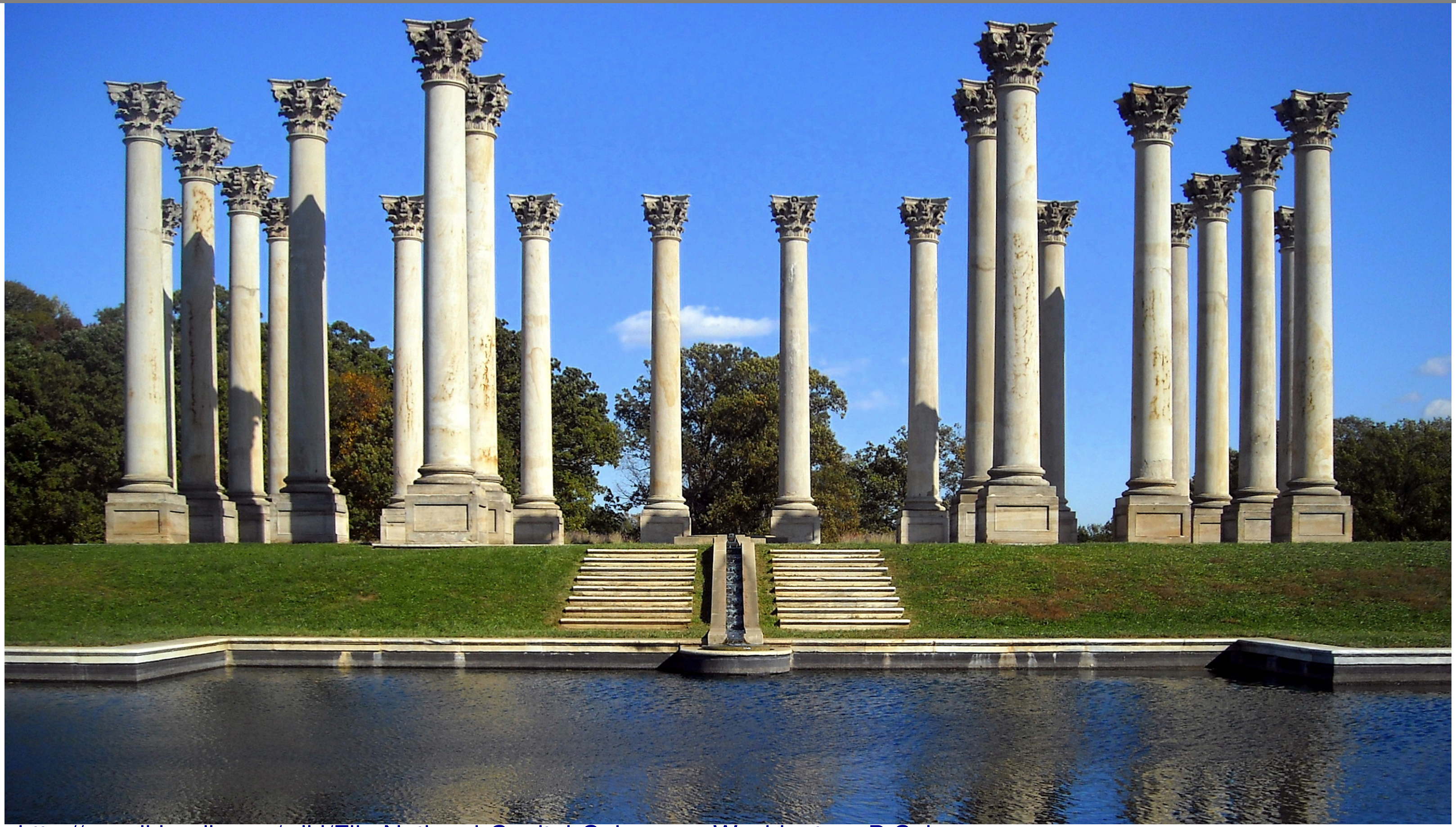
Column-based Graph Layouts

Gregor Betz, *Andreas Gemsa*, Christof Mathies, Ignaz Rutter, Dorothea Wagner

Karlsruhe Institute of Technology (KIT),



http://en.wikipedia.org/wiki/File:National_Capitol_Columns_-_Washington,_D.C..jpg

Motivation

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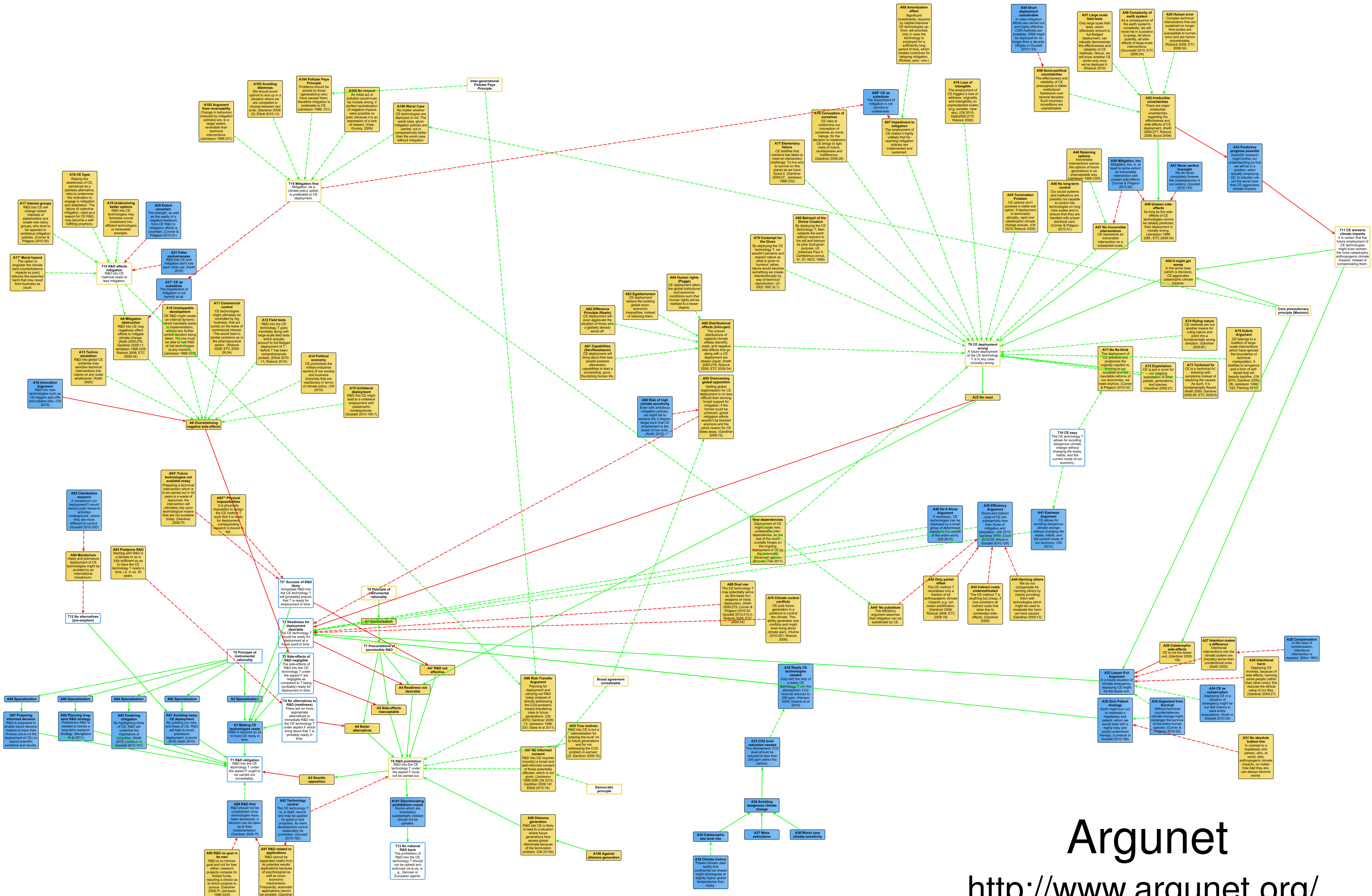


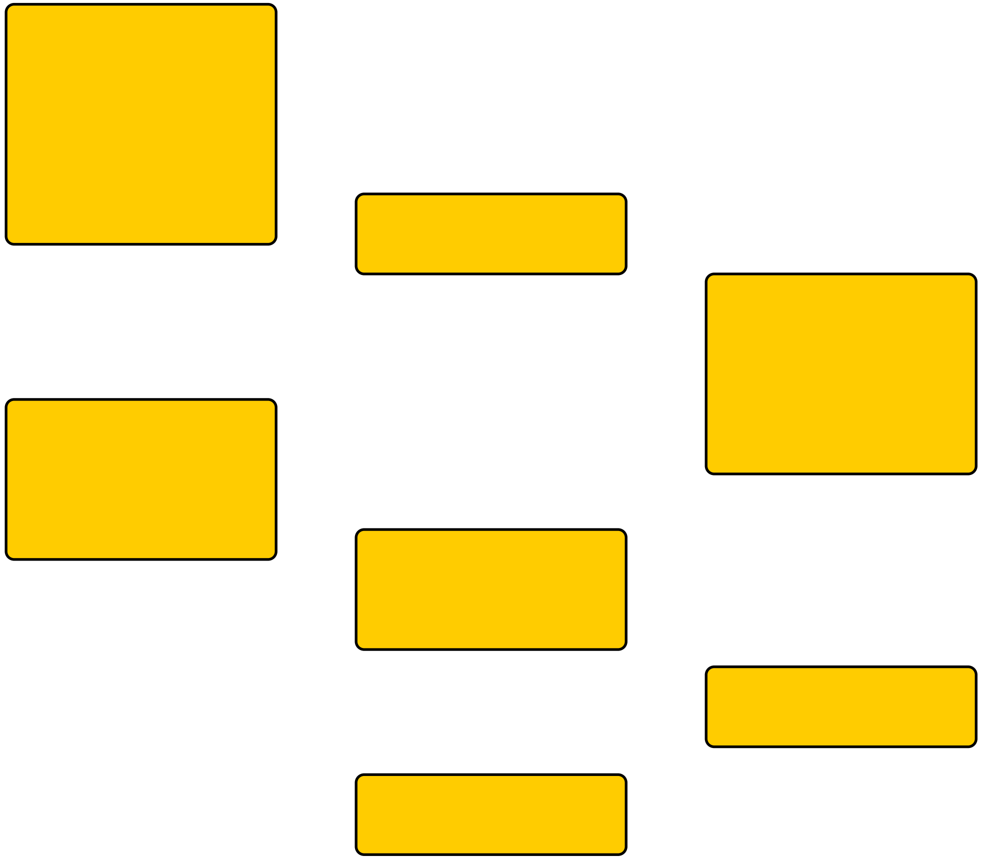
Argunet

http://www.argunet.org/

Introduction

Drawing Style

- boxes of uniform width
- upward drawings
- ingoing edges at the top & outgoing edges at the bottom
- orthogonal edges
- directed acyclic graphs



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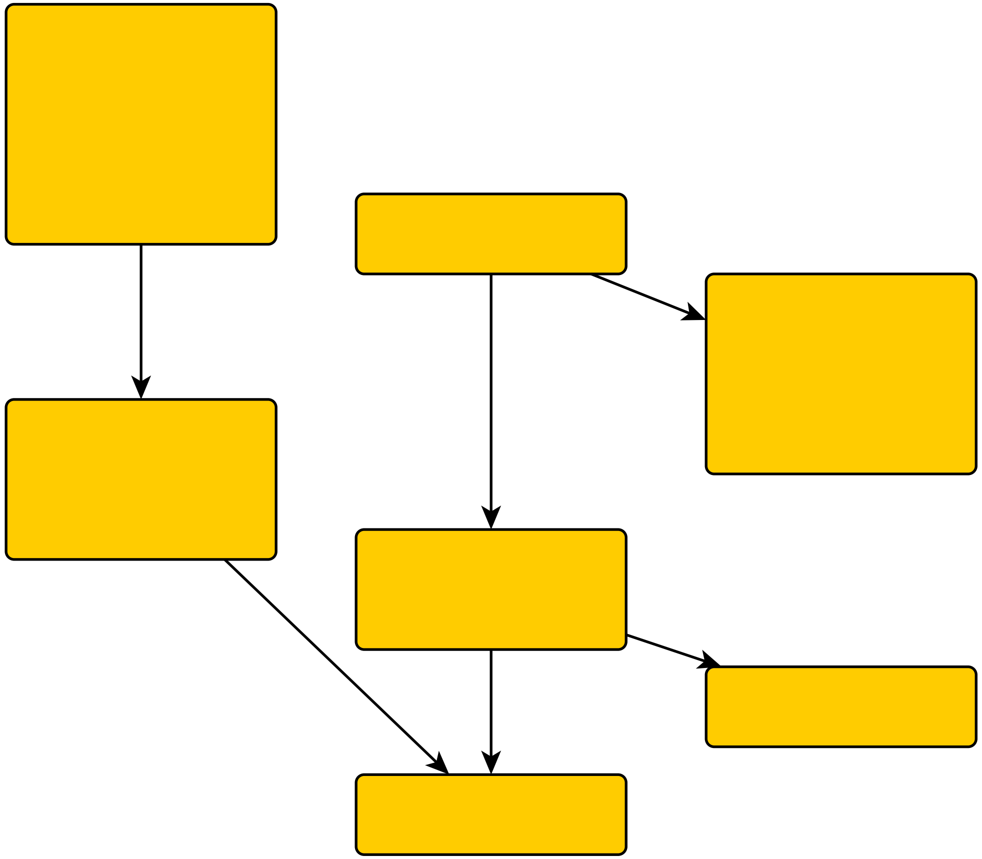

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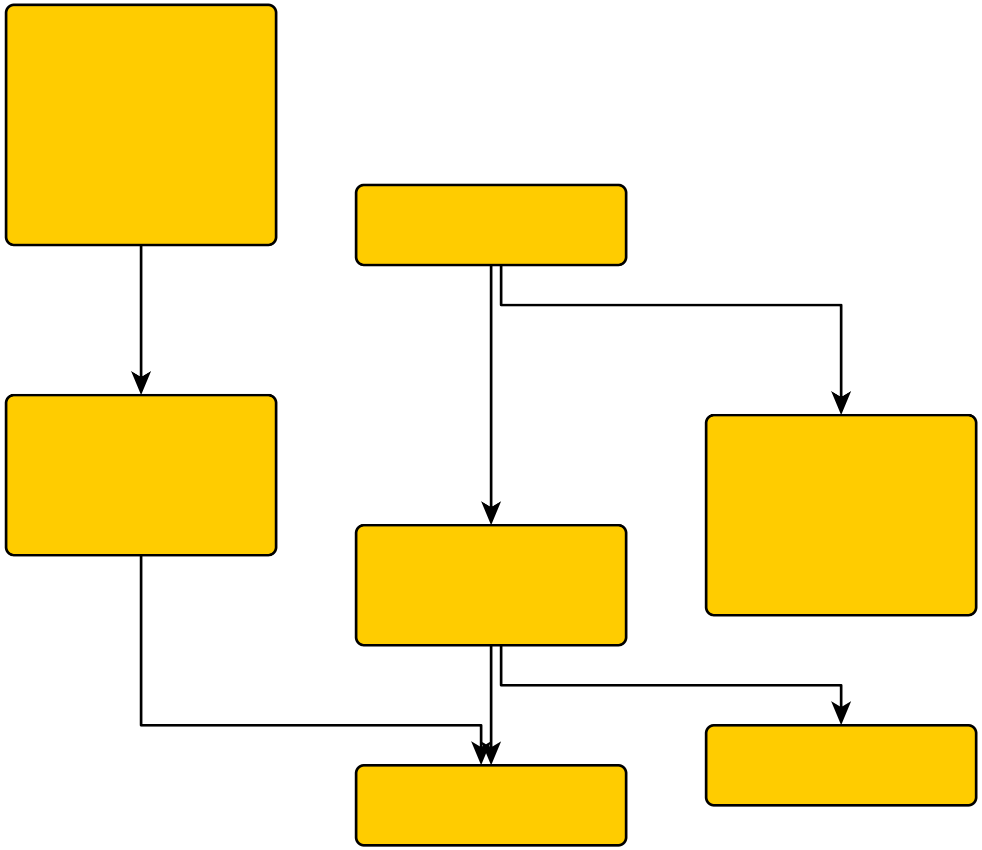

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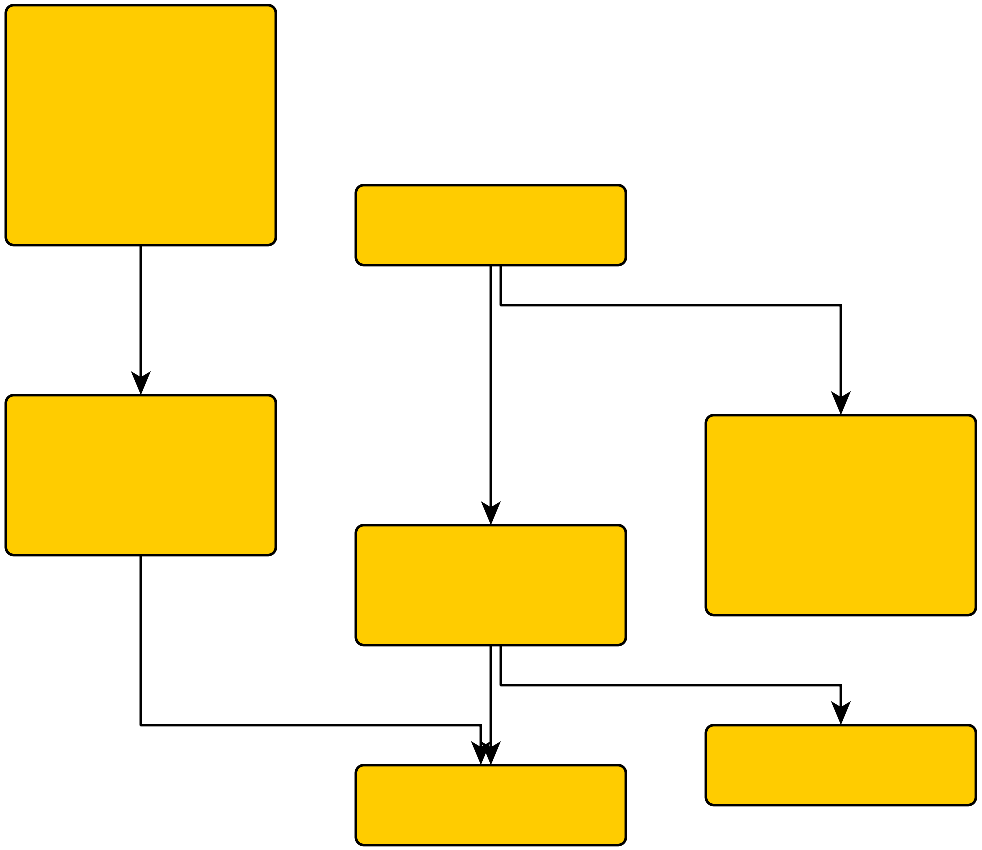

Sugiyama Framework ?

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Sugiyama Framework

three steps:

- layer assignment
- determining positions of nodes in each layer
- positioning nodes and edges

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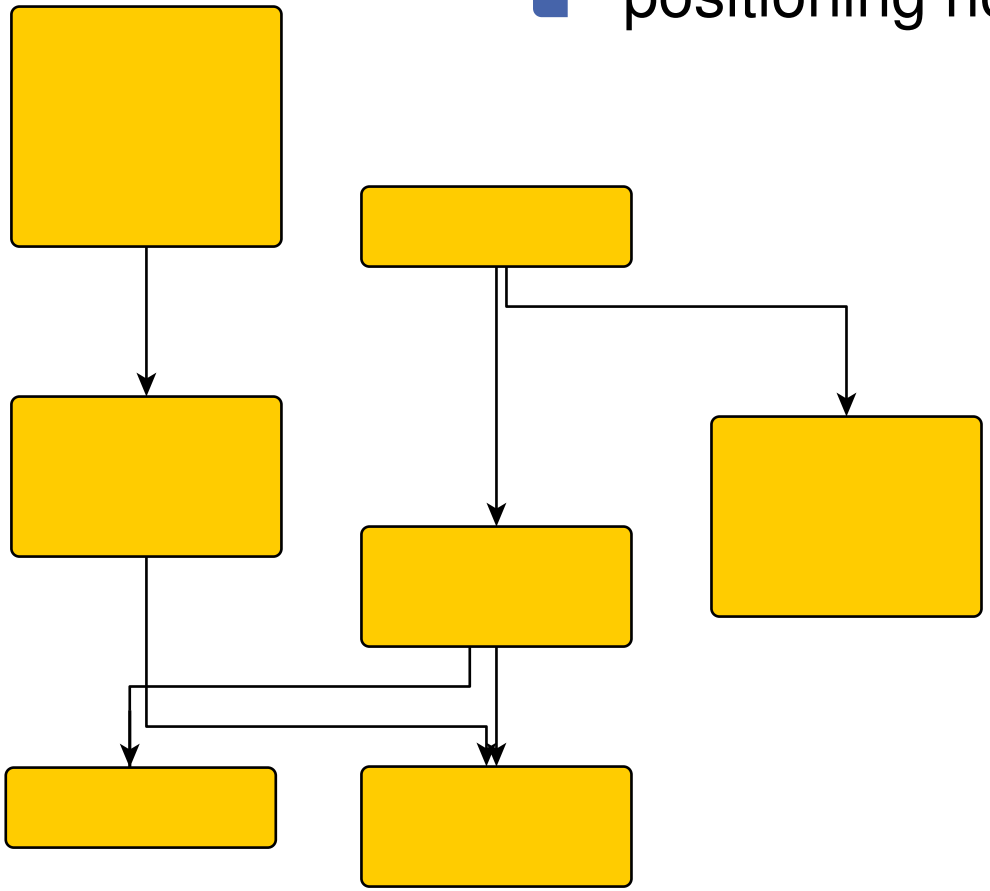

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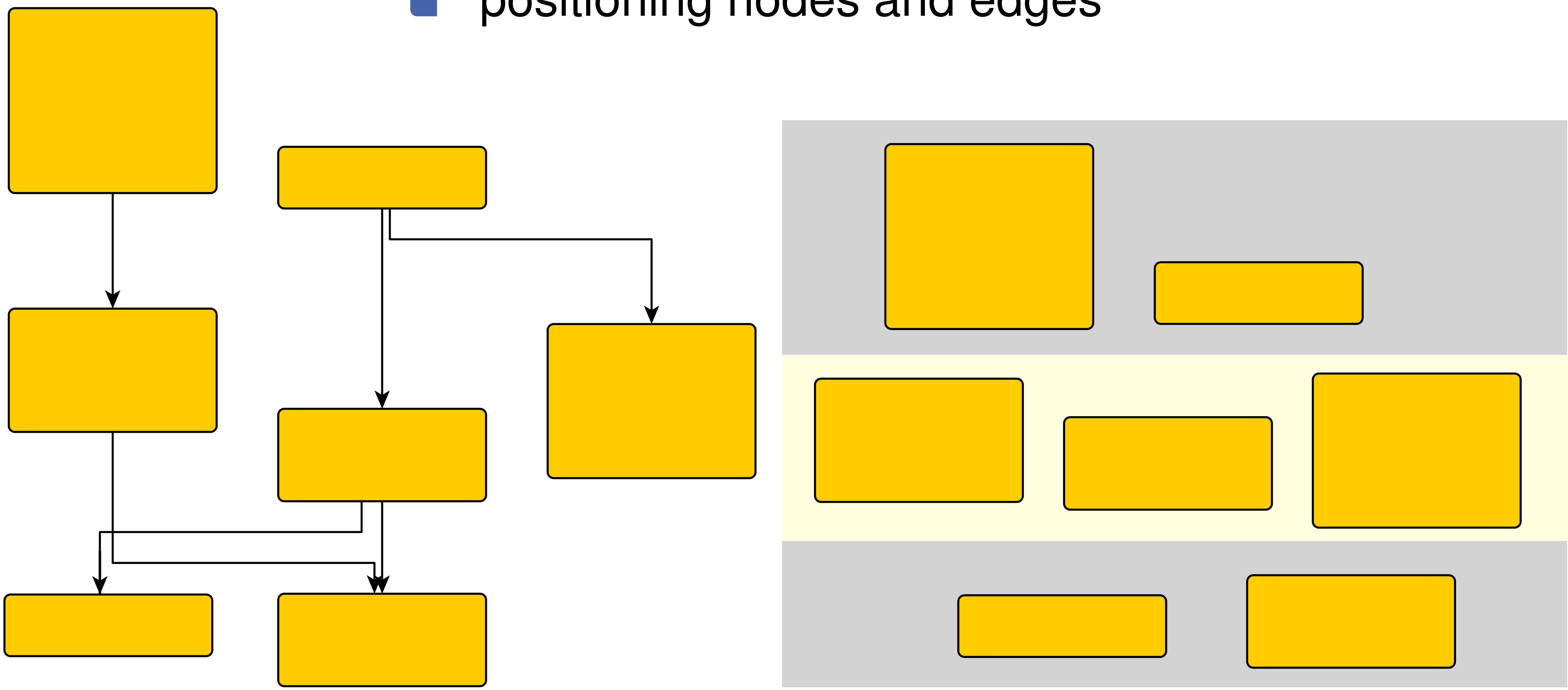

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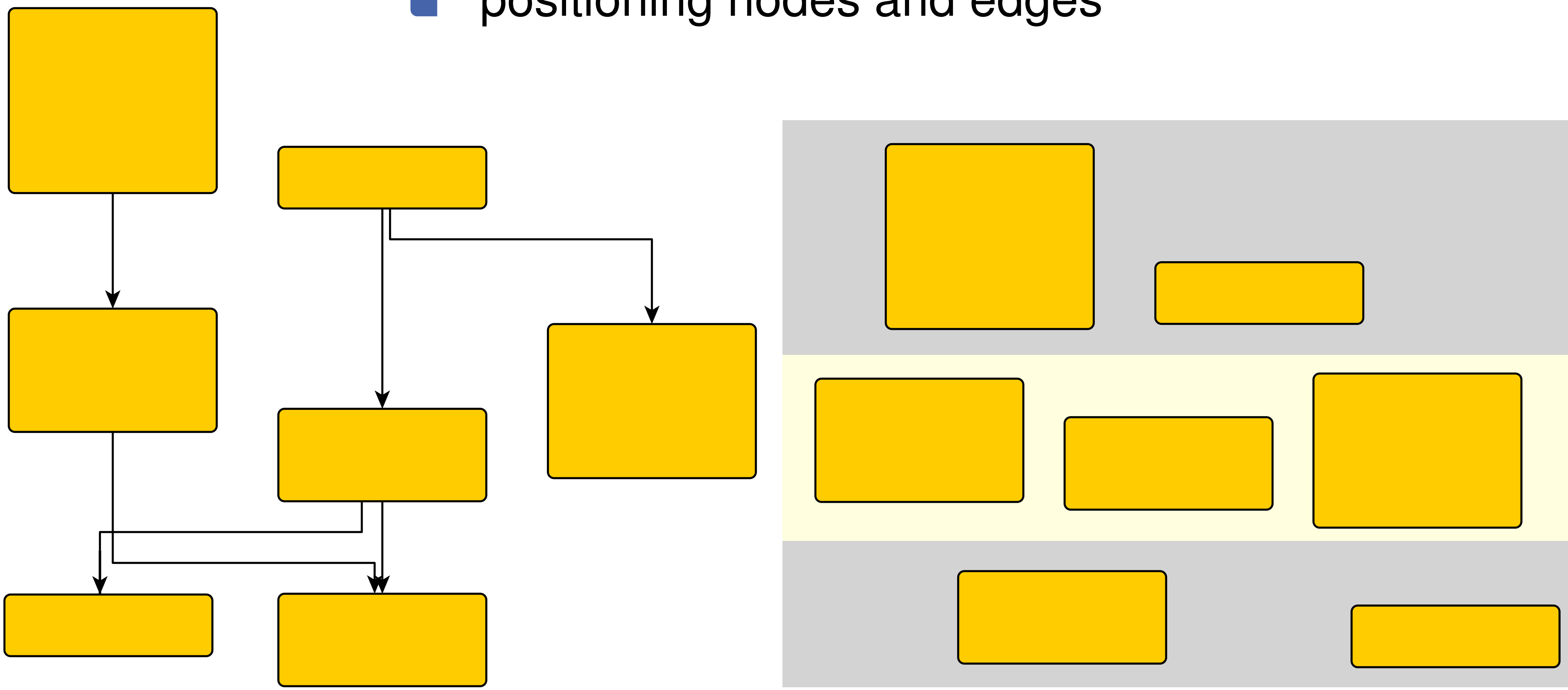

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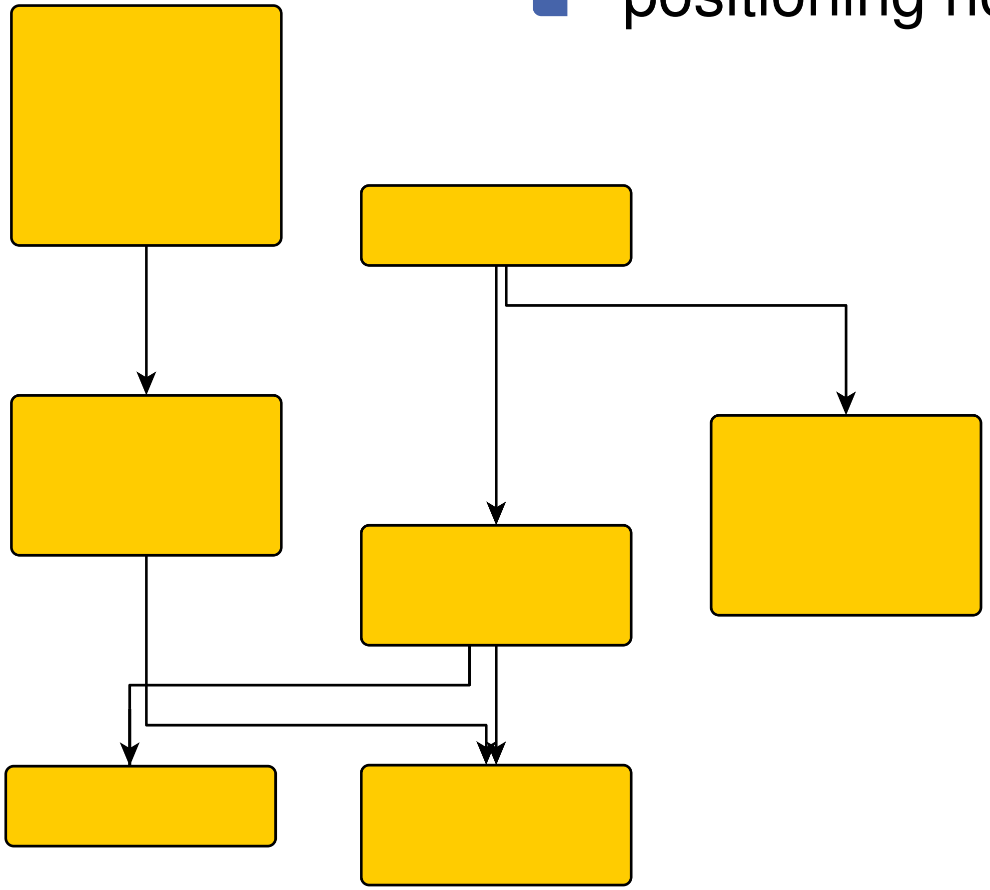

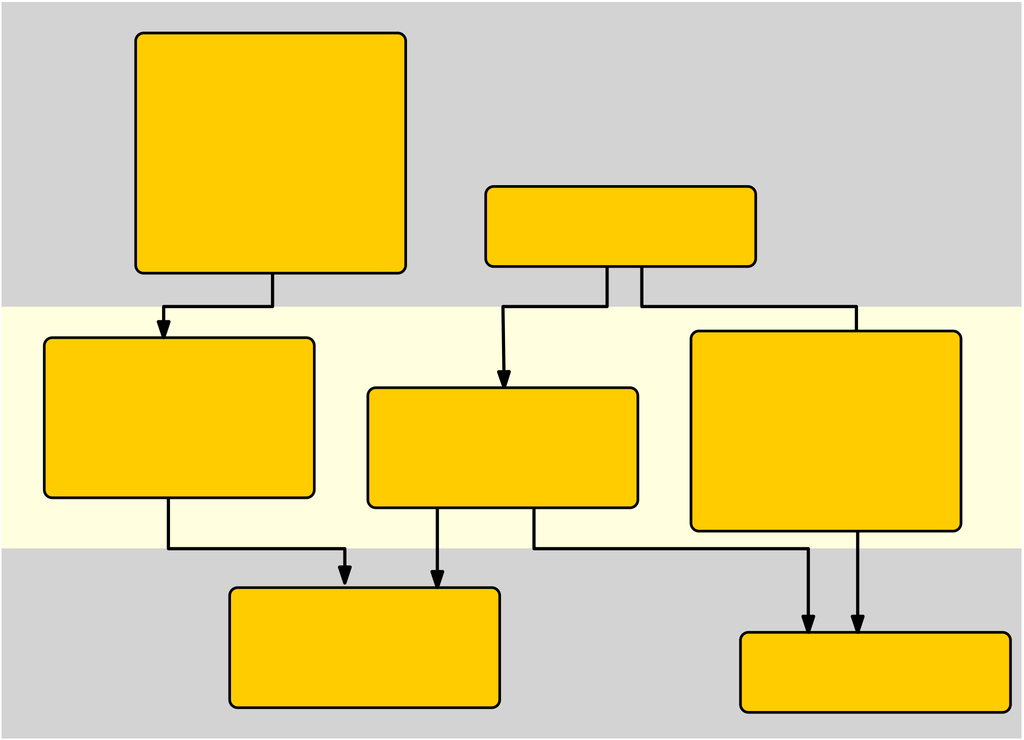

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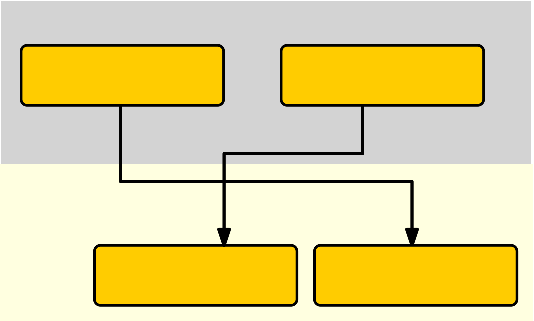
two drawbacks:

- unfortunate layering
  $\Rightarrow$ many unncessary crossings



- tall nodes can lead to non-compact layouts

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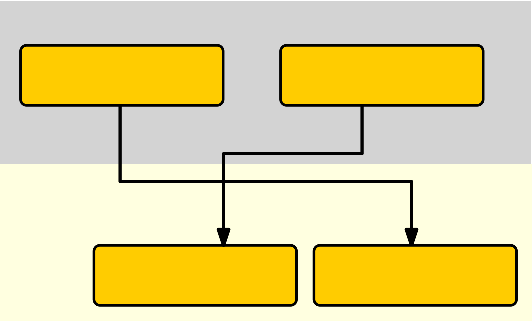

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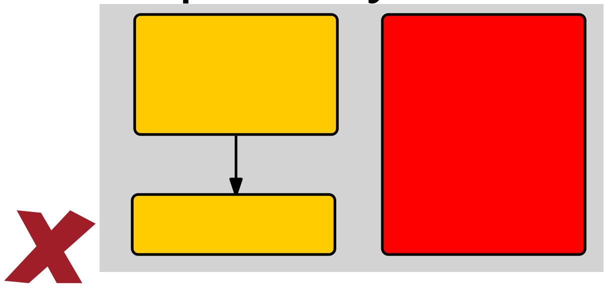

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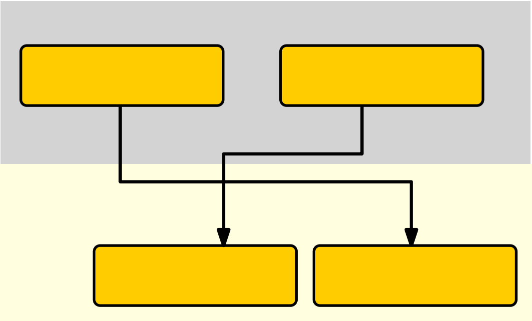

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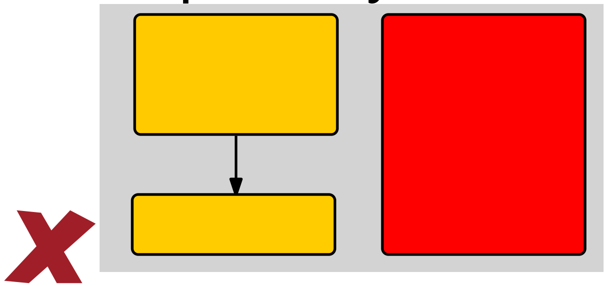

layer-free upward crossing minimization
[Chimani et al. ’10]

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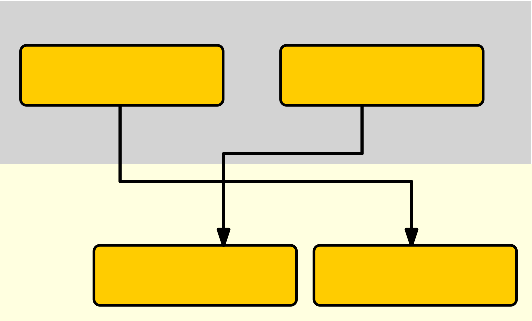

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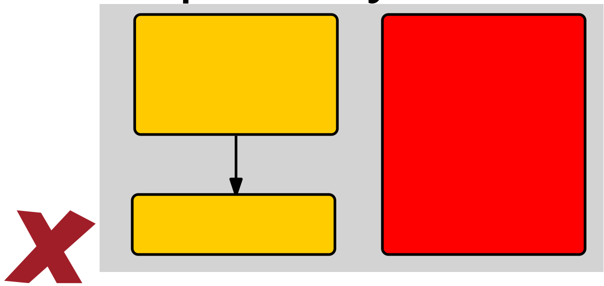

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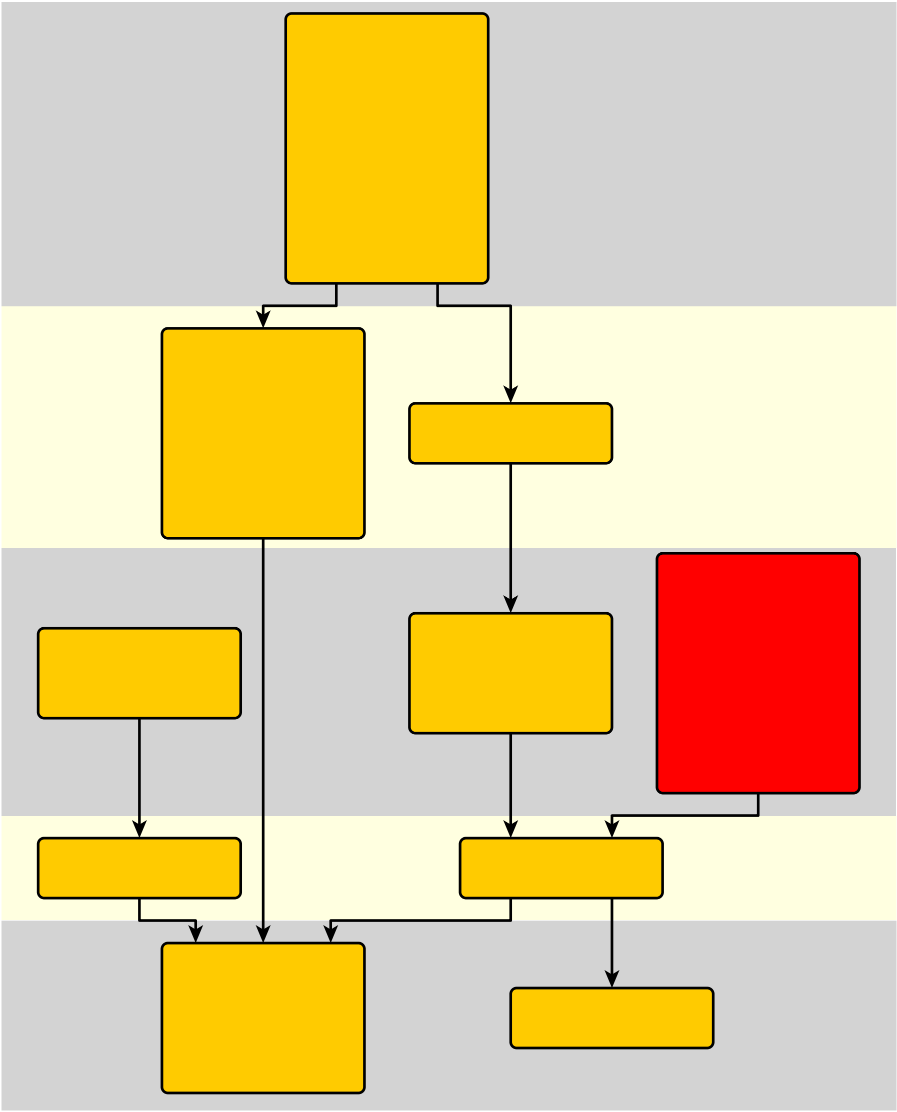

Sugiyama-Framework:
non-compact layouts

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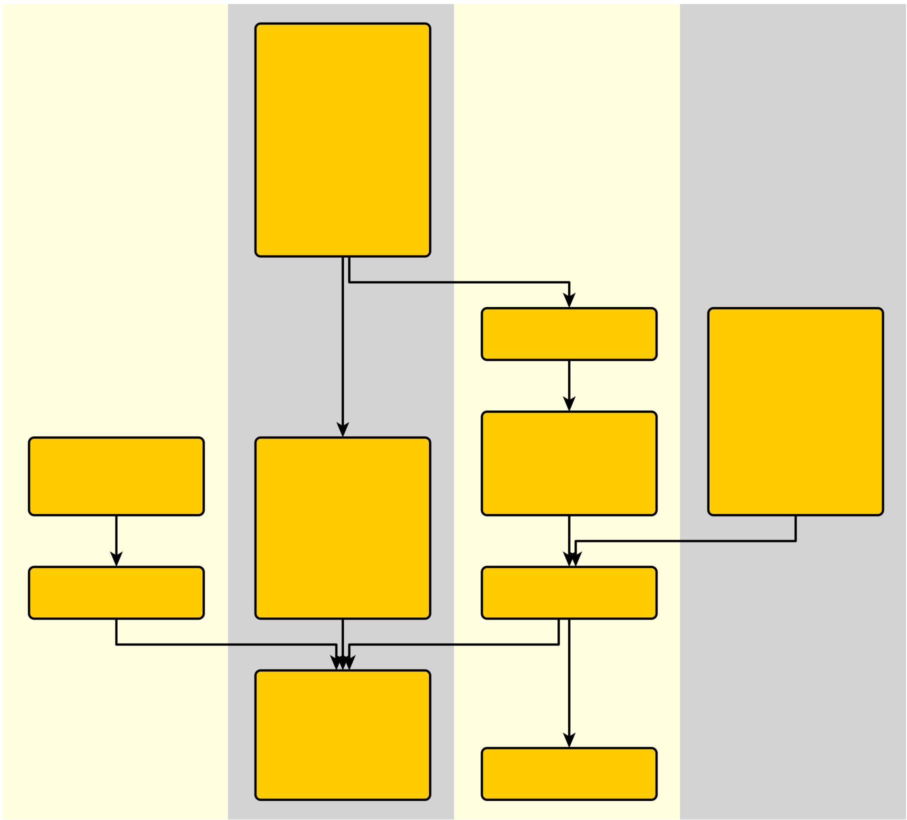
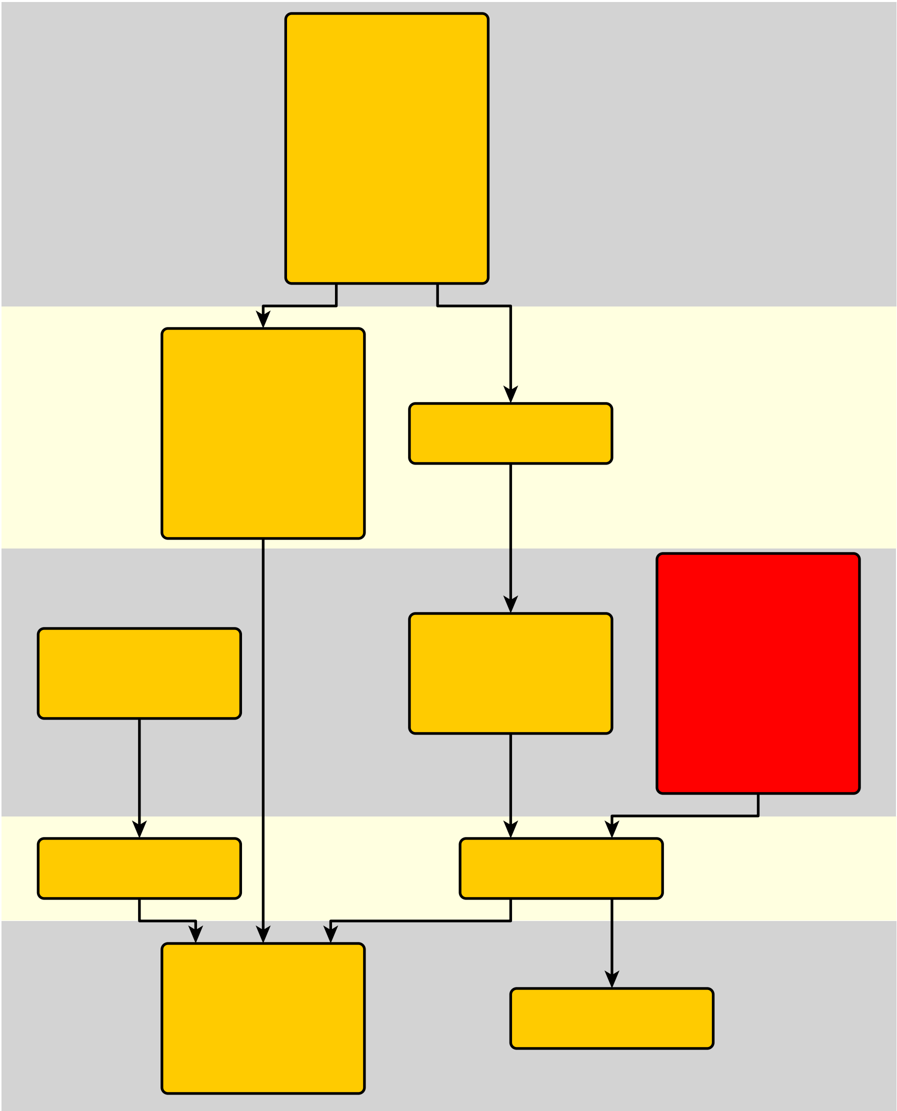


our approach



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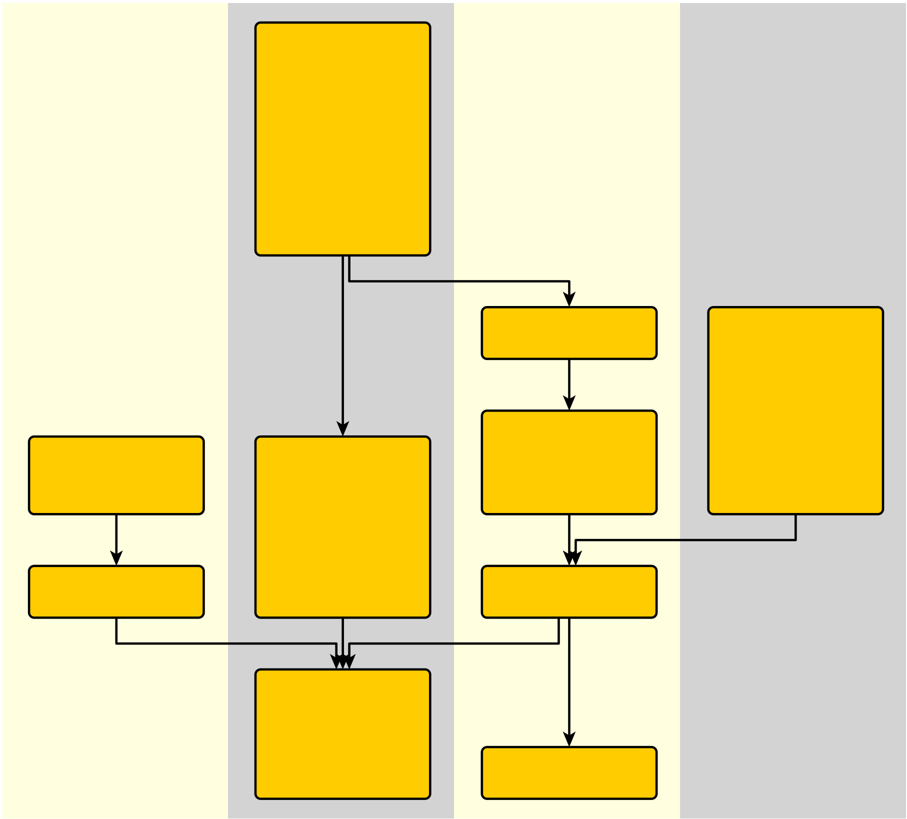

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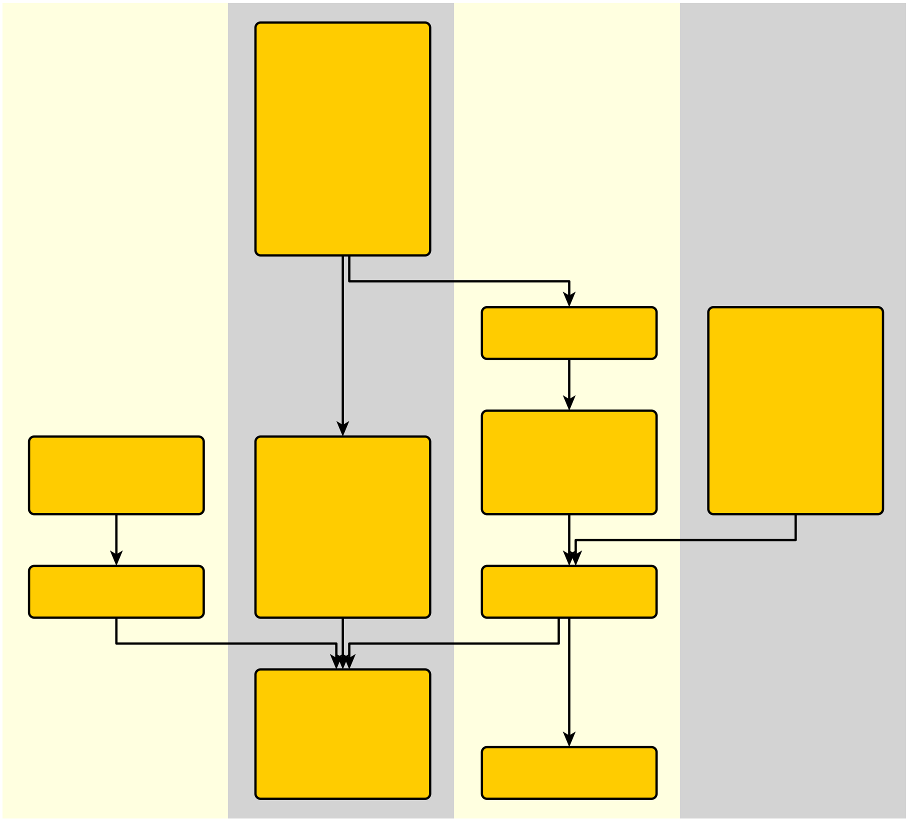

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Quality Guarantees

- Vertical alignment of predecessors
- At most four bends per edge
- Local symmetry
- Spacing constraints
- Edge Bundling

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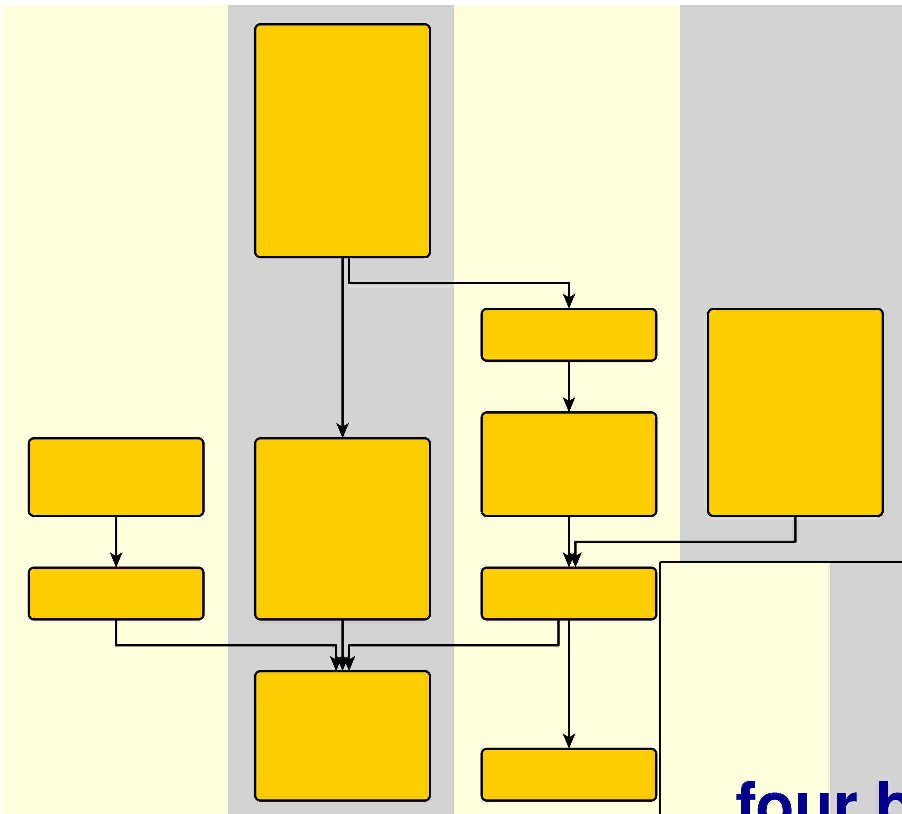

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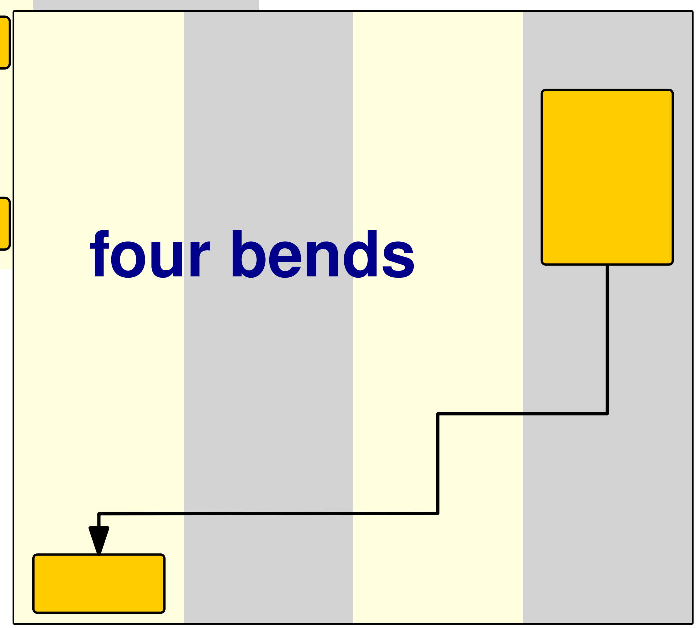

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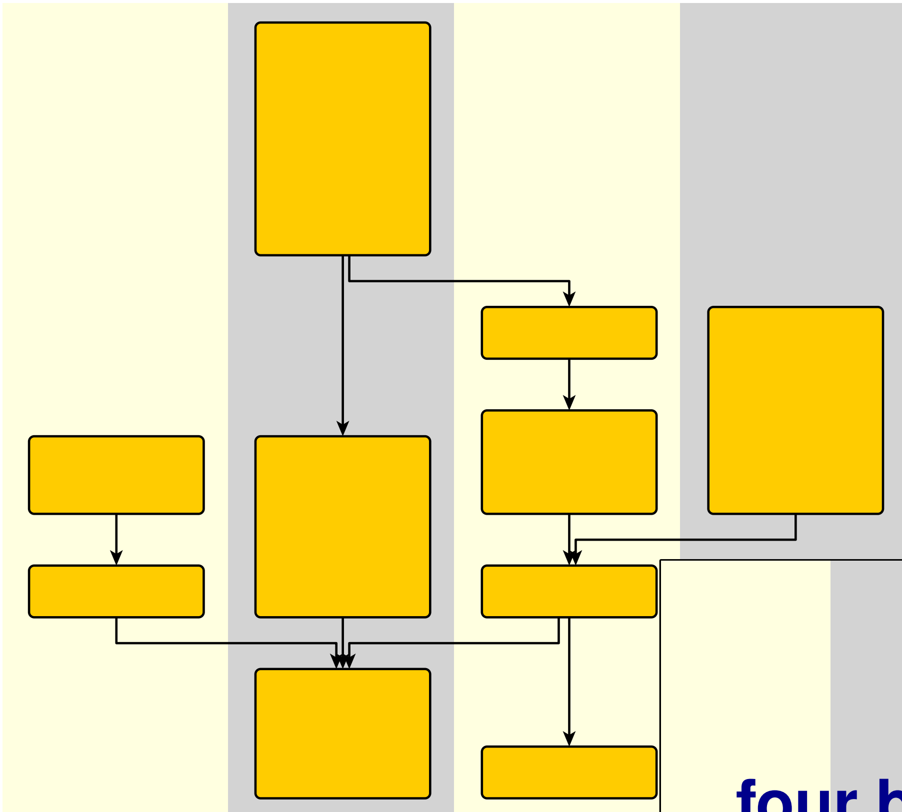

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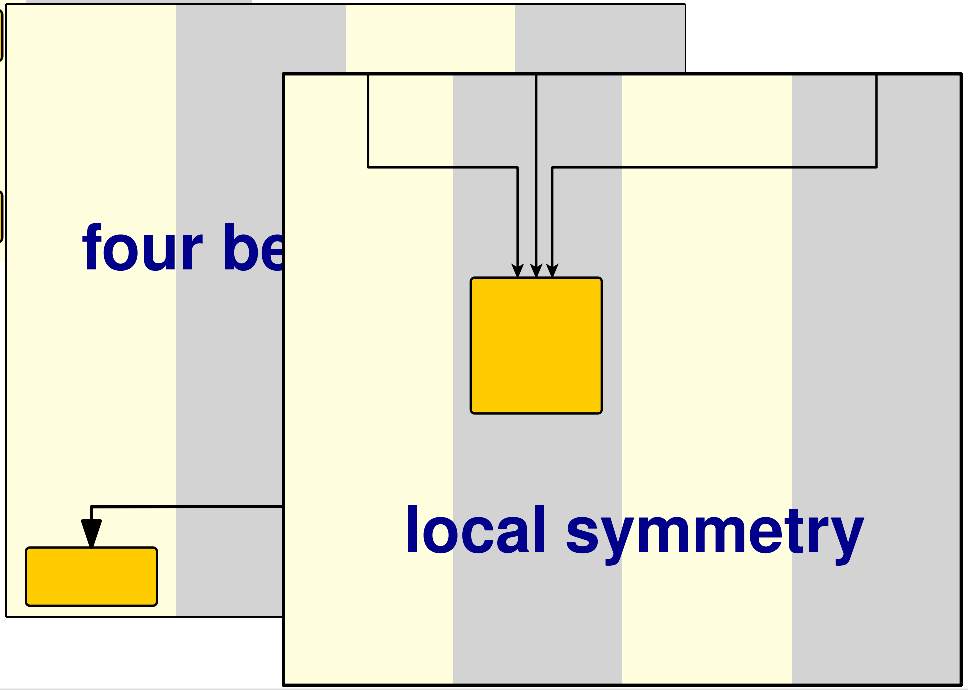

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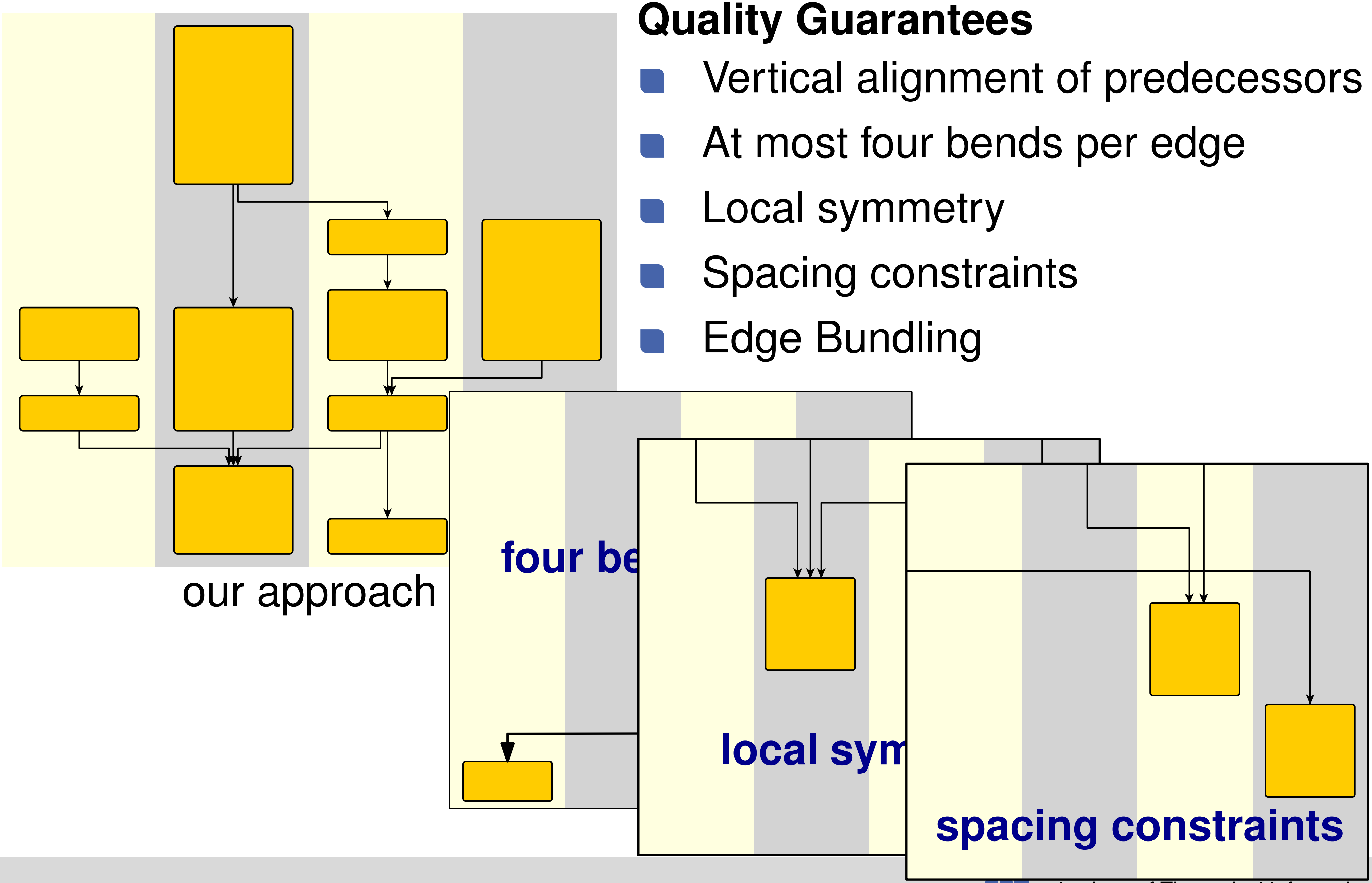

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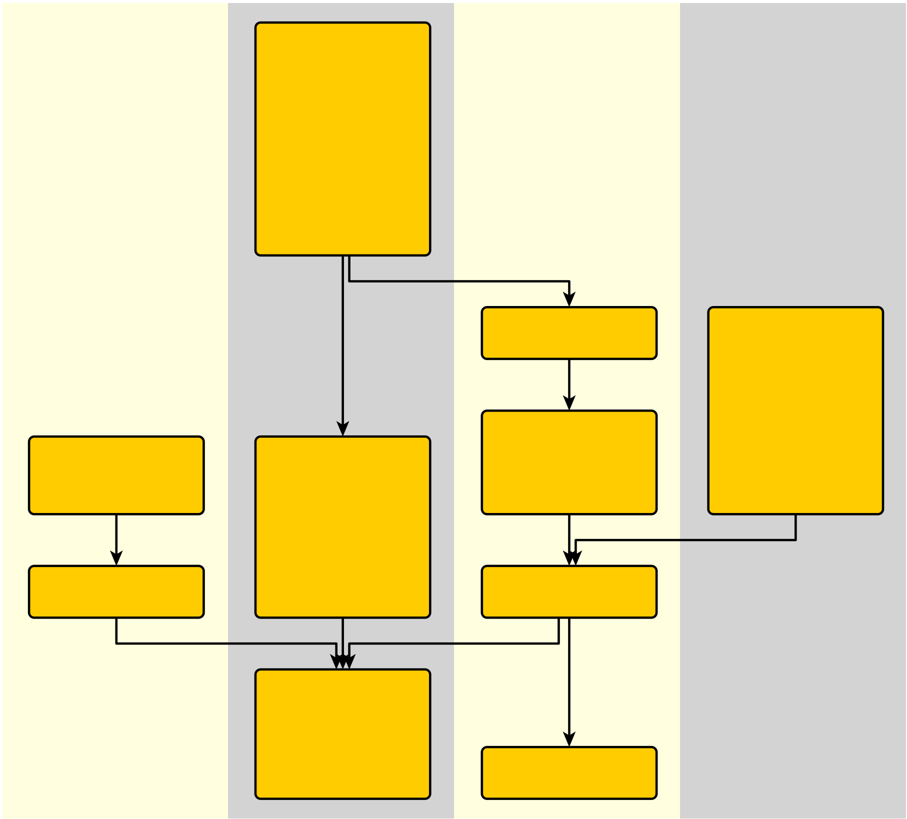

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- minimize number of crossings
- minimize number of bends
- minimize total edge length

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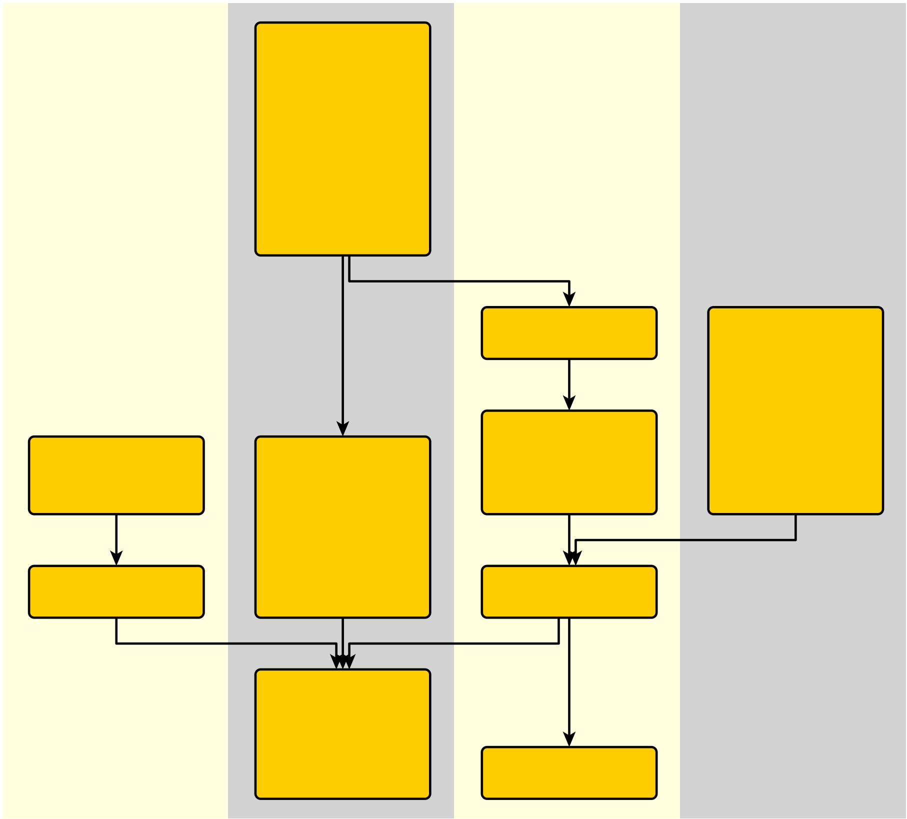

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How do we achieve this?

integrate *layer-free upward crossing minimization* in the *topology-shape-metric framework*

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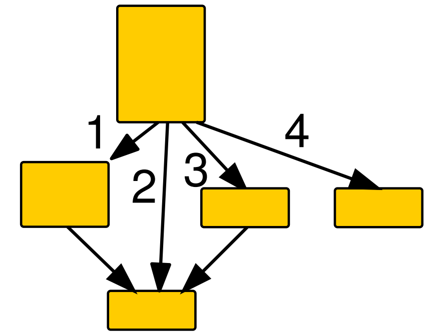
Our Approach - TSM + Layer-free ...

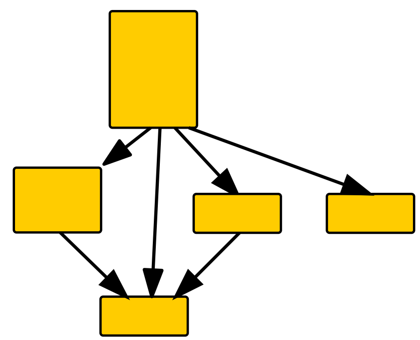
topology compute embedding of the graph

$G = (V, E)$ $\Longrightarrow$

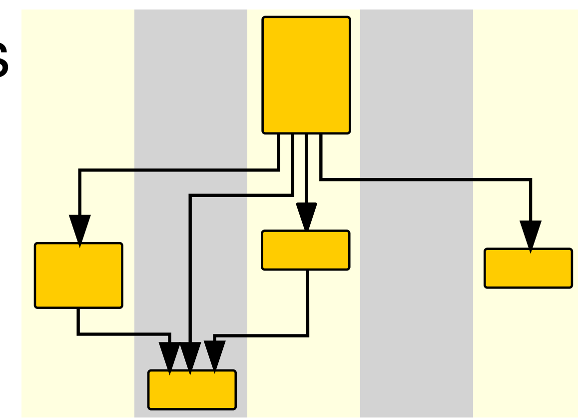


shape assign bends to edges

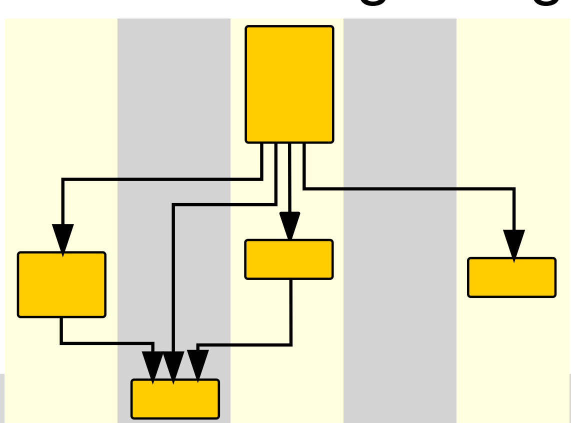
assign nodes/edges/bends to columns



$\Longrightarrow$



metric determine edge lengths & node positions



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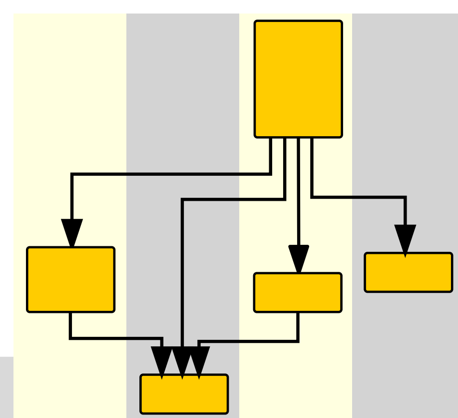


Topology – Compute Embedding

Layer-free upward crossing minimization [Chimani et al. ’10]

two steps:

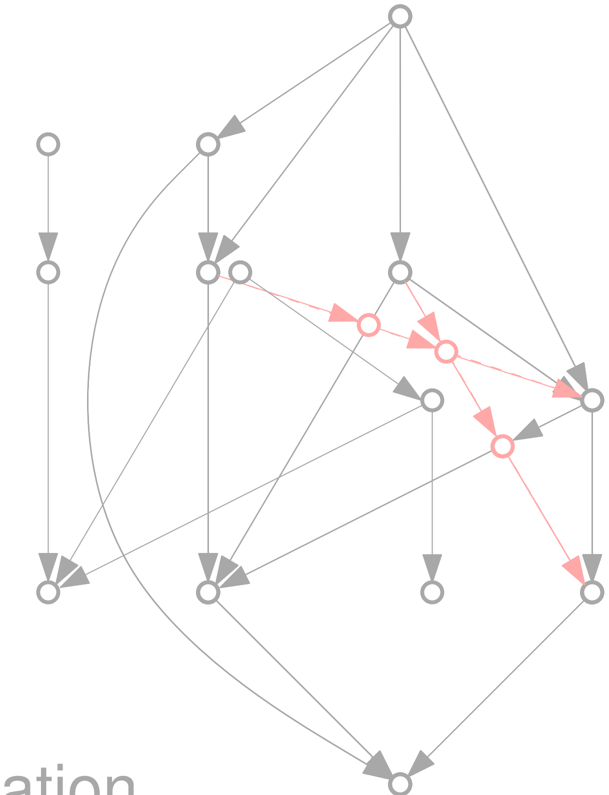
(i) delete edges until the remaining graph is upward planar

(ii) reinsert the deleted edges one by one

- add super source/sink

I’ll skip the details

- add deleted edges
- insert dummy nodes



result:

upward planar represantation

add super source/sink

Shape – Assign Each Node & Edge a Column

goal: find realizeable column assignment for each node and edge

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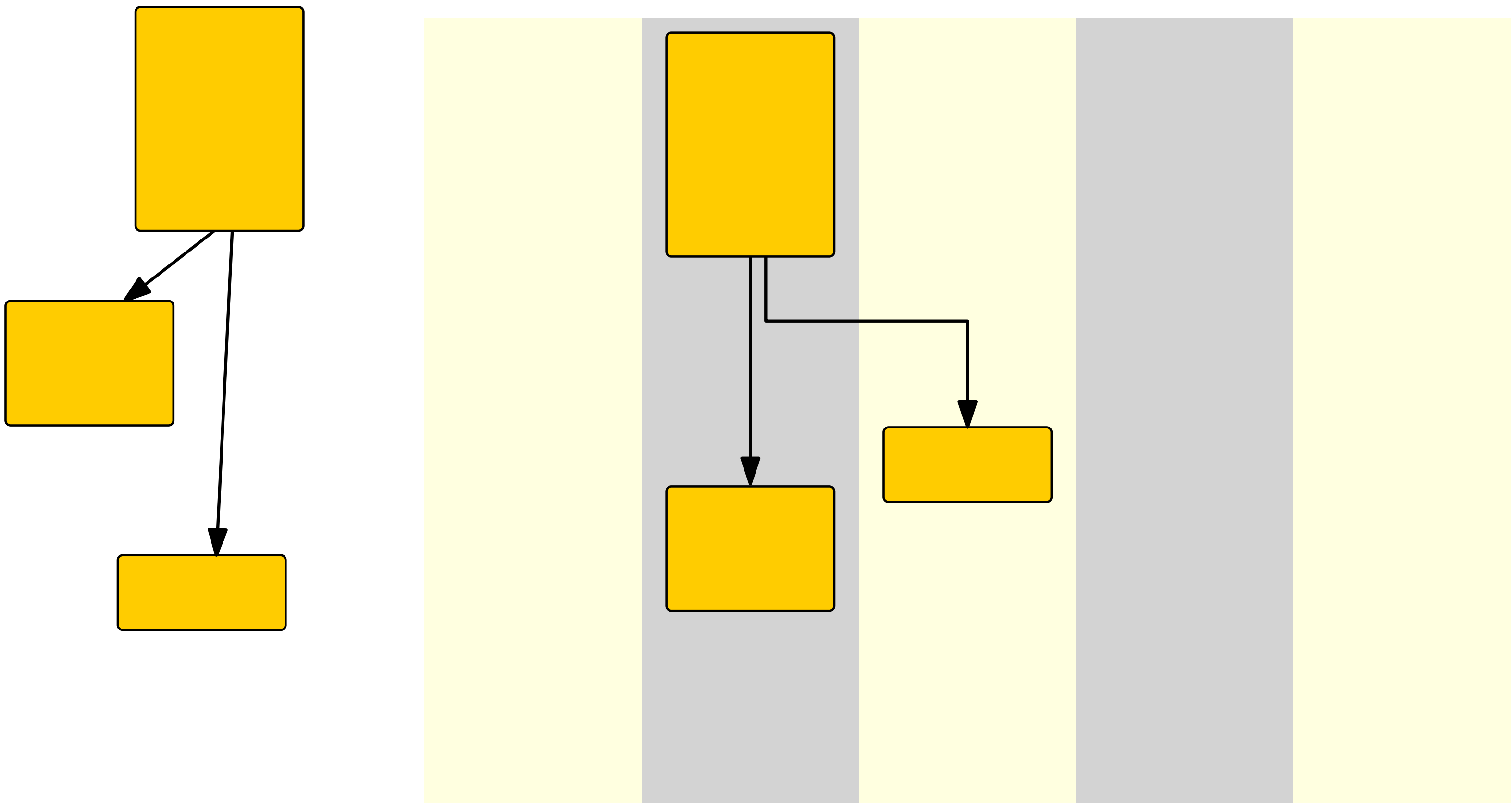

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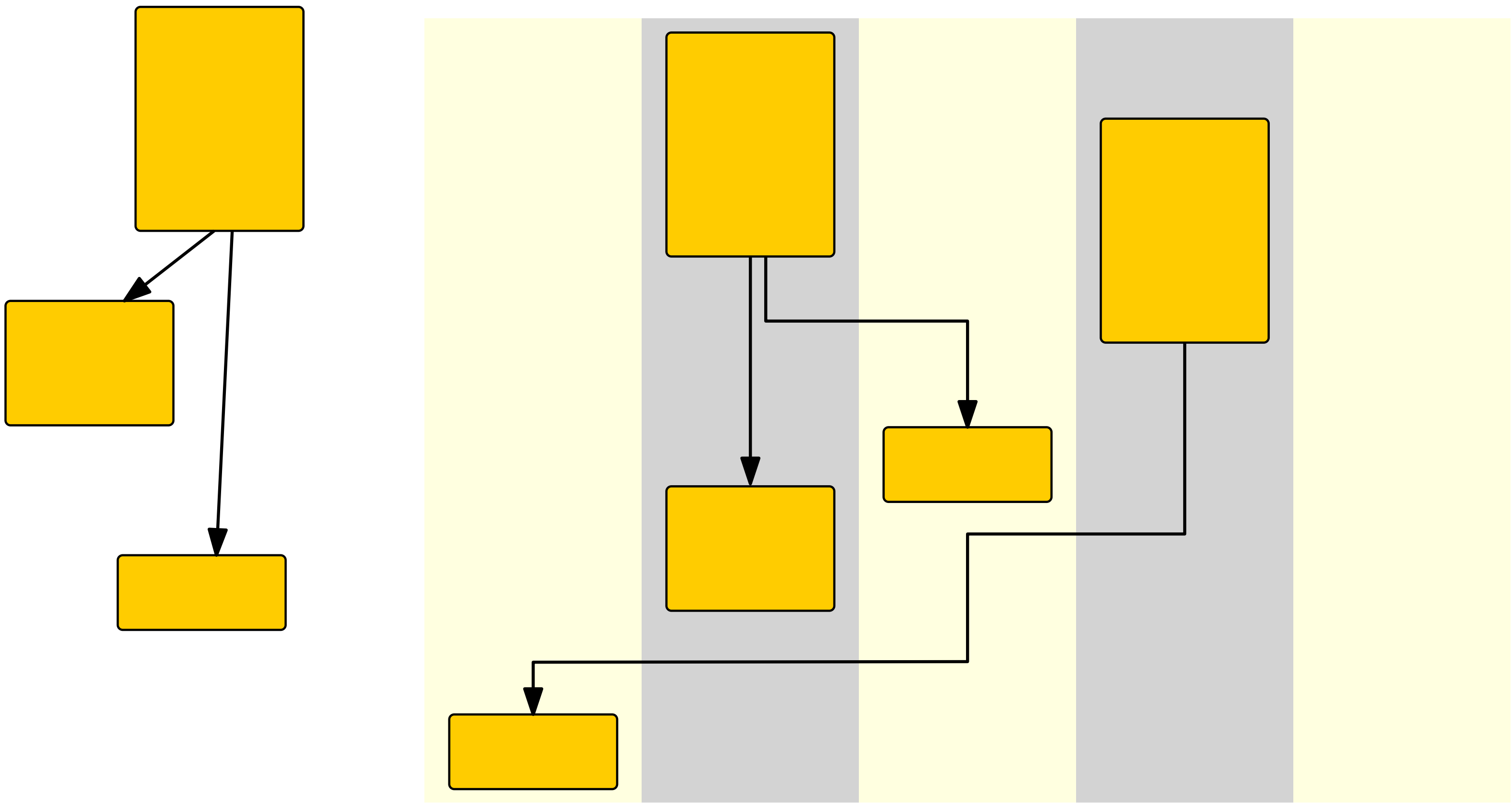

Shape – Assign Each Node & Edge a Column

Better Heuristic for Orthogonal Graph Drawings [Biedl, Kant ’98]

goal: find realizeable column assignment for each node and edge

Theorem 1
There exists an algorithm that computes a realizable column assigment in $O(|E|)$.

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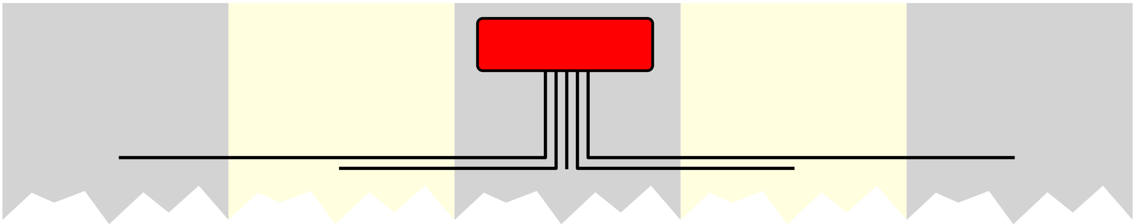
draw from top to bottom



- start with super source
- draw vertical and horizontal part of edges

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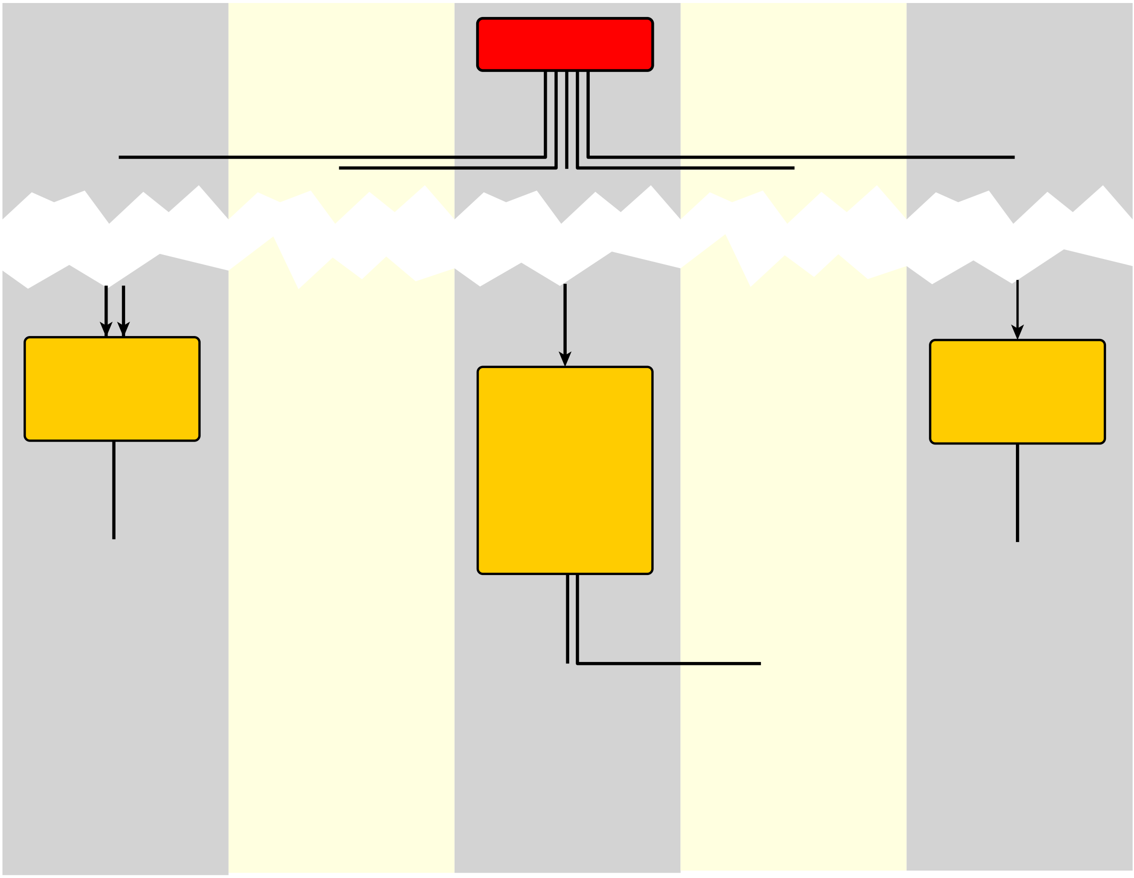

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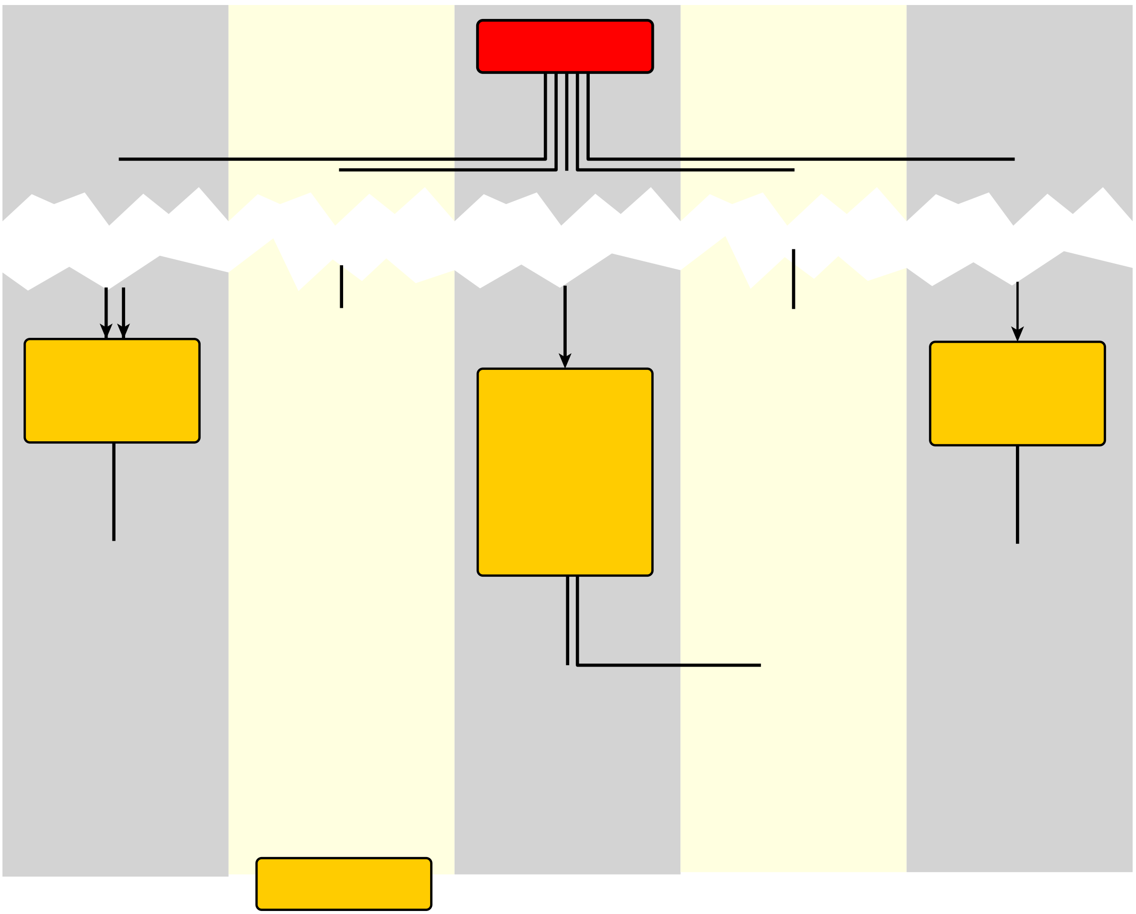

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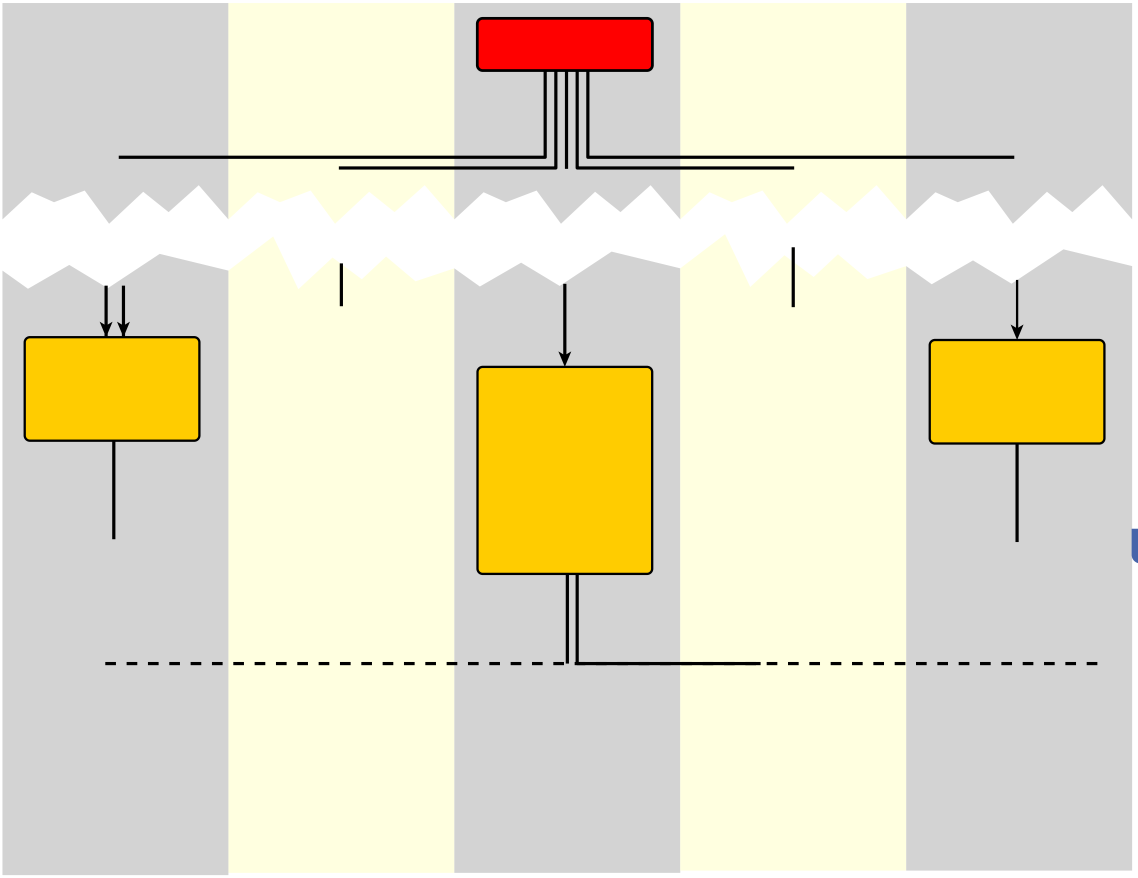

- find highest position

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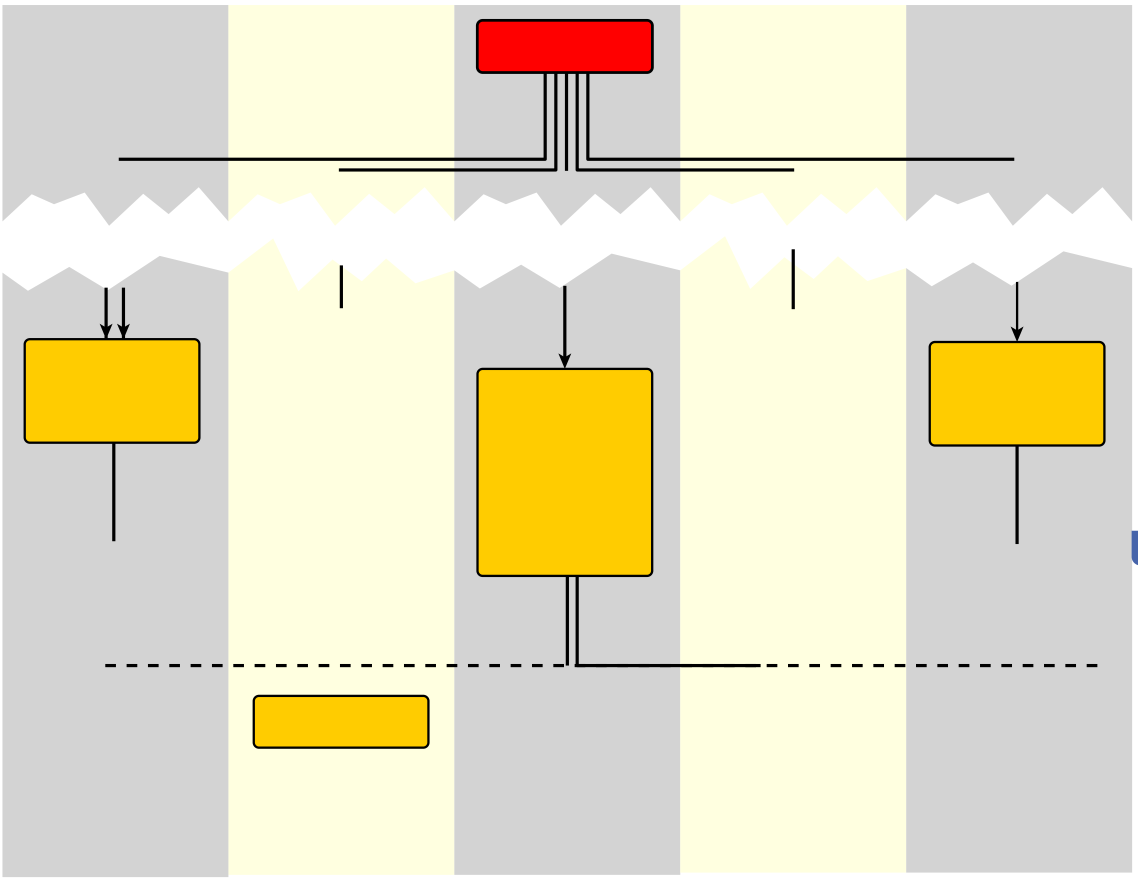

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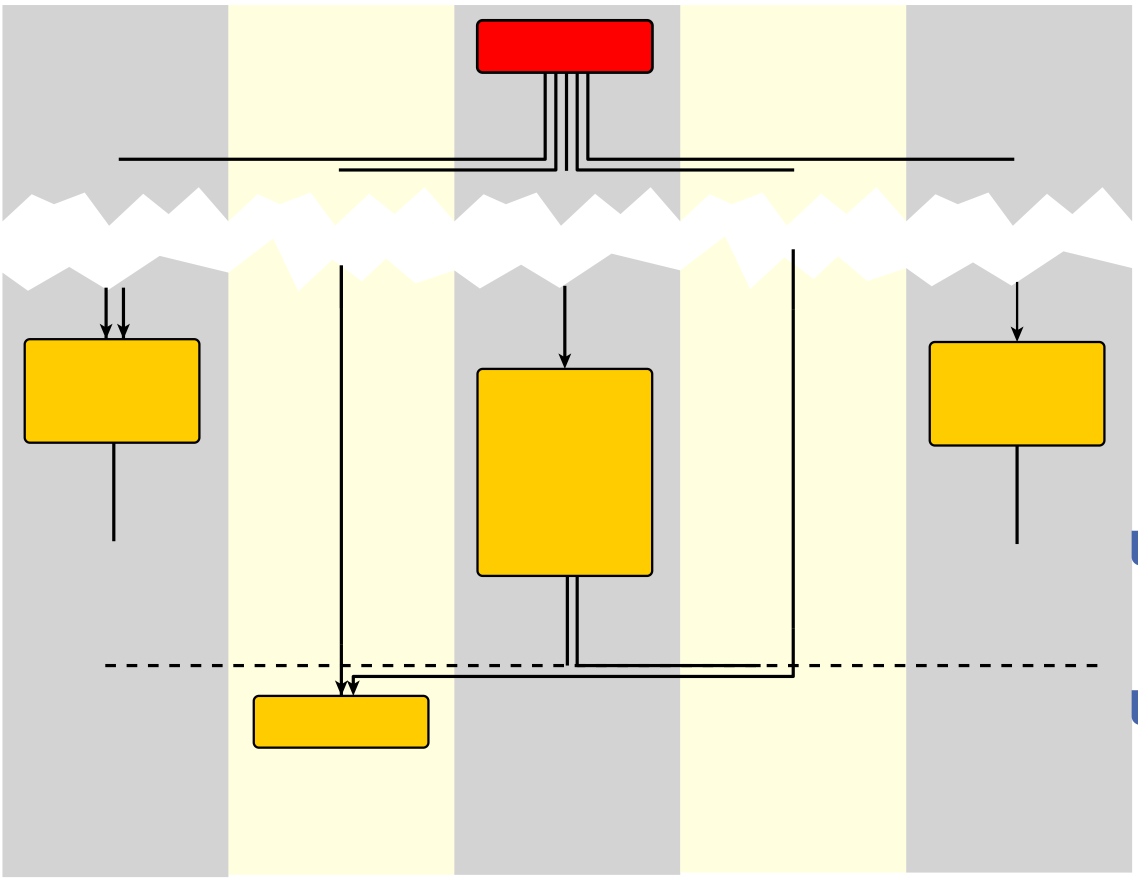

- find highest position
- draw inedges

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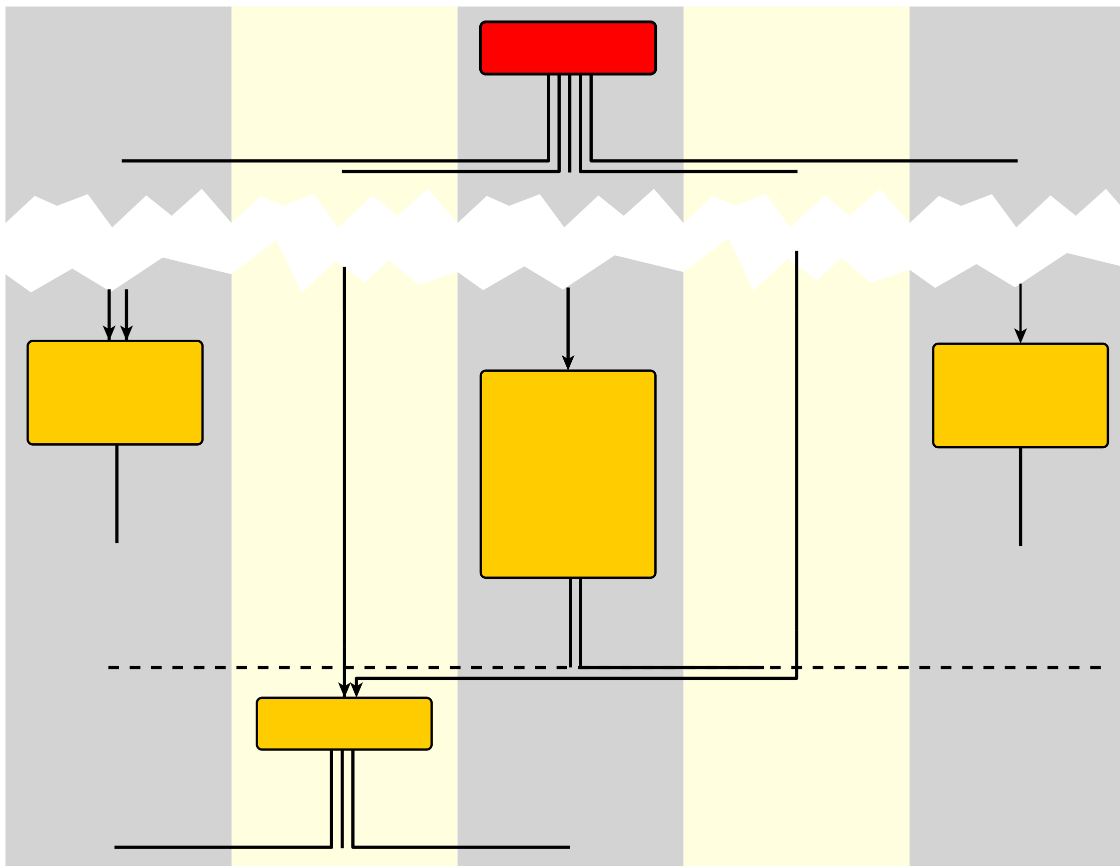

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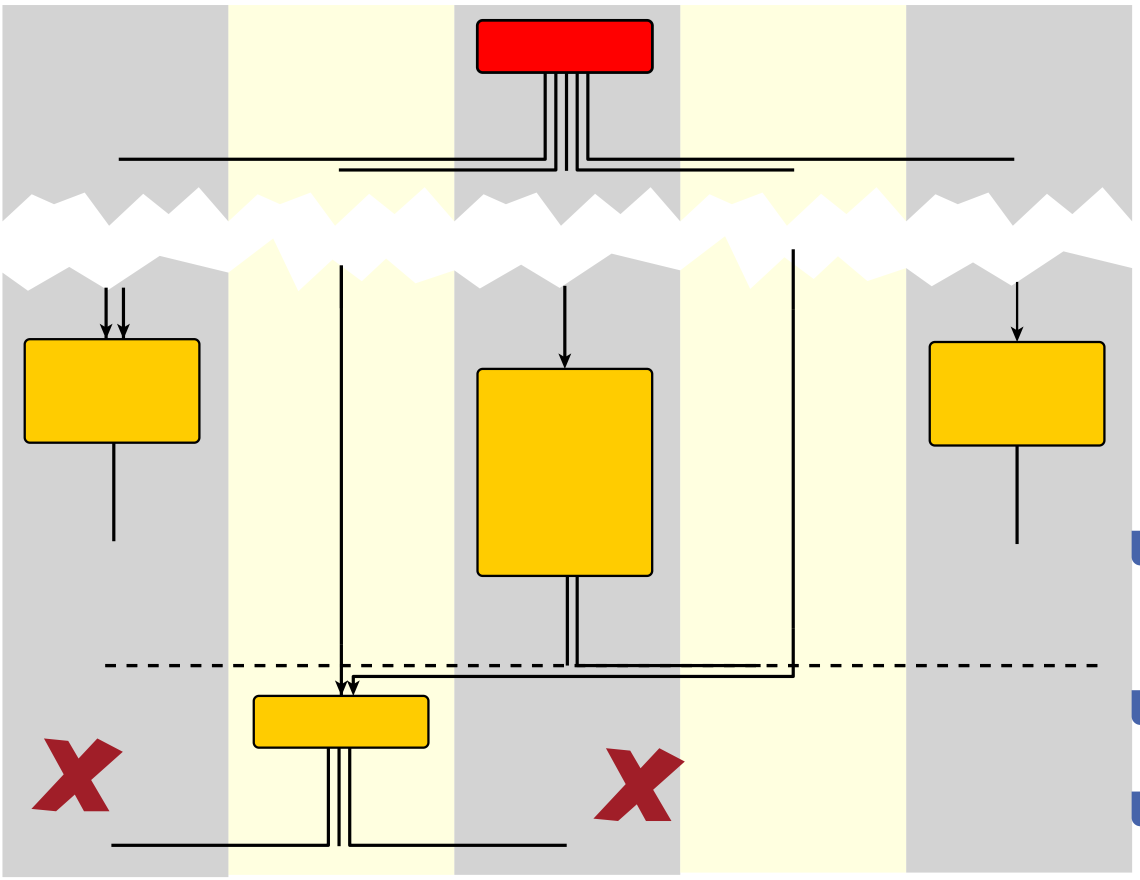

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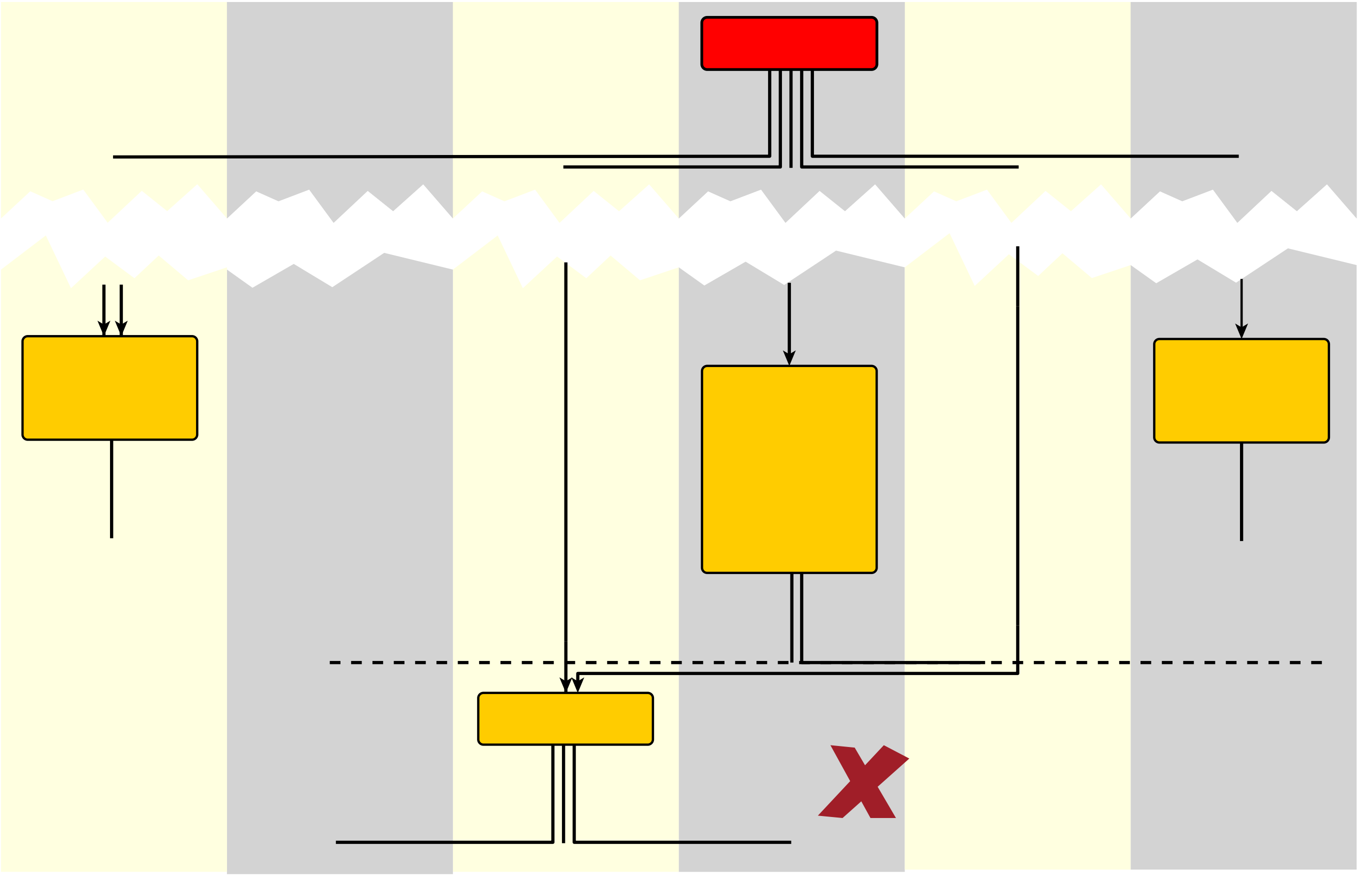

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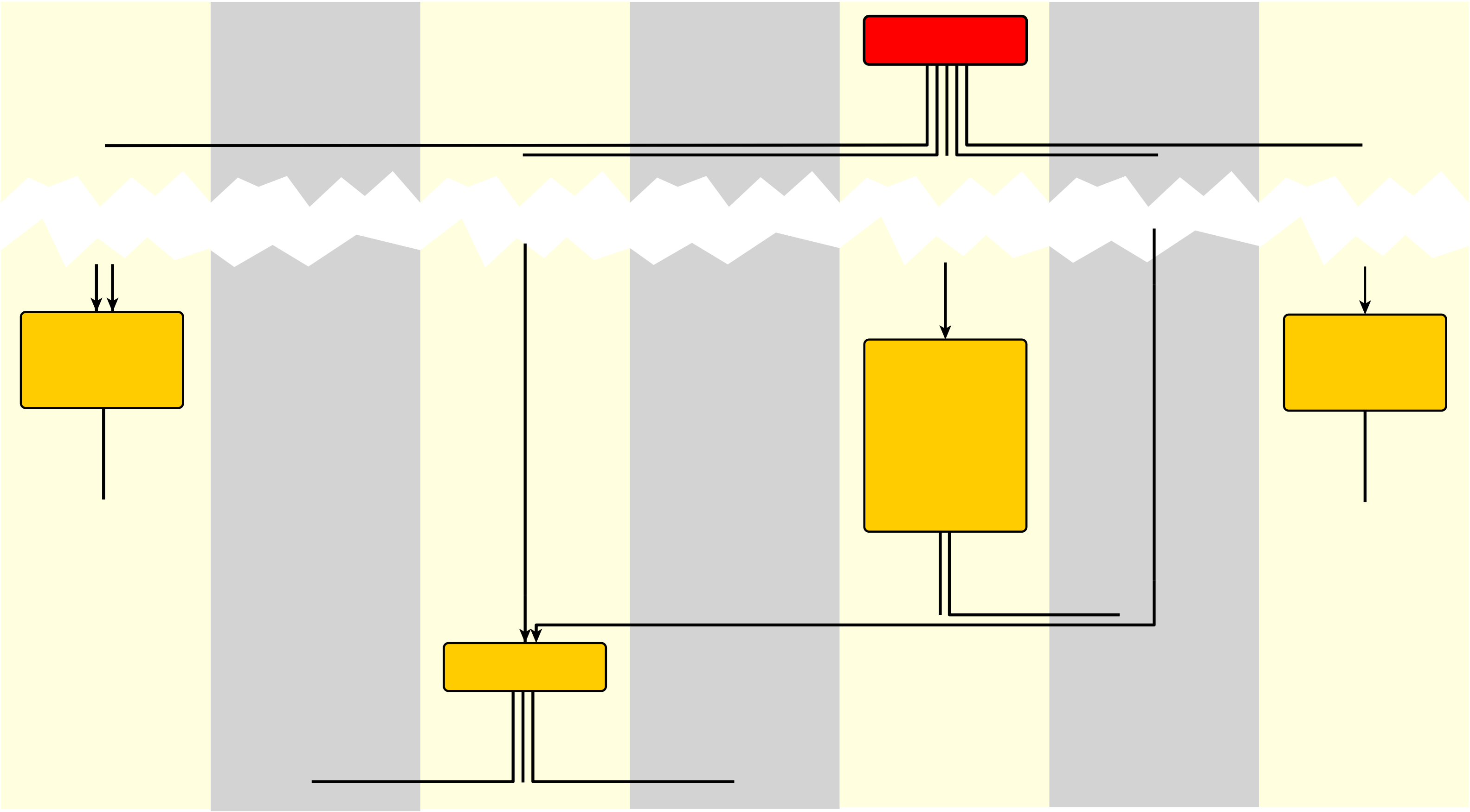

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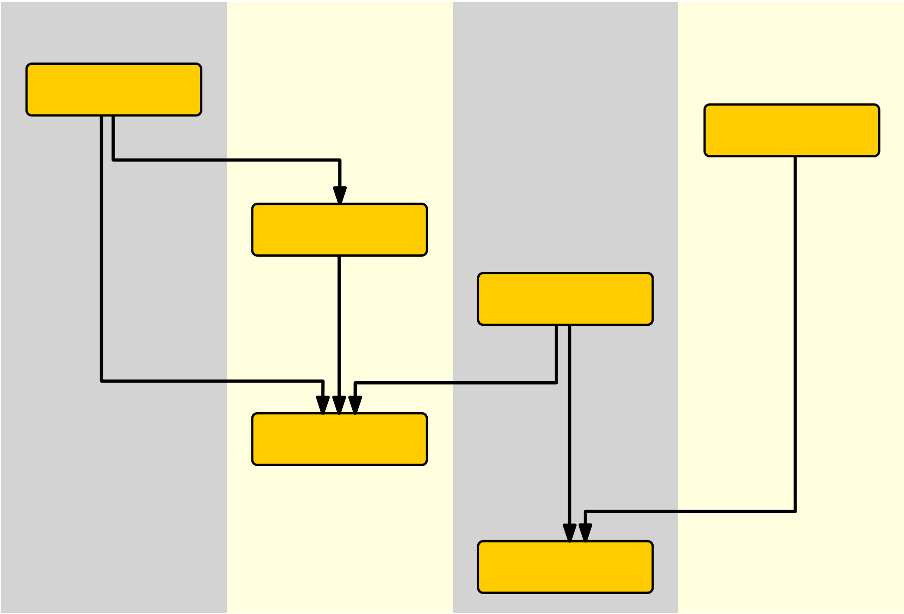
Metrics – Compute final coordinates

goal: minimize total vertical edge length

Theorem 2

Finding a realization of a column assignment with minimum total vertical edge length is $\mathcal{NP}$-complete.



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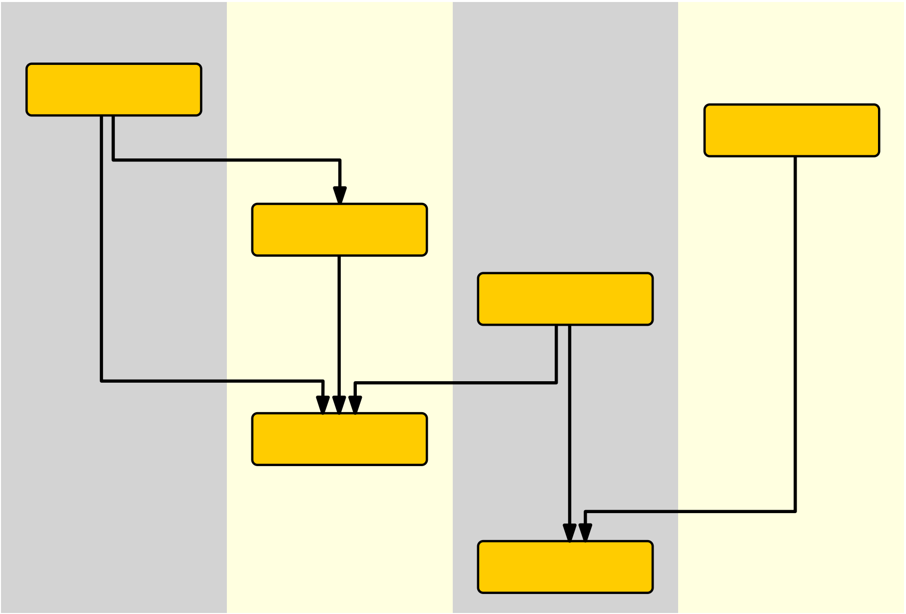

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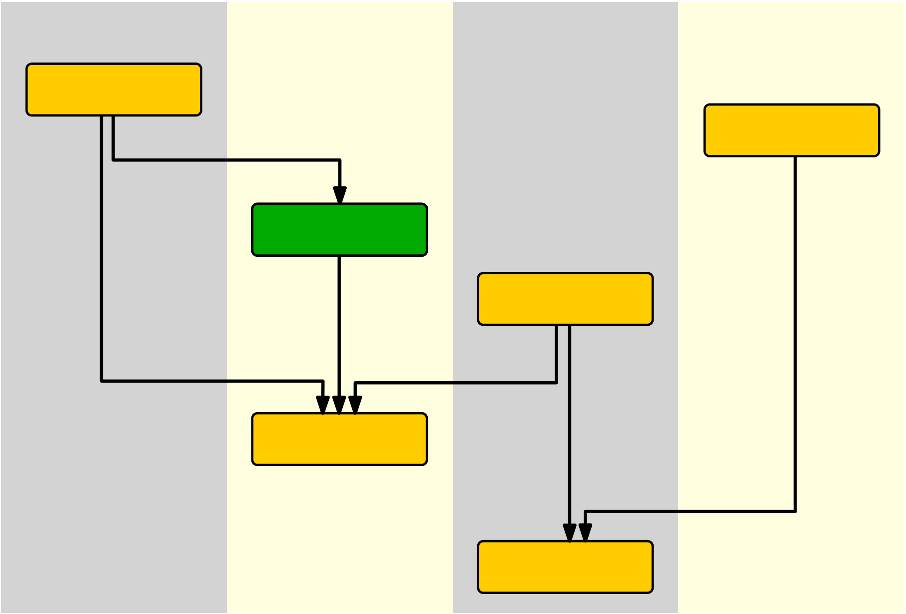

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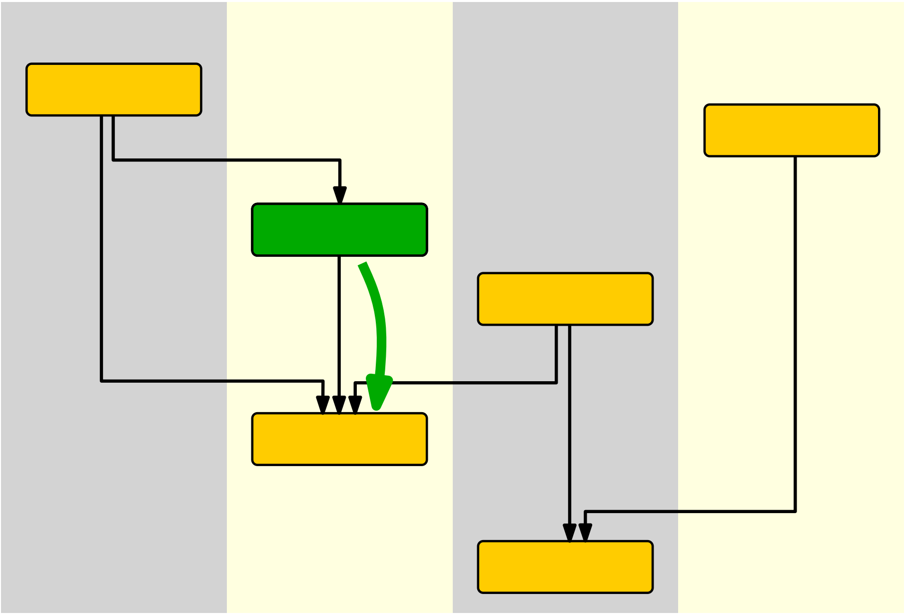

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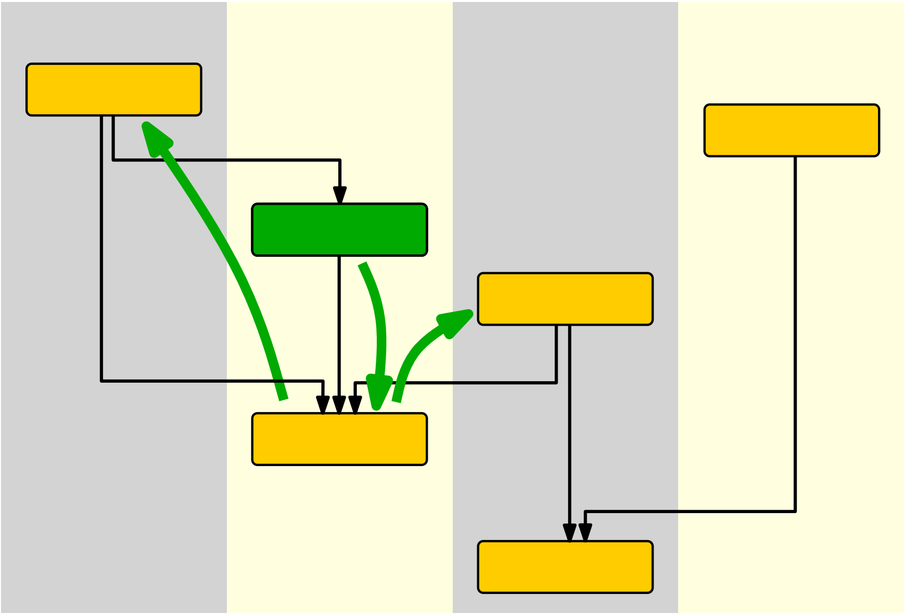
Finding a realization of a column assignment with minimum total vertical edge length is $\mathcal{NP}$-complete.

greedy algorithm:
(i) assign every node to a group [vertical alignment of predecessors]



Metrics – Compute final coordinates

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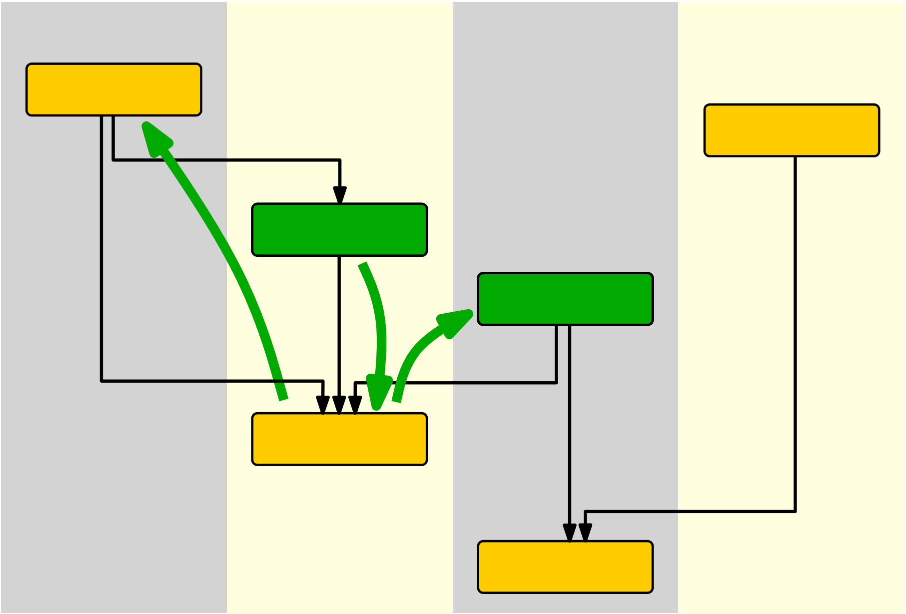

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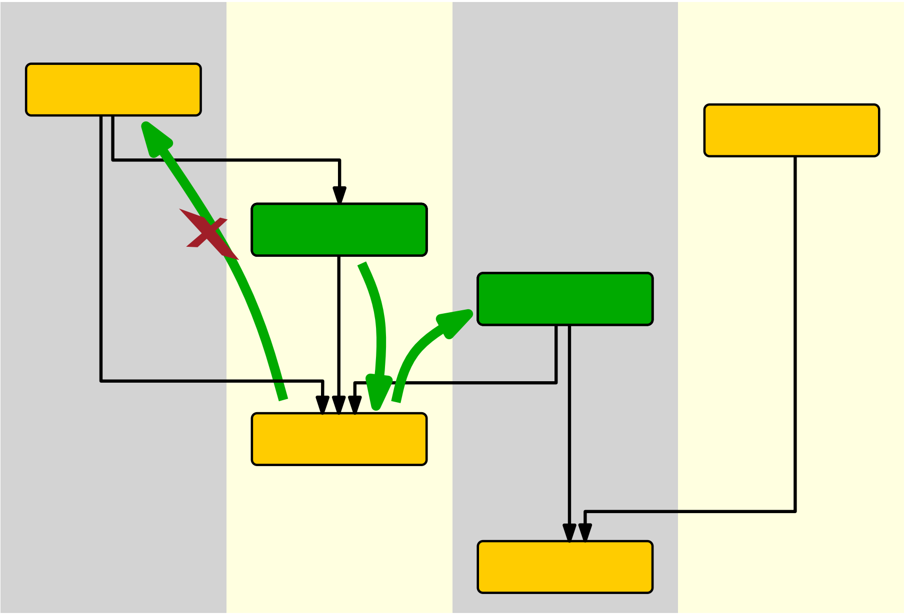

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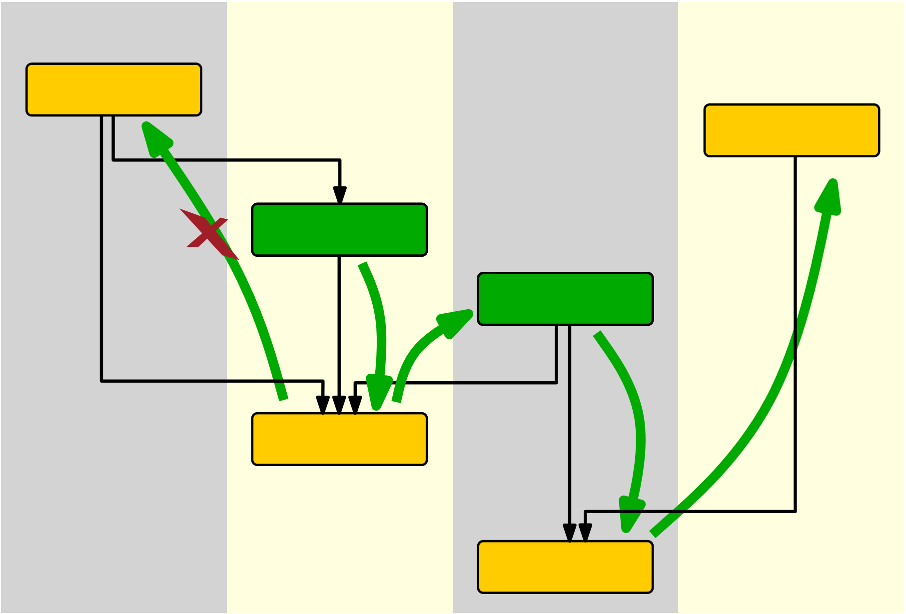

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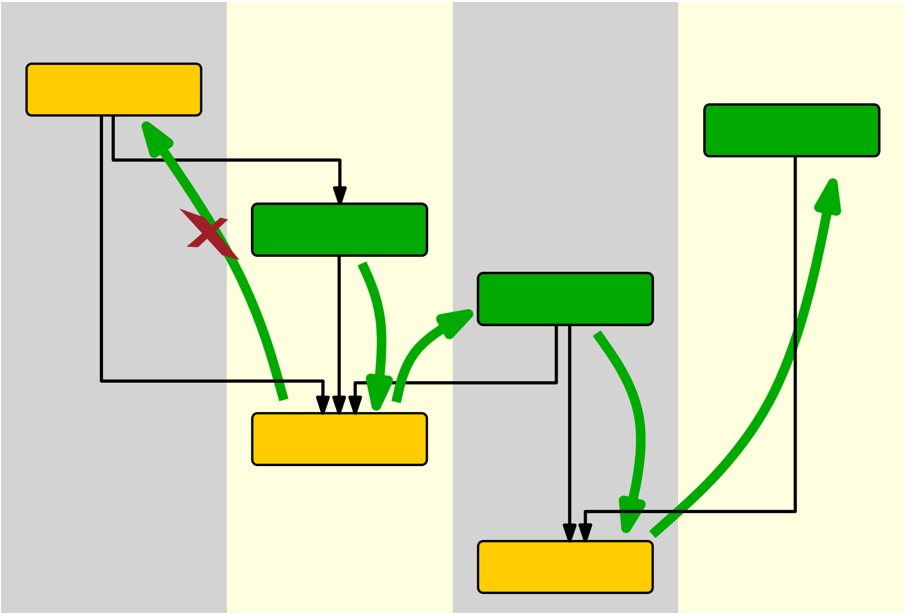

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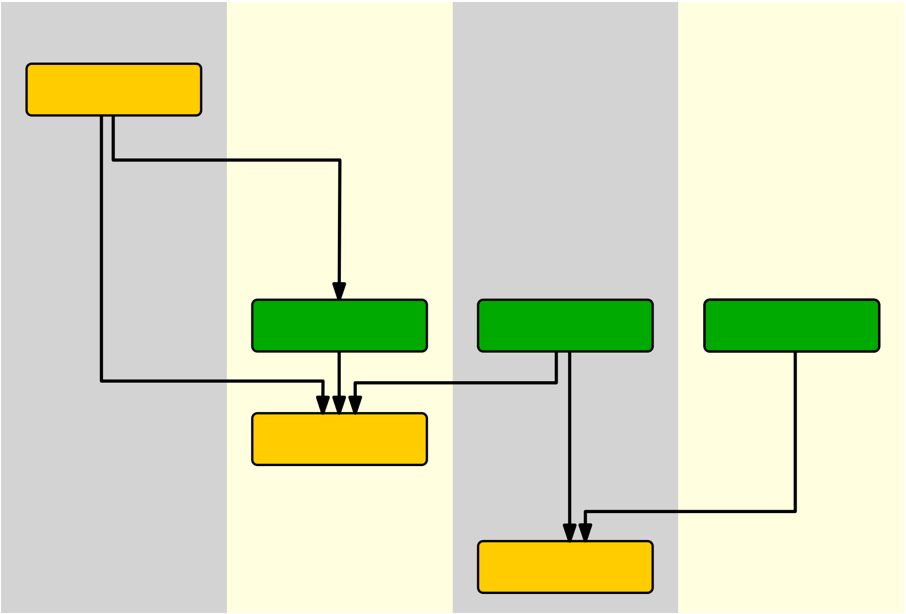

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greedy algorithm:

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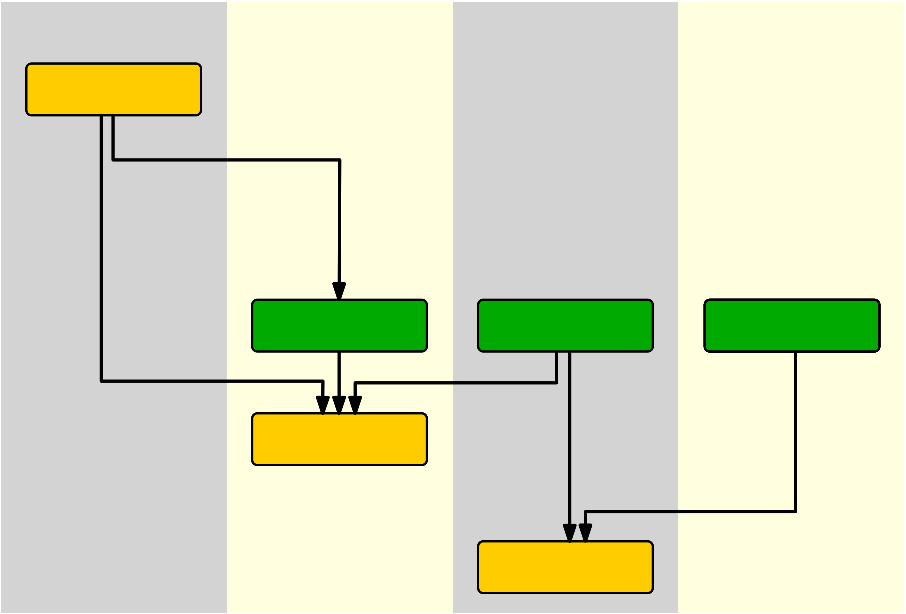

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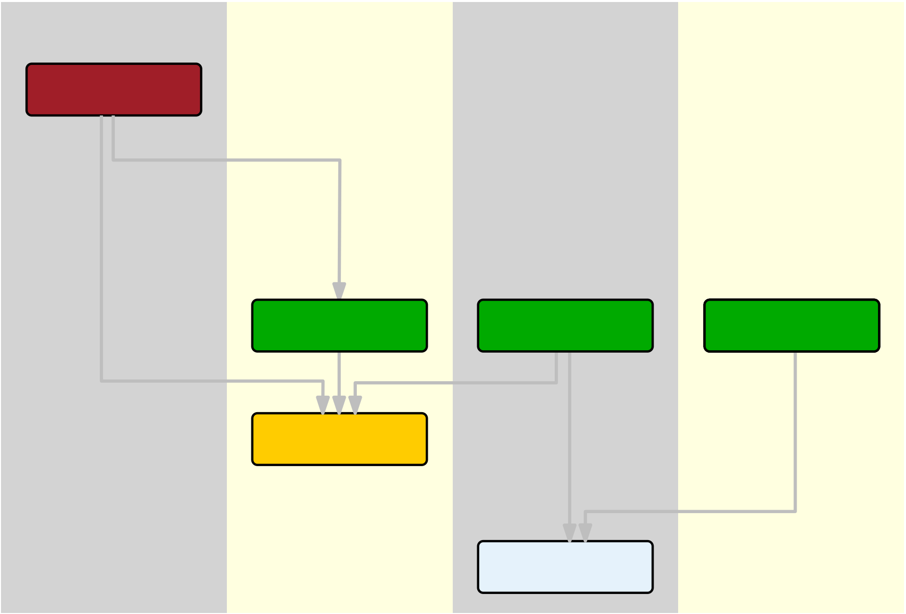

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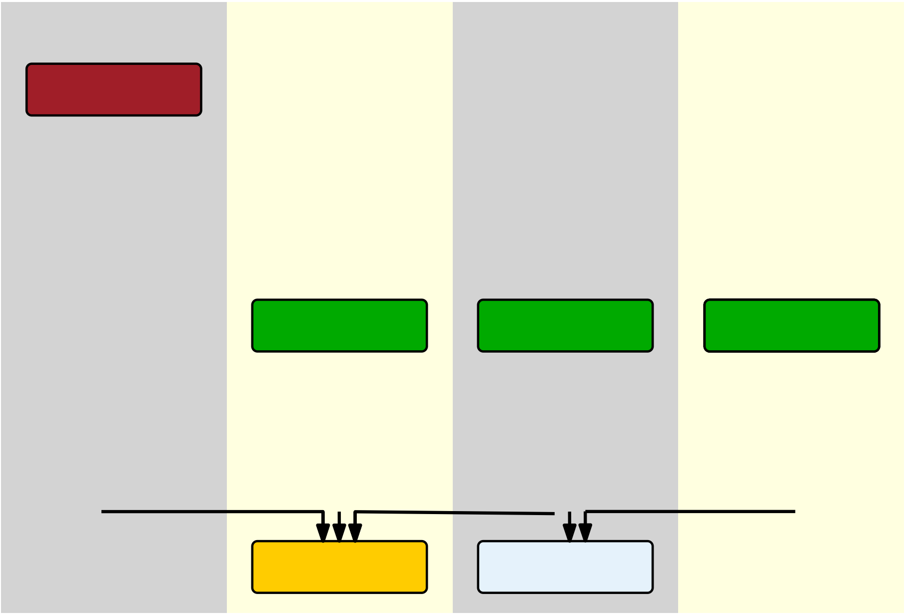
- draw horizontal part of inedges



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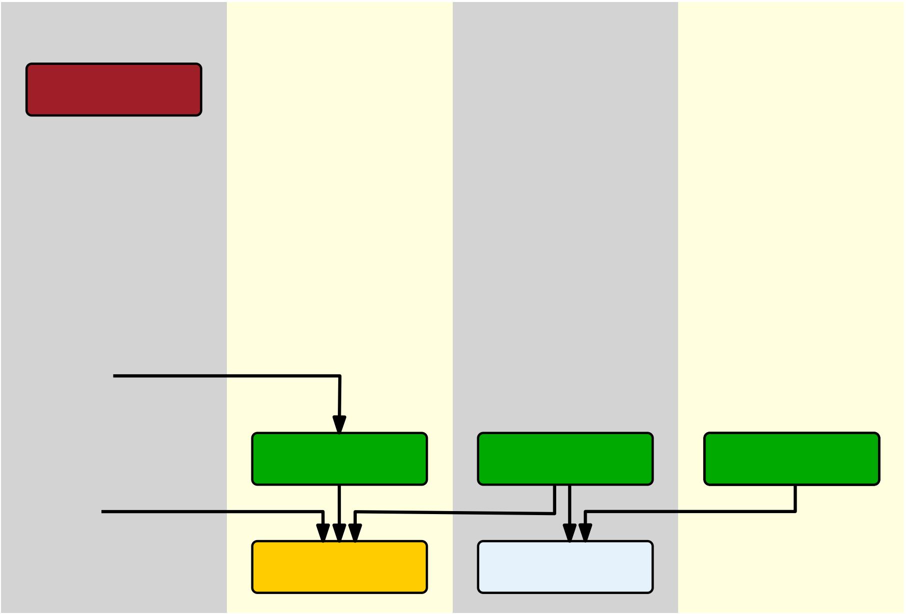

- draw horizontal part of inedges
- complete drawing of outedges

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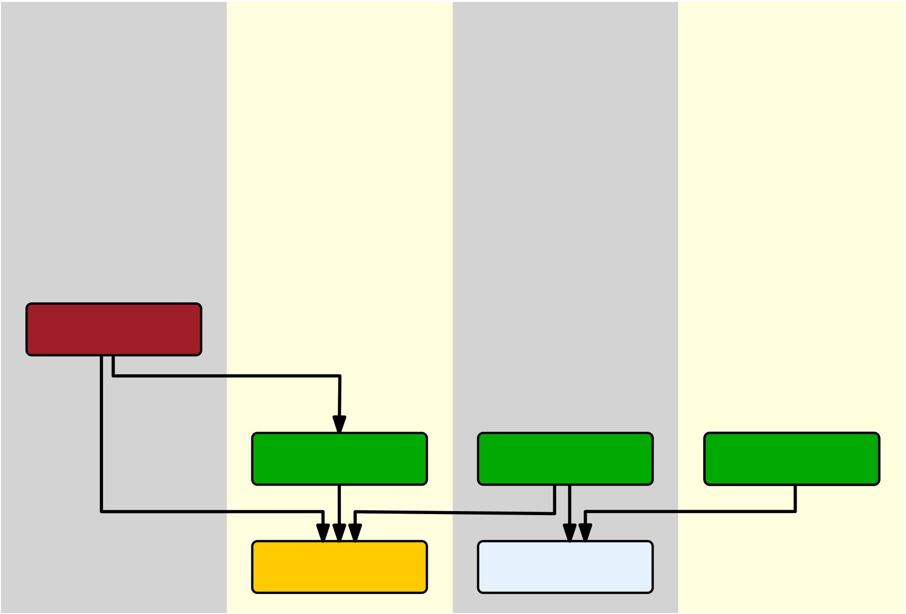

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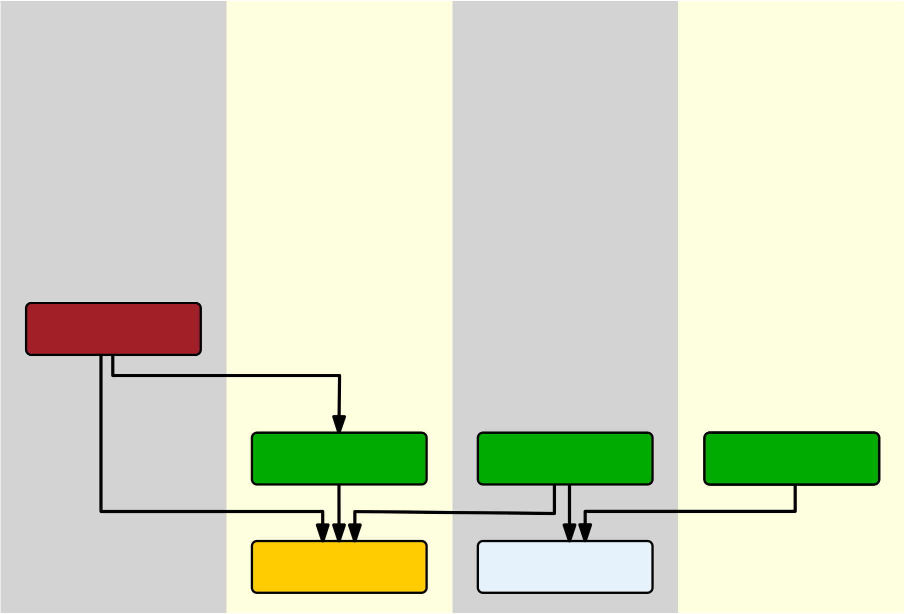
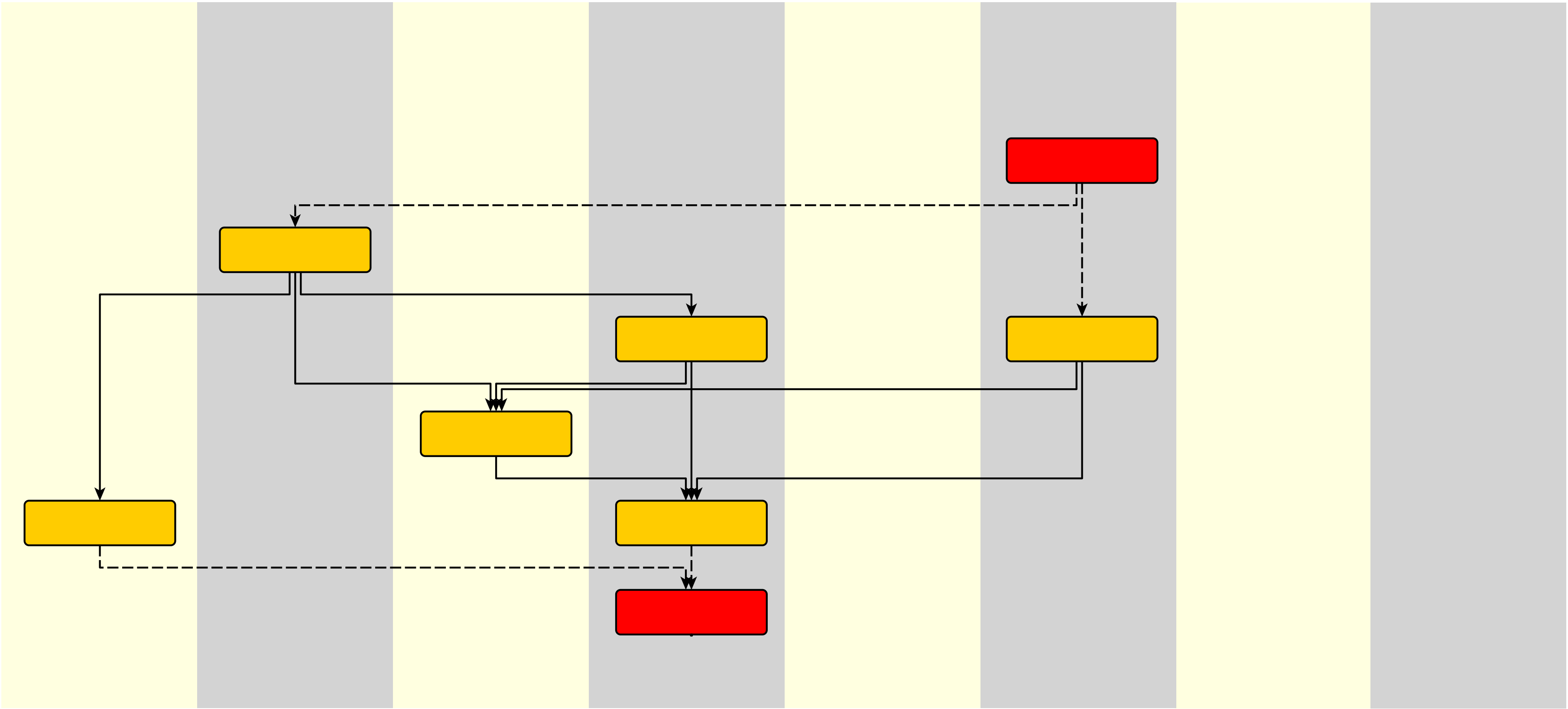


Theorem 3
The greedy algorithm requires $O((n+b)m)$ time.

b = #columns

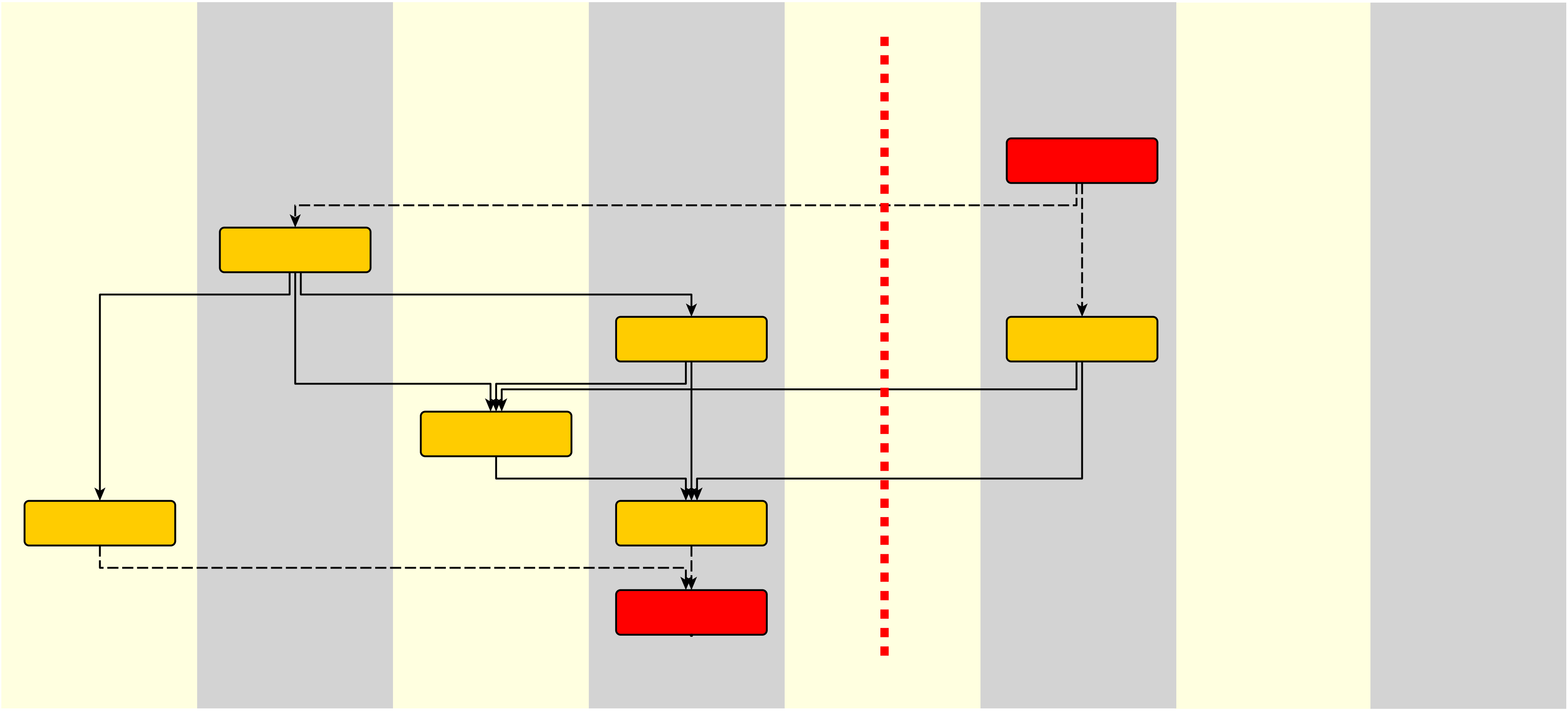
Metrics – Compute final coordinates

goal: minimize total horizontal edge length



Metrics – Compute final coordinates

goal: minimize total horizontal edge length



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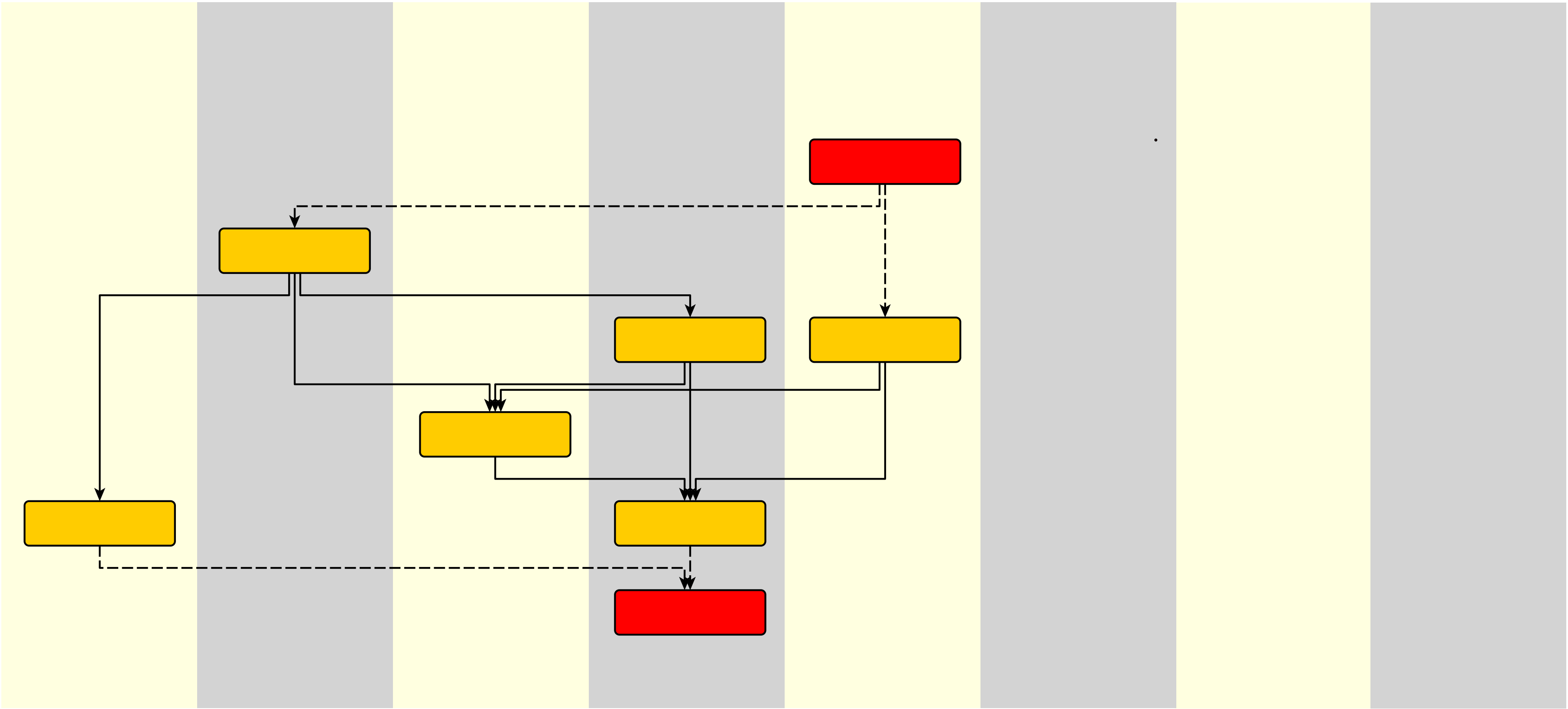
goal: minimize total horizontal edge length



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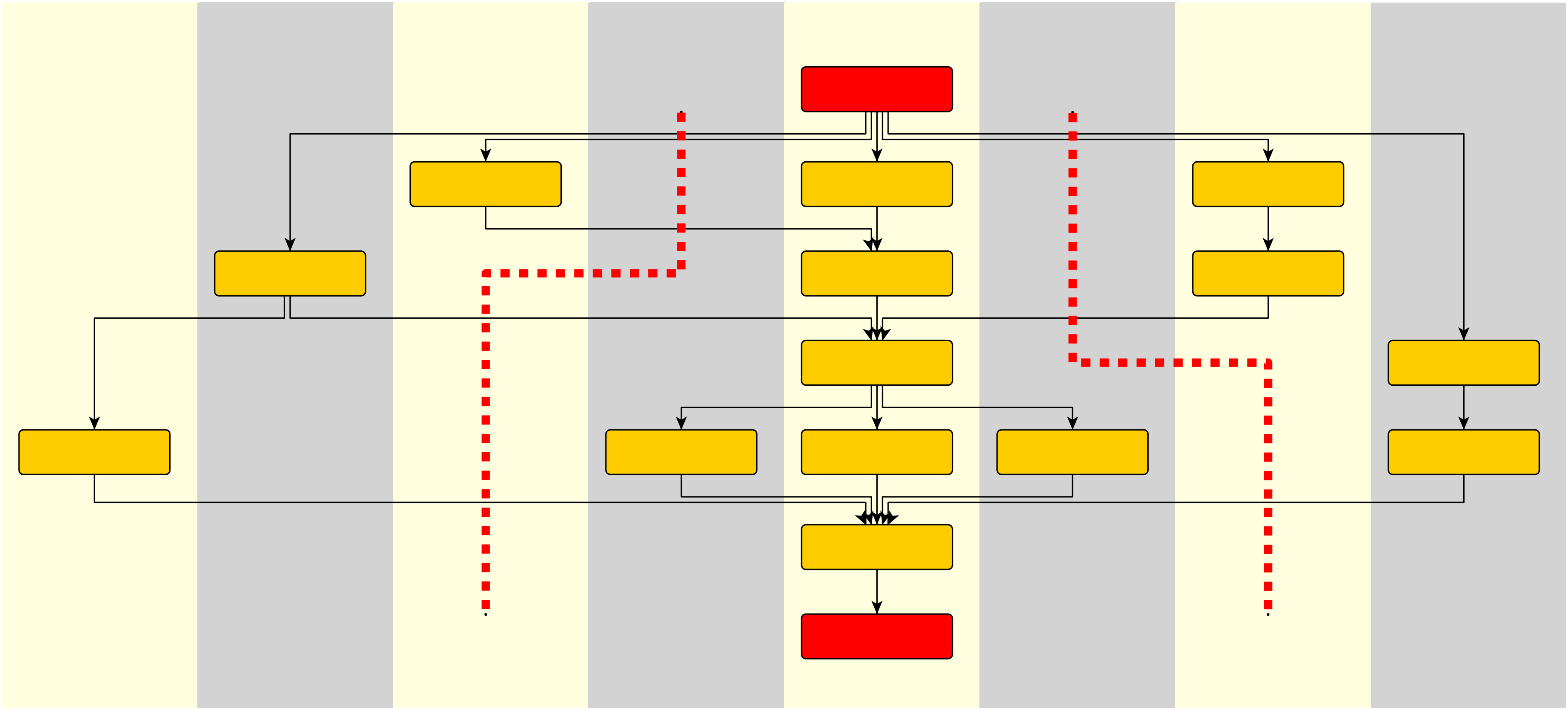

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Metrics – Compute final coordinates

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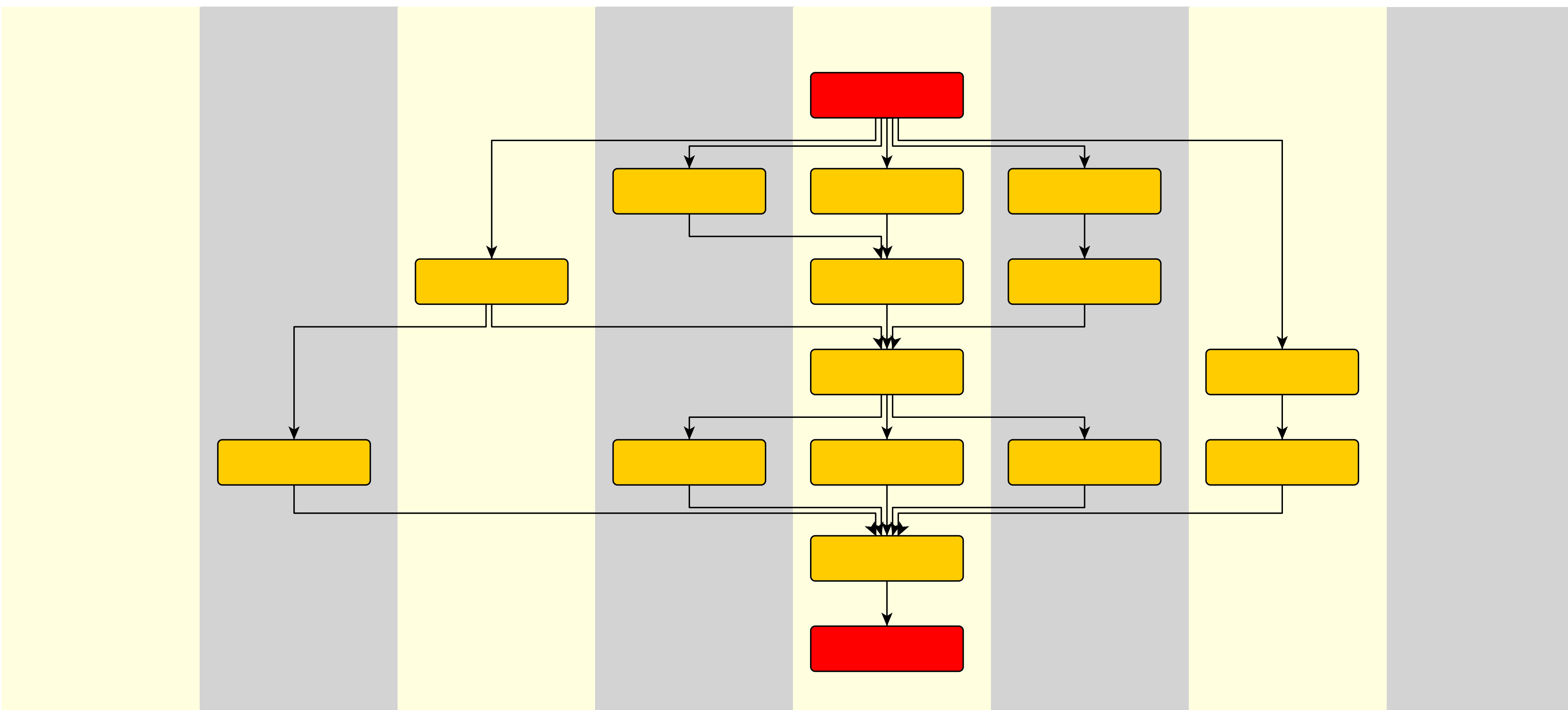

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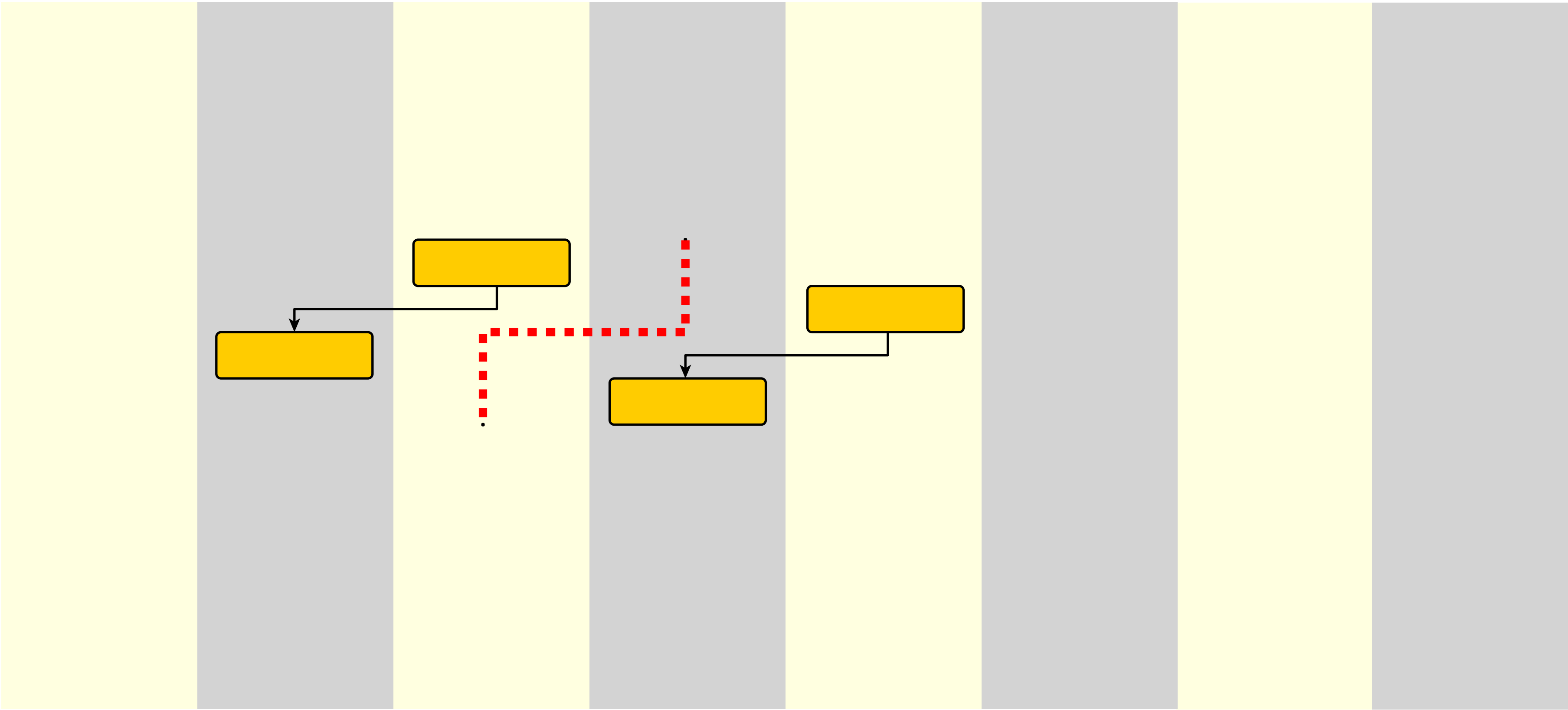
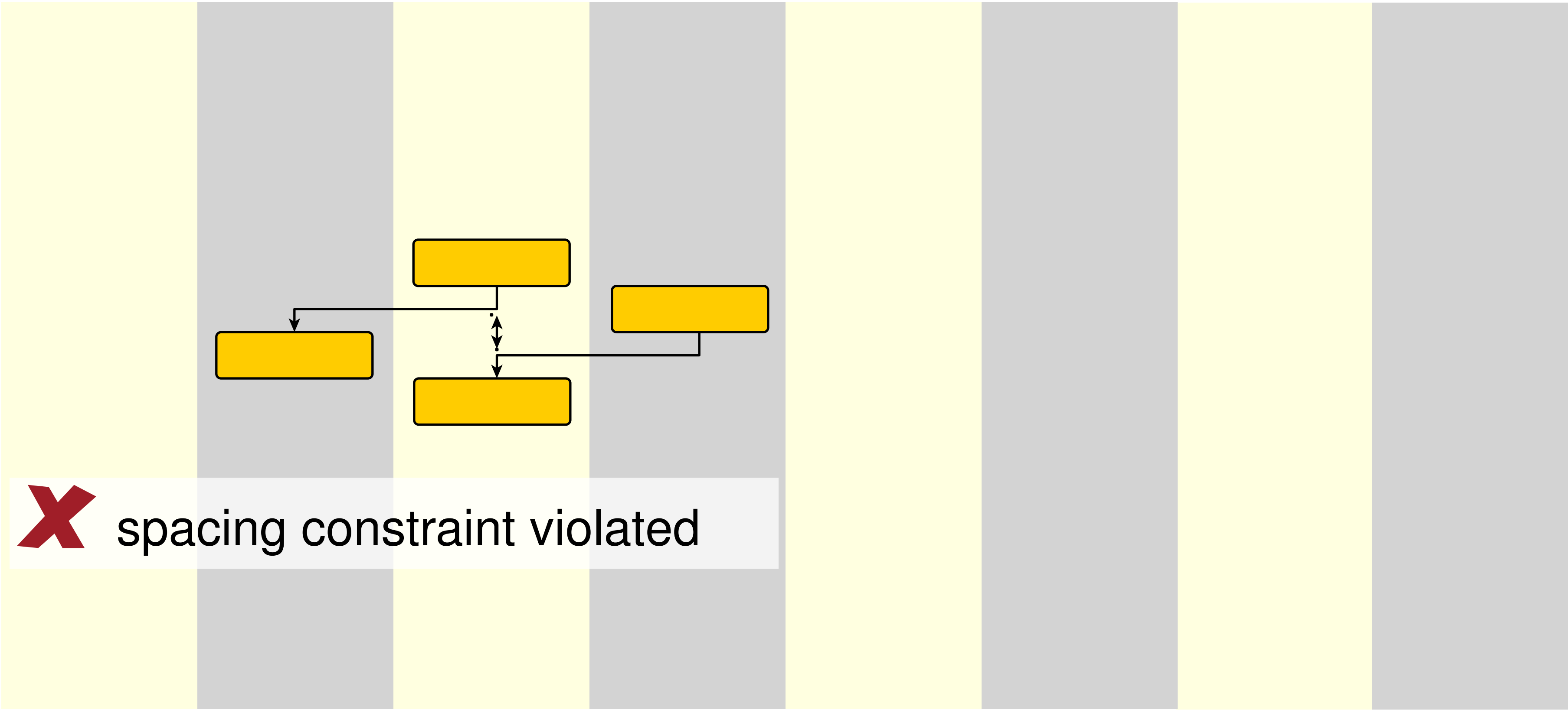
goal: minimize total horizontal edge length



Metrics – Compute final coordinates

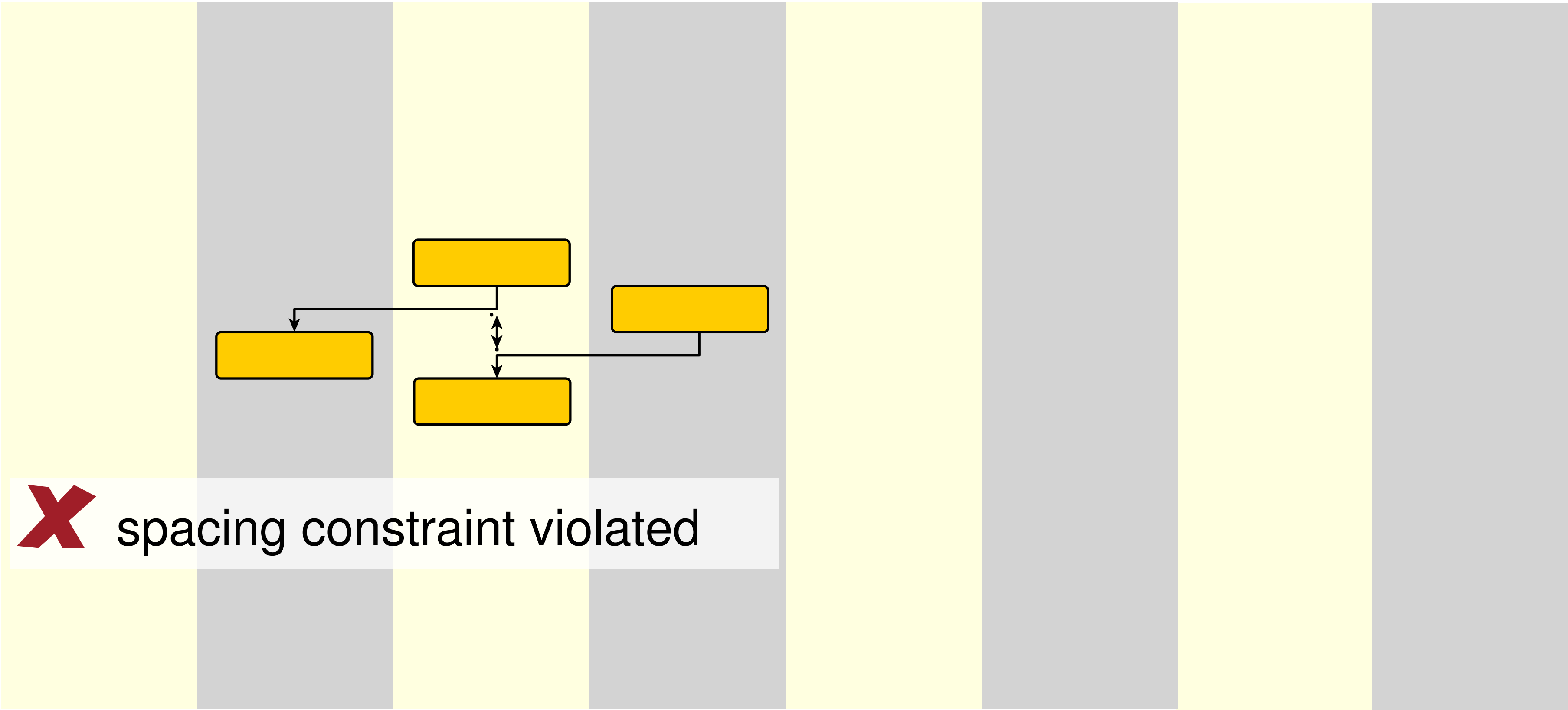
goal: minimize total horizontal edge length



Metrics – Compute final coordinates

goal: minimize total horizontal edge length

idea:
- determine compaction paths
- compact along them



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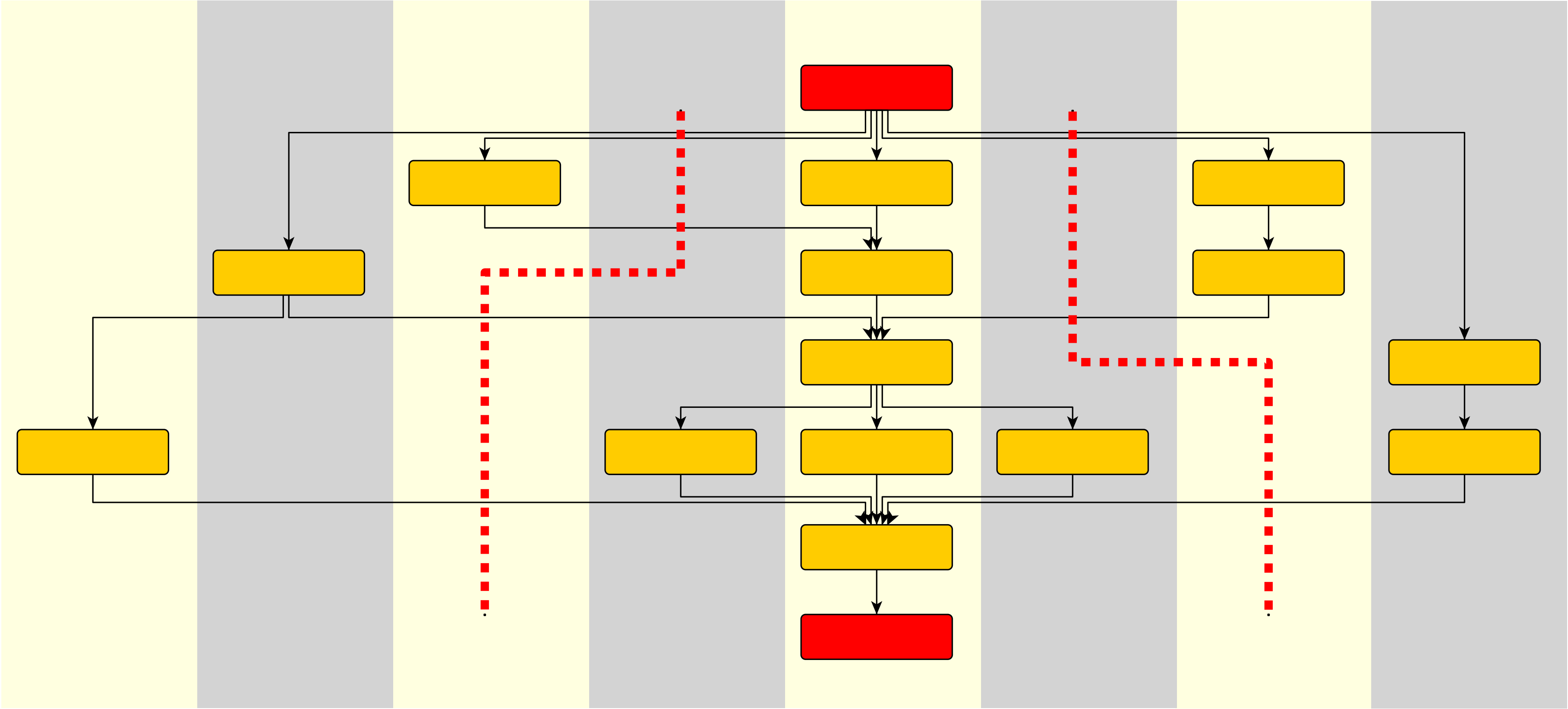

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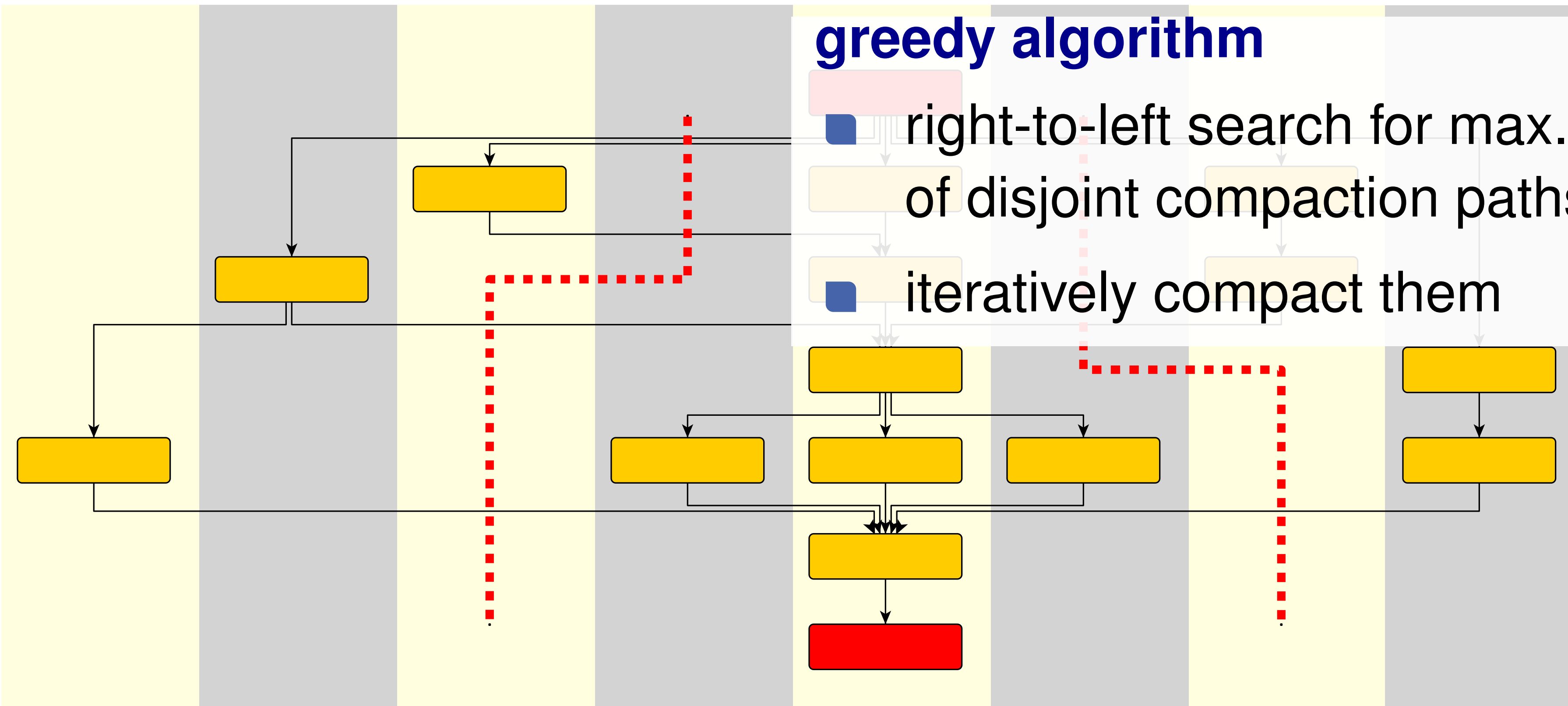

greedy algorithm
- right-to-left search for max. set of disjoint compaction paths
- iteratively compact them

Metrics – Compute final coordinates

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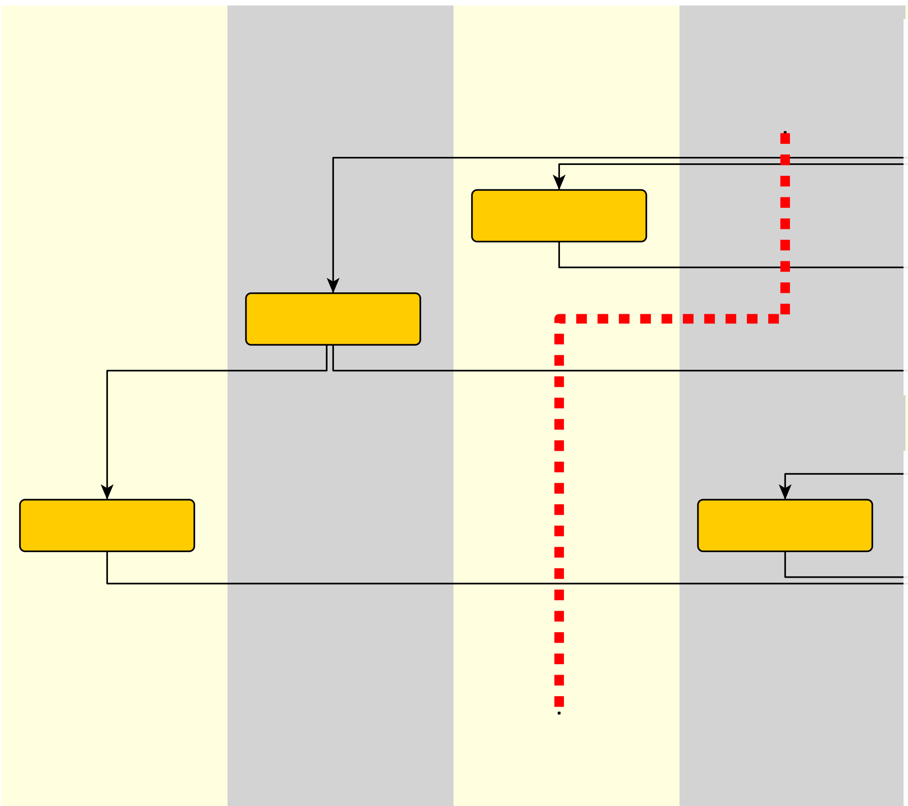

greedy algorithm
- right-to-left search for max. set of disjoint compaction paths
- iteratively compact them

Theorem 4
The greedy algorithm requires $O(mb(b + \log n))$ time.

b = #columns

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Our Approach – Theoretical Results

Quality Guarantees

- Vertical alignment of predecessors ✓
- Local symmetry ✓
- At most four bends per edge ✓
- Spacing constraints ✓
- Edge Bundling ✓

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- Vertical alignment of predecessors ✓
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Theorem 5
There exists an algorithm that computes a column-based layout in $O(m^2(n+b))$ time.

b = #columns

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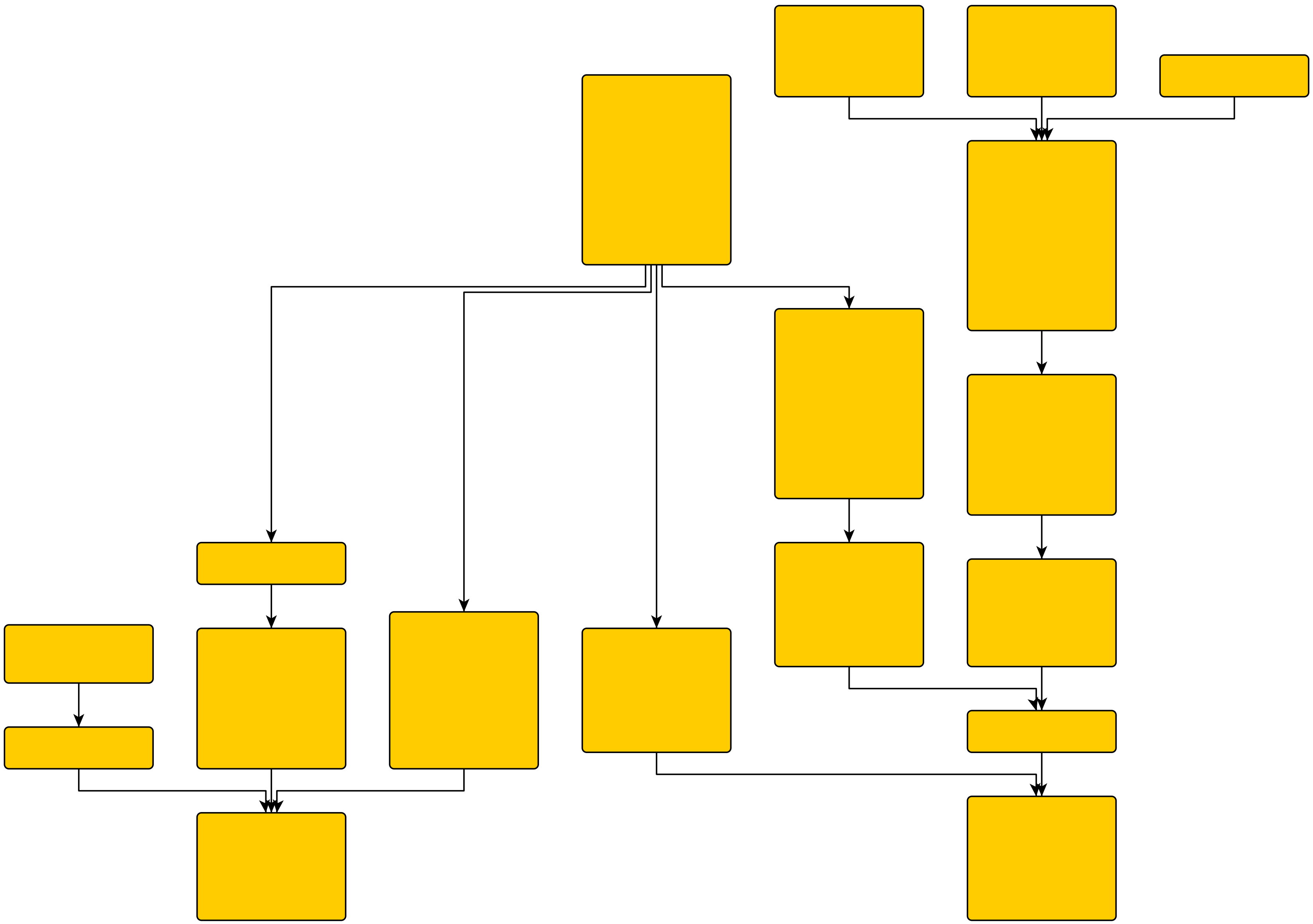
Experimental Evaluation

Experimental evaluation with 51 argument maps

- avg. number of nodes $\leq$ 40 nodes
- max. degree $\leq$ 6
- 93% planar und acyclic graphs

Experimental Evaluation



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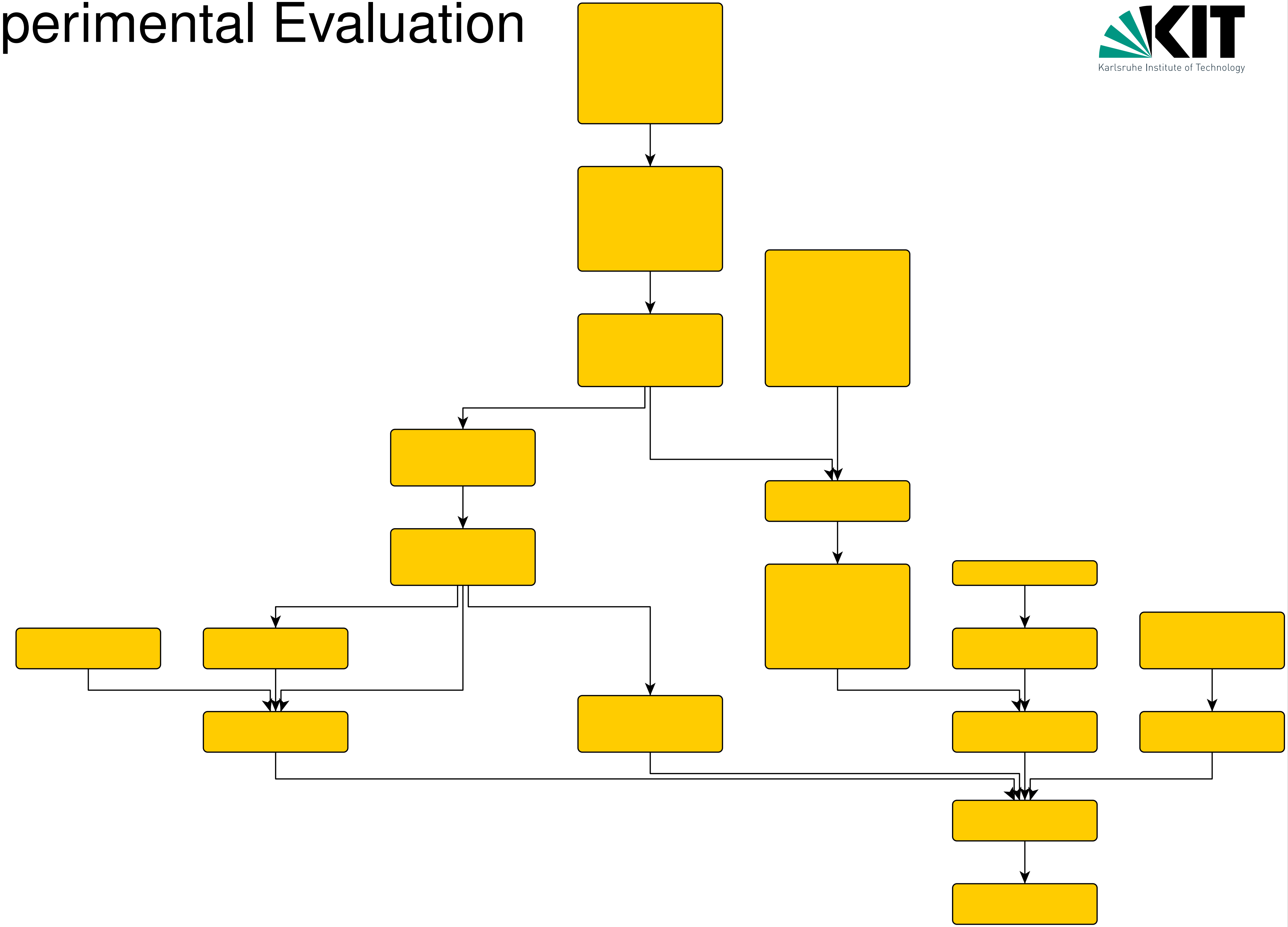

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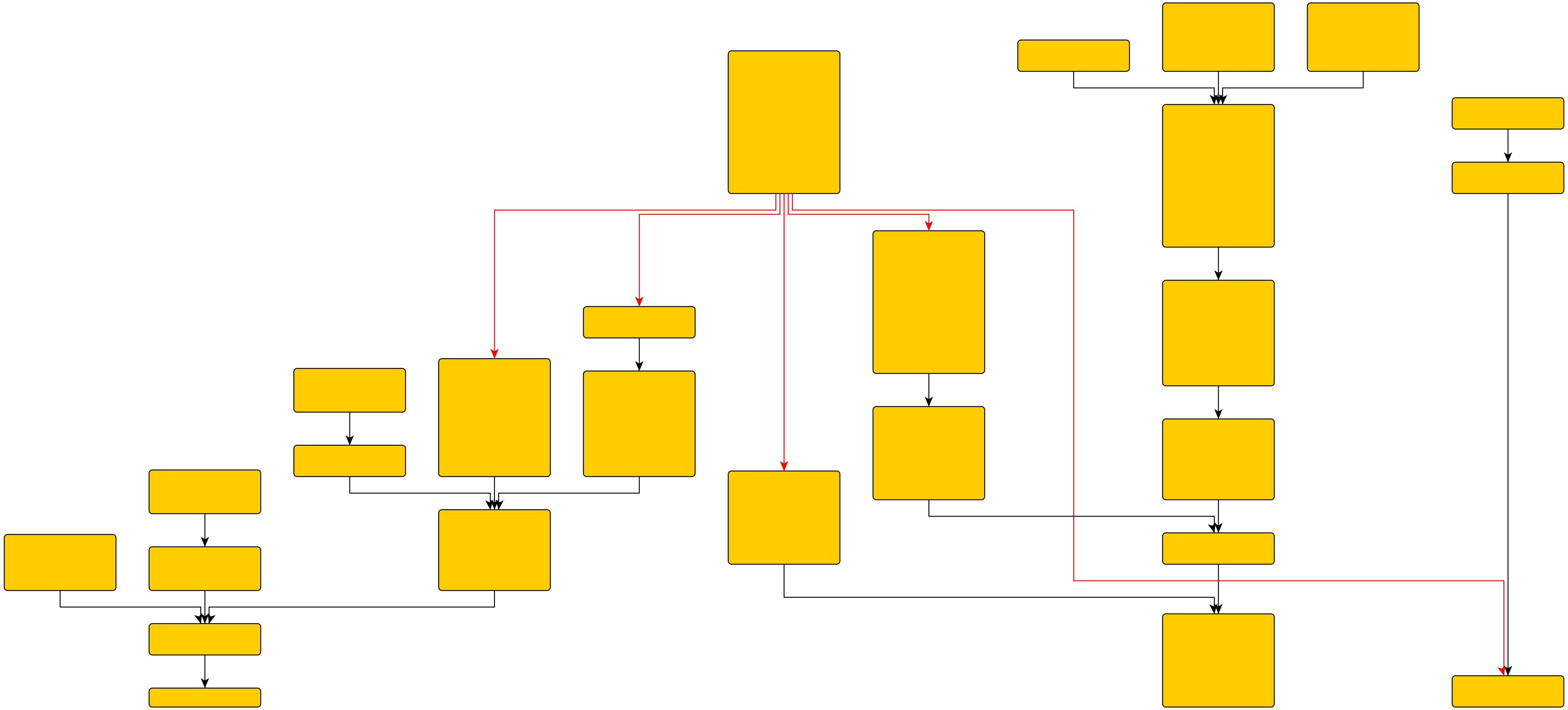

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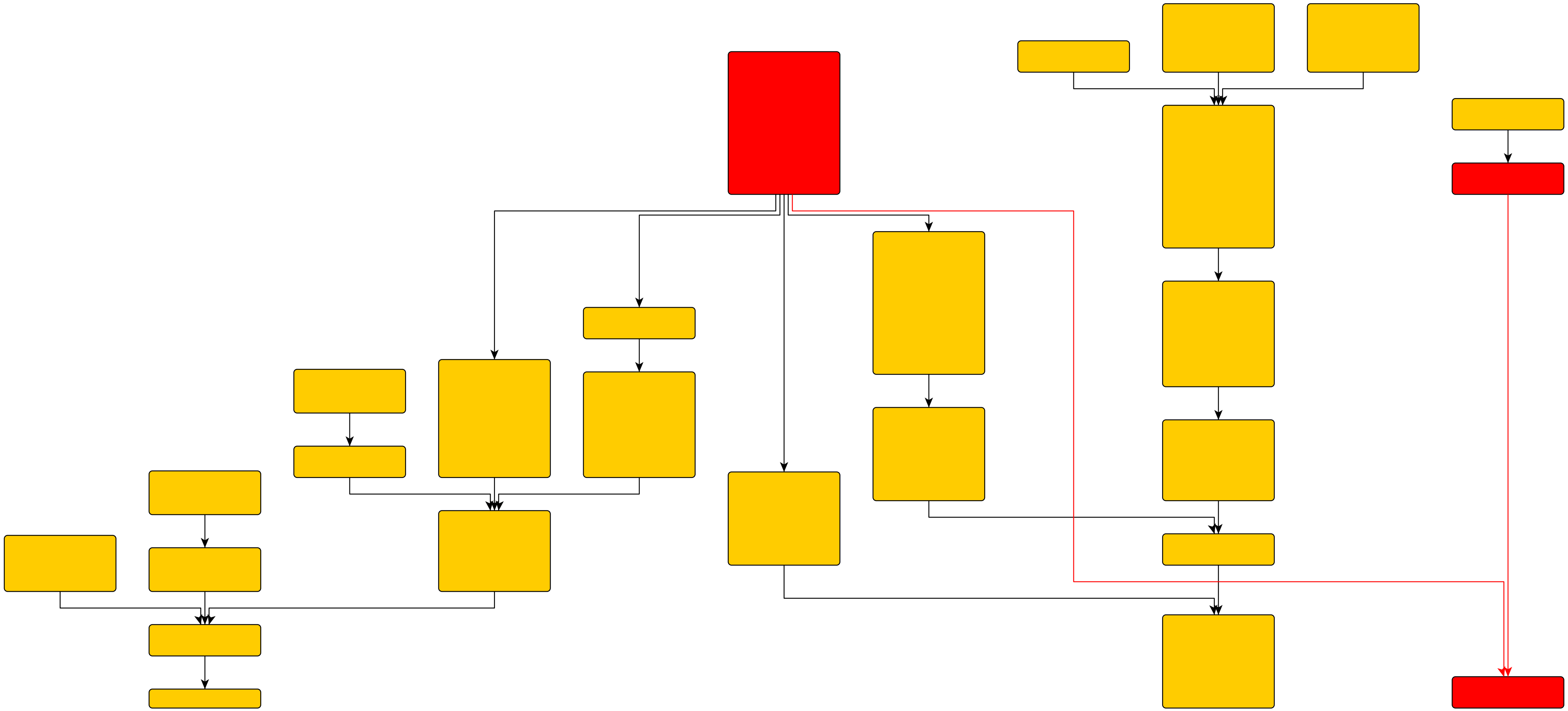

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Statistics:

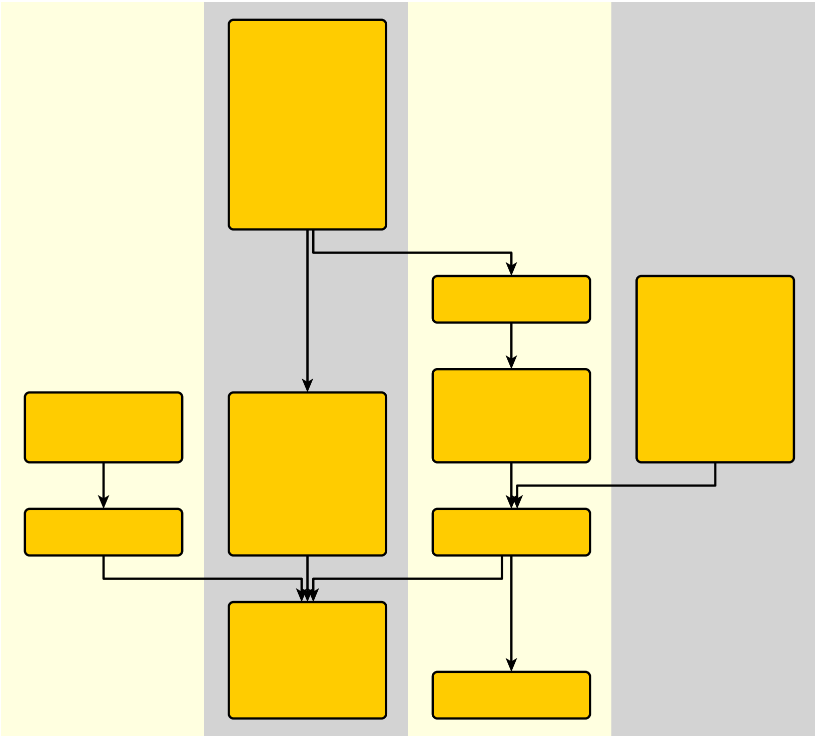
- avg. number of bends per edge: 1.06
- max. runtime: 25s [134 nodes, 158 edges]
- avg. runtime: 0.77s

the topology step requires [on avg.] 99.6% of the runtime

Conclusion

Topology-Shape-Metrics [Tamassia ’87]

topology	compute embedding of the graph
shape	assign bends to edges
metric	determine edge lengths & node positions



Column-based Graph Layouts

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Prof. Dr. Dorothea Wagner

Conclusion

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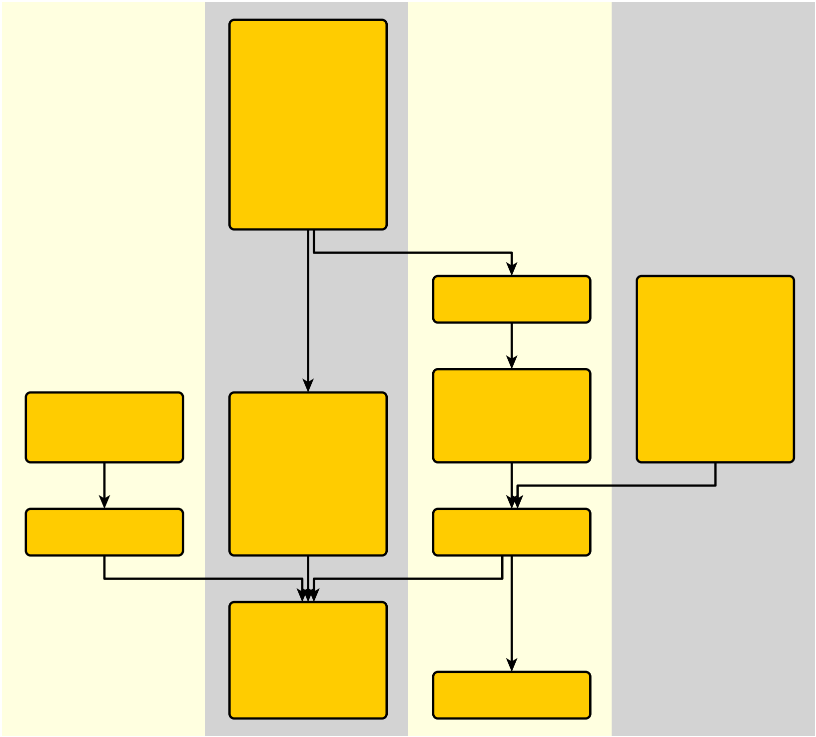
relaxation of topology



Column-based Graph Layouts

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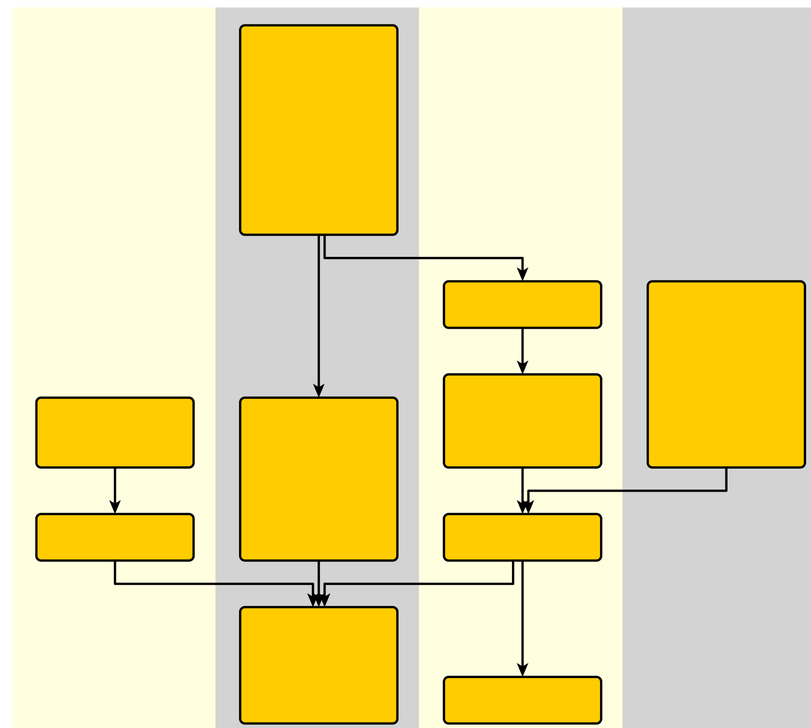
Our Approach

- clean and structured layouts
- reasonable running time



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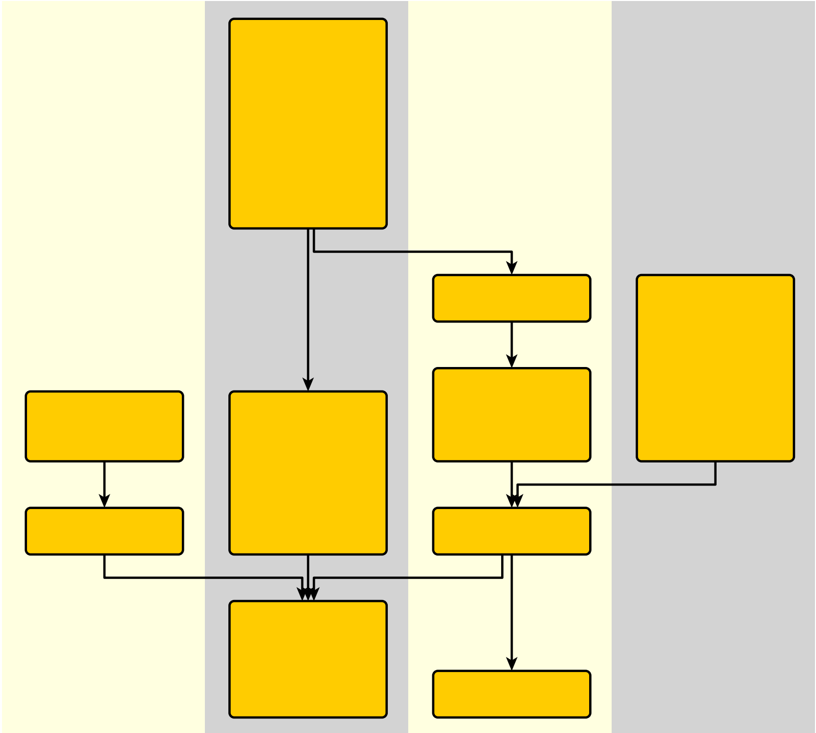
Future Work

- incremental layouts



Column-based Graph Layouts

Conclusion

Karlsruhe Institute of Technology

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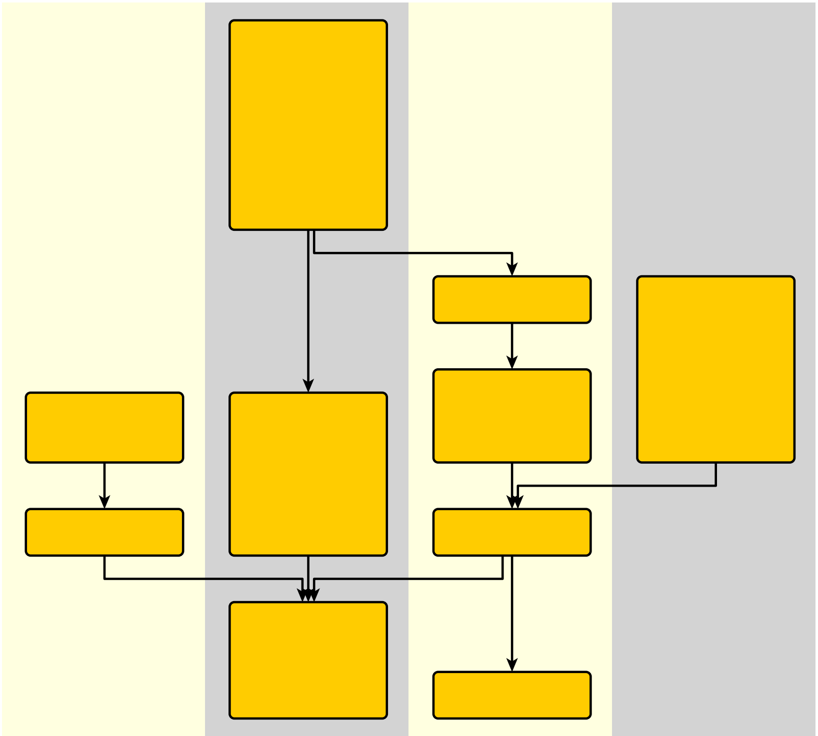
Thank you!



Column-based Graph Layouts