Exercise Sheet 2

Assignment: November 14, 2013
Delivery: None, Discussion on November 20, 2012

1 Canonical Ordering

Let G be a maximal plane graph with vertices v_1, v_2, v_n on the outer face. Let P be a simple path in G connecting vertices v_1 and v_2 and not containing v_n. Let G_P be the subgraph of G bounded by path P and edge (v_1, v_2). Prove that there exists a canonical ordering of G such that all the vertices of G_P appear as initial subsequnce of this ordering.

2 Generalization of Canonical Ordering for Triconnected Planar Graphs

The canonical ordering (which untill now has been defined only for maximal planar graphs) can be generalizaed to triconected planar graphs as follows. Let $G = (V, E)$ be a triconnected planar graph. An ordering $v_1, \dots, v_n$ of the vertices of G is called *canonical* if vertices v_2, v_n are neighbors of v_1, and v_1, v_2, v_n belong to a common face, and for every k, $k > 3$:

(a) Vertex v_k is on the outer face of G_k and has at least two neighbors in G_{k-1}, which are on the outer face of G_{k-1}. Vertex v_k has at least one neighbor in $G - G_k$. Graph G_k is biconnected,
(b) or there exists an $\ell \geq 1$ such that $v_k, \dots, v_{k+\ell}$ is a chain on the outer face of $G_{k+\ell}$ and has exactly two neighbors in G_{k-1}, which are on the outer face of G_{k-1}. Every vertex $v_k, \dots, v_{k+\ell}$ has at least one neighbor in $G - G_{k+\ell}$. Finally, $G_{k+\ell}$ is biconnected.

By G_k we denote the subgraph of G induced by the vertices $v_1, \dots, v_k$ and by $G - G_k$ the subgraph of G resulting from G by removing the vertices and edges of G_k.

Prove that every triconnected planar graph admits a canonical ordering.

3 Visibility Representation of Maximal Planar Graphs

Recall the definition of *visibility representation* from the previous exersize set. Prove that every maximal planar graph admits a visibility representation.

Hint: Use canonical ordering.

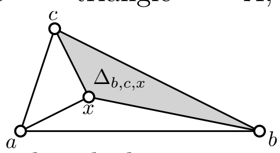
4 Barycentric Coordinates

Let $\Delta_{a,b,c}$ be a triangle on the plane on vertices a, b and c. For each point x laying inside triangle $\Delta_{a,b,c}$ there exists a triple (x_a, x_b, x_c) such that $x_a \cdot a + x_b \cdot b + x_c \cdot c = x$ and $x_a + x_b + x_c = 1$. The triple (x_a, x_b, x_c) is called *barycentric coordinates* of x with respect to $\Delta_{a,b,c}$.

Prove that:

(a) If $A(\Delta)$ denotes the area of the triangle A, then

$$x_a = \frac{A(\Delta_{b,c,x})}{A(\Delta_{a,b,c})},\ x_b = \frac{A(\Delta_{a,c,x})}{A(\Delta_{a,b,c})},\ x_c = \frac{A(\Delta_{a,b,x})}{A(\Delta_{a,b,c})}$$



(b) Equations $x_a = 0$, $x_b = 0$, $x_c = 0$ represent lines through bc, ab and ab, respectively.

(c) Let (x_a, x_b, x_c) be barycentric coordinates of point x in triagnle Δ_{abc}. The set of points $\{(x_a, x'_b, x'_c) : x'_b, x'_c \in \mathbb{R}\}$ represents a line parallel to edge bc passing through point x. Similarly, sets of points $\{(x'_a, x_b, x'_c) : x'_a, x'_c \in \mathbb{R}\}$, $\{(x'_a, x'_b, x_c) : x'_a, x'_b \in \mathbb{R}\}$ represent lines parallel to edges ac, ab, respectively, passing through point x.

5 Baricentric representation

A *Baricentric representation* of a graph G is an injective function $f : v \in V(G) \rightarrow (v_a, v_b, v_c) \in \mathbb{R}^3$ satisfying the following two conditions:

(1) $v_a + v_b + v_c = 1$ for all vertices v; and

(2) for each edge (x, y) there is no vertex $z \notin \{x, y\}$, such that $\max\{x_k, y_k\} > z_k$ for each $k \in \{a, b, c\}$.

Let f be a barycentric representation of graph G, and let a, b, c be non collinear points on the plane. Prove that the function $g : v \in G(V) \rightarrow v_a a + v_b b + v_c c \in \mathbb{R}^2$ yelds a planar straight line drawing of G inside the triangle $\Delta_{a,b,c}$.