Exercise Sheet 3

Assignment: November 26, 2013
Delivery: None, Discussion on December 3, 2013

1 Linear time construction of a Schnyder realizer.

Let G be a maximal planar graph with n vertices. Can a Schnyder labeling and a Schnyder realizer be constructed in time $O(n)$?

Hint: Find a Connection between a canonical ordering and the ordering in which the edge contraction for the construction of a Schnyder labeling is executed.

2 Property of a Schnyder realizer.

Let G be a maximal planar graph with vertices a, b, c on the outer face. Let $T_a,\ T_b,\ T_c$ be the red, the blue and the green trees of a Schnyder realizer, with sinks at vertices $a,\ b,\ c$, respectively. Let v be an internal vertex of G and denote by $P_a(v),\ P_b(v),\ P_c(v)$ the paths connecting v with $a,\ b,\ c$ in $T_a,\ T_b,\ T_c$, respectively. Show that paths $P_a(v)$, $P_b(v)$ and $P_c(v)$ do not have common vertices, except for v.

3 Induced path in a Schnyder realizer.

A path of a graph G is called *induced* if the vertices of this path are connected only by the edges of the path, i.e. path on vertices $v_1, \dots v_k$ is *induced* if for any $1 \leq i, j \leq n$ such that $|i - j| > 1$, edge (v_i, v_j) does not belong to G. Let G be a maximal planar graph and let T_a, T_b, T_c be a Schnyder realizer of G. Assume that the edges of $T_a,\ T_b,\ T_c$ are colored red, blue and green, respectively. Show that a directed monochromatic path in T_a, T_b, T_c is an *induced path* of G.

4 Property of *st*-Ordering

Let $G = (V, E)$ be a biconnected planar graph with a given embedding and let $v_1, \dots, v_n$ be an *st*-ordering of G such that v_1, v_n belong to the outer face of G. Let G_i denote the plane subgraph of G induced by the vertices $v_1, \dots, v_i$. Prove that v_{i+1} belongs to the outer face of G_i.

5 Ear decomposition.

Let $G = (V, E)$ such that for each edge $\{s, t\} \in E$, G has an open ear decomposition that starts with $\{s, t\}$. Show that G is 2-connected. (Recall that the reverse statement was proven in the lecture.)