Karlsruhe Institute of Technology

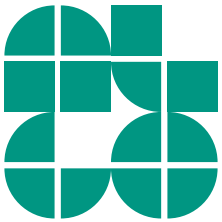
Algorithmen zur Visualisierung von Graphen

Automatisches Zeichnen von Linienplänen

INSTITUT FÜR THEORETISCHE INFORMATIK · FAKULTÄT FÜR INFORMATIK

Tamara Mchedlidze · **Martin Nöllenburg**

28.01.2014



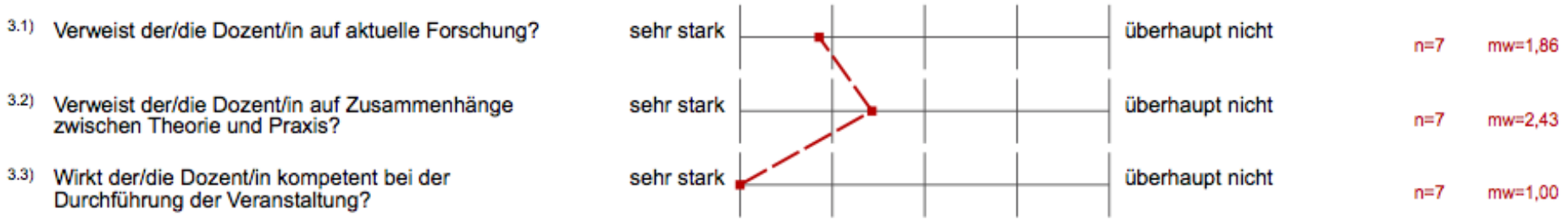
Evaluationsergebnisse

1. Fragen zur Lehrveranstaltung

Evaluationsergebnisse

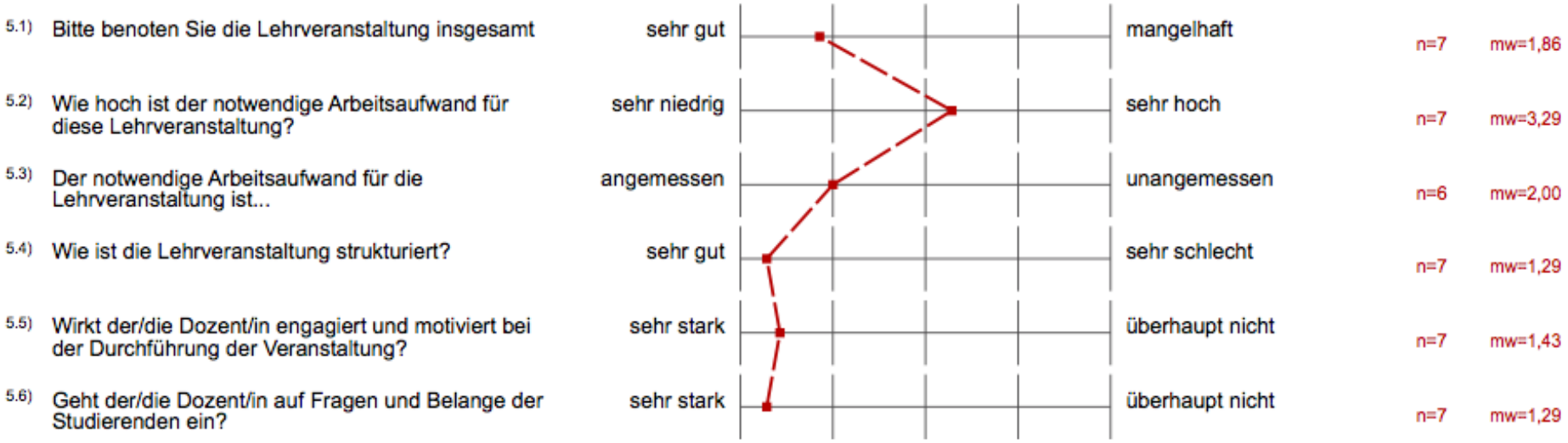
2. Fragen zur Bewertung der Raumbedingungen

3. Fragen zum/zur Dozenten/in



Evaluationsergebnisse

5. Monitoring



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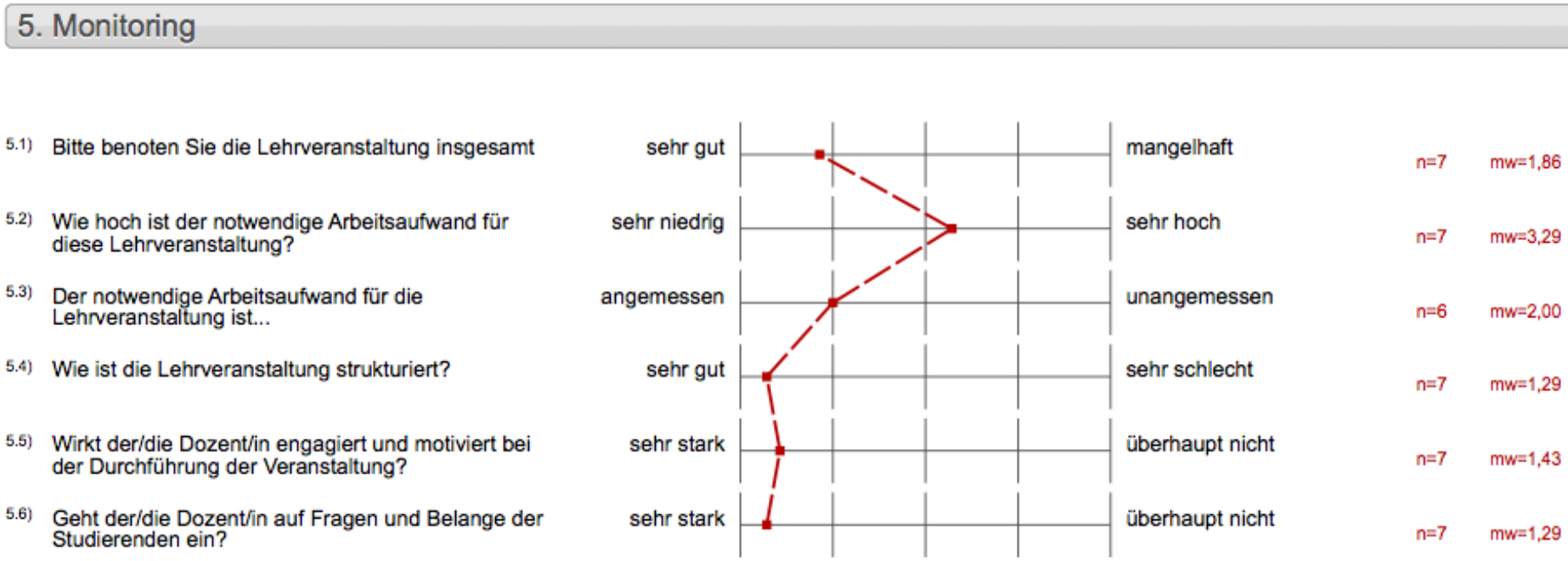

gut gefallen:

- Projekt (2-mal)
- Folien + Tafel
- Folien + Skript
- Beweise

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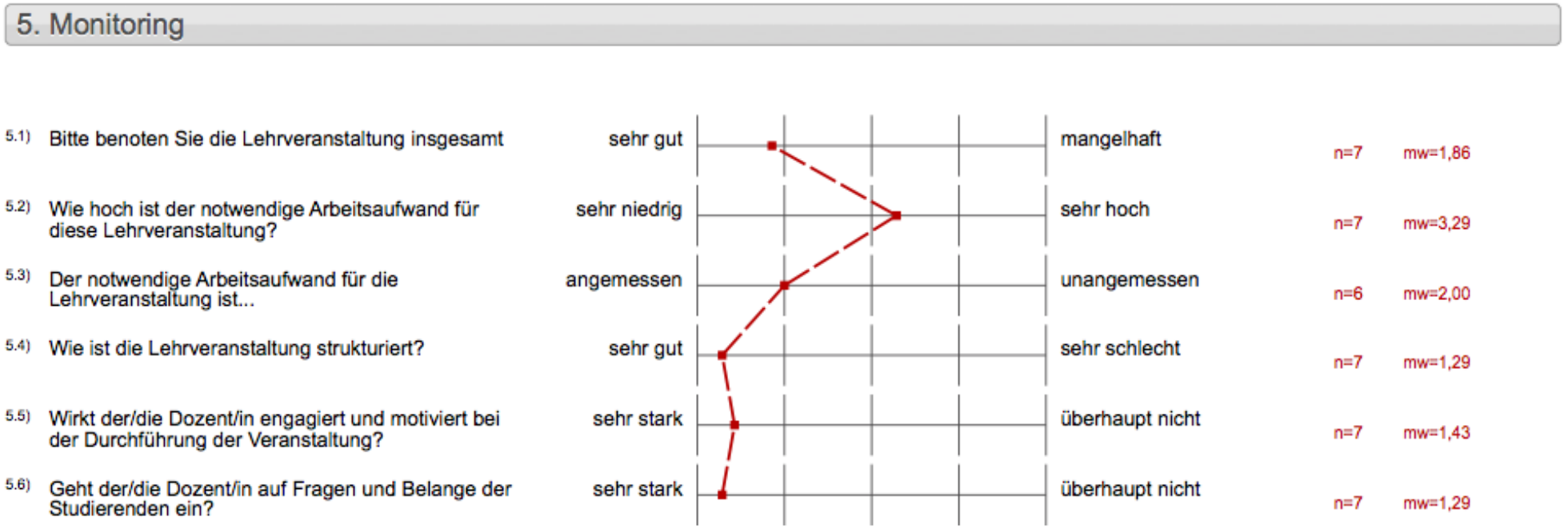

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weniger gut gefallen:

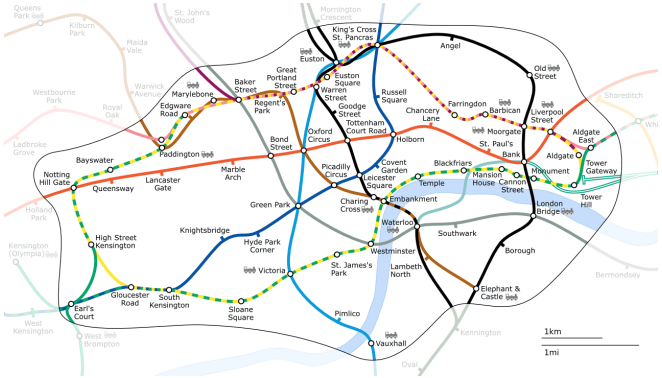
- Musterlösungen für Übungsblätter (2-mal)
- Stoff sehr theoretisch
- Übungstermin

Metro Maps Outline

1. Modeling the Metro Map Problem
   - What is a Metro Map?
   - Hard and Soft Constraints
2. NP-Hardness: Bad News—Nice Proof
   - Rectilinear vs. Octilinear Drawing
   - Reduction from PLANAR 3-SAT
3. MIP Formulation & Experiments
   - Mixed-Integer Programming Formulation
   - Labeling
   - Experiments

Metro Maps Outline

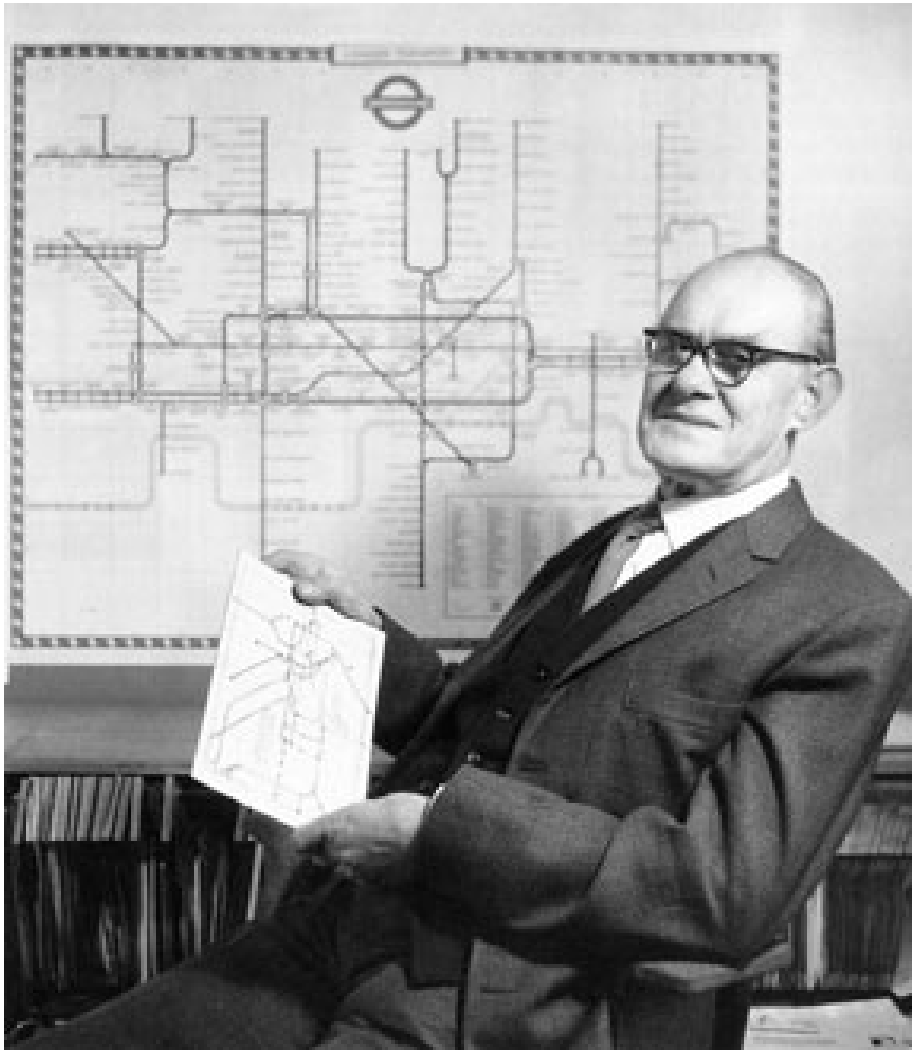
What is a Metro Map?



- schematic diagram for public transport
- visualizes lines and stations
- goal: ease navigation for passengers
  - “How do I get from A to B?”
  - “Where to get off and change trains?”
- distorts geometry and scale
- improves readability
- compromise between
  schematic road map $\leftrightarrow$ abstract graph

Why Automate Drawing Metro Maps?

- current maps designed manually



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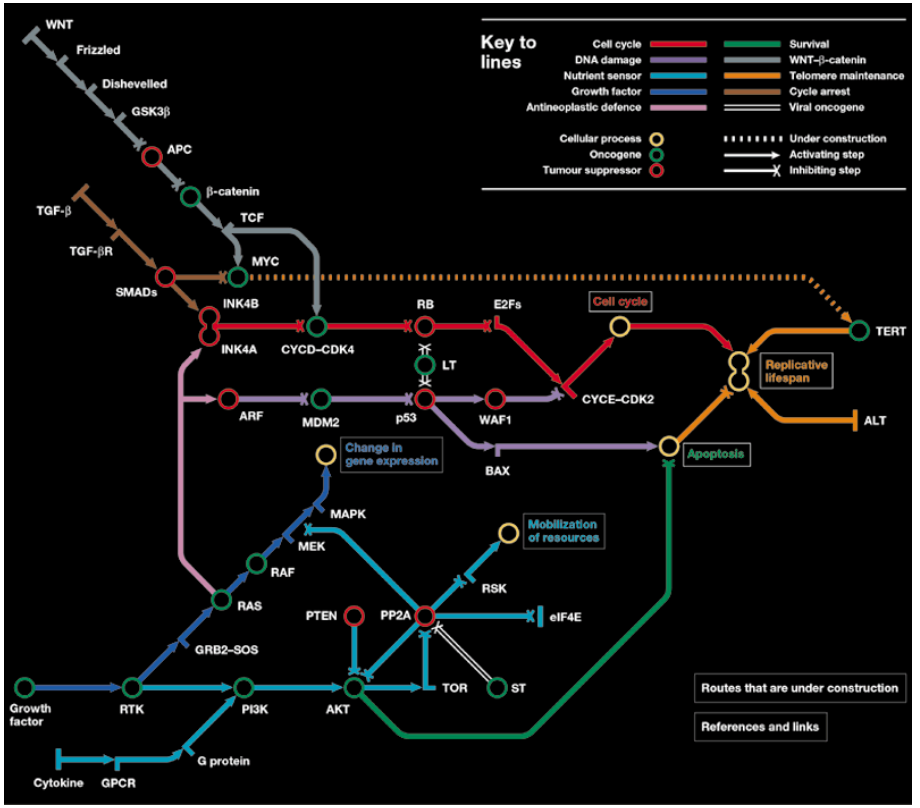

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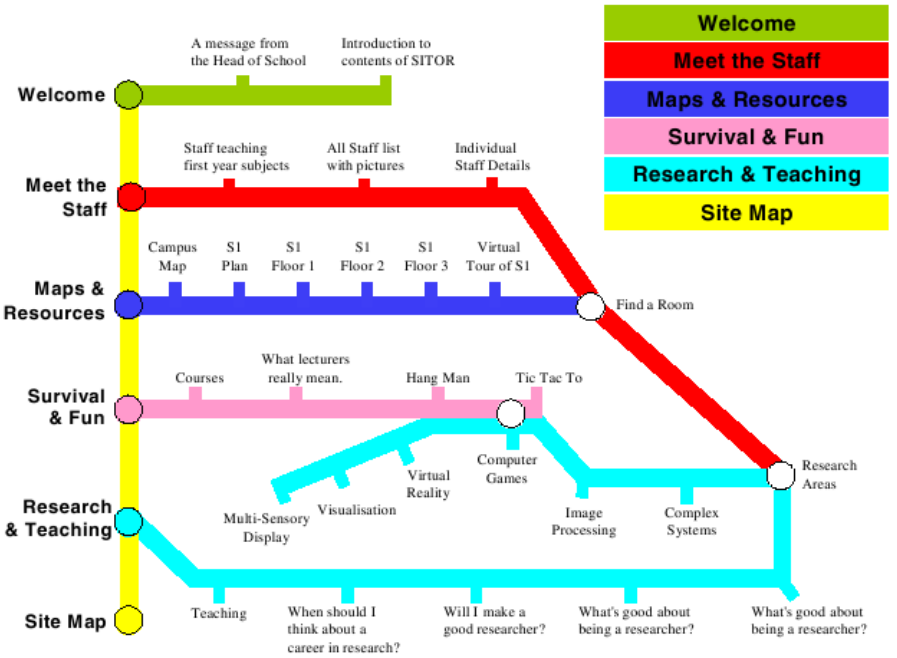

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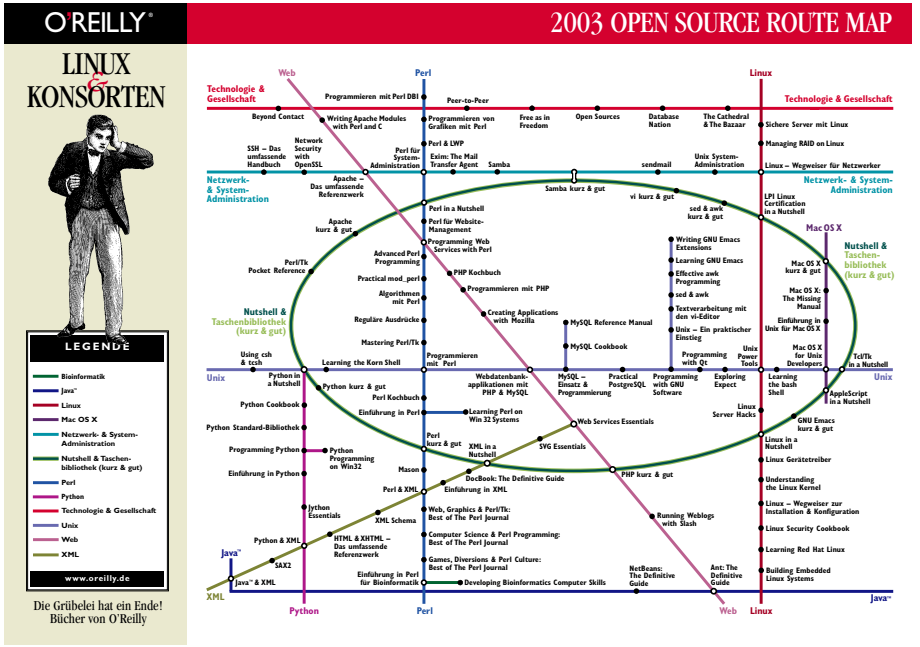

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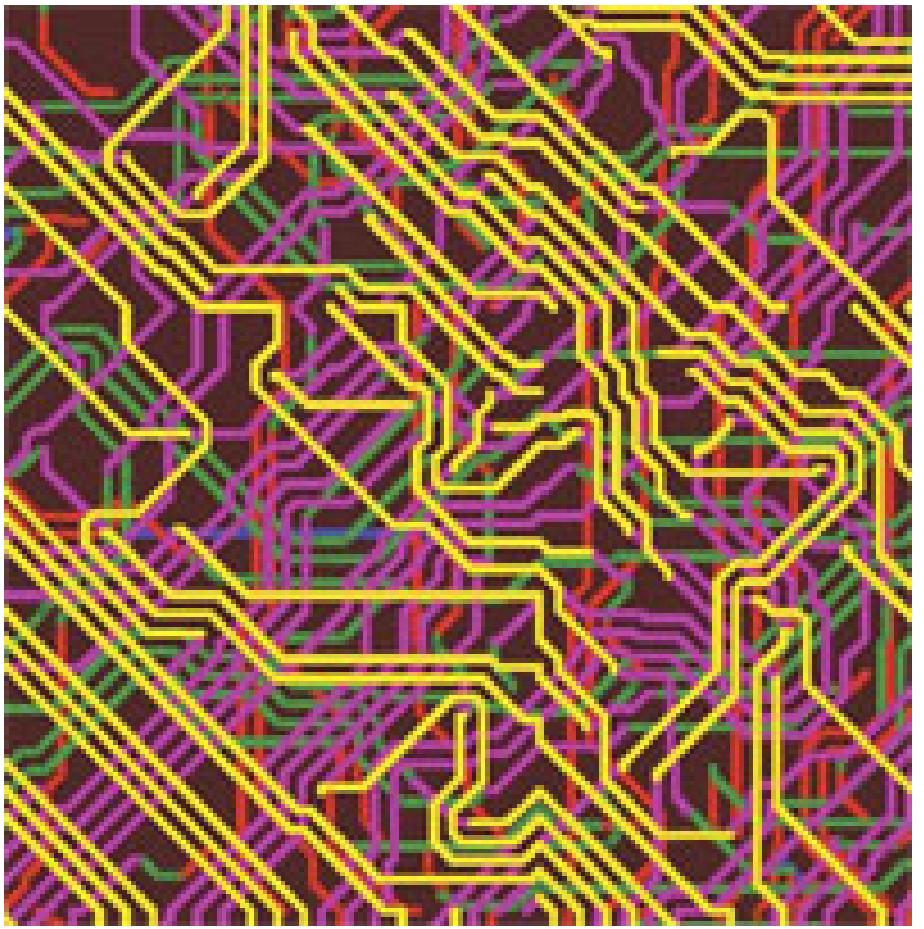
- VLSI: X-architecture



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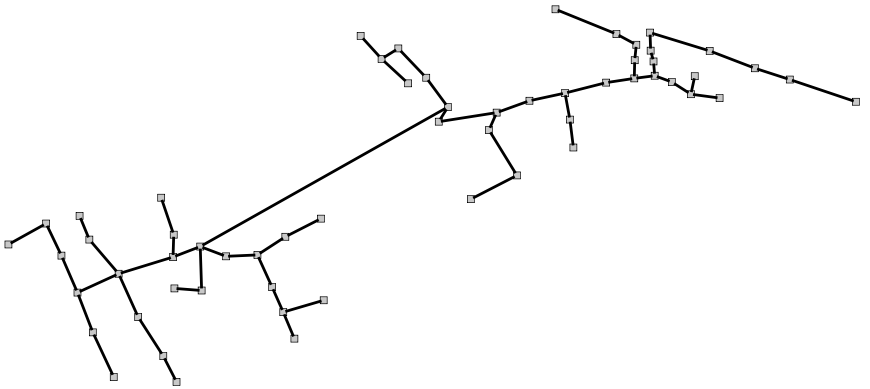
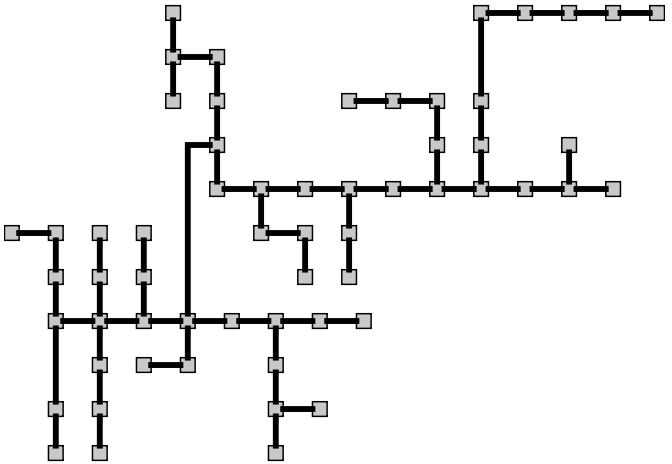
- VLSI: X-architecture
- redrawing sketches [Brandes et al. '03]





More Formally

The Metro Map Problem

Given: planar embedded graph $G = (V, E)$, $V \subset \mathbb{R}^2$,
line cover $\mathcal{L}$ of paths or cycles in G (the metro lines),

Goal: draw G and $\mathcal{L}$ nicely.

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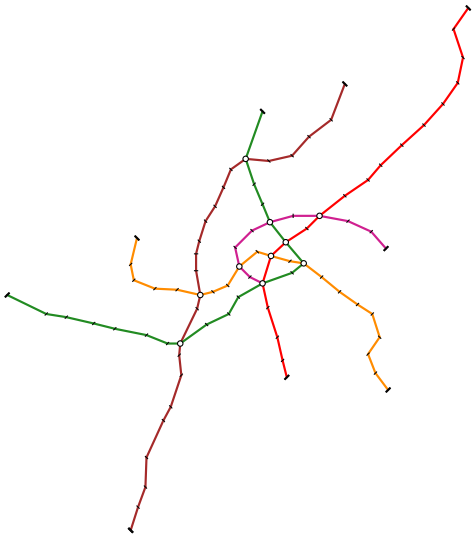
- What is a nice drawing?
- Look at real-world metro maps drawn by graphic designers and model their design principles as
  - *hard* constraints – must be fulfilled,
  - *soft* constraints – should hold as tightly as possible.

Hard Constraints

(H1) preserve embedding of G



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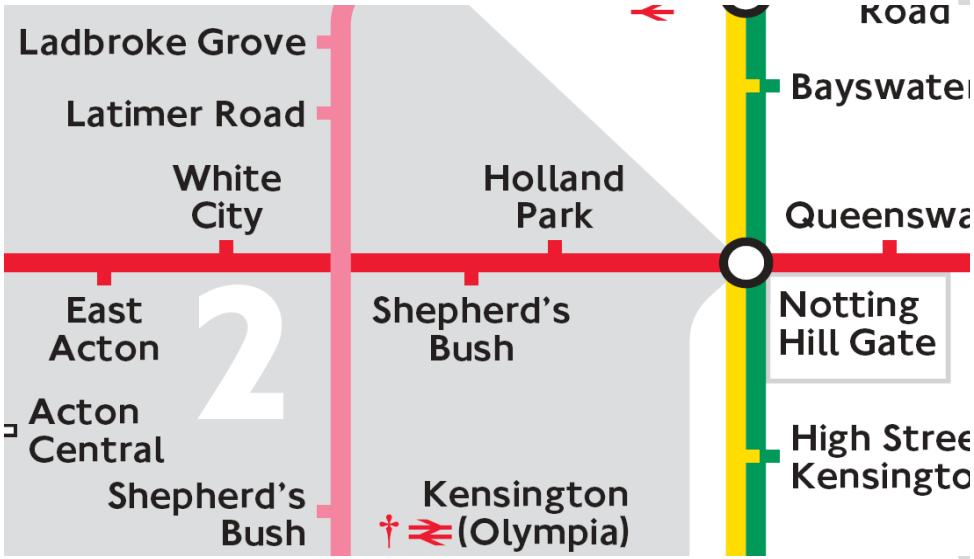
(H4) keep edges d_{min} away from non-incident edges ($\rightarrow$ no crossings)

Soft Constraints

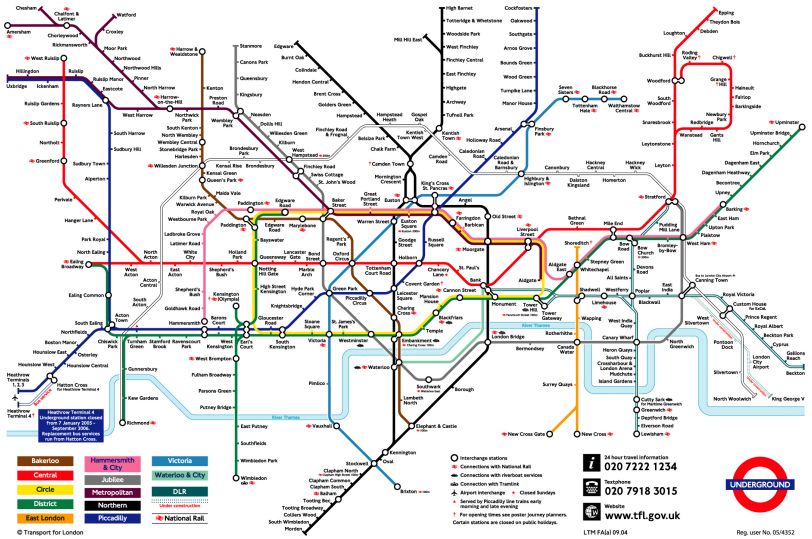
(S1) draw metro lines with few bends



Soft Constraints

(S1) draw metro lines with few bends

(S2) keep total edge length small

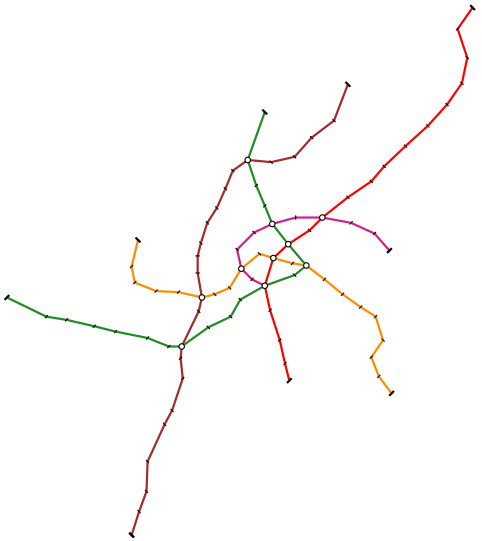


Soft Constraints

(S1) draw metro lines with few bends

(S2) keep total edge length small

(S3) draw each octilinear edge similar to its geographical orientation:
keep relative position of adjacent vertices



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A Related Problem

RectilinearGraphDrawing Decision Problem

Given a planar embedded graph G with max degree 4.
Is there a drawing of G that

- preserves the embedding,
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Our Problem

MetroMapLayout Decision Problem

Given a planar embedded graph G with max degree 8.
Is there a drawing of G that

- preserves the embedding,
- uses straight-line edges,
- is octilinear?

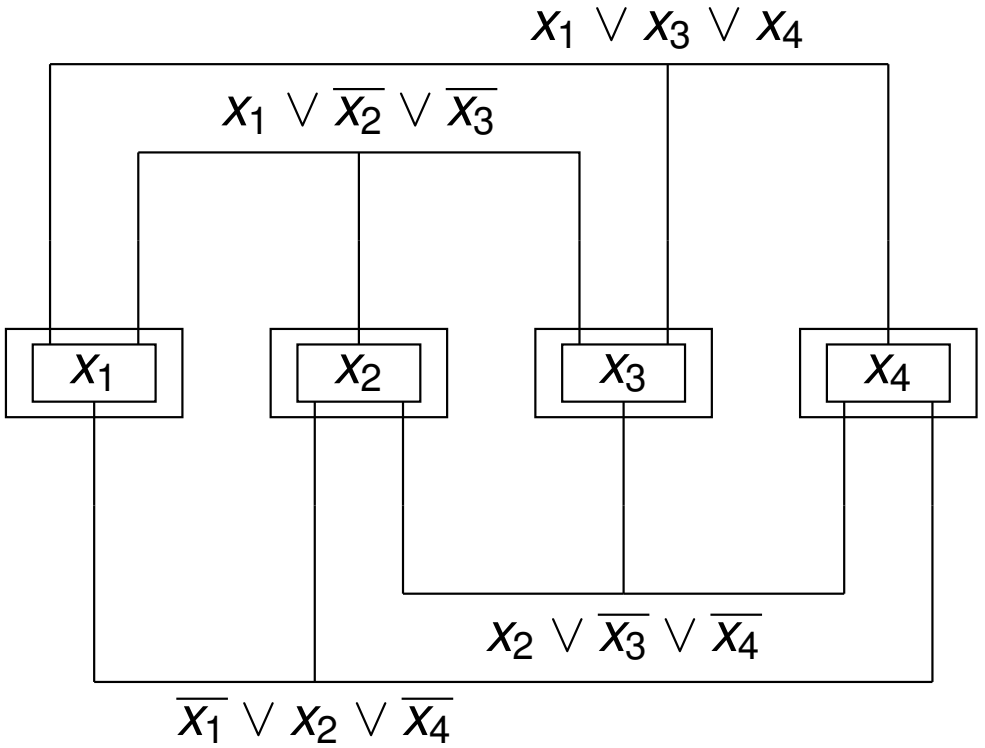
Theorem

MetroMapLayout *is NP-hard.*

Proof.

By Reduction from Planar 3-Sat to MetroMapLayout. □

Outline of the Reduction



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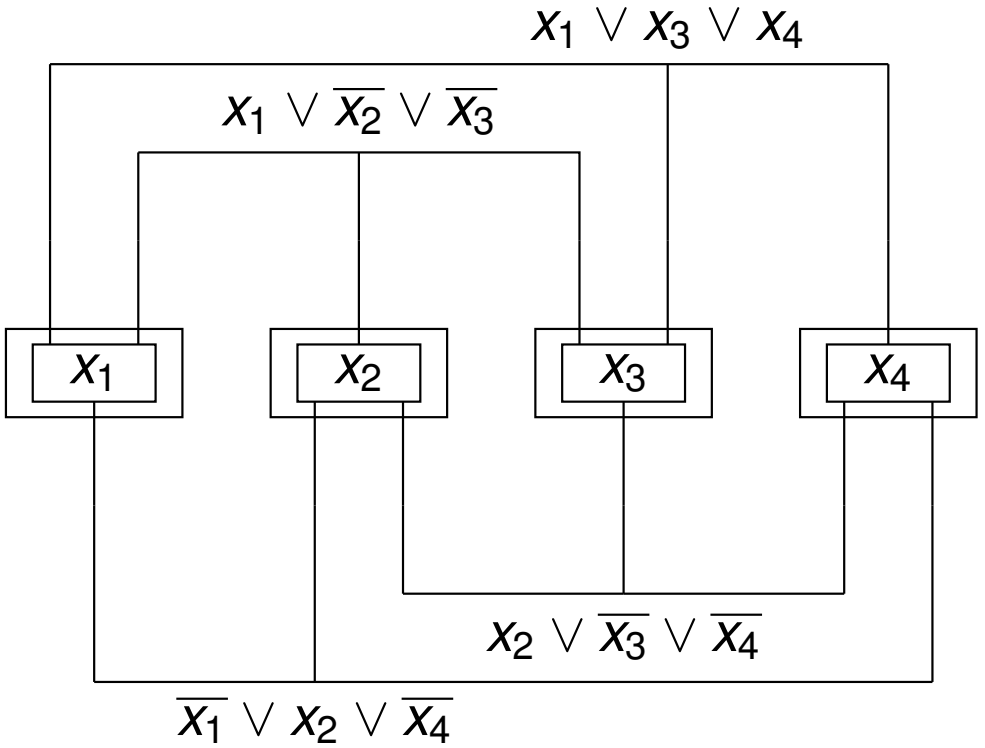

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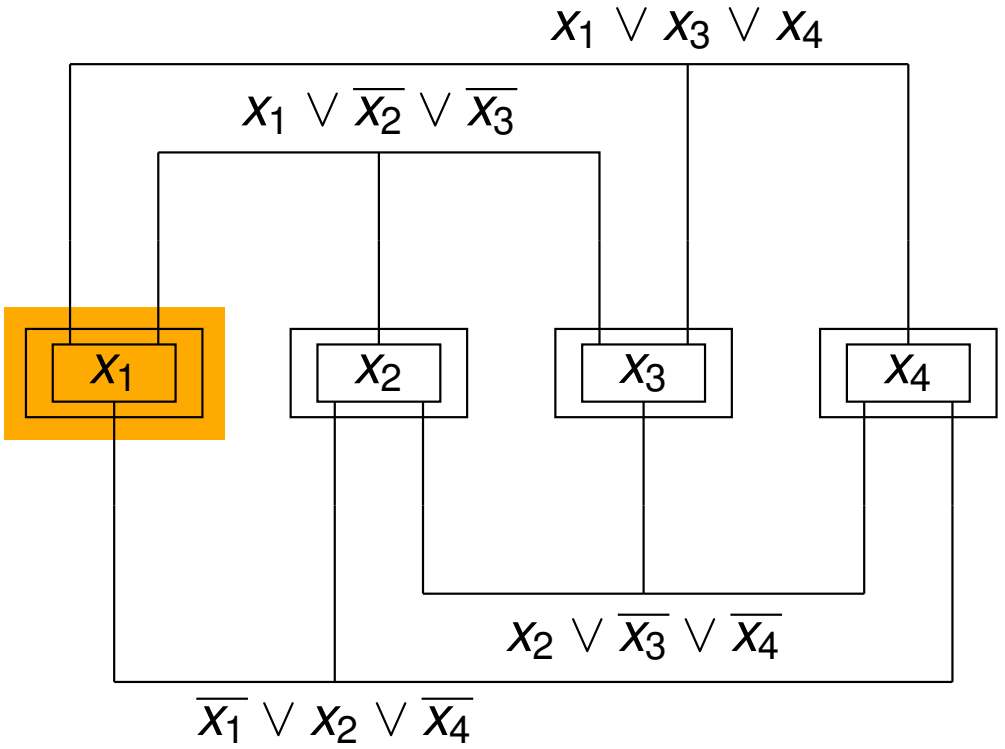
Goal: planar embedded graph G_φ with:

G_φ has a metro map drawing $\Leftrightarrow$ φ satisfiable.

Outline of the Reduction



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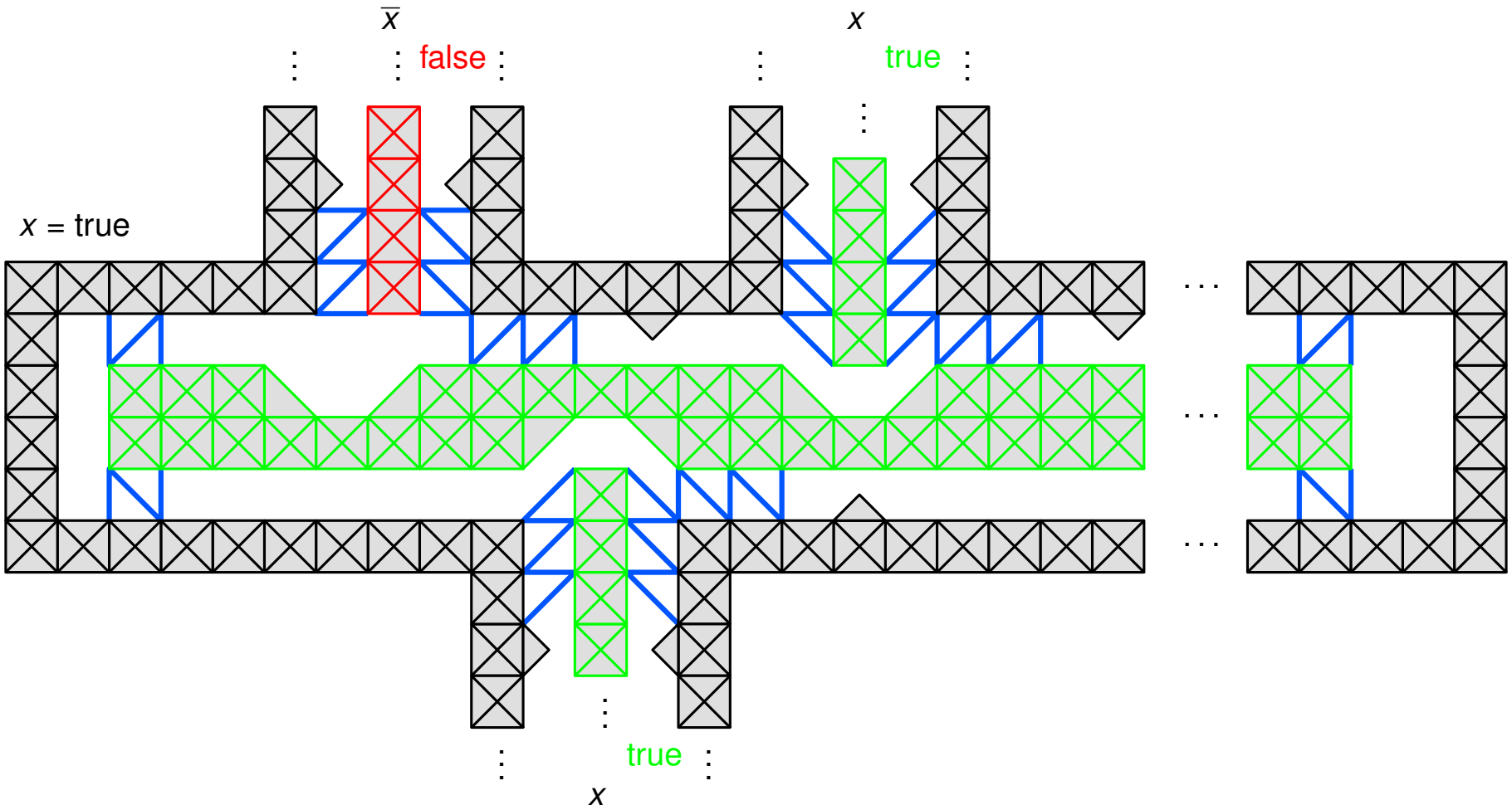

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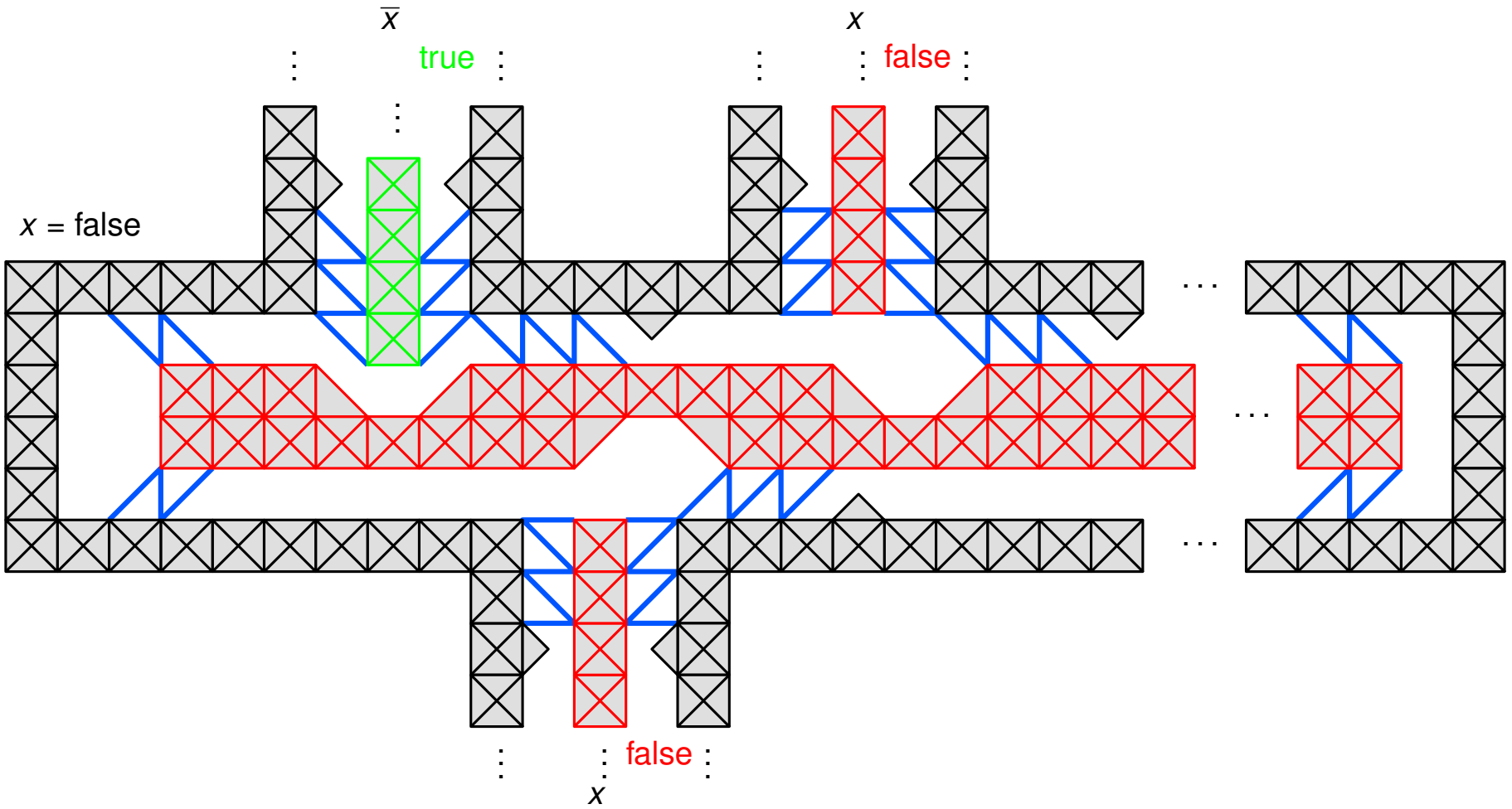

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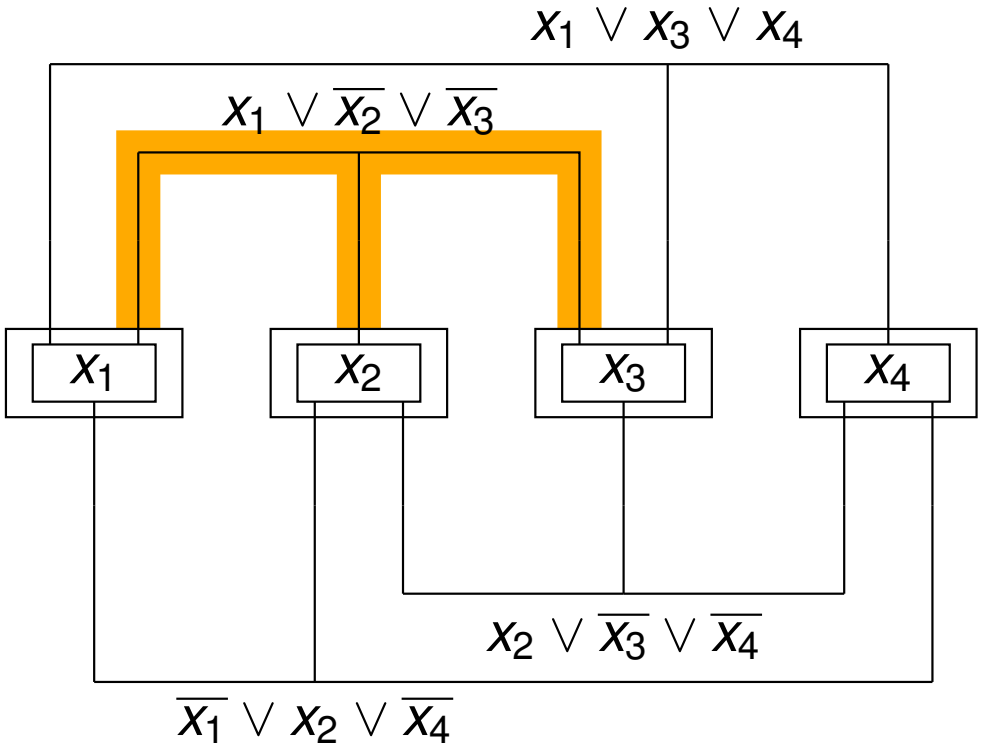

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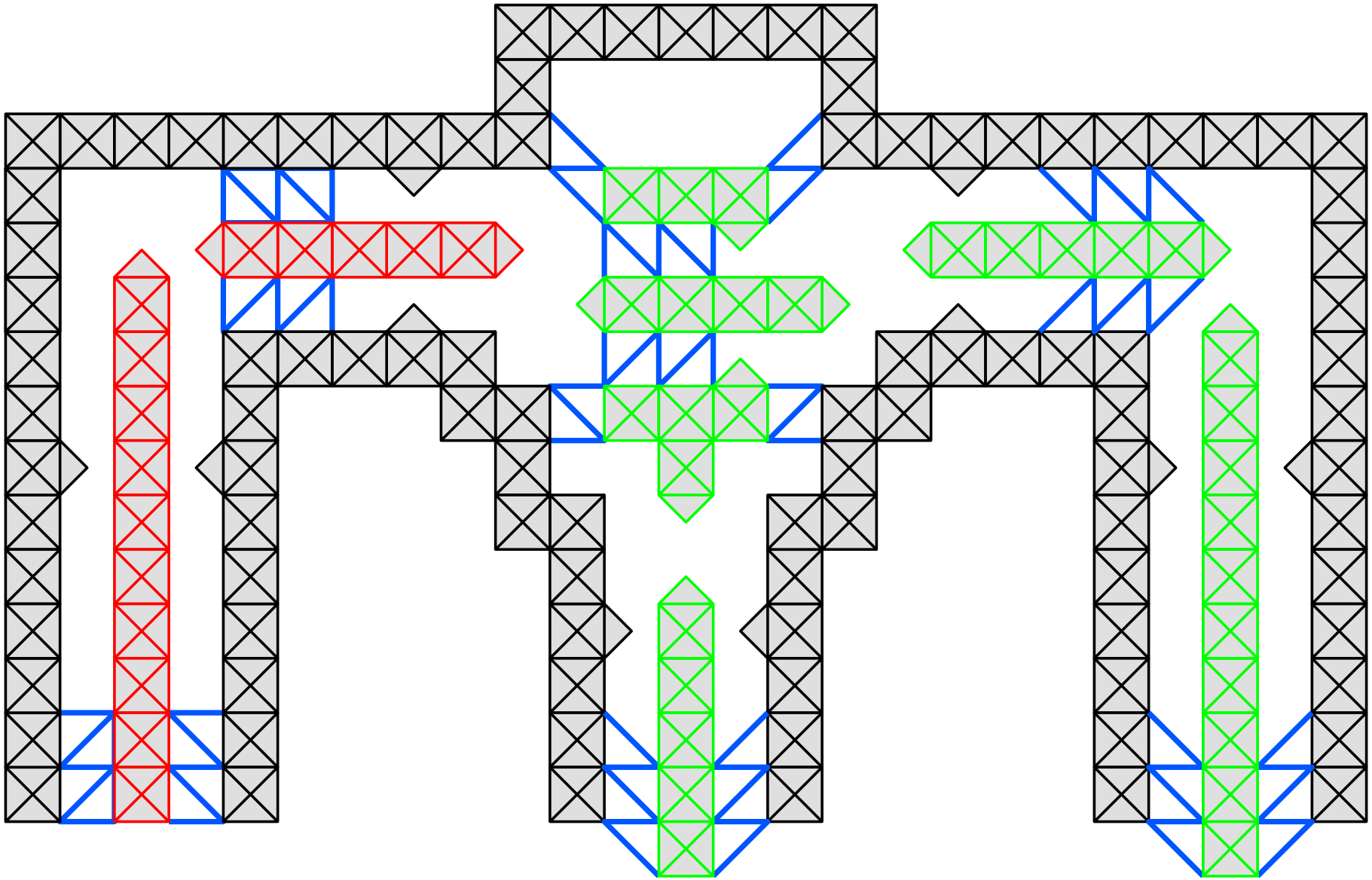

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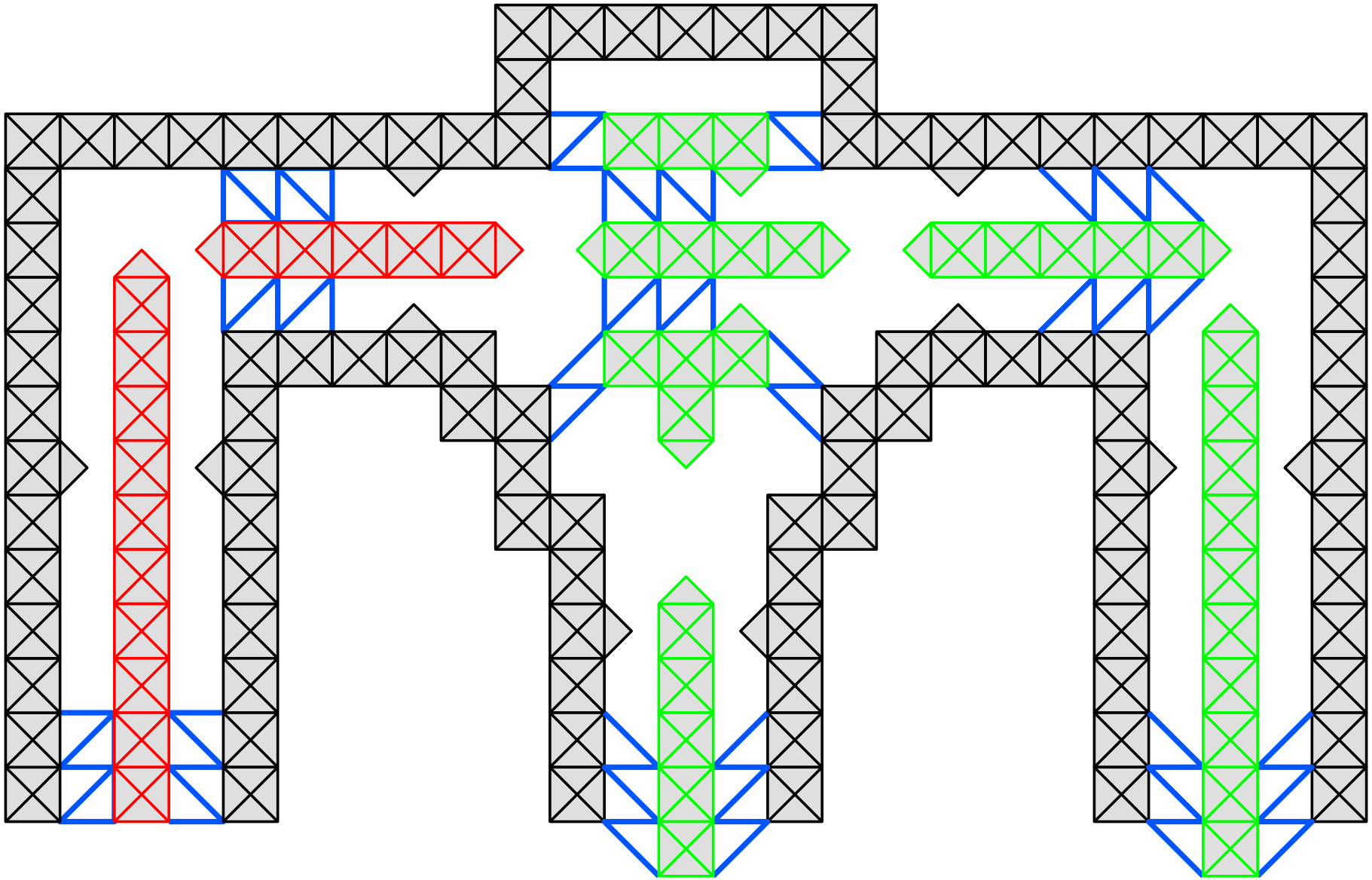

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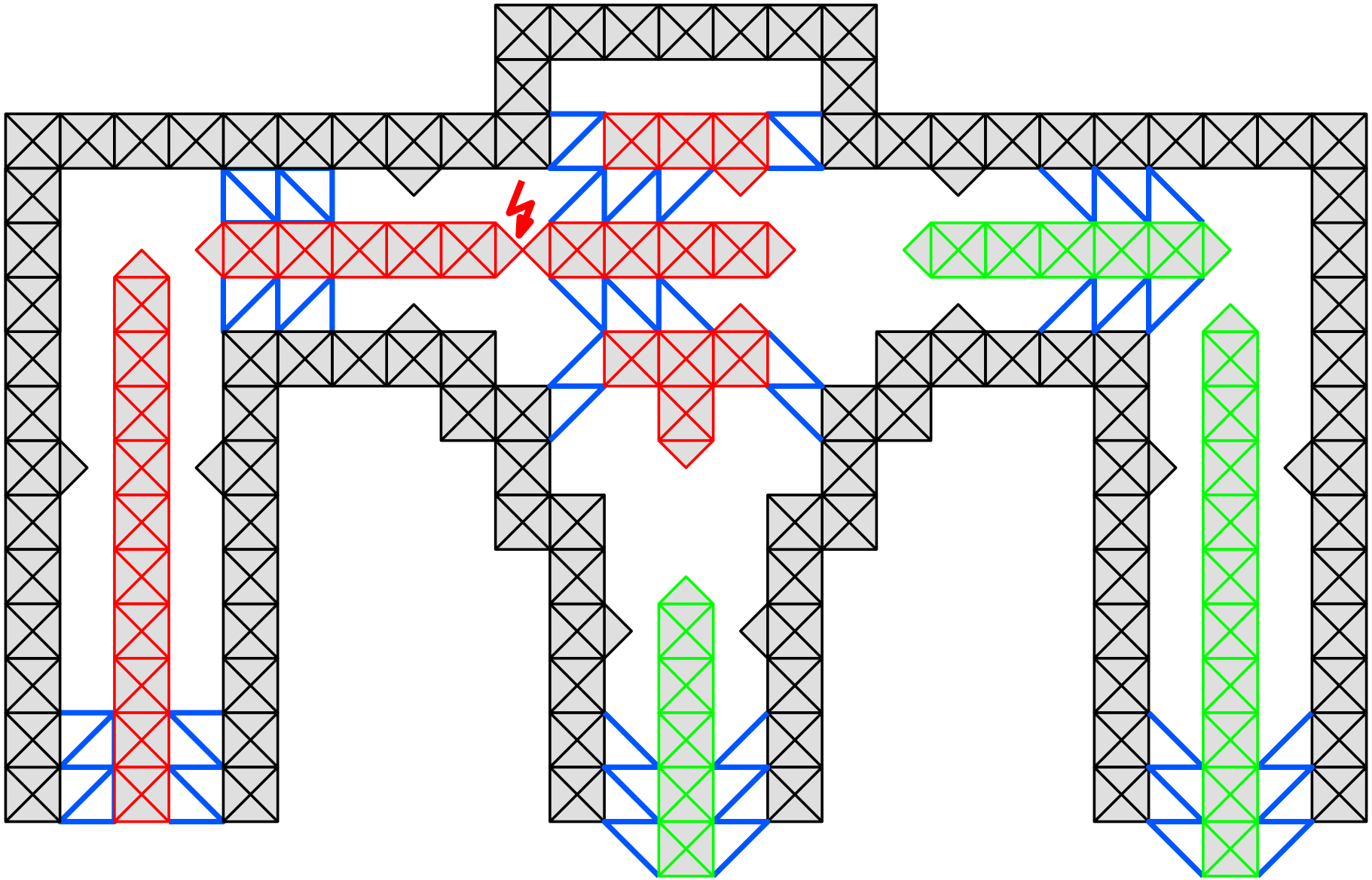

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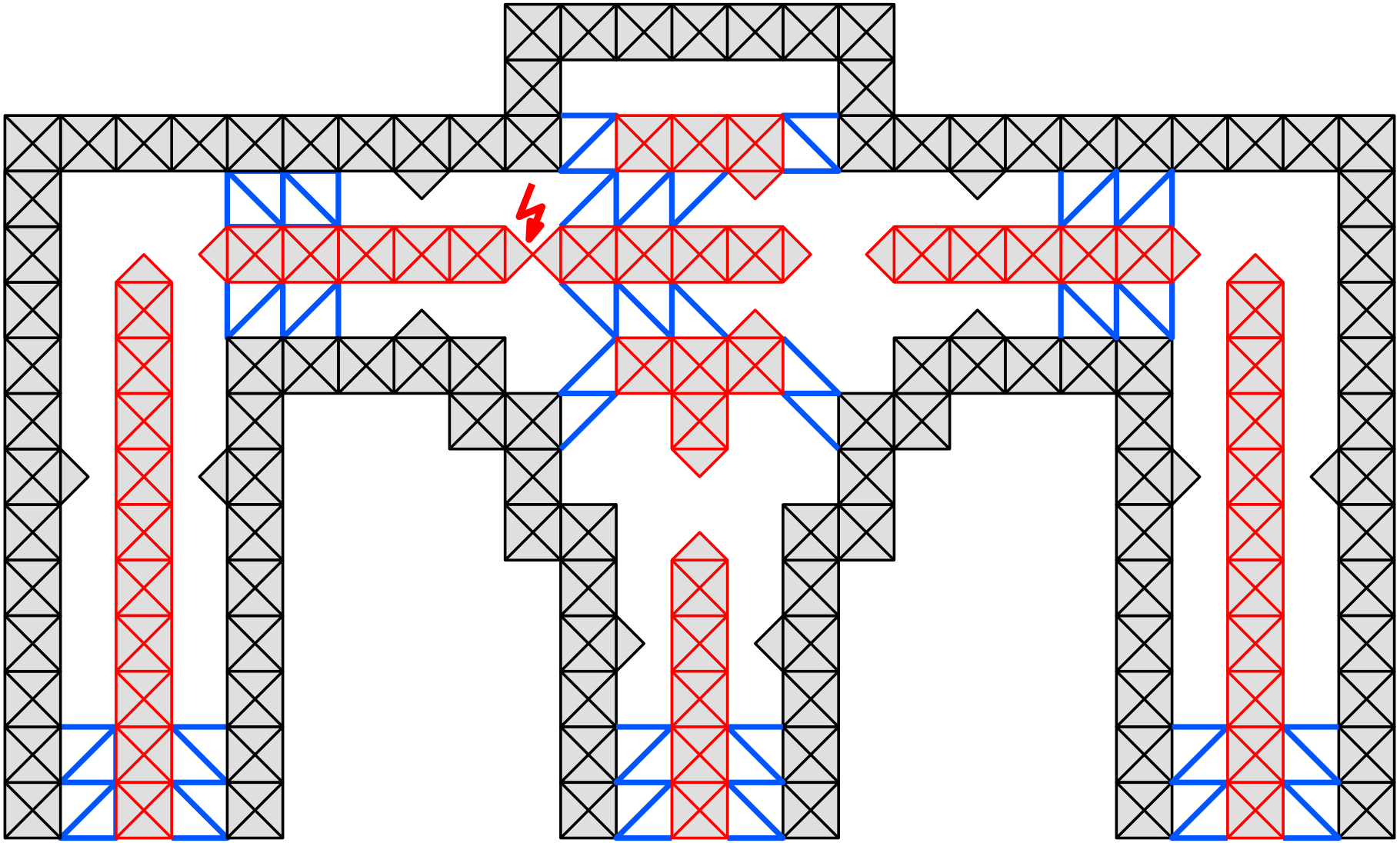

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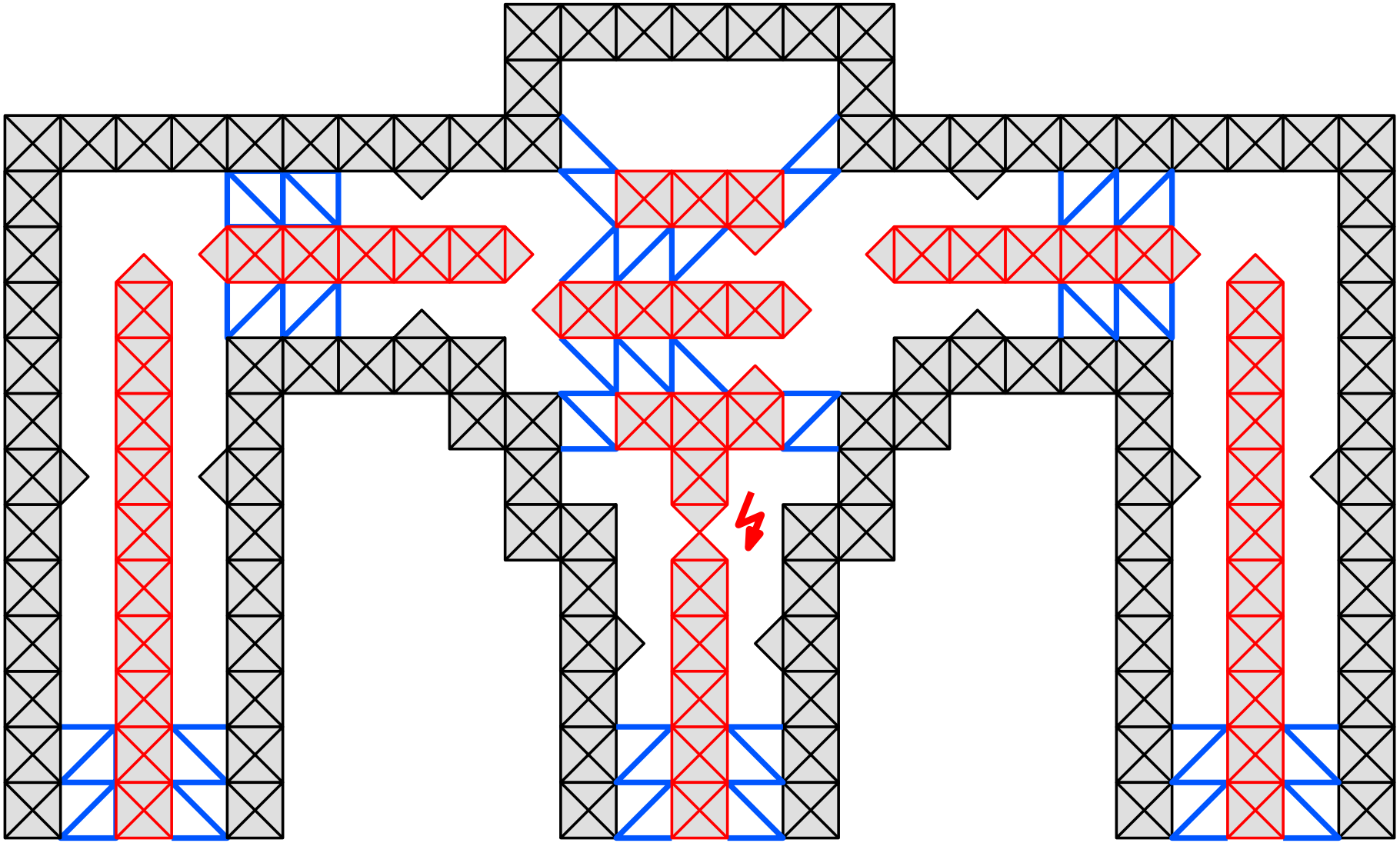

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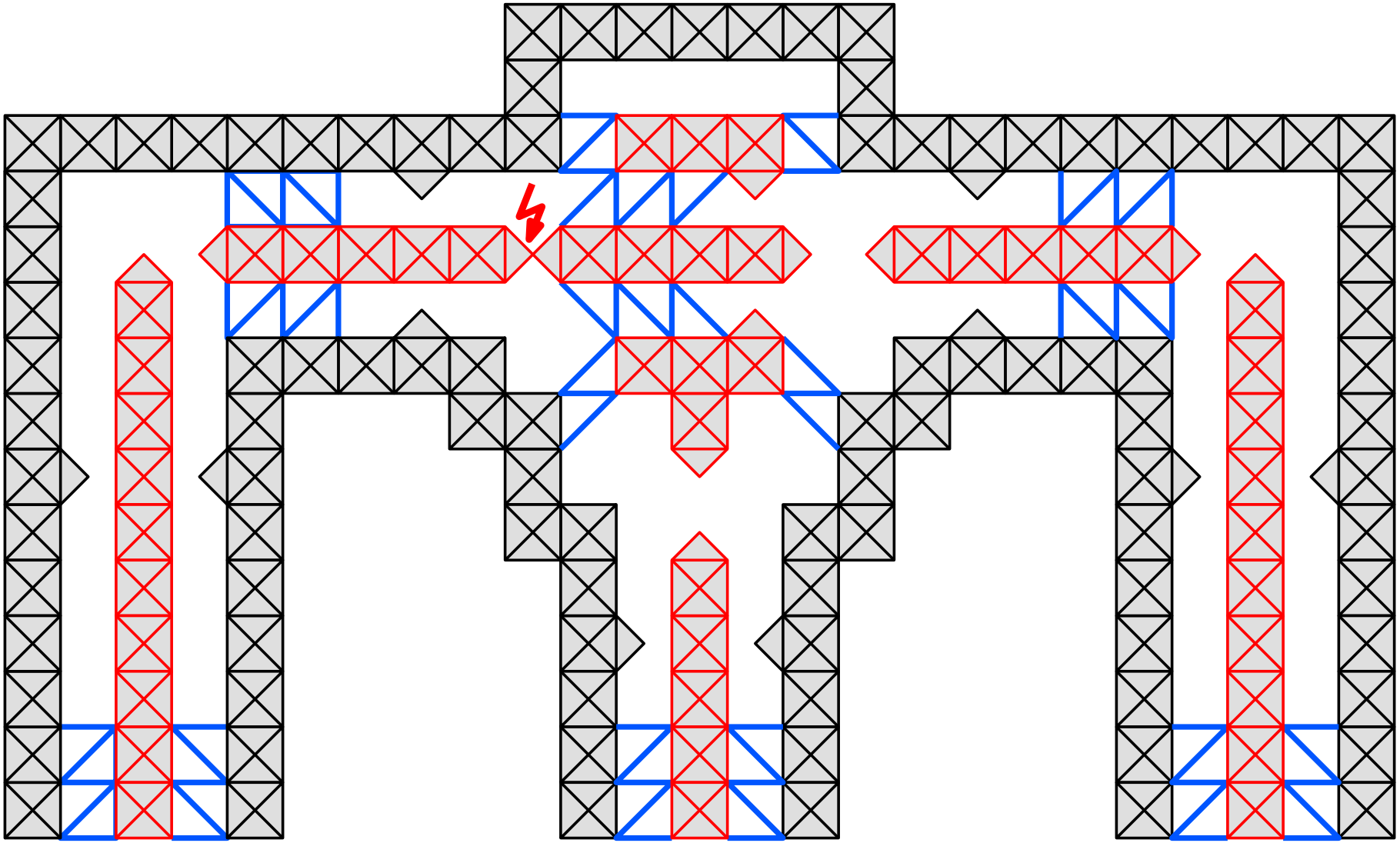

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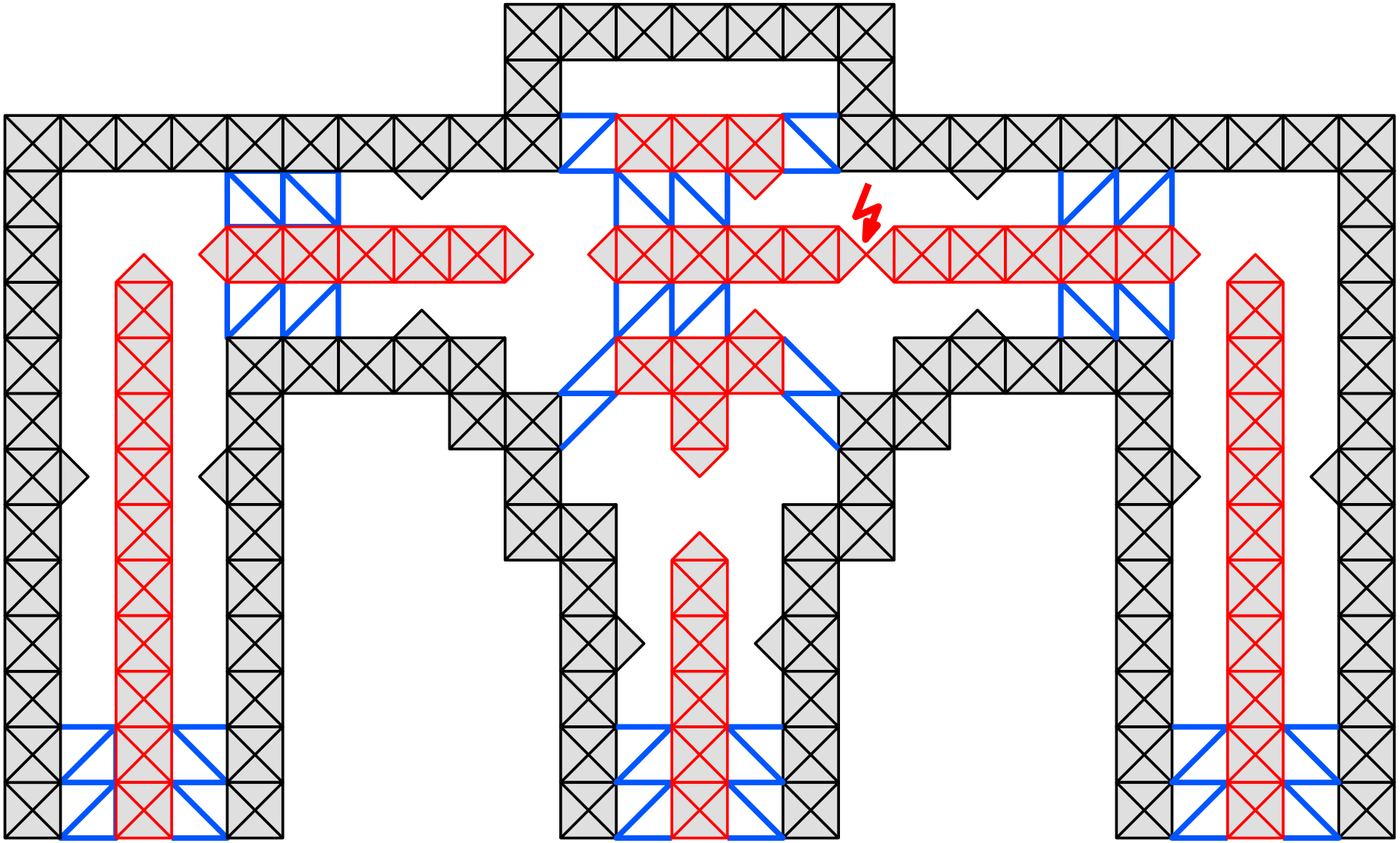
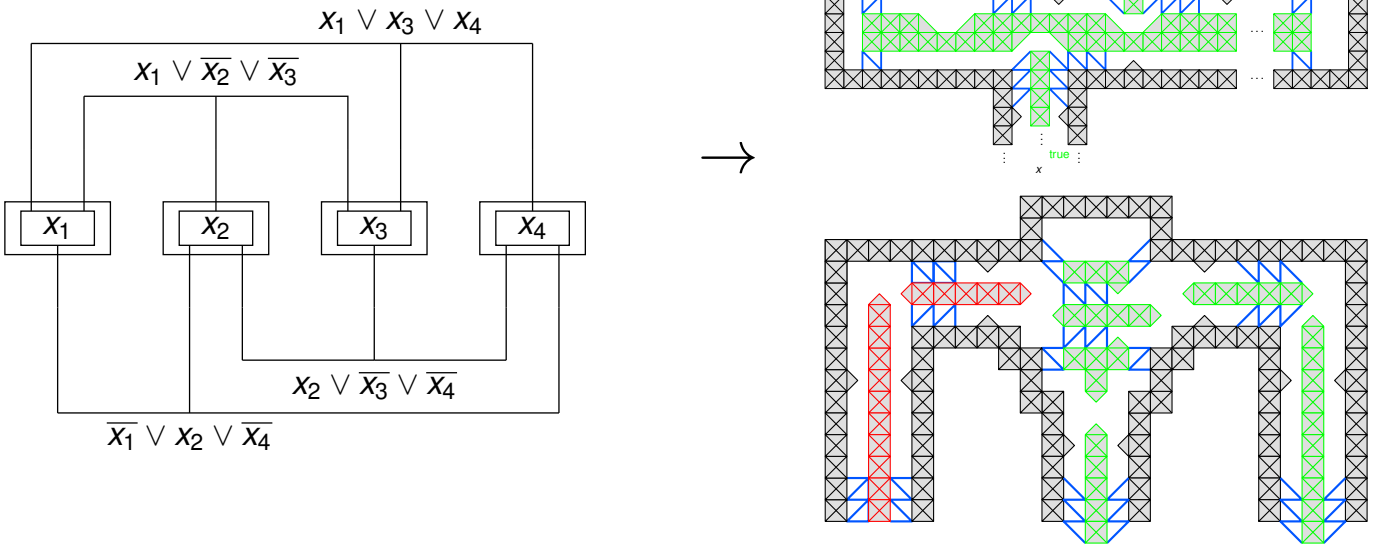


Summary of the Reduction



- Indeed we have:
  - φ satisfiable $\Rightarrow$ corresponding MM drawing of G_φ
  - G_φ has MM drawing $\Rightarrow$ satisfying truth assignment of φ

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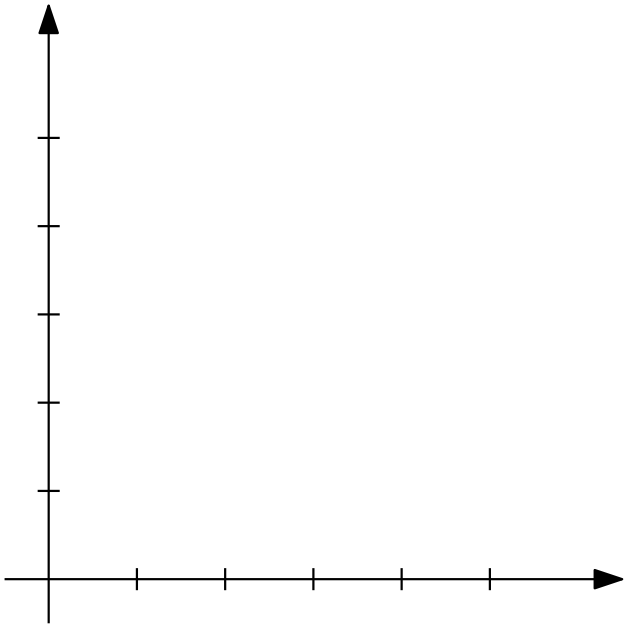
Mathematical Programming

- Linear Programming: efficient optimization method for
  - linear constraints
  - linear objective function
  - real-valued variables



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    maximize $x + 2y$
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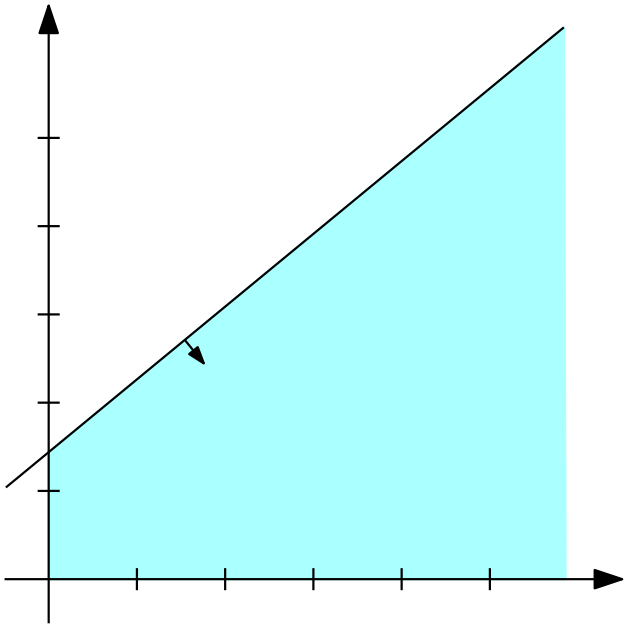

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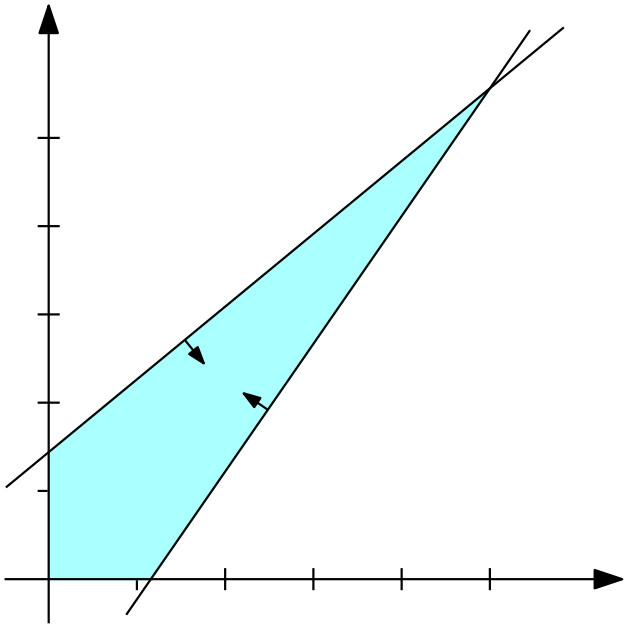

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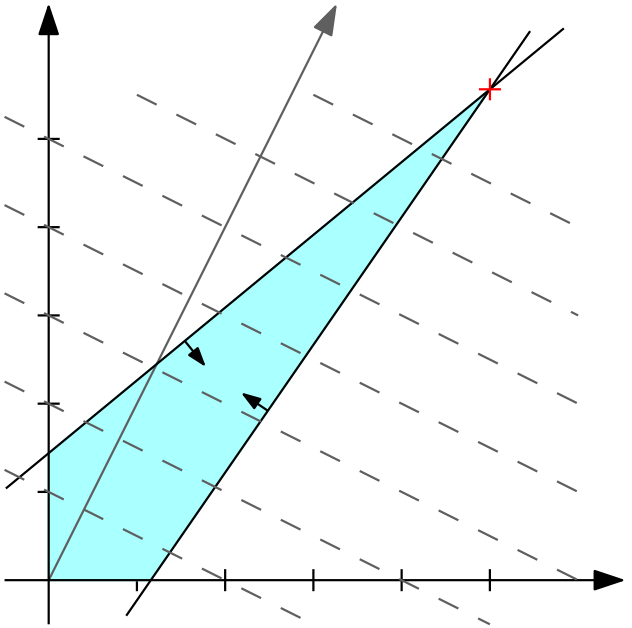

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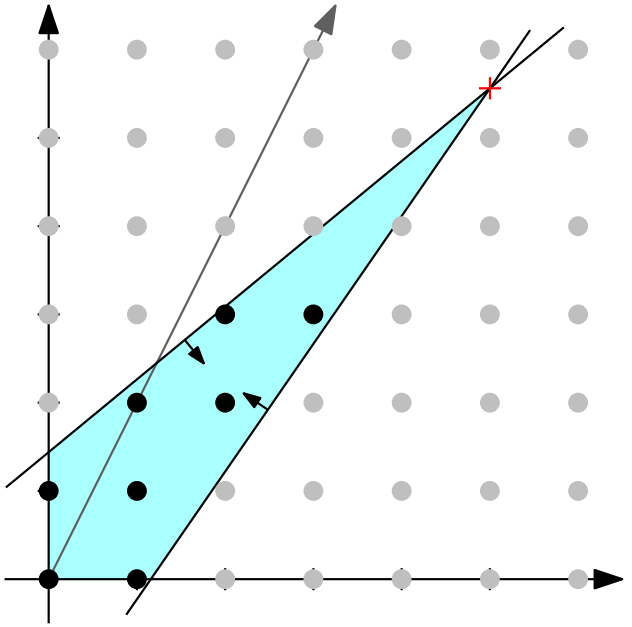
- Mixed-Integer Programming (MIP)
  - allows also integer variables
  - NP-hard in general
  - still a practical method for many real-world optimization problems



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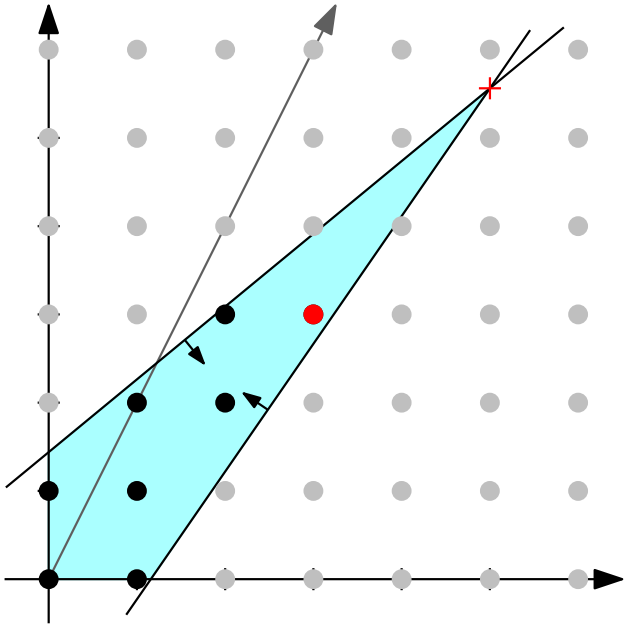

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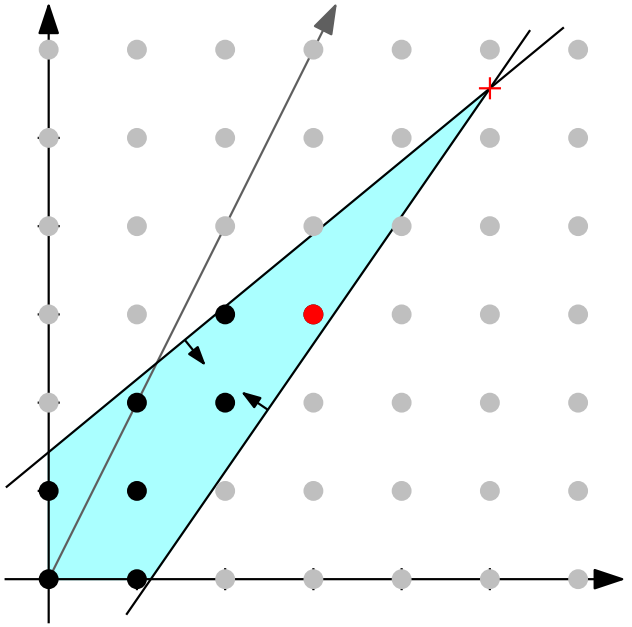
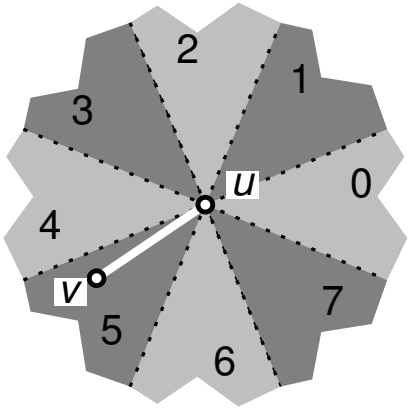


Theorem

The metro map layout problem can be formulated as a MIP s.th.

$$\textit{hard constraints} \rightarrow \textit{linear constraints}$$
$$\textit{soft constraints} \rightarrow \textit{objective function}$$

Definitions: Sectors and Coordinates

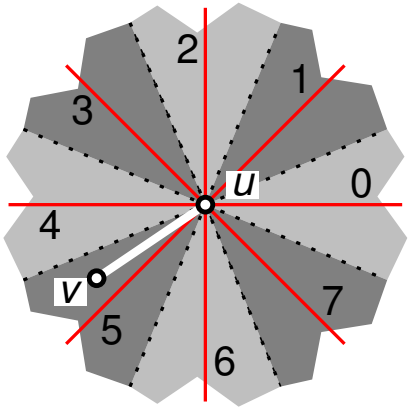


Sectors

– for each vtx. u partition plane into sectors 0–7

- here: $\mathrm{sec}(u, v) = 5$ (input)

Definitions: Sectors and Coordinates



Sectors

– for each vtx. u partition plane into sectors 0–7

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– number octilinear edge directions accordingly

- e.g. $\mathrm{dir}(u, v) = 4$ (output)

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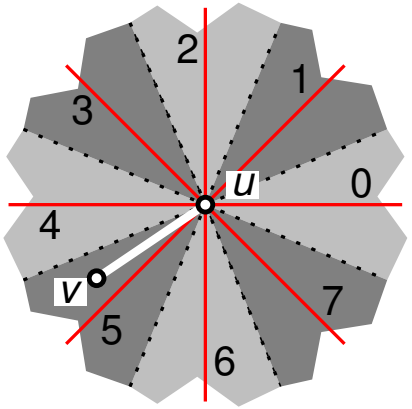

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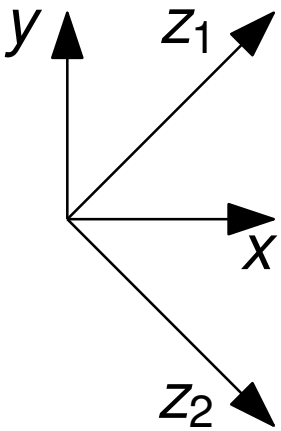


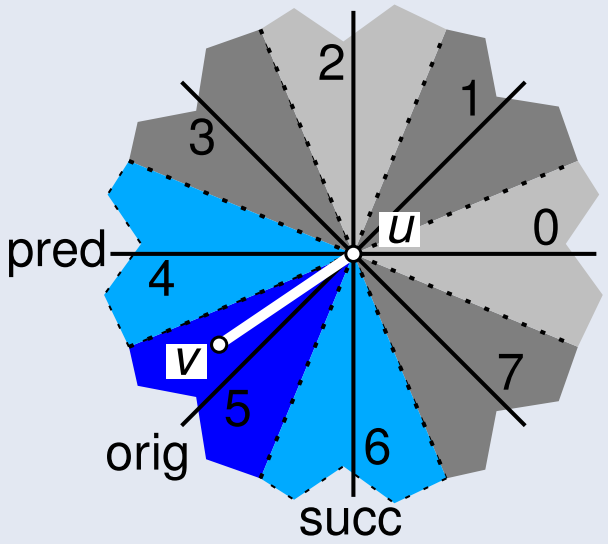
Coordinates

assign z_1- and z_2-coordinates to each vertex v:

- $z_1(v) = x(v) + y(v)$
- $z_2(v) = x(v) - y(v)$

Octilinearity and Relative Position

Goal



Draw edge uv

- octilinearly
- with minimum length ℓ_{uv}
- restricted to 3 directions

Octilinearity and Relative Position

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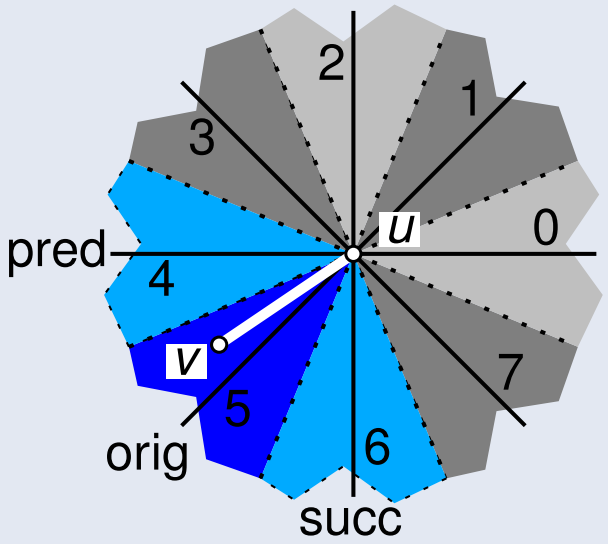

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How to model this using linear constraints?

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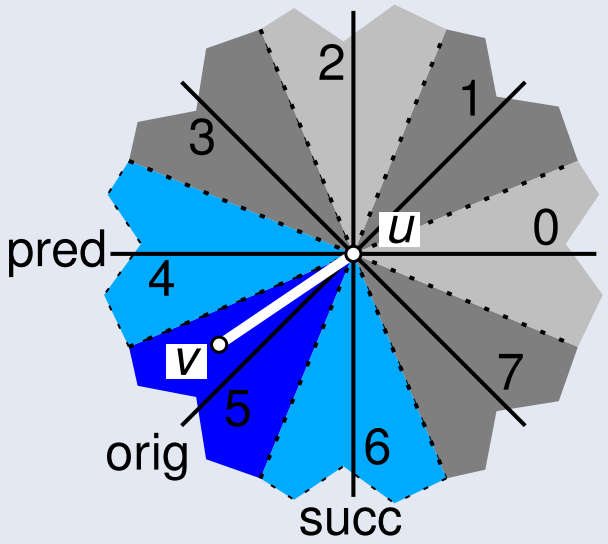

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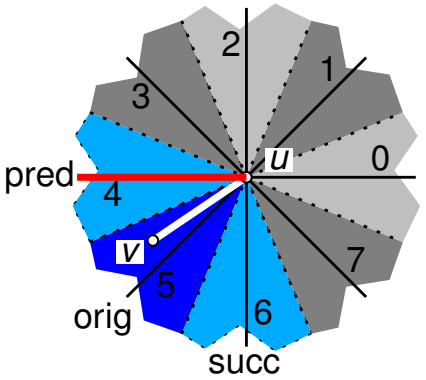
How to model this using linear constraints?

Binary Variables

$$\alpha_{\text{pred}}(u,v) + \alpha_{\text{orig}}(u,v) + \alpha_{\text{succ}}(u,v) = 1$$

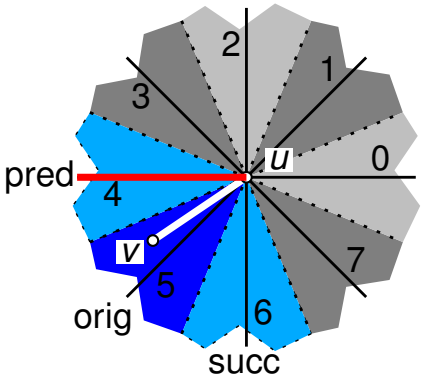
Octilinearity and Relative Position



Predecessor Sector

$$
\begin{aligned}
y(u) - y(v) &\leq M(1 - \alpha_{\text{pred}}(u, v)) \\
-y(u) + y(v) &\leq M(1 - \alpha_{\text{pred}}(u, v)) \\
x(u) - x(v) &\geq -M(1 - \alpha_{\text{pred}}(u, v)) + \ell_{uv}
\end{aligned}
$$

Octilinearity and Relative Position



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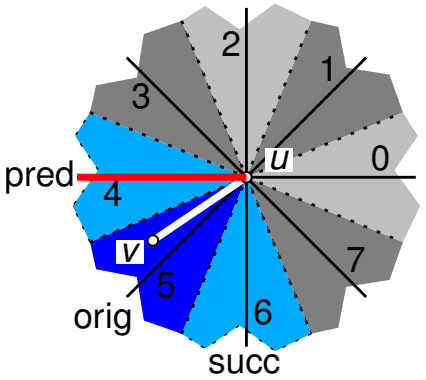
Octilinearity and Relative Position



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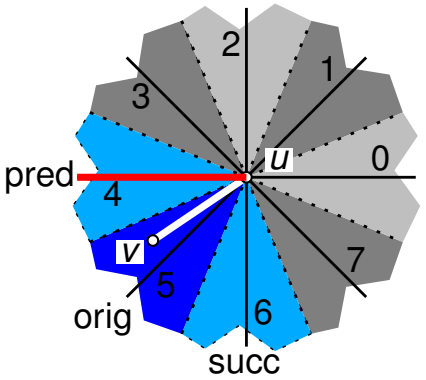
How does this work?

Case 1: $\alpha_{\text{pred}}(u,v) = 0$

$$\begin{aligned} y(u) - y(v) &\leq M \\ -y(u) + y(v) &\leq M \\ x(u) - x(v) &\geq \ell_{uv} - M \end{aligned}$$

Octilinearity and Relative Position



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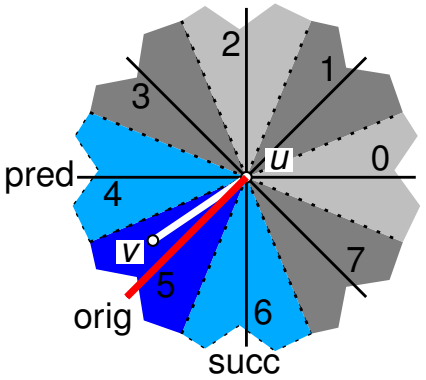
How does this work?

Case 2: $\alpha_{\text{pred}}(u, v) = 1$

$$\begin{aligned} y(u) - y(v) &\leq 0 \\ -y(u) + y(v) &\leq 0 \\ x(u) - x(v) &\geq \ell_{uv} \end{aligned}$$

Octilinearity and Relative Position



Original Sector

$$
\begin{aligned}
z_2(u) - z_2(v) &\leq M(1 - \alpha_{\text{orig}}(u, v)) \\
-z_2(u) + z_2(v) &\leq M(1 - \alpha_{\text{orig}}(u, v)) \\
z_1(u) - z_1(v) &\geq -M(1 - \alpha_{\text{orig}}(u, v)) + 2\ell_{uv}
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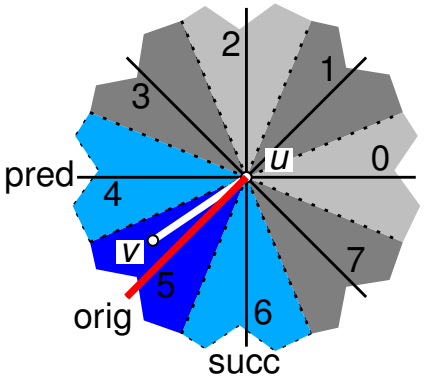
Octilinearity and Relative Position



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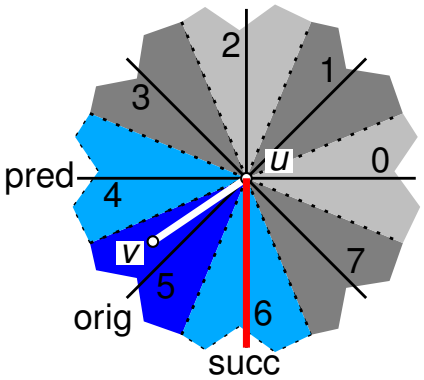


Successor Sector

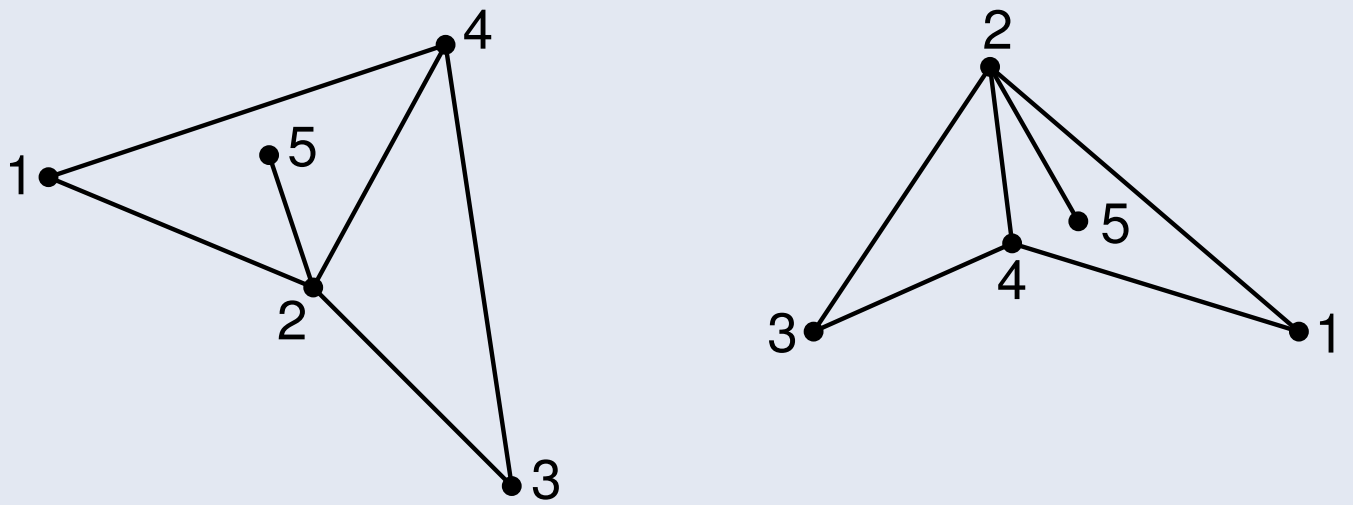
$$
\begin{aligned}
x(u) - x(v) &\leq M(1 - \alpha_{\text{succ}}(u,v)) \\
-x(u) + x(v) &\leq M(1 - \alpha_{\text{succ}}(u,v)) \\
y(u) - y(v) &\geq -M(1 - \alpha_{\text{succ}}(u,v)) + \ell_{uv}
\end{aligned}
$$

Preserving the Embedding (H1)

Definition

Two planar drawings of G have the same *embedding* if the induced orderings on the neighbors of each vertex are equal.

Same Embedding



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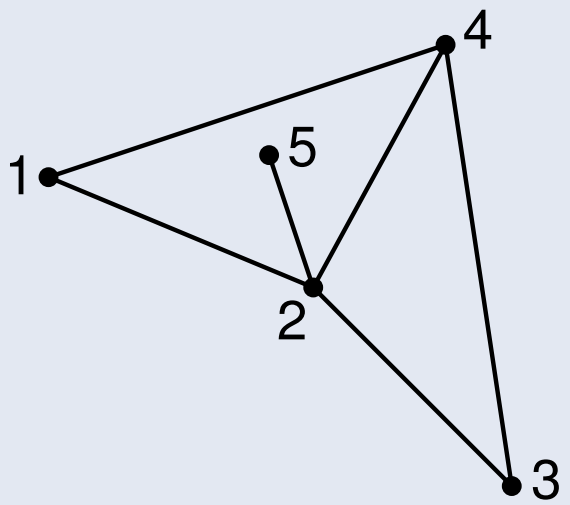
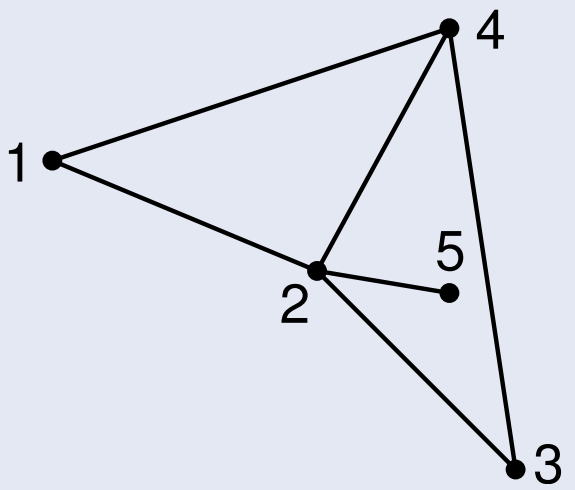
Different Embeddings





Preserving the Embedding (H1)

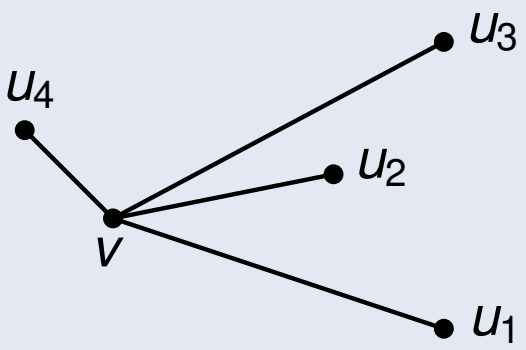
Constraints (Example)

- $N(v) = \{u_1, u_2, u_3, u_4\}$
- circular input order: $u_1 < u_2 < u_3 < u_4 < u_1$

All but one of the following inequalities must hold

$$\text{dir}(v, u_1) < \text{dir}(v, u_2) < \text{dir}(v, u_3) < \text{dir}(v, u_4) < \text{dir}(v, u_1)$$

Input



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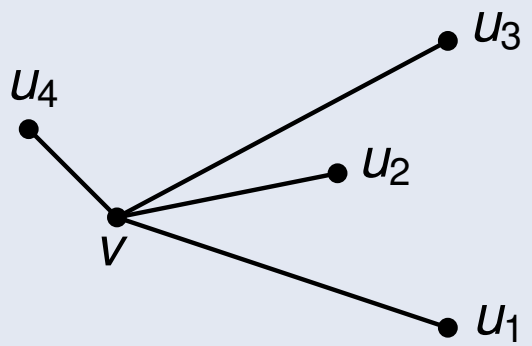
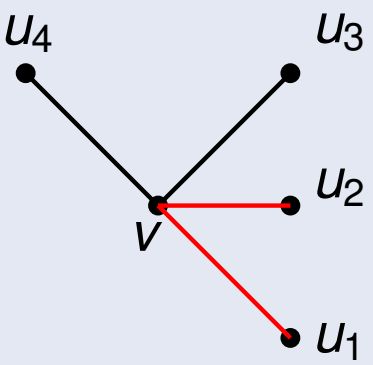
$$\text{dir}(v, u_1) \not< \text{dir}(v, u_2) < \text{dir}(v, u_3) < \text{dir}(v, u_4) < \text{dir}(v, u_1)$$

Input



Output

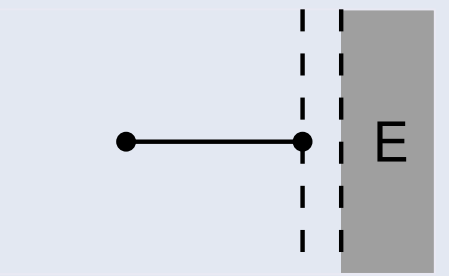


Planarity (H4)

Observation

For octilinear, straight edge e_1 non-incident edge e_2 must be placed d_{min} to the

- east, northeast, north, northwest, west, southwest, south, or southeast



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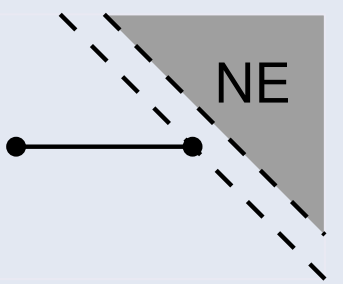

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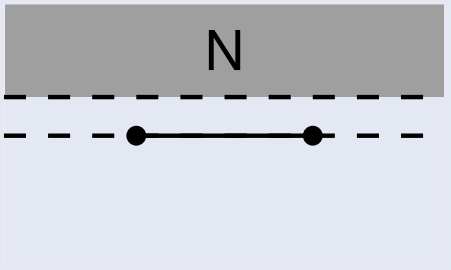

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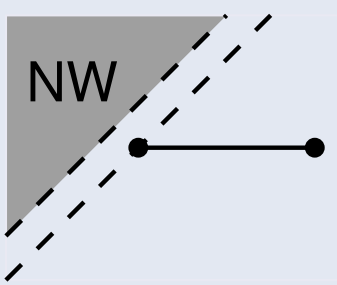

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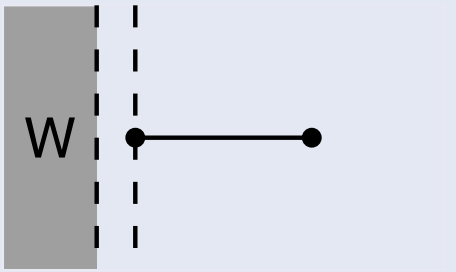

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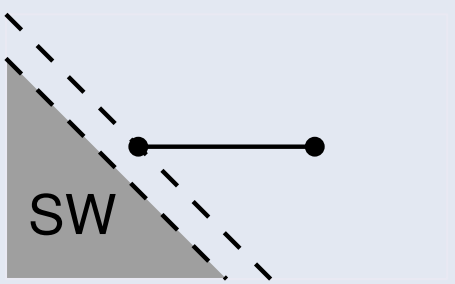

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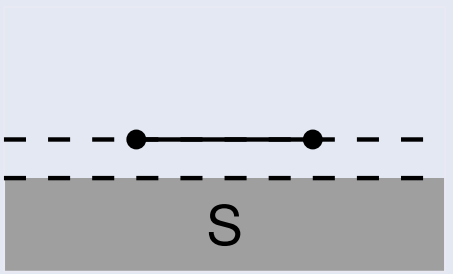

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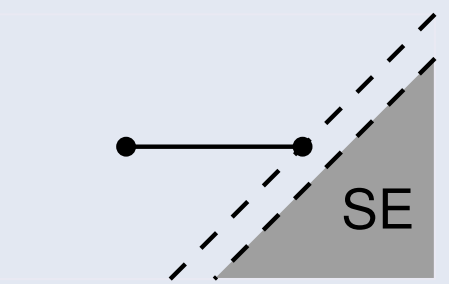

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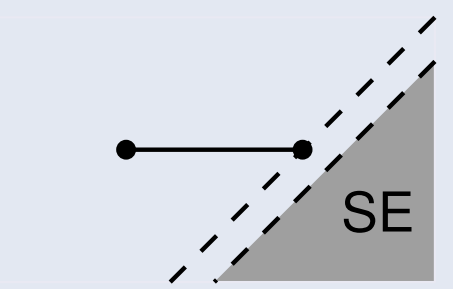

Constraints

- model as MIP with binary variables

$$\alpha_E + \alpha_{NE} + \alpha_N + \alpha_{NW} + \alpha_W + \alpha_{SW} + \alpha_S + \alpha_{SE} \geq 1$$

- required for each pair of non-incident edges

Objective Function

Objective Function

- corresponds to soft constraints (S1)–(S3)
- weighted sum of individual cost functions

$$\text{minimize } \lambda_{\text{bends}}\,\text{cost}_{\text{bends}} + \lambda_{\text{length}}\,\text{cost}_{\text{length}} + \lambda_{\text{relpos}}\,\text{cost}_{\text{relpos}}$$

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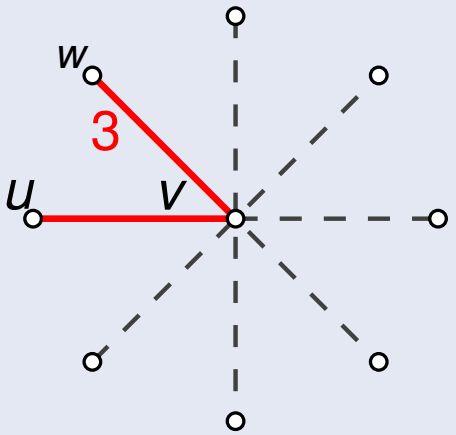
Line Bends (S1)



Edges uv and vw on a metro line $L \in \mathcal{L}$

- draw as straight as possible
- increase cost bend(u, v, w) for increasing acuteness of $\angle(\overline{uv}, \overline{vw})$

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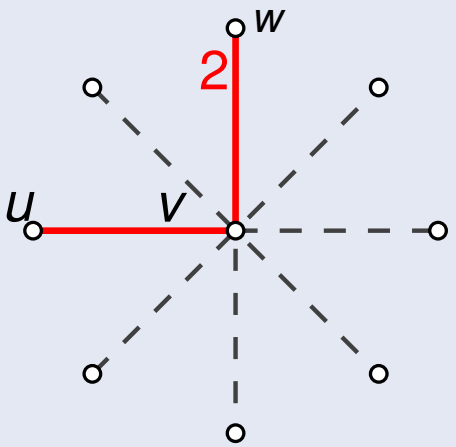

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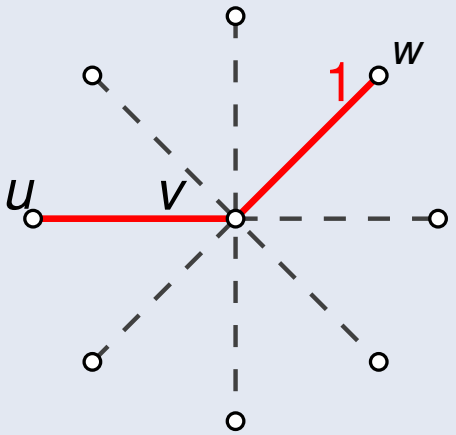

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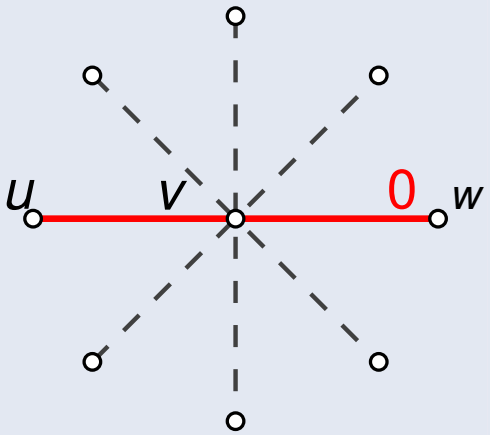

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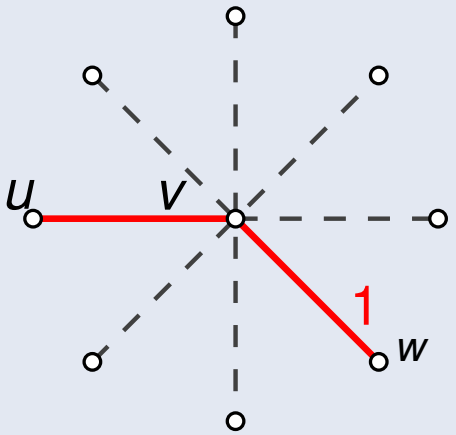

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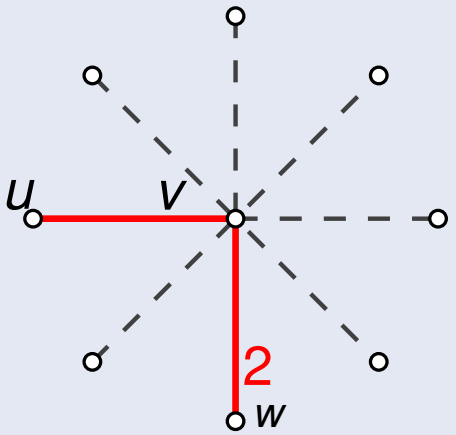

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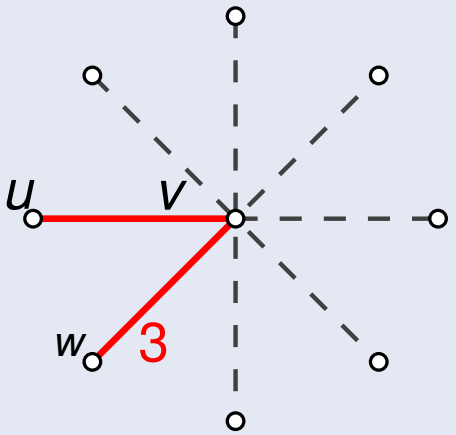

Edges uv and vw on a metro line $L \in \mathcal{L}$

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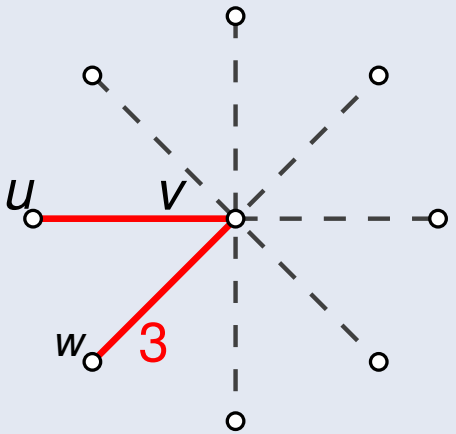

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$$\text{cost}_{\text{bends}} = \sum_{L \in \mathcal{L}} \sum_{uv, vw \in L} \text{bend}(u, v, w)$$

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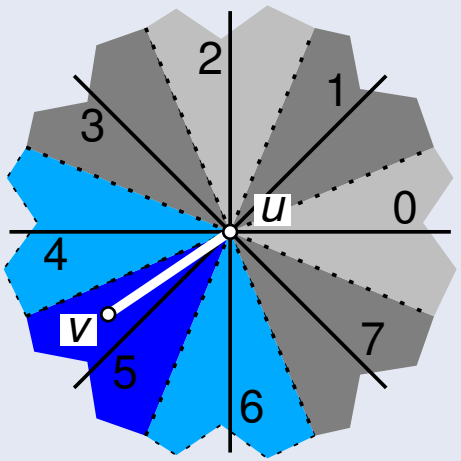

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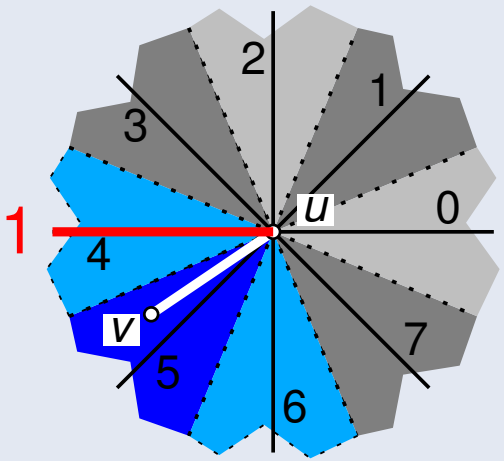

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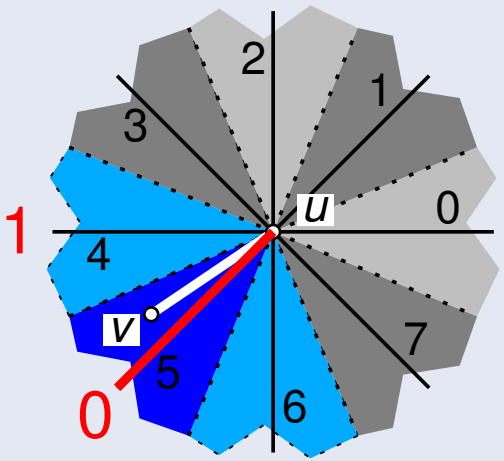

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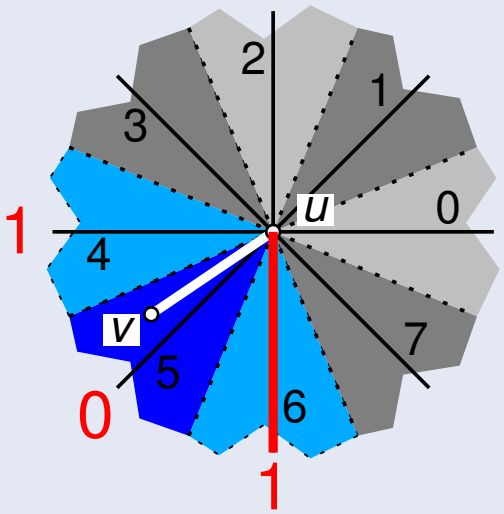

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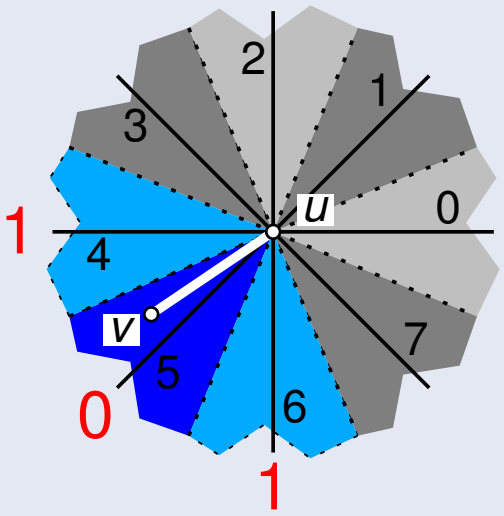

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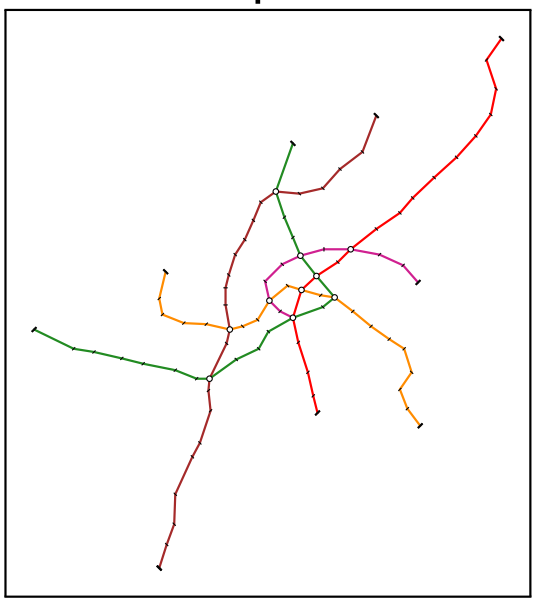

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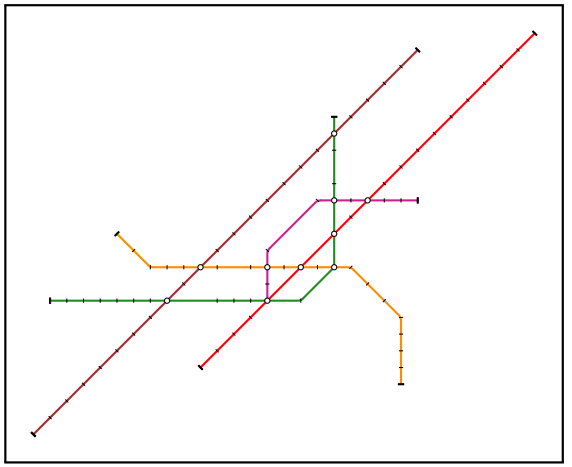

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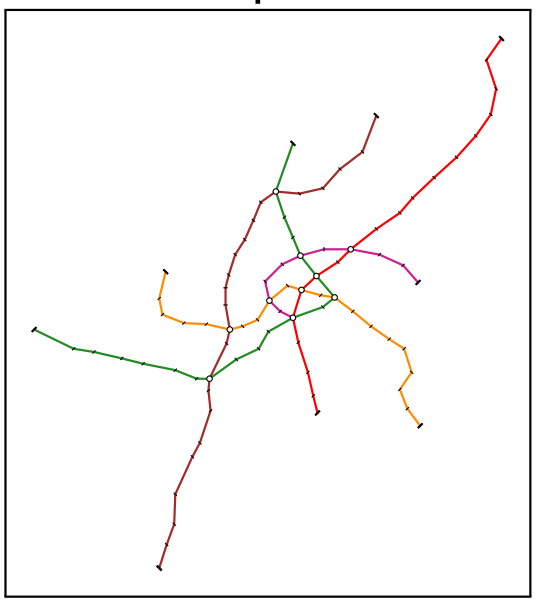

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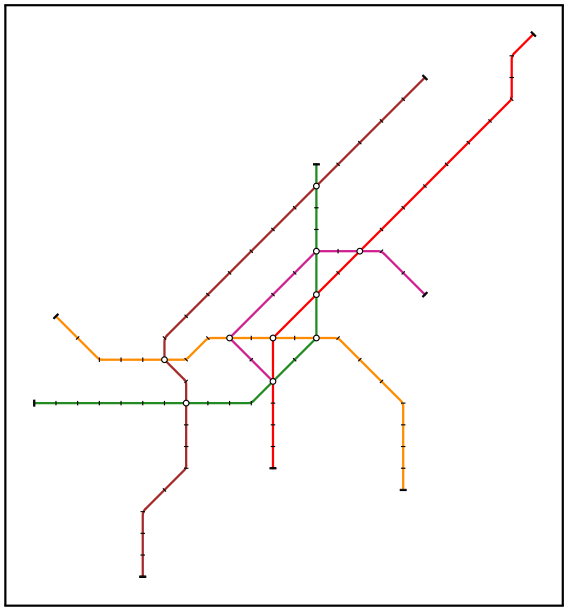

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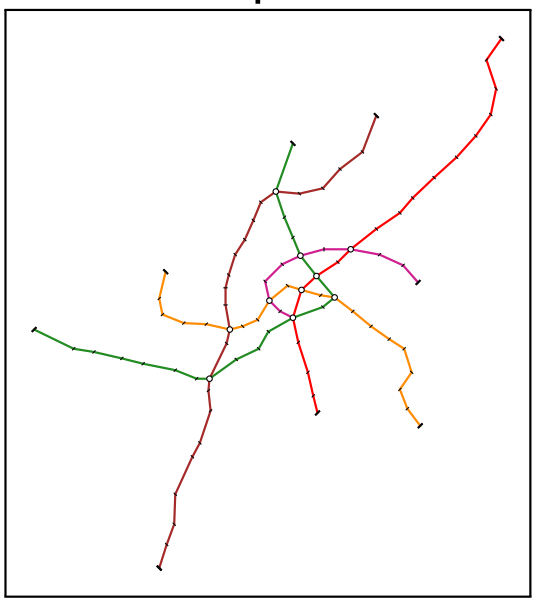

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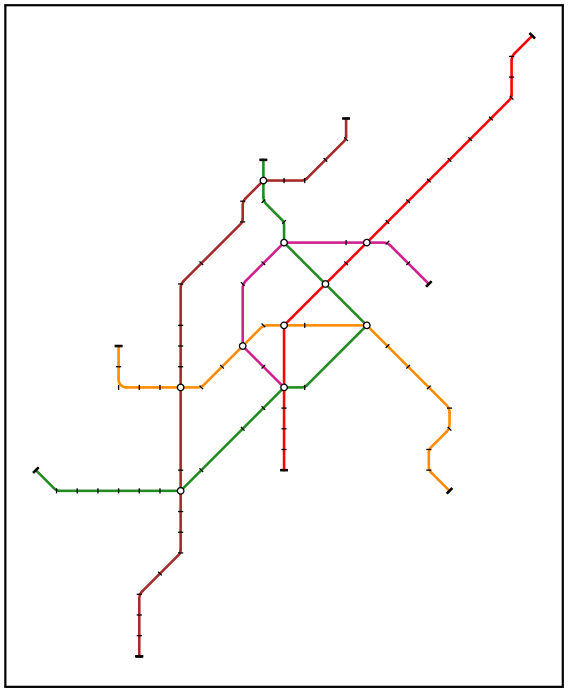

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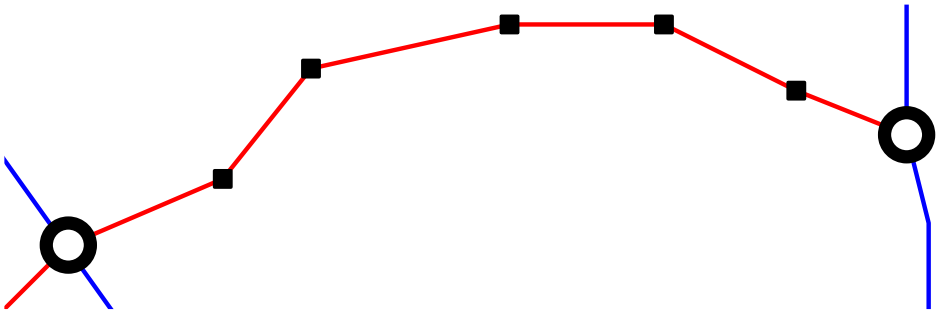

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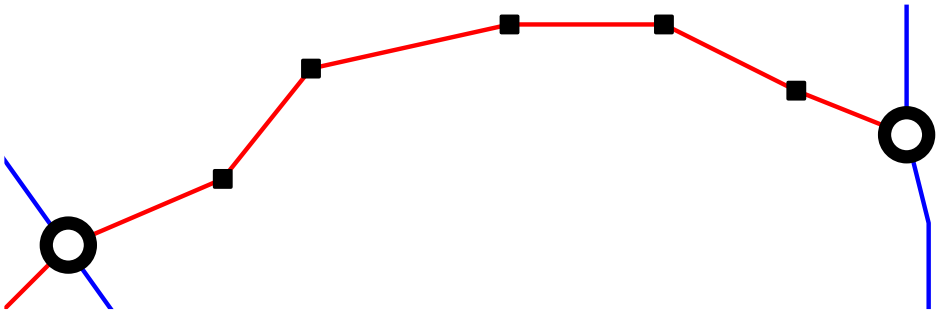

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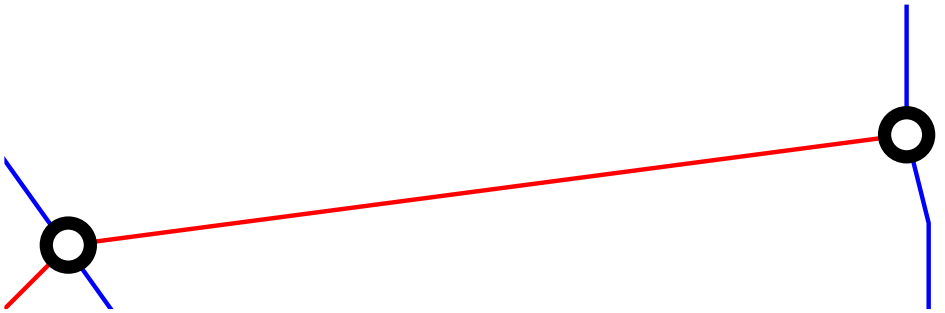

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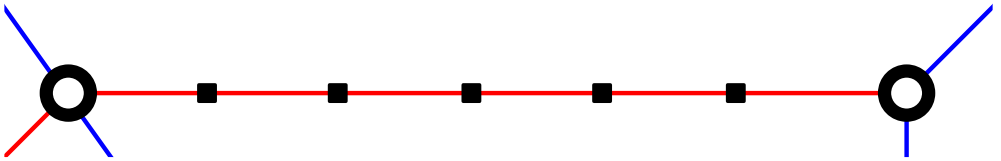

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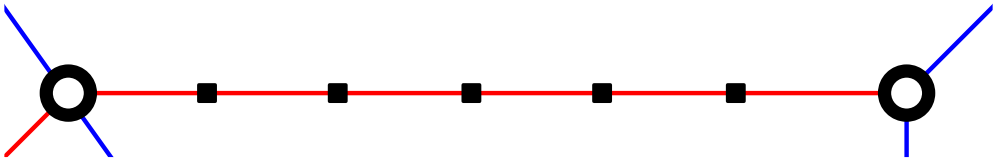

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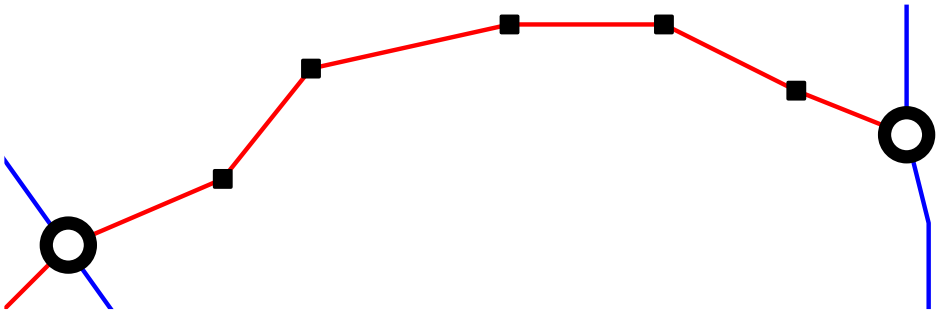

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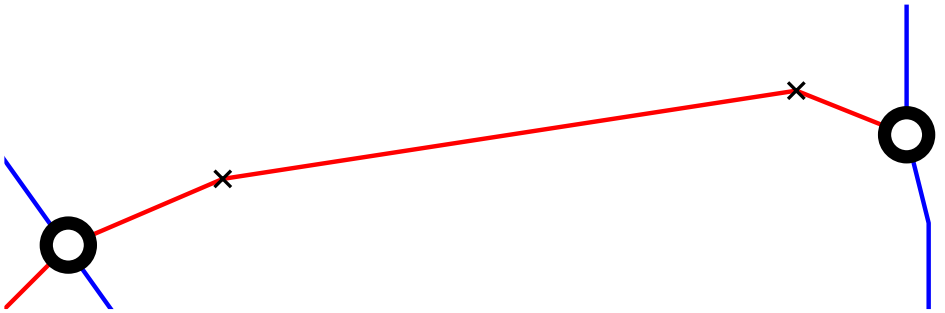

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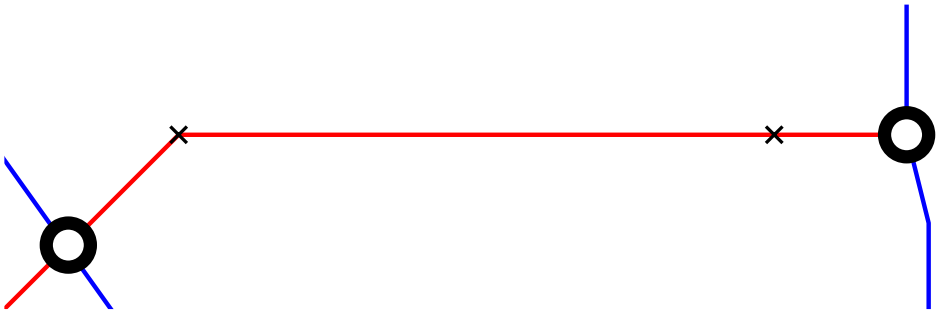

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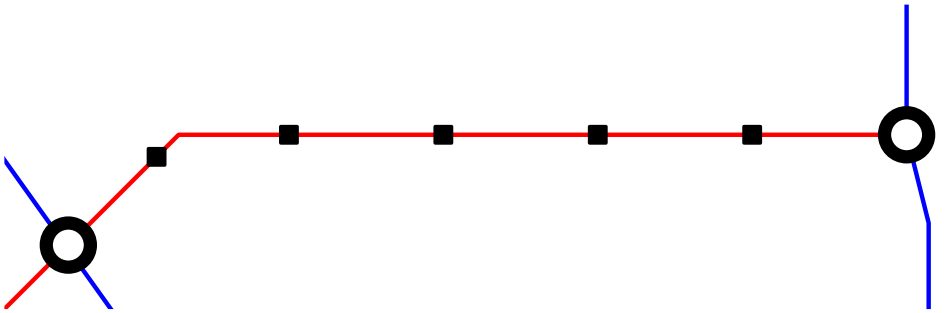

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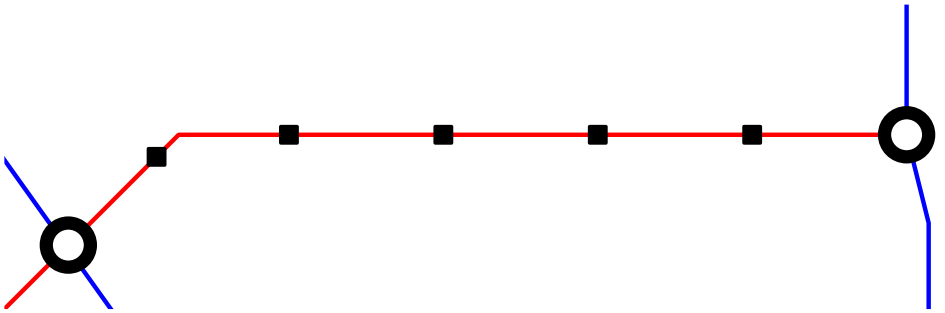

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  - more similar to input

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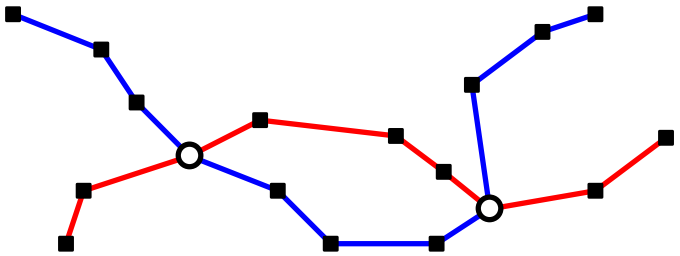

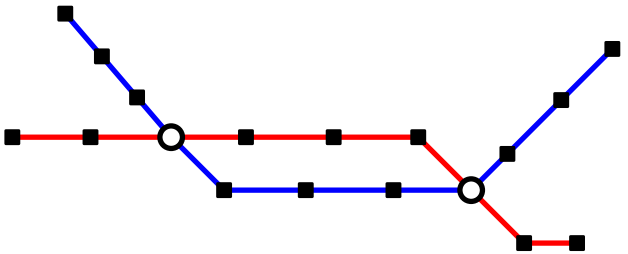

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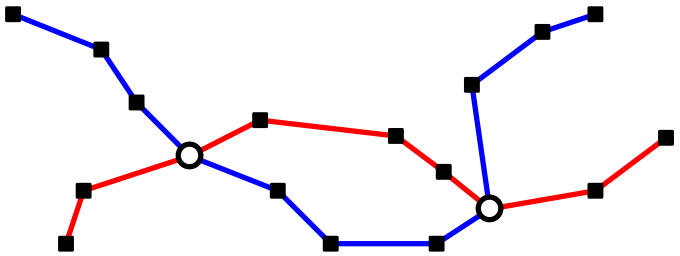
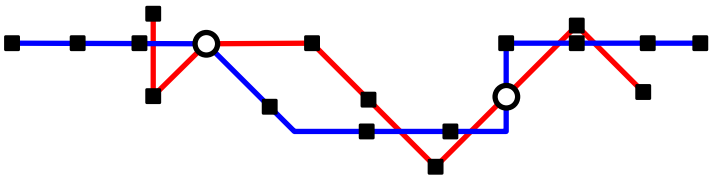




Speeding Up: Callback Functions

- MIP optimizer CPLEX offers advanced callback functions
- add required planarity constraints on the fly

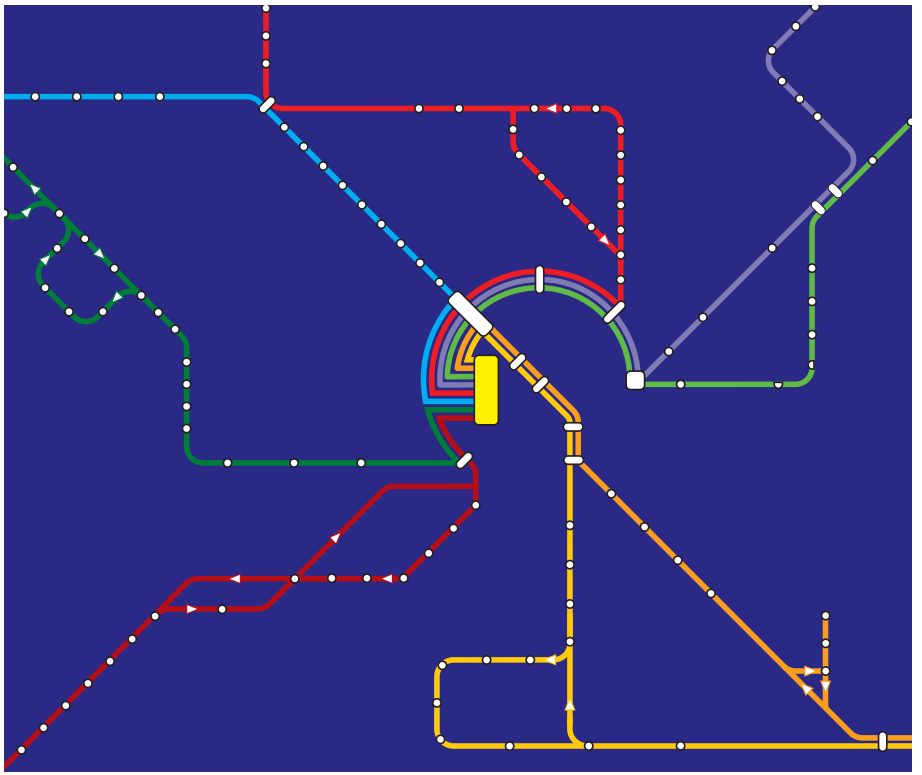
Algorithm

1. start solving MIP without planarity constraints
2. for each new solution
   1. interrupt CPLEX
   2. if solution is not planar
      - add planarity constraints for intersecting edges
      - reject solution

      else
      - accept solution
   3. continue solving the MIP (until optimal)

Labeling

- unlabeled metro map of little use in practice



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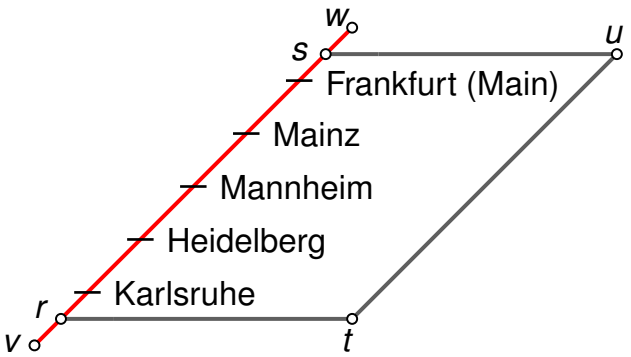
- combine layout and labeling for better results

Modeling Labels

Model labels as special metro lines:

- put all labels between each pair of interchange stations into one parallelogram,



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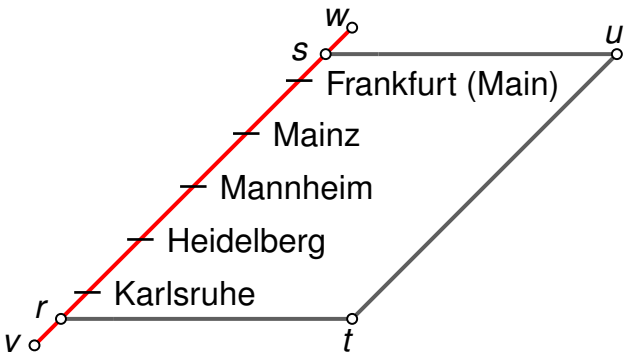

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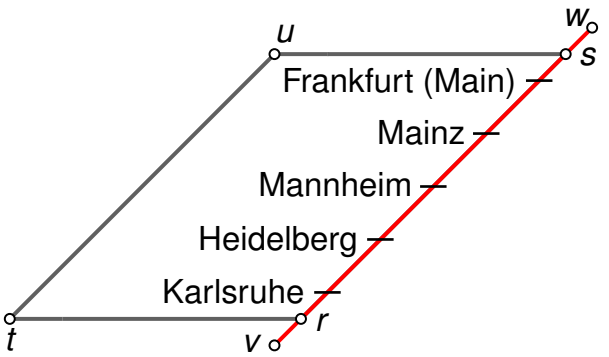

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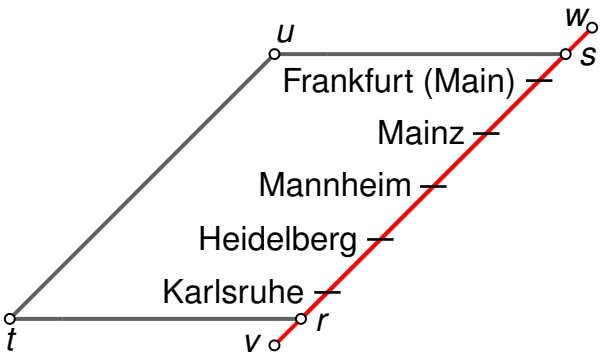

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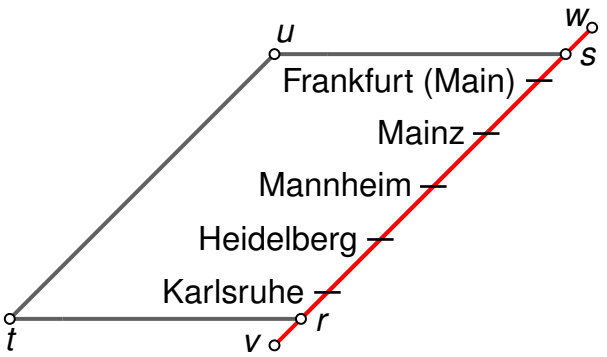

Results – Sydney unlabeled

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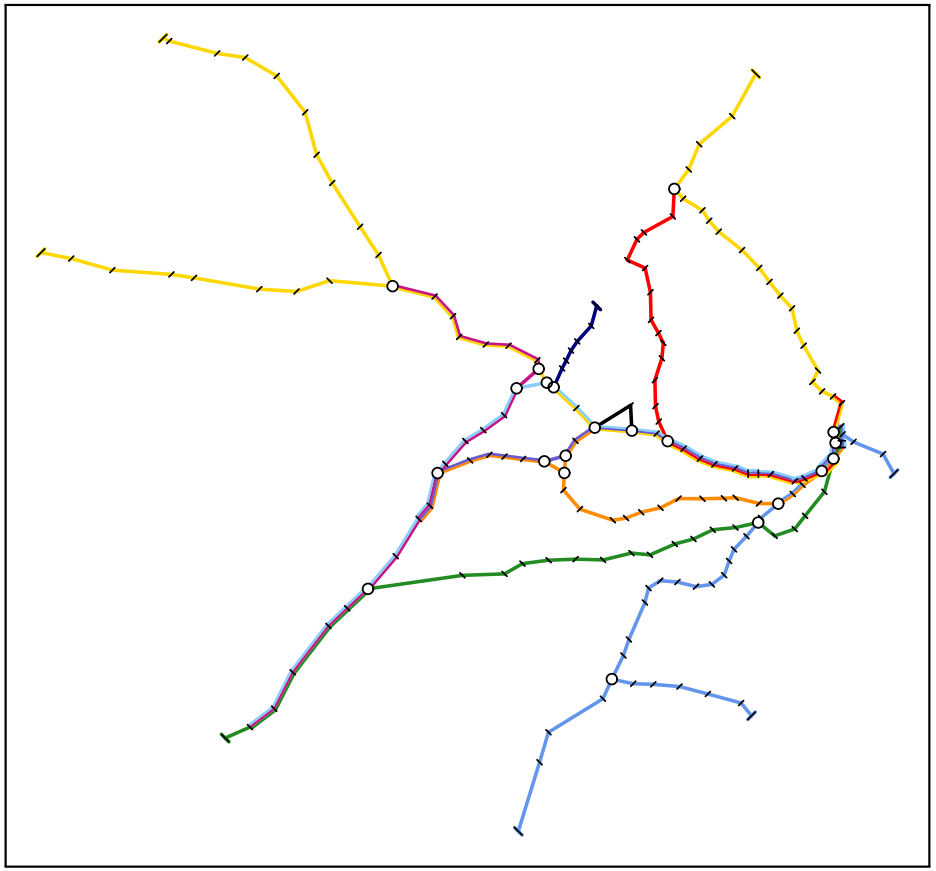

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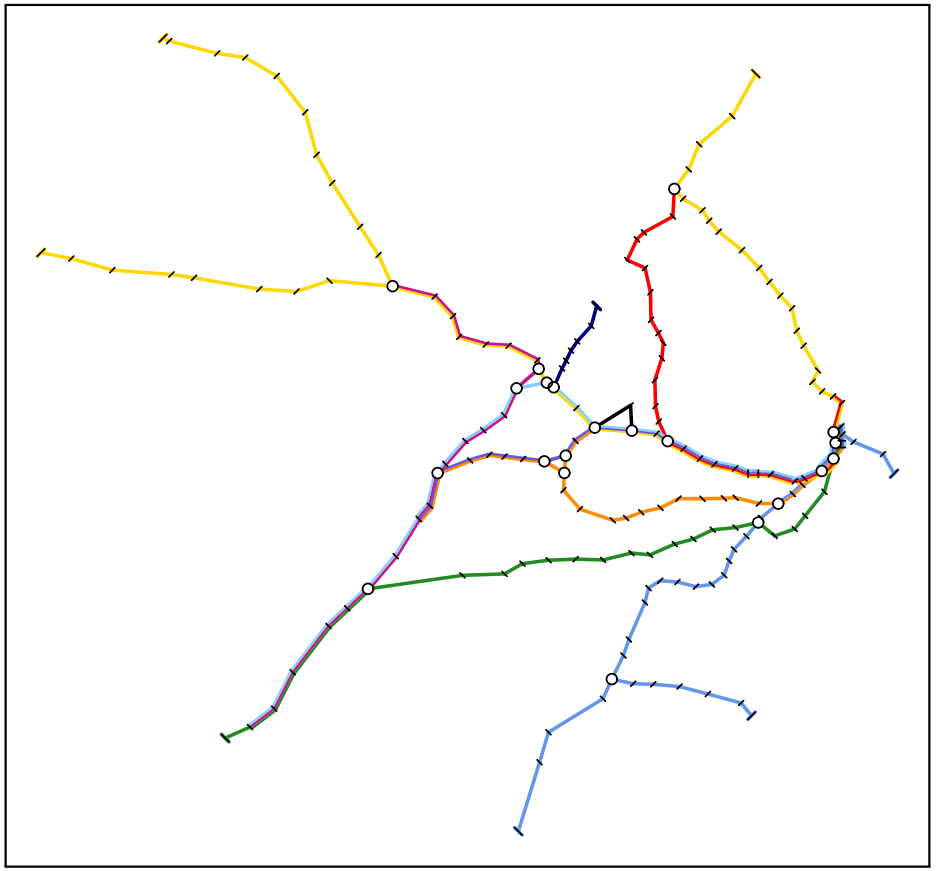

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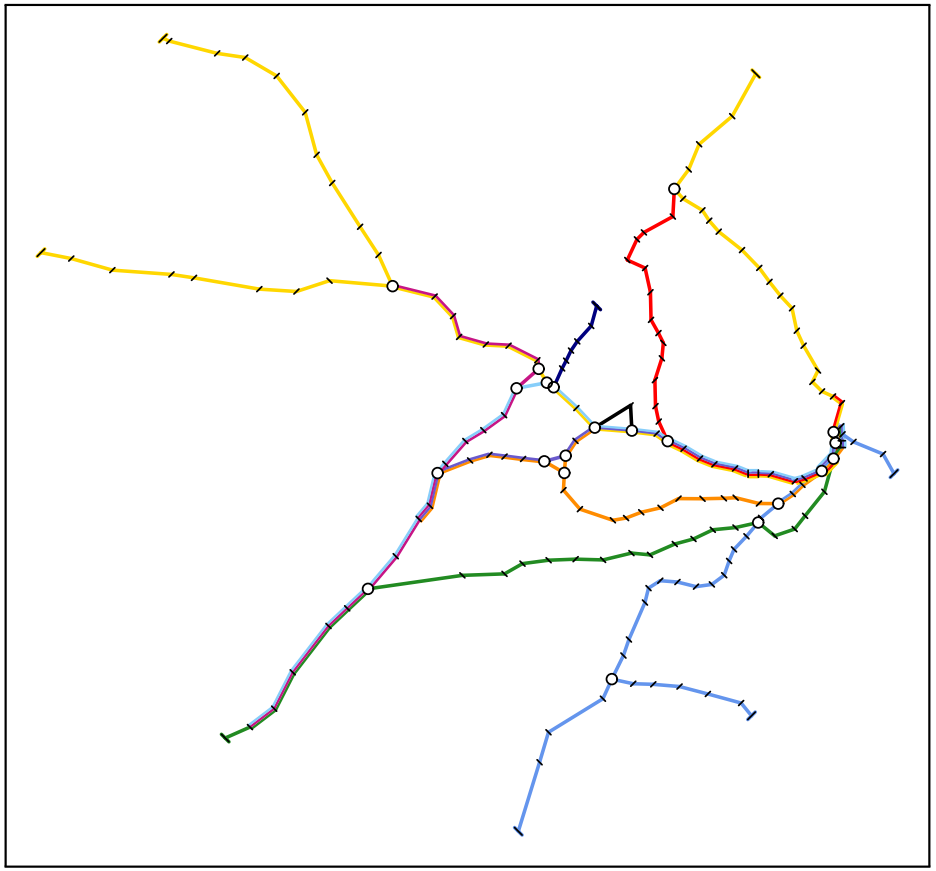

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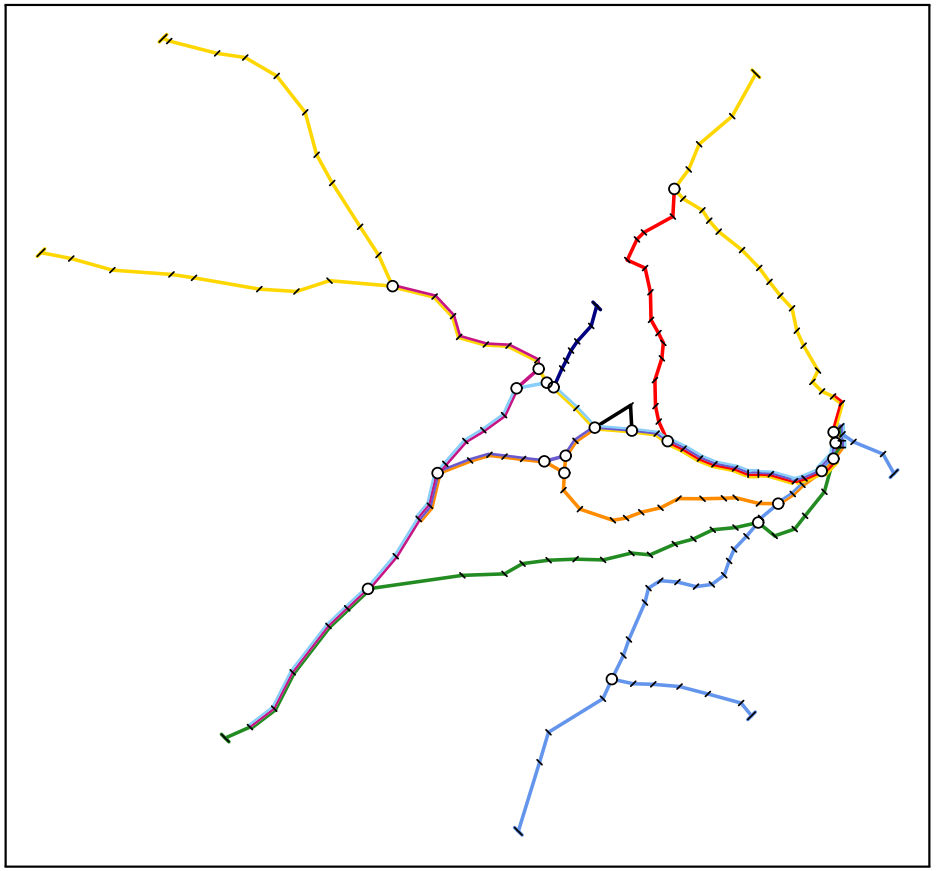
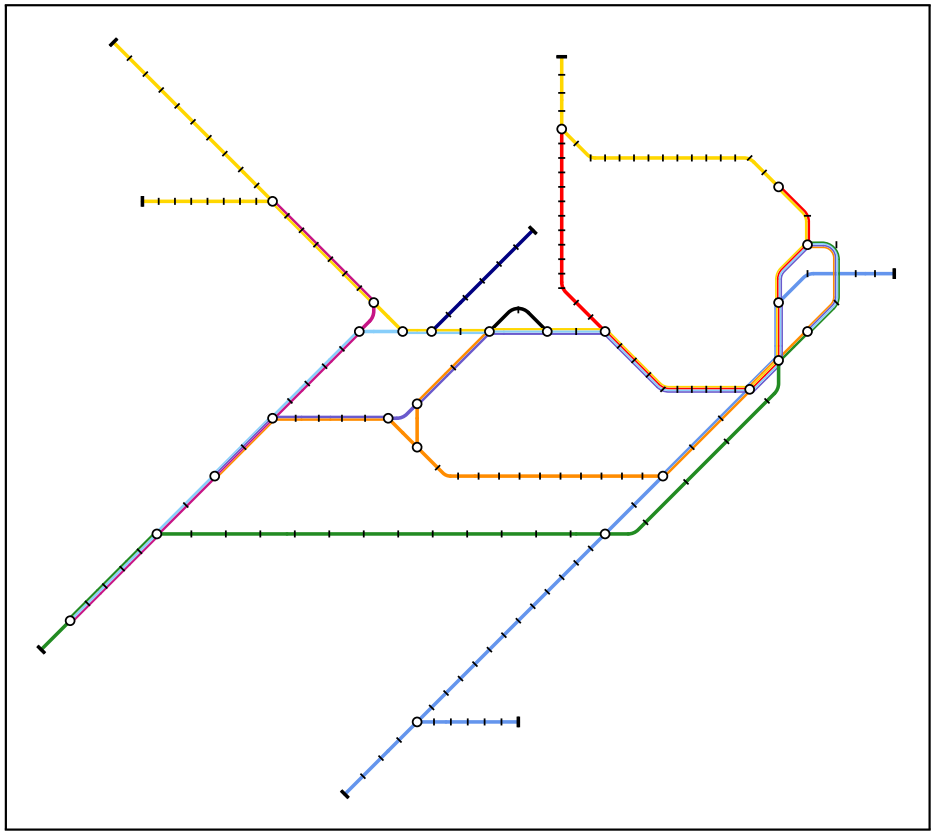
*) 23 minutes w/o proof of opt.
constr. of 3 edge pairs added

Results – Sydney unlabeled

Input

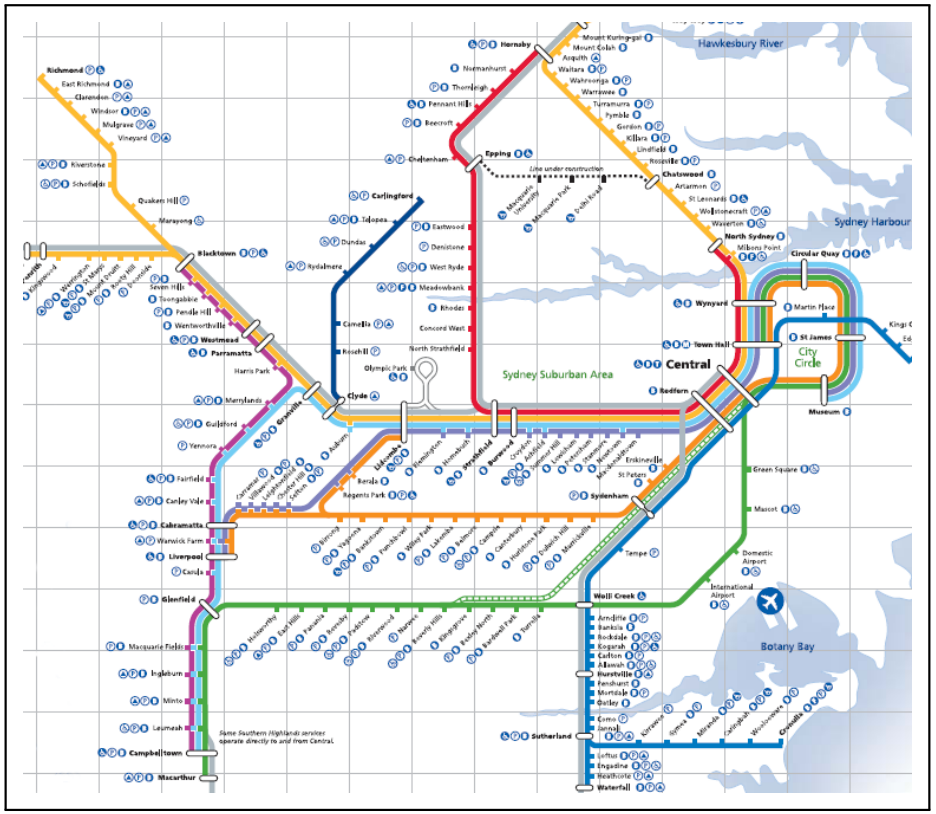


Output (23 min.)

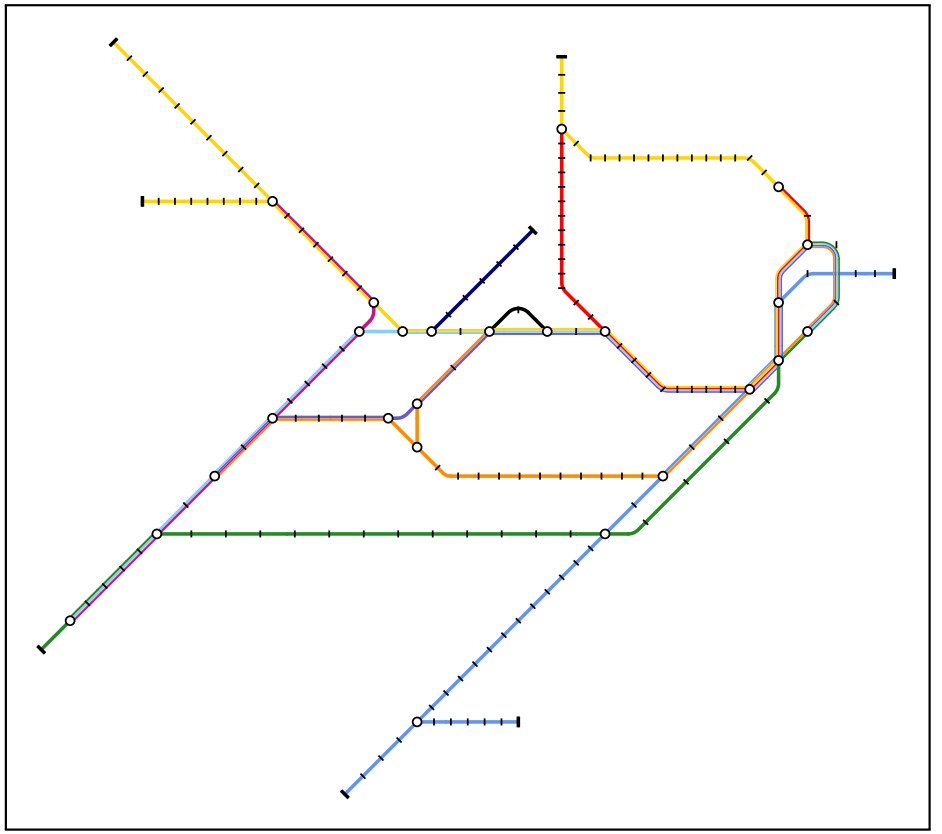


Results – Sydney unlabeled

Official map

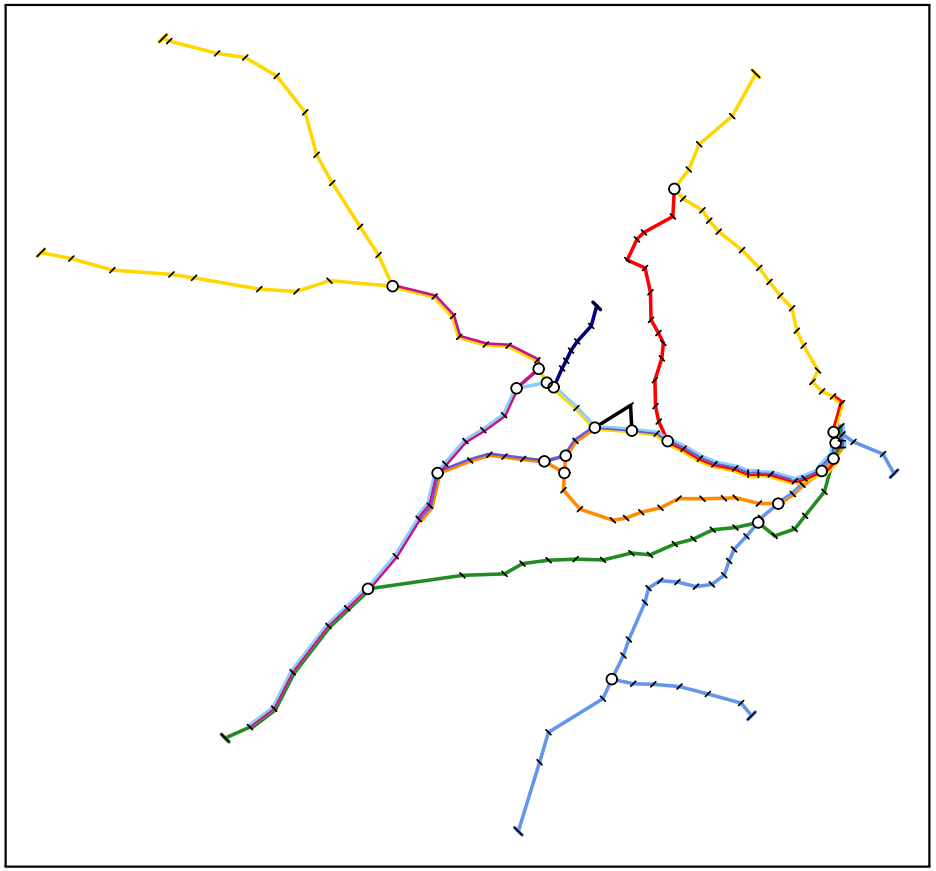


Output (23 min.)



Results – Sydney labeled

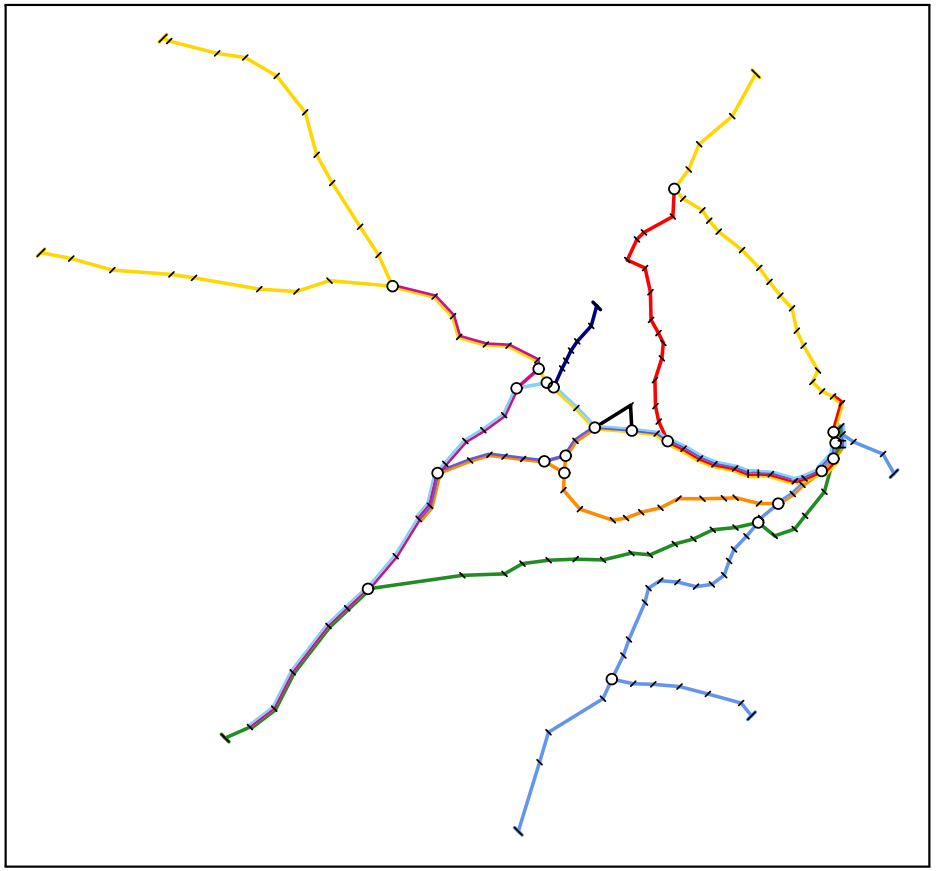
Input



Input	$\lvert V\rvert$	$\lvert E\rvert$	fcs.	$\lvert\mathcal{L}\rvert$
full	174	183	11	
reduced	88	97		10
labeled	242	270	30	

Results – Sydney labeled

Input



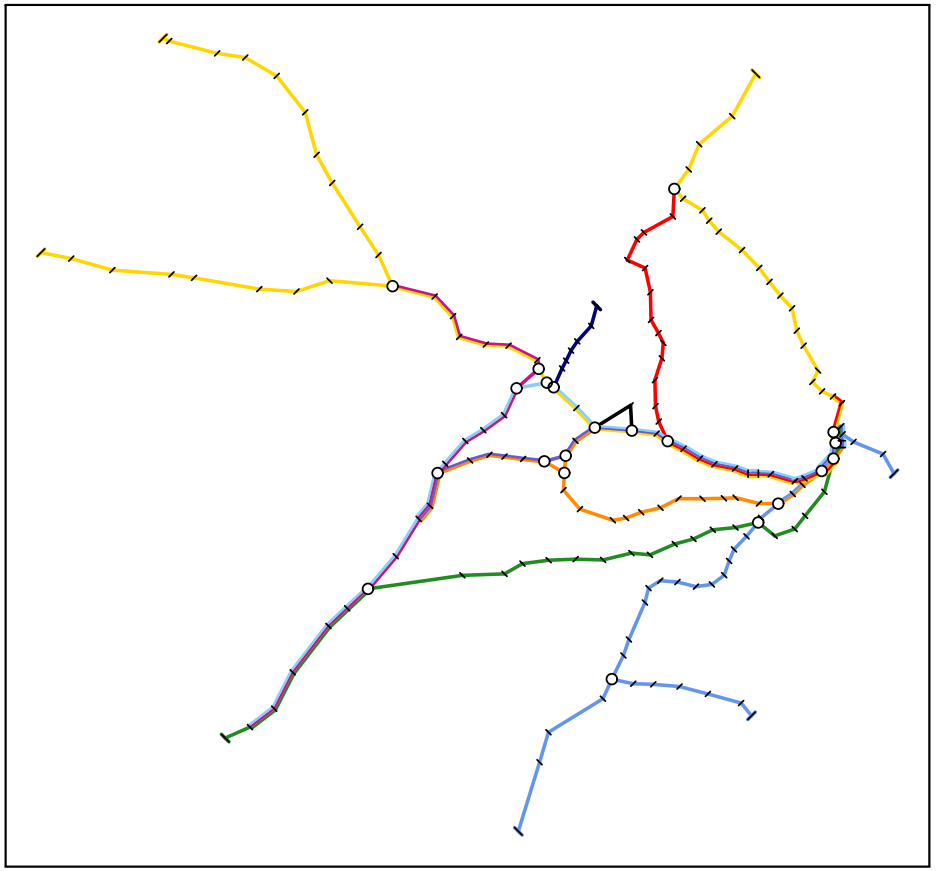
Input	$\lvert V\rvert$	$\lvert E\rvert$	fcs.	$\lvert\mathcal{L}\rvert$
full	174	183	11	
reduced	88	97		10
labeled	242	270	30	

↓

MIP	constr.	var.
full	1,191,406	290,137
callback	21,988	92,681
skipped	6,838	2,969

Results – Sydney labeled

Input



Input	$\lvert V\rvert$	$\lvert E\rvert$	fcs.	$\lvert\mathcal{L}\rvert$
full	174	183	11	
reduced	88	97		10
labeled	242	270	30	

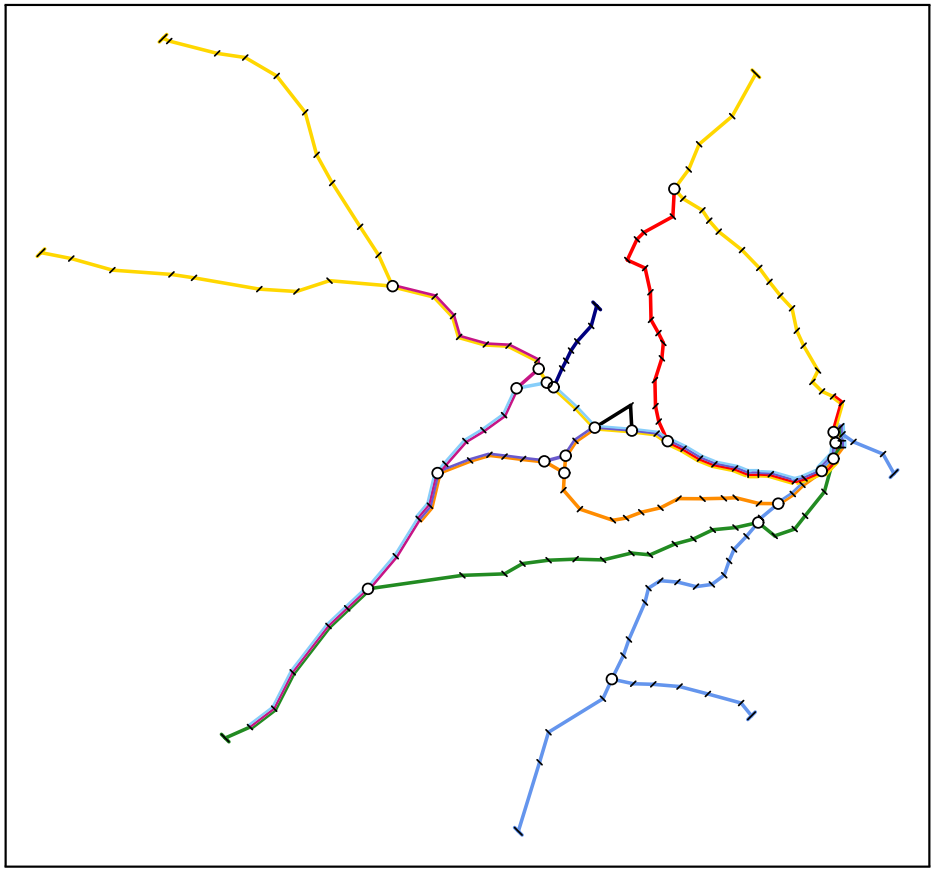
↓

MIP	constr.	var.
full	1,191,406	290,137
callback*	21,988	92,681
skipped	6,838	2,969

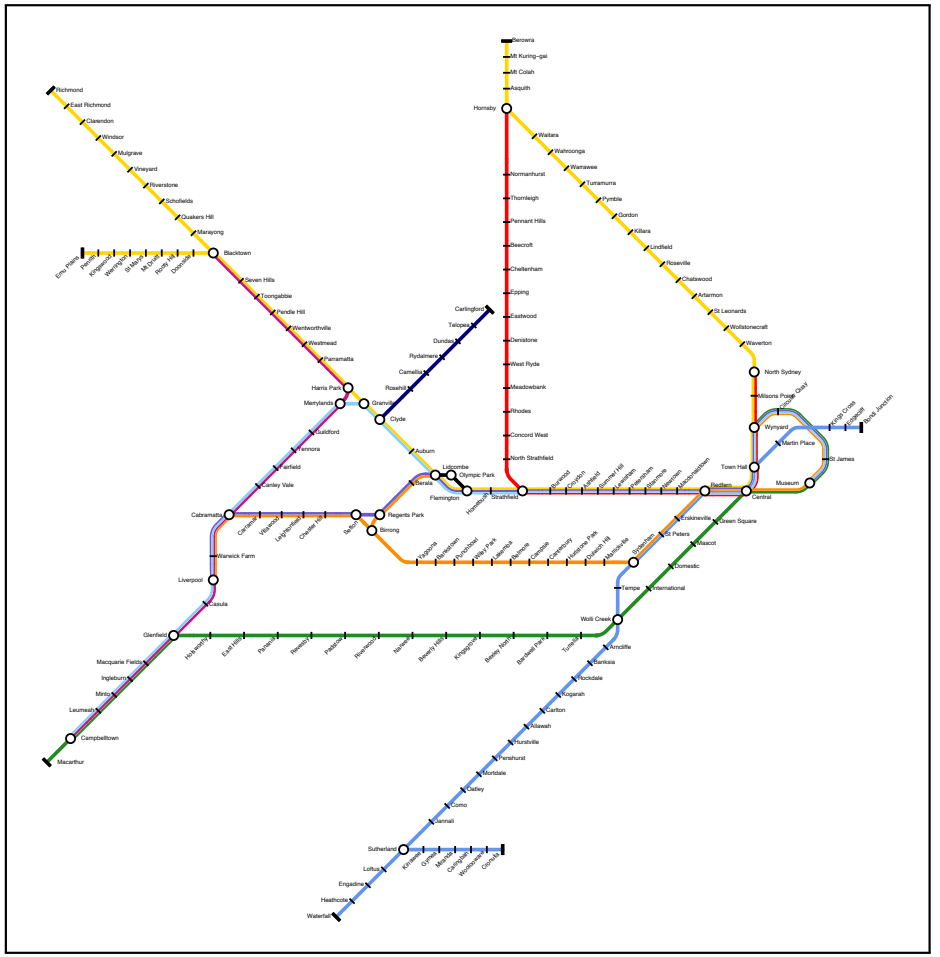
*) 10:30 hours w/o proof of opt.
add constr. of 123 edge pairs

Results – Sydney labeled

Input

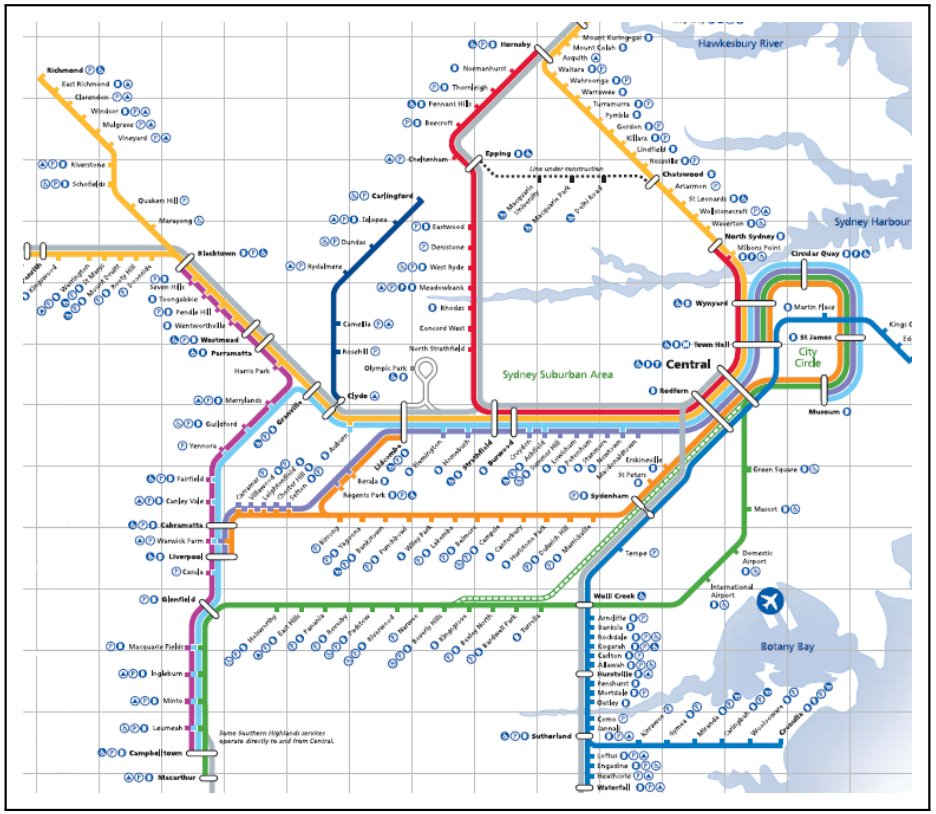


Output (10:30 hrs.)

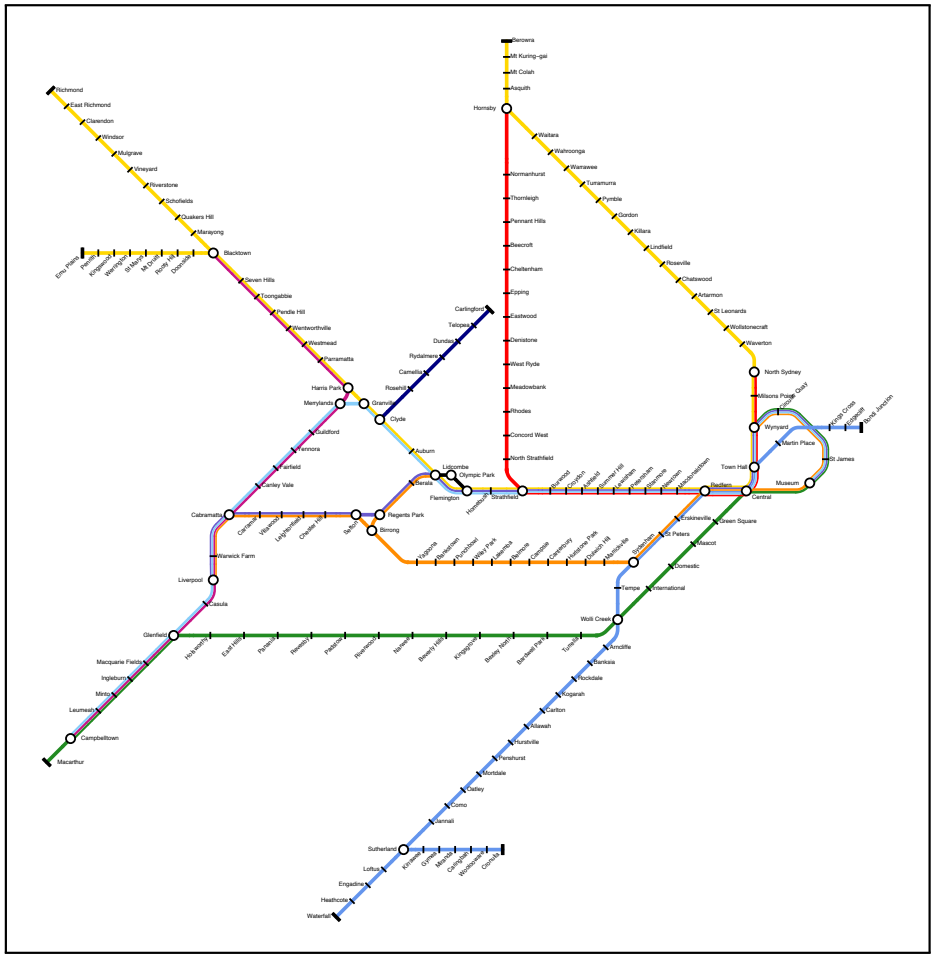


Results – Sydney labeled

Official map

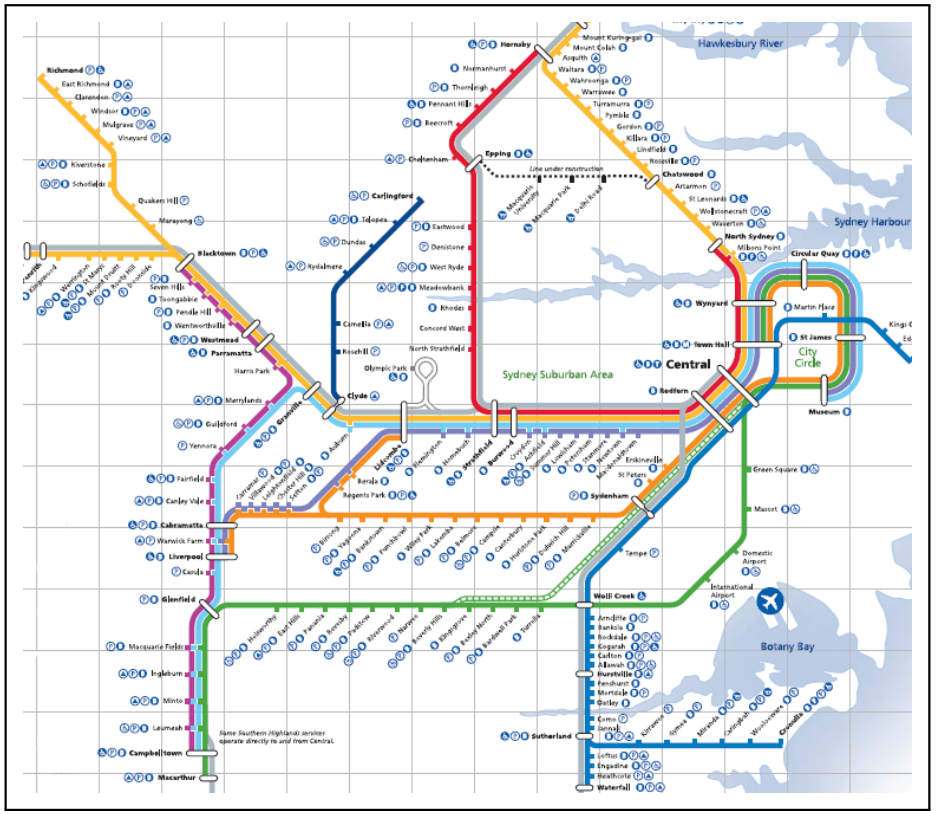


Output (10:30 hrs.)

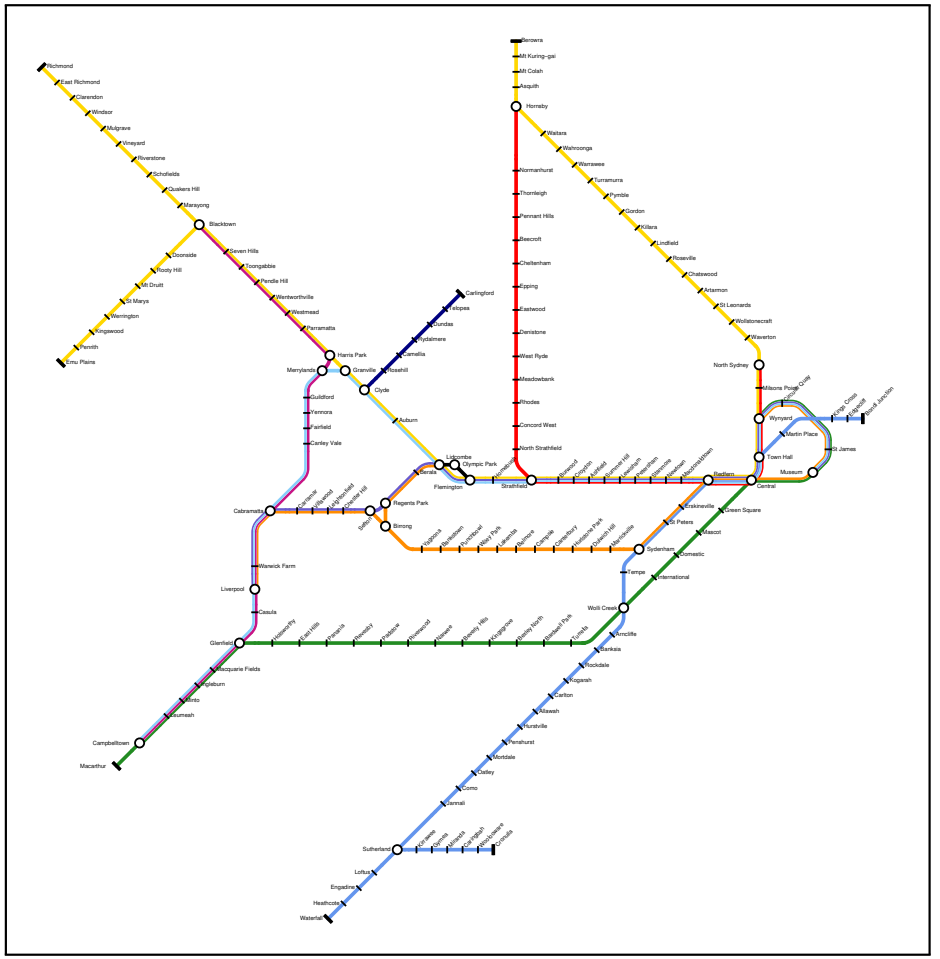


Results – Sydney labeled

Official map

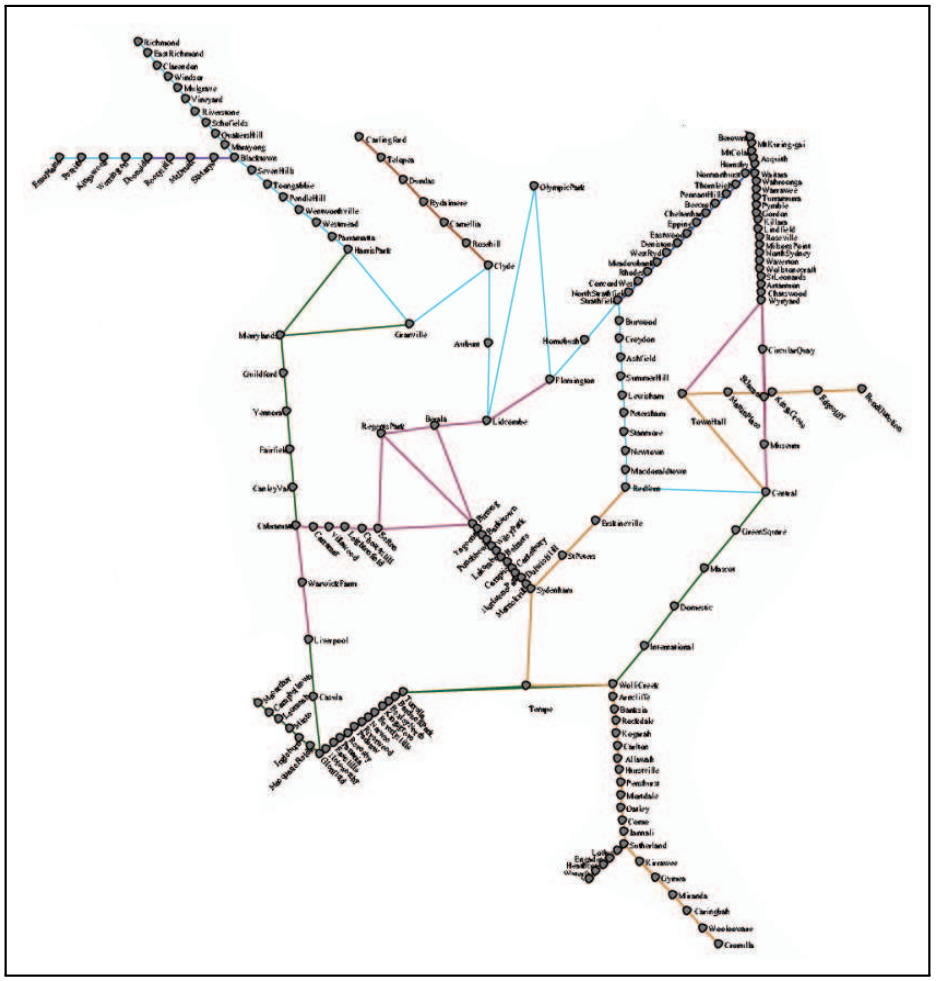


Output (1:40 hrs.)

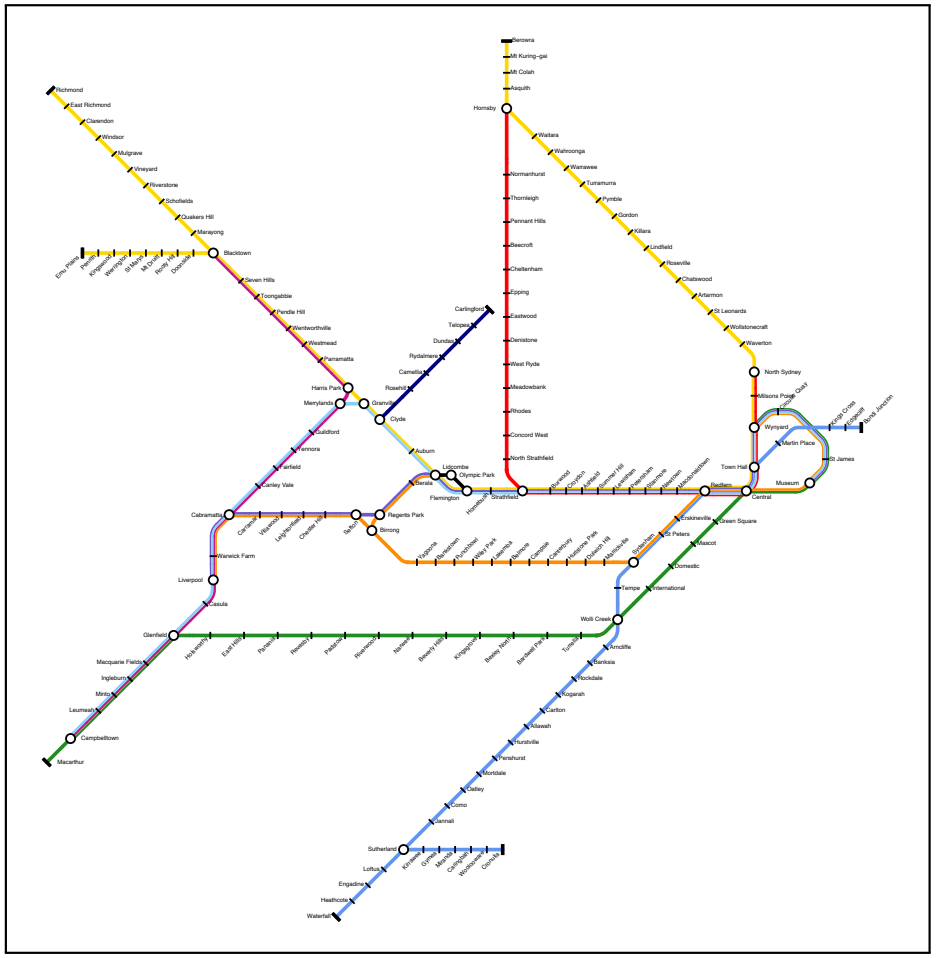


Sydney: Related Work

[Hong et al. GD'04] (7.6 sec.)

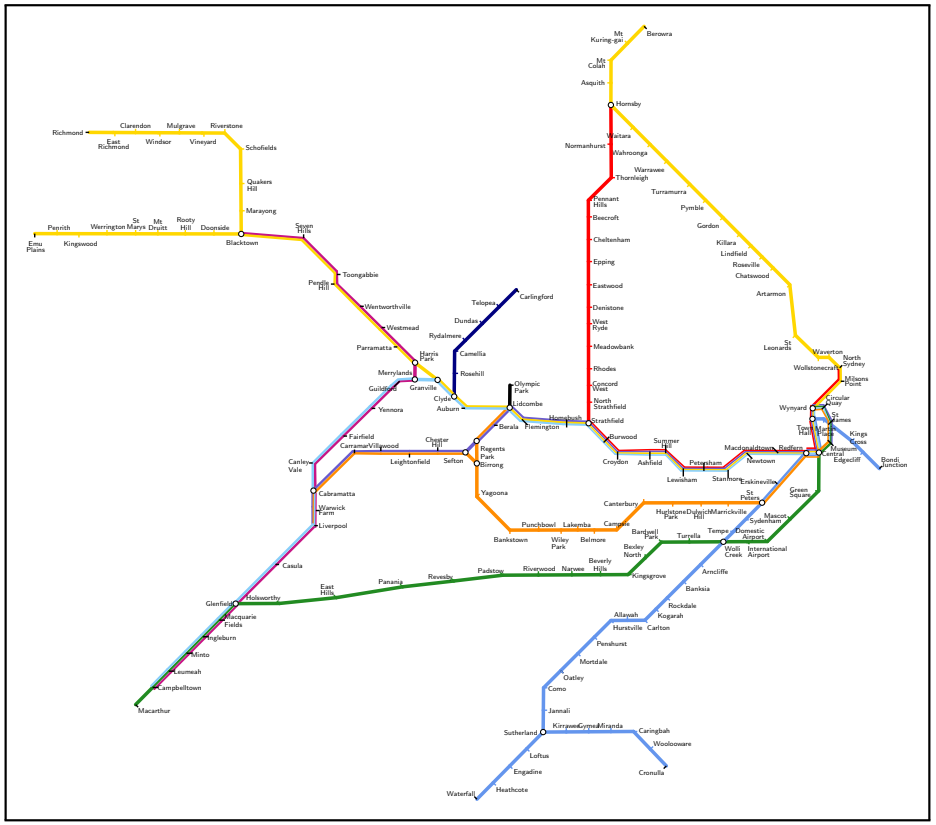


Our output (10:30 hrs.)

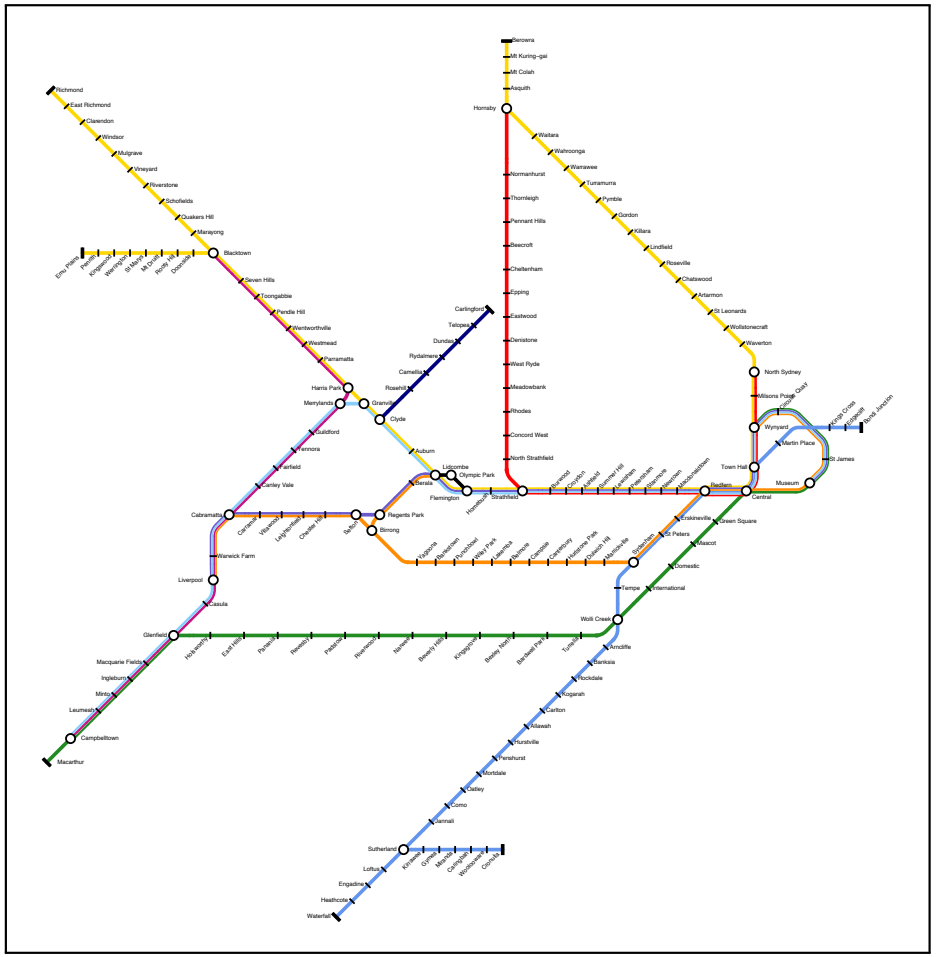


Sydney: Related Work

[Stott, Rodgers TVCG'10] (2 hrs.)

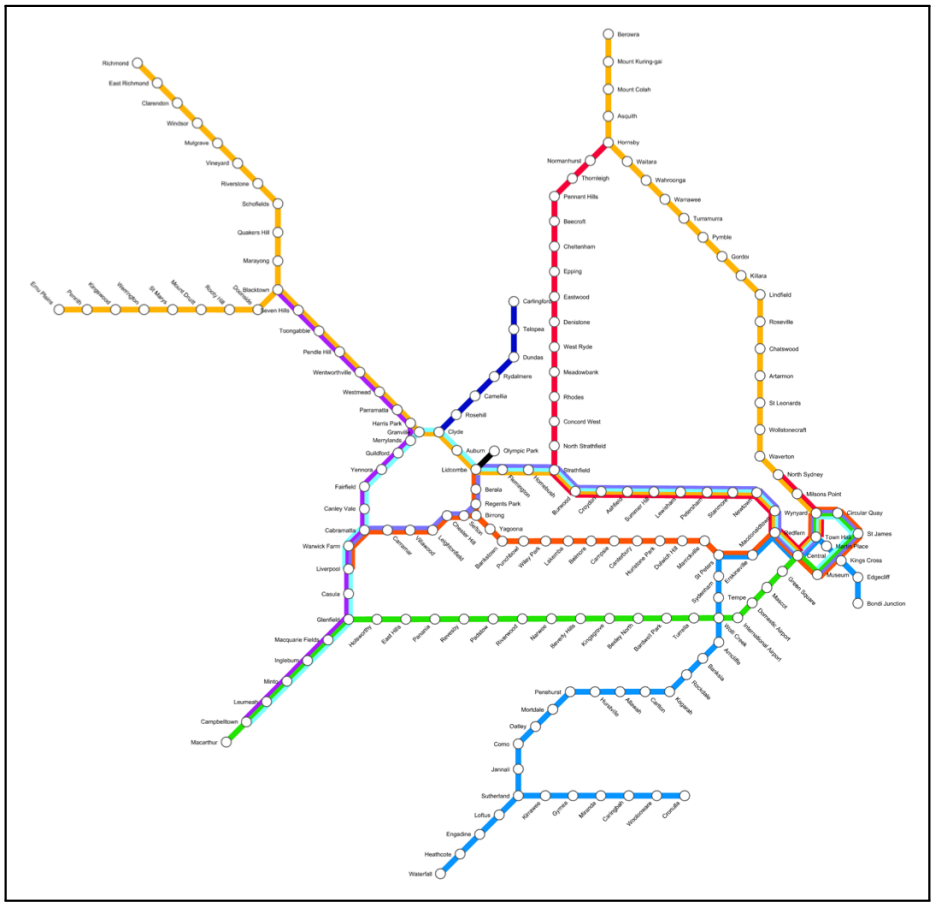


Our output (10:30 hrs.)

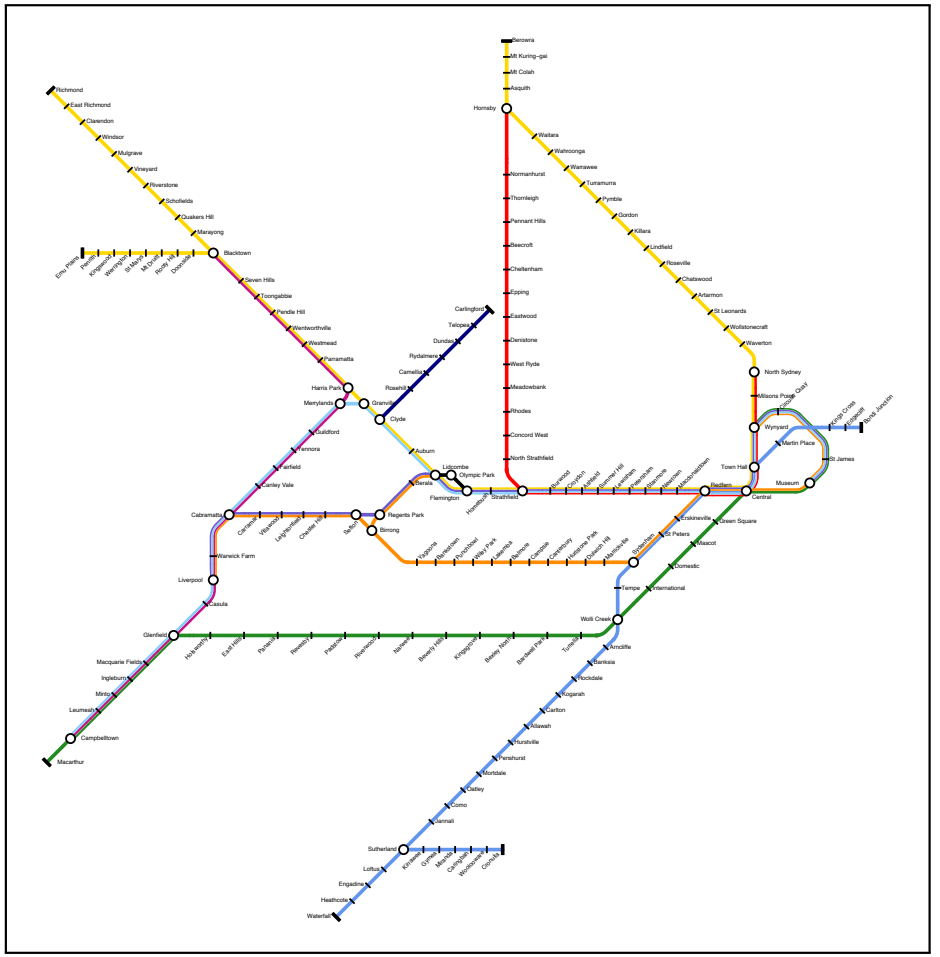


Sydney: Related Work

[Wang, Chi TVCG'11] (1 sec.)



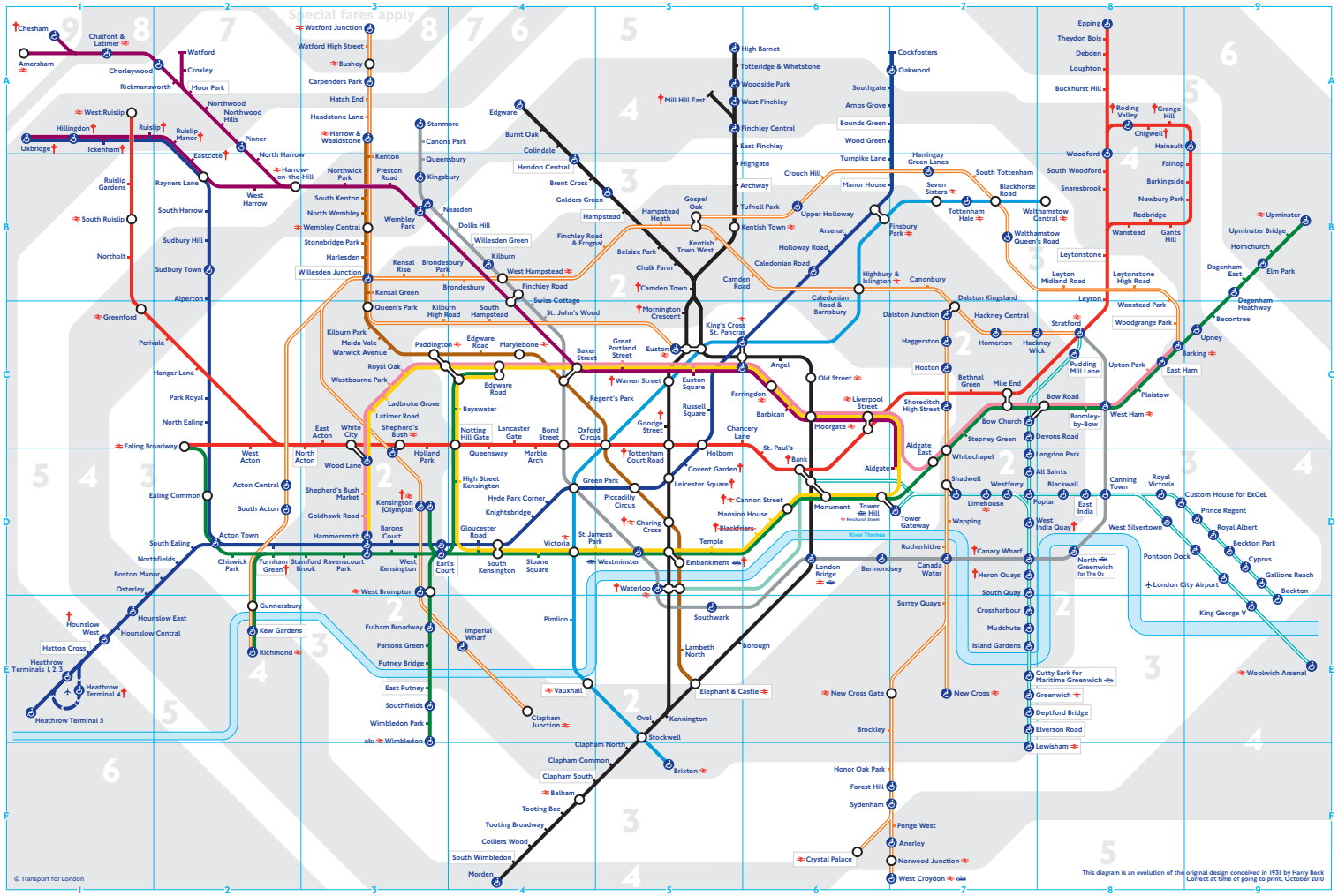
Our output (10:30 hrs.)



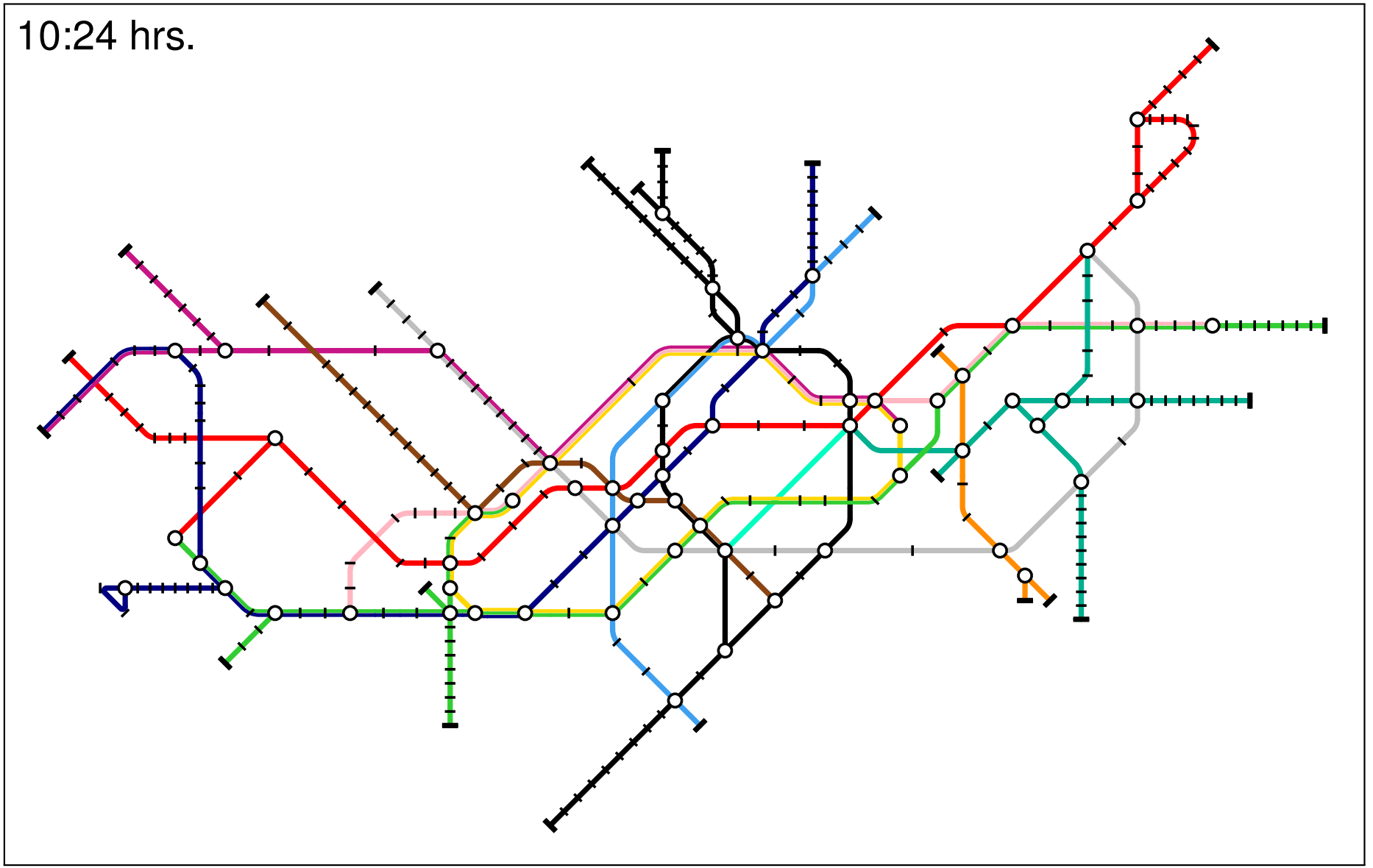
Large Example: London

Karlsruhe Institute of Technology

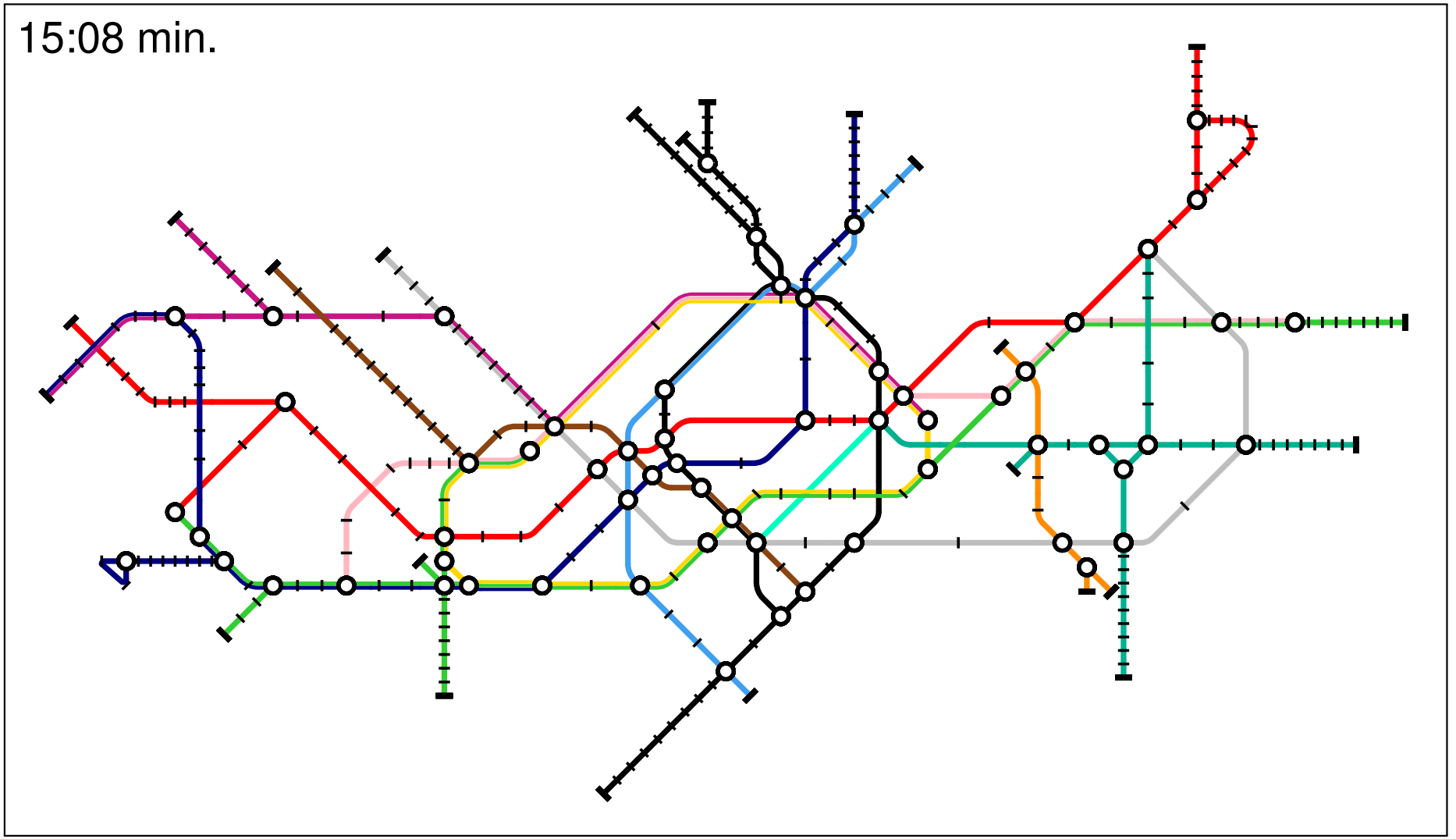
Tube map



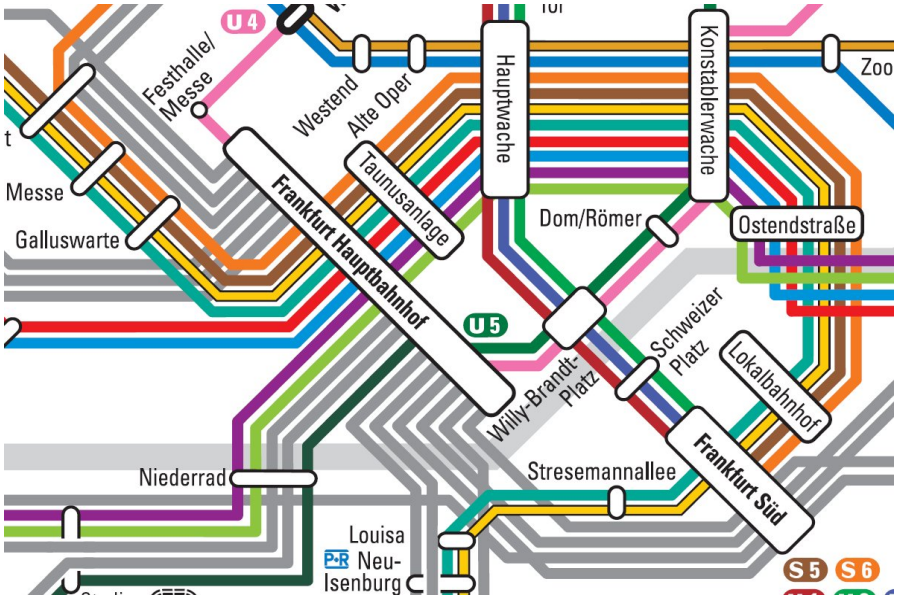
Large Example: London

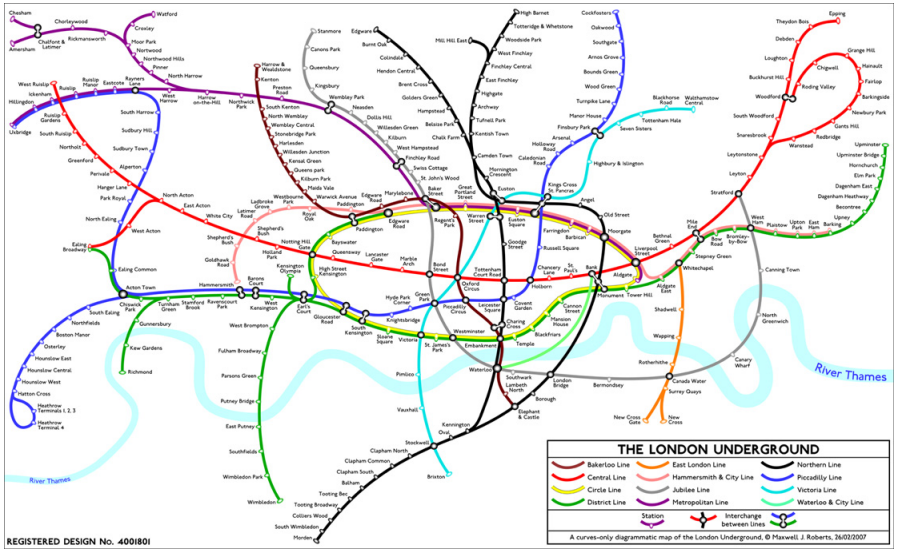


Large Example: London



Problem solved?





Open questions

- more user interaction
- how to handle large stations and many parallel lines?
- formulate global aesthetics like symmetry and balance
- use of curves for metro layouts (see [Fink et al. GD'12])

Summary

- METROMAPLAYOUT is NP-hard
- formulation of hard and soft constraints as MIP
- combined layout and labeling
- MIP size & runtime reductions
- high-quality results
- MIP can schematize *any* kind of graph sketch

Summary

- MetroMapLayout is NP-hard
- formulation of hard and soft constraints as MIP
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For more info see:

M. Nöllenburg and A. Wolff. *Drawing and labeling high-quality metro maps by mixed-integer programming.* IEEE Trans. Visualization and Computer Graphics 17(5):626–641, 2011.