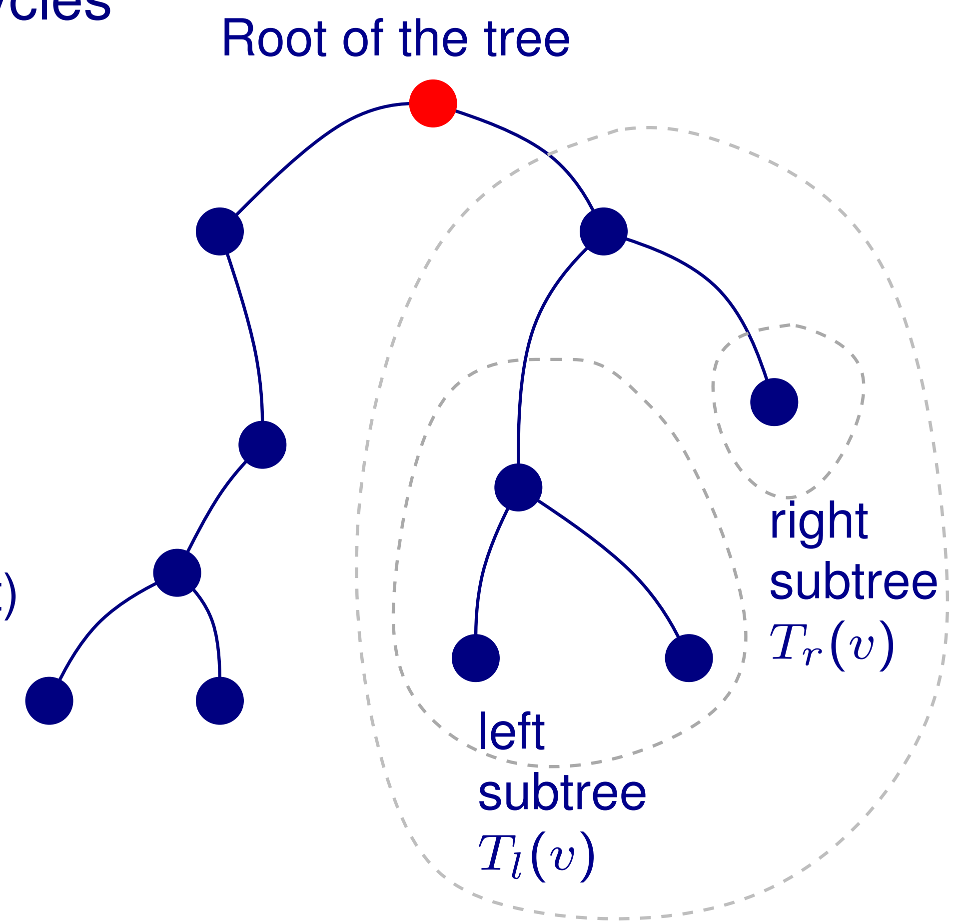
- Pre-order (First parent, then subtrees)
- In-order (Left child, parent, right child)
- Post-order (First subtrees, then parent)



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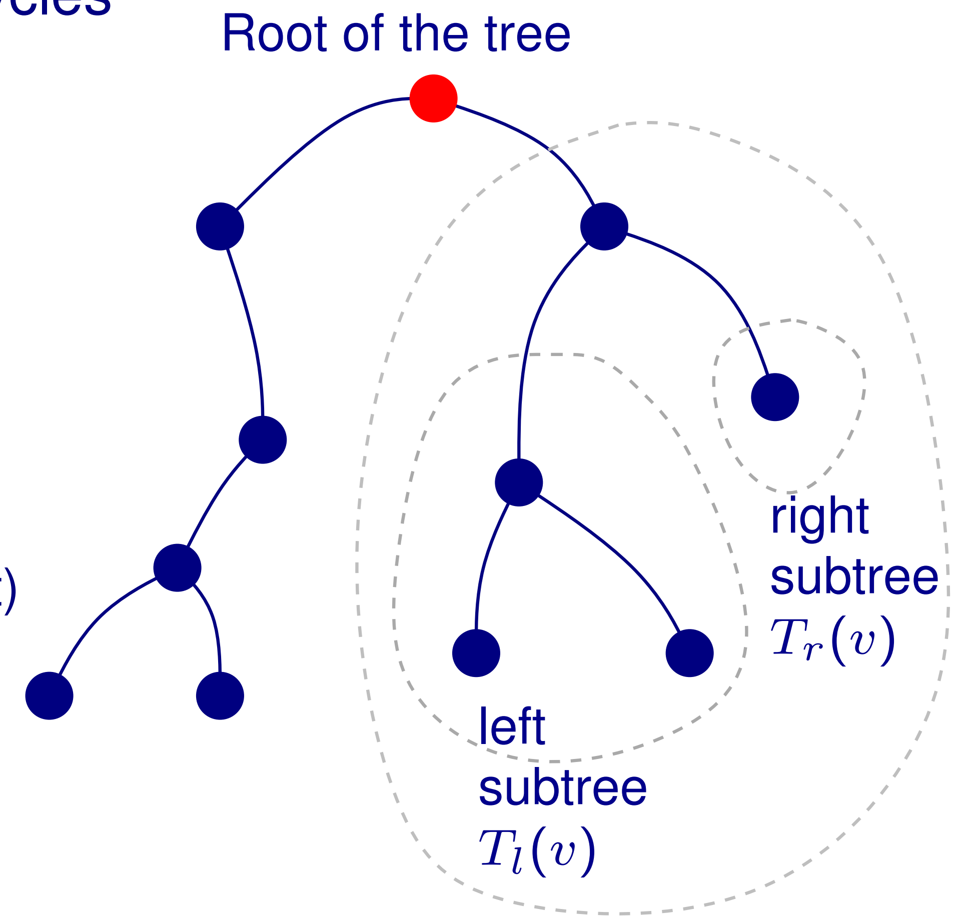
Breadth-first search



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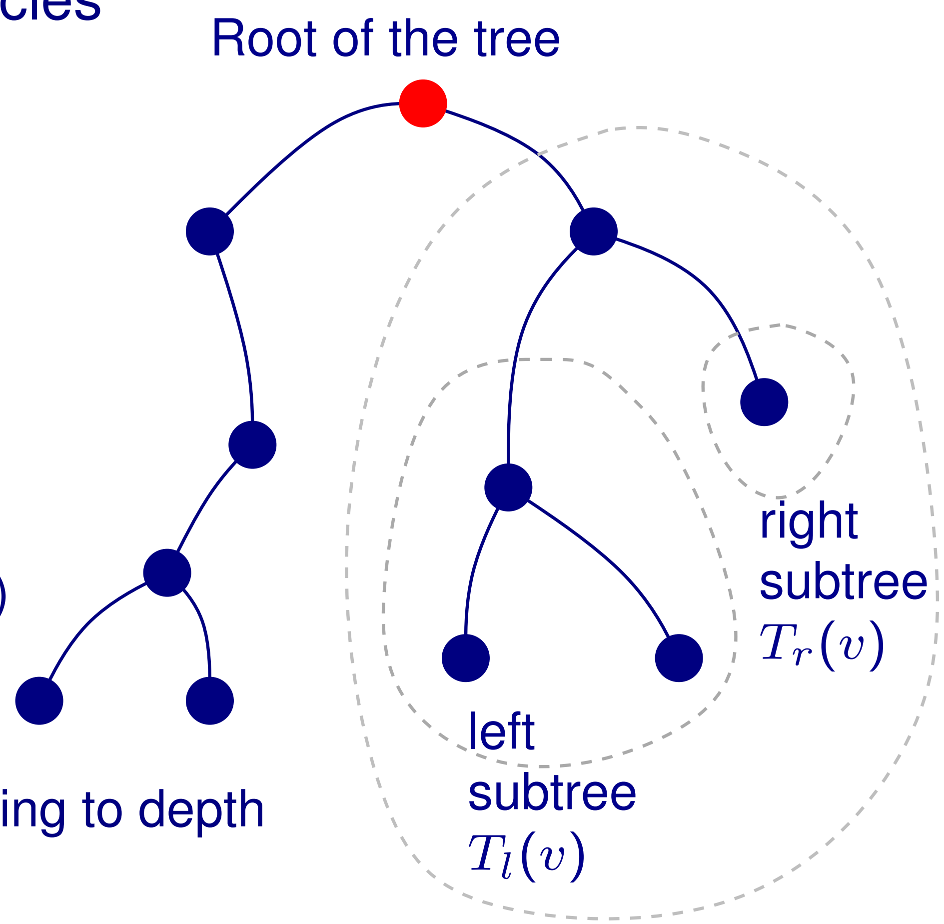
Breadth-first search

- Assignes vertices to levels corresponding to depth



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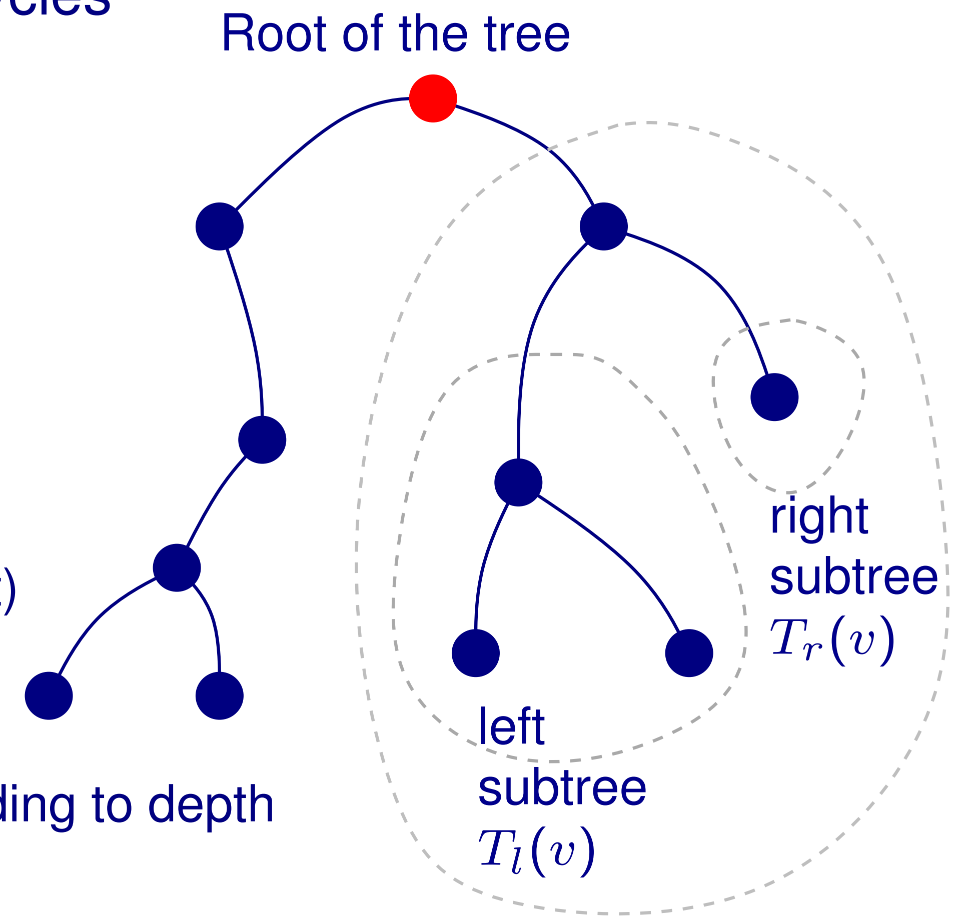

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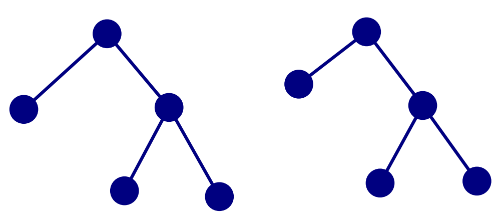
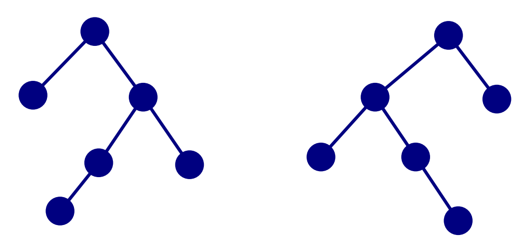
Isomorphism

Simple



Axial



Drawing of a Tree

Given: A rooted binary tree

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Institut für Theoretische Informatik
Lehrstuhl Algorithmik I

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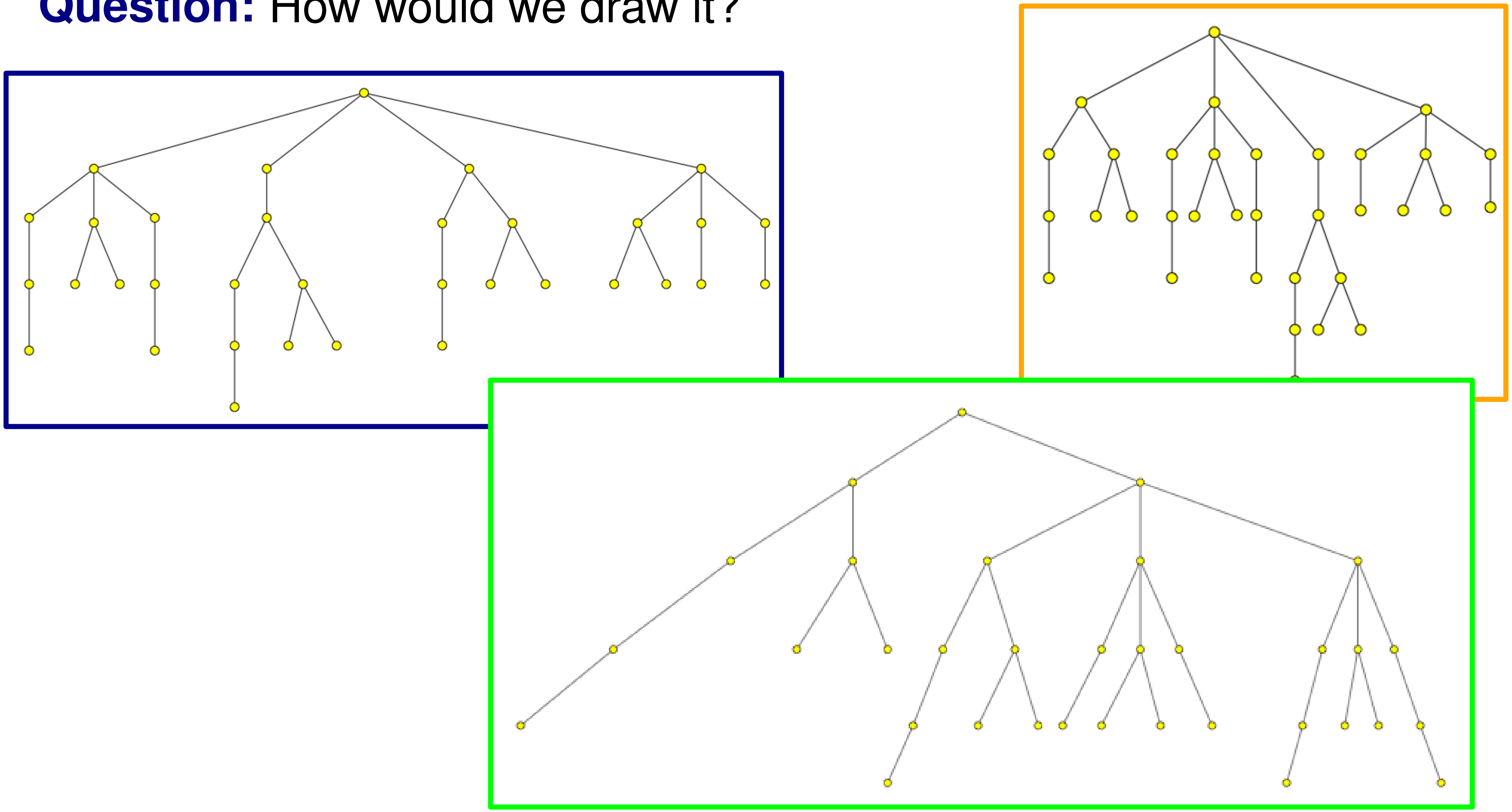

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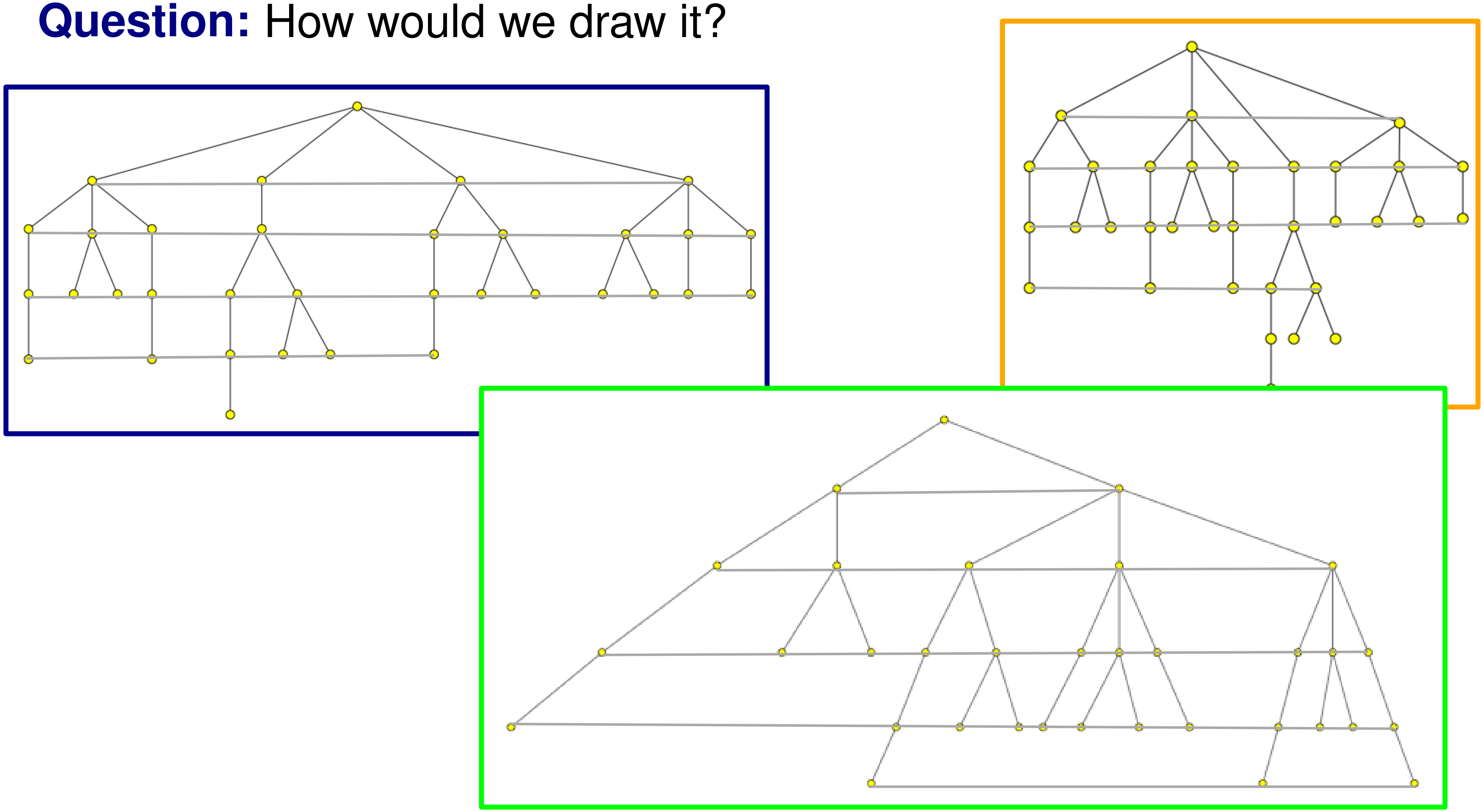

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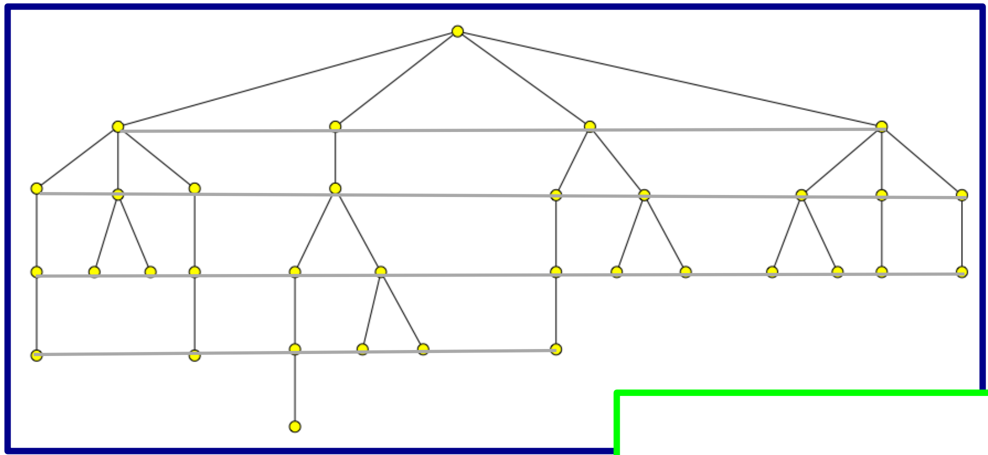

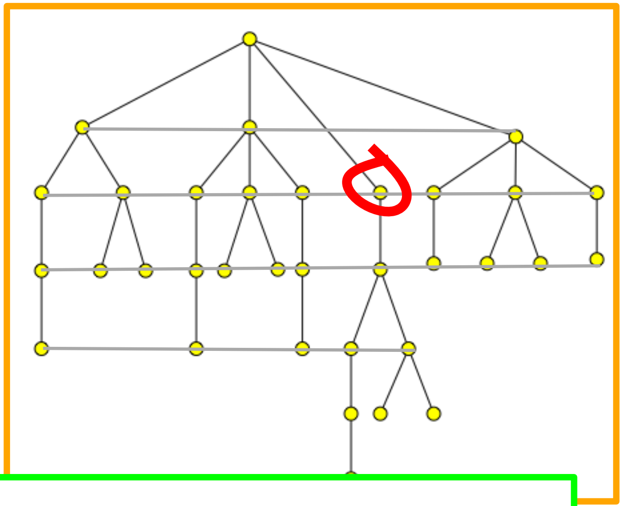

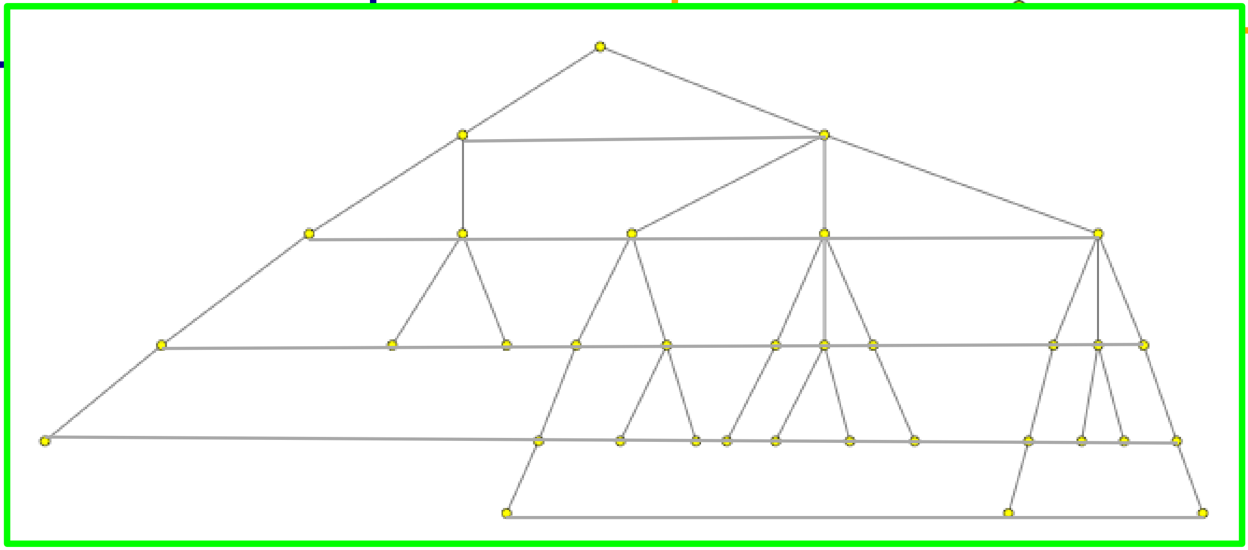

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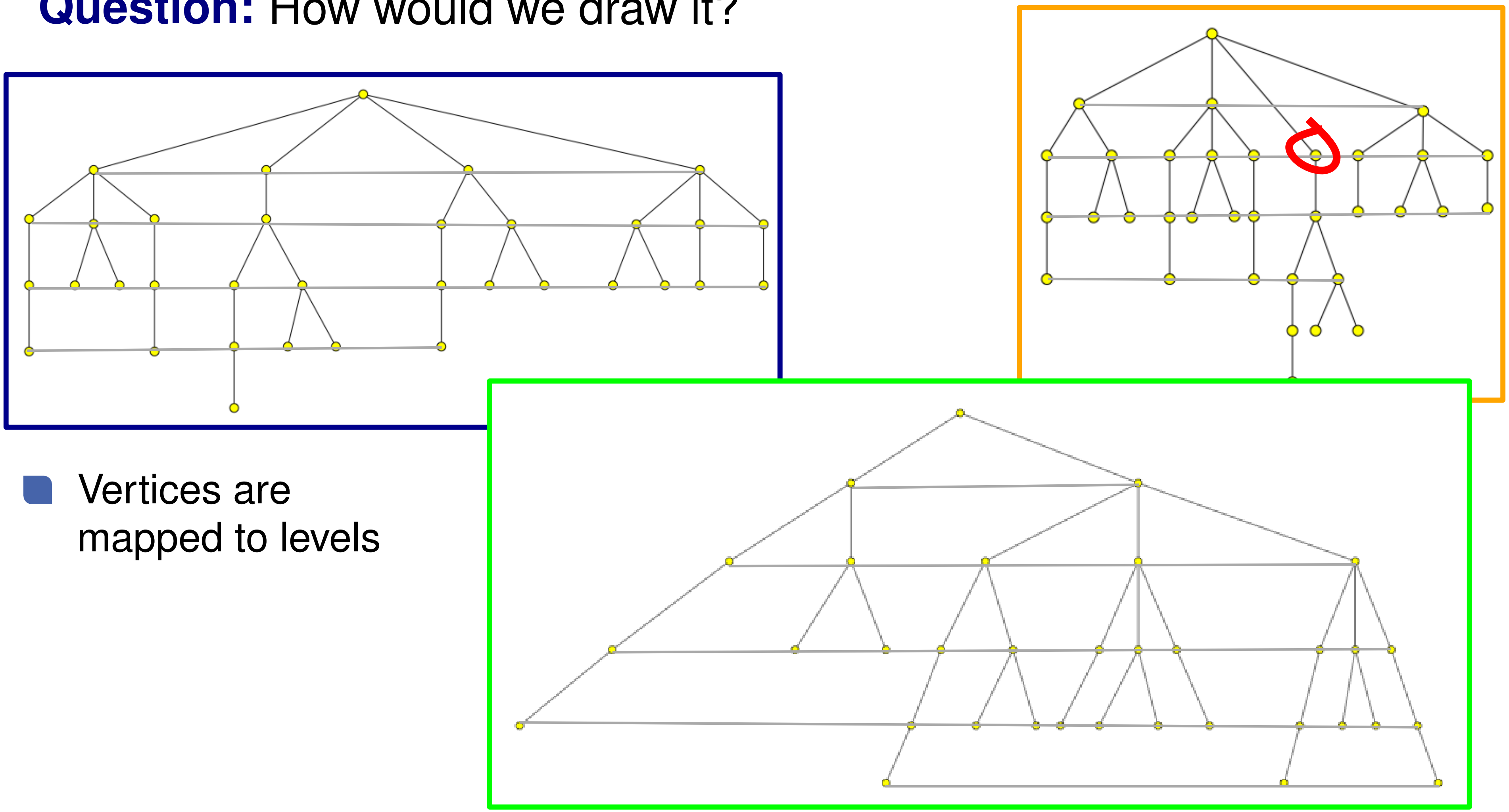

- Vertices are mapped to levels

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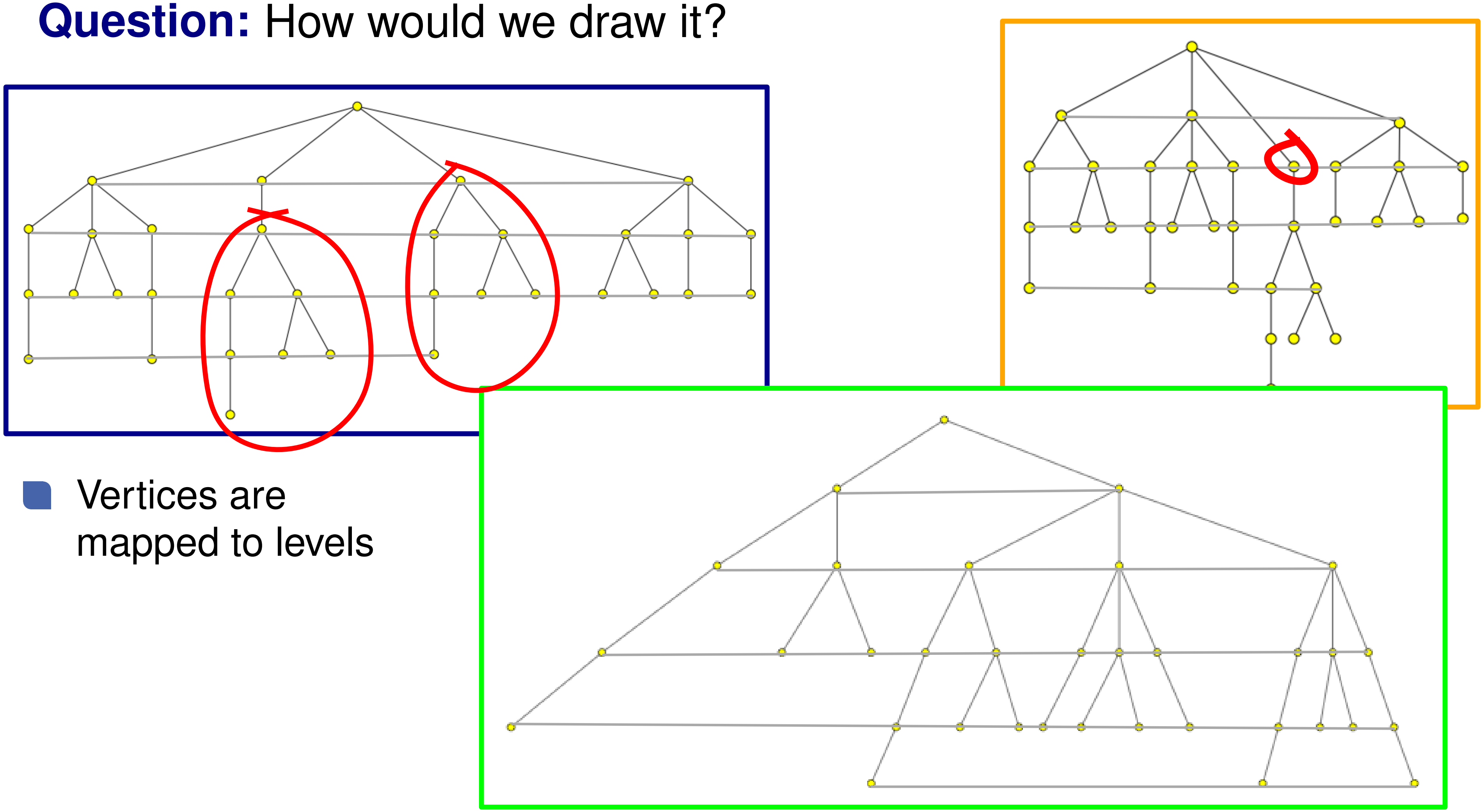

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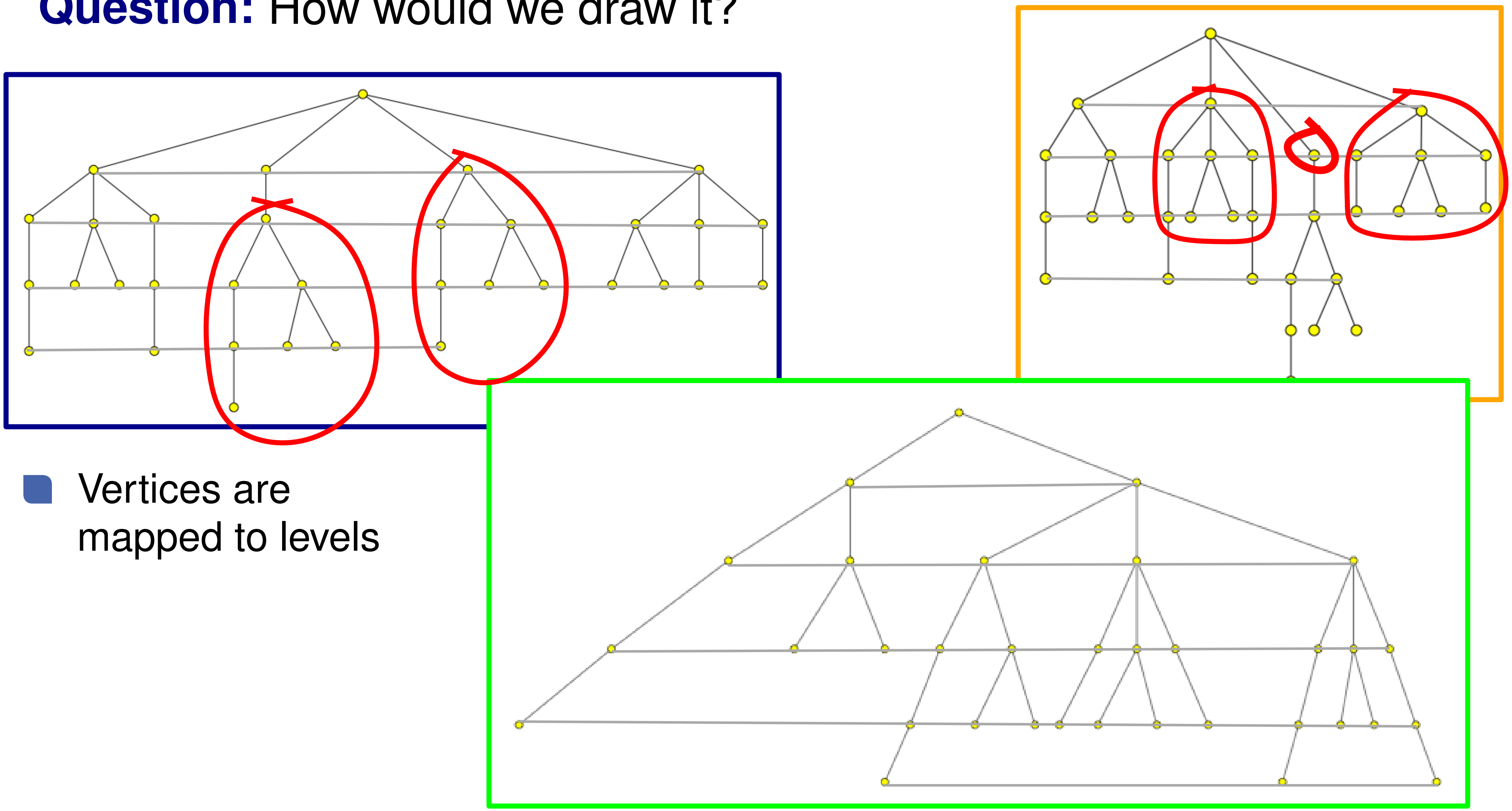

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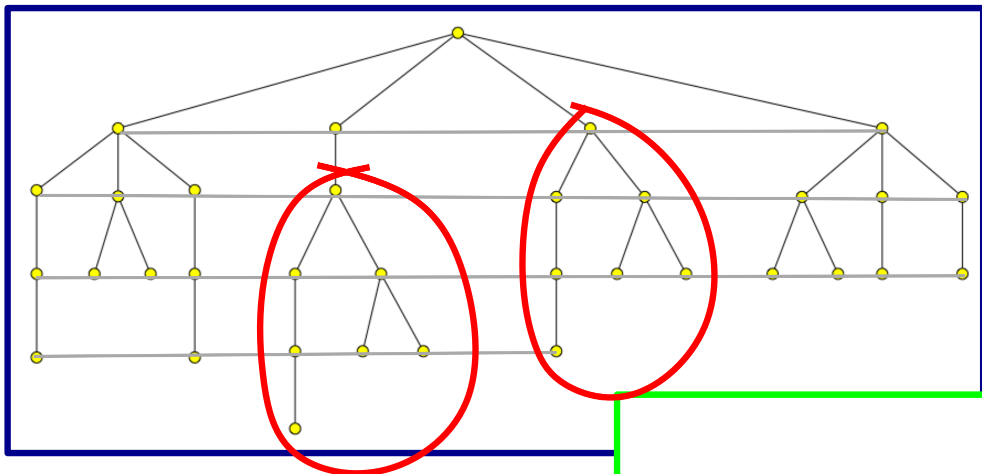

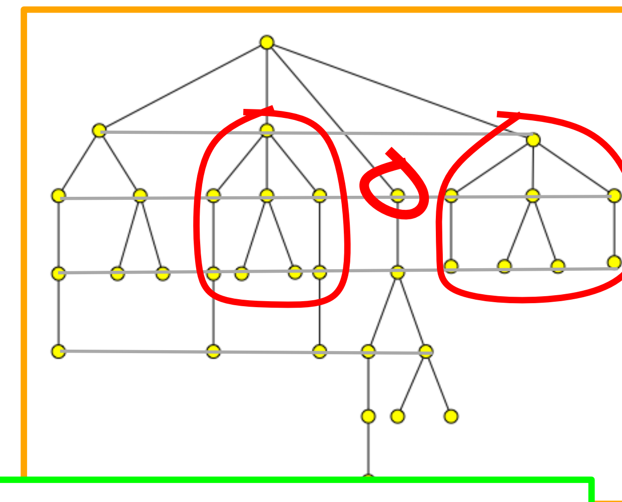

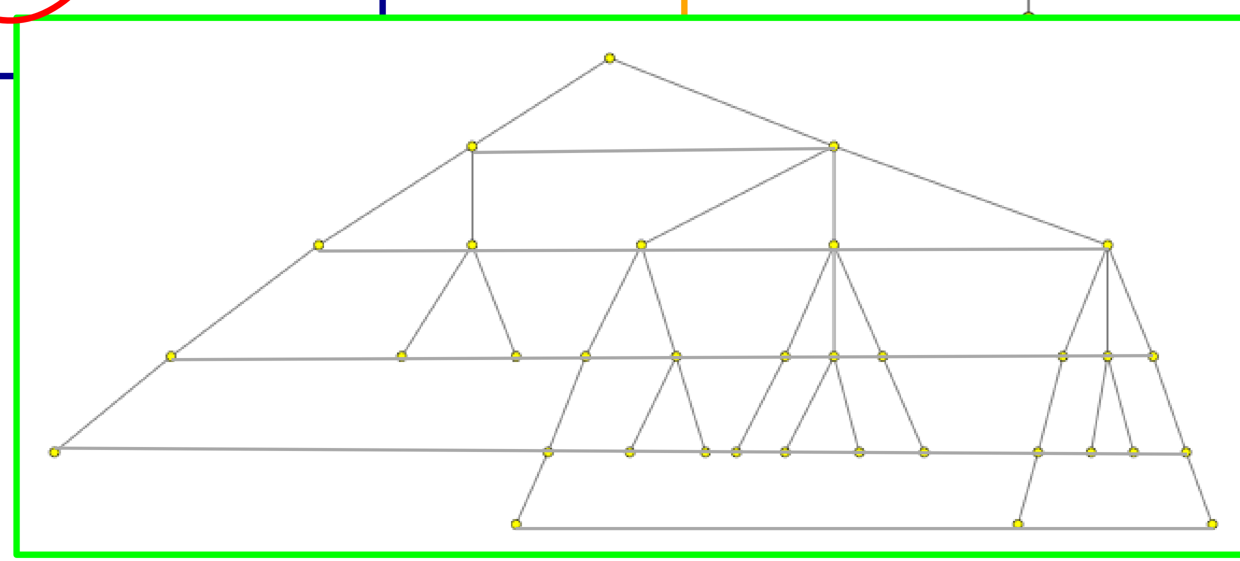

- Vertices are mapped to levels
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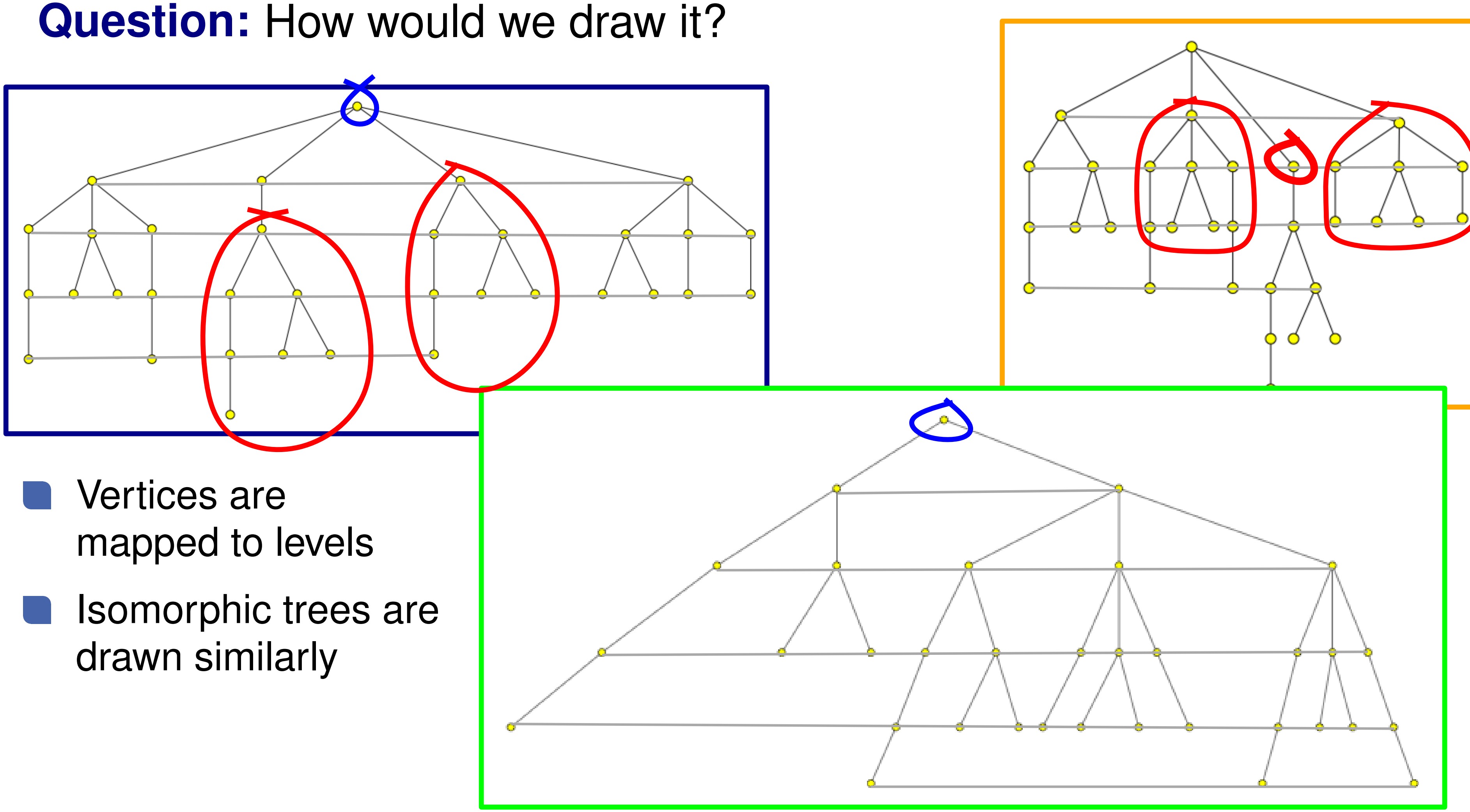

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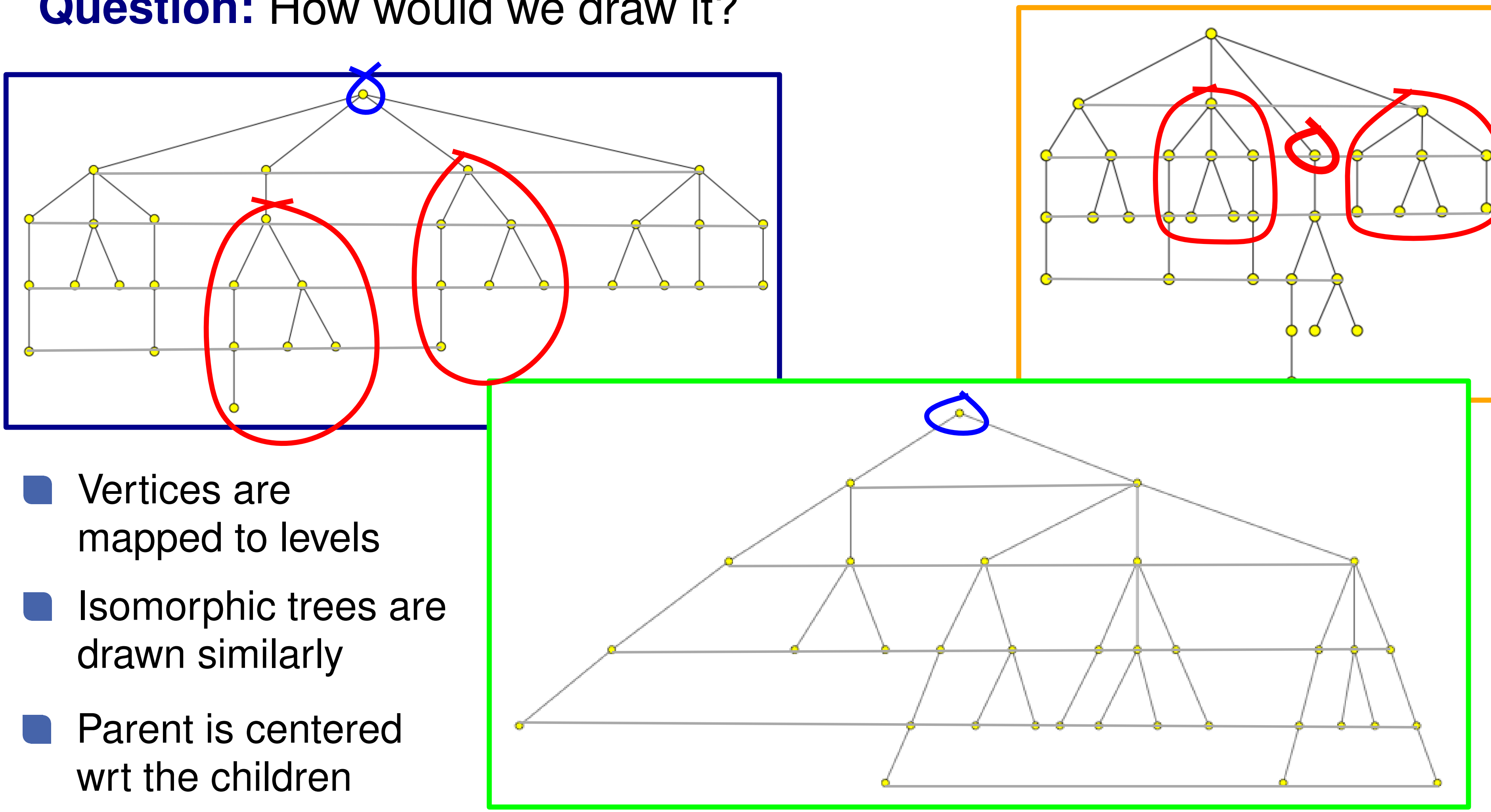


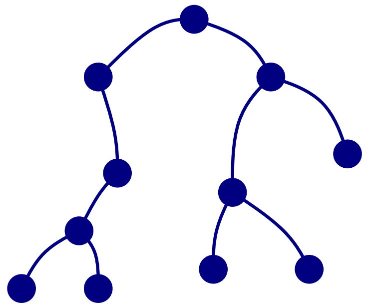
- Vertices are mapped to levels
- Isomorphic trees are drawn similarly
- Parent is centered wrt the children

Level-based Layout

Algorithm Outline:

Input: A binary tree

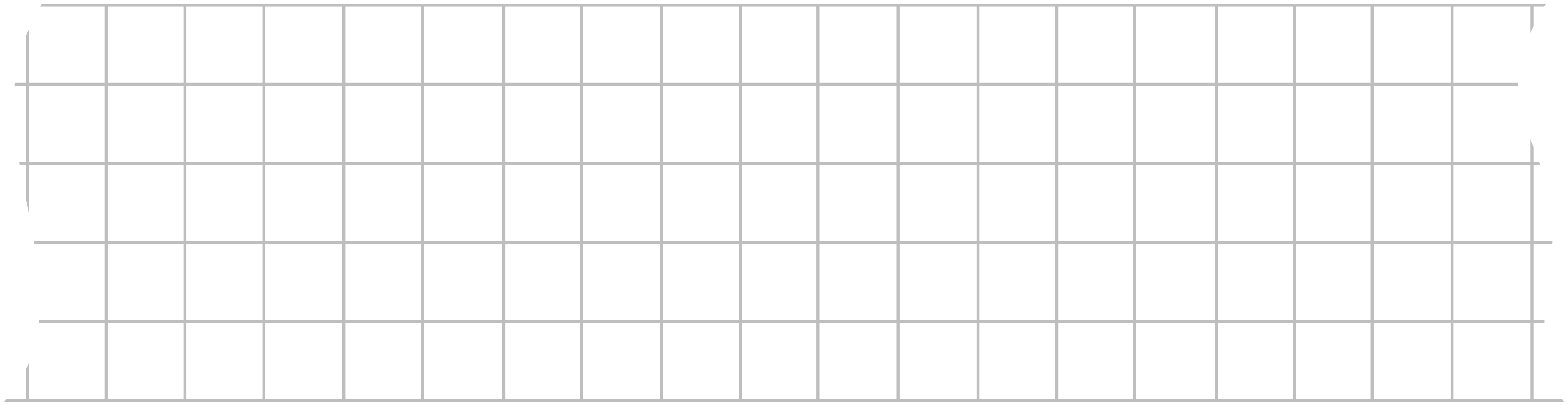
Output: A leveled drawing of T



Base case: A single vertex

Divide: Recursively apply the algorithm to draw the left and the right subtrees of T

Conquer:



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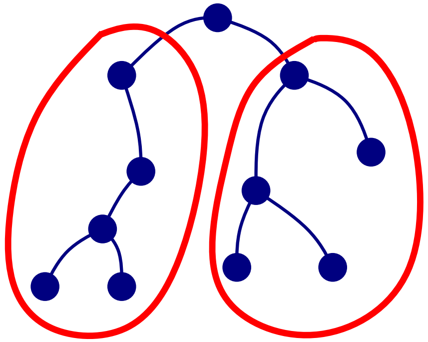

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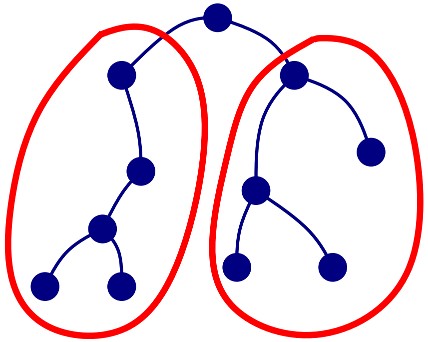

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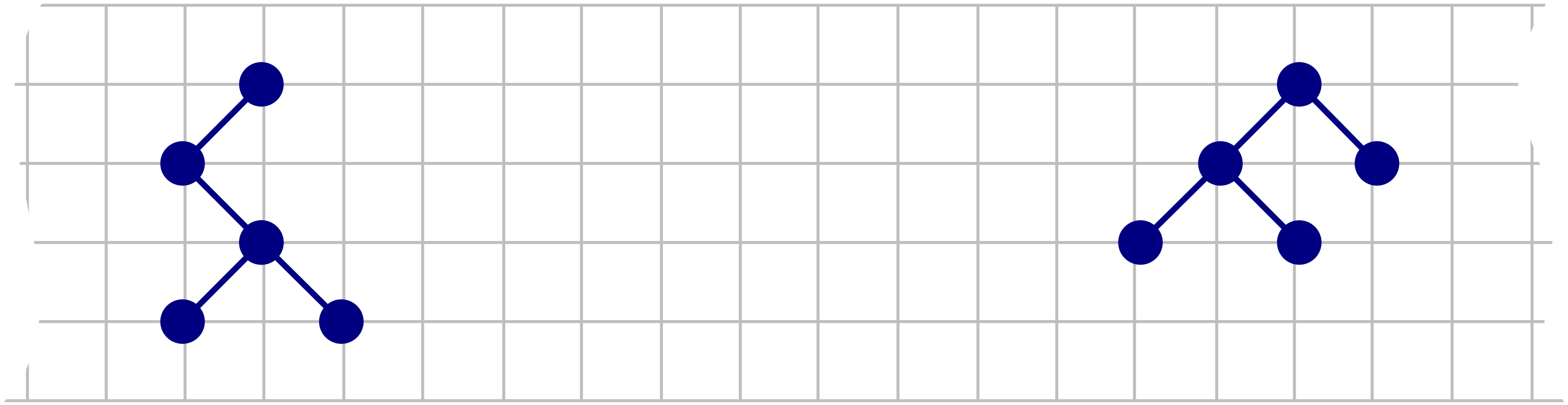

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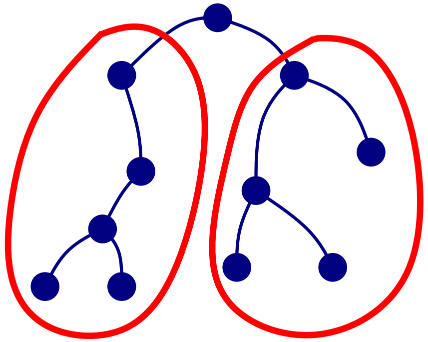

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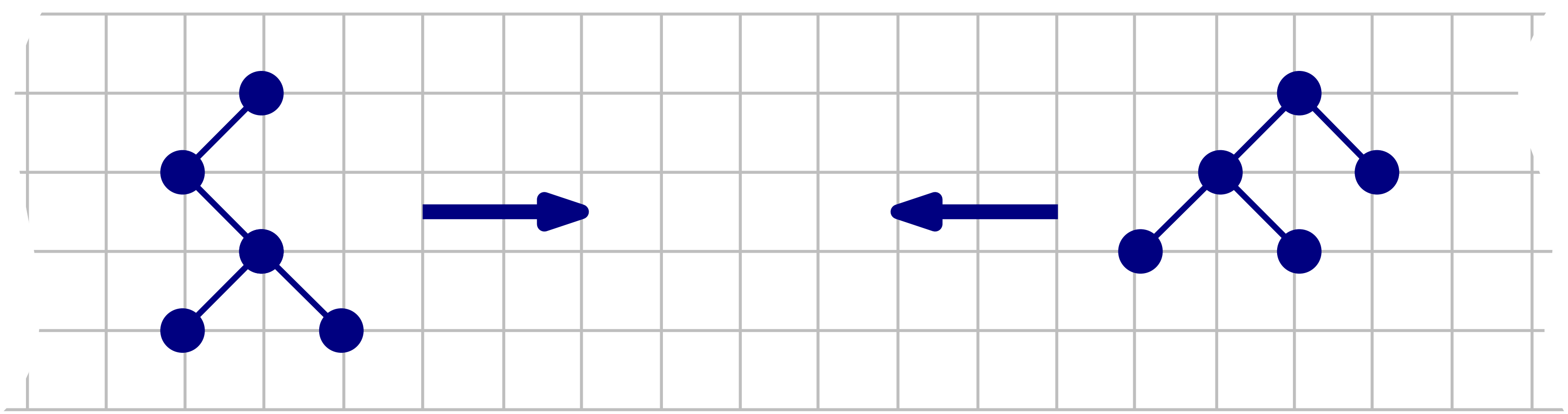

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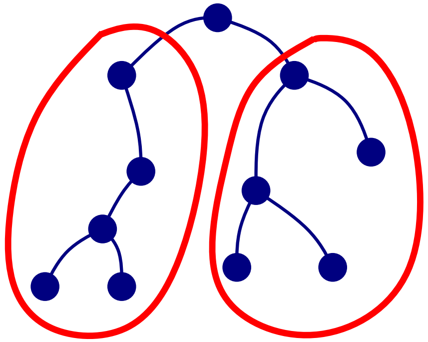

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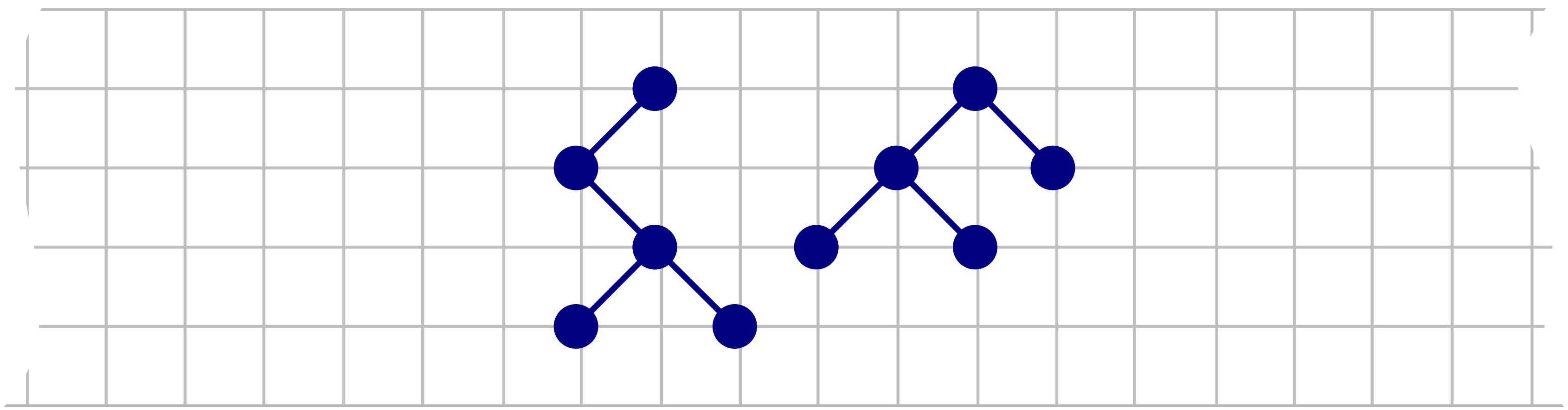

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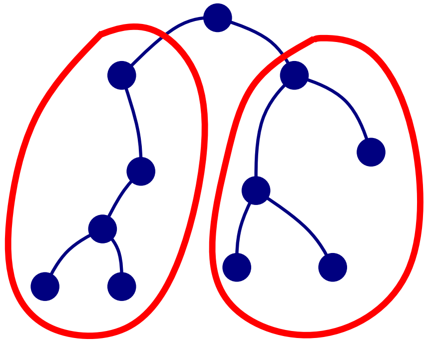

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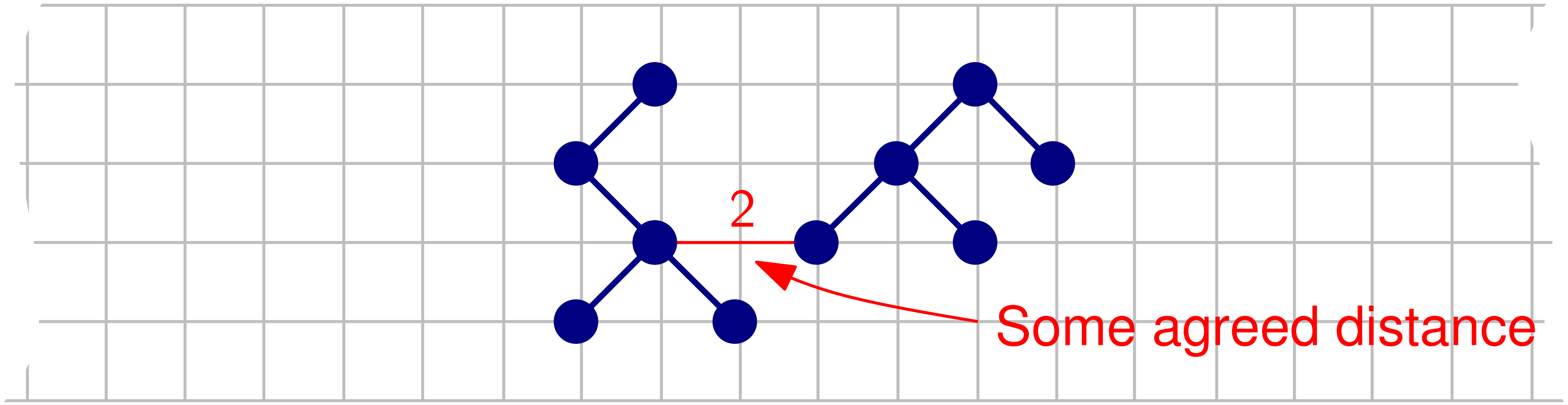

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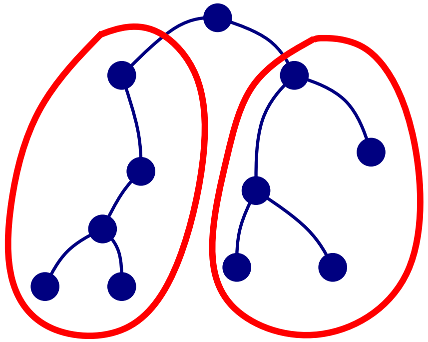

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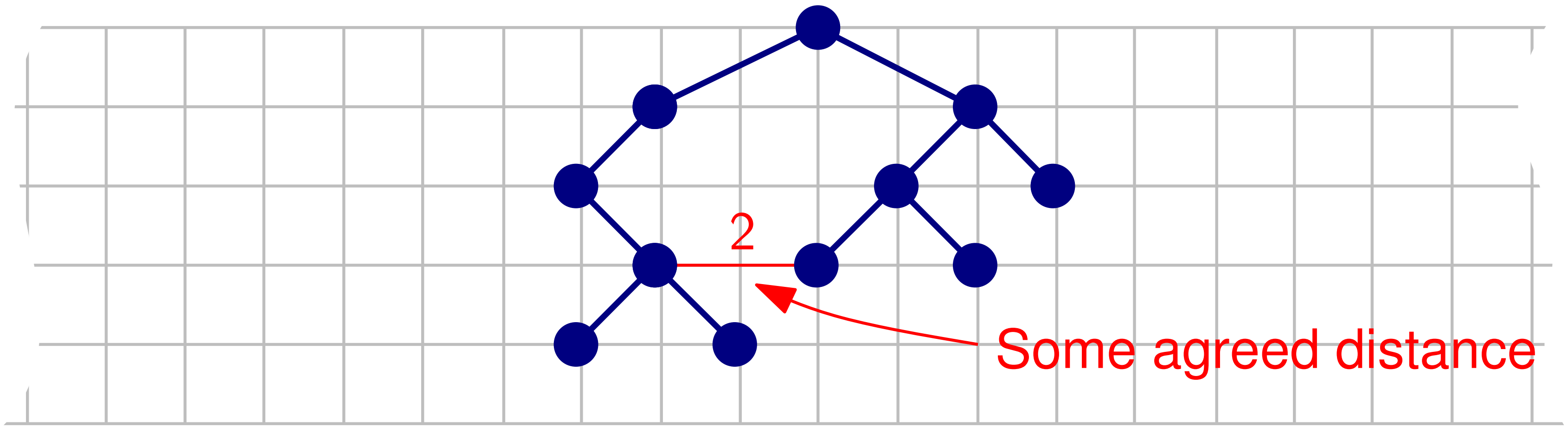

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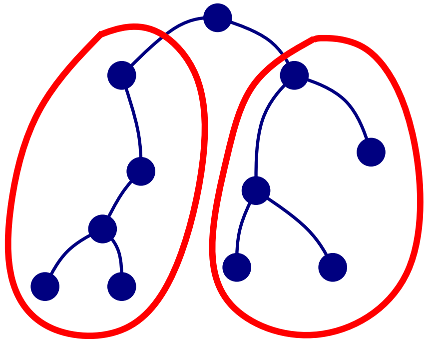

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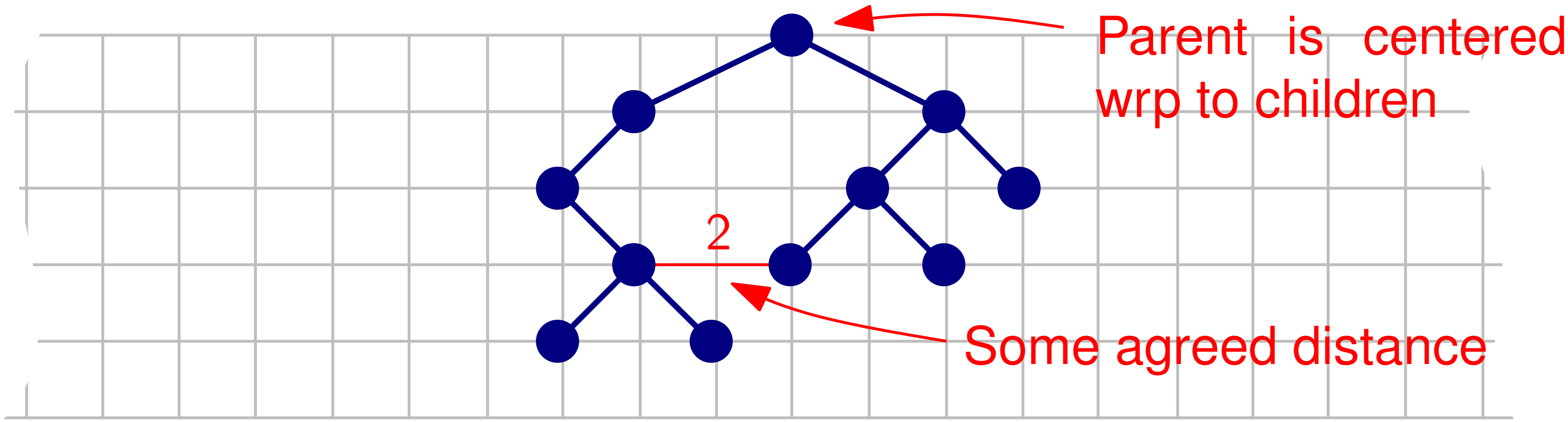

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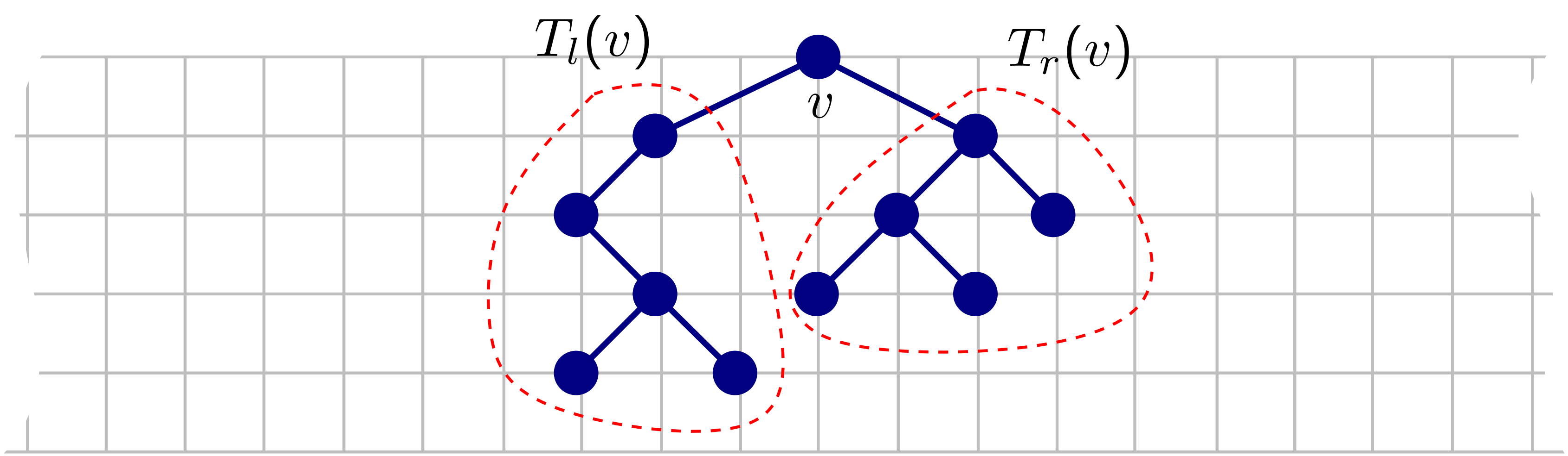
Implementation Details (postorder and preorder traversals)

Postorder traversal: For each vertex v compute horizontal displacement of the left and the right child



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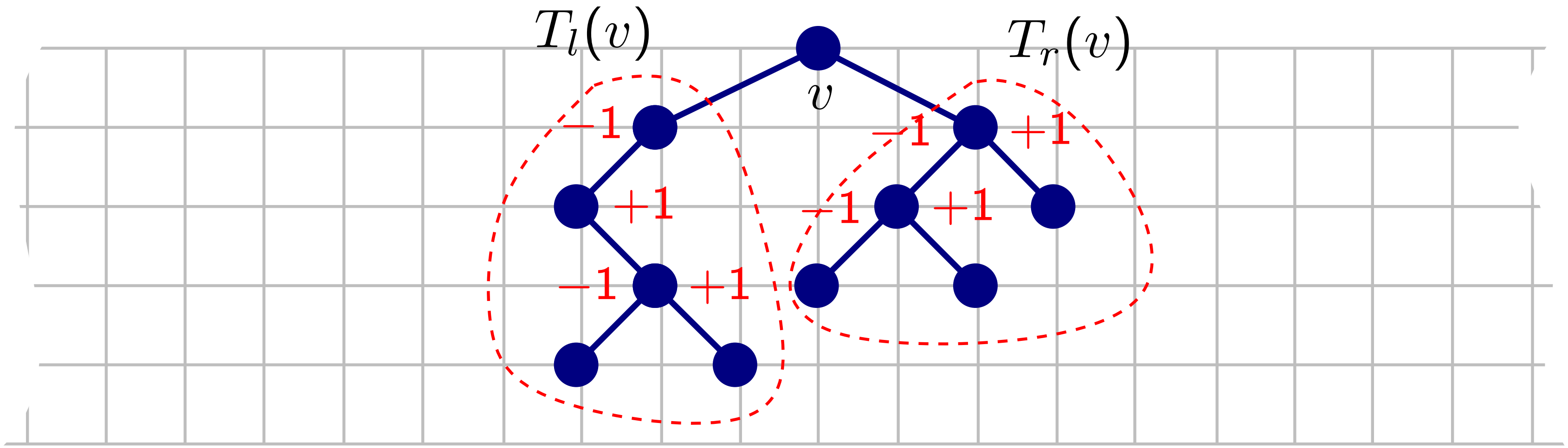

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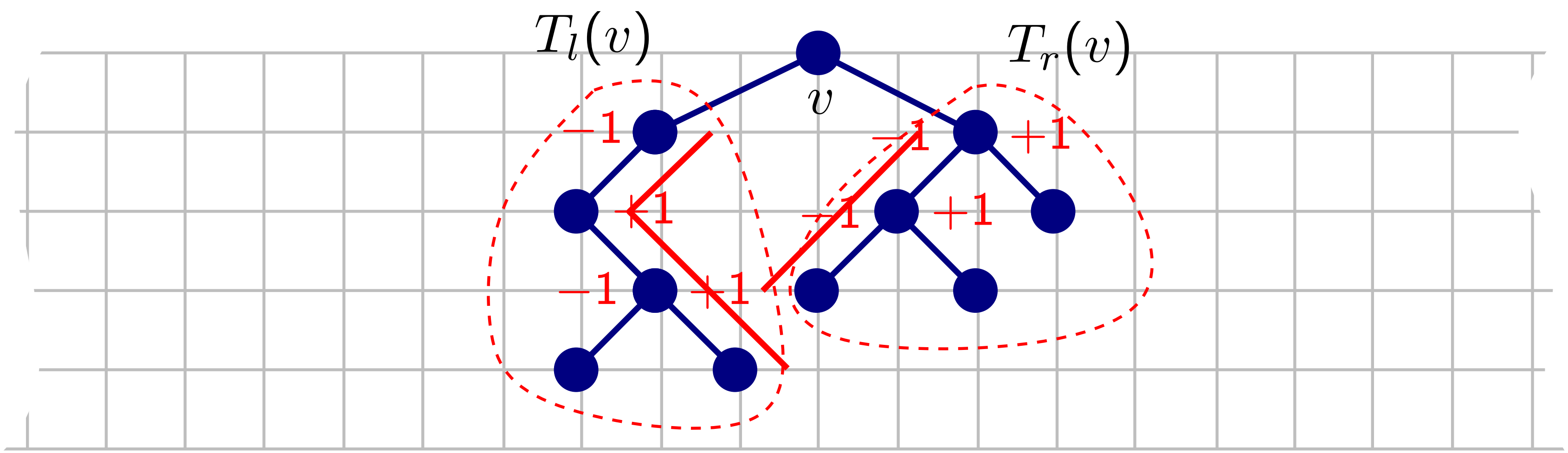

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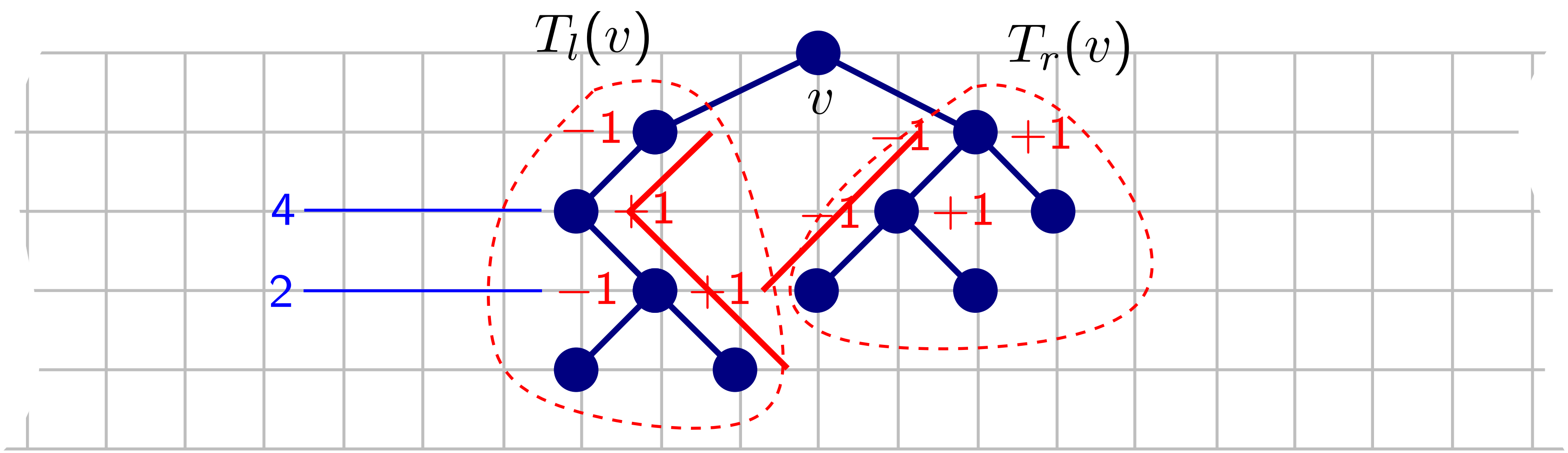

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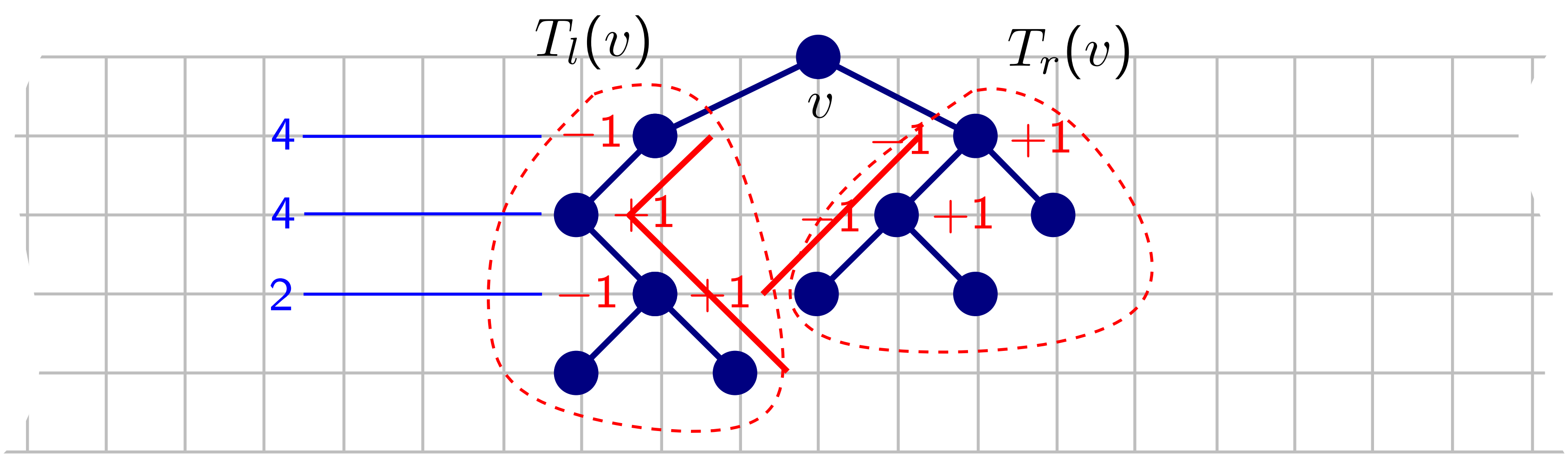

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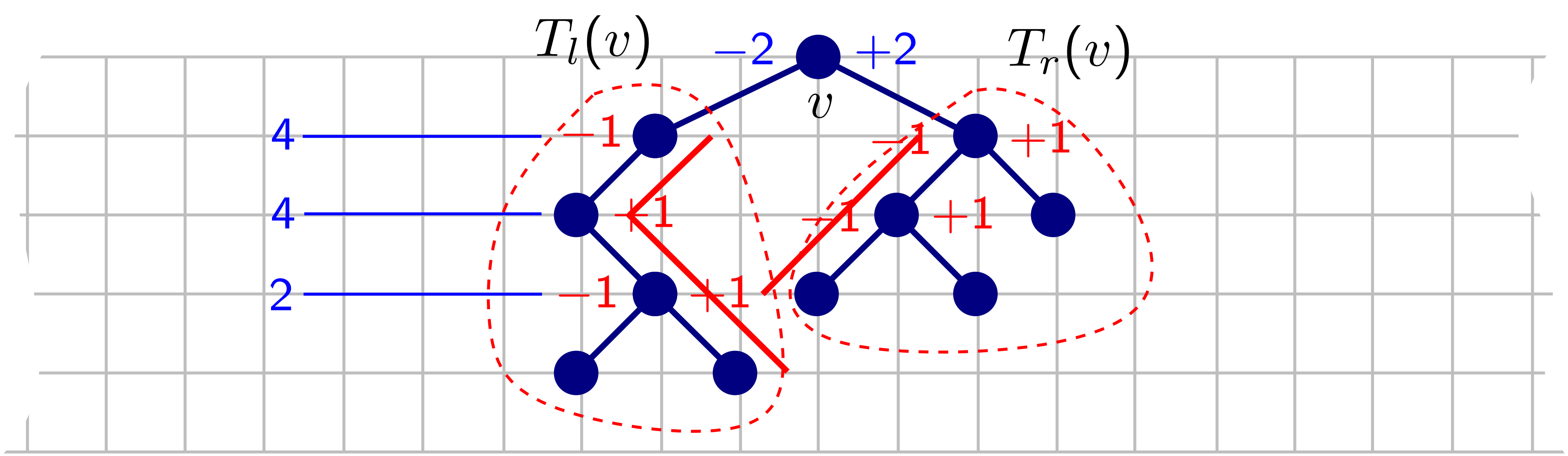

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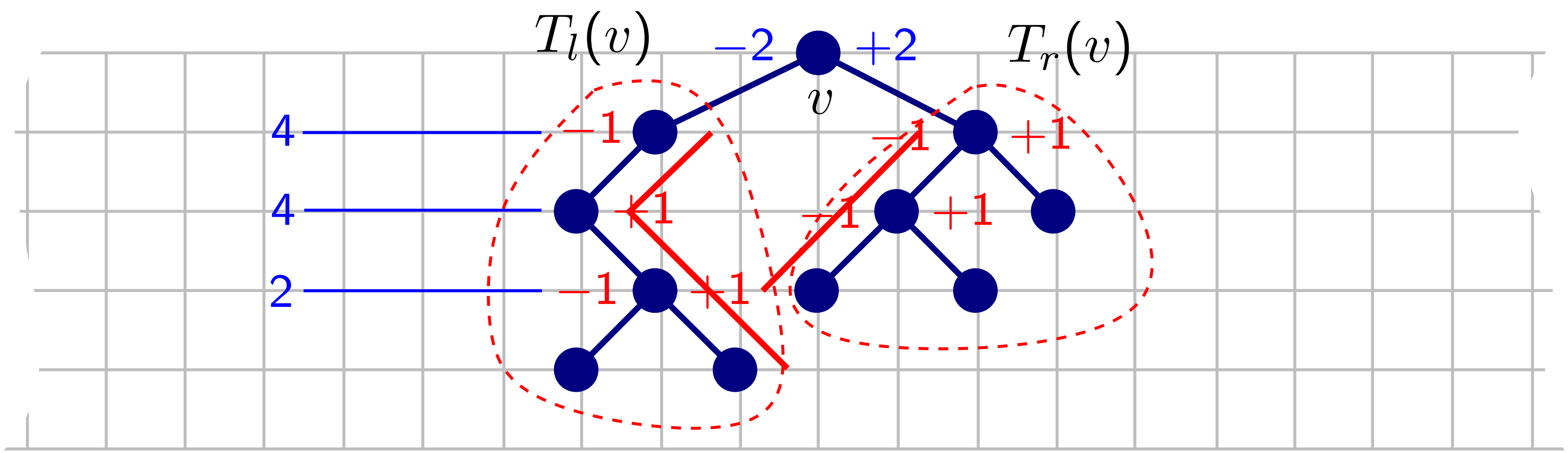
- Assume at each vertex u (below v) we have stored the left and the right boundary of the subtree $T(u)$
- “Summ up” the horizontal displacements of the right boundary of $T_l(v)$ and the left boundary of $T_r(v)$ to obtain the displ. of the children of v



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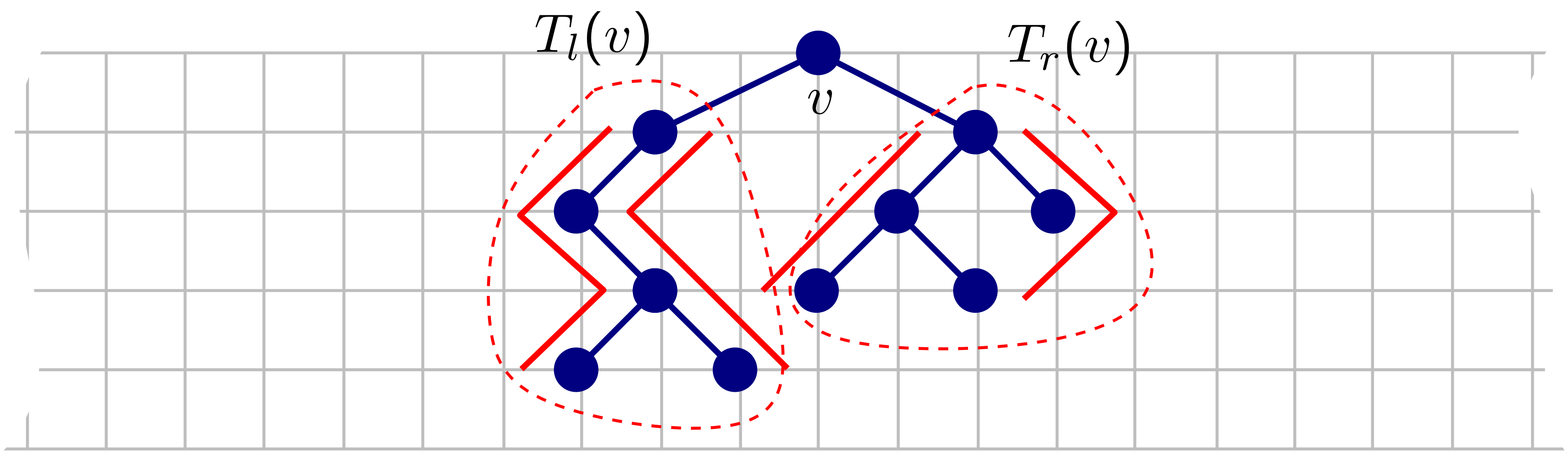

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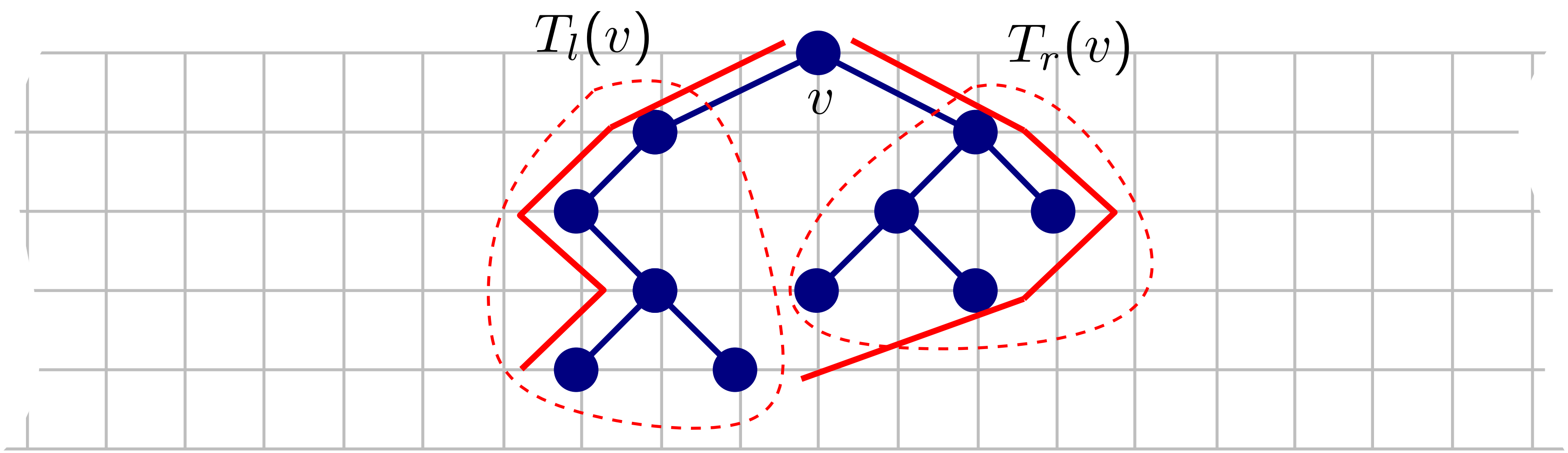

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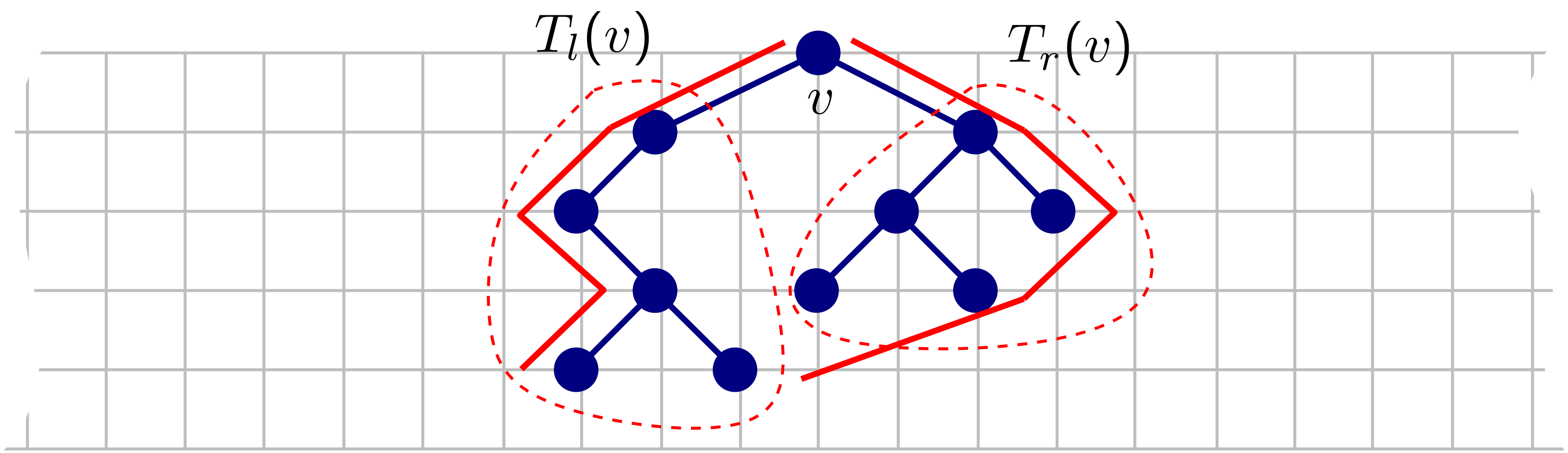
- Store at v the left and the right boundaries of $T(v)$



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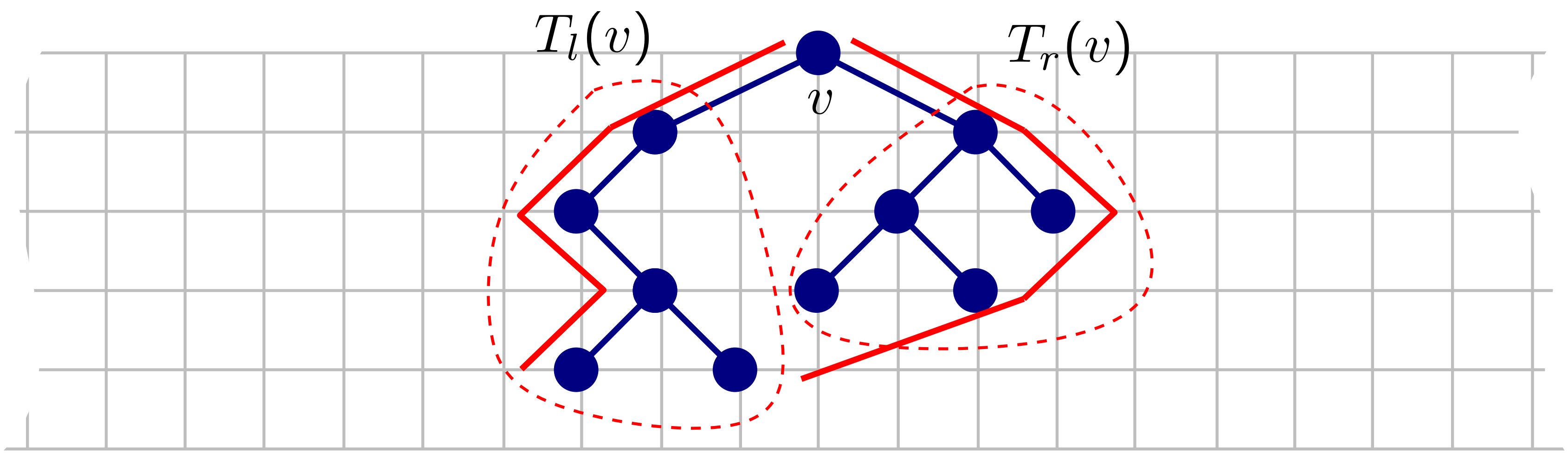
Implementation Details (postorder and preorder traversals)

Postorder traversal: For each vertex v compute horizontal displacement of the left and the right child

Preorder traversal: Compute x- and y-coordinates.



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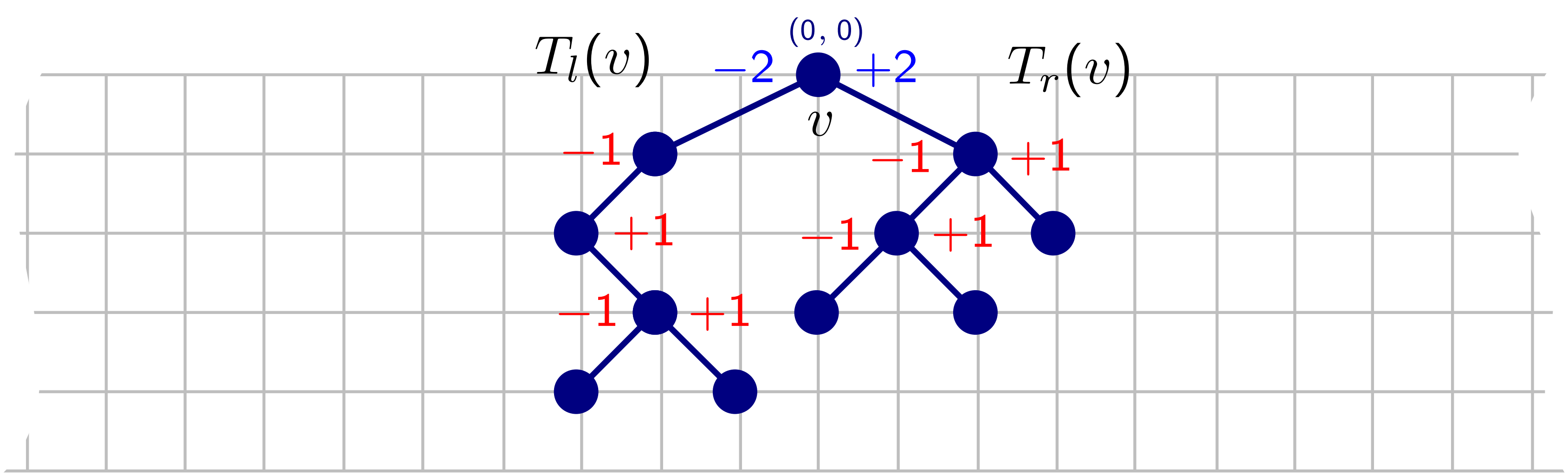

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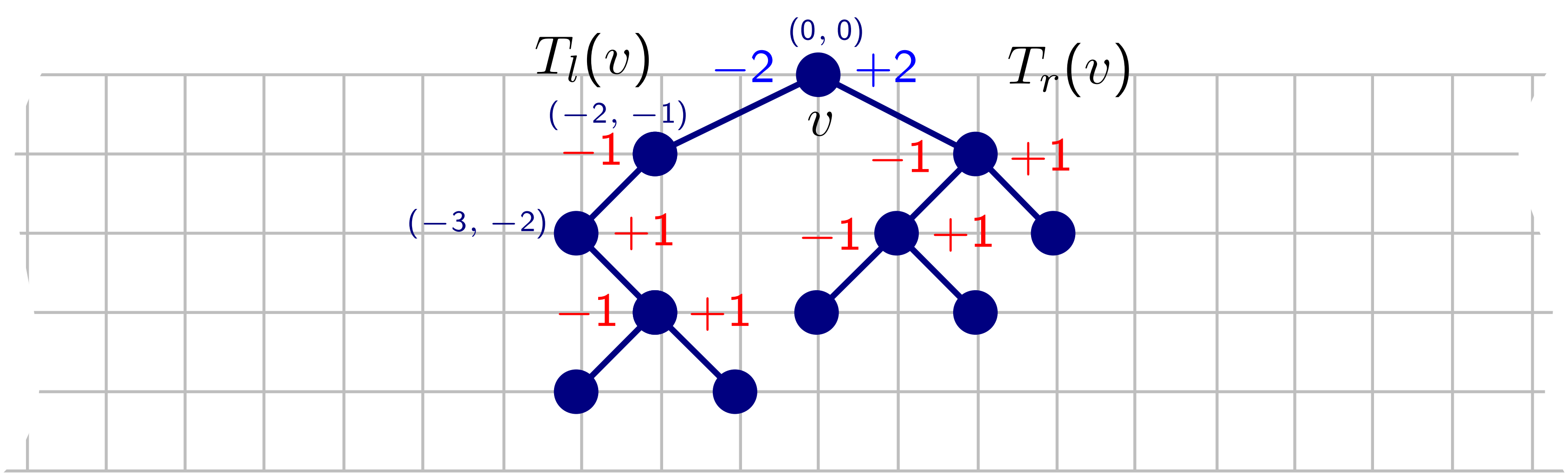

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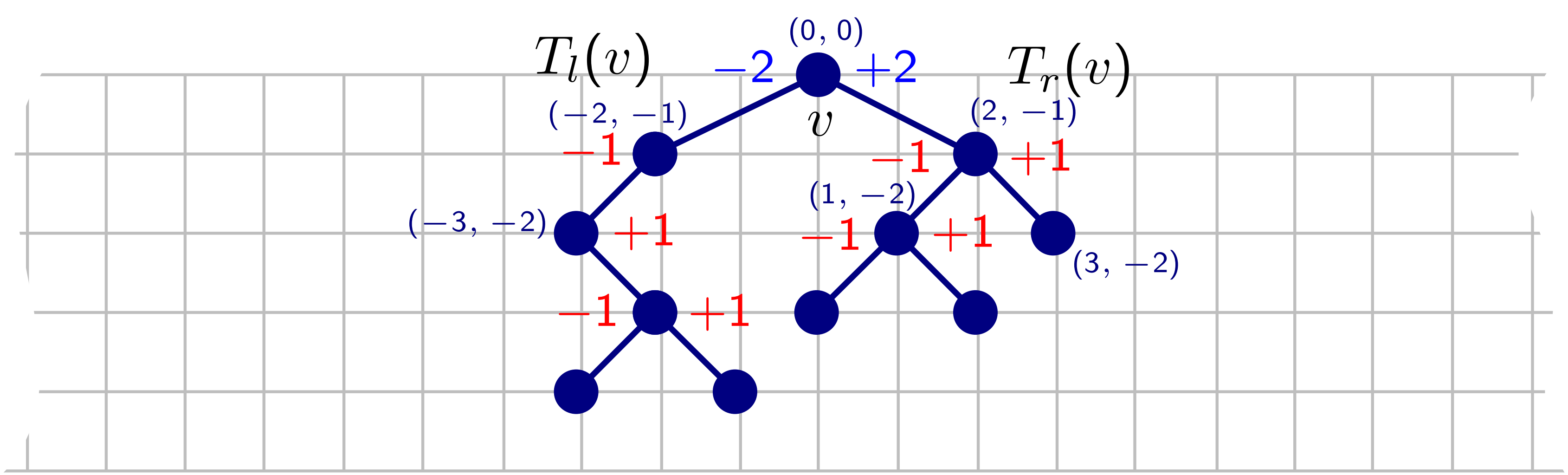

Level-based Layout

Time Complexity

Postorder traversal: For each vertex v compute horizontal displacement of the left and the right child

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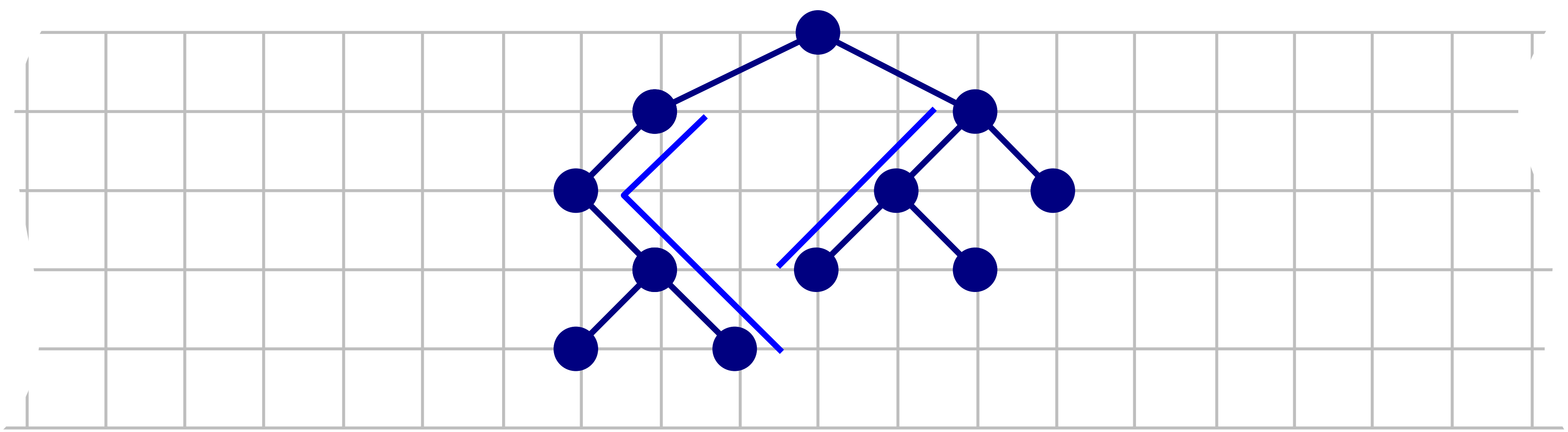
- Store at v the left and the right boundaries of $T(v)$

Preorder traversal: Compute x- and y-coordinates.



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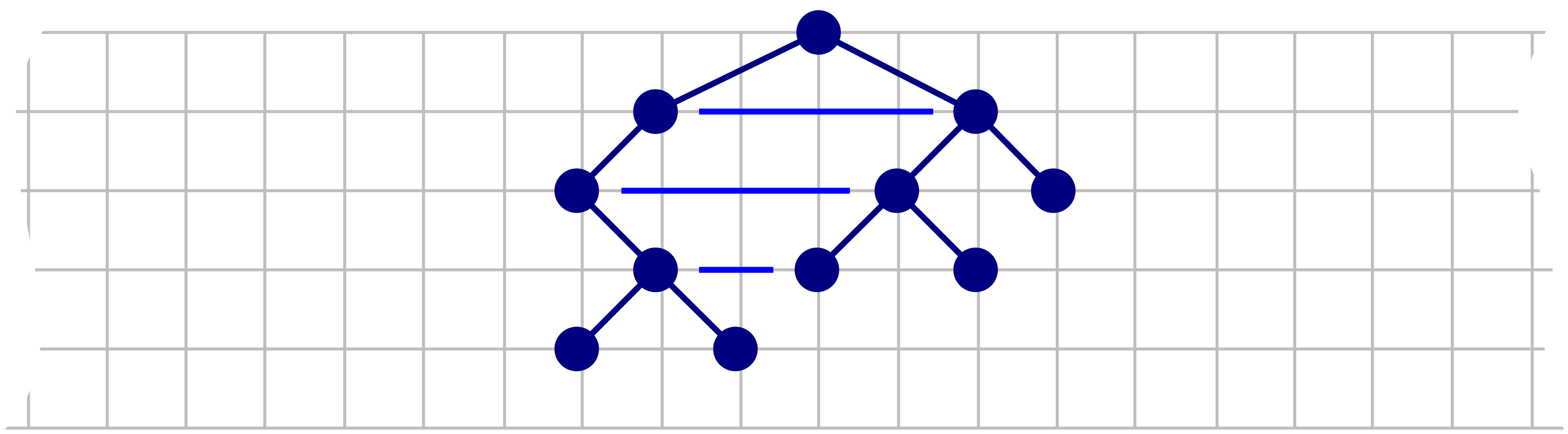

Level-based Layout

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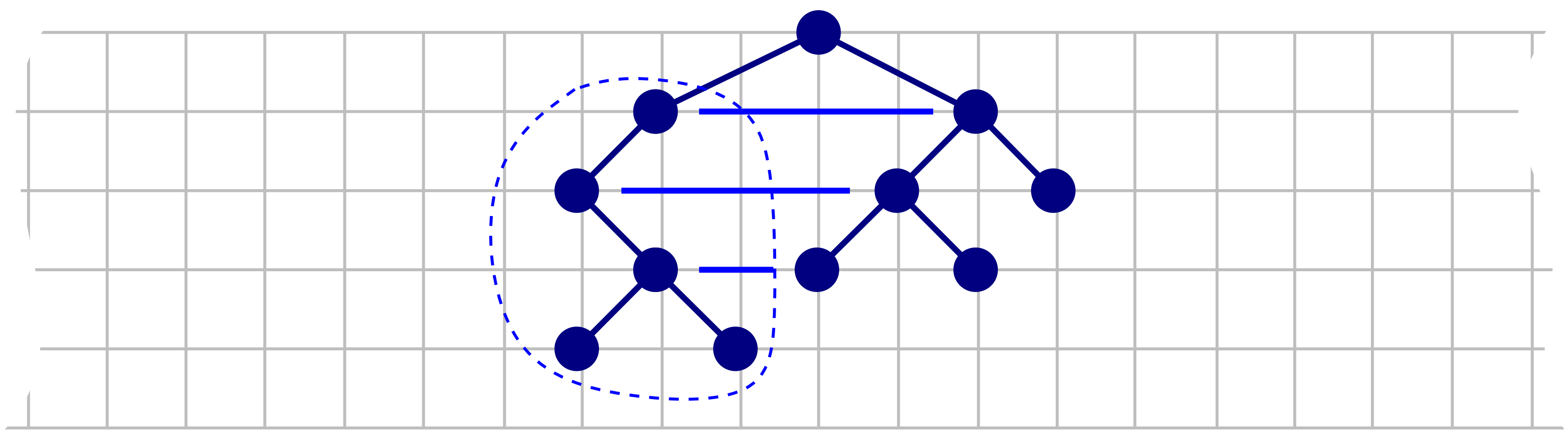

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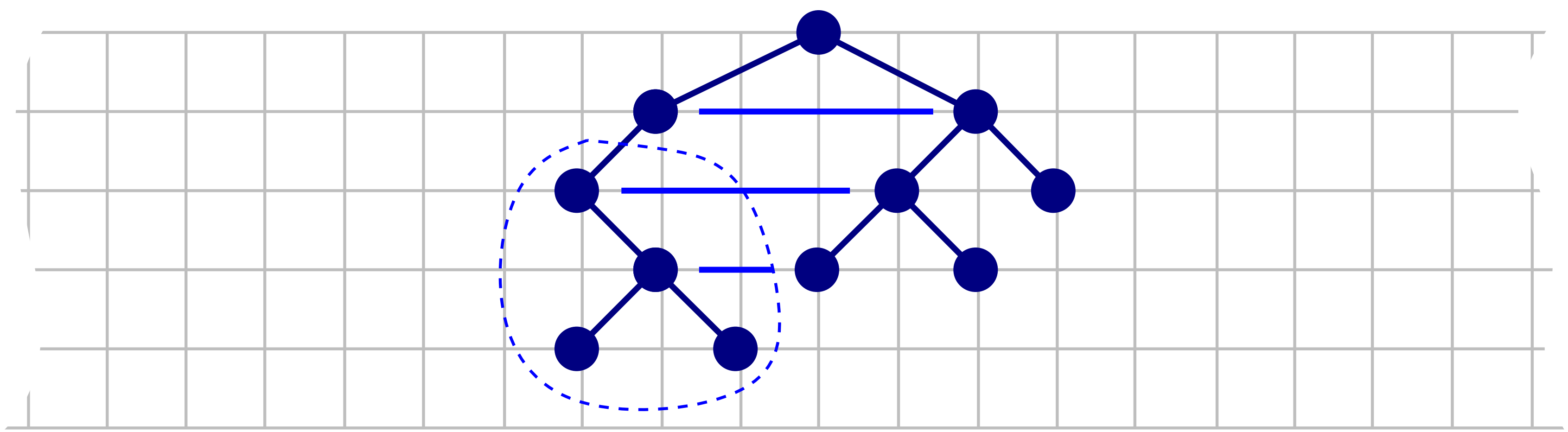

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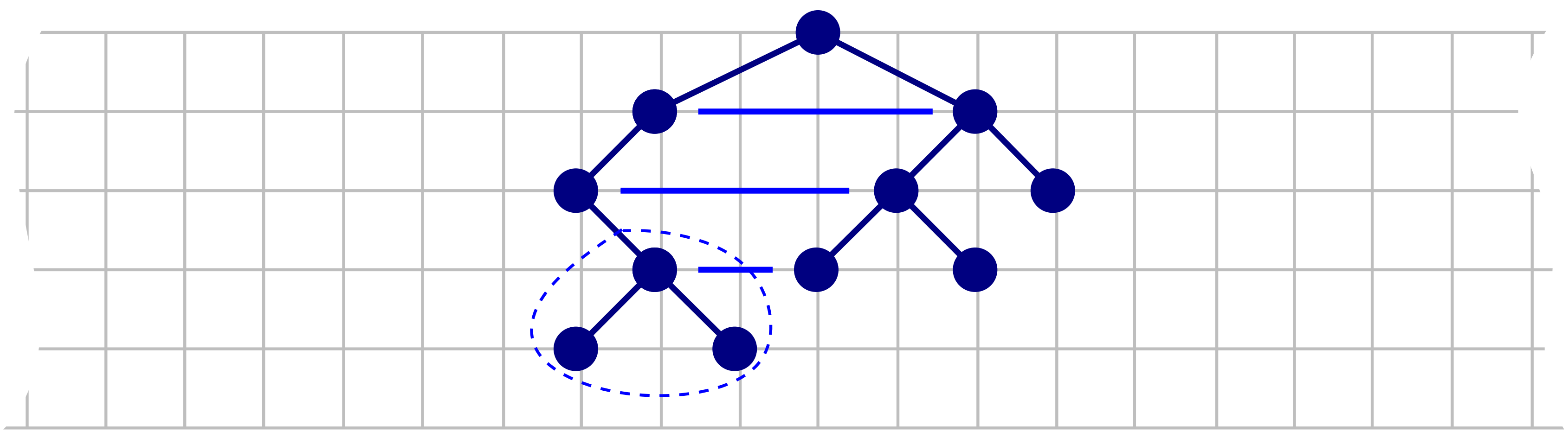

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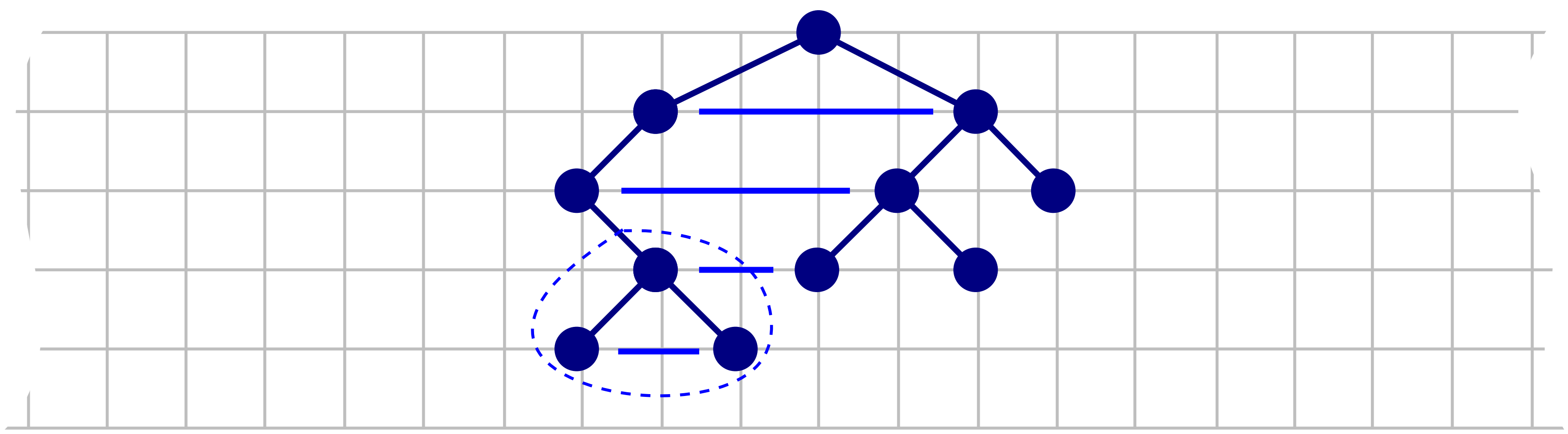

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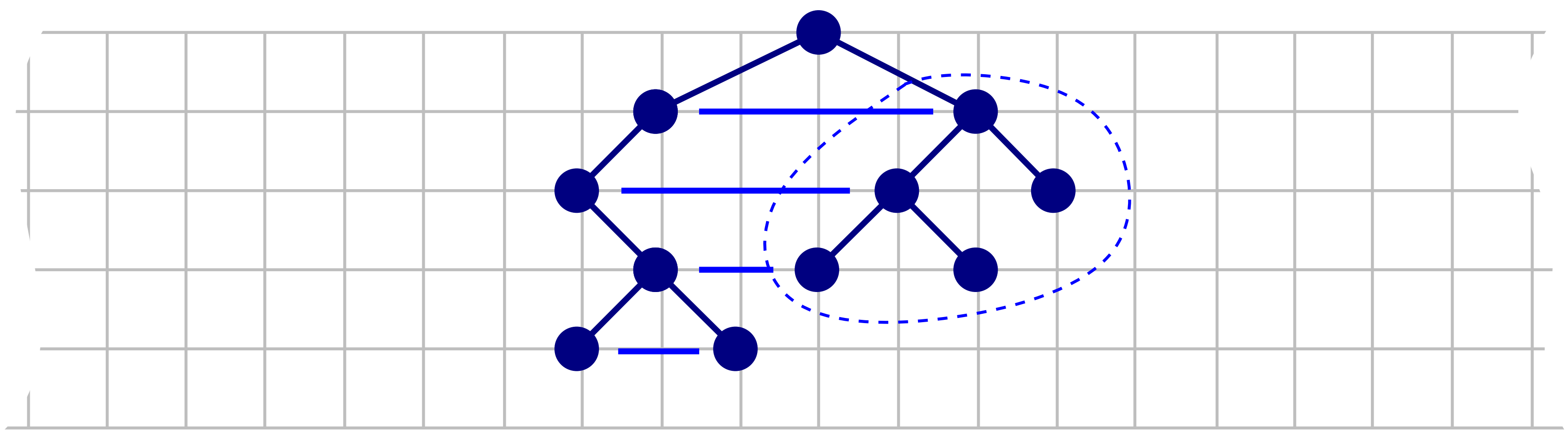

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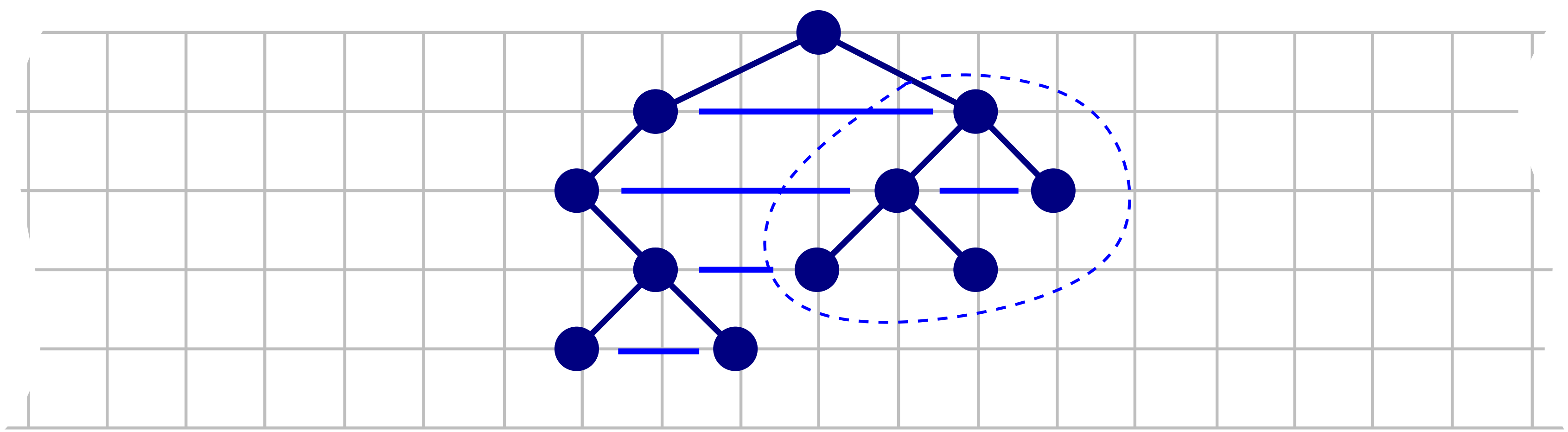

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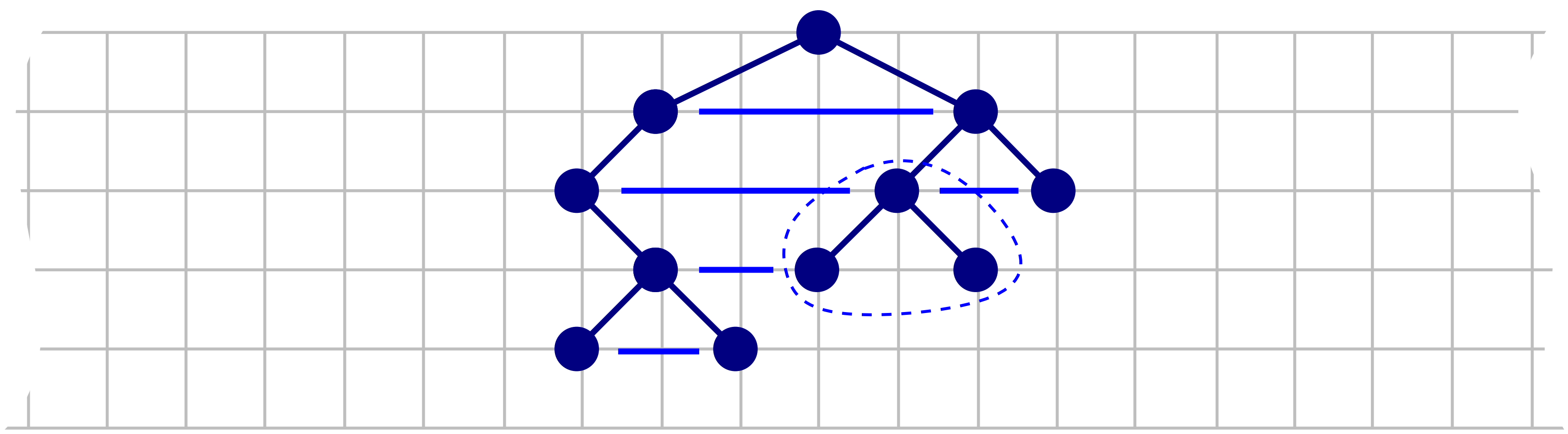

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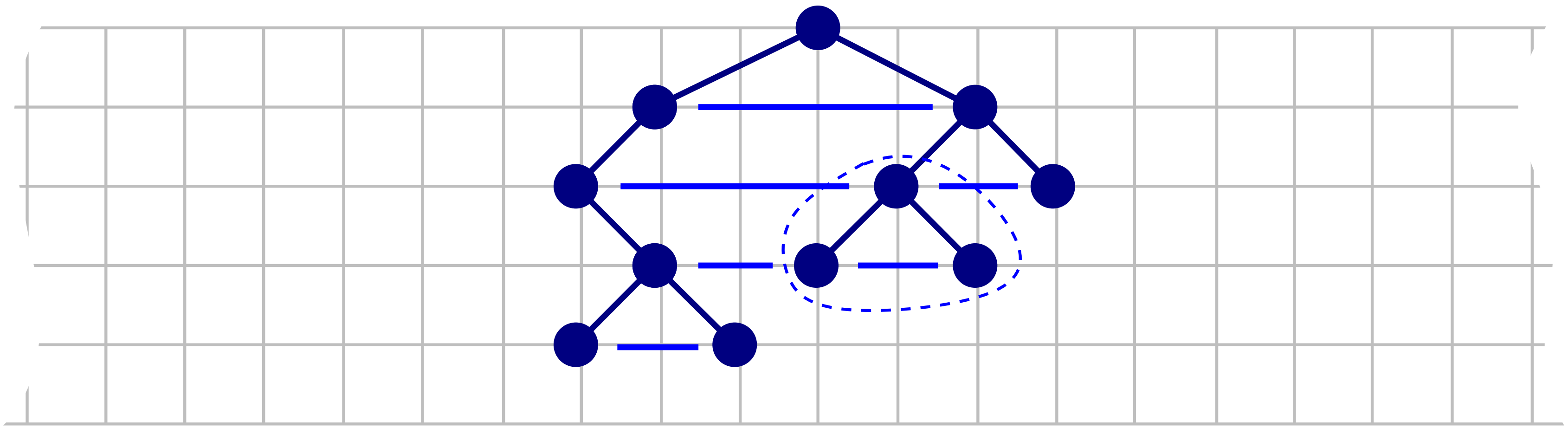

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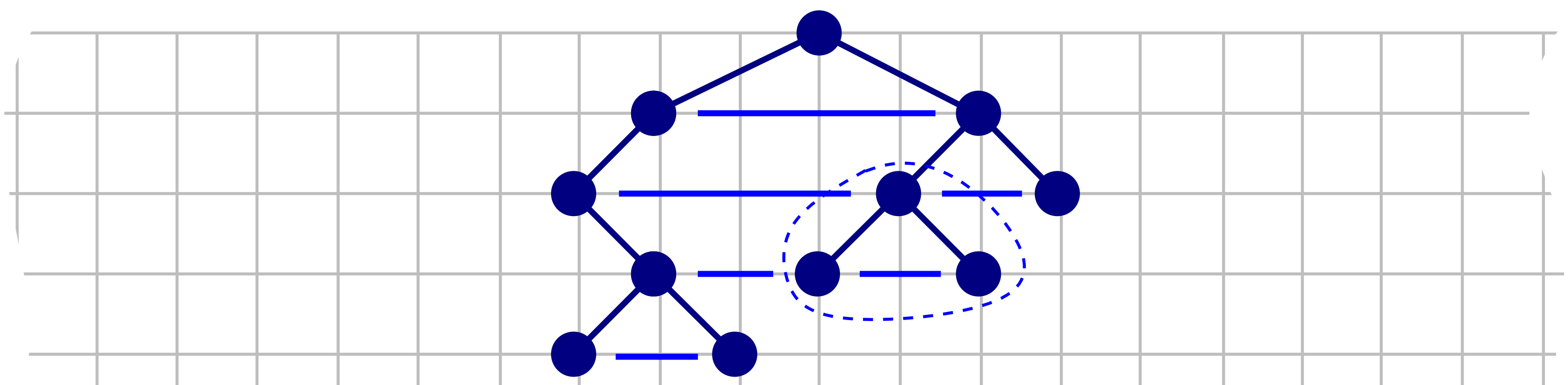

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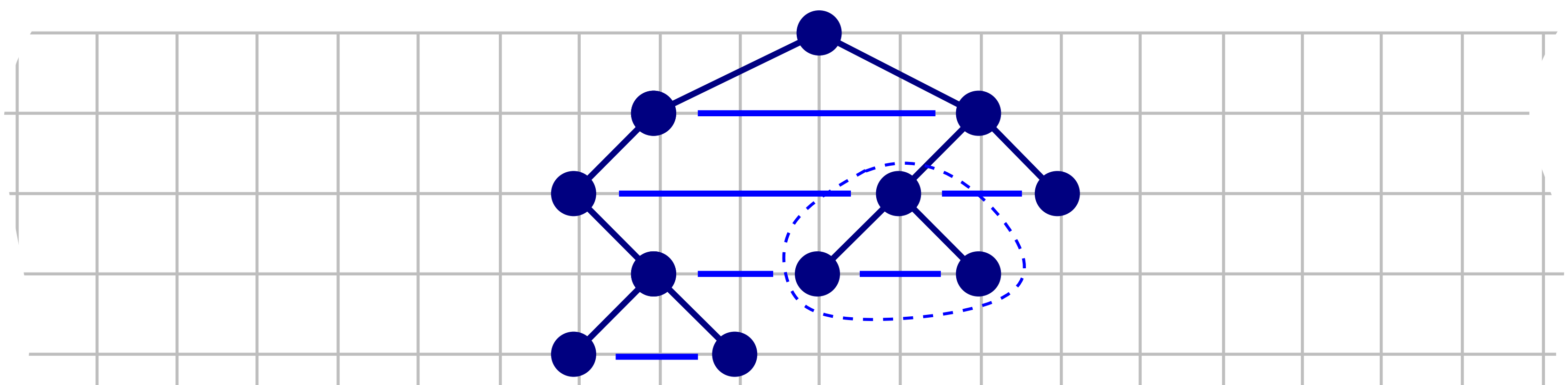

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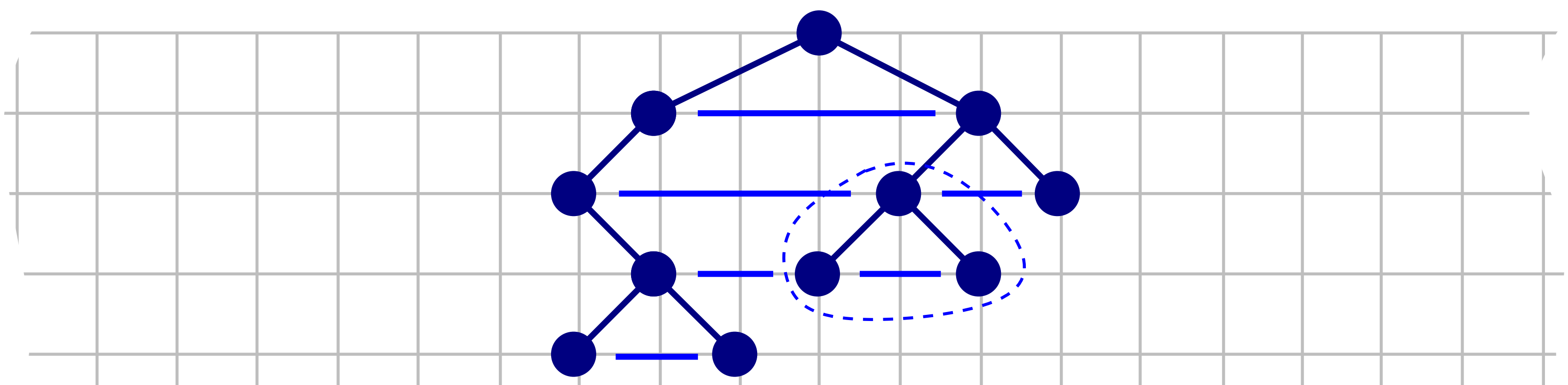

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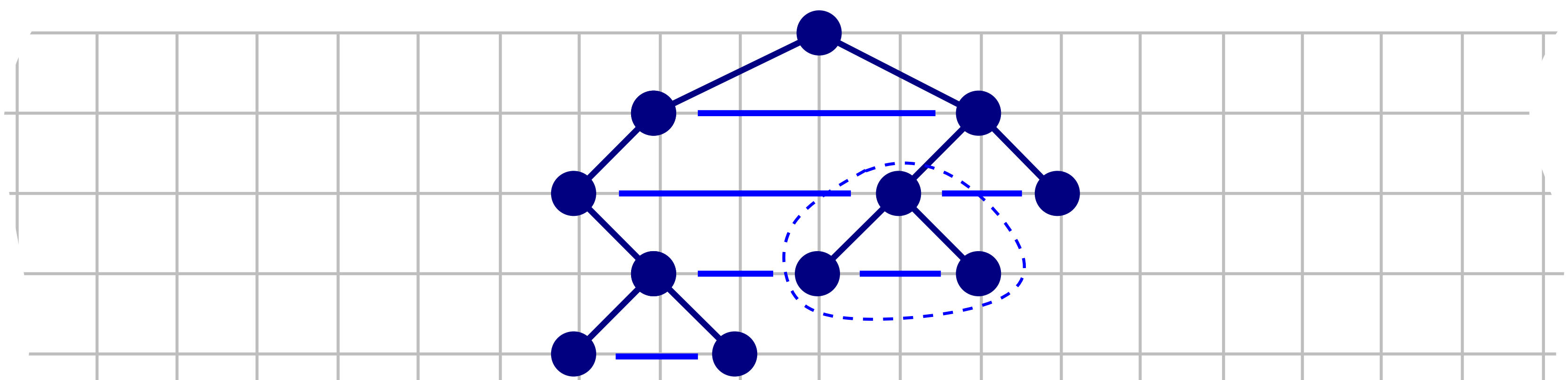

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Theorem (Reingold & Tilford)

Let T be a binary tree with n vertices. Algorithm (R & T) constructs a drawing Γ of T in $O(n)$ time, such that:

- Γ is planar and straight-line
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Level-based Layout

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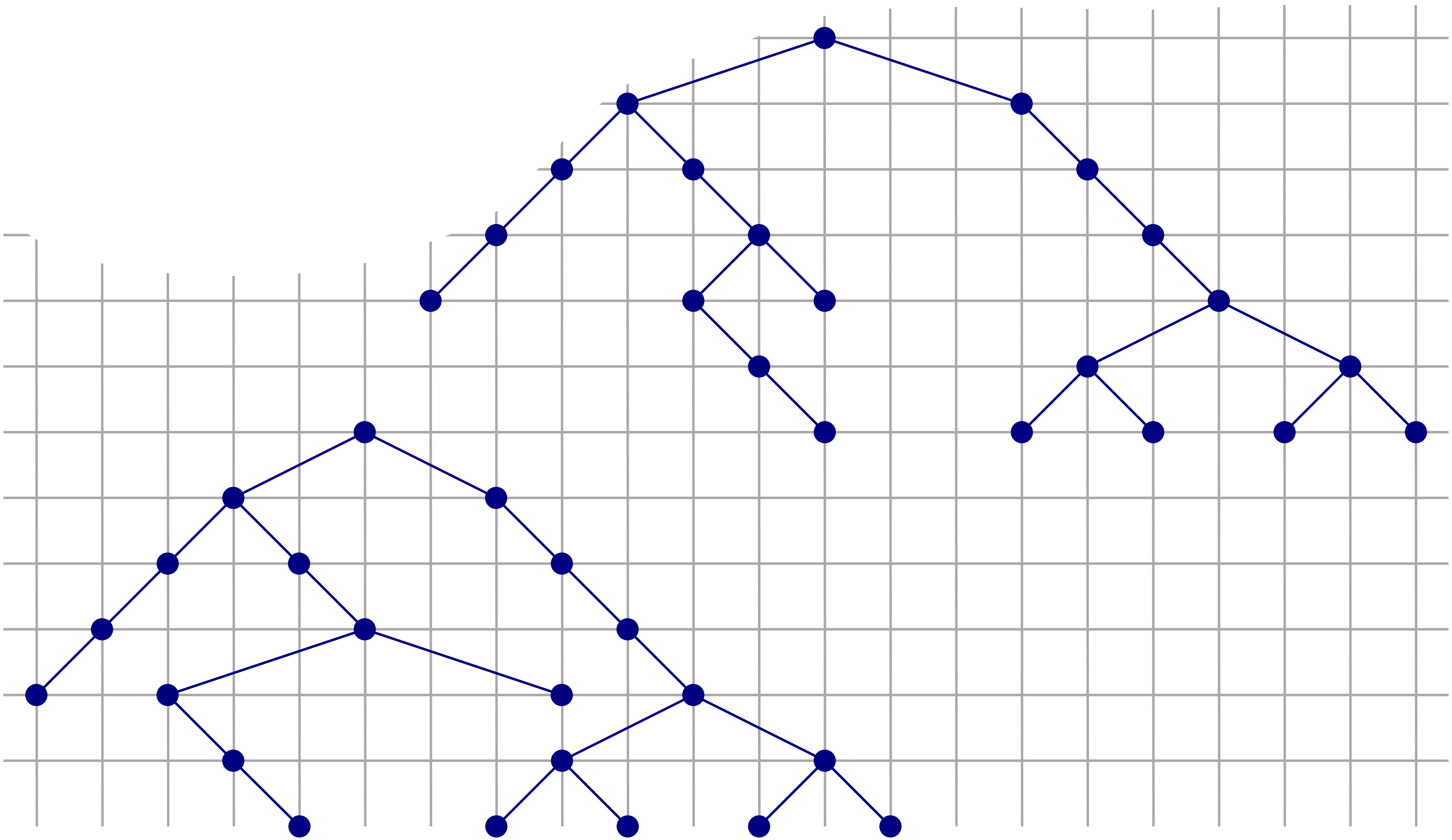
- Simply isomorphic subtrees have congruent (coincident) drawing, up to translation
- Axially isomorphic trees have congruent drawing, up to translation and reflection around y-axis

Level-based Layout

- The presented algorithm tries to minimize width



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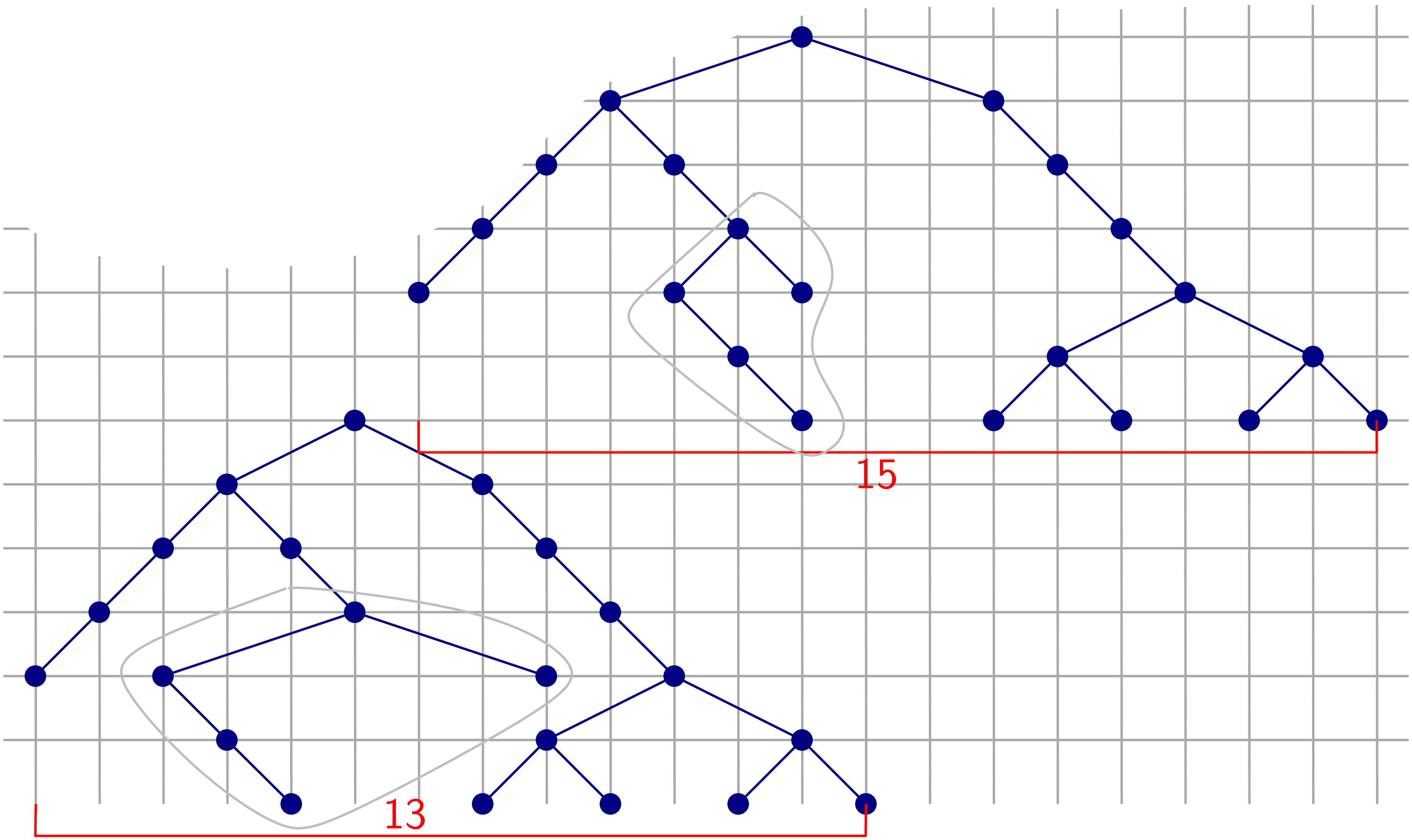

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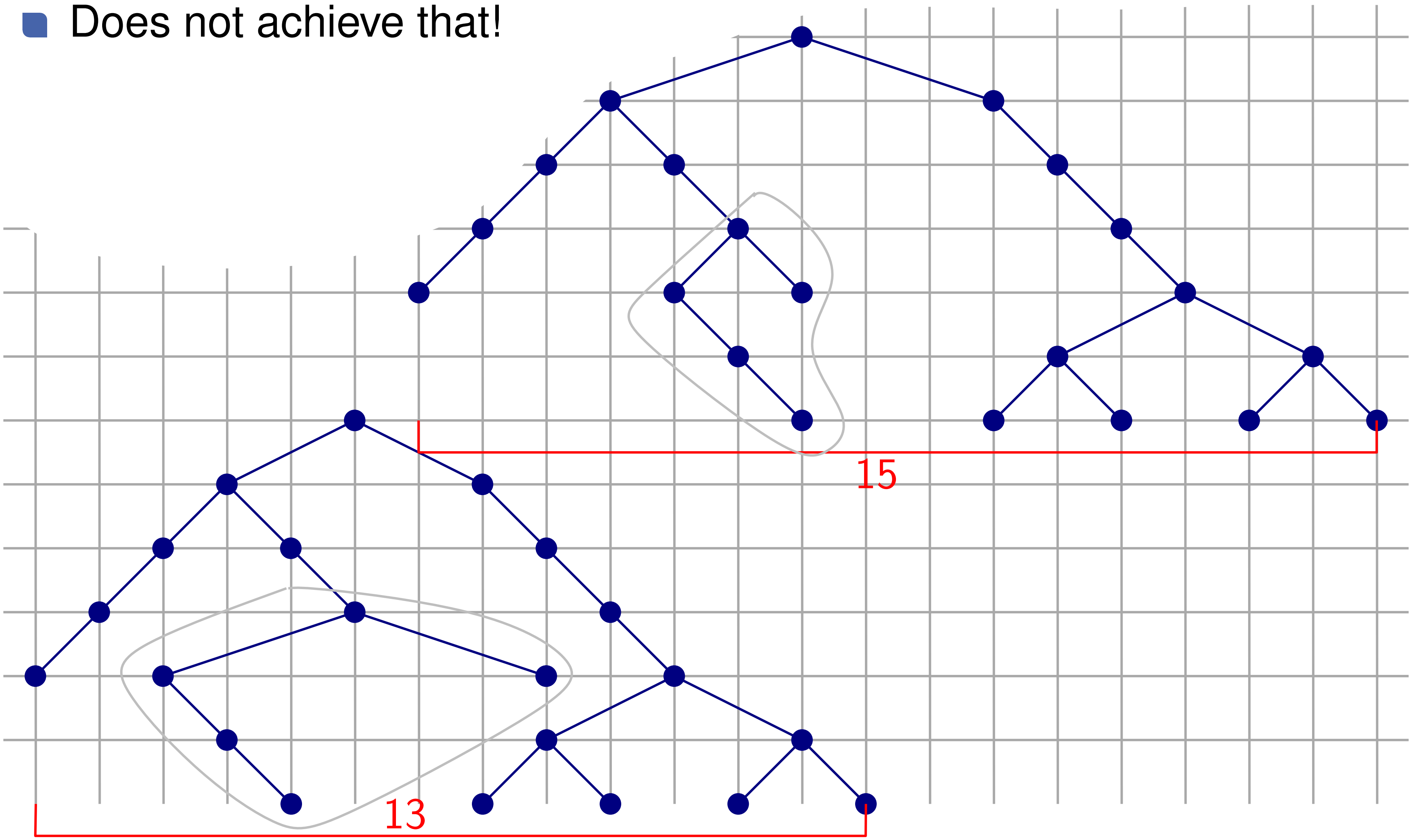

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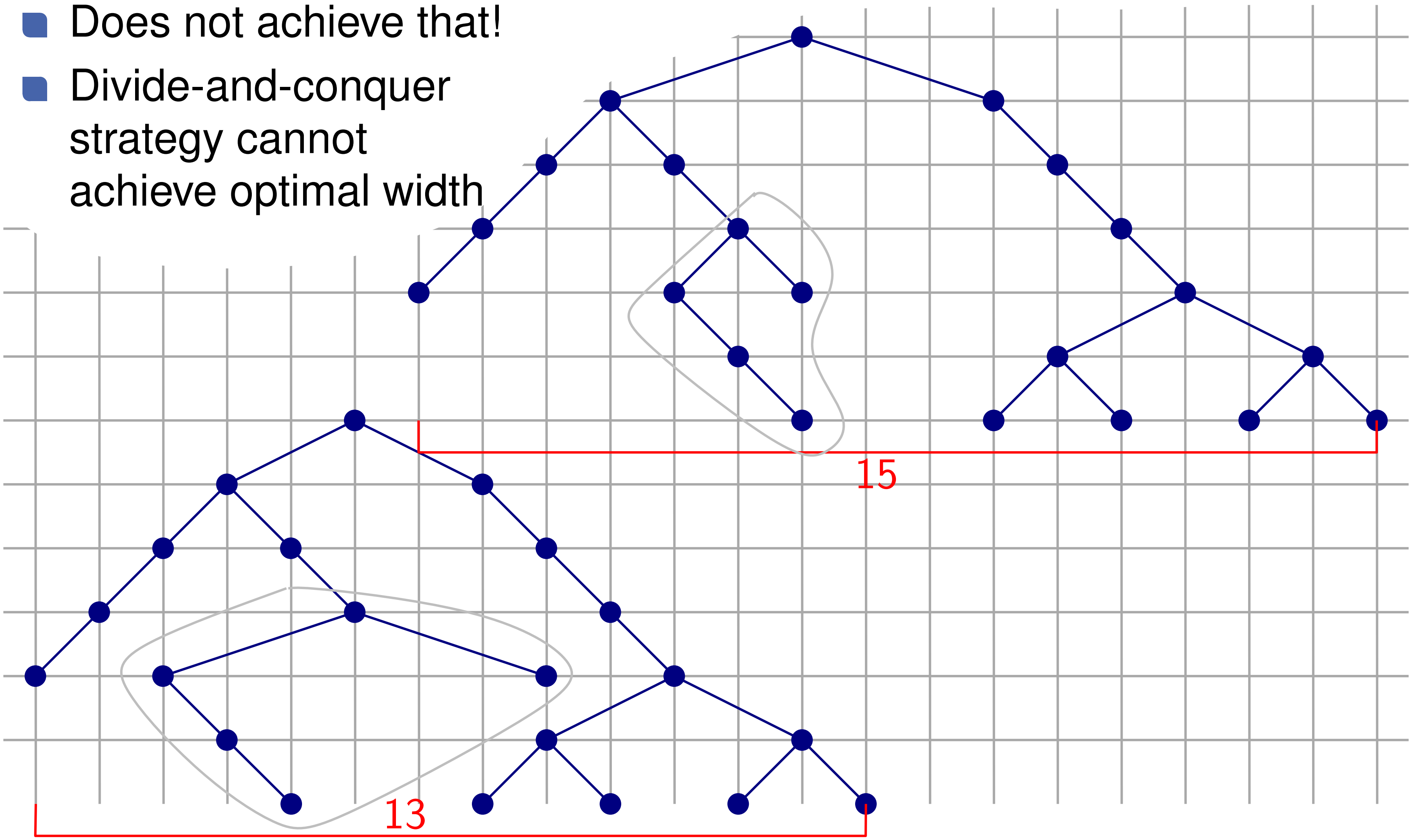

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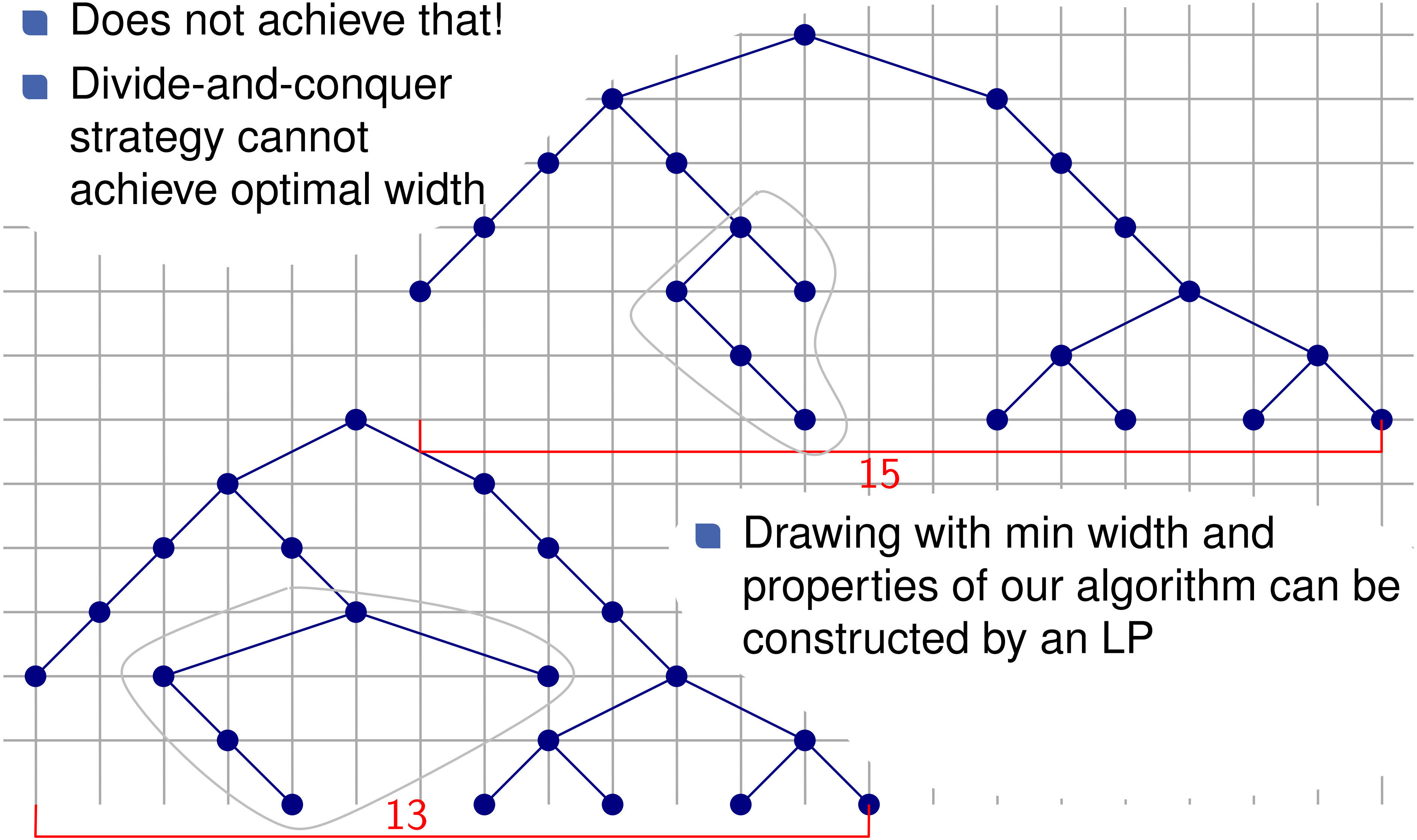

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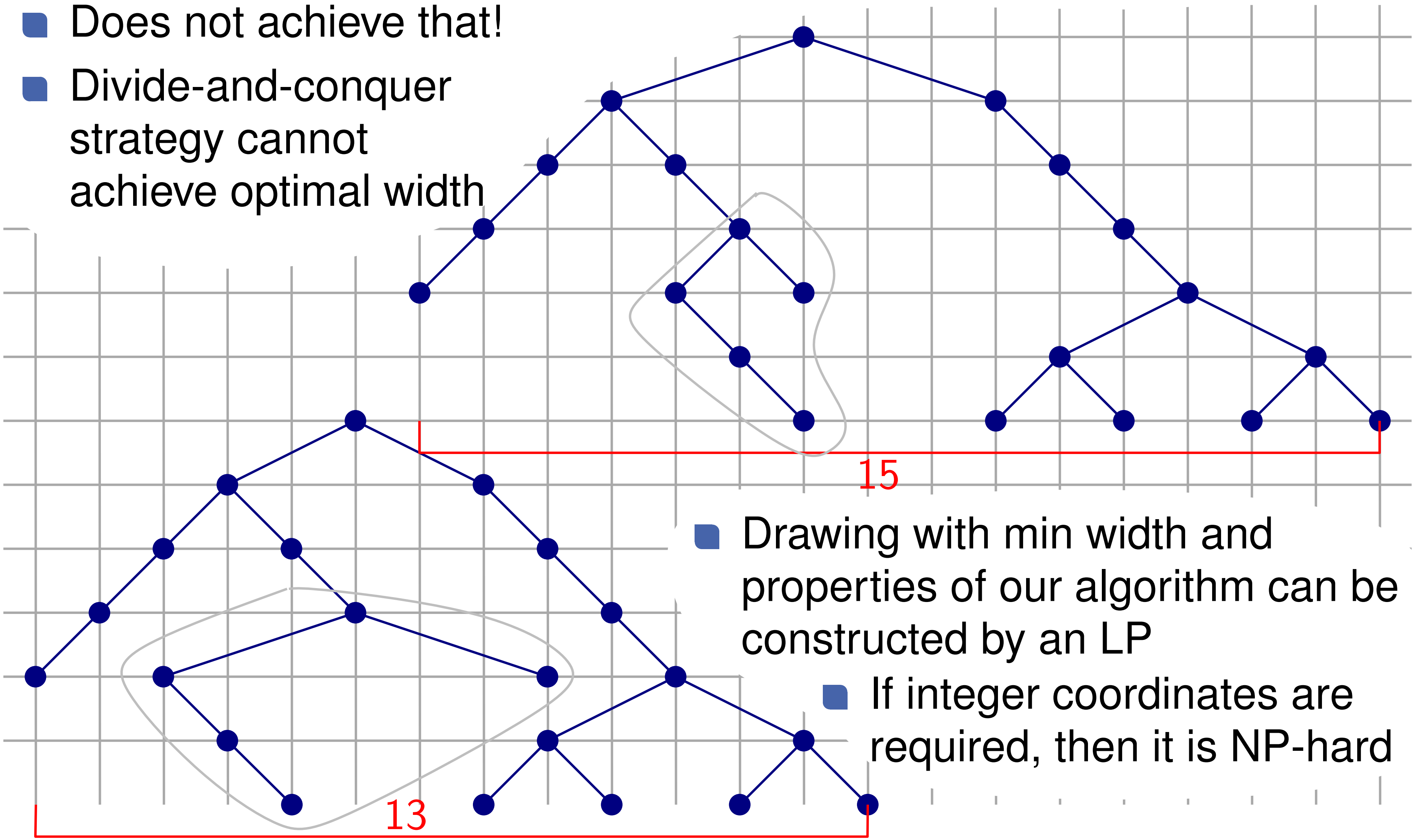


- Drawing with min width and properties of our algorithm can be constructed by an LP
  - If integer coordinates are required, then it is NP-hard

Level-based Layout - General trees

Algorithm Outline:

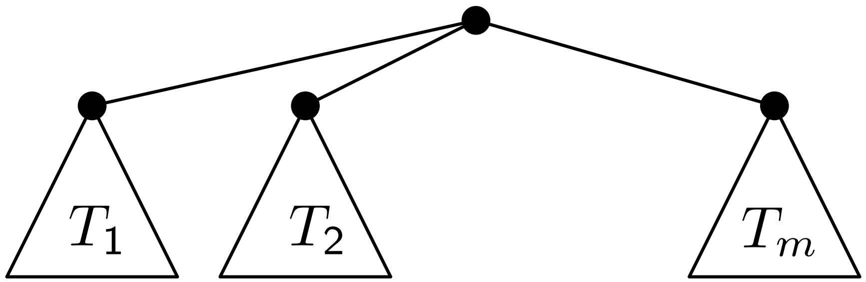
Input: A rooted tree

Output: A layered drawing of T

Base case: A single vertex

Divide: Assume that T has subtrees $T_1, \ldots T_m$. Draw each T_i recursively.

Conquer:



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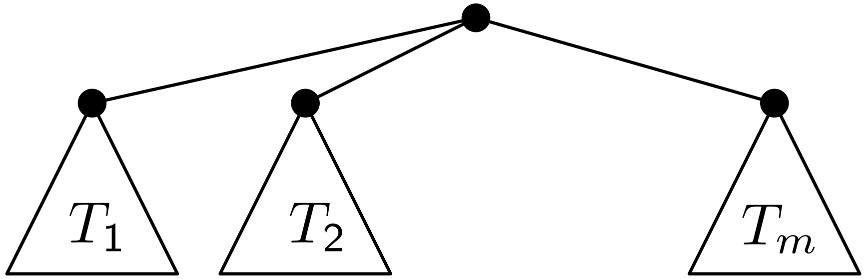
- Position the root half-way between the roots of T_1 and T_m.



Applications of Level-based Layout

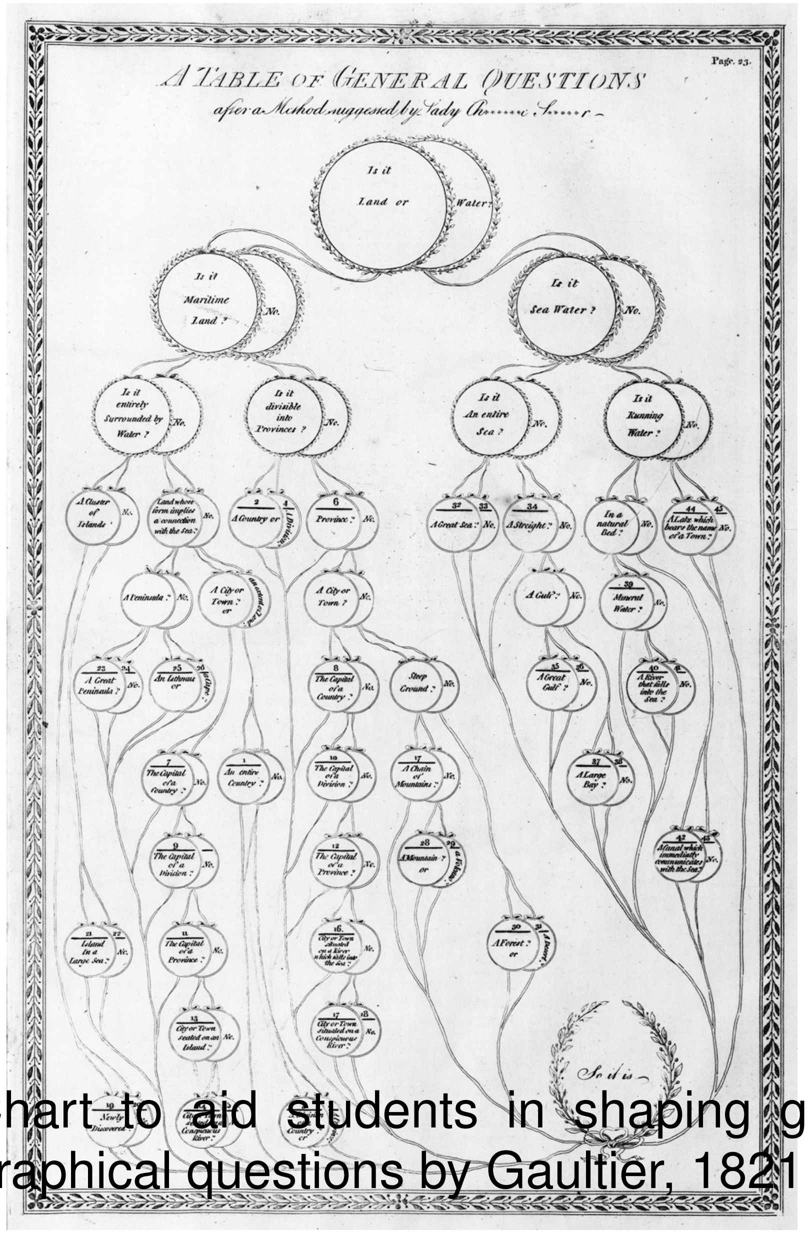


Chart to aid students in shaping geographical questions by Gaultier, 1821

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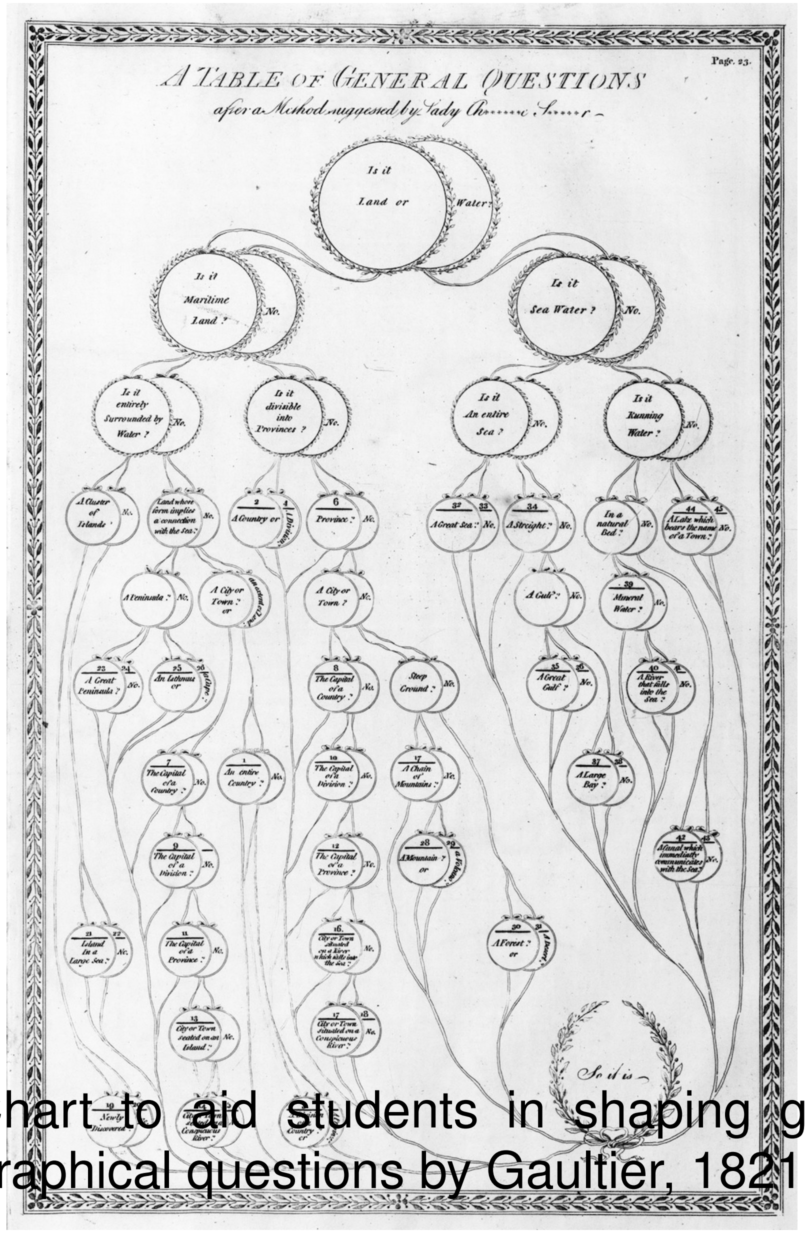

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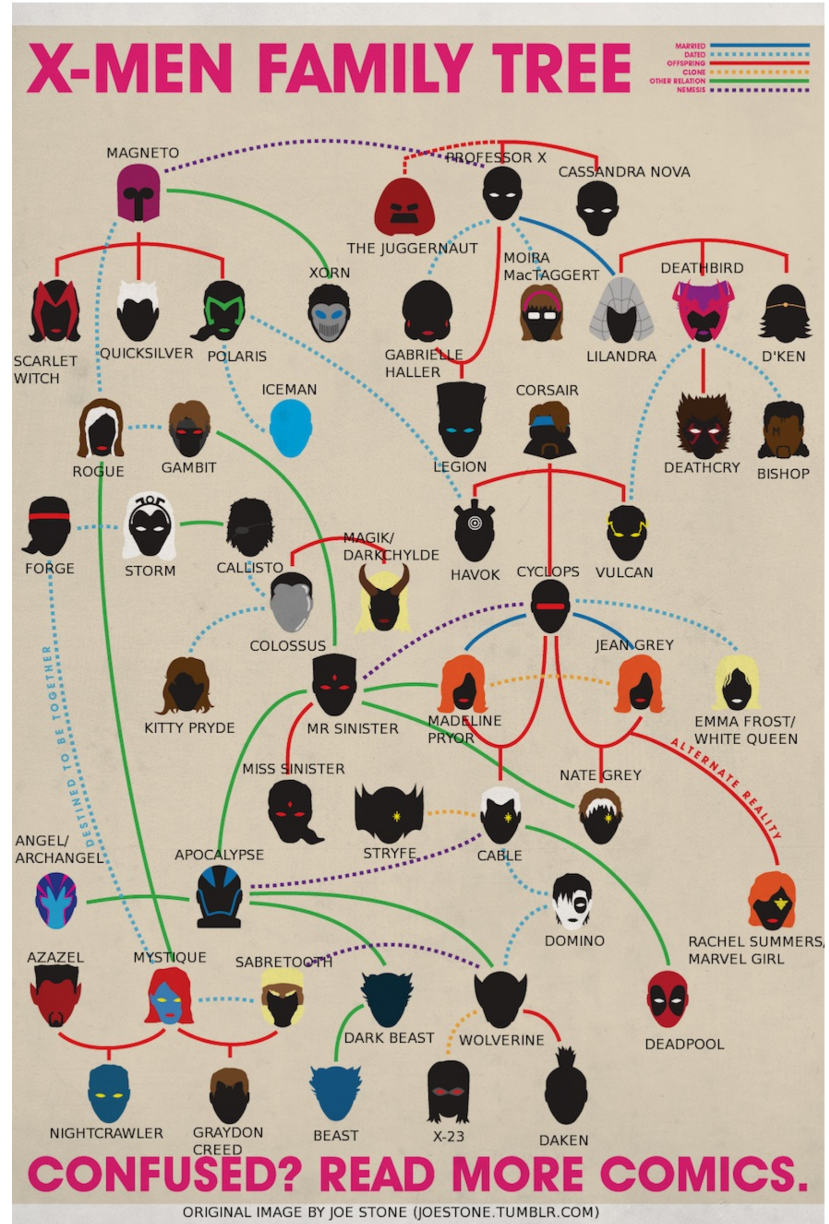

Can we draw trees differently?

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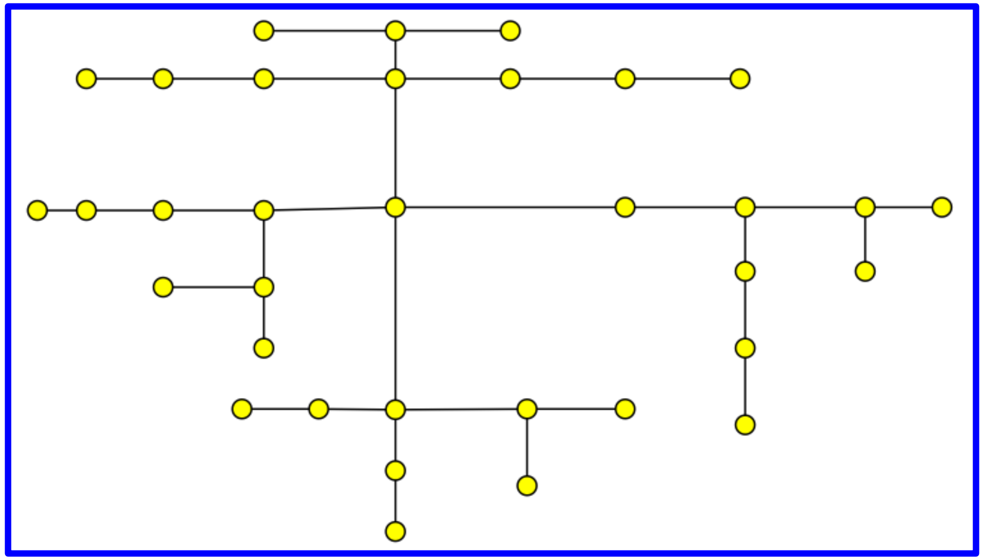

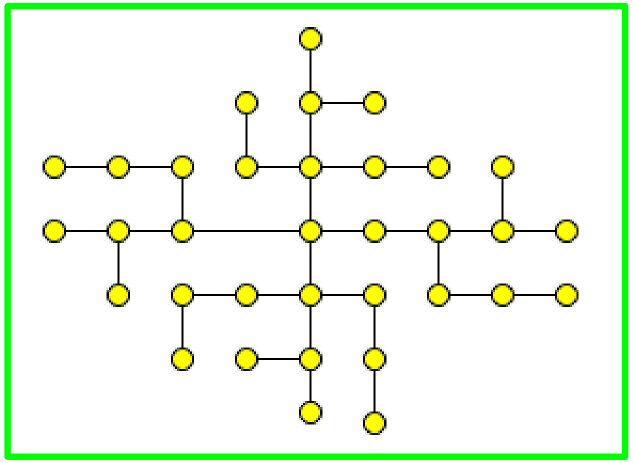

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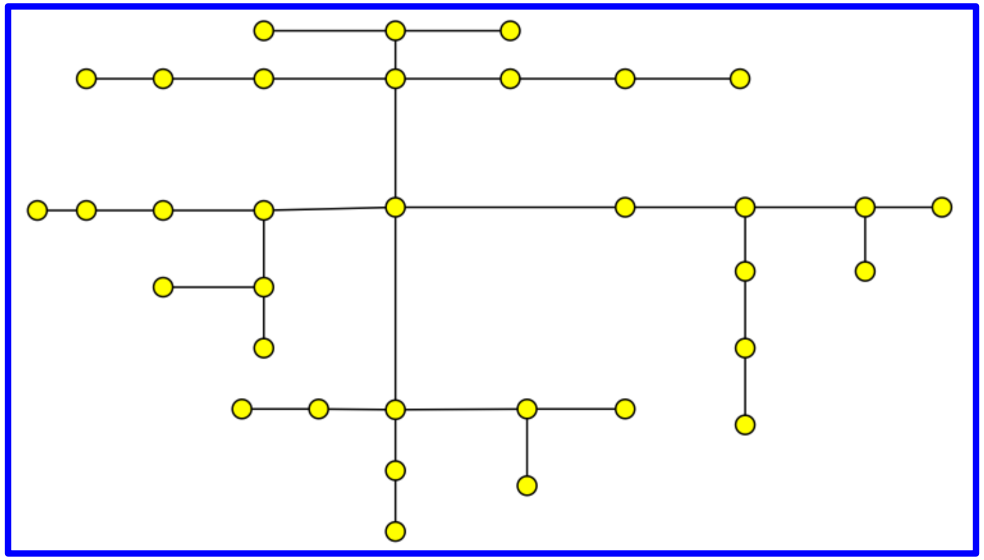
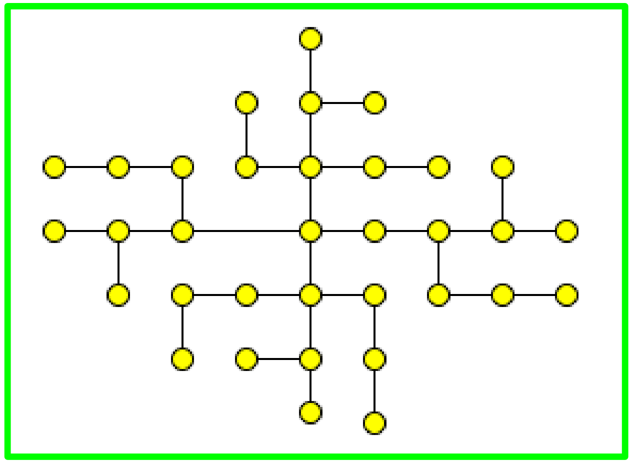
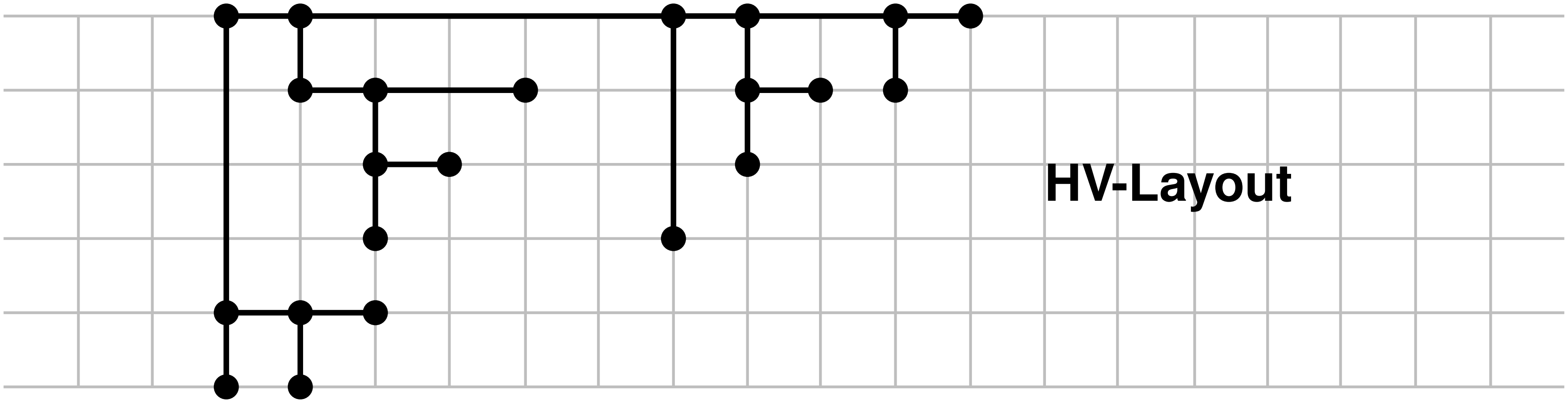




Divide & Conquer Approach:



HV-Layout

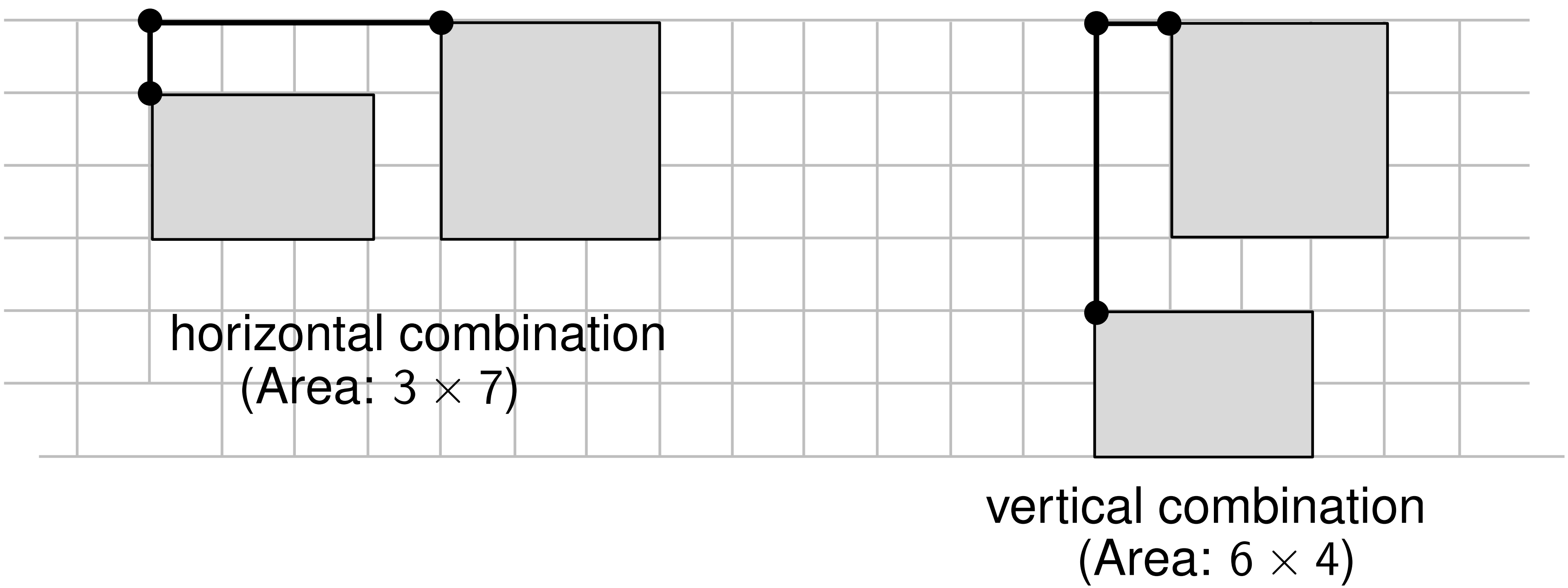
HV-Layout

Idea for binary trees:

- Children are vertically and horizontally aligned with the root
- The bounding boxes of the children do not intersect

Induction base: 

Induction step: combine layouts



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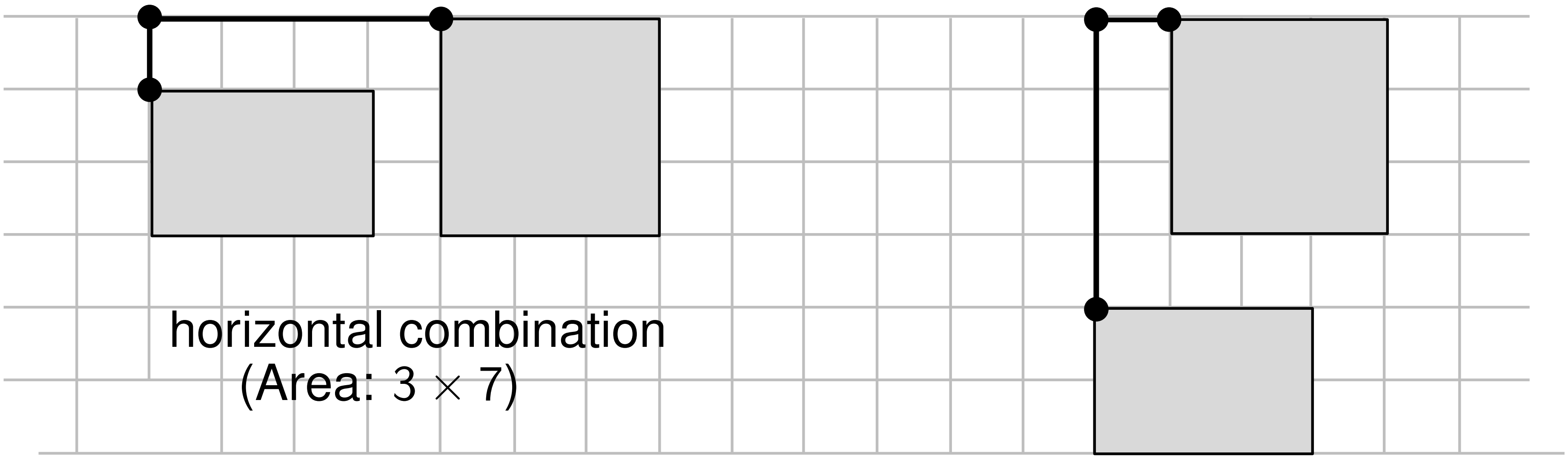

Compute minimum area using Dynamic Programming

Right-Heavy HV-Layout

Right-Heavy approach:

- At every induction step apply horizontal combination
- Place the larger subtree to the right

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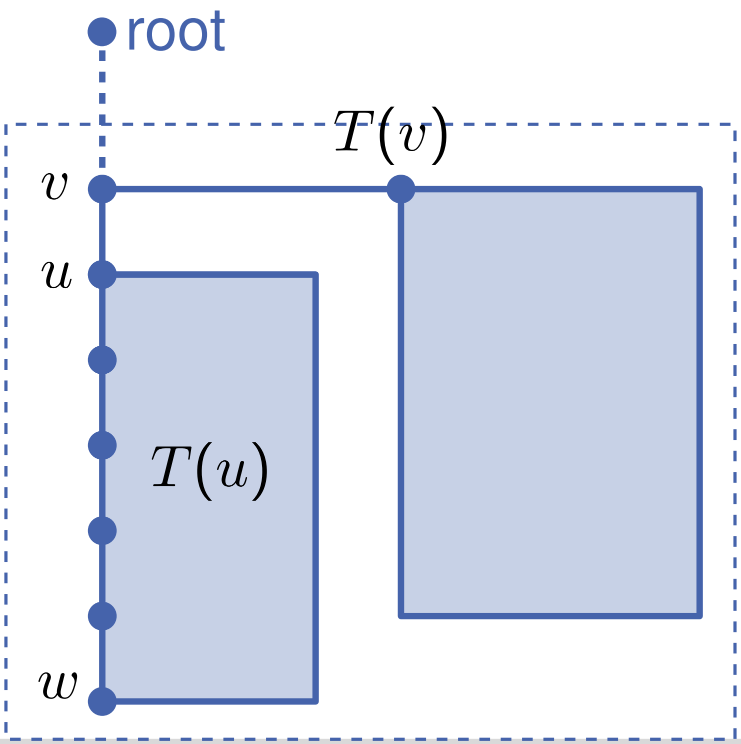

Proof:

- Each vertical edge has length 1
- Let w be the lowest node in the drawing
- Let P be a path from w to the root of T
- For every edge (u, v) in P: $|T(v)| > 2|T(u)|$
- $\Rightarrow P$ contains at most $\log n$ edges

Right-Heavy HV-Layout

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Let T be a binary tree with n vertices. The Right-Heavy algorithm constructs in $O(n)$ time a drawing Γ of T such that:

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Right-Heavy HV-Layout

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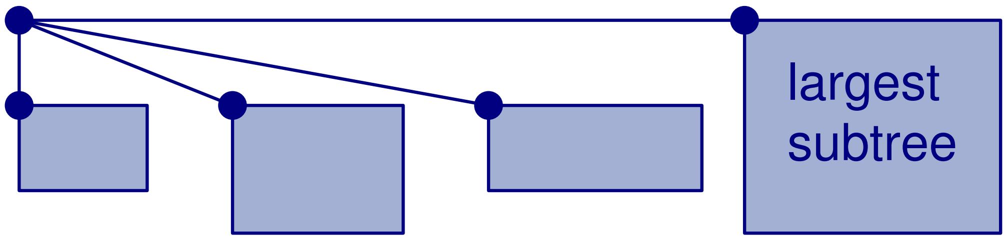
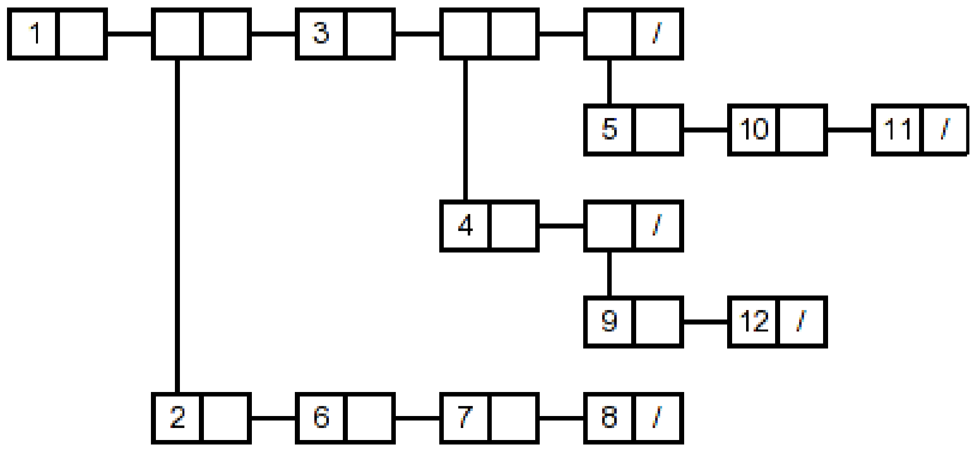


Application of HV-Layout

Cons cell diagram in LISP



`http://gajon.org/`

More tree drawings...

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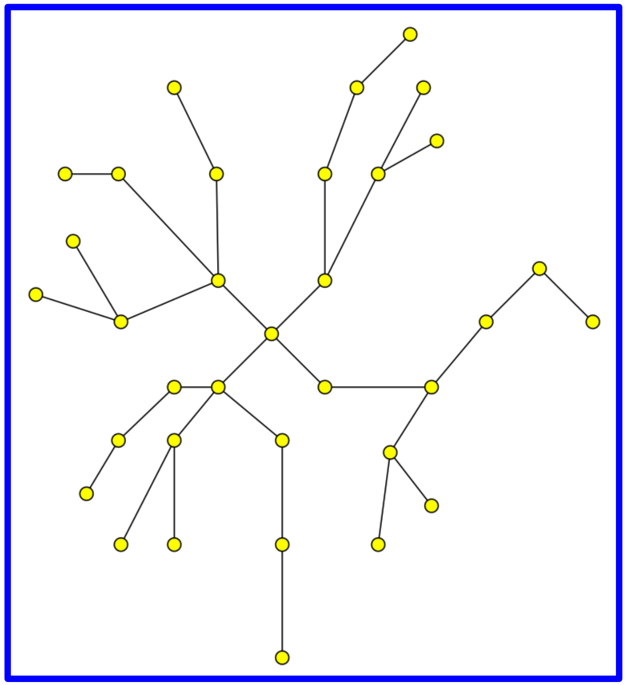

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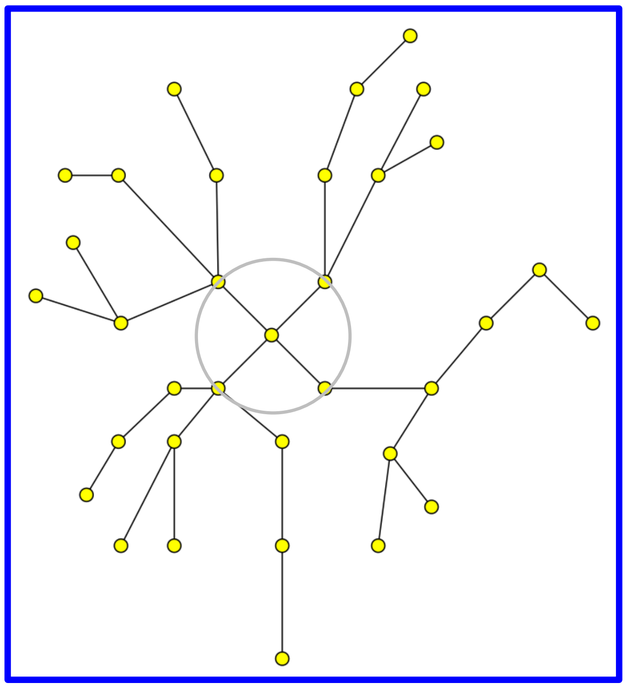

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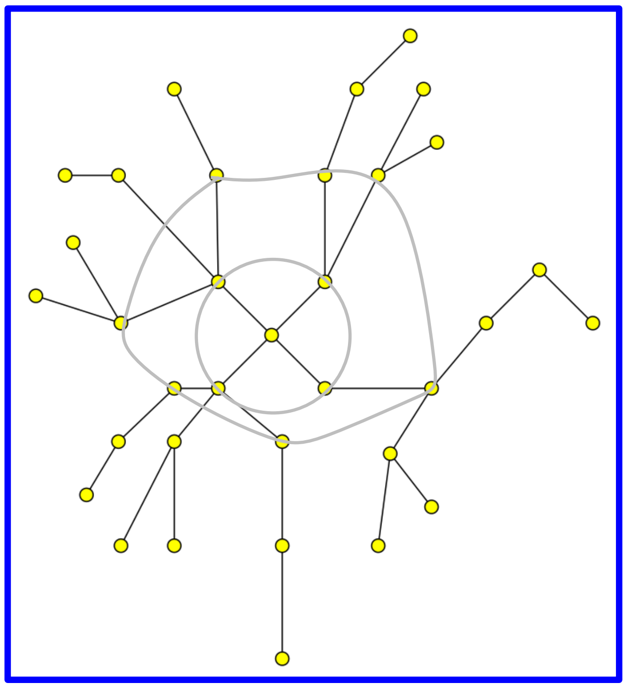

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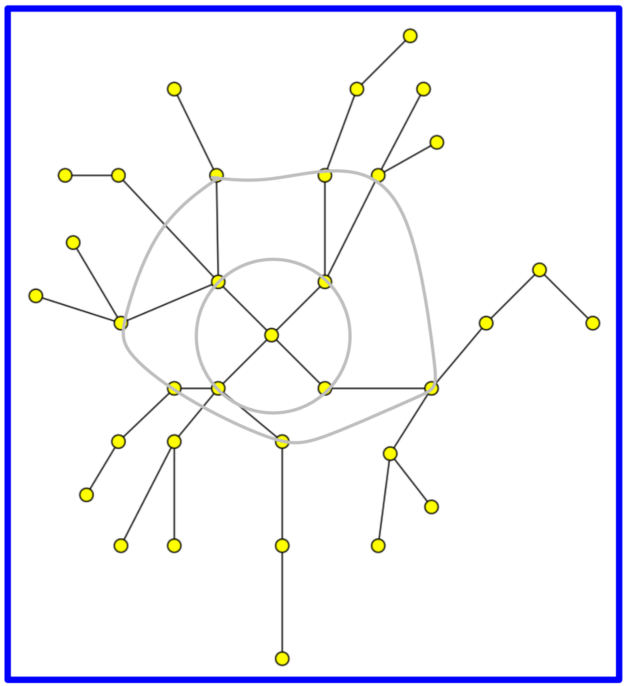
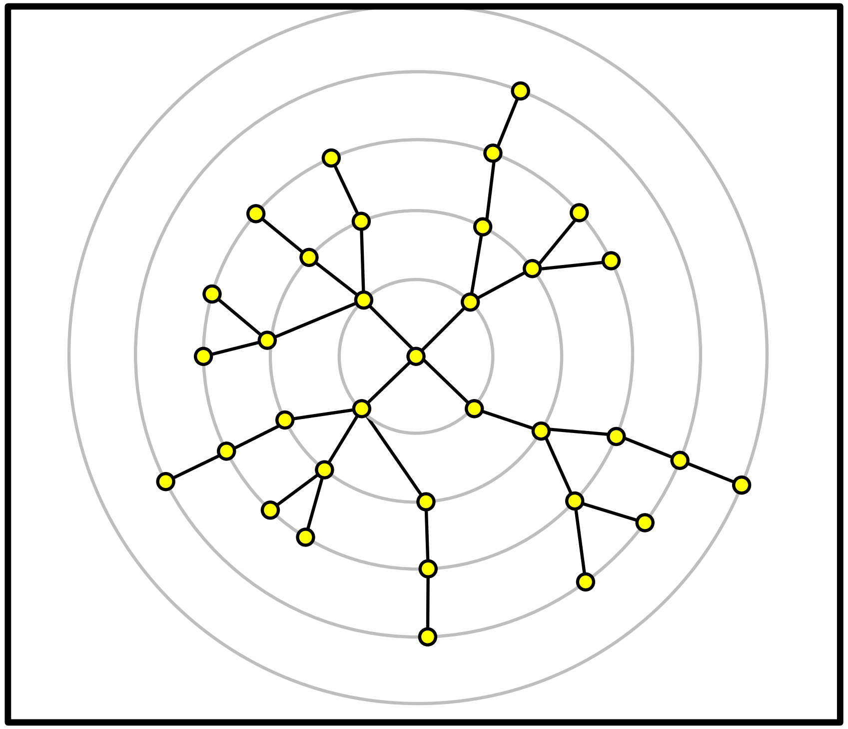
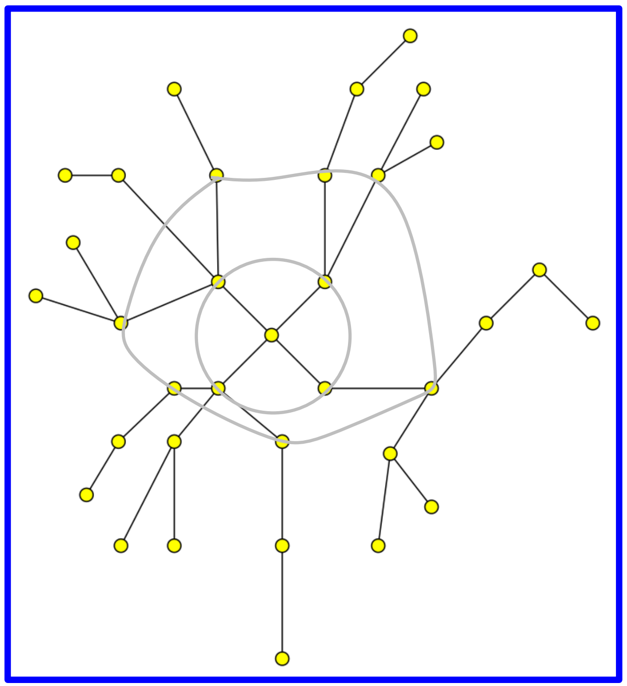
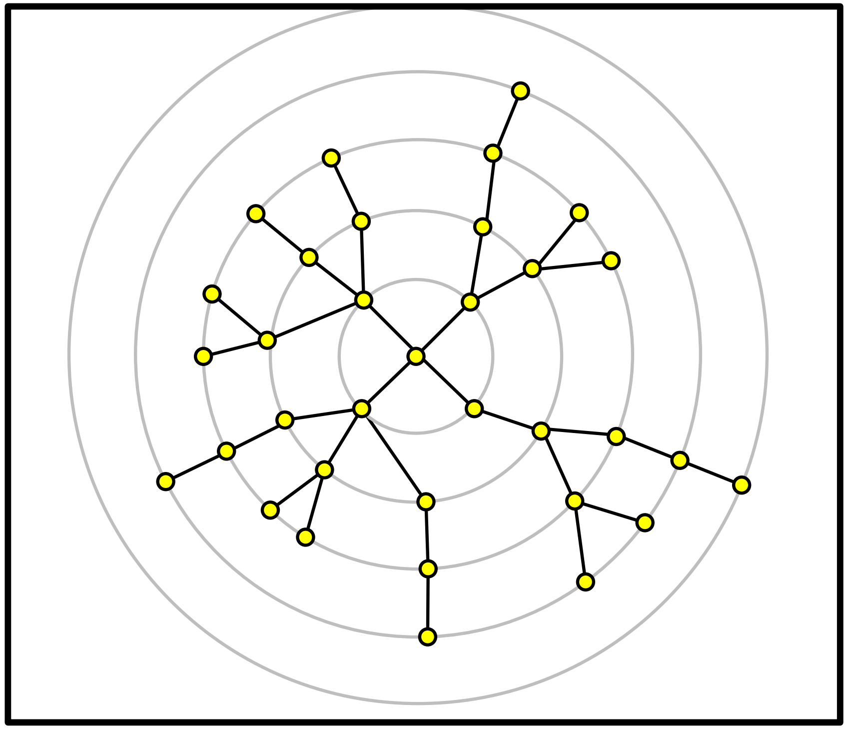




More tree drawings...



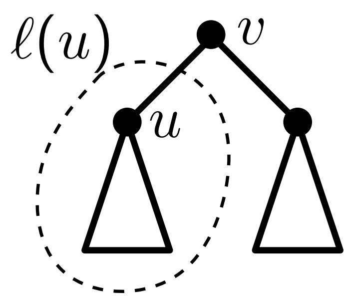


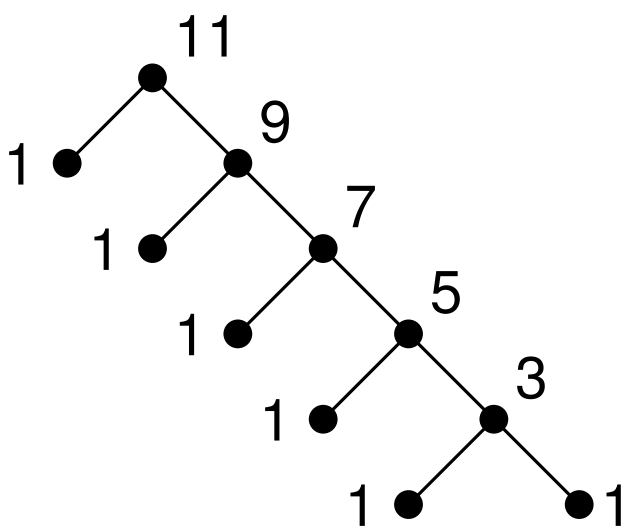
Radial Layout

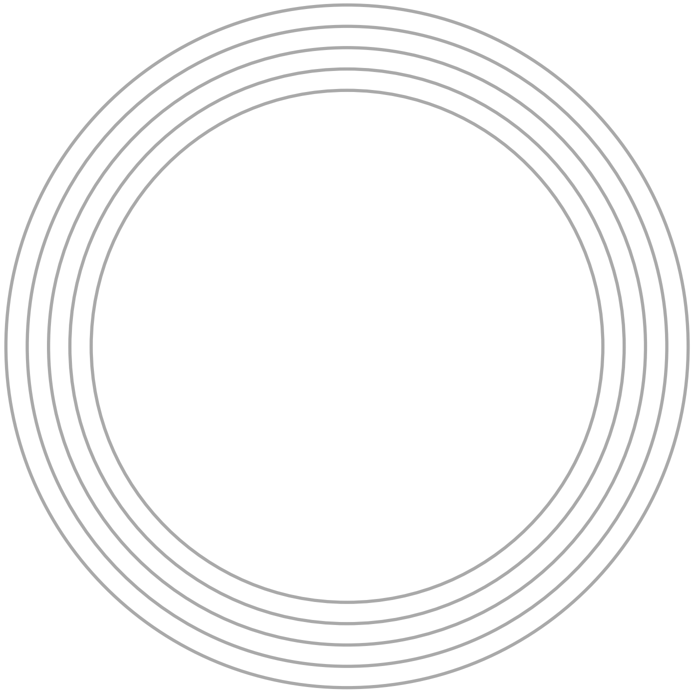
Radial Layout

Example:

- Angle corresponding to the subtree rooted at u: $\tau_u = \frac{\ell(u)}{\ell(v)-1}$







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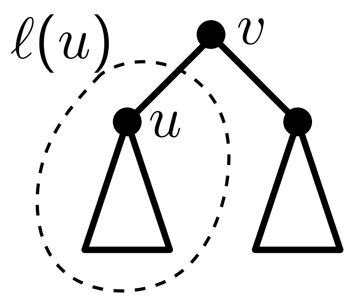

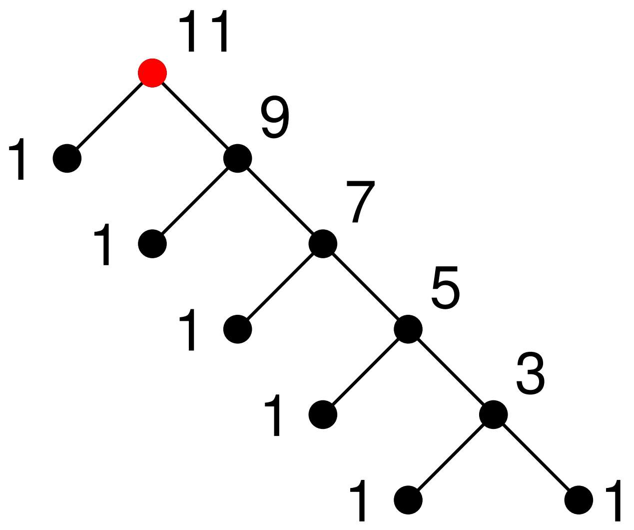

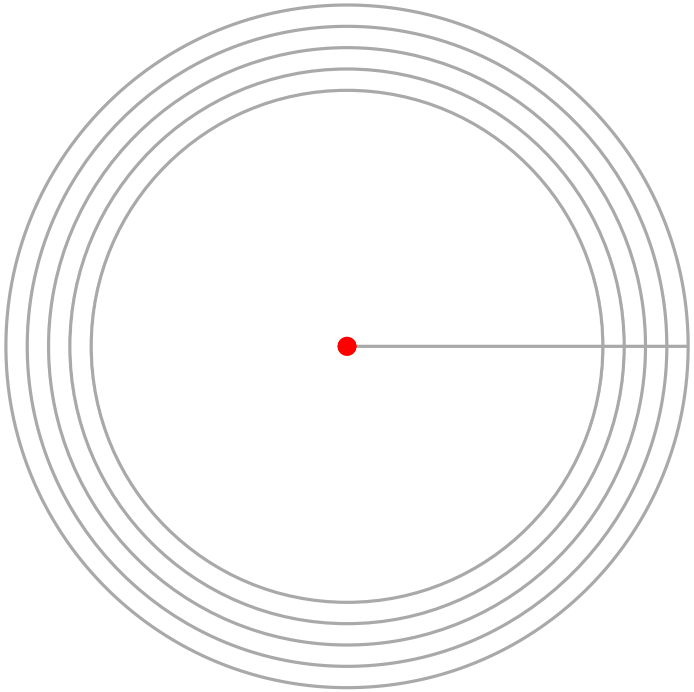

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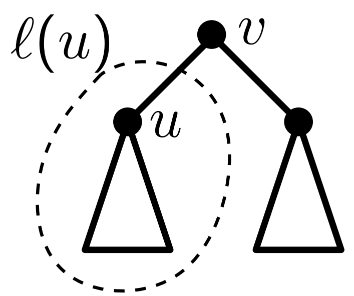

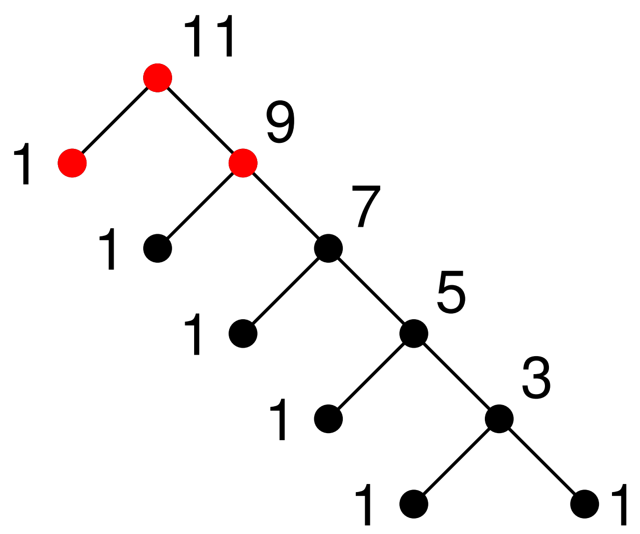

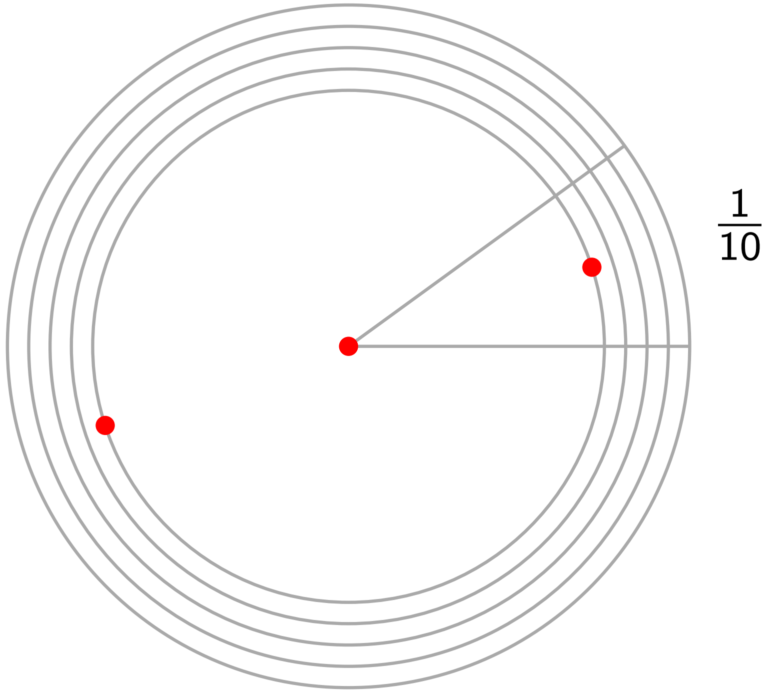

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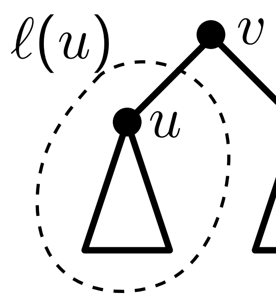

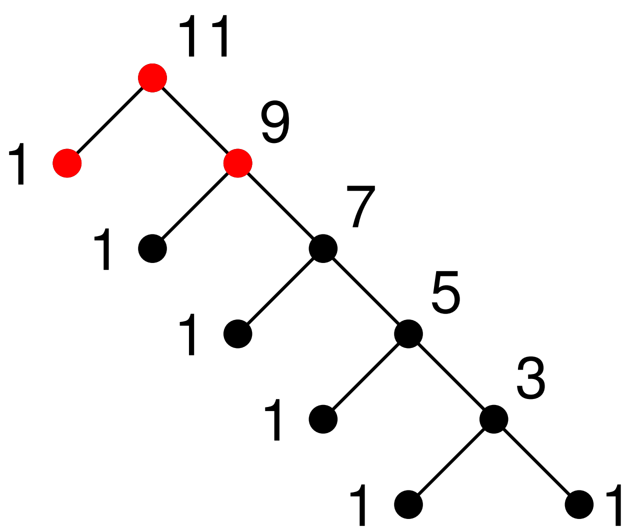

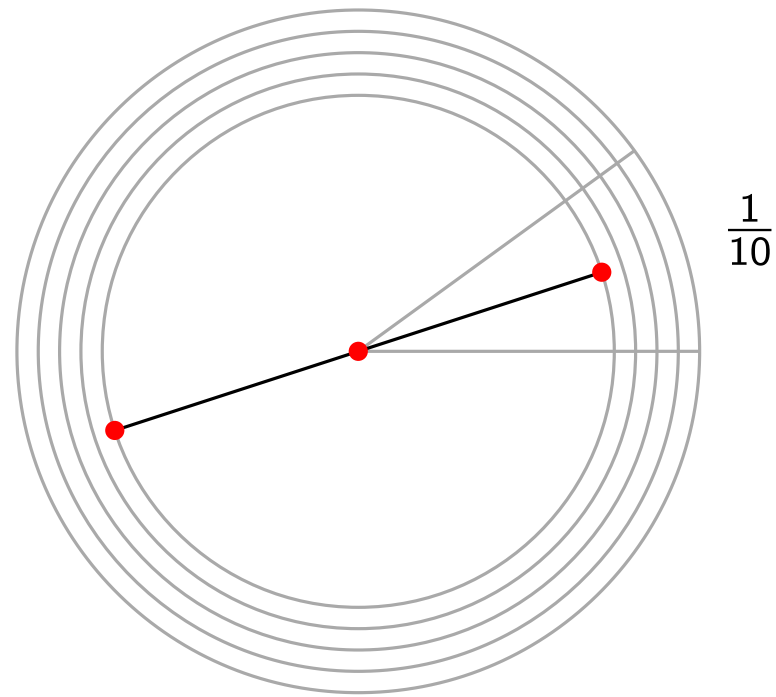

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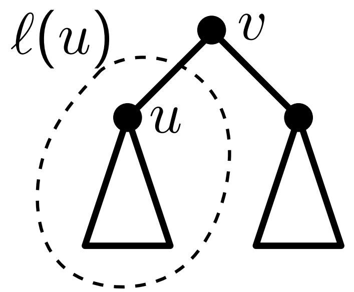

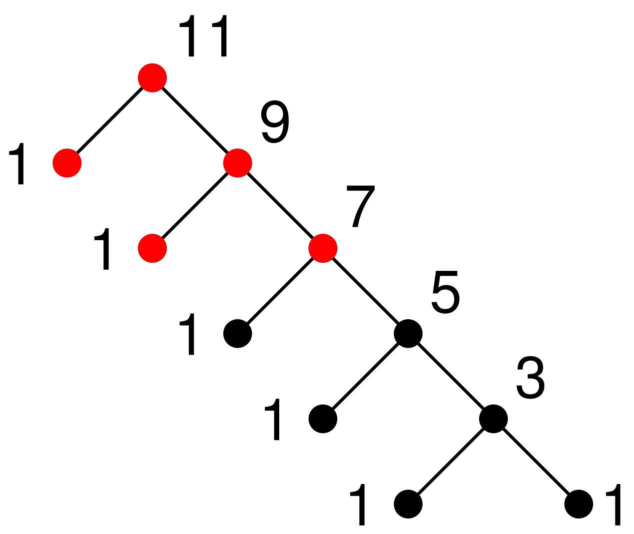

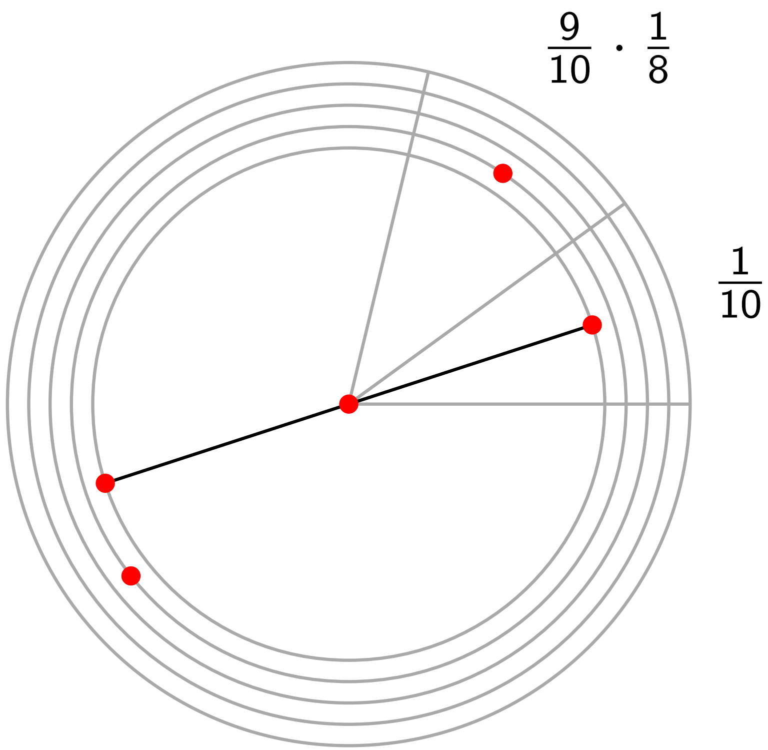

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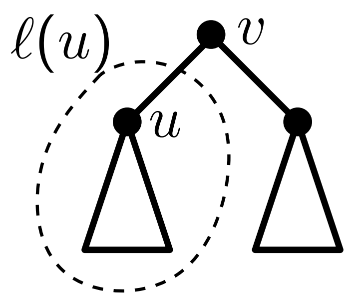

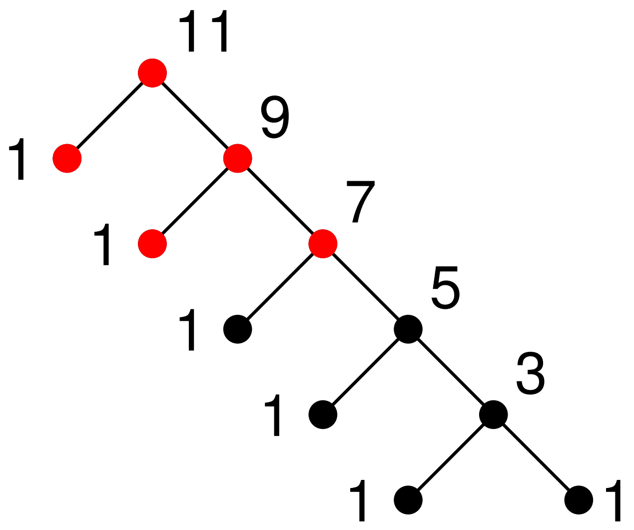

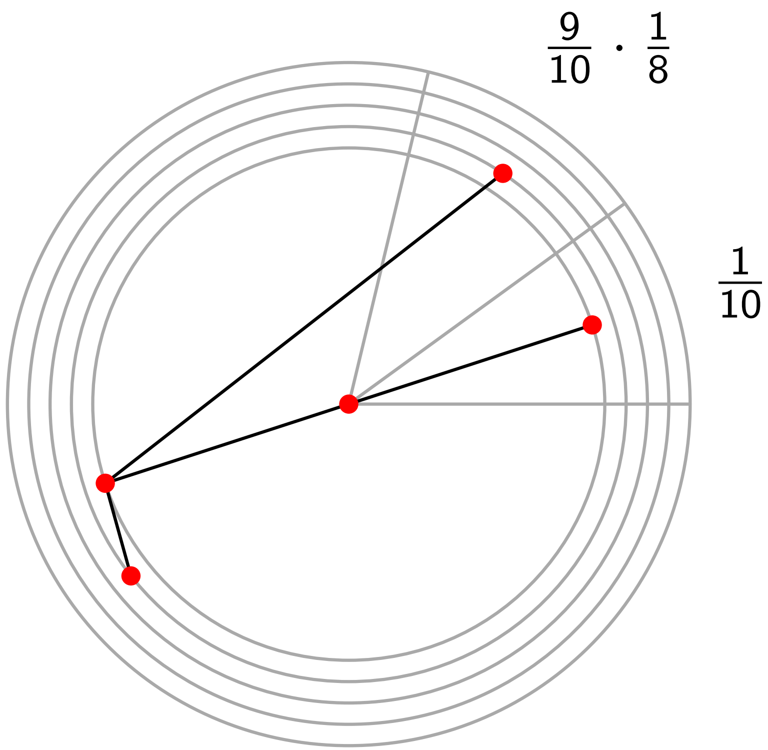

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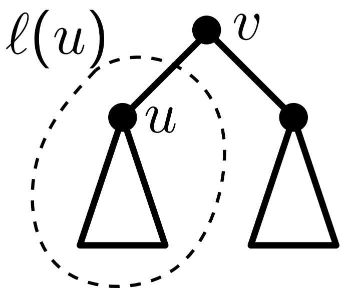

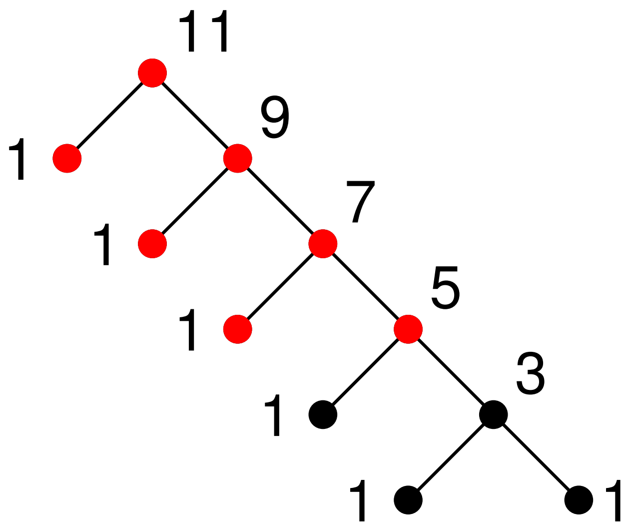

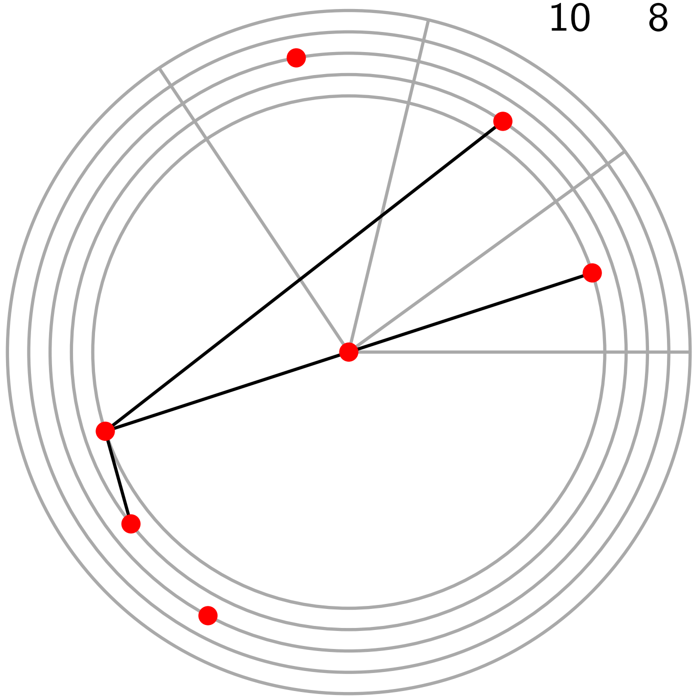

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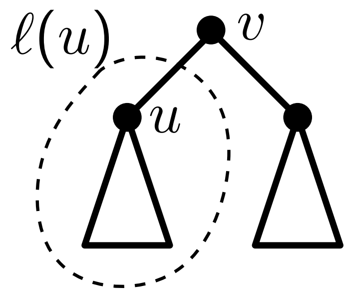
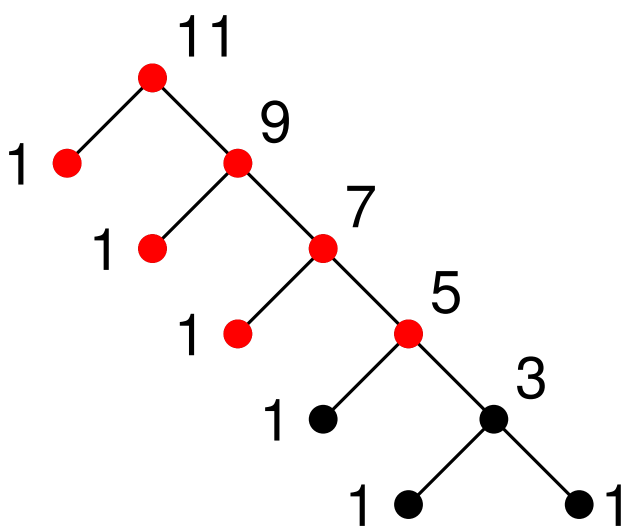
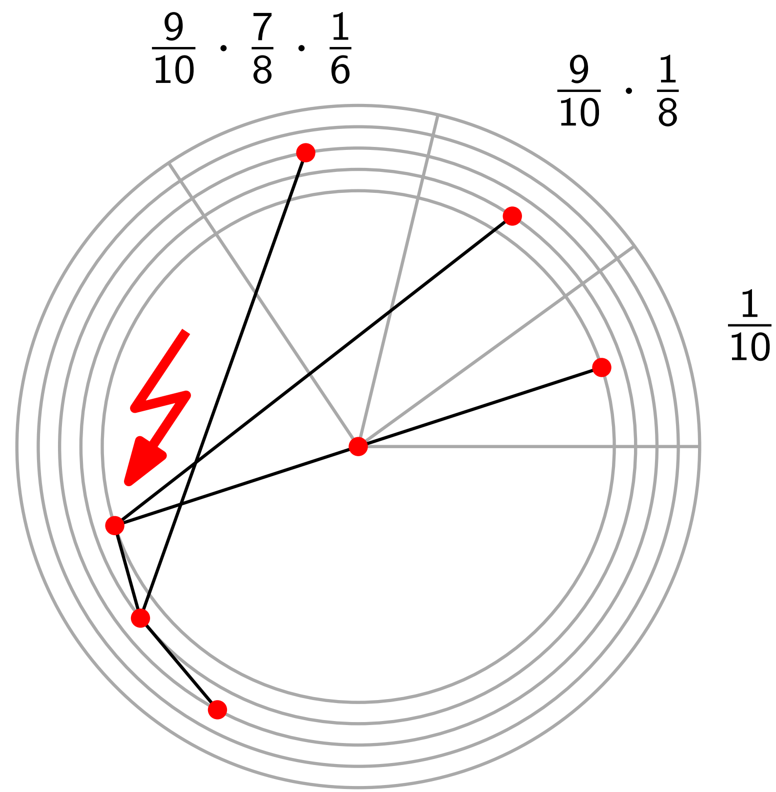
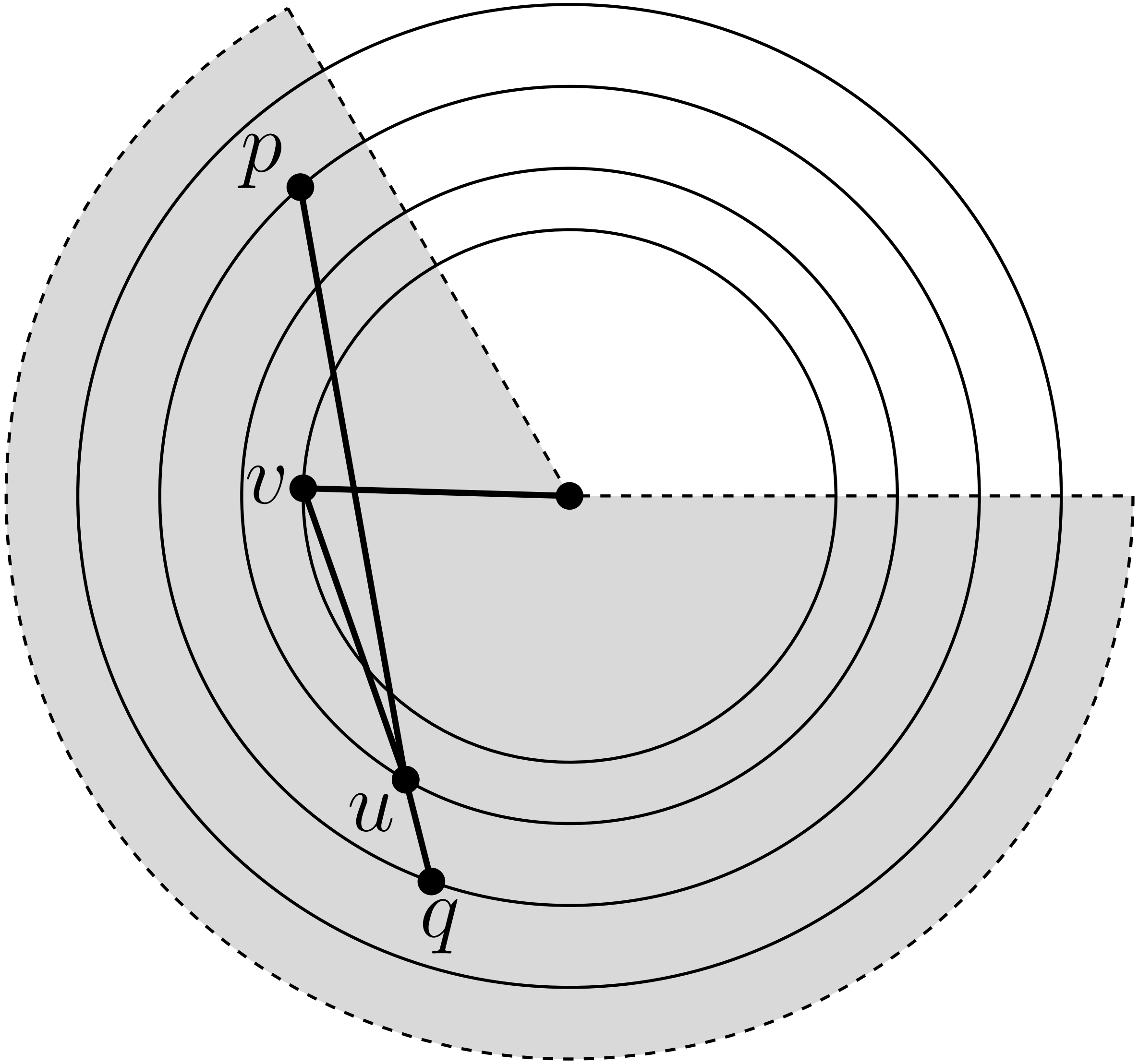






Radial Layout

How to avoid crossings:



Radial Layout

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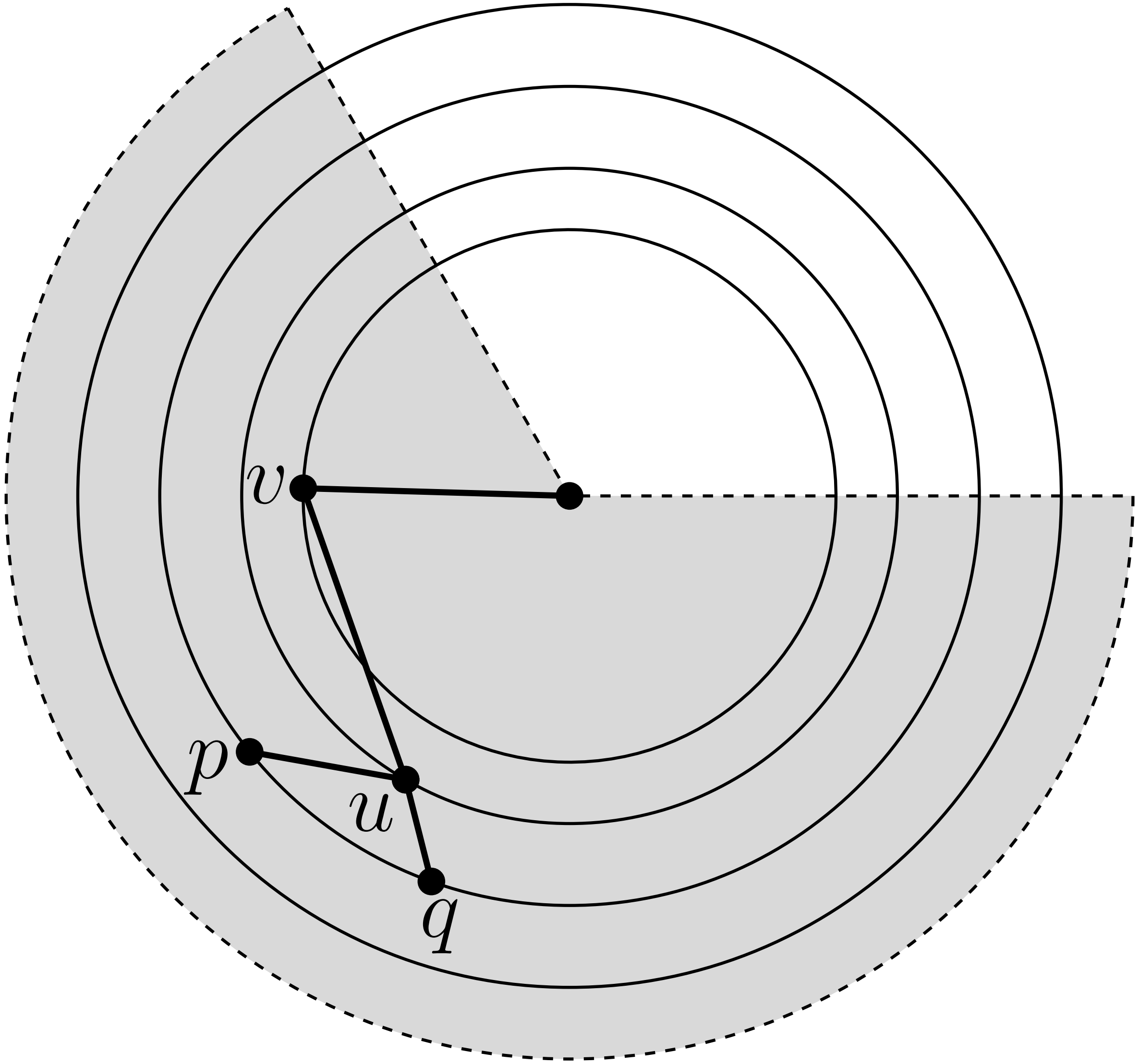

Radial Layout

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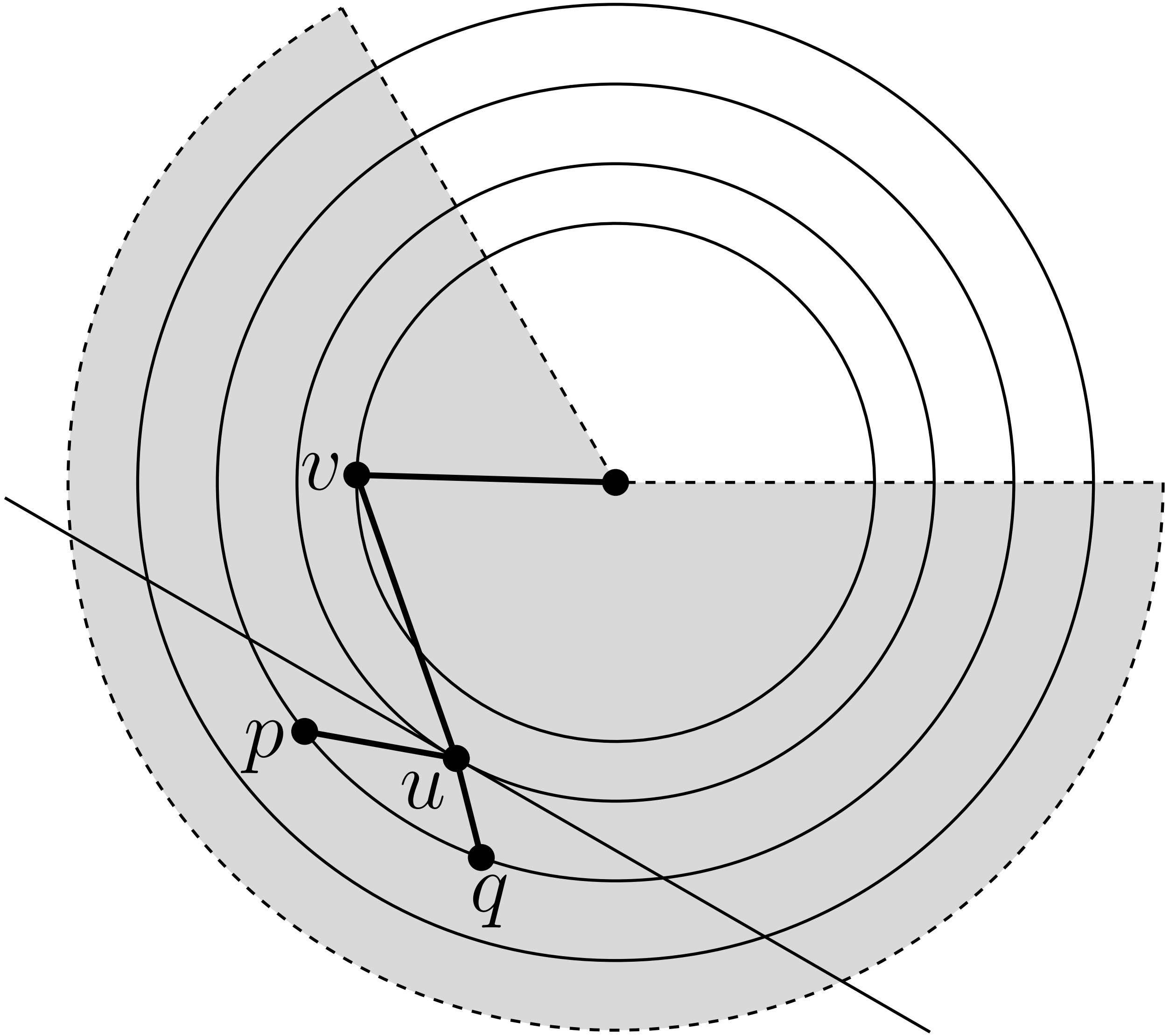

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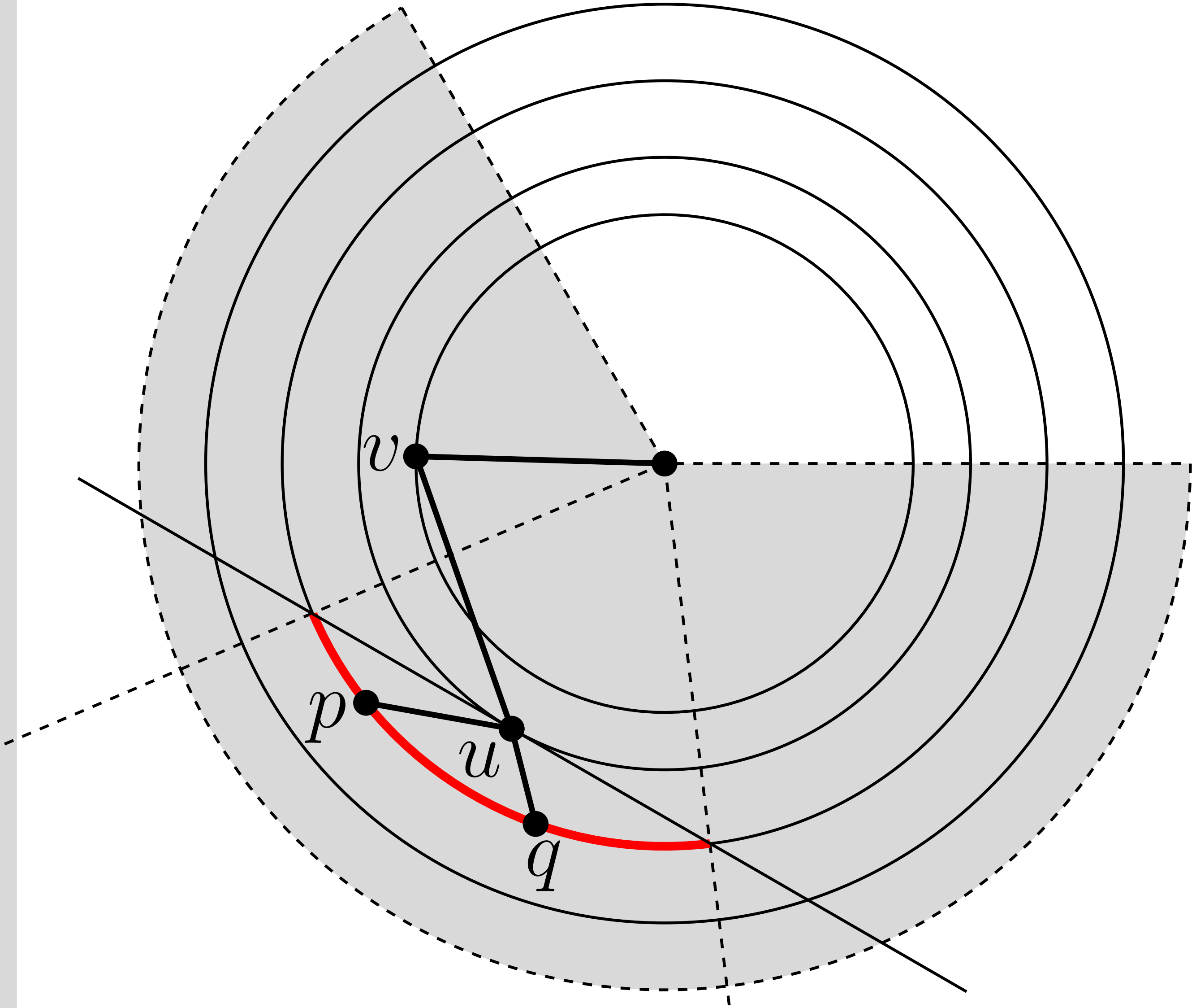

Radial Layout

Karlsruhe Institute of Technology

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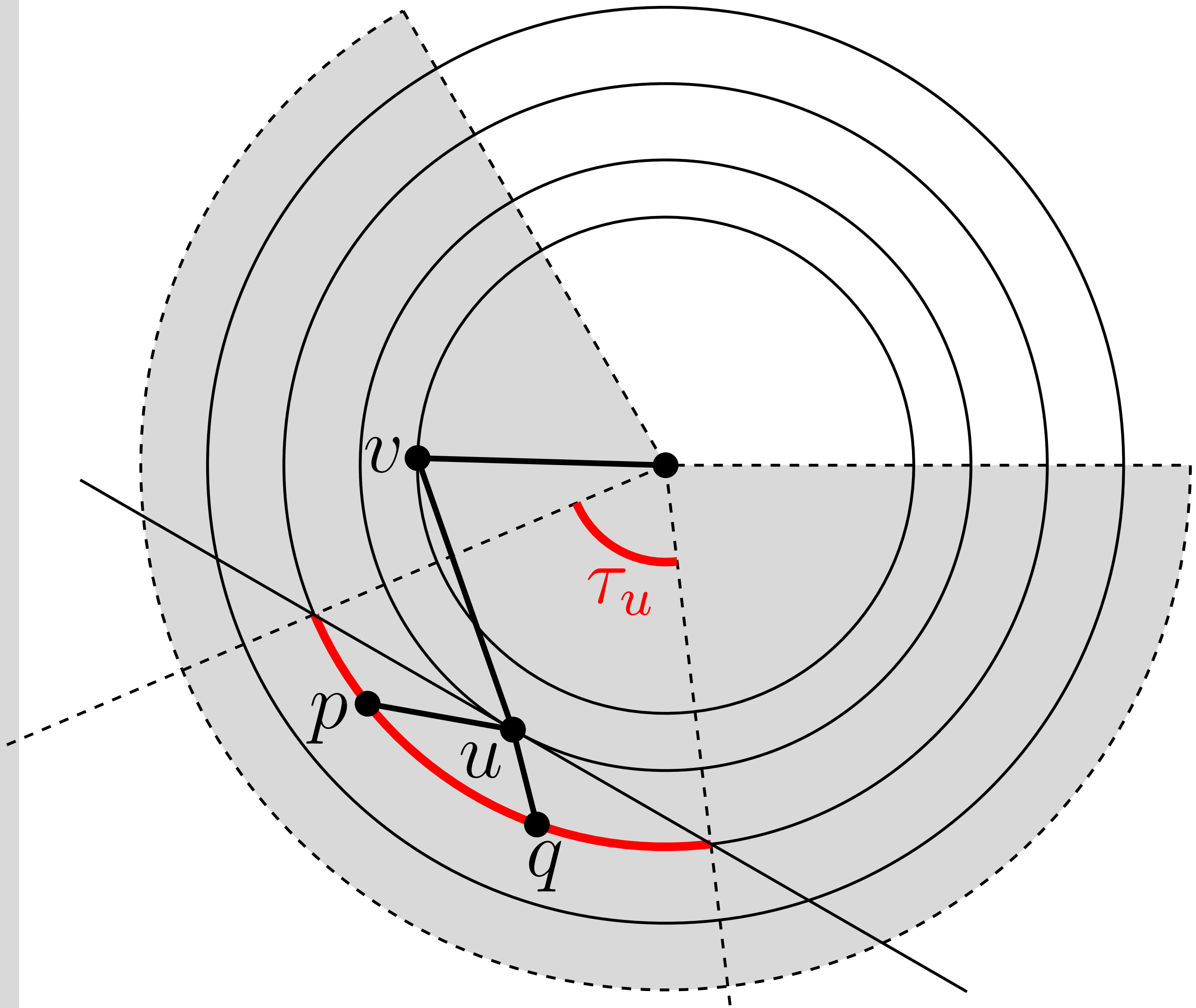

Radial Layout

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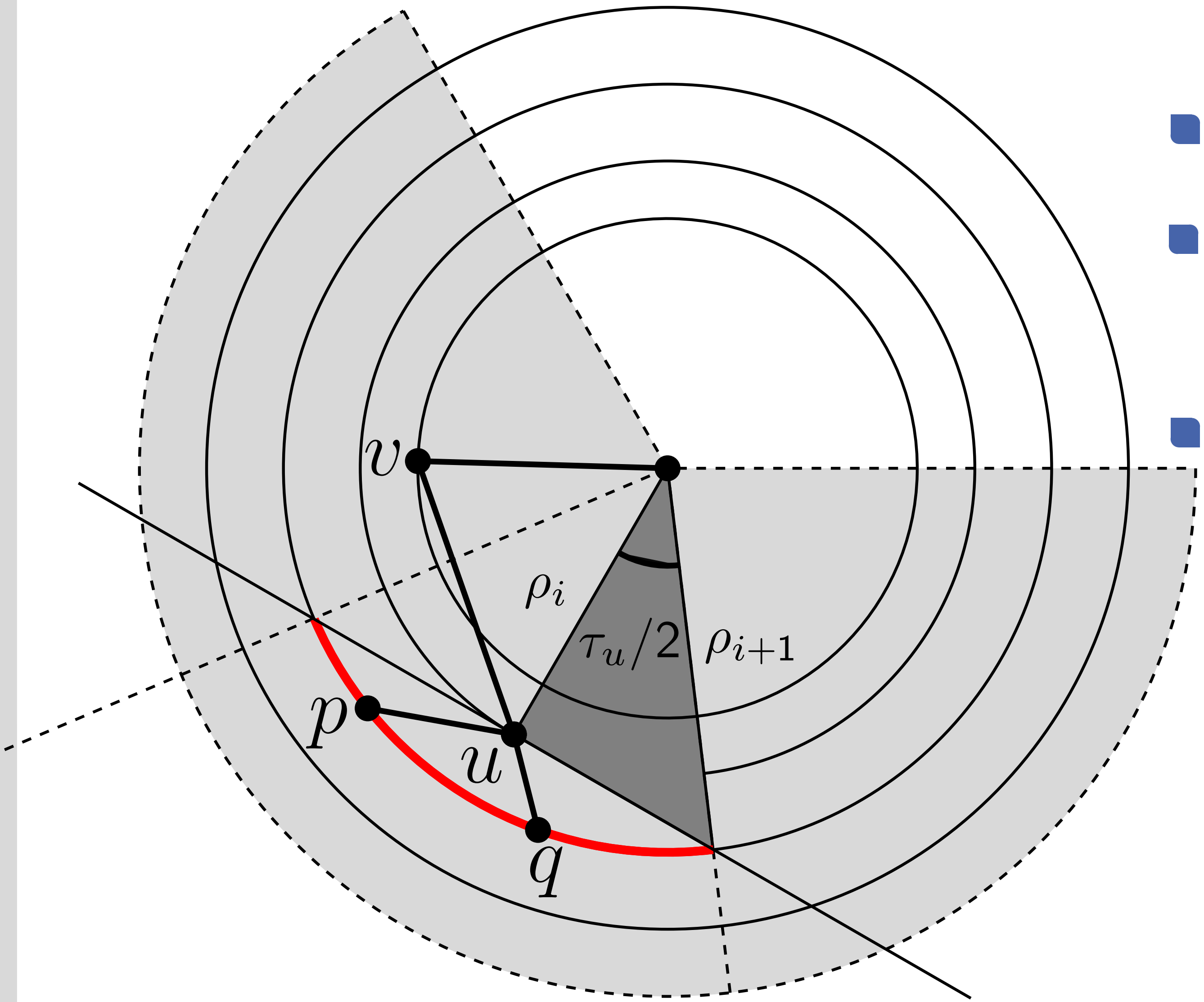

- τ_u - angle of the wedge corresponding to vertex u
- ρ_i - raduis of layer i
- $\ell(v)$-number of nodes in the subtree rooted at v
- $\cos \frac{\tau_u}{2} = \frac{\rho_i}{\rho_{i+1}}$

Radial Layout

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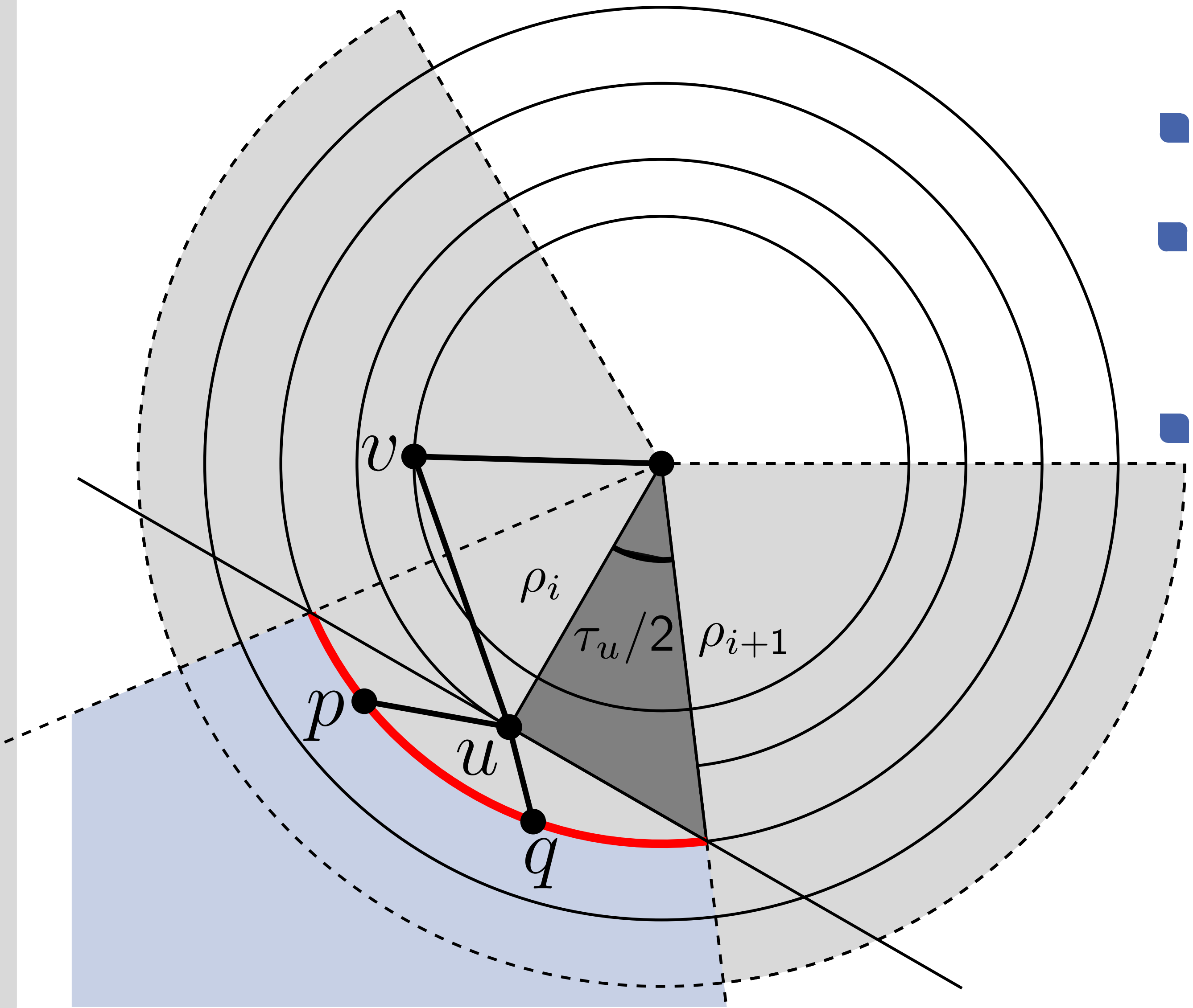

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Radial Layout

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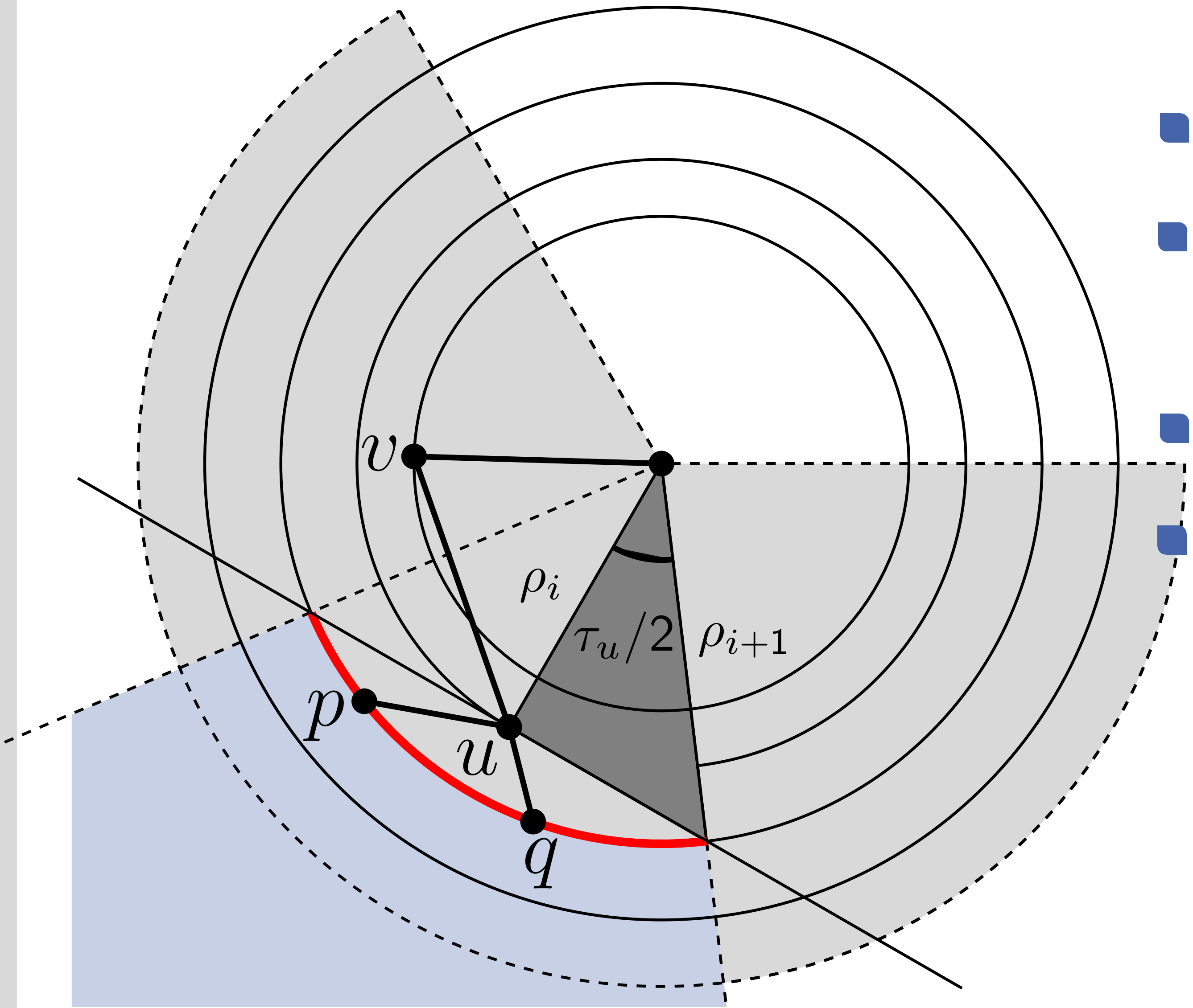

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- $\tau_u = 2 \arccos \frac{\rho_i}{\rho_{i+1}}$ (correction)

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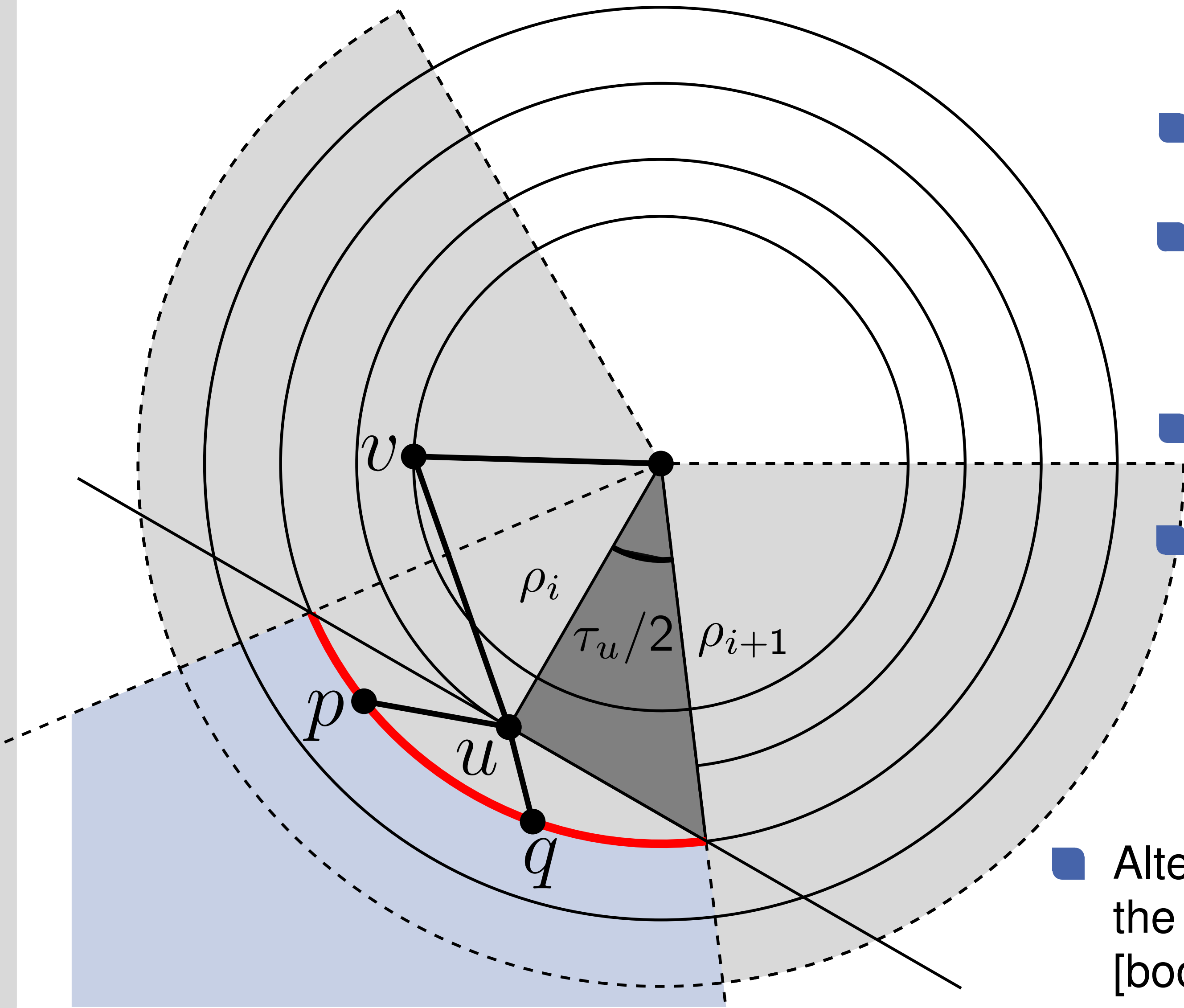

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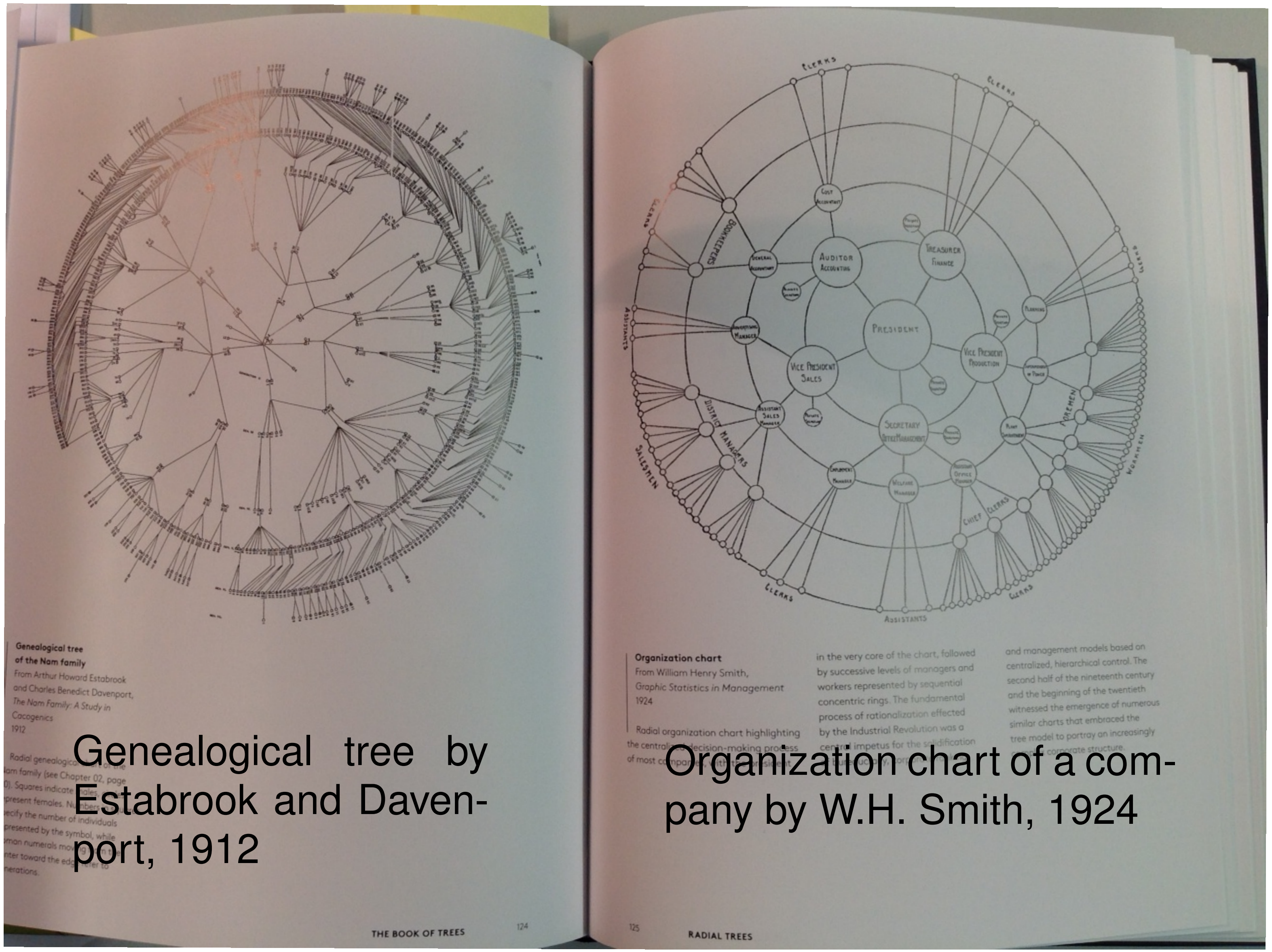
- $\cos \frac{\tau_u}{2} = \frac{\rho_i}{\rho_{i+1}}$
- $\tau_u = 2 \arccos \frac{\rho_i}{\rho_{i+1}}$ (correction)
- Alternatively use number of leaves in the subtree to subdivide the angles [book Di Battista et al.]

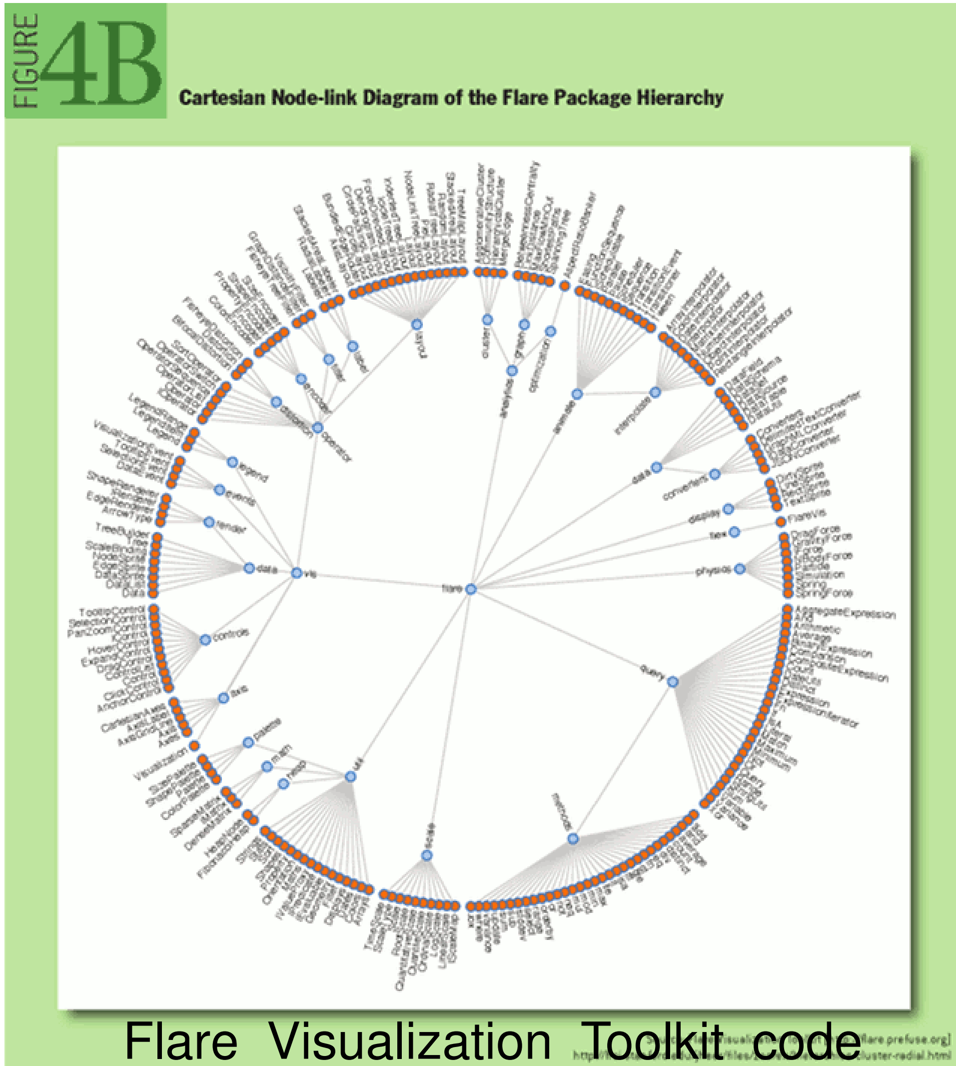
Applications of Radial Layout



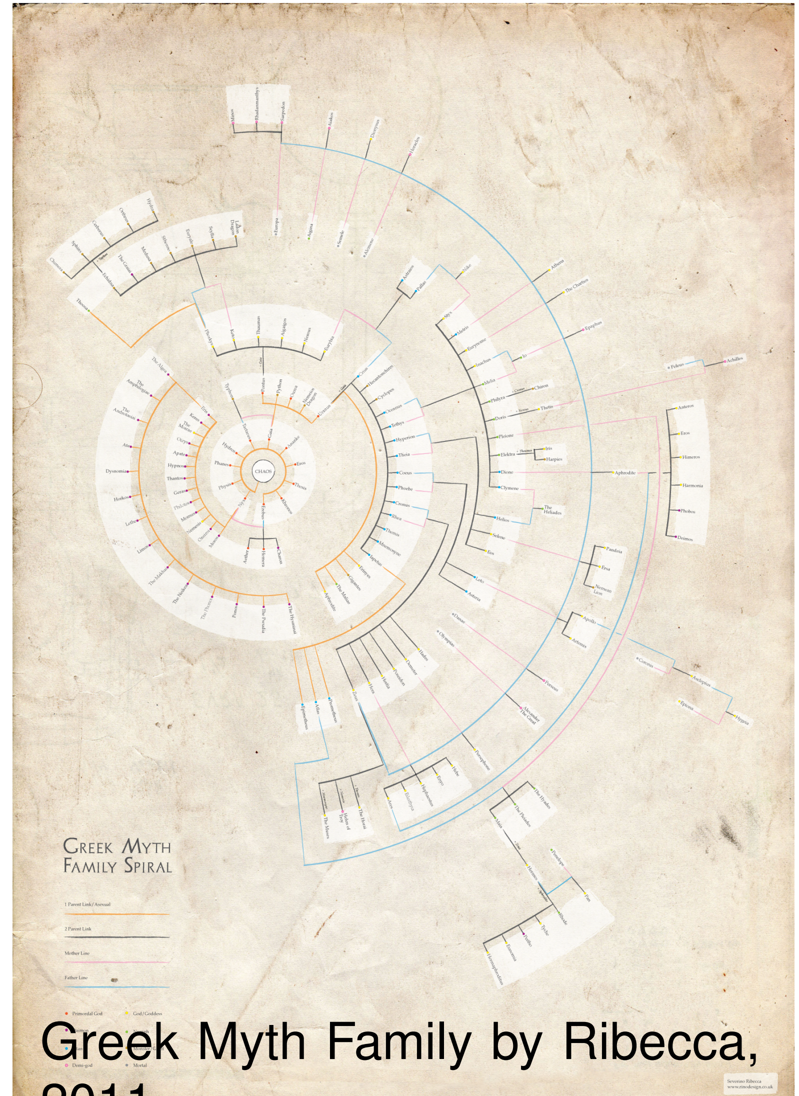
Genealogical tree by Estabrook and Davenport, 1912

Organization chart of a company by W.H. Smith, 1924

Applications of Radial Layout



Flare Visualization Toolkit code structure by Heer, Bostock and Ogievetsky, 2010



Greek Myth Family by Ribecca, 2011