

Exercise Sheet 5

Assignment: January 8, 2014
Delivery: None, Discussion on January 8, 2014

1 Extended Canonical Ordering for 4-Connected Graphs

A planar graph $G = (V, E)$ is called *proper triangular planar* (PTP, for short) if the following conditions hold:

- Every interior face of G is a triangle and the exterior face of G is a quadrangle;
- G has no separating triangles.

Let $G = (V, E)$ be a PTP graph with vertices a, b, c, d on the outer face. A labeling $v_1 = a, v_2 = c, v_3, \dots, v_n = d$ of the vertices of G is called an *extended canonical ordering* of G if for every $4 \leq k \leq n$:

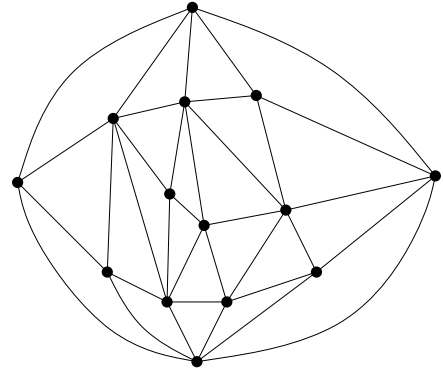
- The subgraph G_{k-1} induced by v_1, \dots, v_{k-1} is biconnected and the boundary C_{k-1} of G_{k-1} contains the edge (a, b) ;
- v_k is in the exterior face of G_{k-1} , and its neighbors in G_{k-1} form (at least 2-element) subinterval of the path $C_{k-1} \setminus (a, b)$. If $k \leq k - 2$, v_k has at least 2 neighbors in $G \setminus G_{k-1}$.

Let $G = (V, E)$ be a PTP graph with vertices a, b, c, d on the outer face.

- Prove that the graph obtained from G by removal of vertices c, d and all edges incident to them is biconnected.
- Let $C = \{a = u_1, \dots, u_k = b, a\}$ be a simple cycle of G such that $c, d \notin C$. Let G_C denote the graph induced by the vertices of G laying inside C (including the vertices of C). Let $v_i \in C$, $2 \leq i \leq k - 1$ such that no internal chord of C is incident to v_i . Show that $G \setminus \{v_i\}$ is biconnected.
- Let C be as above and let (v_i, v_j) , $1 \leq i < j \leq k$, be an internal chord of C . Show that there exists a vertex v_l , $i < l < j$, which is adjacent to at least two vertices of $G \setminus G_C$.
- Using (a-c) prove that G has an extended canonical ordering such that $v_1 = a, v_2 = b, v_{n-1} = c, v_n = d$.

2 Construction of Rectangular Dual

Consider the graph G of the figure. Check whether G satisfies the necessary conditions to have a rectangular dual. In affirmative, construct a rectangular dual of G .



3 Contact Representation of Maximal Planar Graphs

Prove that a maximal planar graph admits a contact representation with T-shapes. An example of such a representation is shown in the figure.

Hint: Use canonical ordering in the way similar to the construction of a visibility representation (Exercise set 2).

