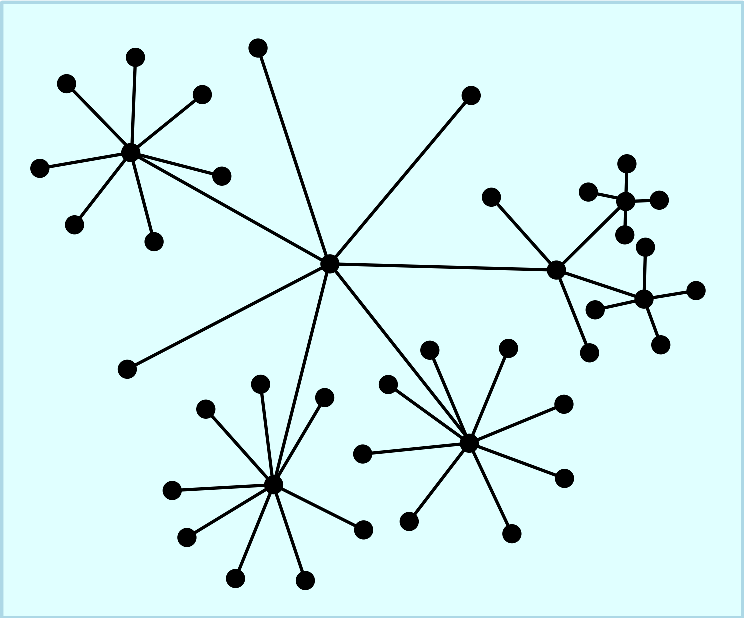
Karlsruhe Institute of Technology

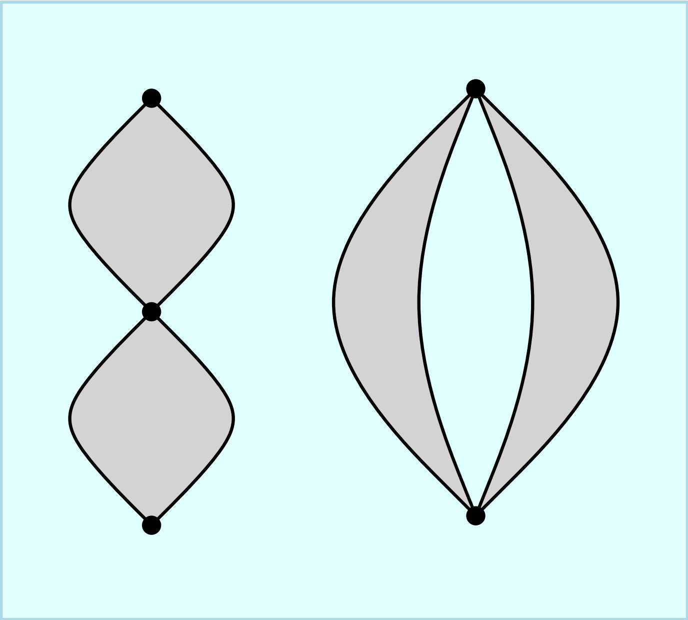
# Algorithms for graph visualization

## Divide and Conquer - Trees and Series-Parallel Graphs

WINTER SEMESTER 2014/2015

**Tamara Mchedlidze** – MARTIN NÖLLENBURG





# Baloon Layout

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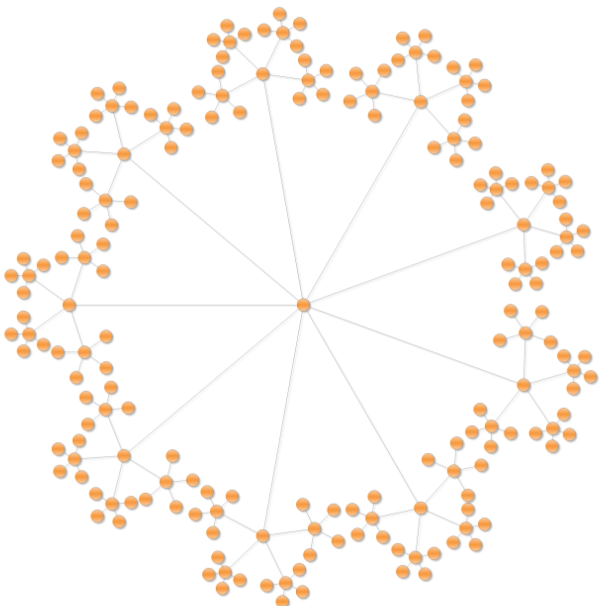

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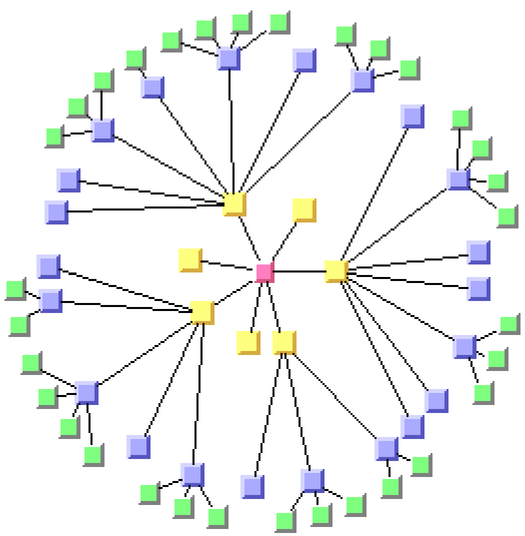
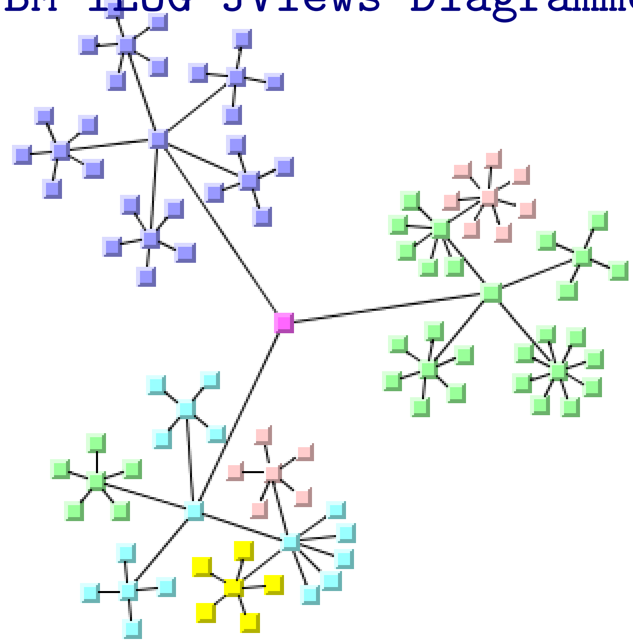
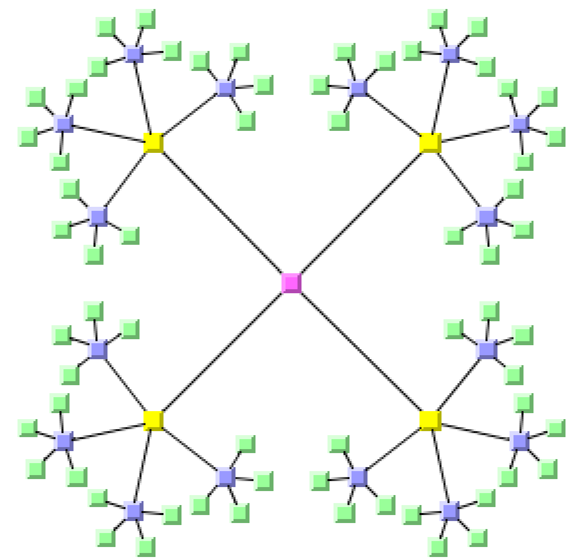
IBM ILOG JViews Diagrammer: www.ibm.com







# Baloon Layout

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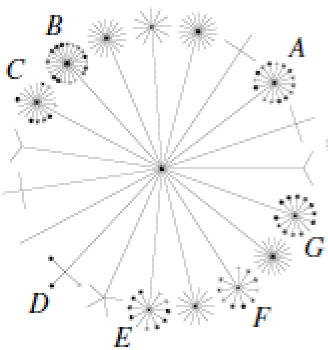
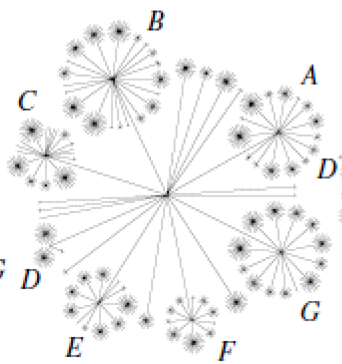
## A baloon drawing has the following properties:

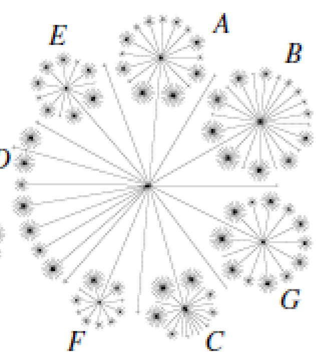
- All the children of the same parent lie on circle centered at their parent
- The drawing is planar
- The further an edge from the root is, the shorter it becomes




All subtrees at the same depth have the same size.




Subtrees may have different size, the tree is ordered.




Subtrees may have different size, the tree is unordered. Drawn by Lin& Yen Algorithm.

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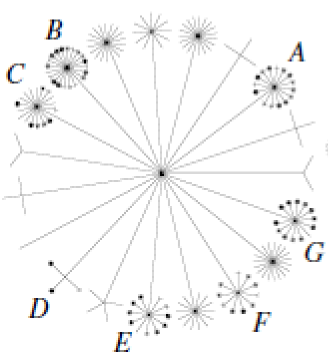


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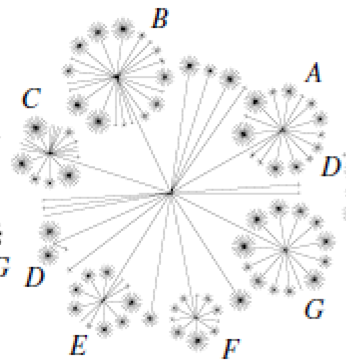


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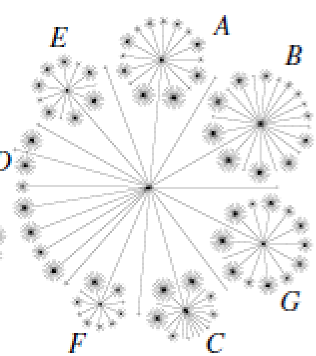


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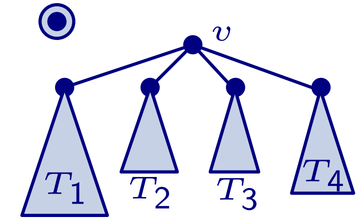
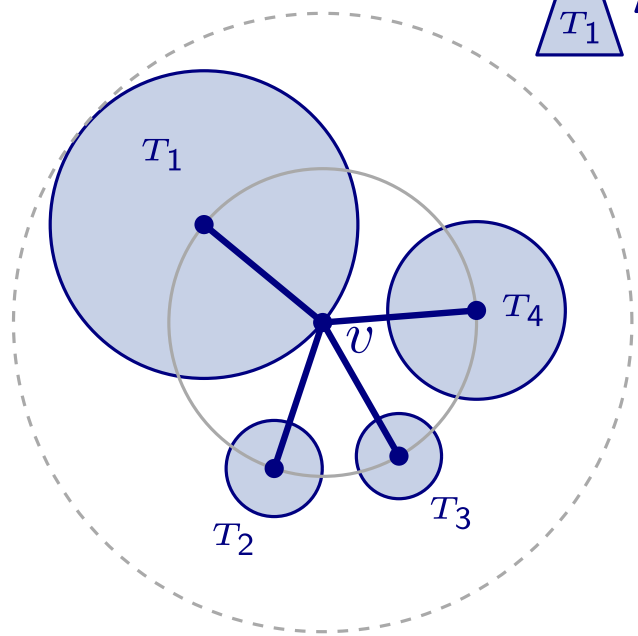
**Induction base:** 

**Induction step:**



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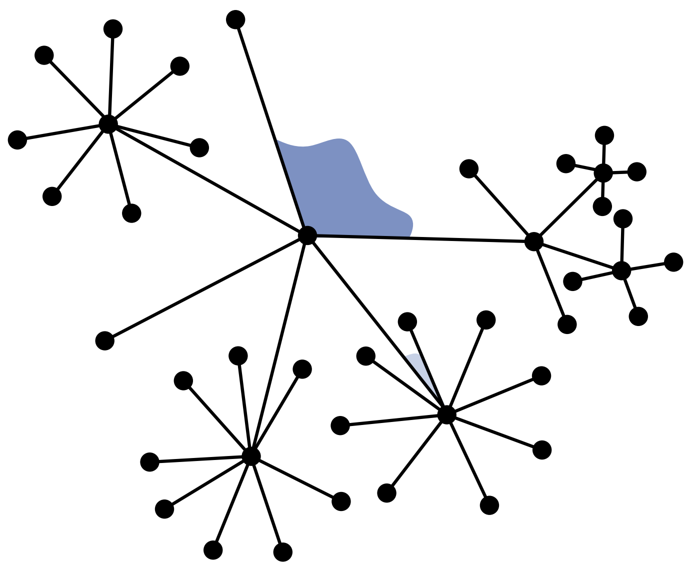

**Aesthetics:**

- Aspect ratio $= \frac{\text{largest angle}}{\text{smallest angle}}$
- Angular resolution $=$ $\min\{\text{angle between two adjacent edges}\}$

**Question:** Can we find a baloon drawing with max angular resolution and min aspect ratio in an unordered tree? (Algorithm by Lin & Yen)

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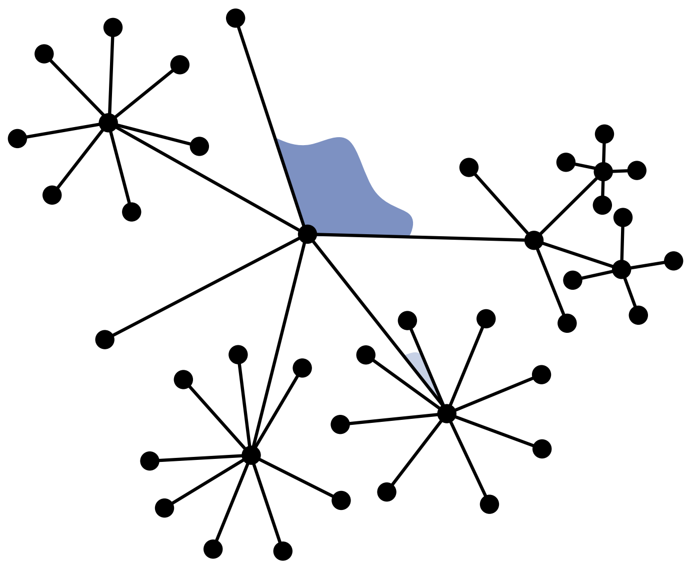

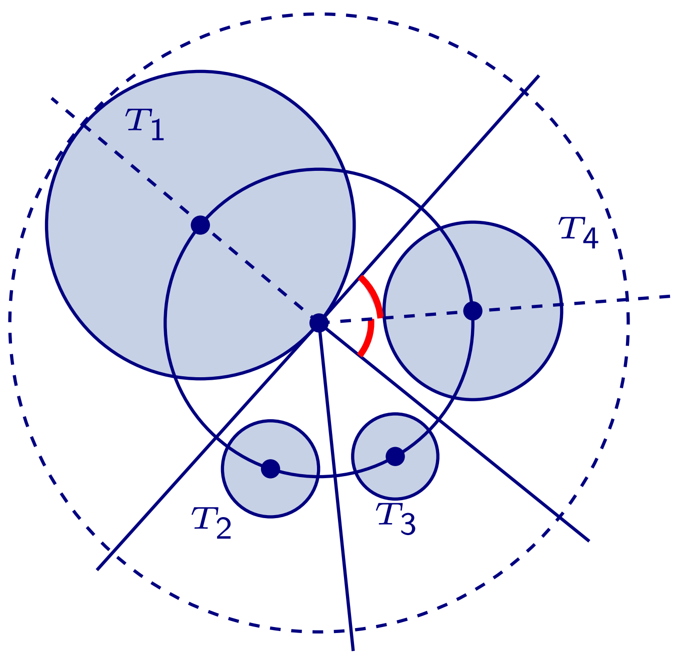


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- We investigate drawing with **even angles** (drawing with uneven angles might have a better area)

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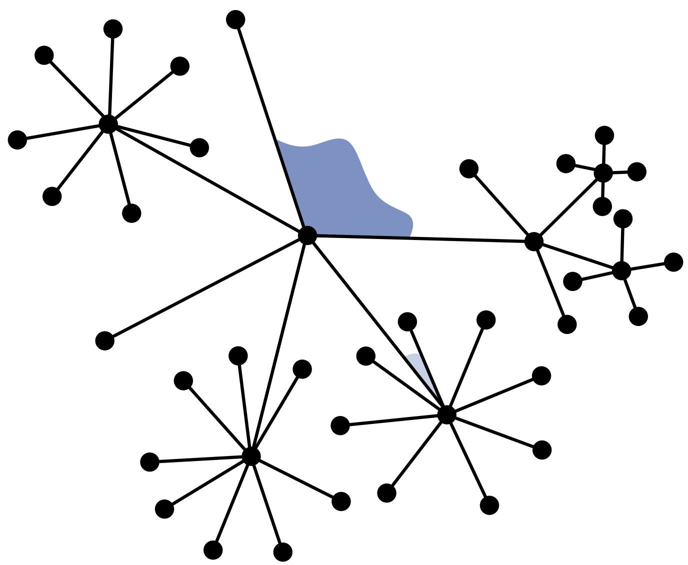

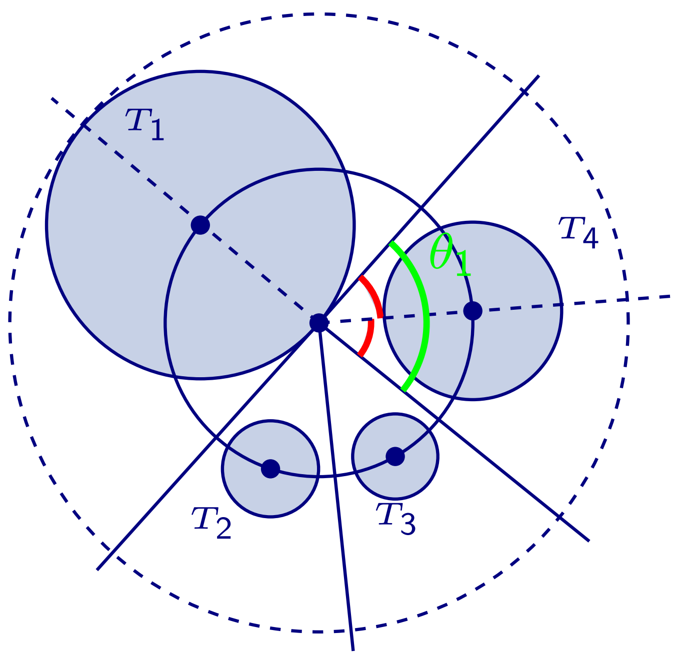


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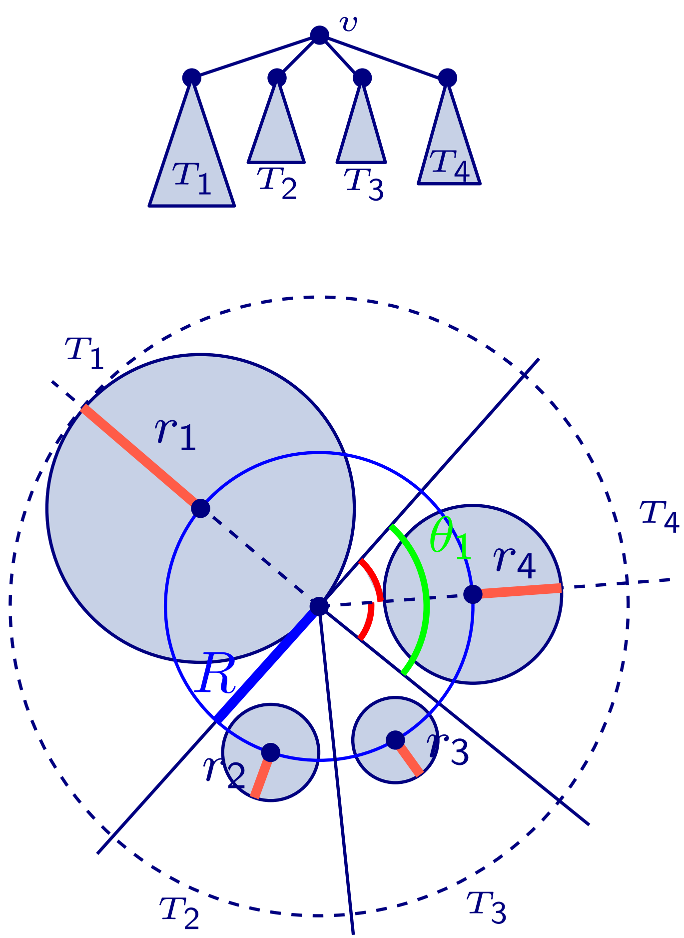
- We investigate drawing with **even angles** (drawing with uneven angles might have a better area)
- An arrangement of the subtree at a level can be describeed by a permutation $\sigma = \{1, 4, 2, 3\}$
- $\theta_i$ - angle of the wedge containing the subtree $T_i$

# Baloon Layout. Algorithm by Lin & Yen.

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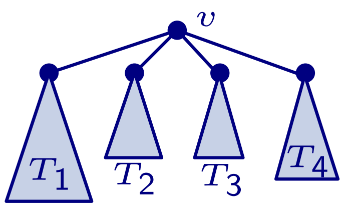


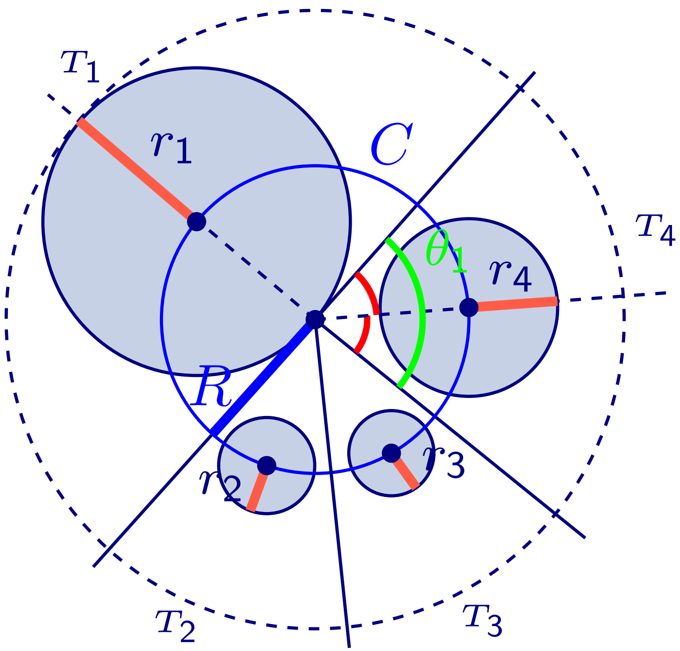


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- Let $C \approx 2 \sum r_i$

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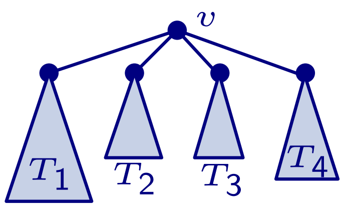


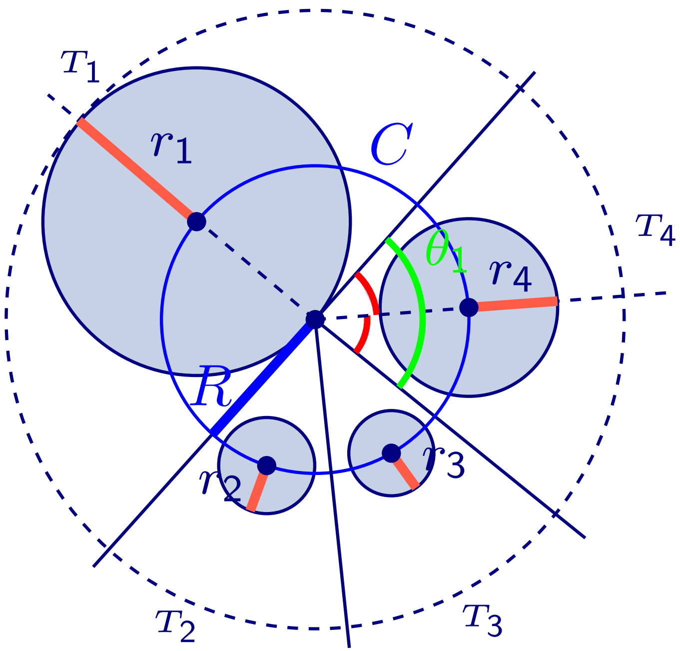


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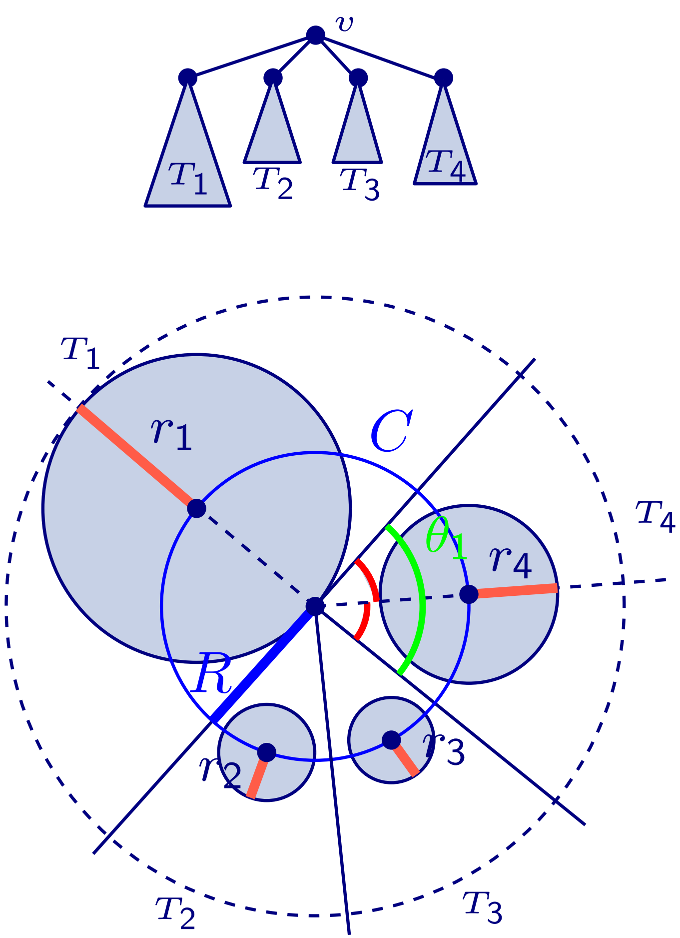


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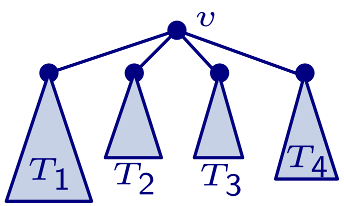


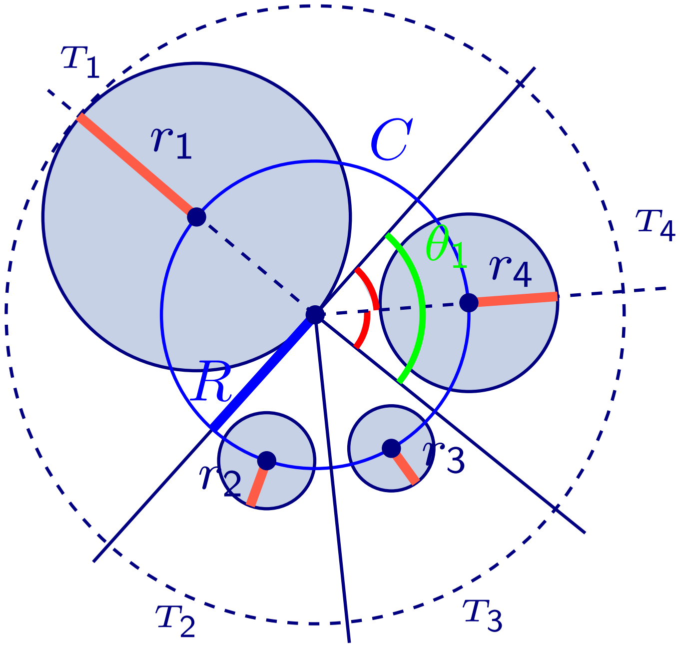


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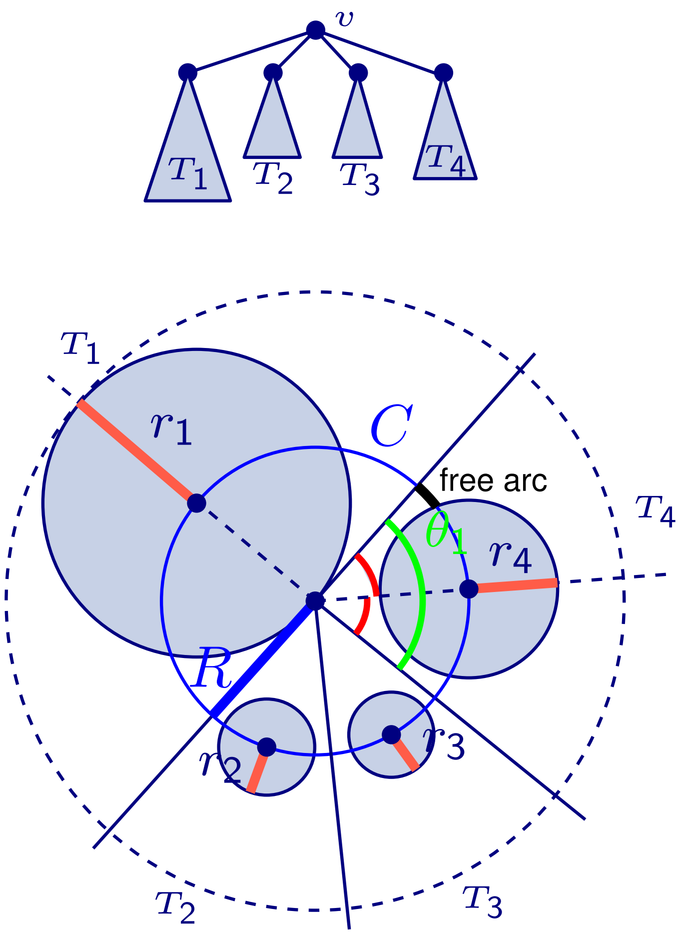


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- Set $\theta_i = 2(r_i + \text{free arc})/R$

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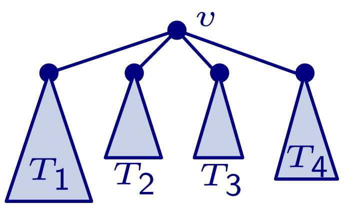


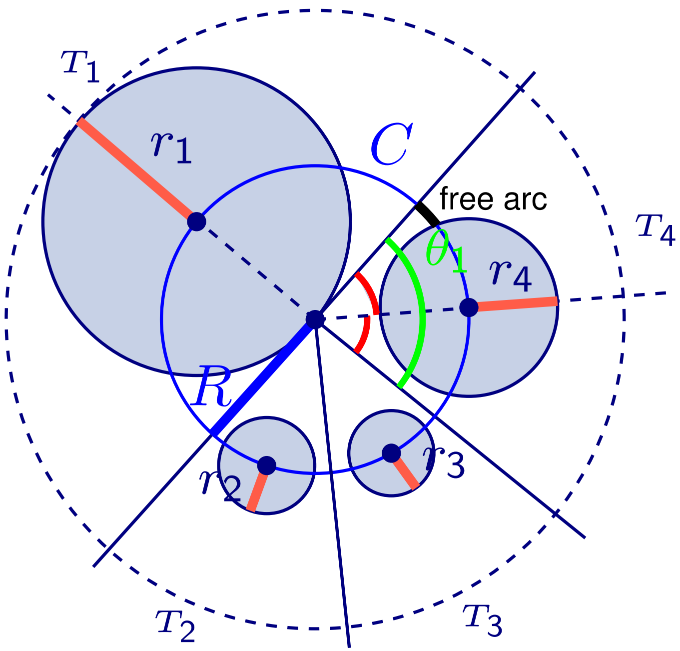


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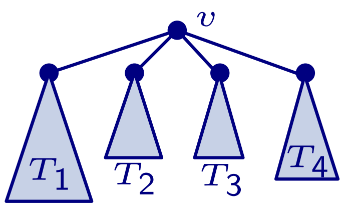


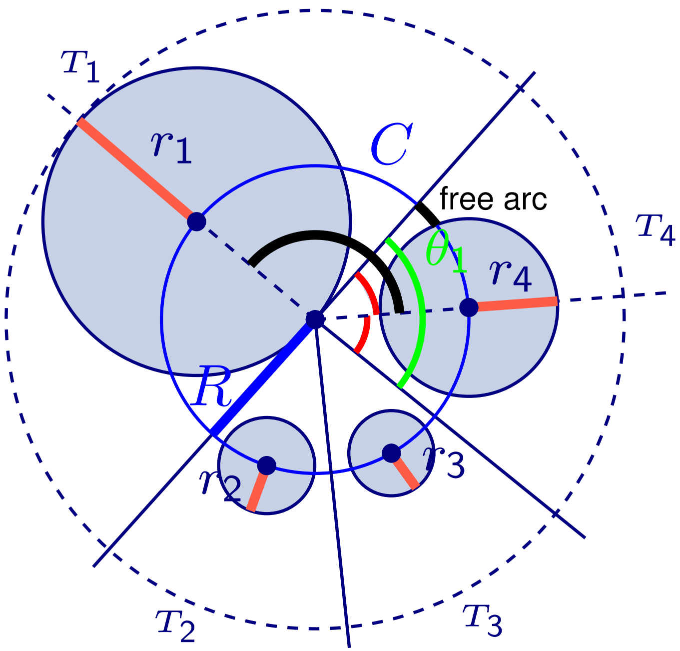


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- $\frac{\theta_{\sigma_i} + \theta_{\sigma_{i+1}}}{2}$ - angle between two consecutive edges

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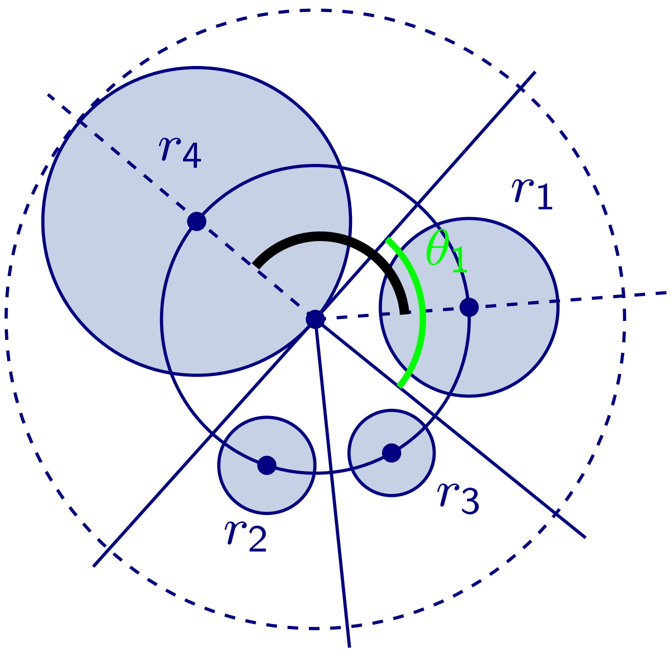


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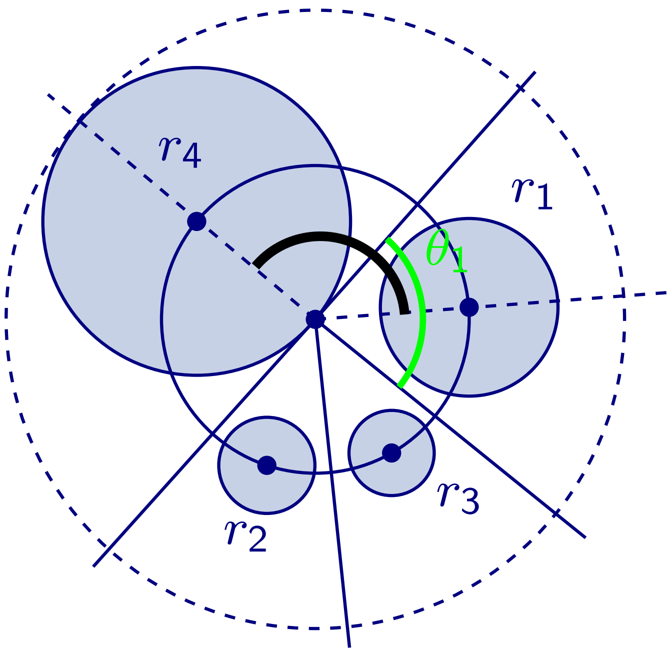


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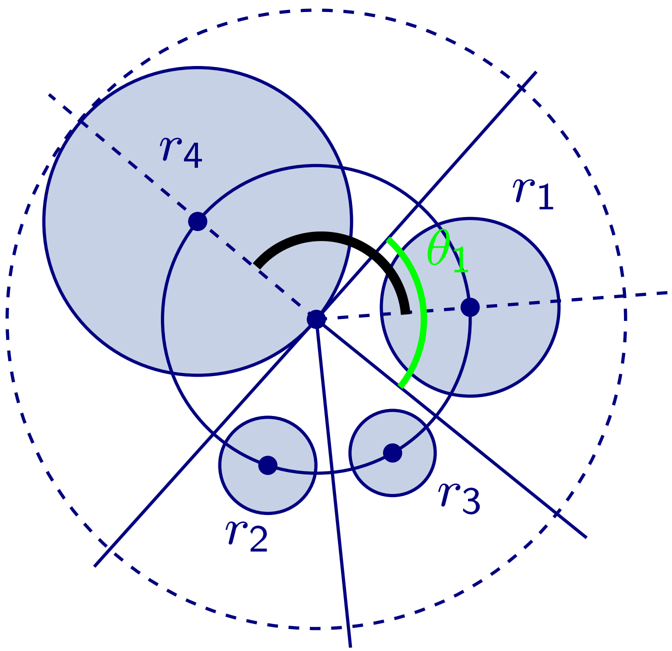


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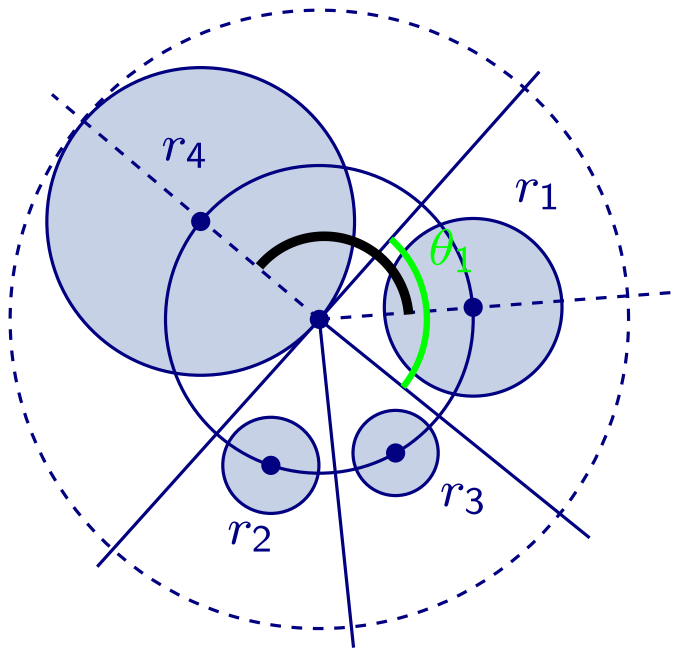


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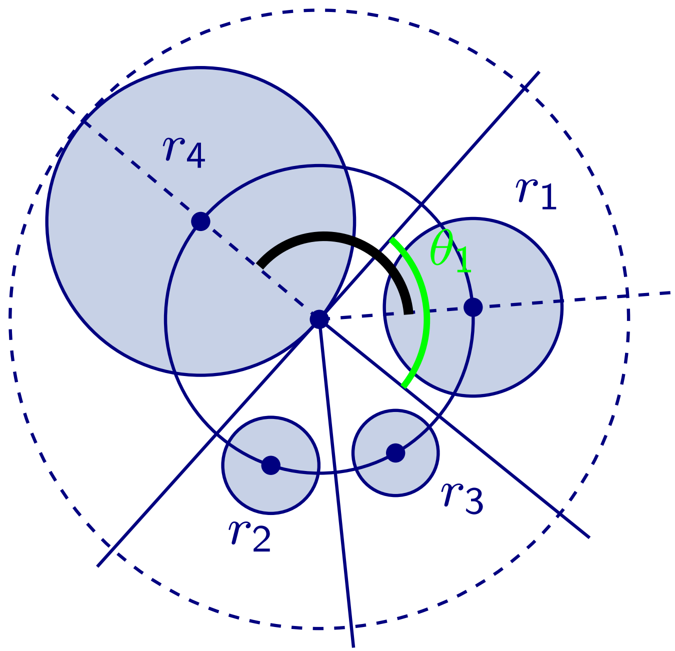


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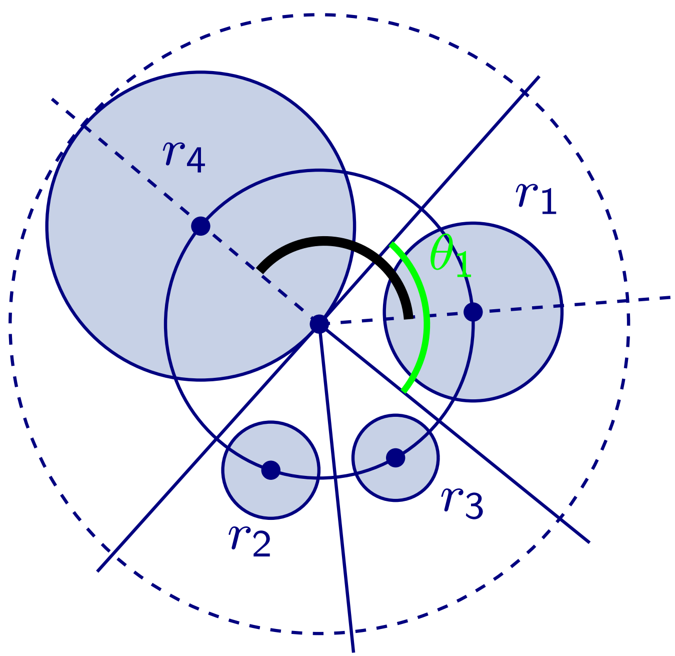


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- Let $\alpha_{ij} = \frac{M_i+m_j}{2}$. Angles $\frac{mid+m_k}{2}, \alpha_{(i-1)i}, \frac{M_k+mid}{2}, \alpha_{i(i-1)}$ are in $\sigma$.

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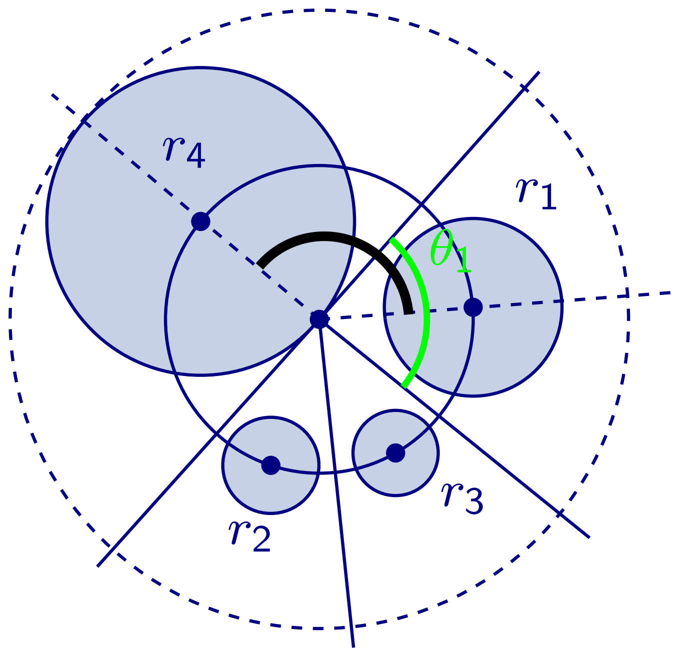


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- Relations among $\alpha_{ij}$:

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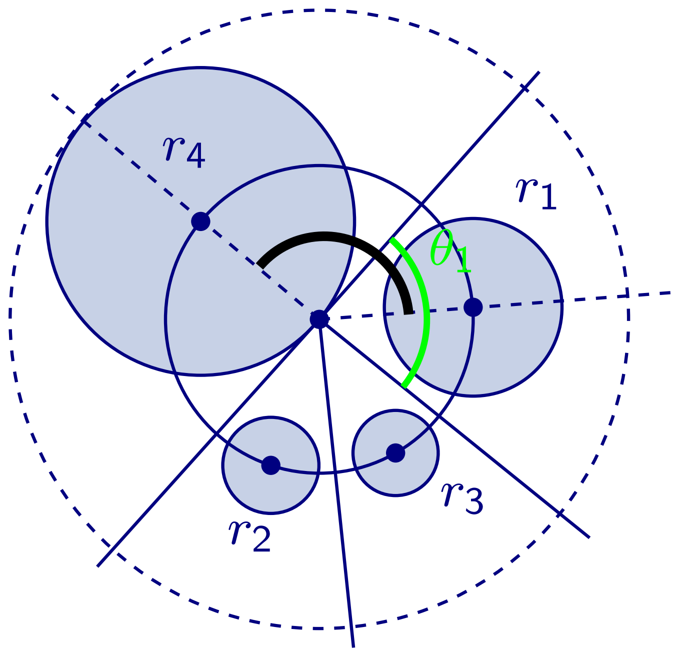


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- Relations among $\alpha_{ij}$:

$$\alpha_{12} > \alpha_{32} < \alpha_{34} > \cdots > \alpha_{j(j-1)} < \cdots > \alpha_{k(k-1)} < \frac{M_k+mid}{2} > \frac{mid+m_k}{2} < \alpha_{(k-1)k} > \cdots > \alpha_{43} < \alpha_{23} > \alpha_{21} < \alpha_{12}.$$

# Baloon Layout. Algorithm by Lin & Yen.

- Recall $\alpha_{ij} = \frac{M_i + m_j}{2}$
- Relations among $\alpha_{ij}$:

$\alpha_{12} > \alpha_{32} < \alpha_{34} > \cdots > \alpha_{j(j-1)} < \cdots > \alpha_{k(k-1)} < \frac{M_k + mid}{2} > \frac{mid + m_k}{2} < \alpha_{(k-1)k} > \cdots > \alpha_{43} < \alpha_{23} > \alpha_{21} < \alpha_{12}$.

- Smallest angle in $\sigma$ is either: $\frac{mid + m_k}{2}$ or $\alpha_{j(j-1)}$, while the size of the biggest angle is either $\frac{M_k + mid}{2}$ or $\alpha_{(l-1)l}$, $j, l \in \{2, \ldots, k\}$.

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- If they do not, let $x, y$ be the neighbors of $m_{i-1}$ in $\delta$, then:

$$\underbrace{m_1 < \cdots < m_{i-1}}_{i-1} < \dots < M_i < \underbrace{\dots < x < \dots < y < M_1}_{i-1}$$

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- Thus, $M_i$ and $m_{i-1}$ neighbor in $\delta$ and therefore $AngRes_\sigma = AngRes_\delta$, i.e. $\sigma$ maximizes the size of the smallest angle.

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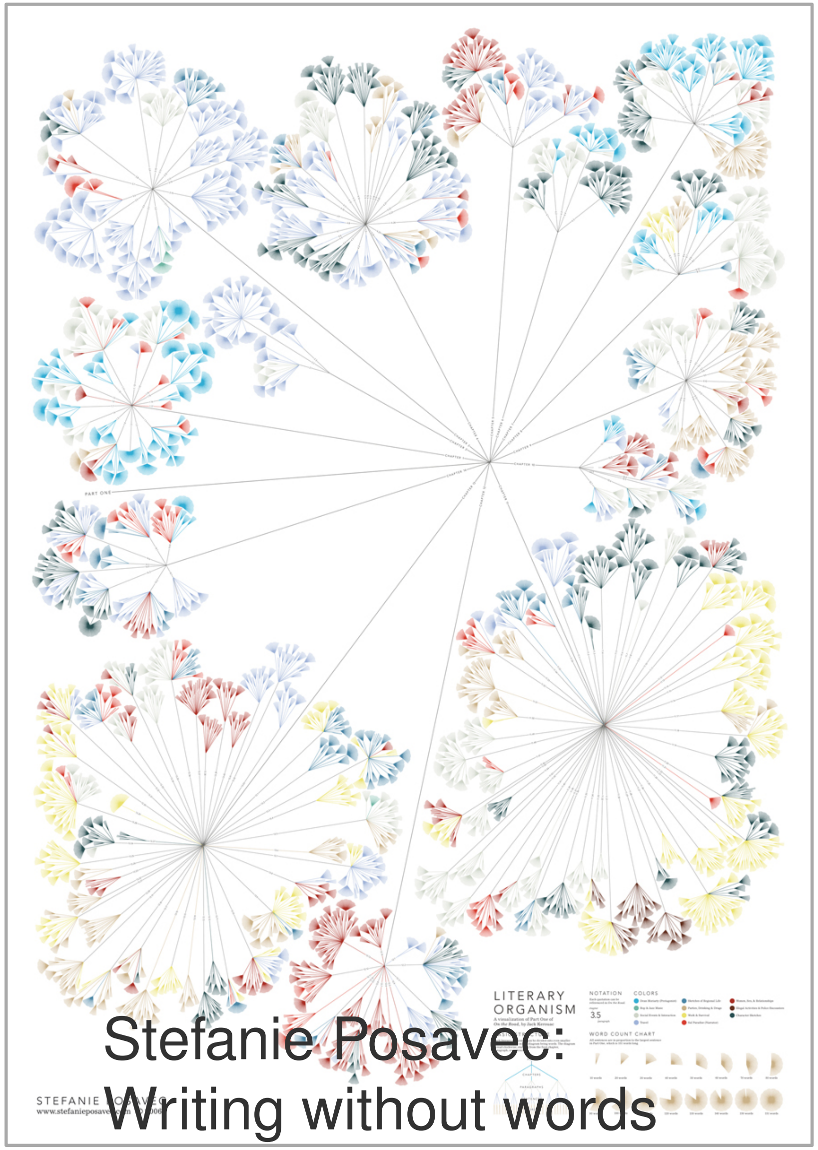
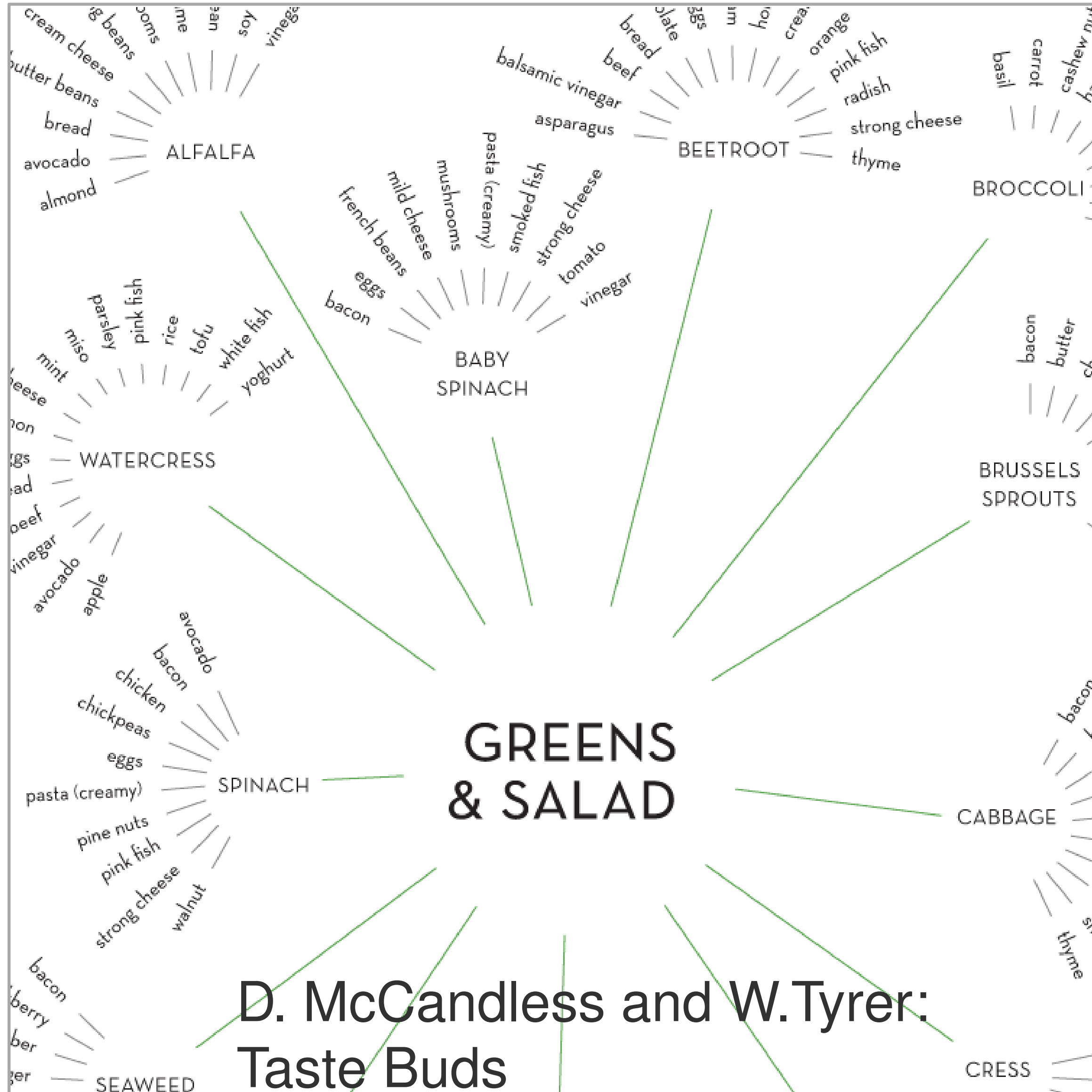
- The radii and therefore the angles are independent on each level
- Therefore, if we apply $\sigma$ at each level, we obtain an optimal aspect ratio. $\square$

# Applications of (almost) Ballon Layout




Stefanie Posavec:
Writing without words

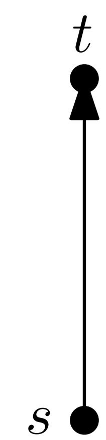



D. McCandless and W.Tyrer:
Taste Buds

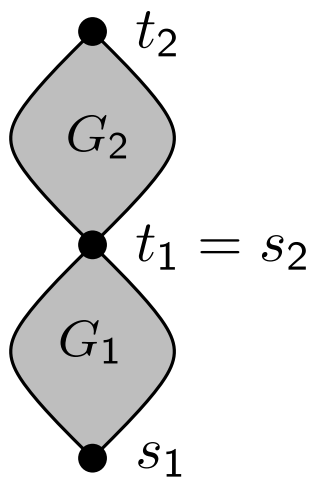
# Series-parallel Graphs

Graph $G$ is **series-parallel**, if

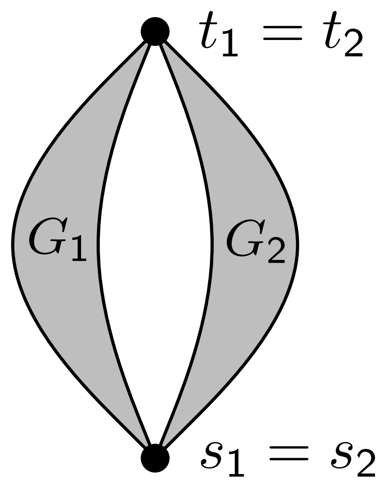
- It contains a single edge $(s, t)$ ($s$-source, $t$-sink)
- It consists of two series-parallel graphs $G_1, G_2$ with sources $s_1, s_2$ and sinks $t_1, t_2$ which are combined using one of the following rules:




**Series composition:**
Identify $t_1$ and $s_2$,
$s_1$ is the source of $G$, $t_2$ is the sink of $G$




**Parallel composition:**
Identify $s_1, s_2$ and set it to be source of $G$
Identify $t_1, t_2$ and set it to be sink of $G$



# Series-parallel Graphs. Decomposition Tree.



**Lemma**

Series-parallel graphs are acyclic and planar.

In order to proof this statement we can use a **decomposition tree** of $G$, which is a binary tree $T$ with nodes of three types: S,P and Q-type.

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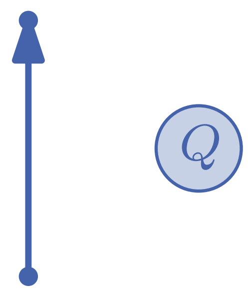

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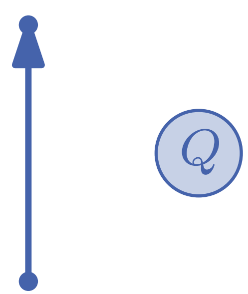


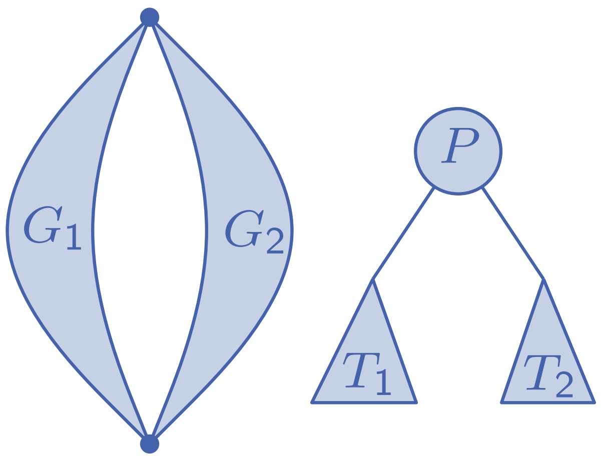

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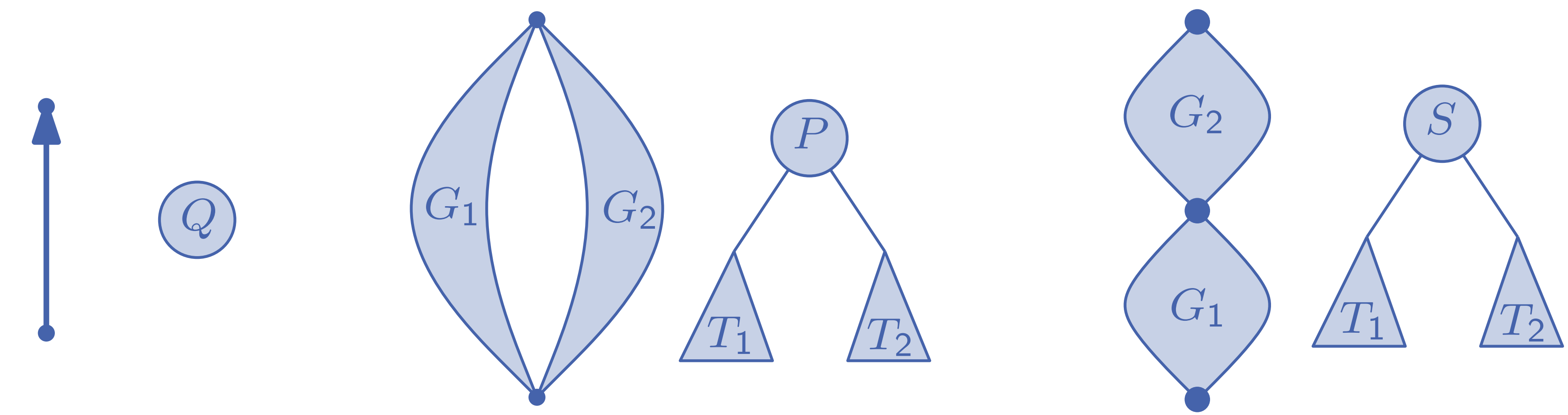
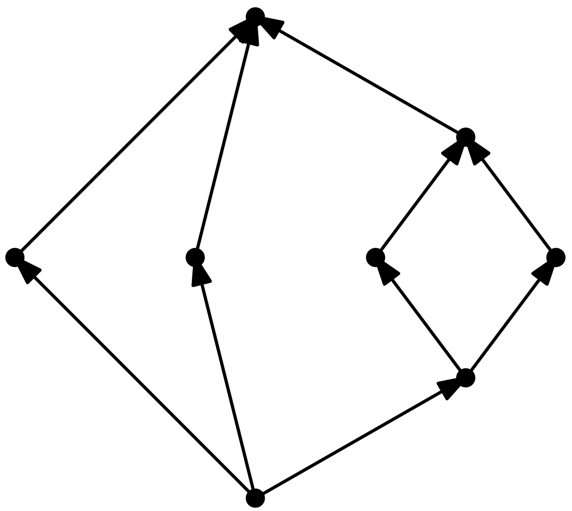


# Series-parallel Graphs. Decomposition Example.



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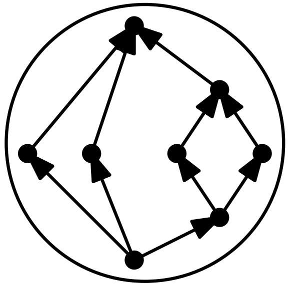

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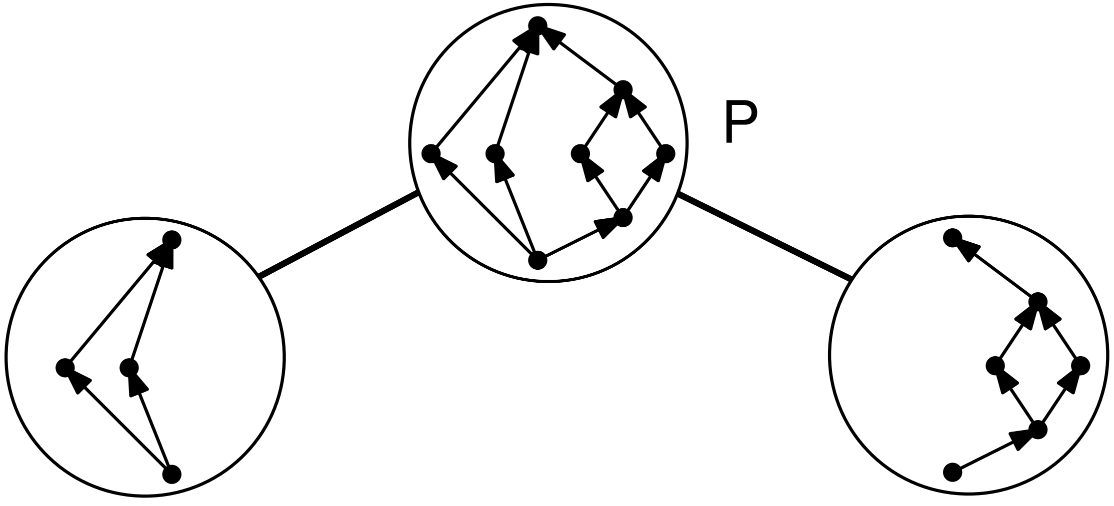

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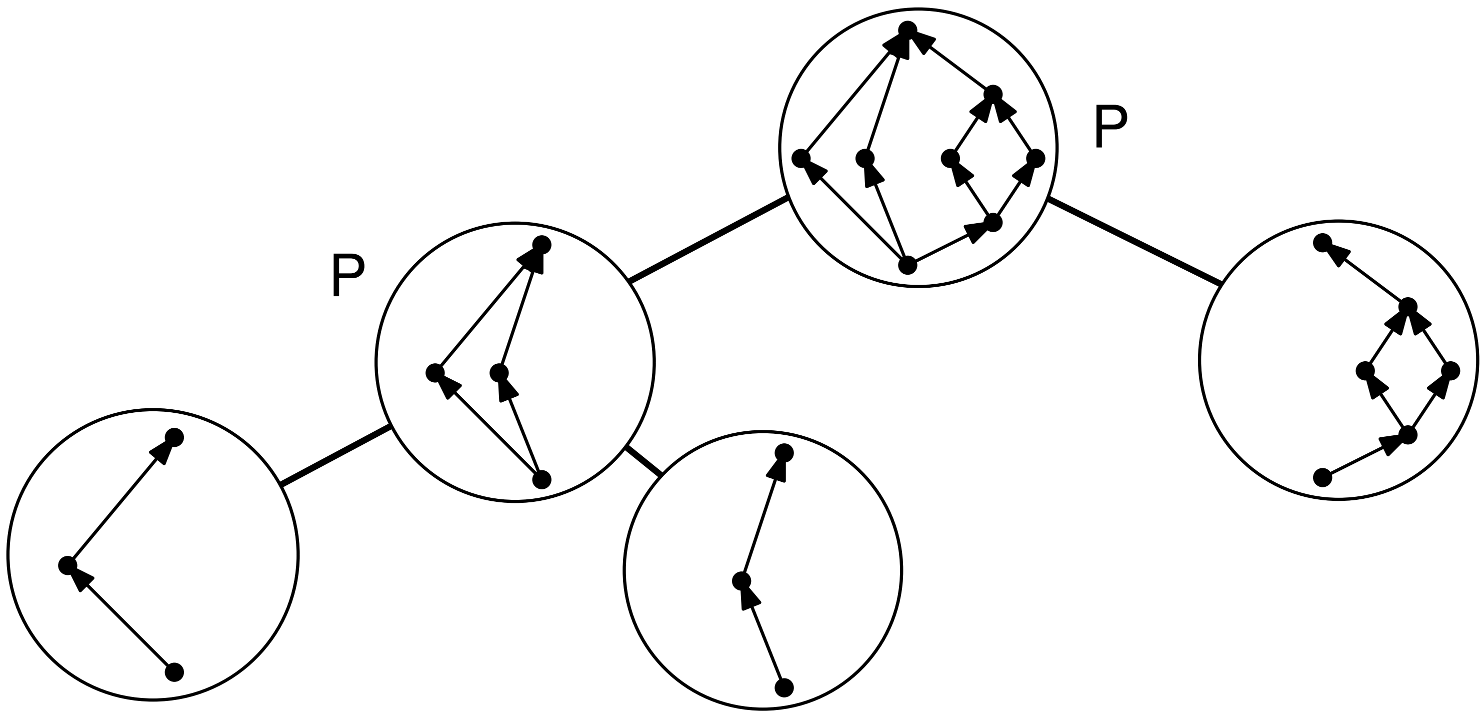

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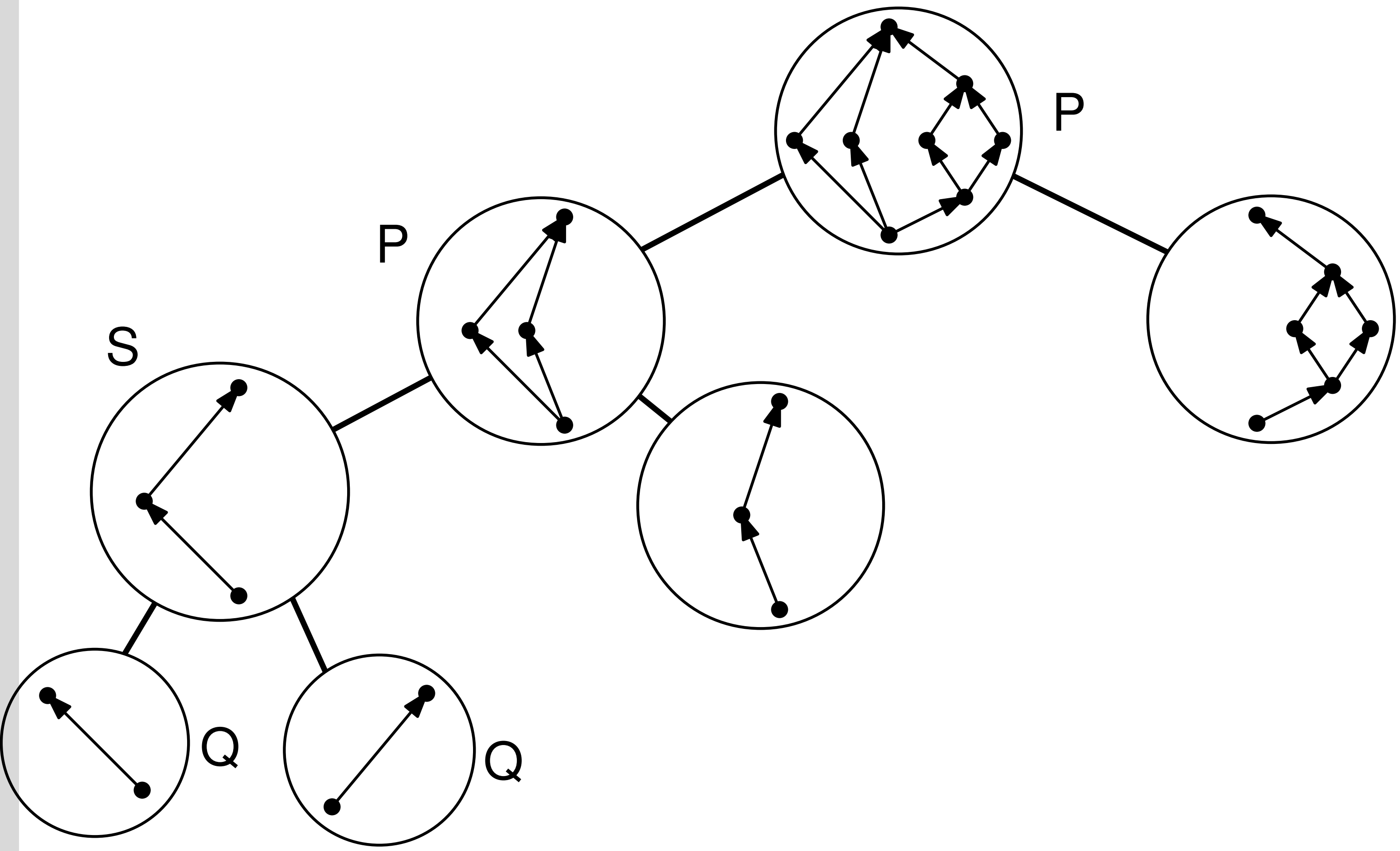

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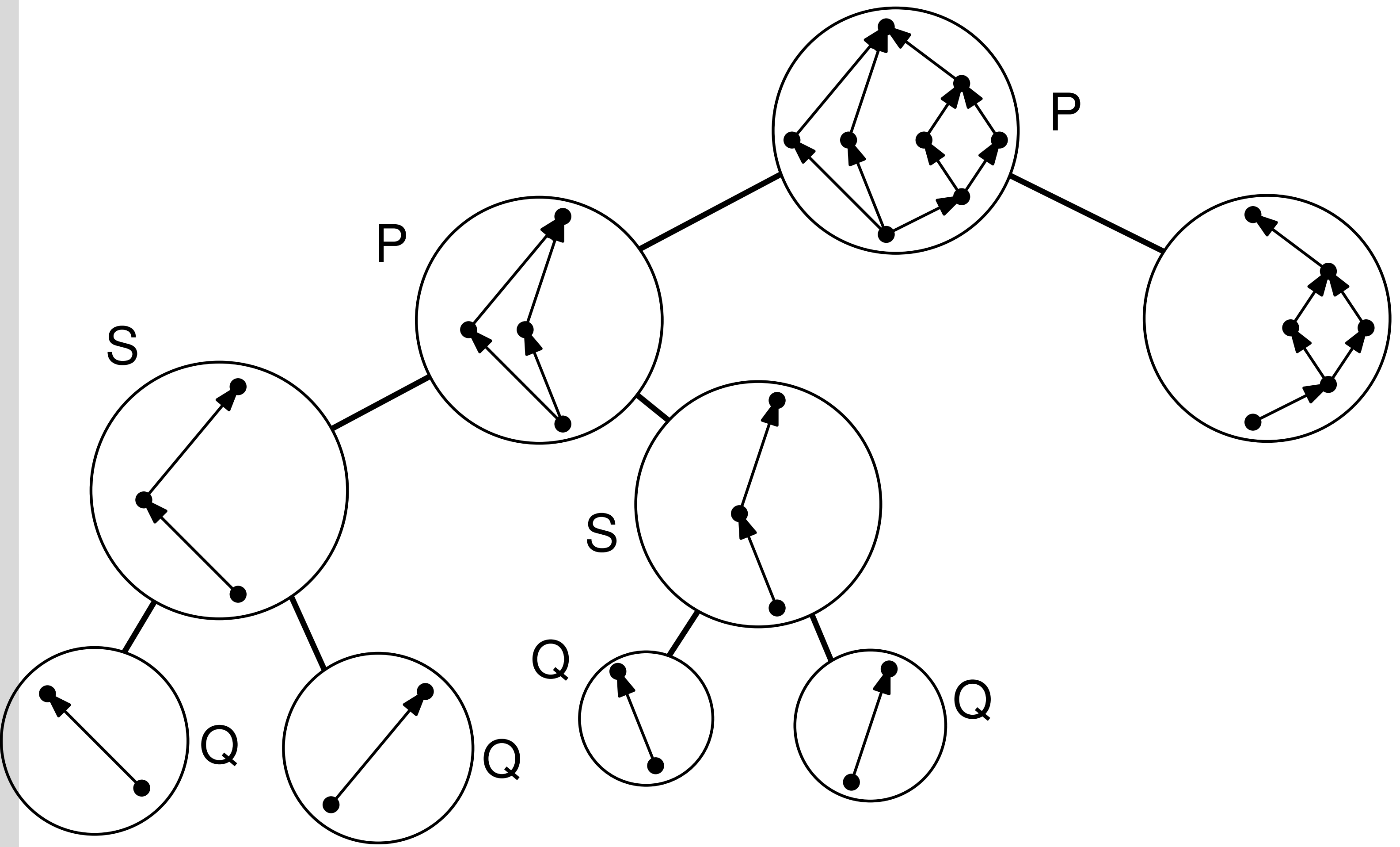

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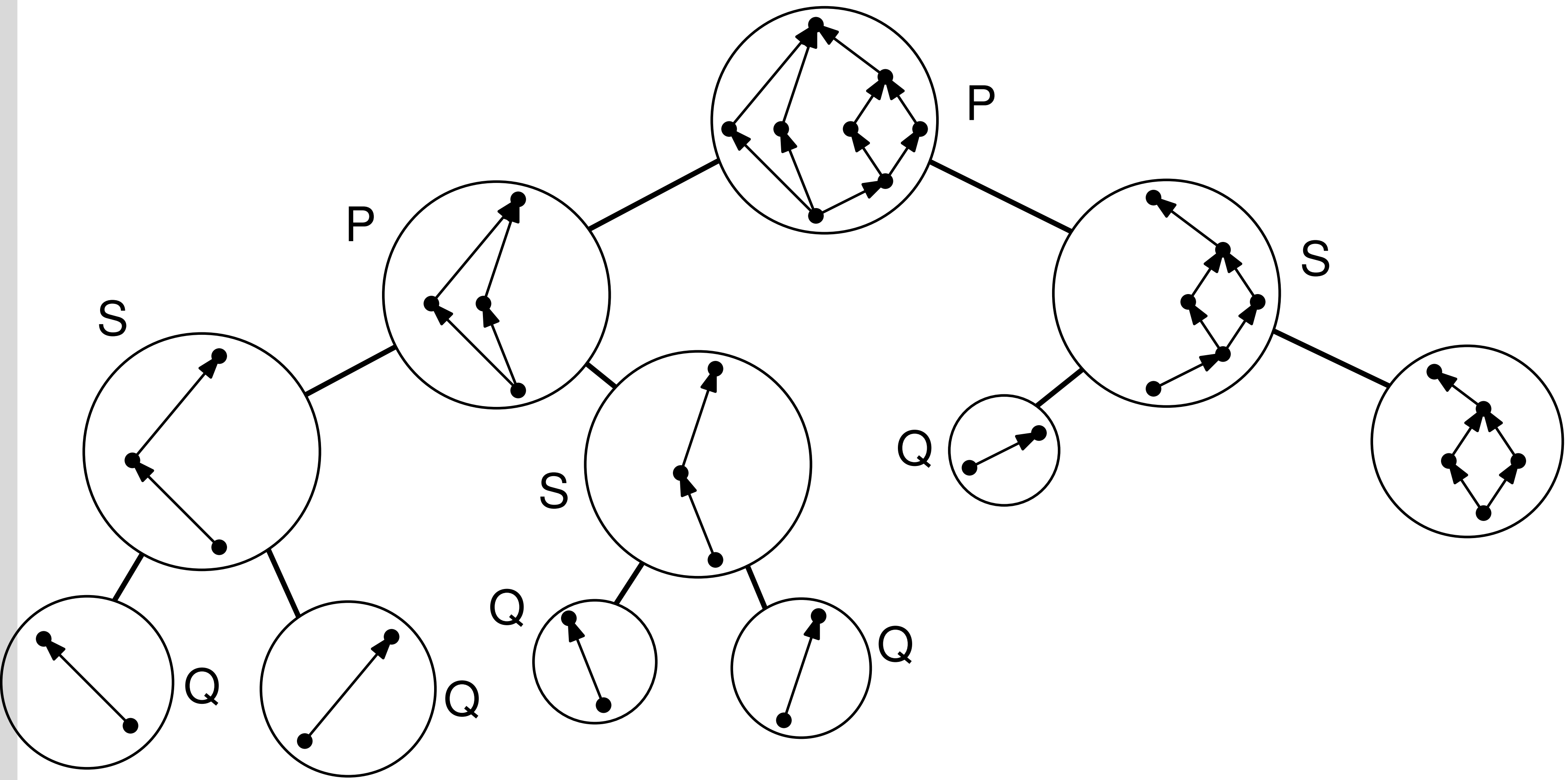

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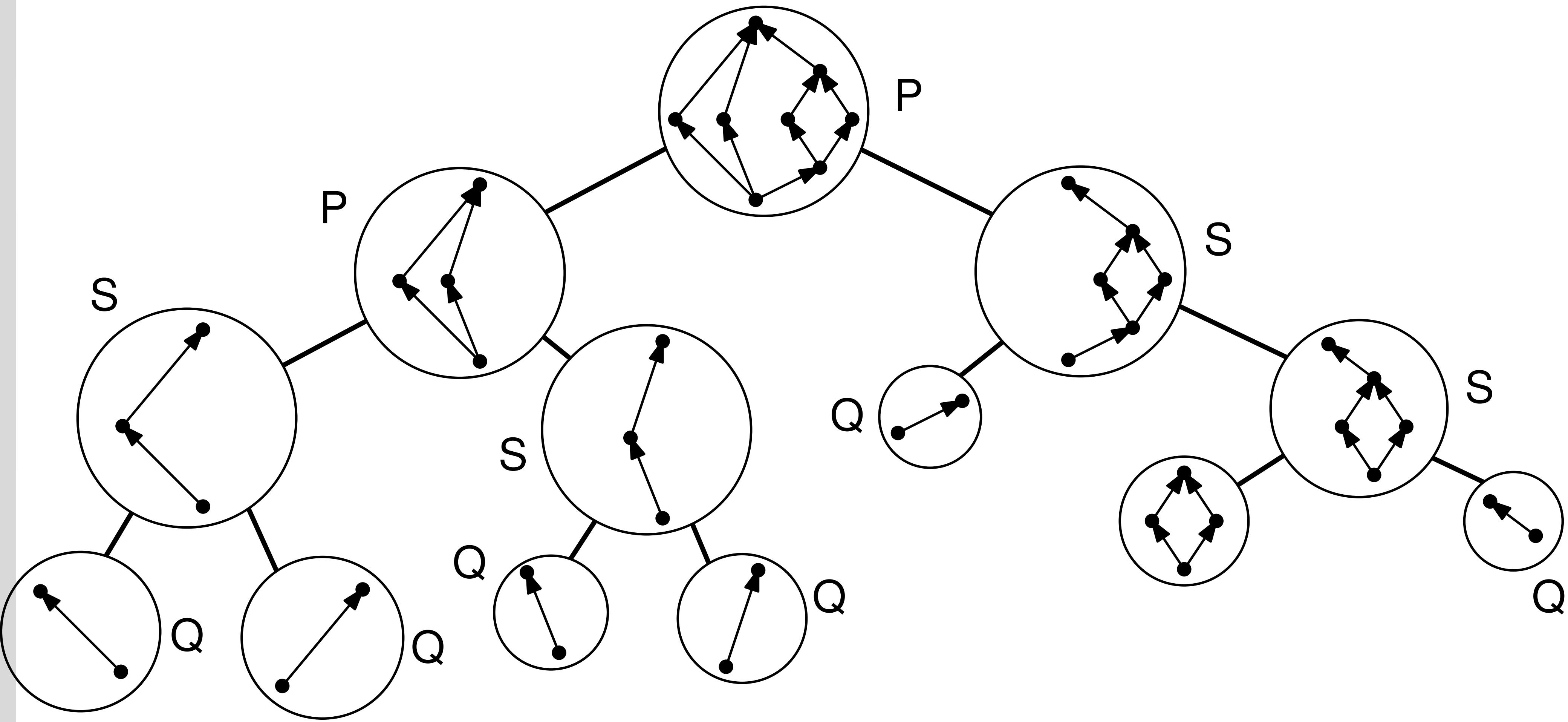

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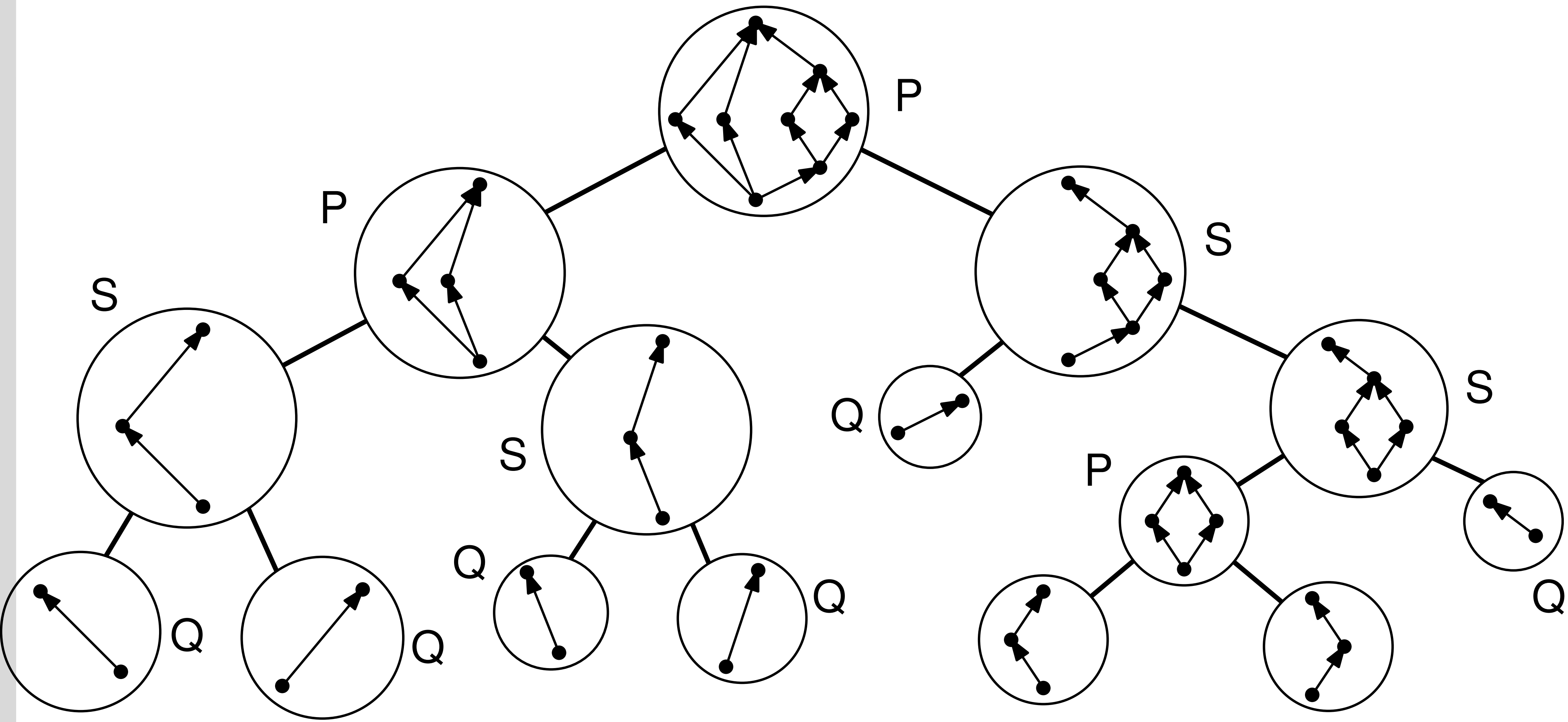

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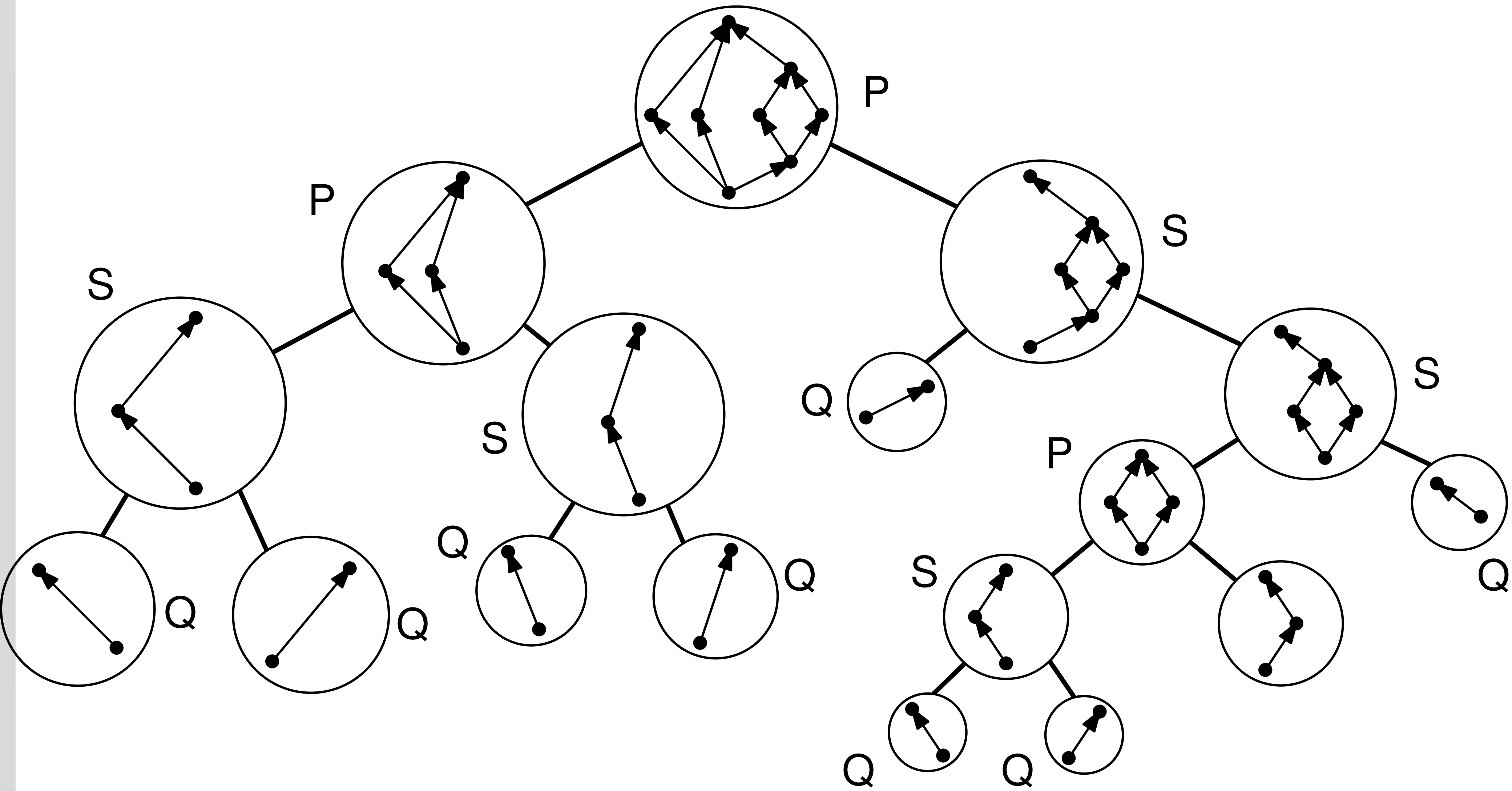

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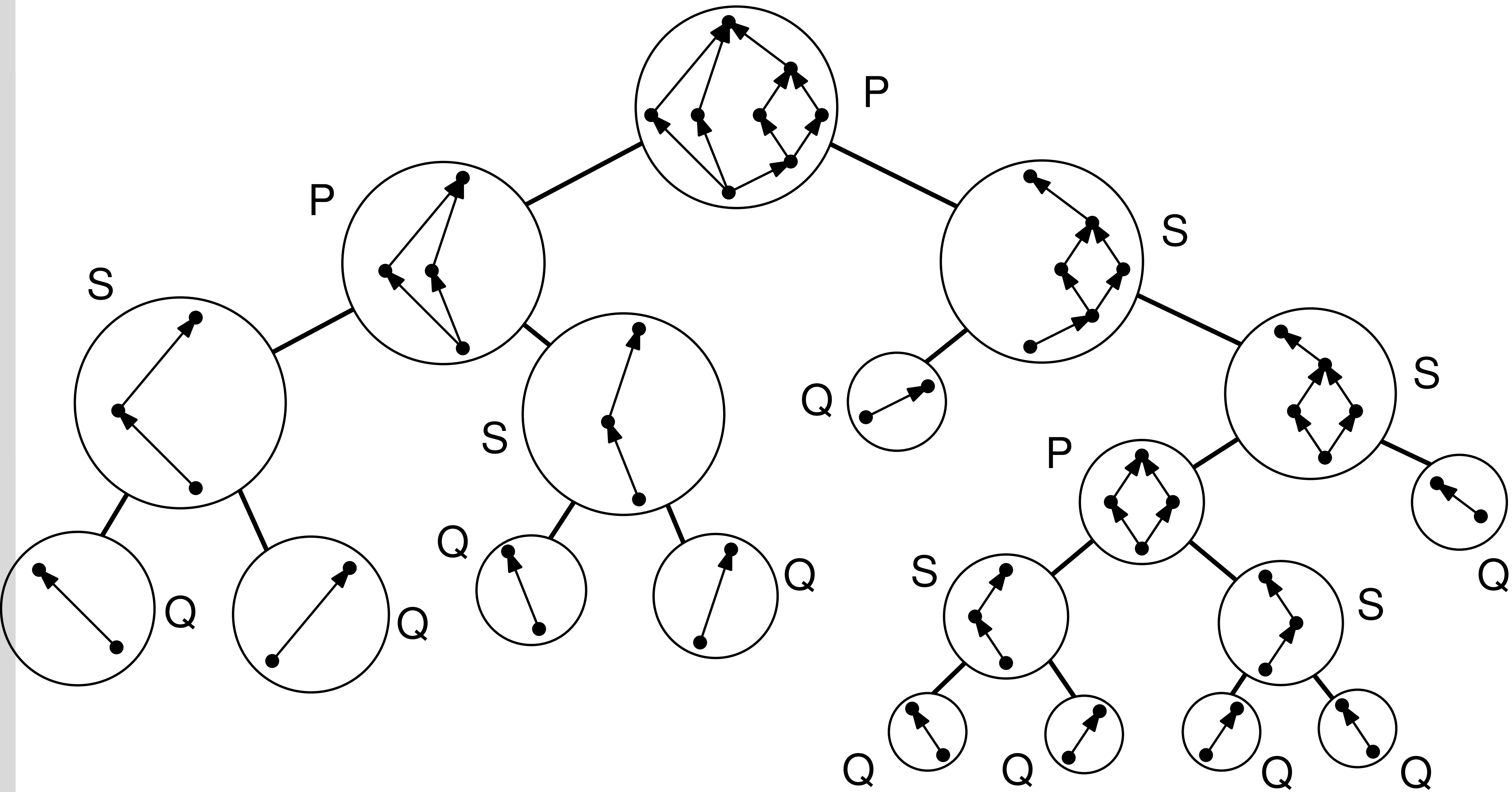
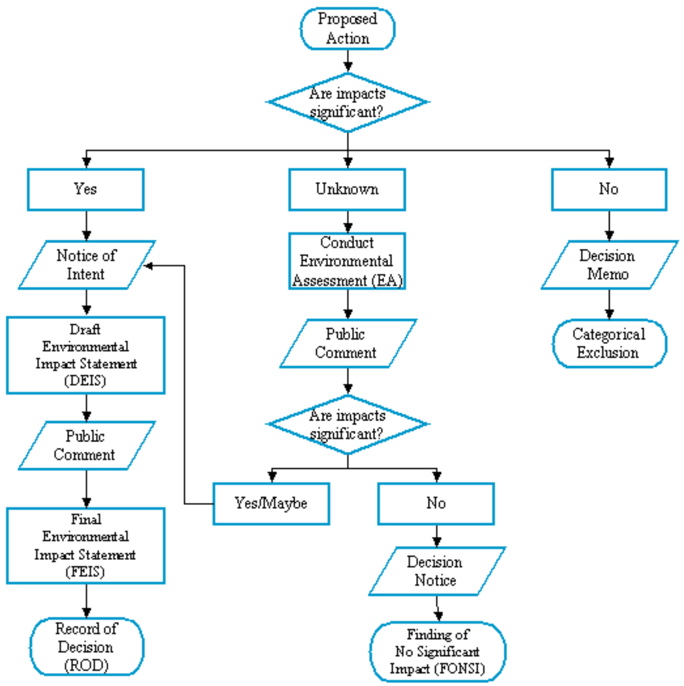
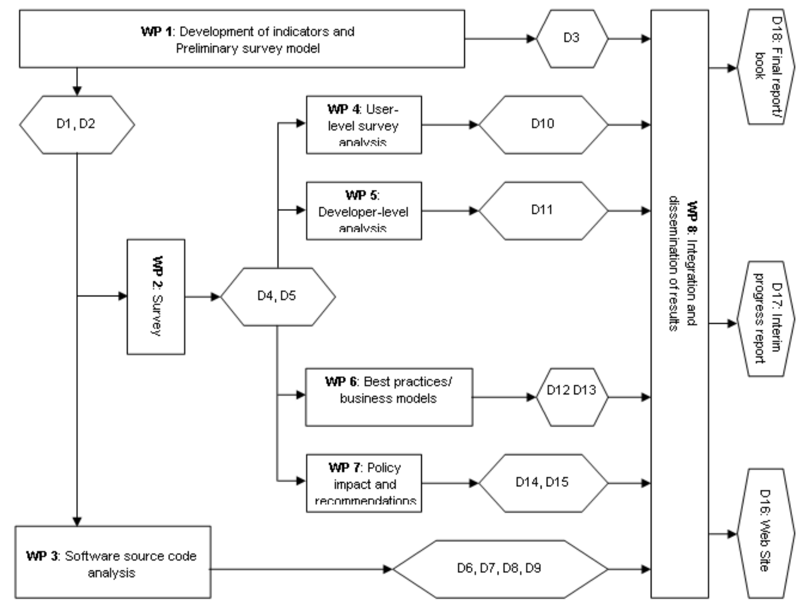


# Series-parallel Graphs. Applications.




Flowcharts




PERT-Diagrams

(Program Evaluation and Review Technique)

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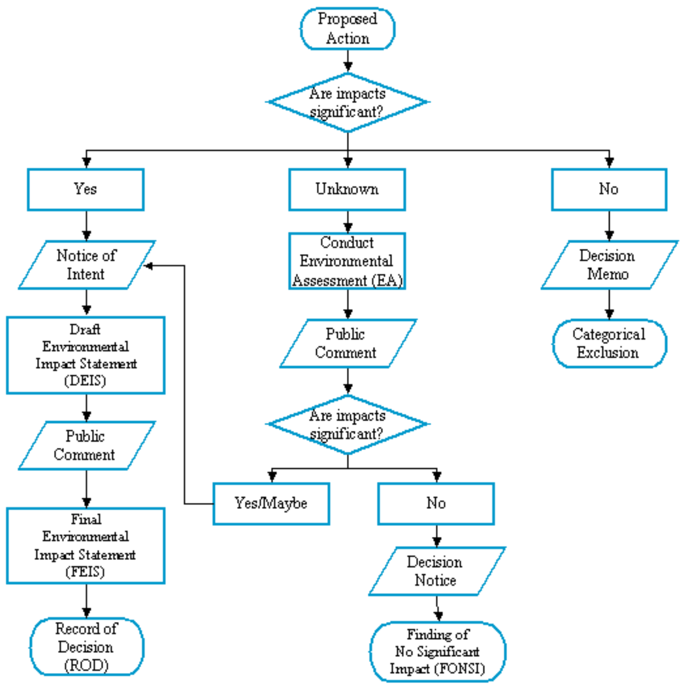


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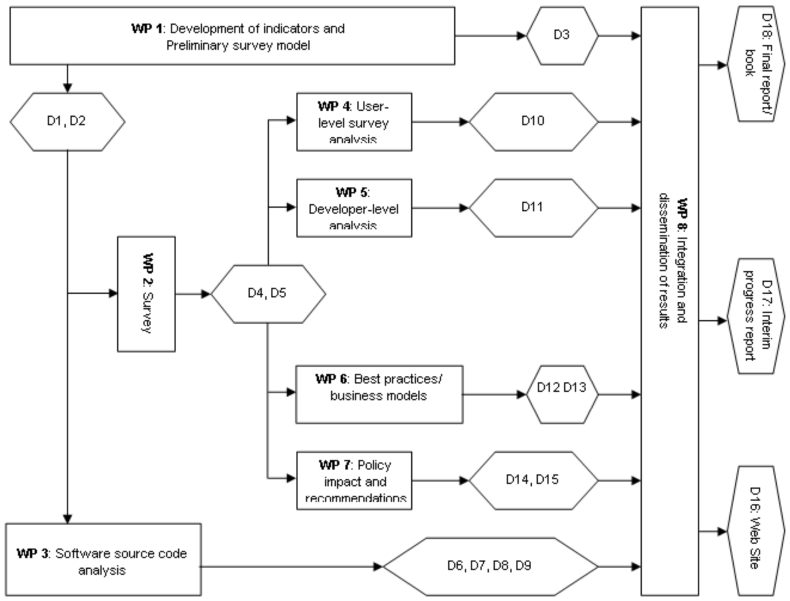


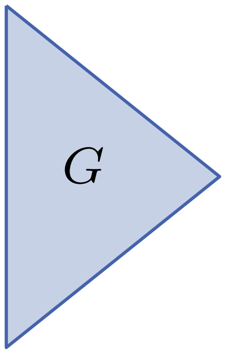

PERT-Diagrams

(Program Evaluation and Review Technique)

**Computational Complexity:** Linear time algorithms for $\mathcal{NP}$-hard problems (e.g. Maximum Matching, Maximum Independent Set, Hamiltonian Completion)

# Straight-line Drawing of SP-Graphs

- Draw graph $G$ inside a right-angled isosceles bounding triangle $\Delta(G)$



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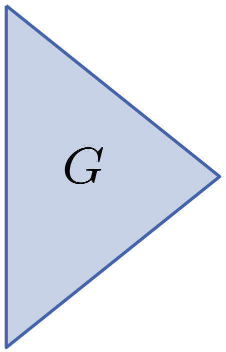


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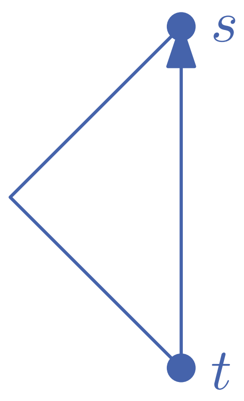

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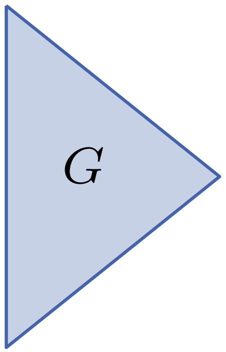


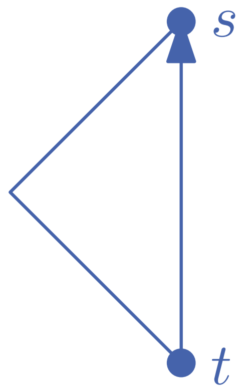


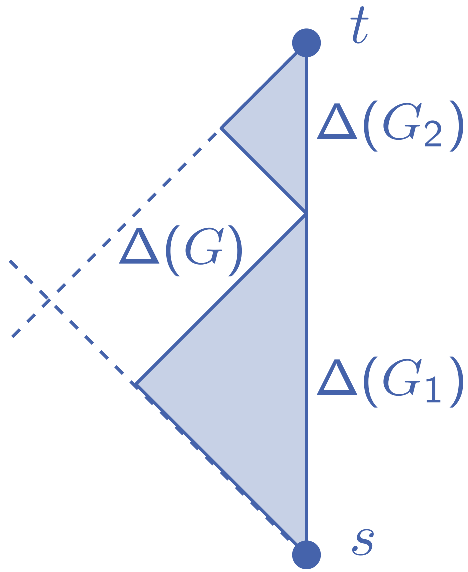

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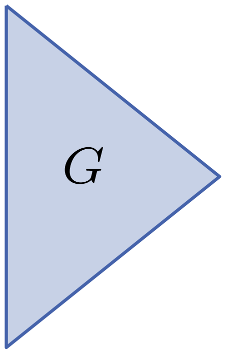


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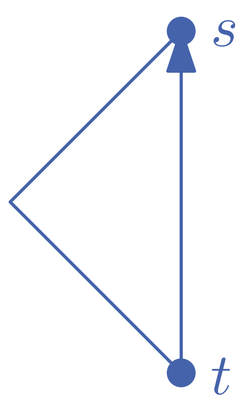


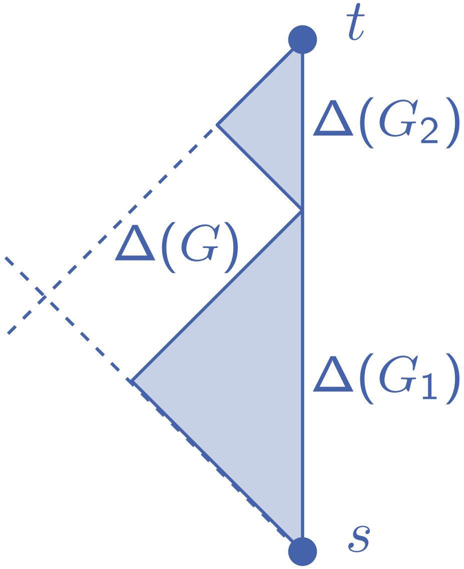


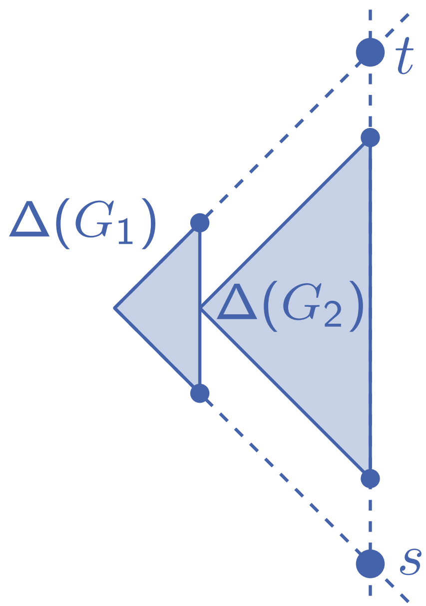

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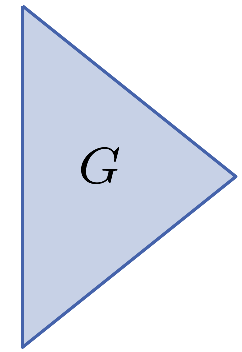


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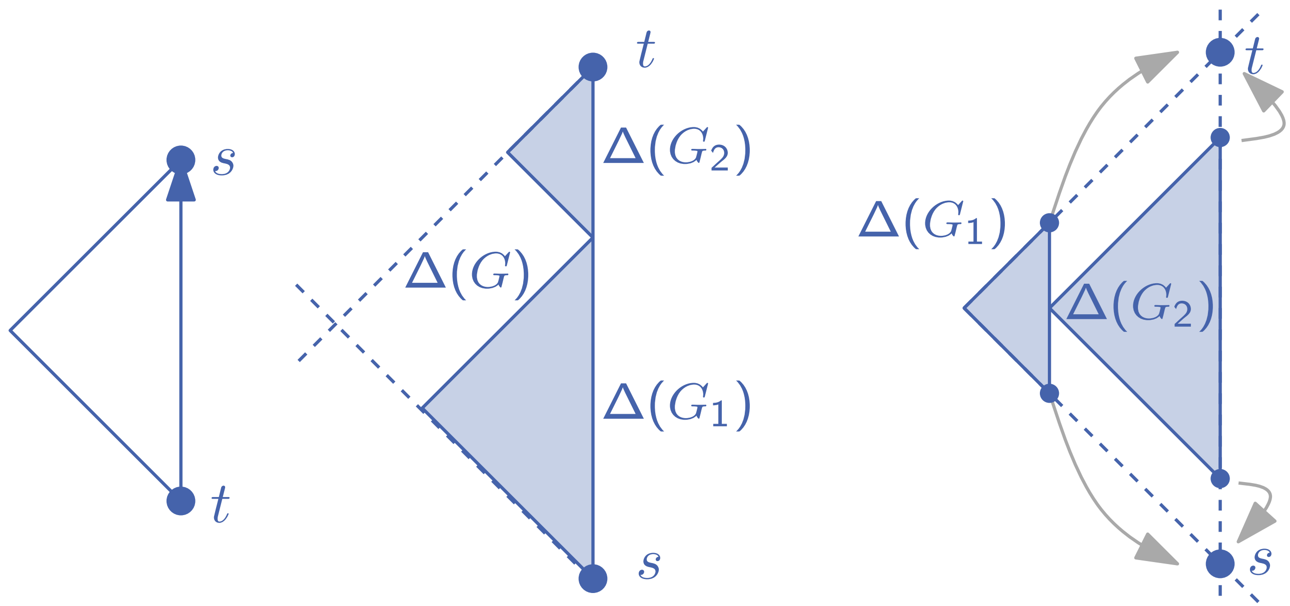

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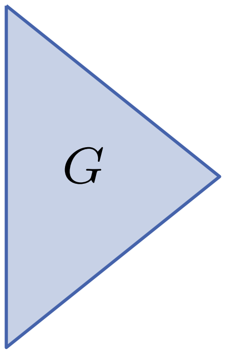


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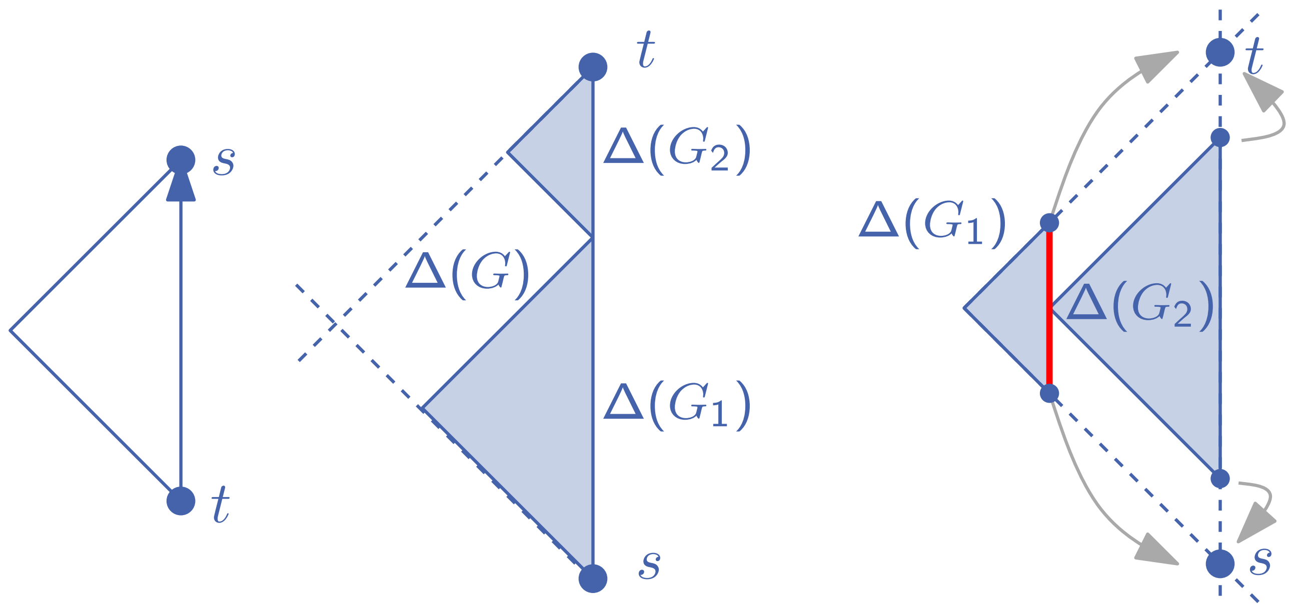

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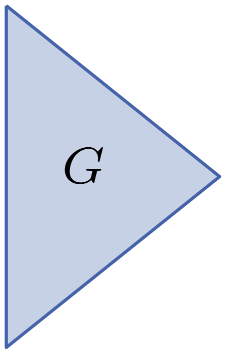


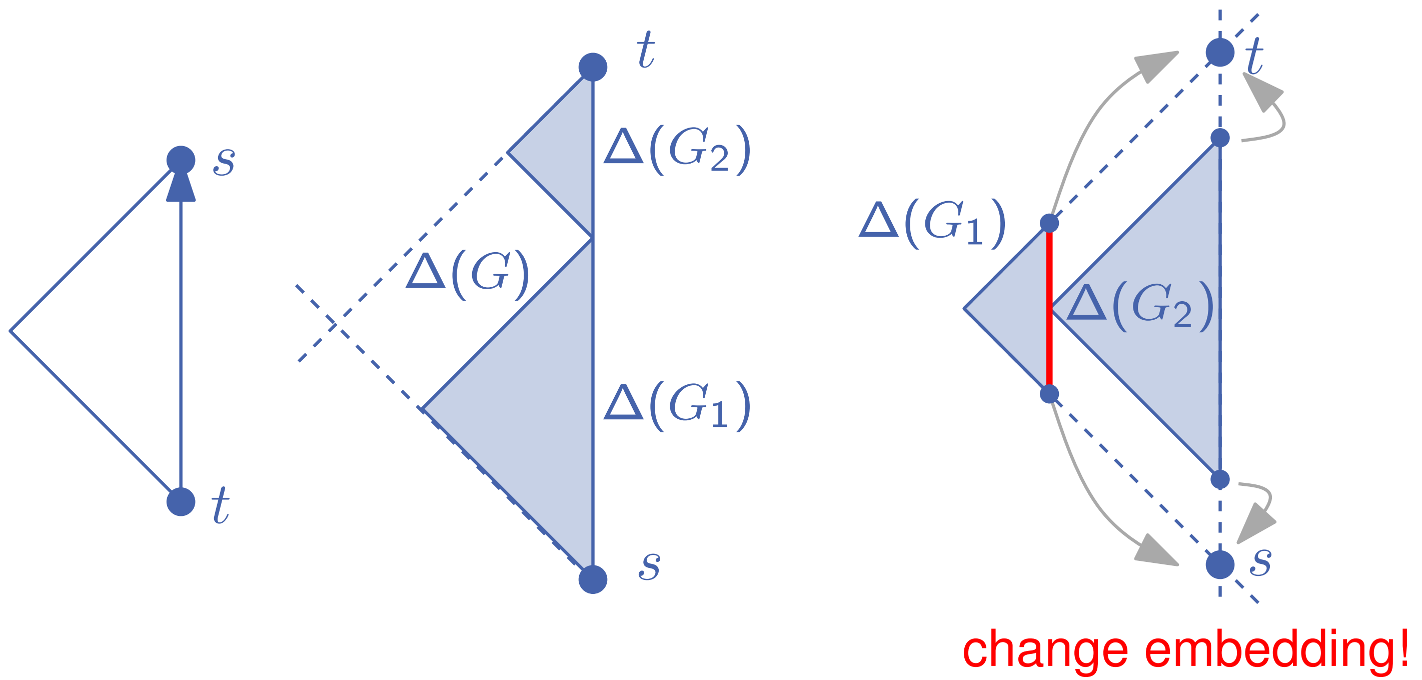

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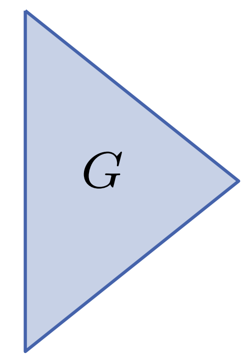


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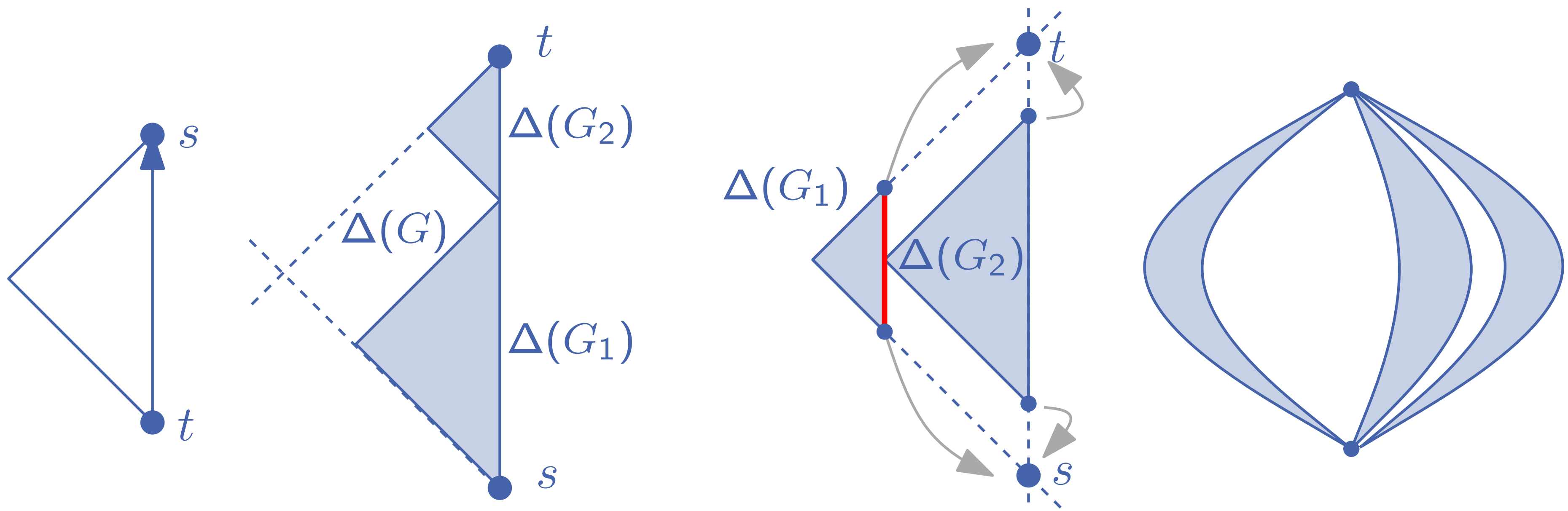


change embedding!

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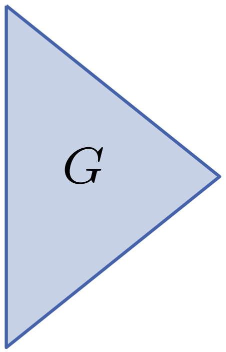


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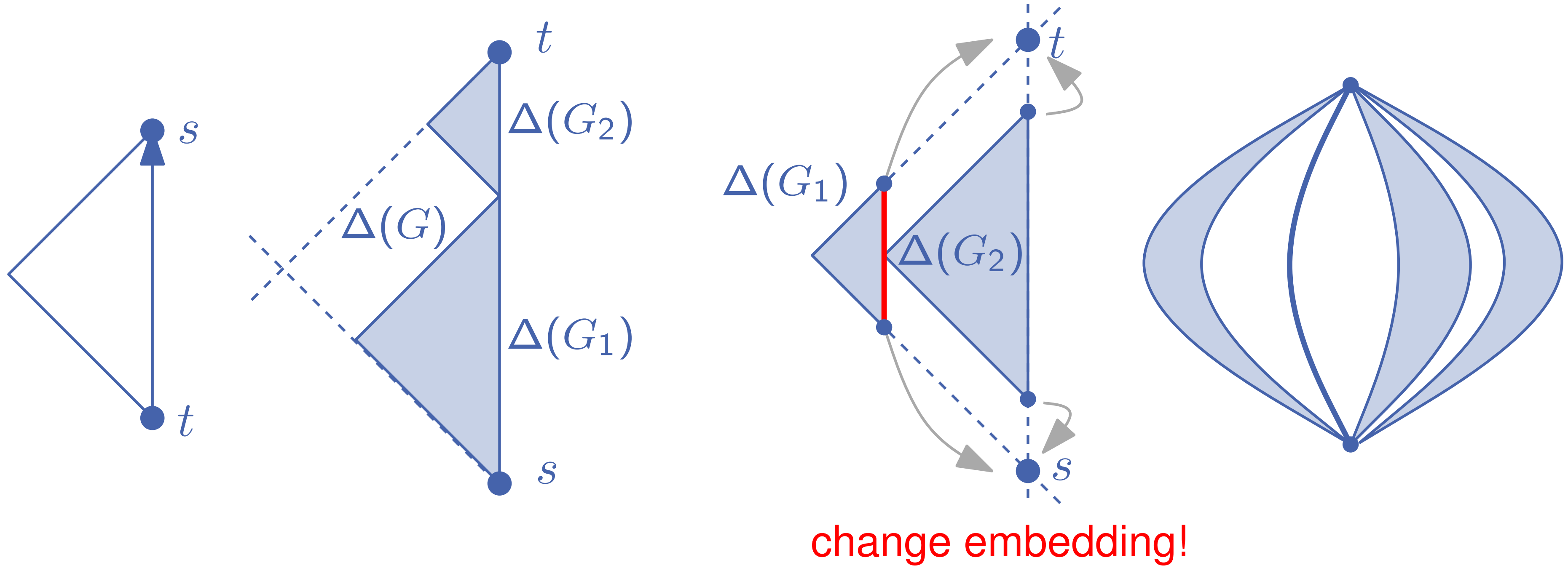
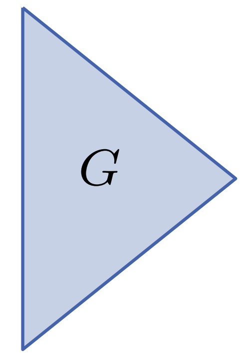
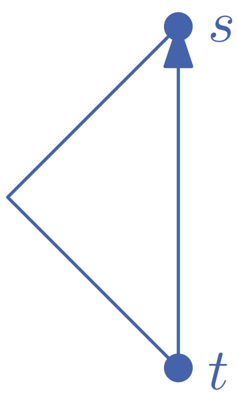
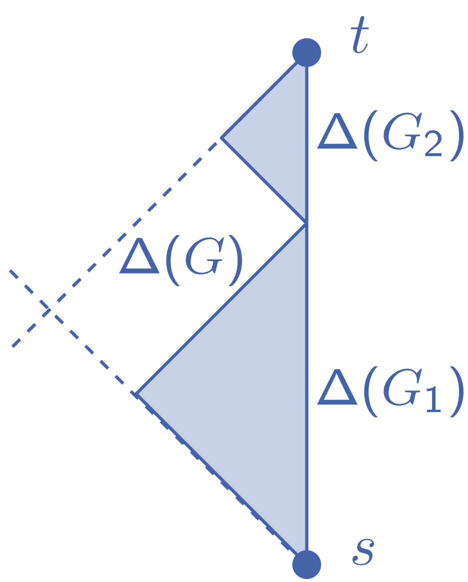


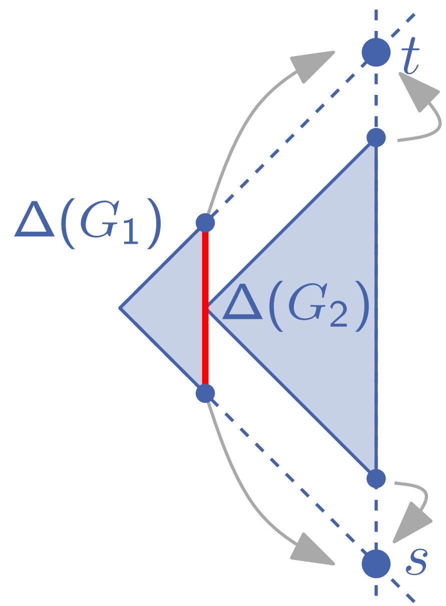
# Straight-line Drawing of SP-Graphs



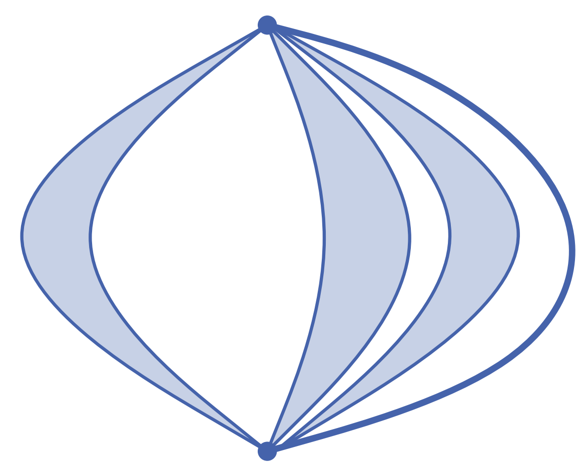

- Draw graph $G$ inside a right-angled isosceles bounding triangle $\Delta(G)$
- Q-Nodes (Induction base):
- S-Nodes (series composition)
- P-Nodes (parallel composition)



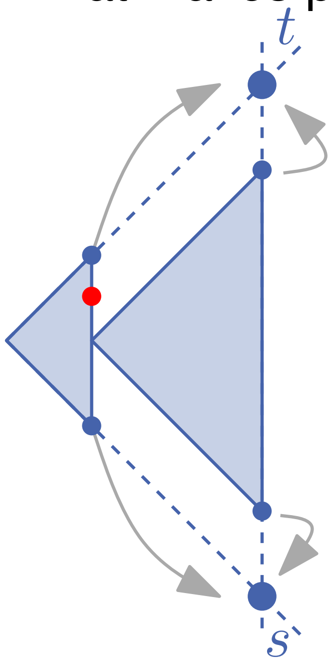




change embedding!



# Straight-line Drawing of SP-Graphs

- What makes parallel composition possible without creating crossings?



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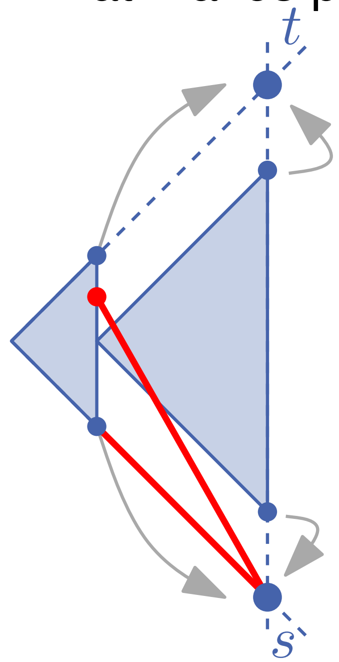

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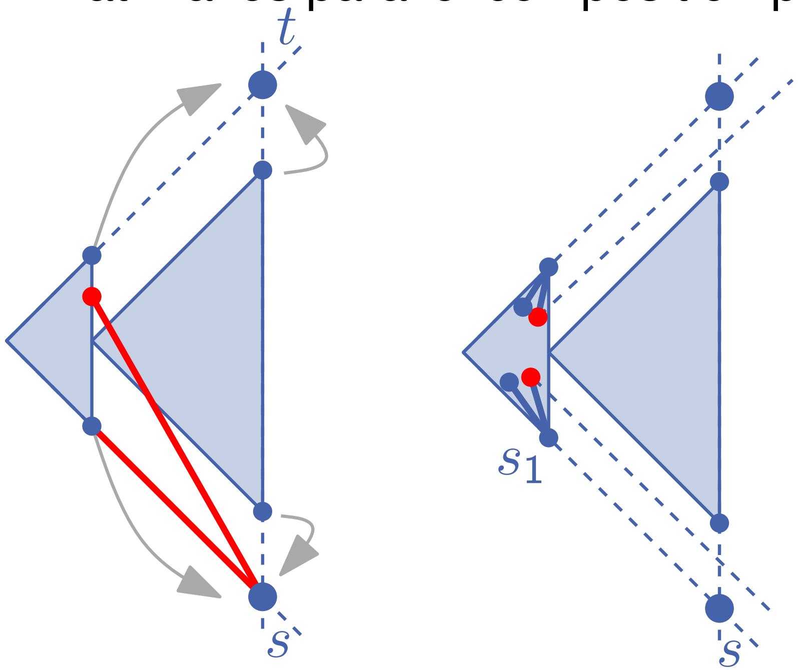

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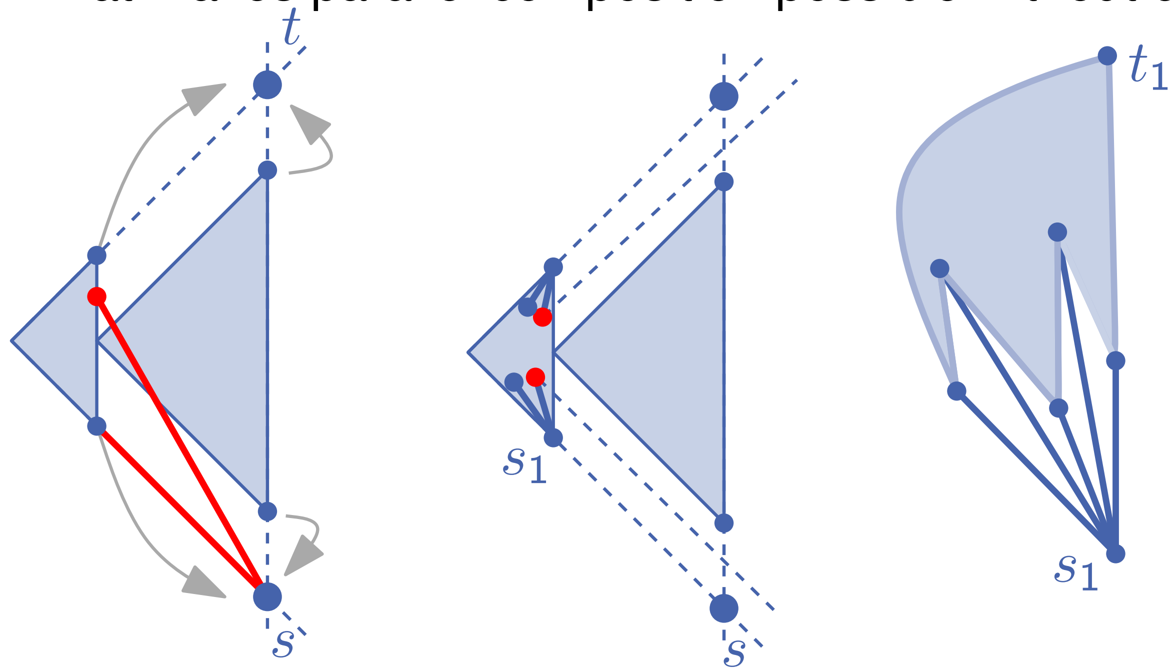

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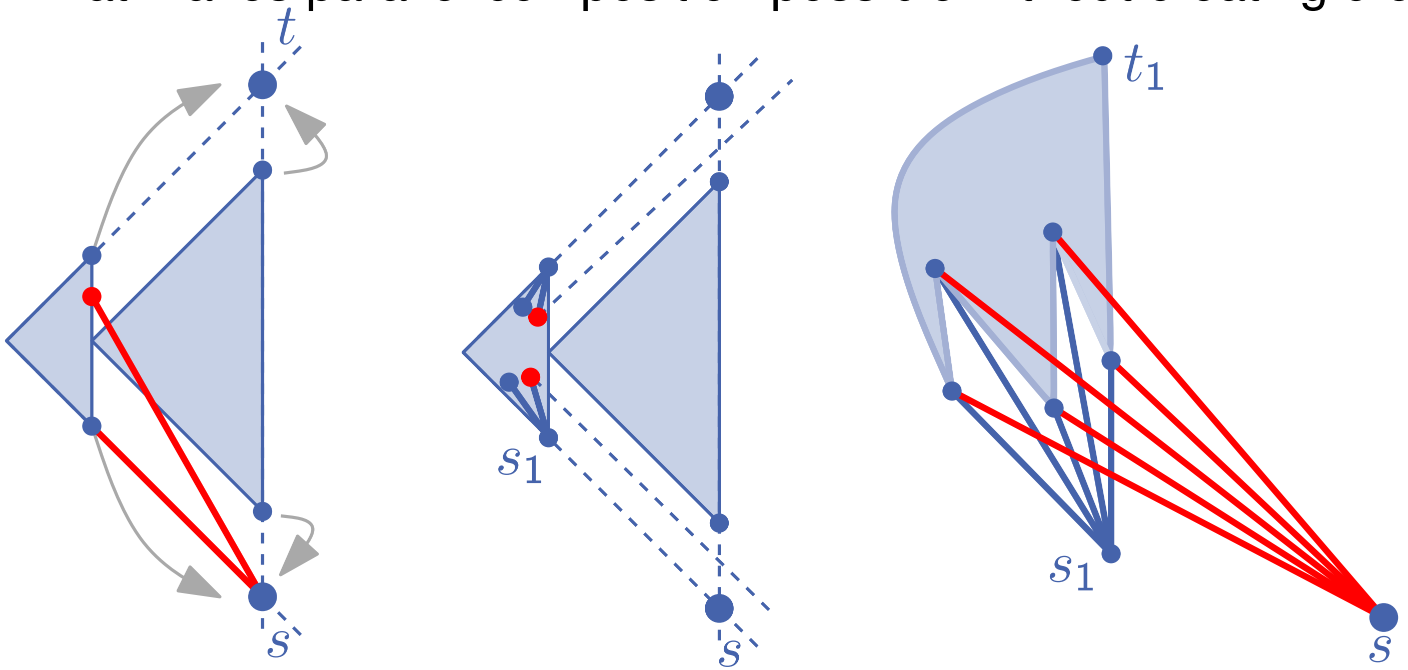

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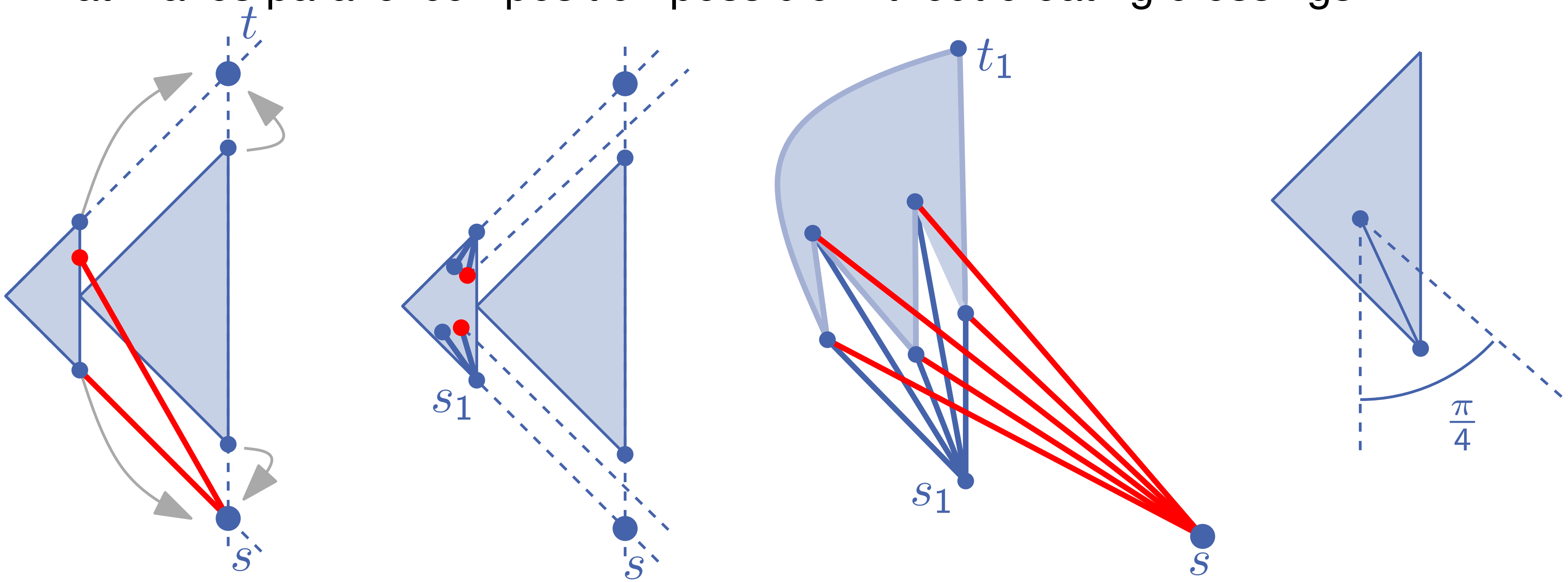

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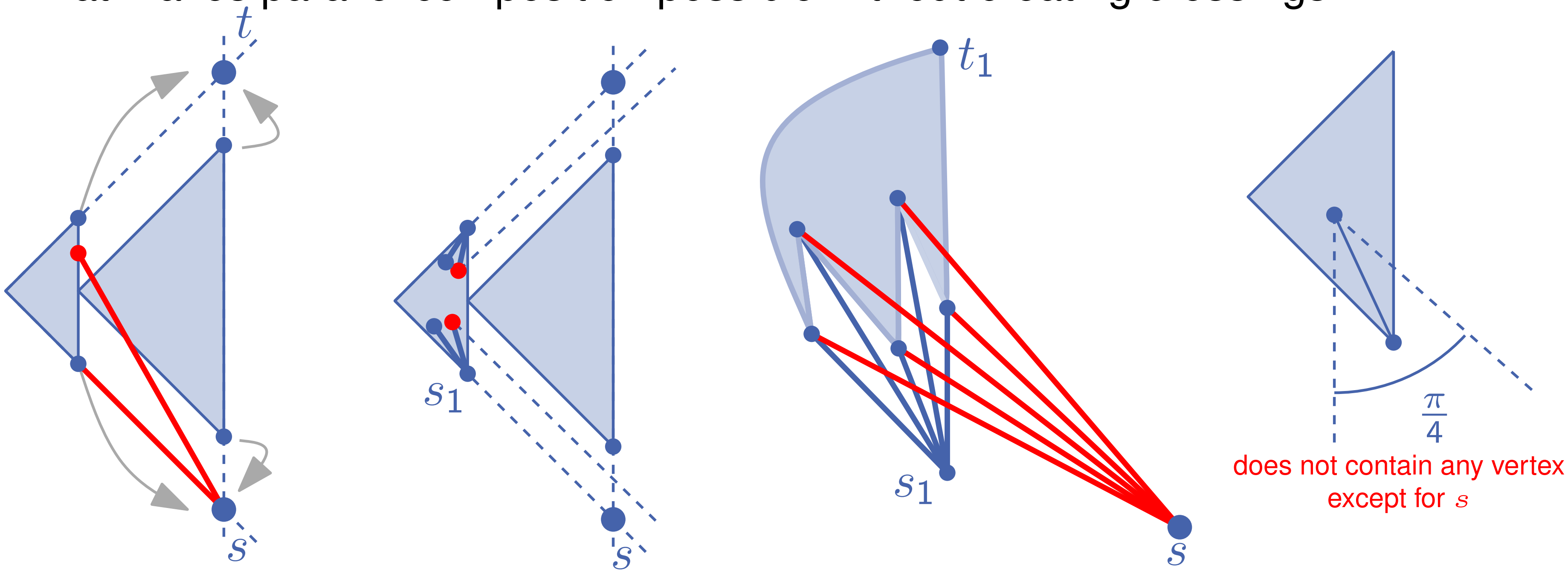

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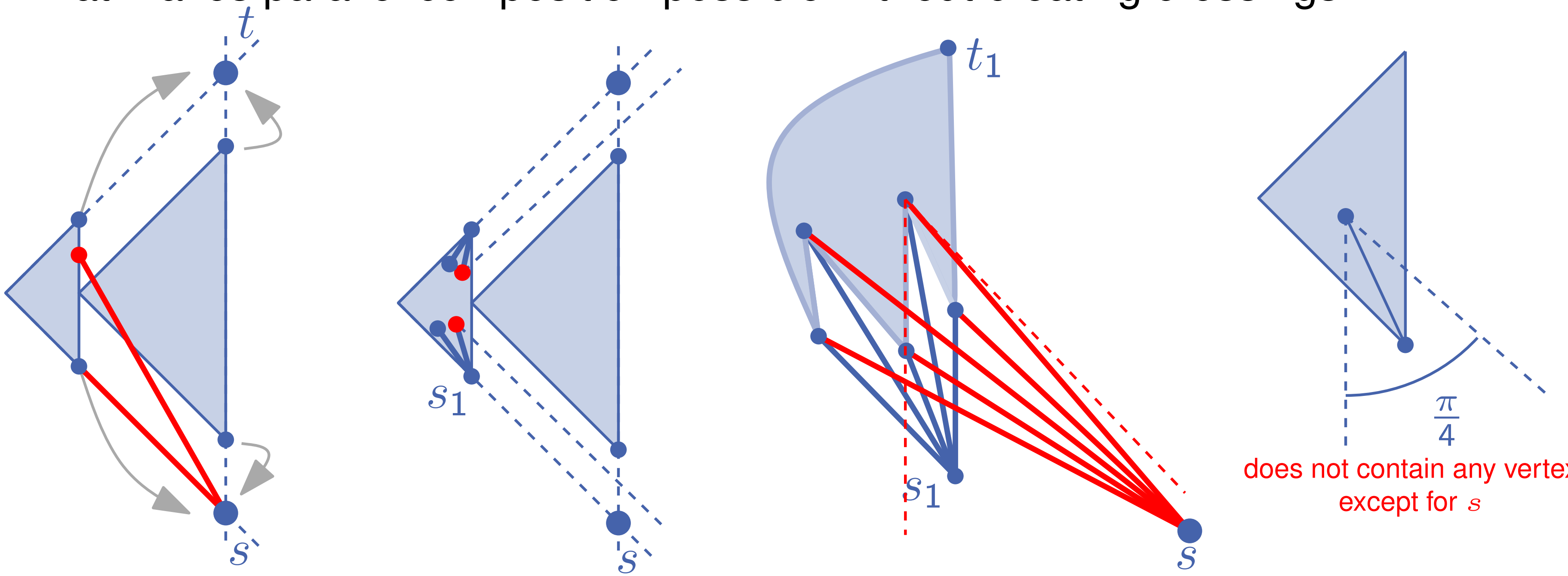

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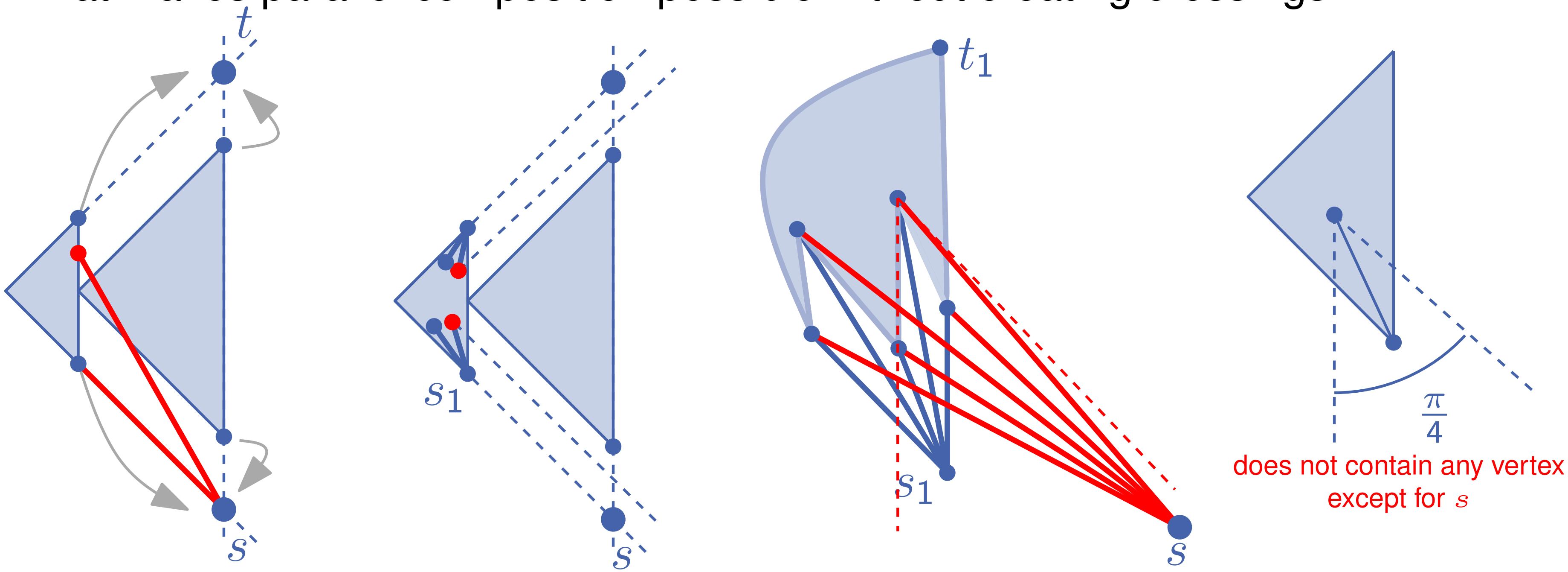


**Lemma** If this condition holds then parallel composition results in a planar drawing.

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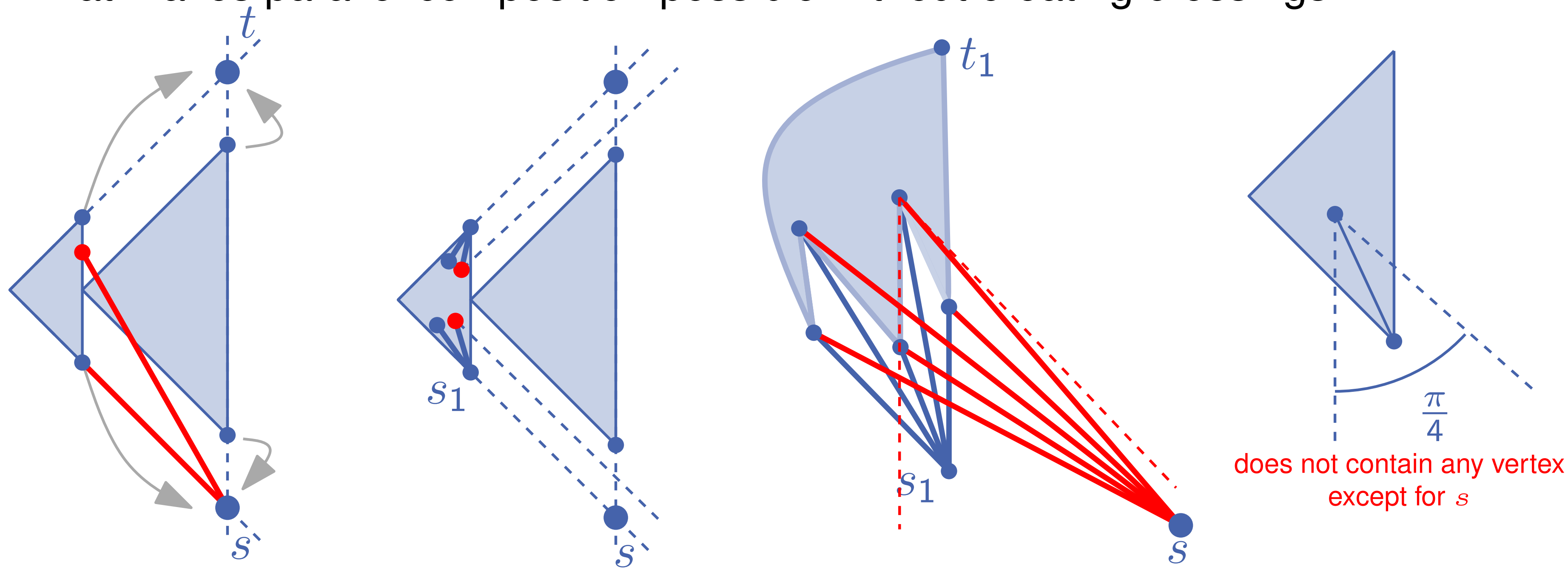


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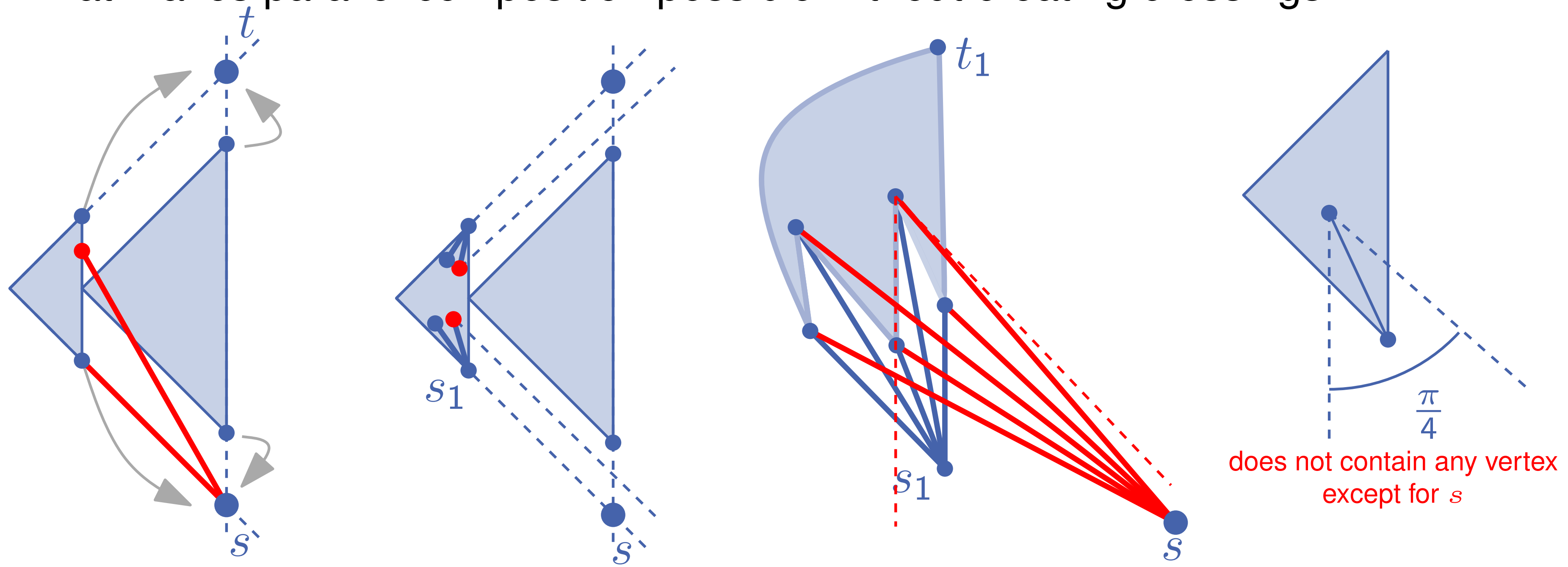


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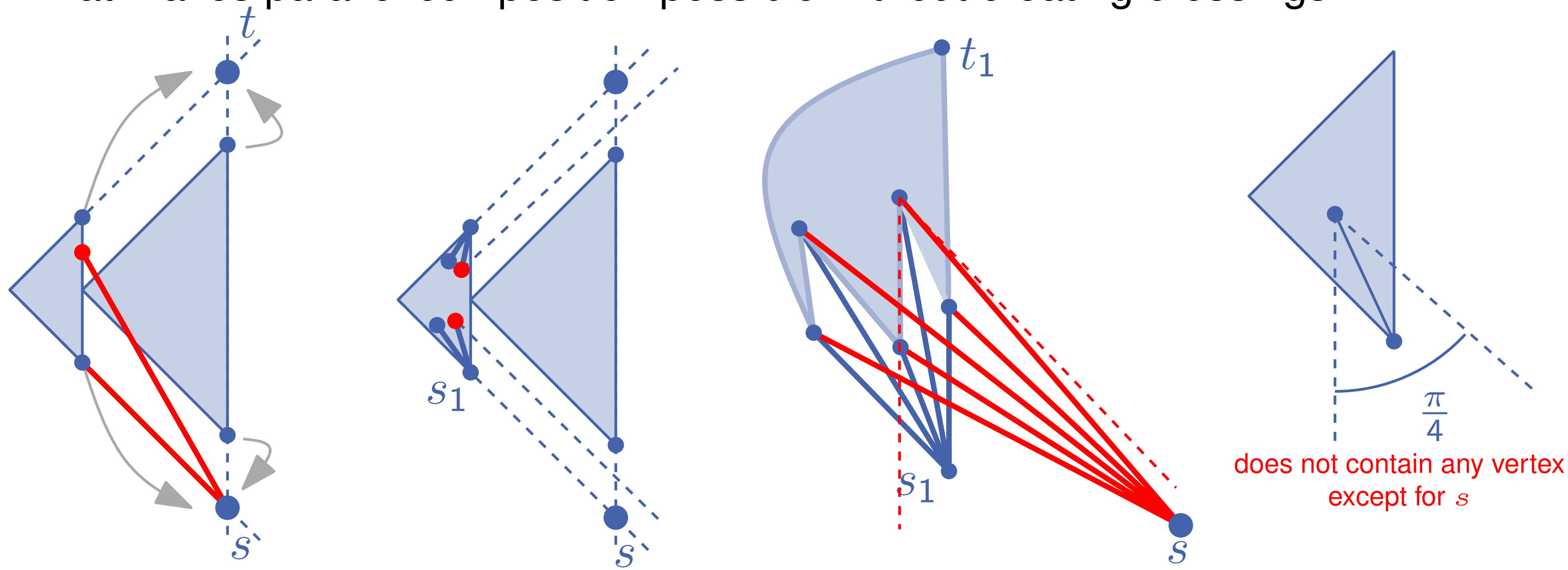


**Lemma** If this condition holds then parallel composition results in a planar drawing.

- The condition can be preserved during the induction step.
- The area of the drawing is? $O(m^2), m$ is the number of edges

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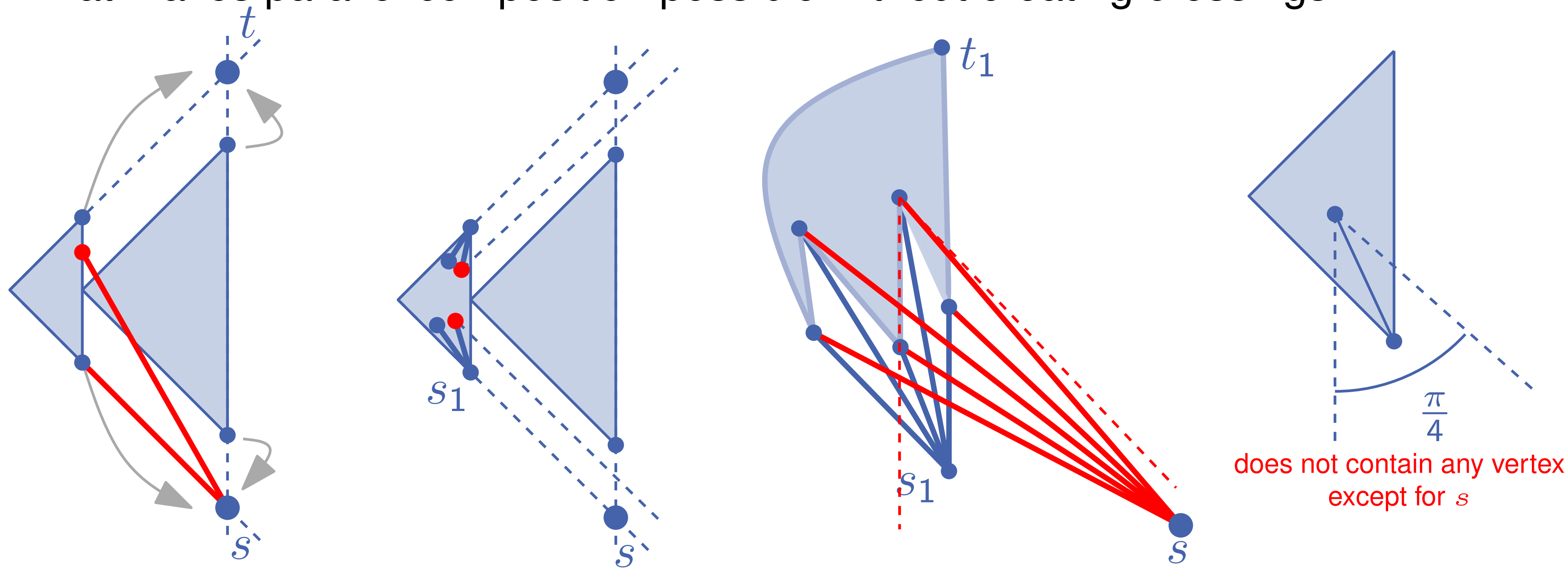


## Theorem

A series-parallel graph $G$ (**with variable embedding**) admits an **upward planar** straigh-line drawing with $O(n^2)$ area.

# Lower Bound for the Area

**Theorem [Bertolazzi et al. 94]**

There exists a $2n$-vertex series-parallel graph $G_n$ such that any upward planar drawing of $G_n$ respecting embedding requires area $\Omega(4^n)$.

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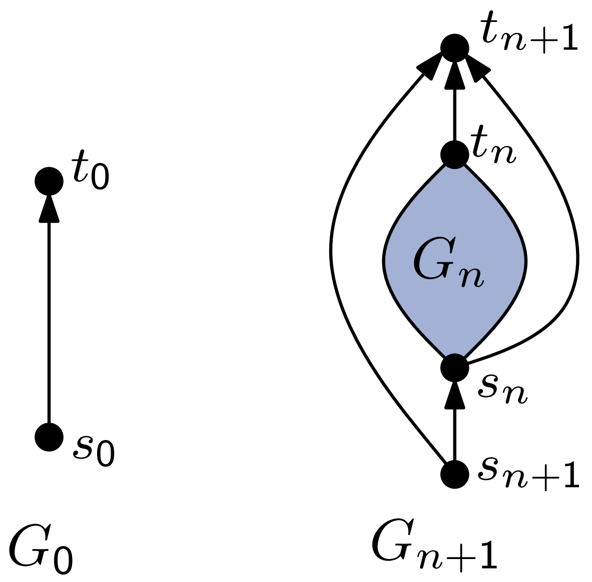

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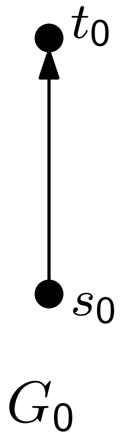


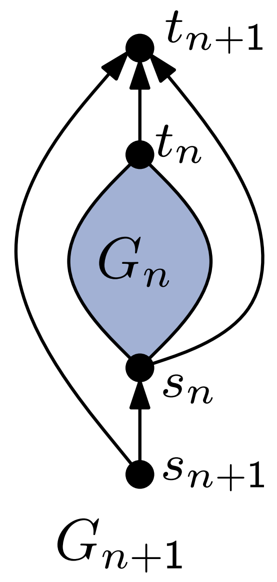


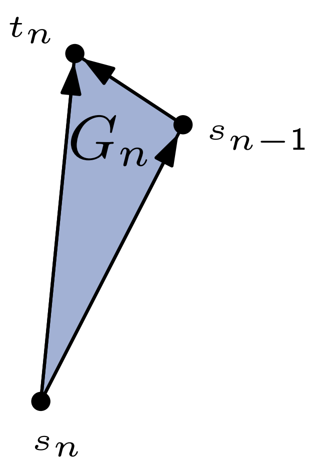

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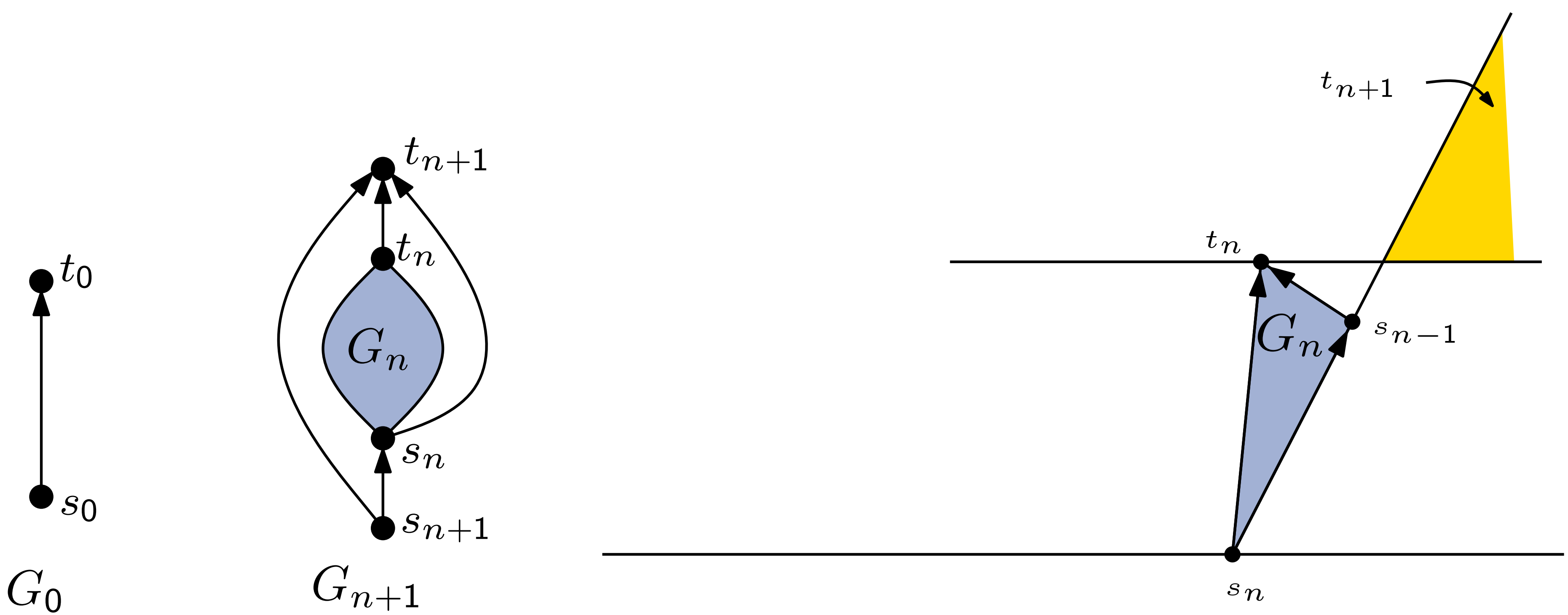

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Karlsruhe Institute of Technology

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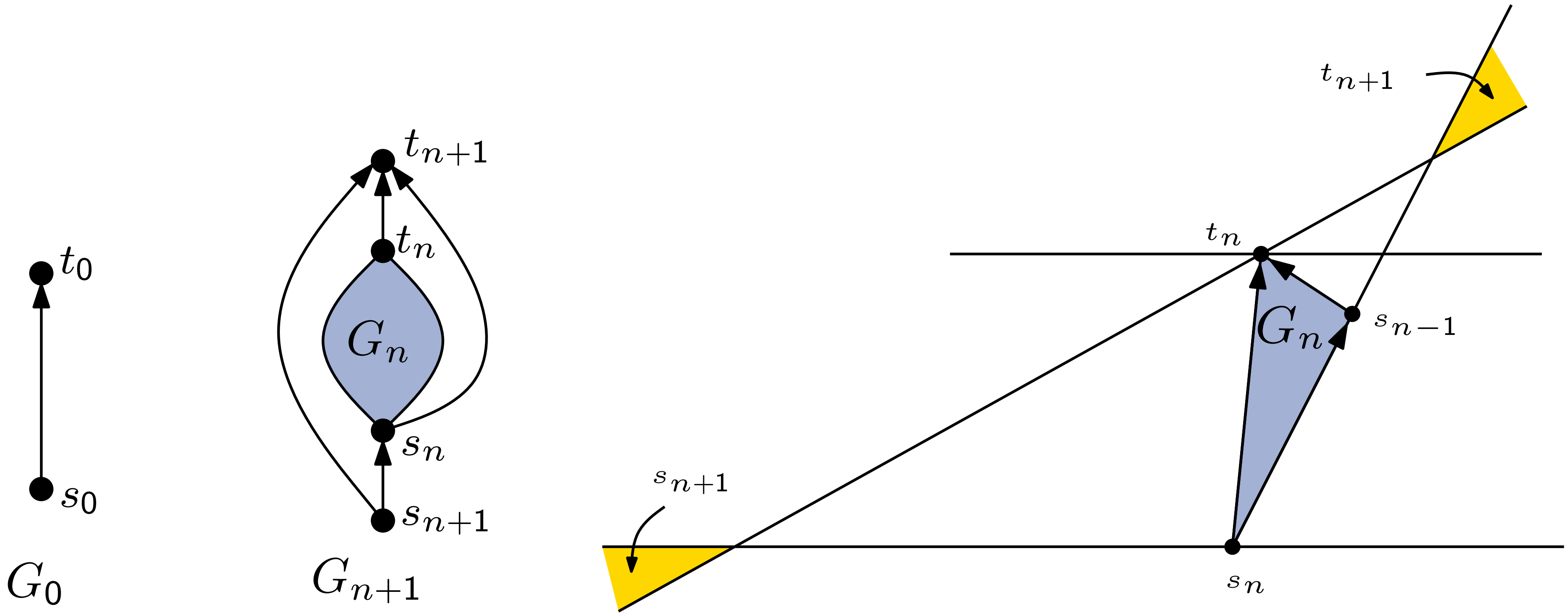




Institut für Theoretische Informatik
Lehrstuhl Algorithmik I

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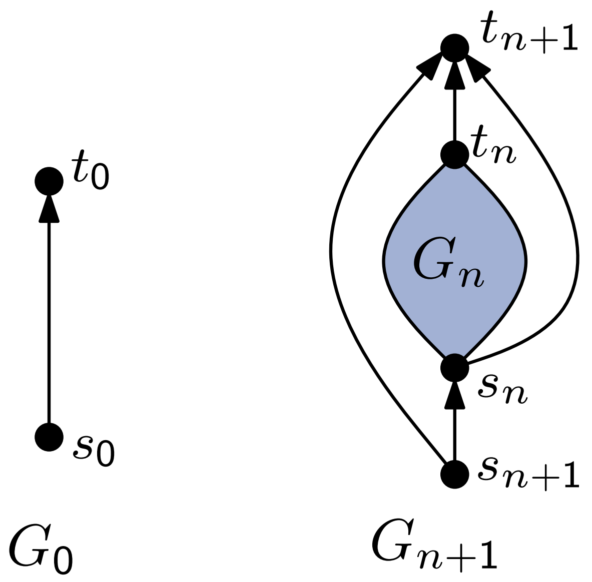


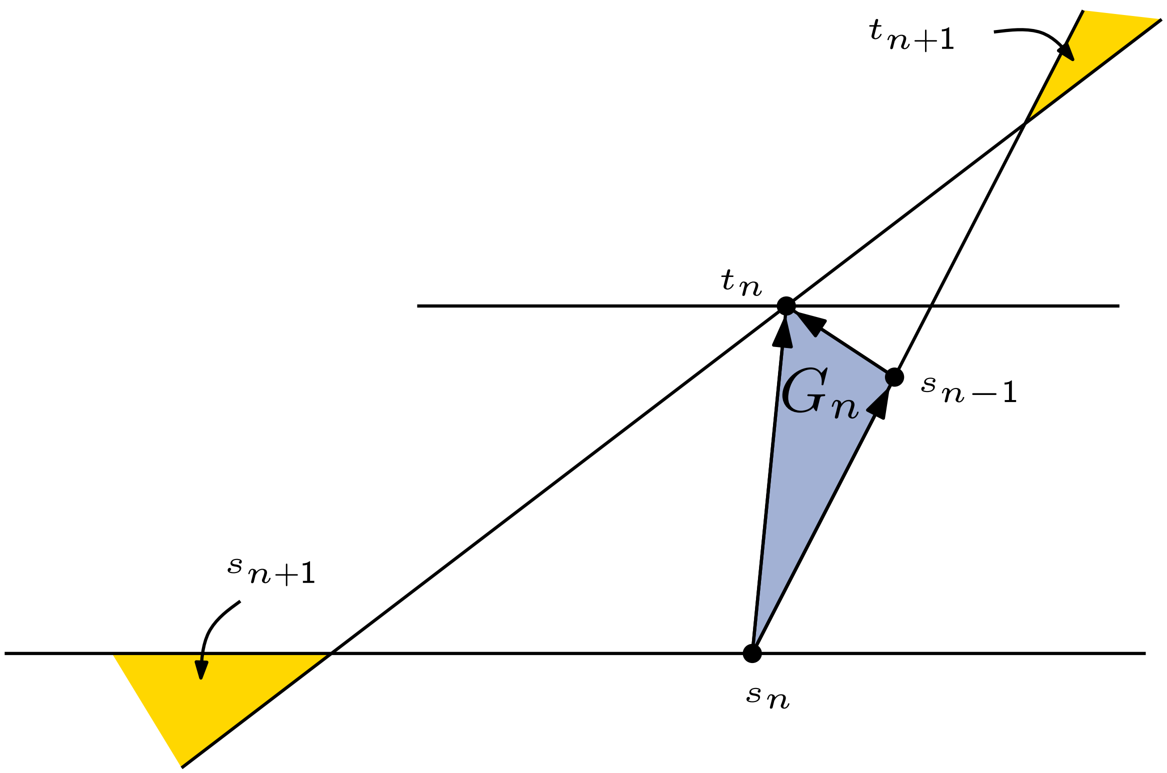

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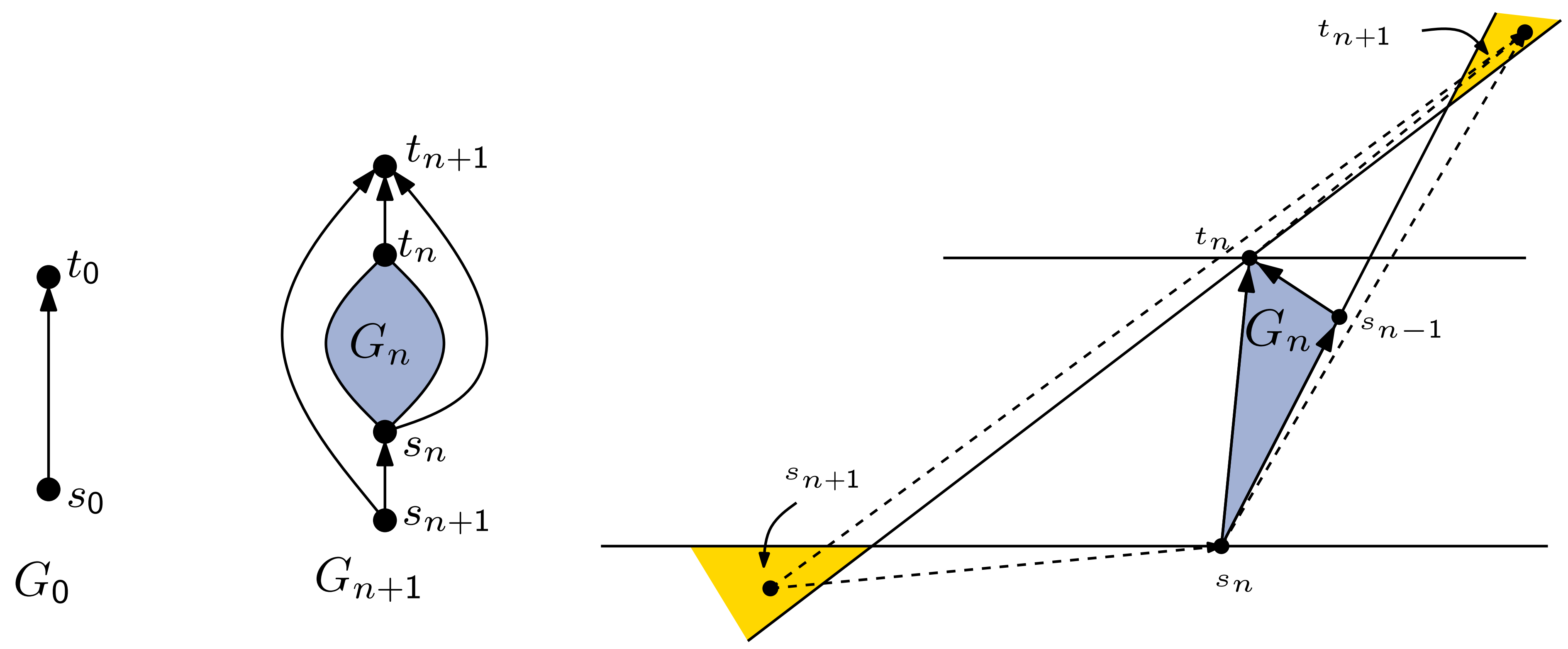

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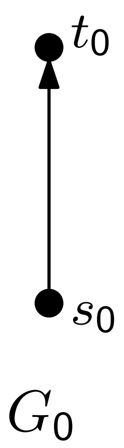


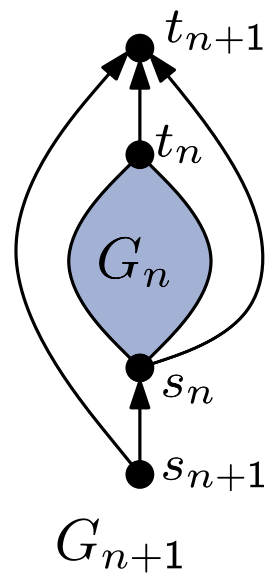


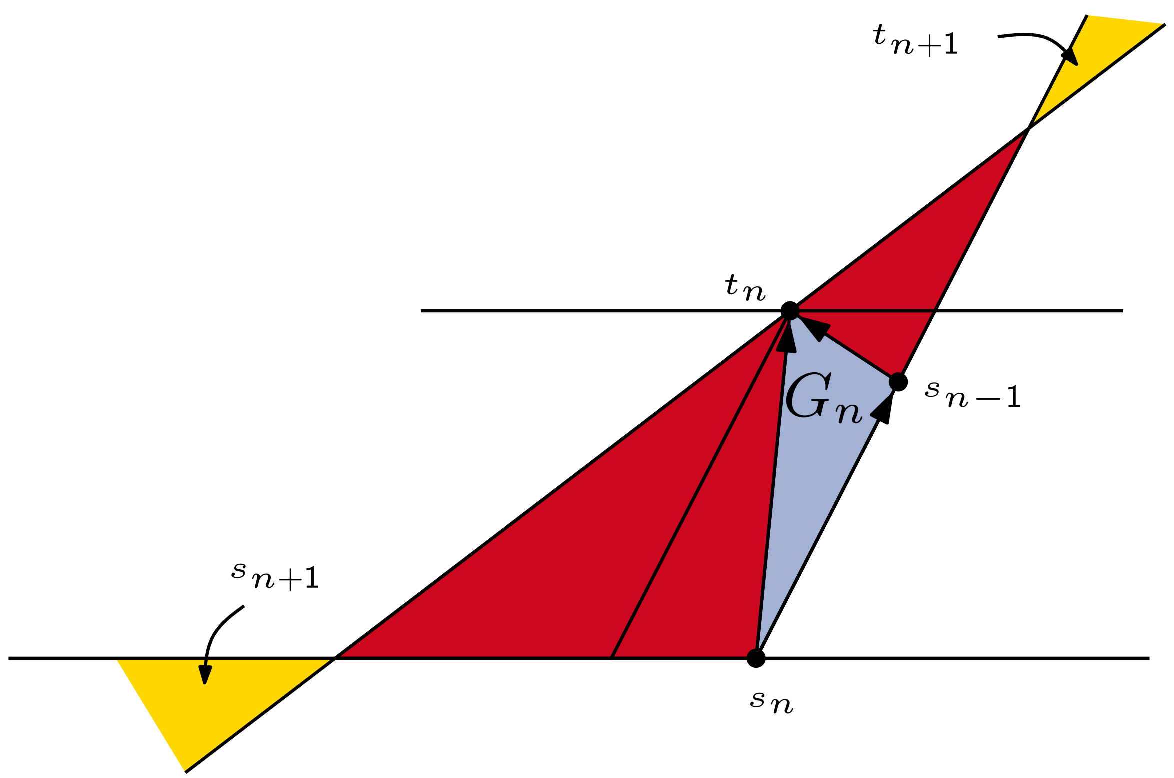

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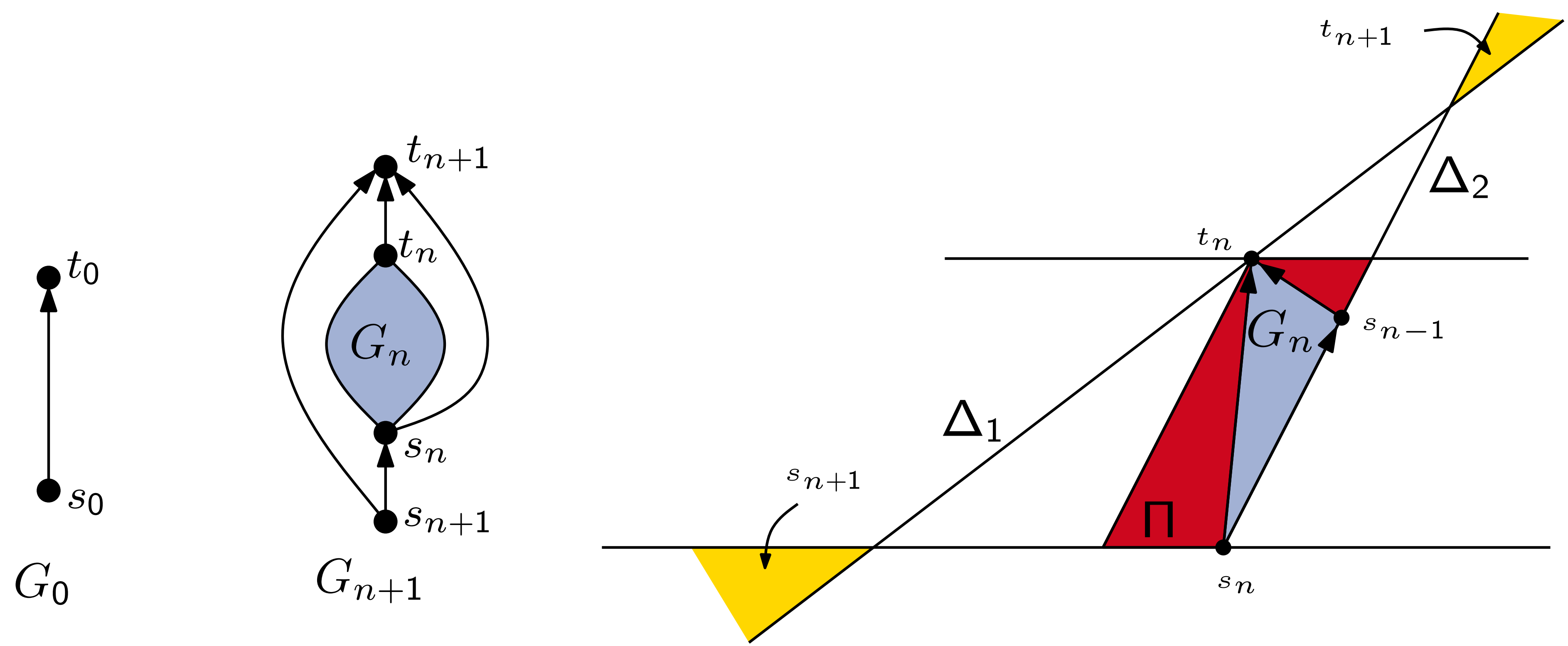

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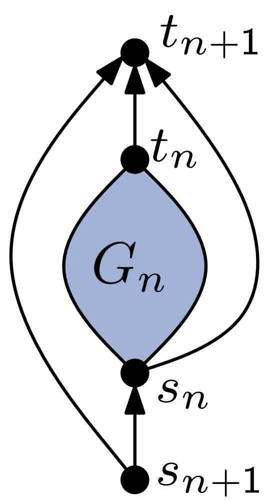
- We have that: $Area(\Pi) > 2 \cdot Area(G_n)$


$G_0$

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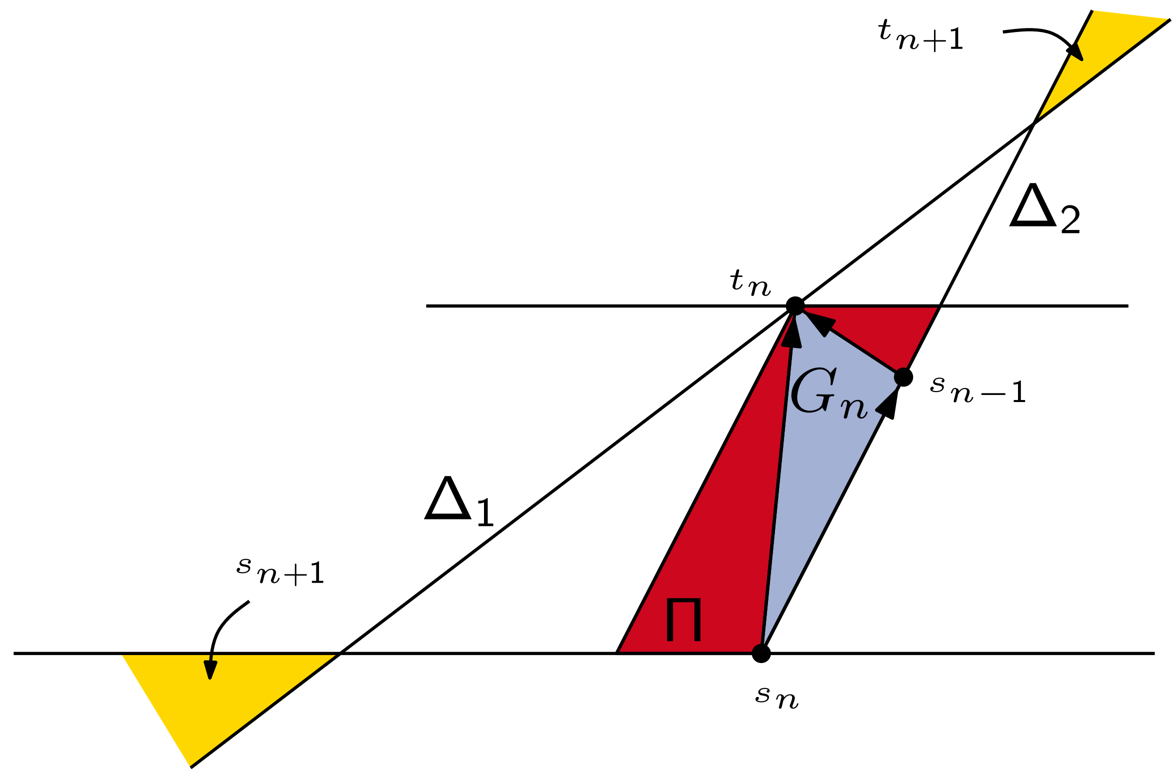

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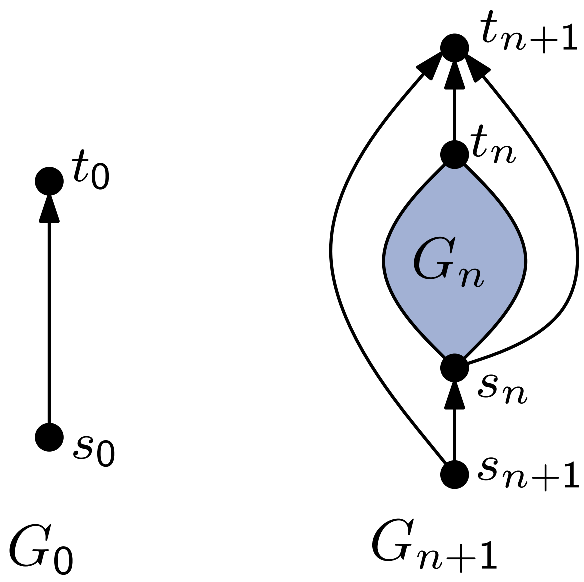


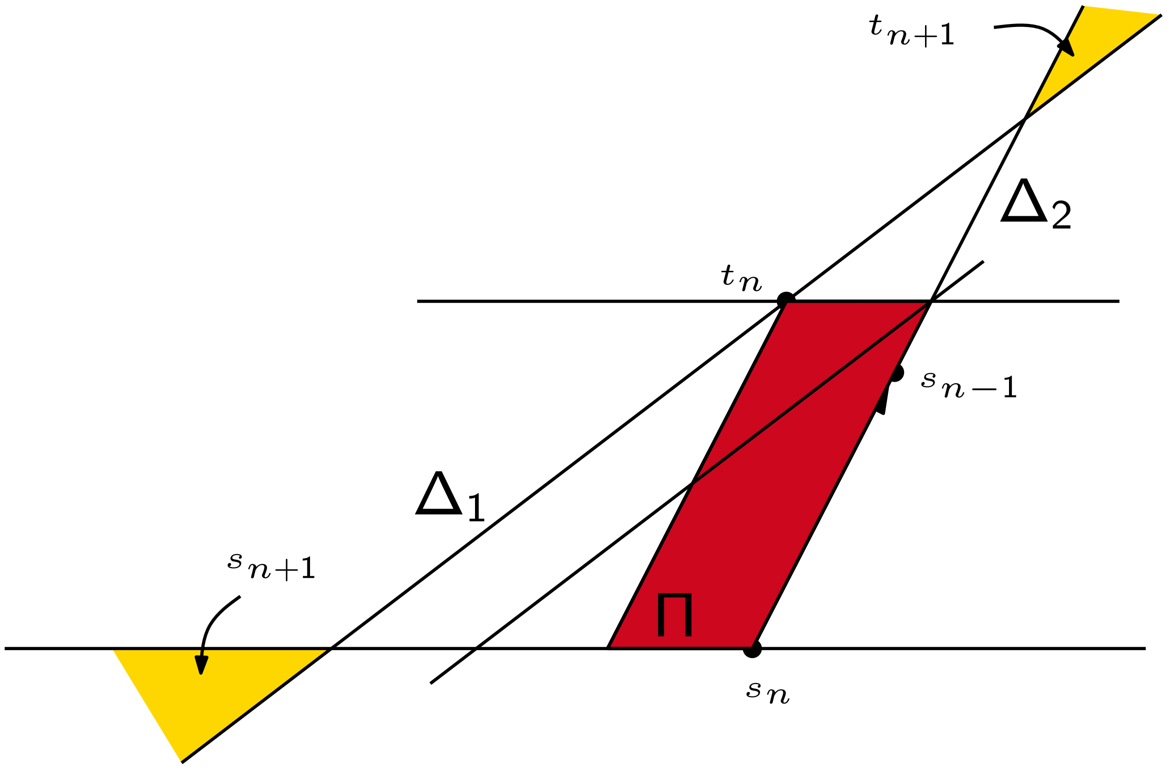

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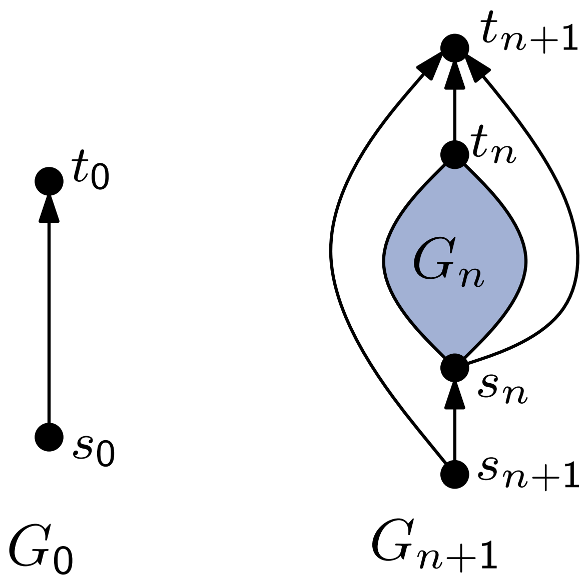


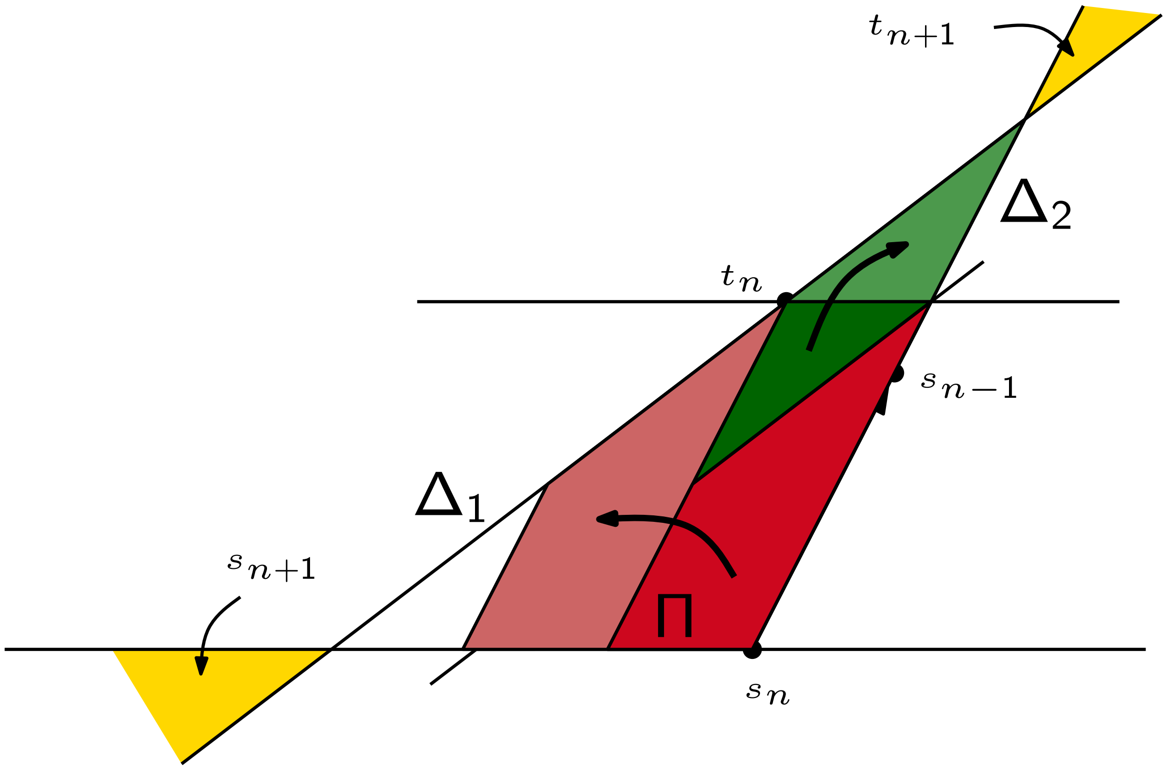

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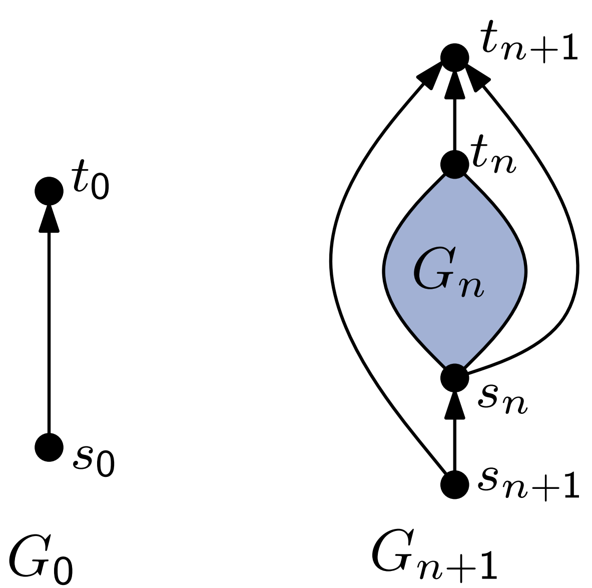


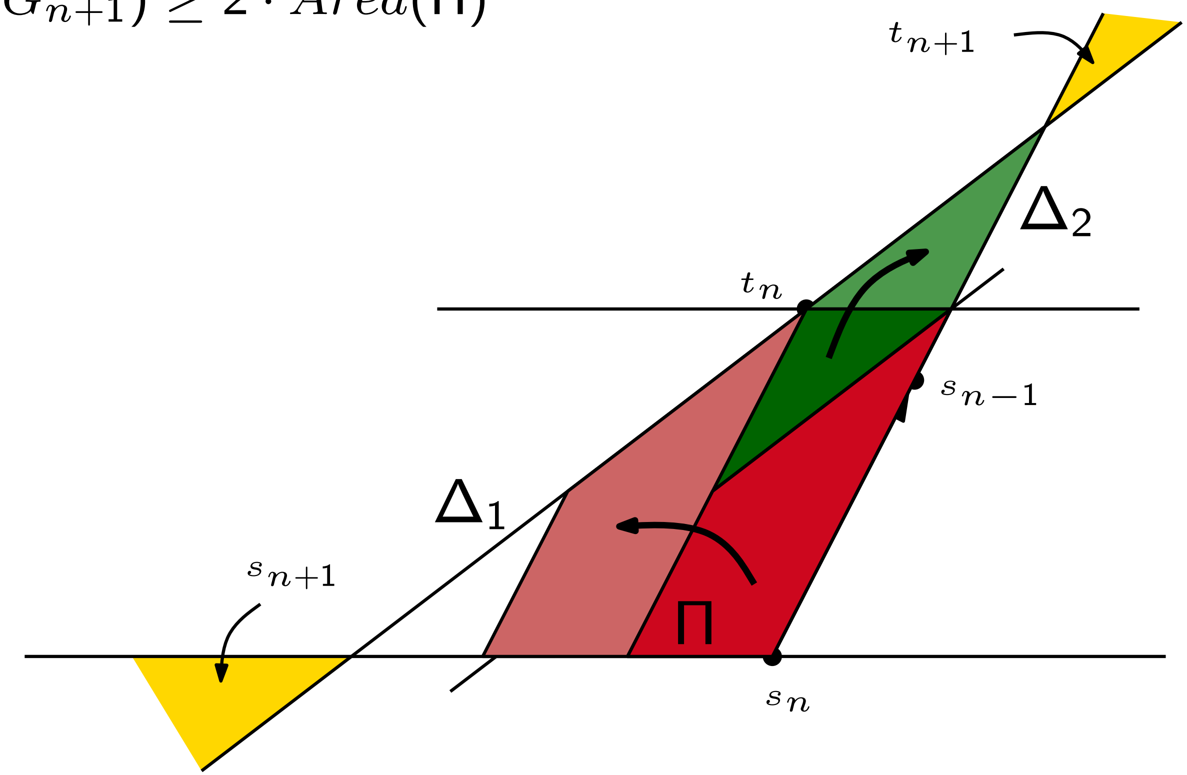

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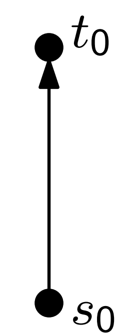


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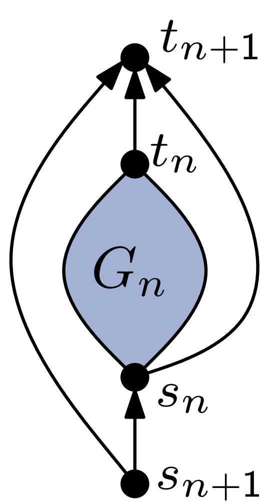


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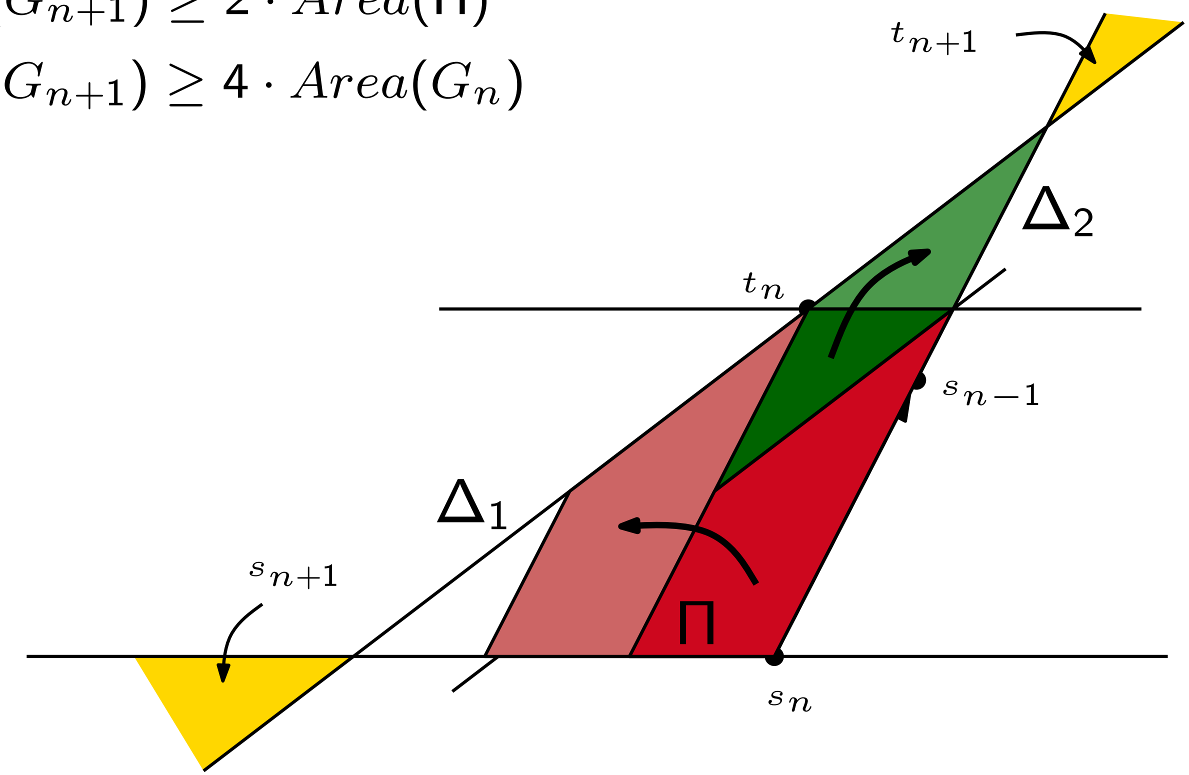
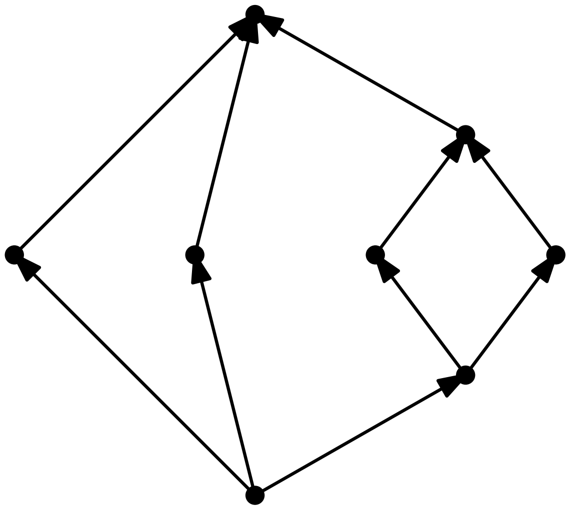
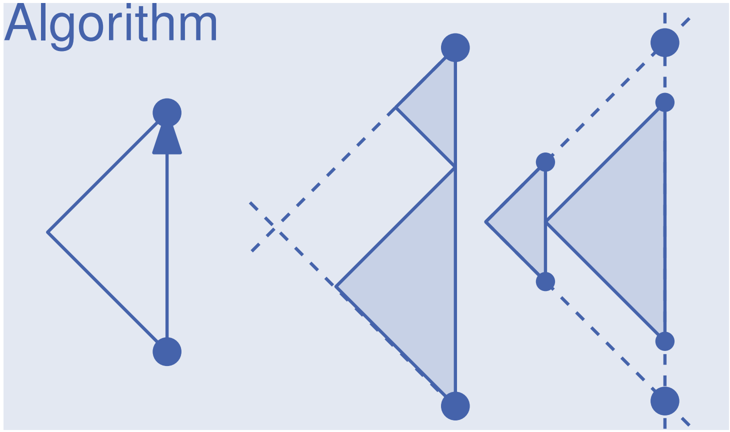
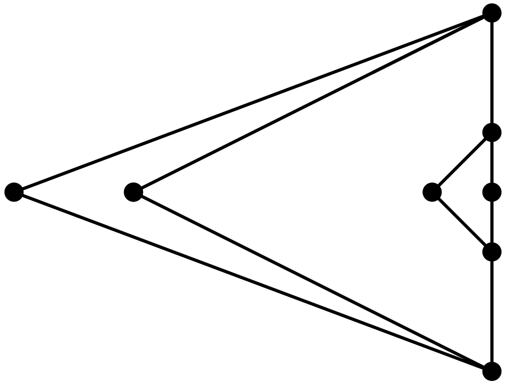


# Property of the Algorithm





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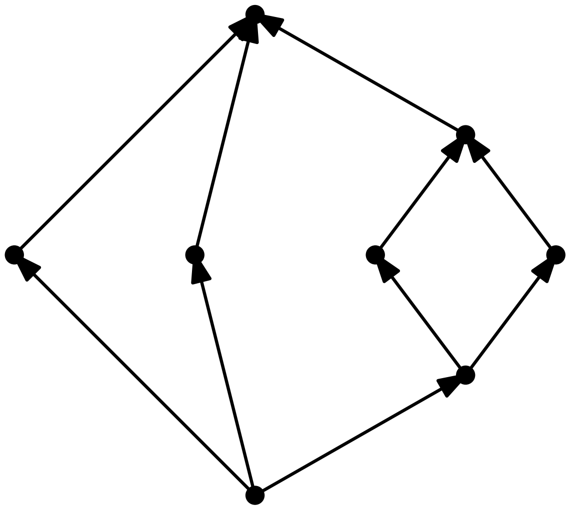

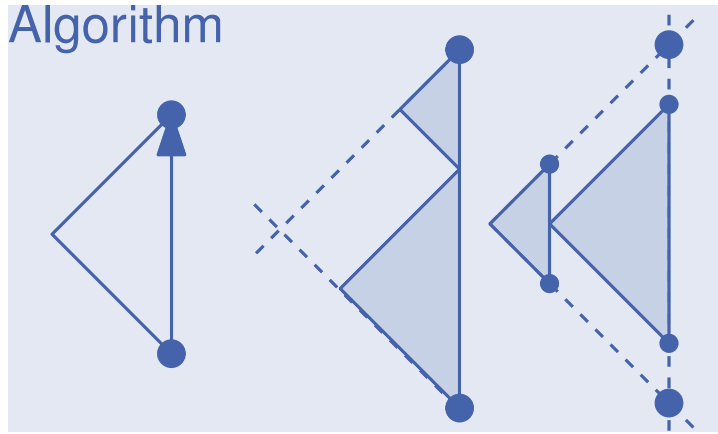


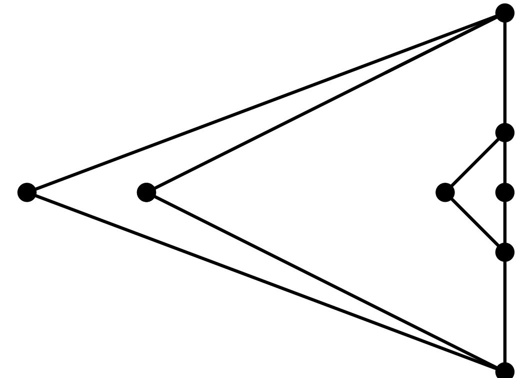

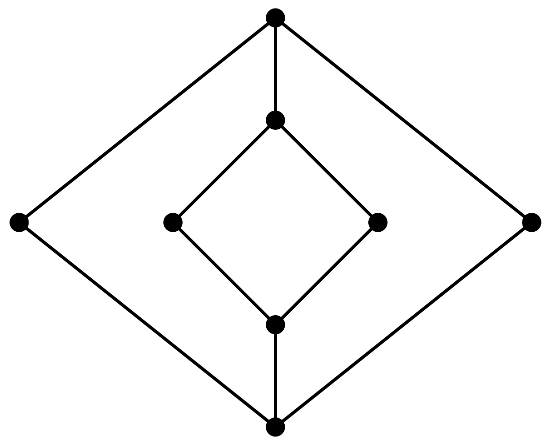

**nicer???**

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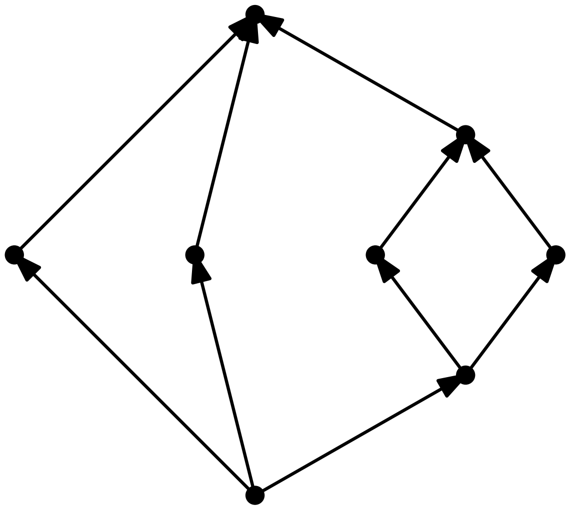

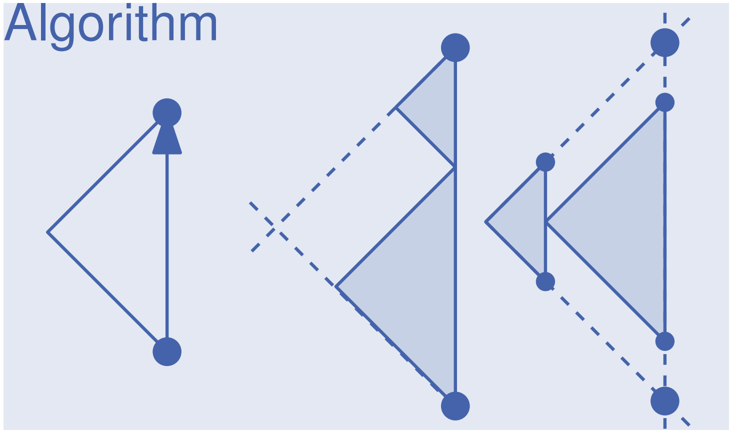


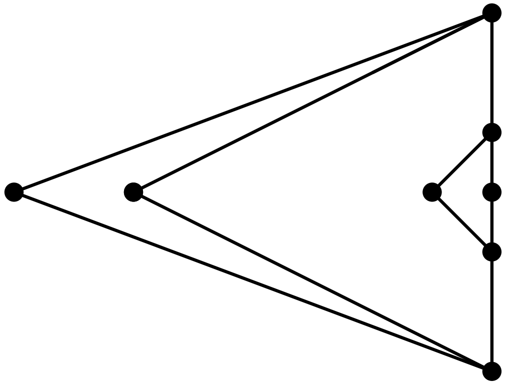

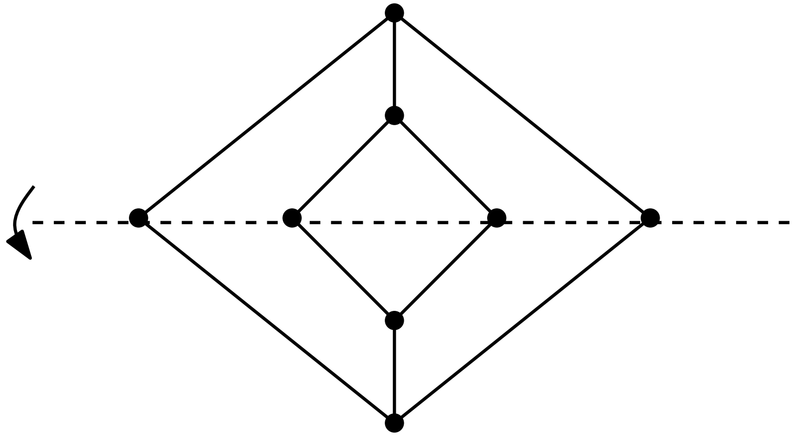

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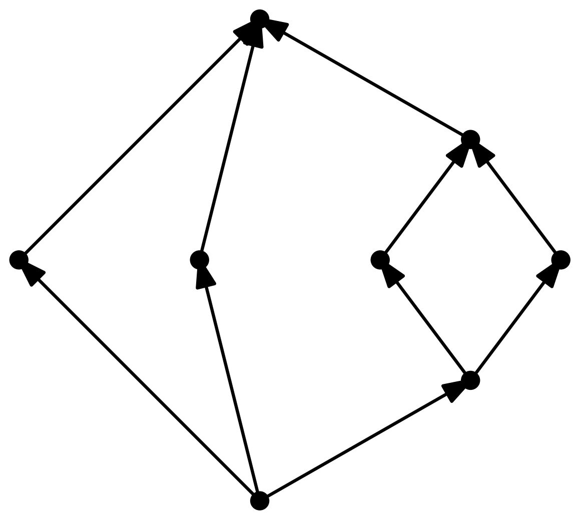

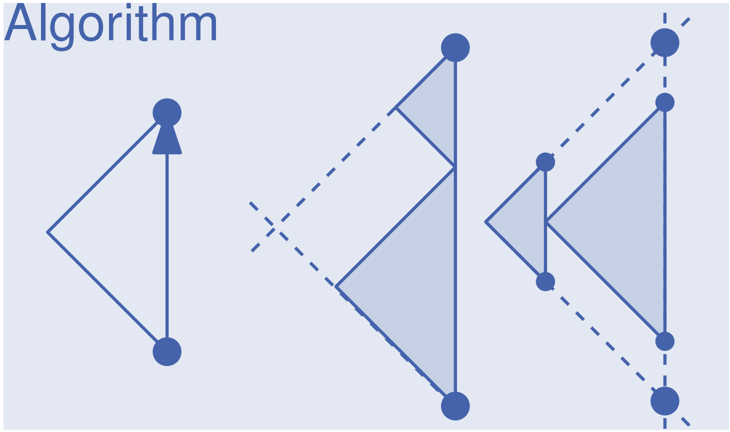


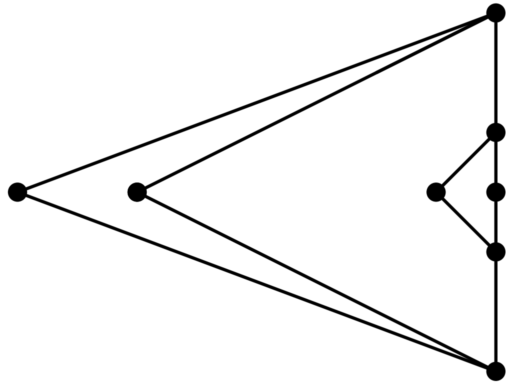

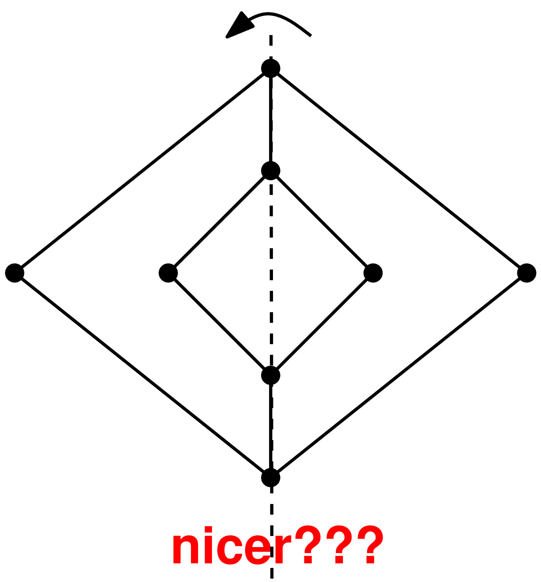

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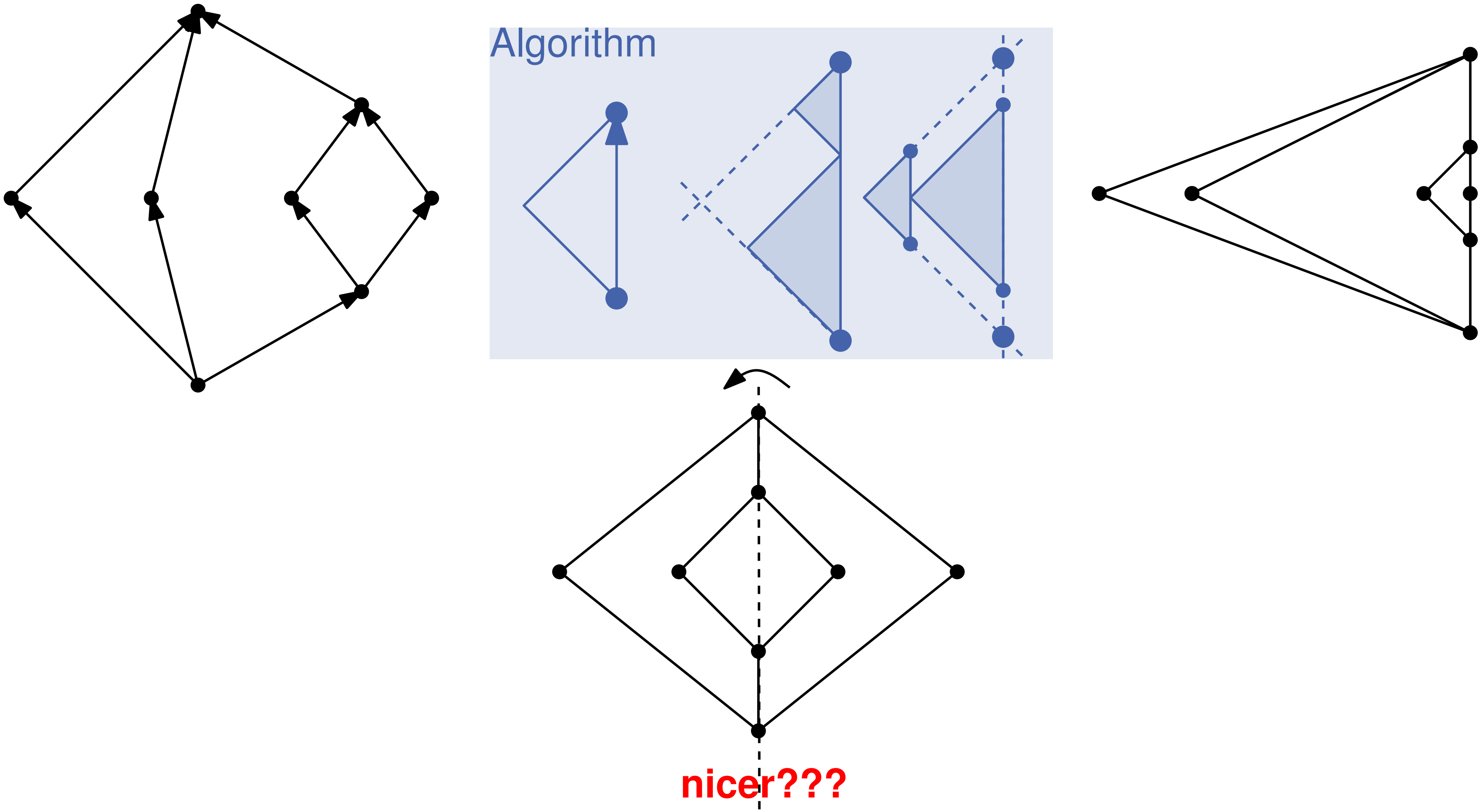


- Algorithm by Hong, Eades and Lee (2000) creates symmetrical drawings of series-parallel graphs.