Drawing Trees with Perfect Angular Resolution and Polynomial Area

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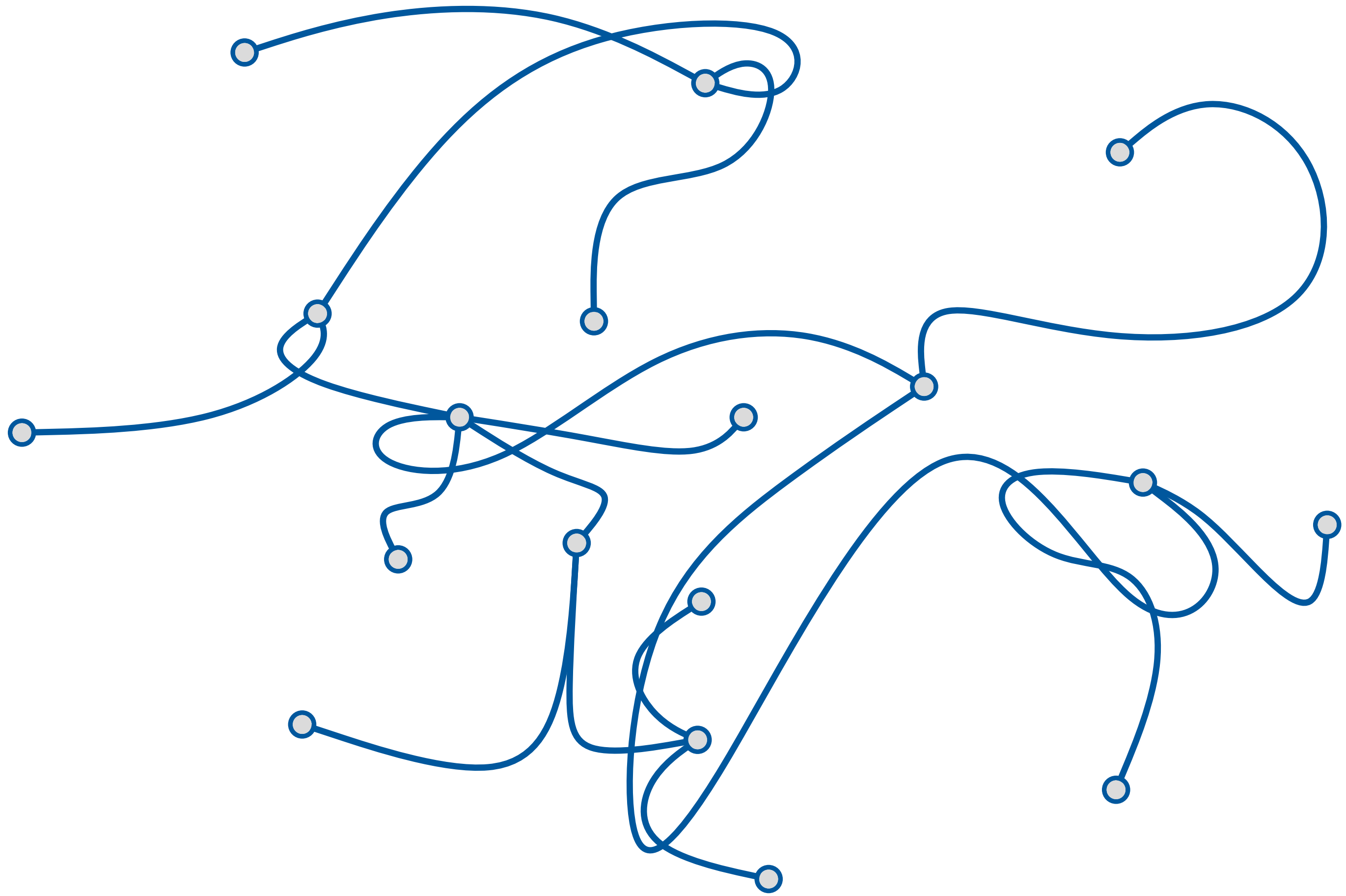
Vorlesung *Algorithmen zur Visualisierung von Graphen* · 05.02.2013

What makes a good tree drawing?



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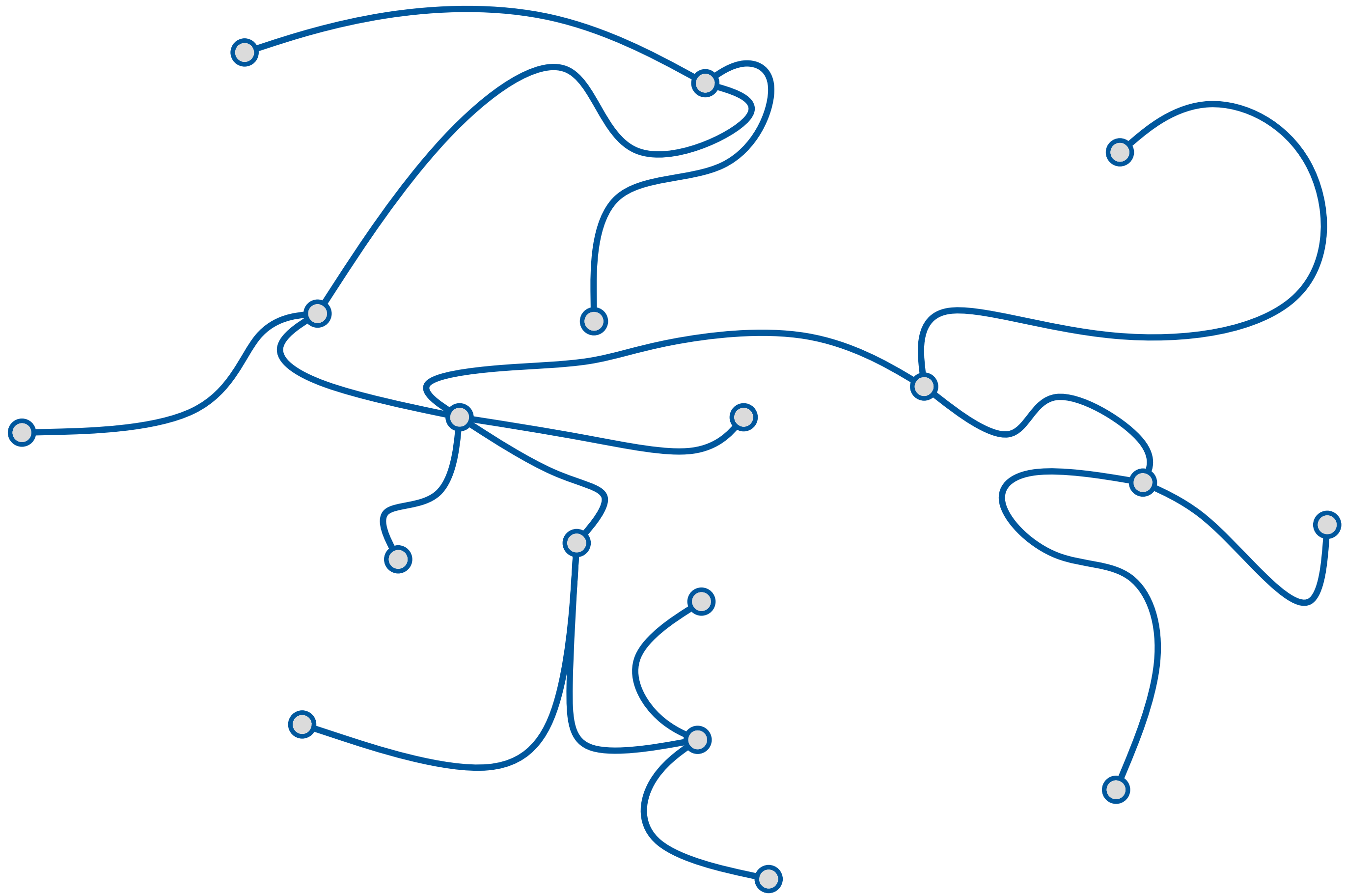

- crossing-free edges

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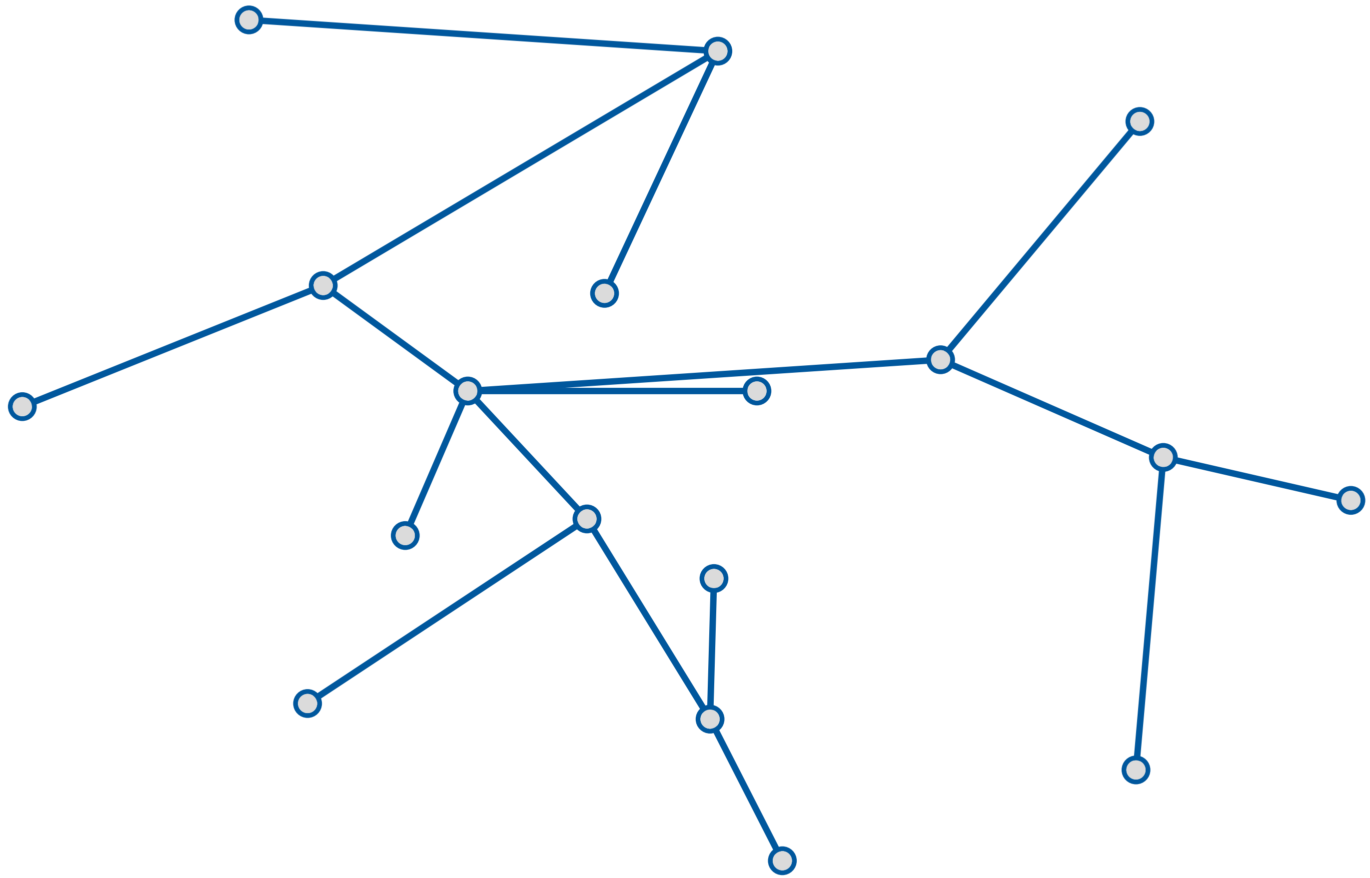

- crossing-free edges
- easy-to-follow edges

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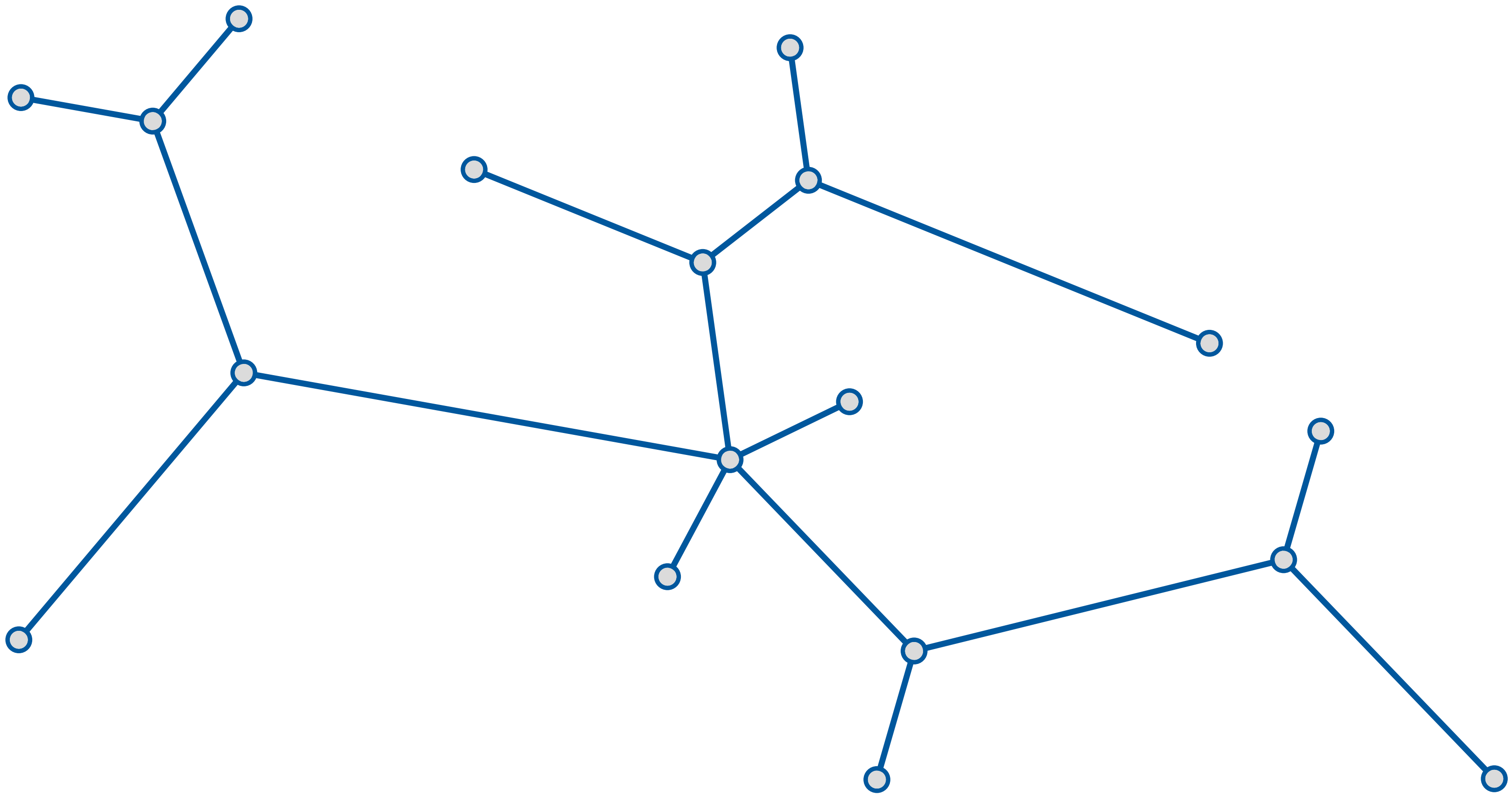

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- easy-to-follow edges
- good angular resolution

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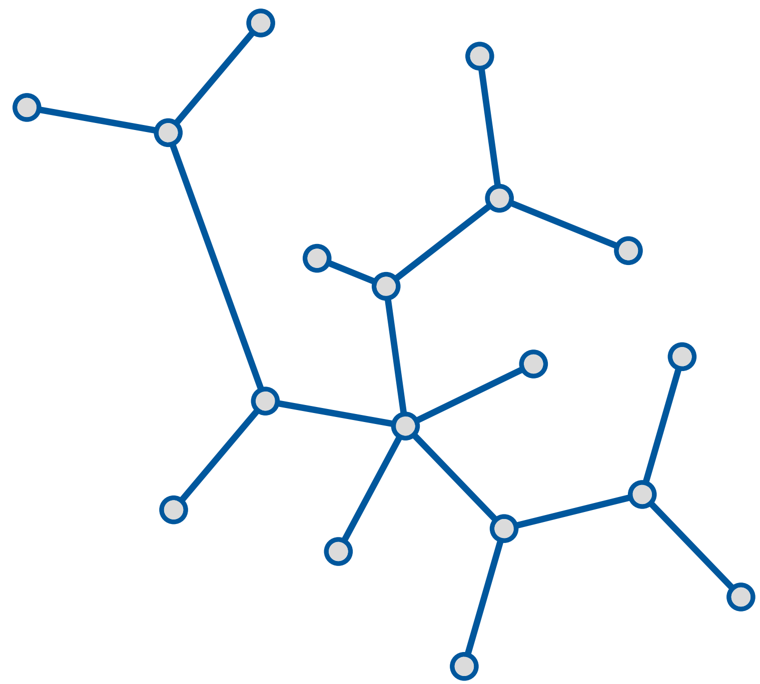
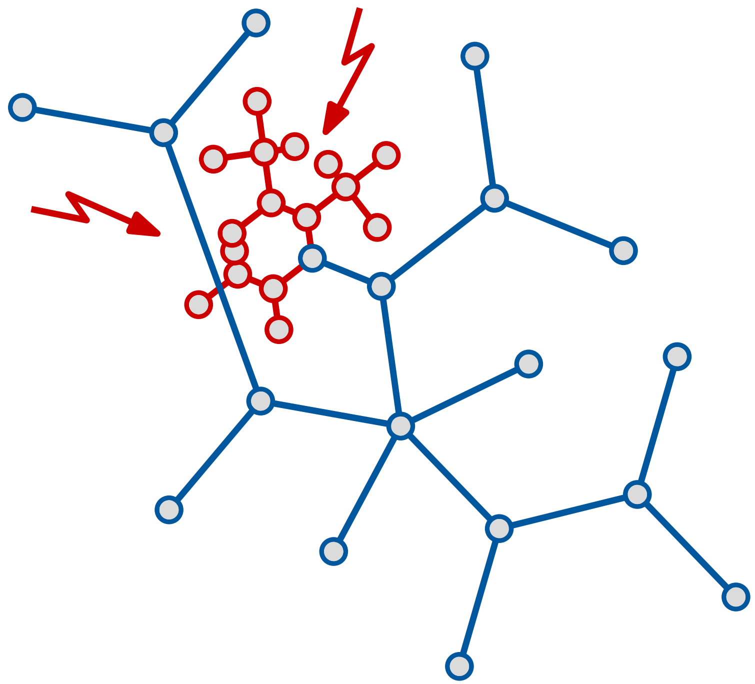


- crossing-free edges
- easy-to-follow edges
- good angular resolution
- small space consumption

What makes a good tree drawing?



Can we achieve all these goals simultaneously?

- crossing-free edges
- easy-to-follow edges
- good angular resolution
- small space consumption

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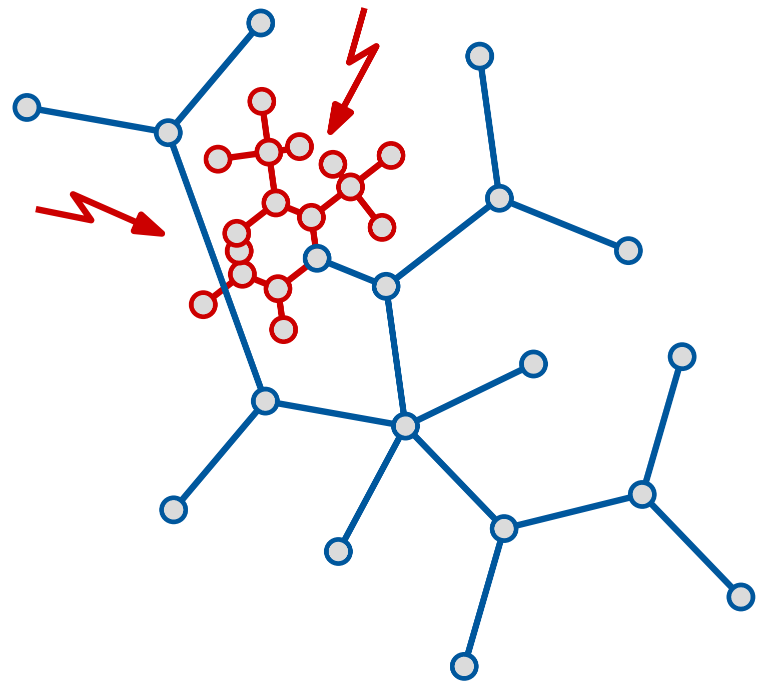

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It depends...

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Our results

Any tree has a drawing with

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- perfect angular resolution
- polynomial area

	unordered trees	ordered trees
straight edges	✓	✗

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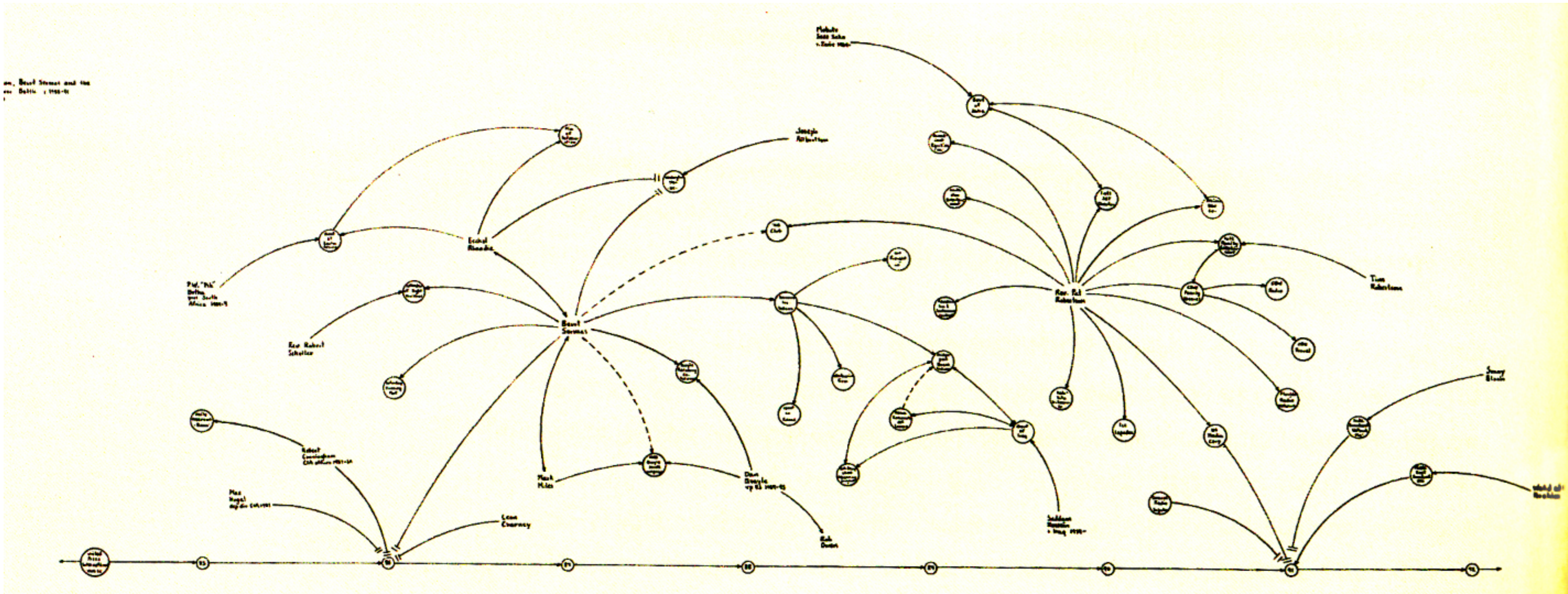
Inspiration from fine arts



Mark Lombardi “Pat Robertson, Beurt Servaas, and the UPI Takeover Battle, ca. 1985–91”

Edges in Mark Lombardi’s network drawings

- easy-to-follow circular arcs
- generalization of straight-line edges

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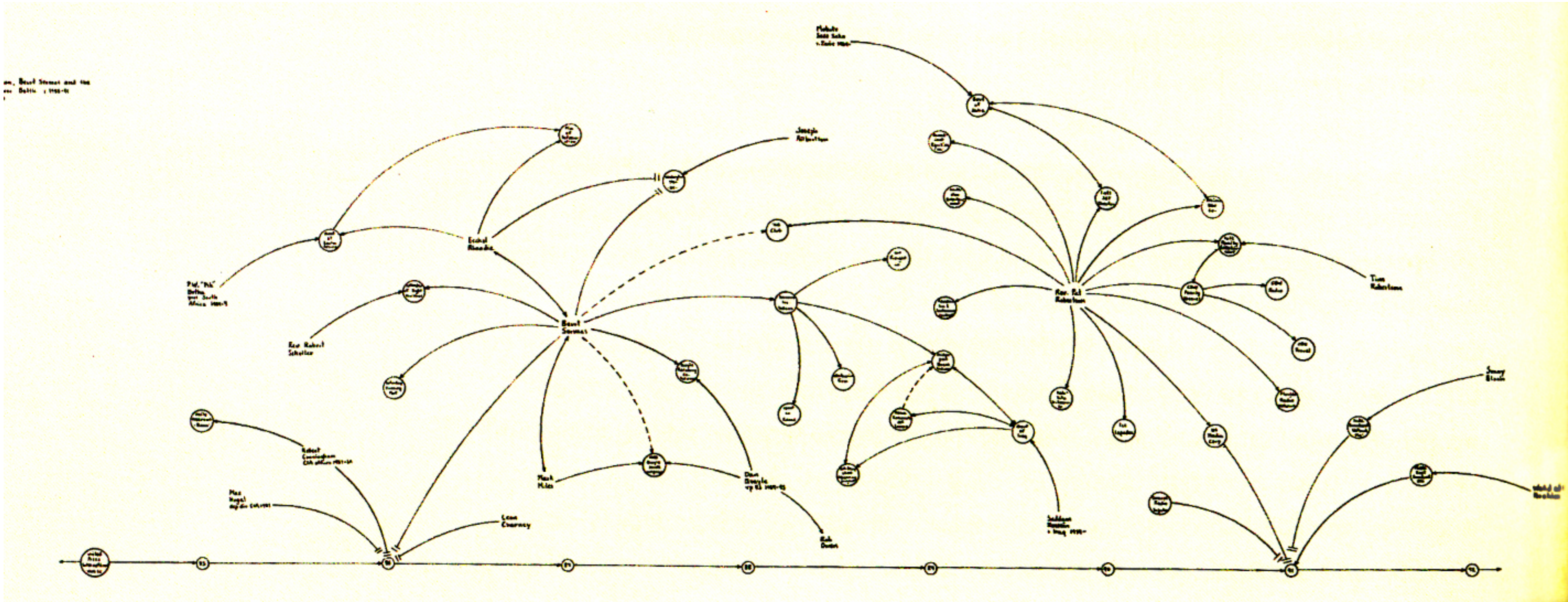

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Edges in Mark Lombardi's network drawings

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- generalization of straight-line edges

Draw edges as circular arcs!

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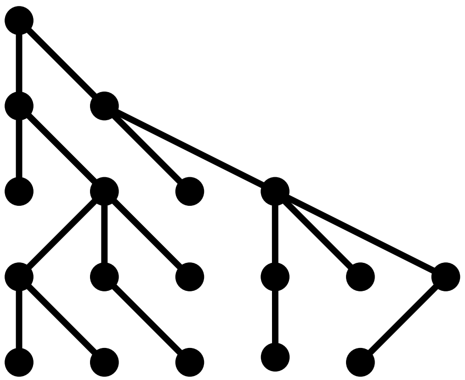
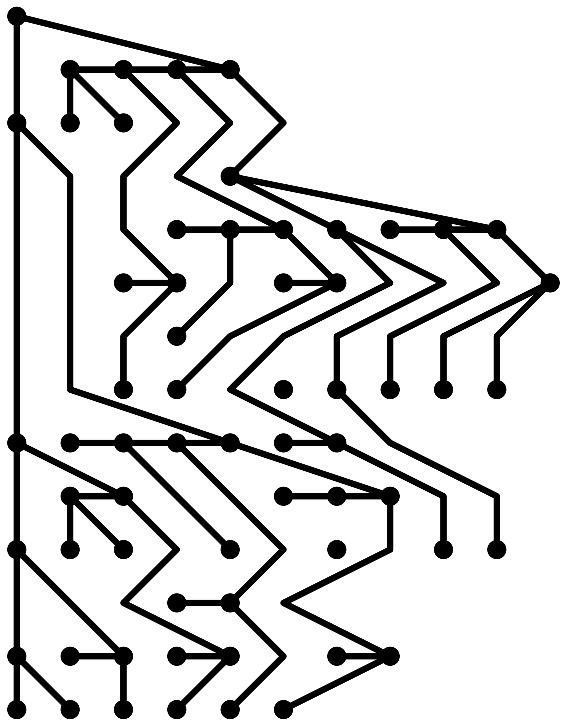
Related work

Straight-line drawings of trees

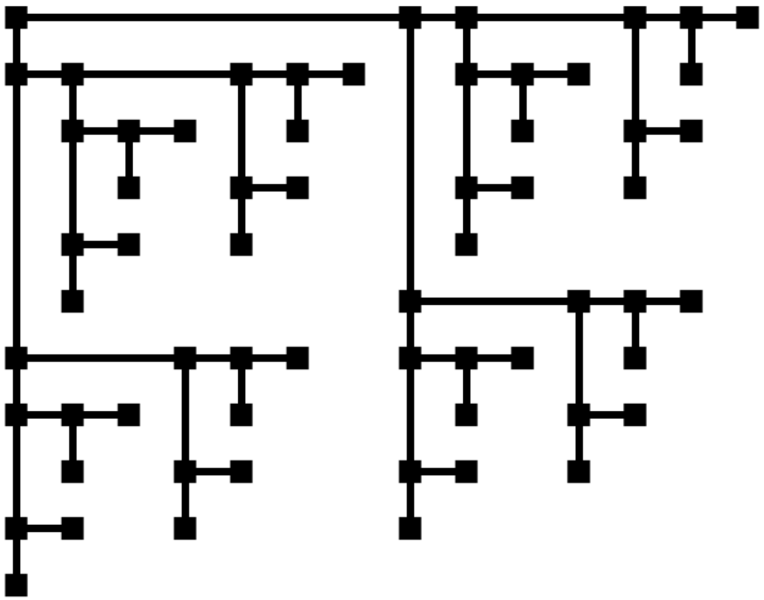
- old topic in graph drawing [Wetherell & Shannon, 1979]
- optimize area but not angular resolution



[Wetherell & Shannon, 1979]



[Garg, Goodrich, Tamassia, 1994]



[Garg & Rusu, 2004]

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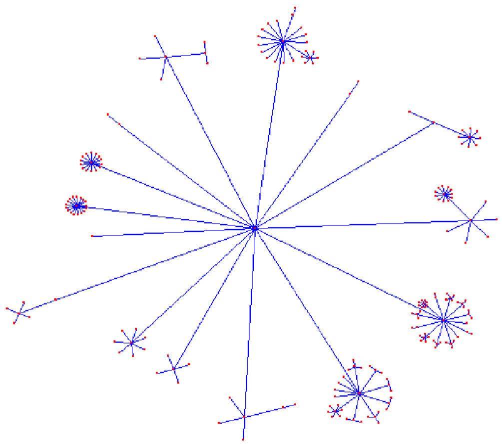
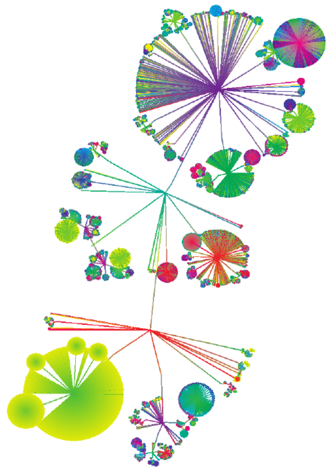
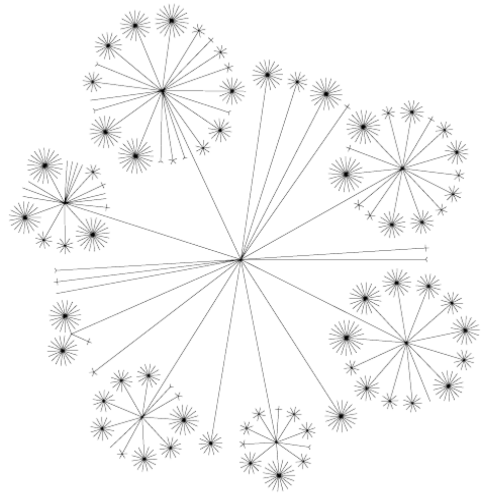
Angular-resolution-aware tree drawings

- circular drawings for rooted trees [Melançon & Herman, 1998]
- bubble drawings for ordered trees [Grivet et al., 2004]
- balloon drawings for (un)ordered trees [Lin & Yen, 2007]
- closer to our goal but no guarantee of *all* constraints







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Drawing graphs with smooth curves

- circular arcs for planar graphs with bounded angular resolution [Cheng et al., 2001]
- force-directed curvilinear drawings to improve angular resolution [Finkel & Tamassia, 2004]

Overview

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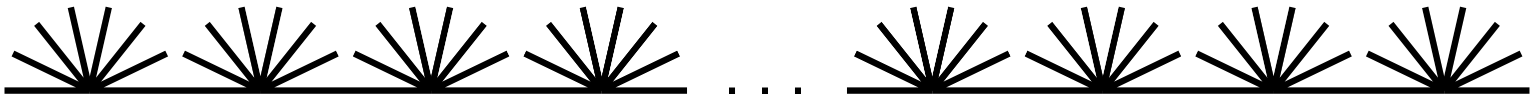
	unordered trees	ordered trees
straight edges	✓	1 ✗
Lombardi edges	✓	2 ✓

Ordered trees and straight-line edges

Fibonacci caterpillars

There is an infinite family of trees that require exponential area in any straight-line drawing with perfect angular resolution.



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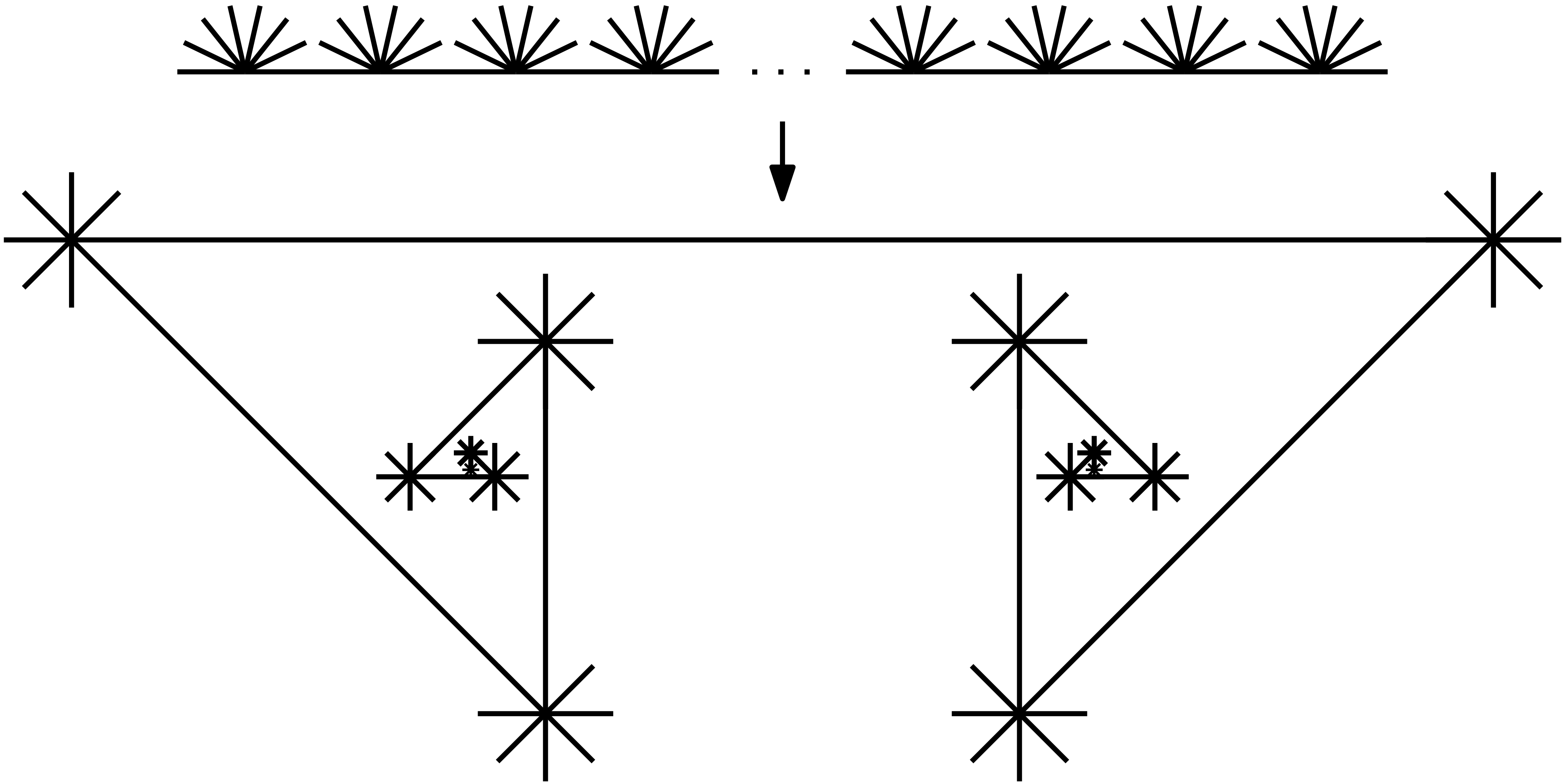

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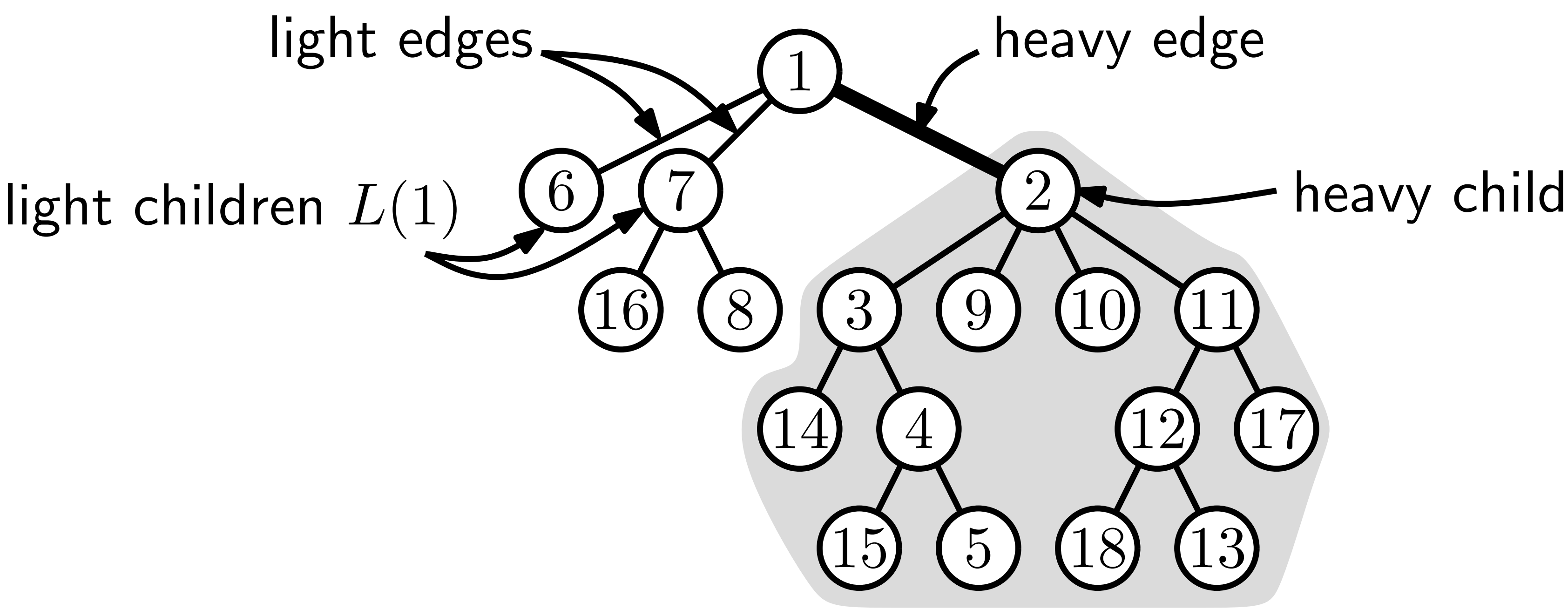
Heavy path decomposition (I)

Definition

- choose arbitrary root r of tree T
- u is the **heavy child** of v if subtree T_u is largest among children of v
- all other children of v are **light children** in a set $L(v)$
- edge (u, v) from heavy child u to parent v is a **heavy edge**
- edges from light children to parent v are **light edges**



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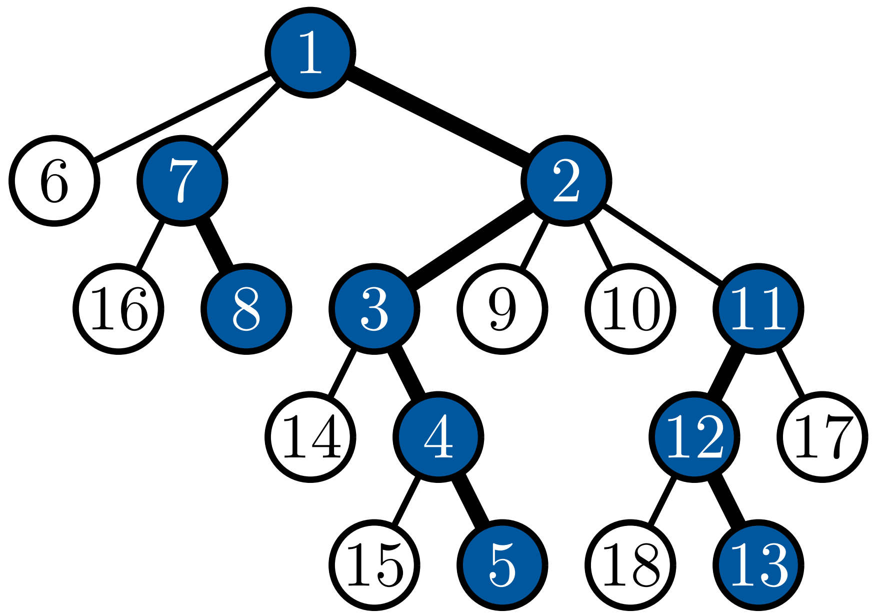


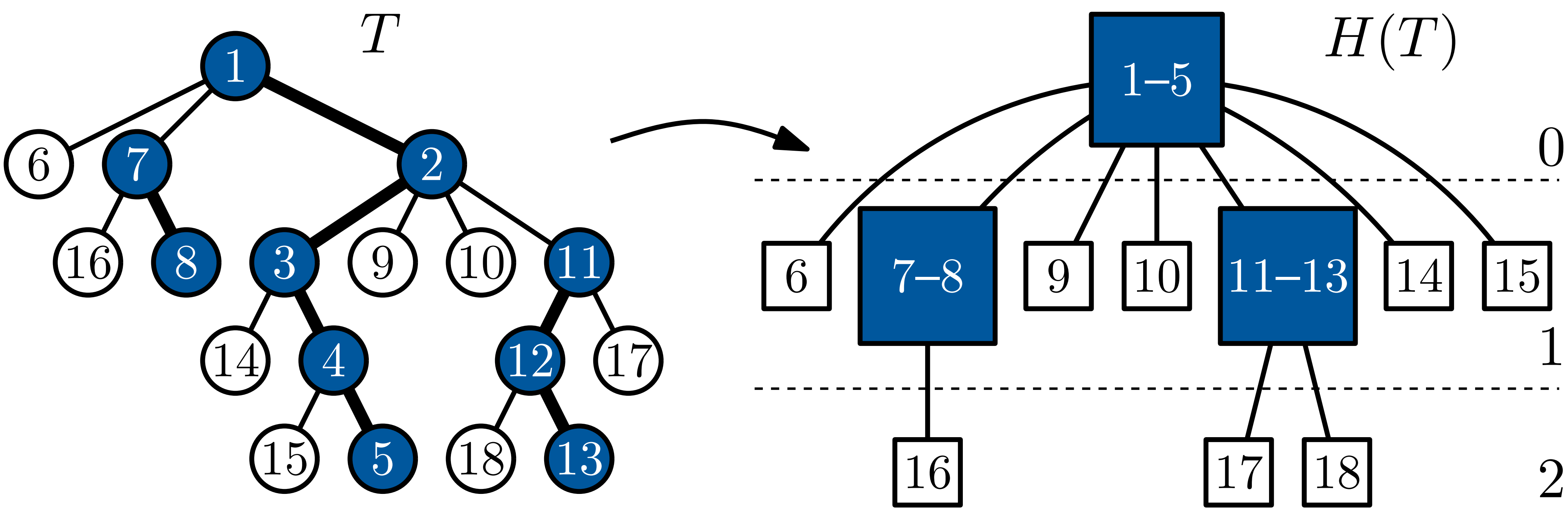
Heavy path decomposition (II)

Definition

The set of heavy edges induces the **heavy path decomposition** (HPD) of T into heavy paths and singleton nodes. [Harel & Tarjan, 1984]

Property

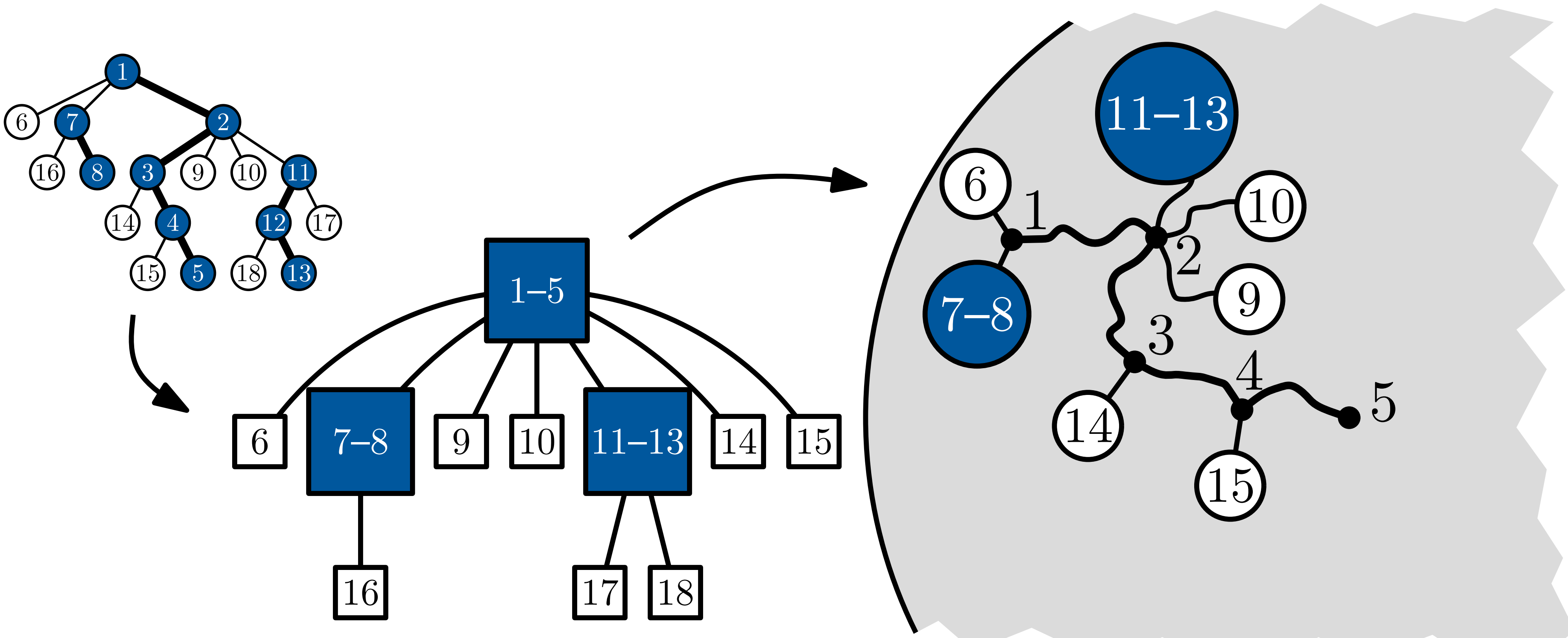
The HPD induces a decomposition tree $H(T)$ of height $h \leq \log_2 n$.



Drawing ordered trees

Sketch of the algorithm

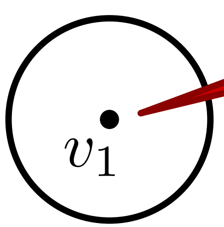
- draw each heavy path P of T within a disk of polynomial area
- each heavy path at level $j \geq 1$ of $H(T)$ is a light child of a heavy path at level $j-1$
- given drawings of all heavy paths at level j recursively construct drawings of all heavy paths at level $j-1$



Step 1: Drawing heavy paths

Input

- heavy path $P = (v_1, v_2, \ldots, v_k)$ rooted at v_k
- required angles $\alpha_i = c \cdot \frac{2\pi}{\deg(v_i)}$ between $v_{i-1}v_i$ and v_iv_{i+1} $(c \in \mathbb{N})$
- disks D_i containing v_i and all subtrees of light children of v_i



place leaf v_1 and disk D_1 in the center

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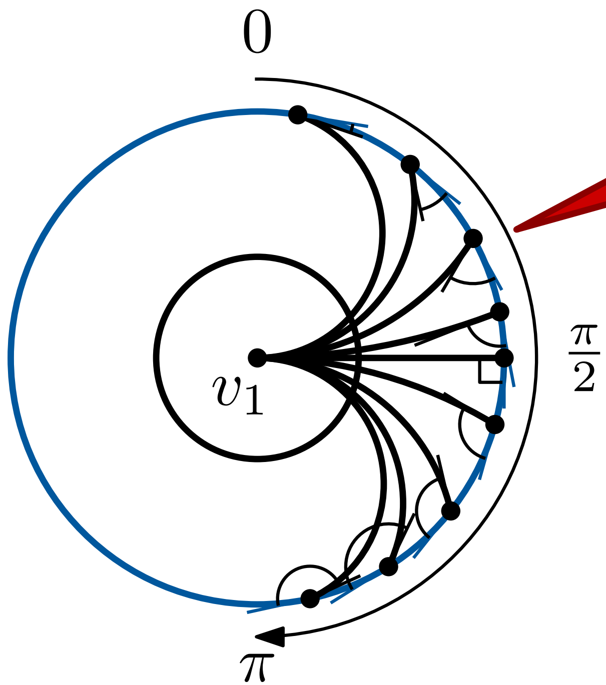

place v_2 (and D_2) on a concentric circle based on $\frac{\alpha_2}{2} \in [0, \pi]$

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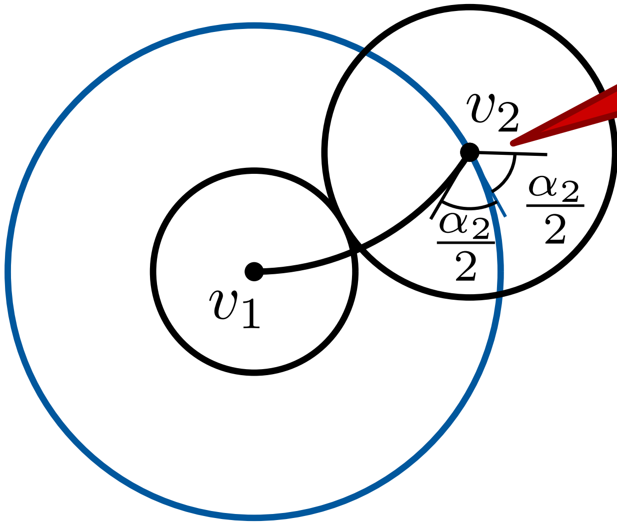

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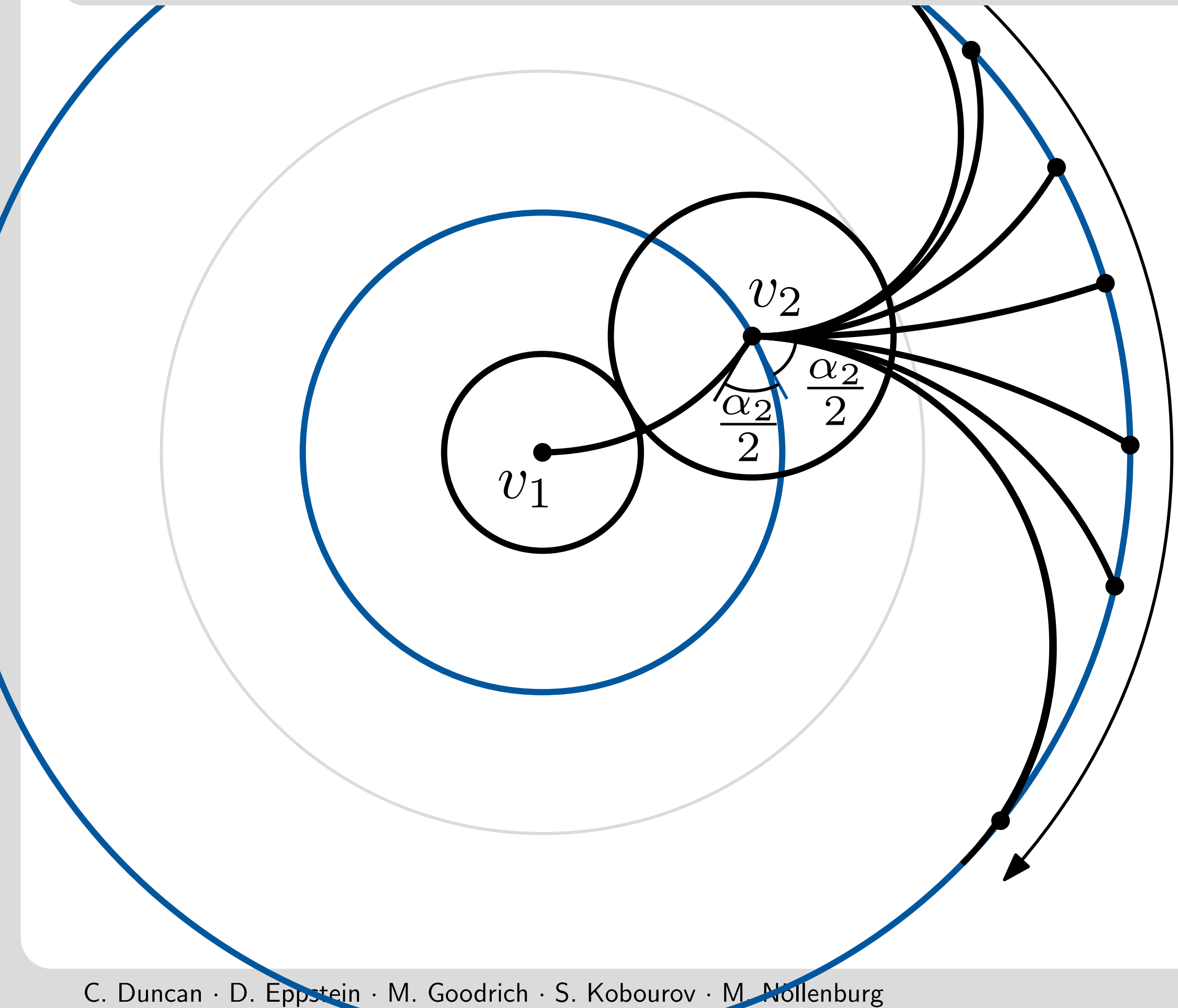

place v_3 (and D_3) on a concentric circle based on $\frac{\alpha_3}{2} \in [0, \pi]$

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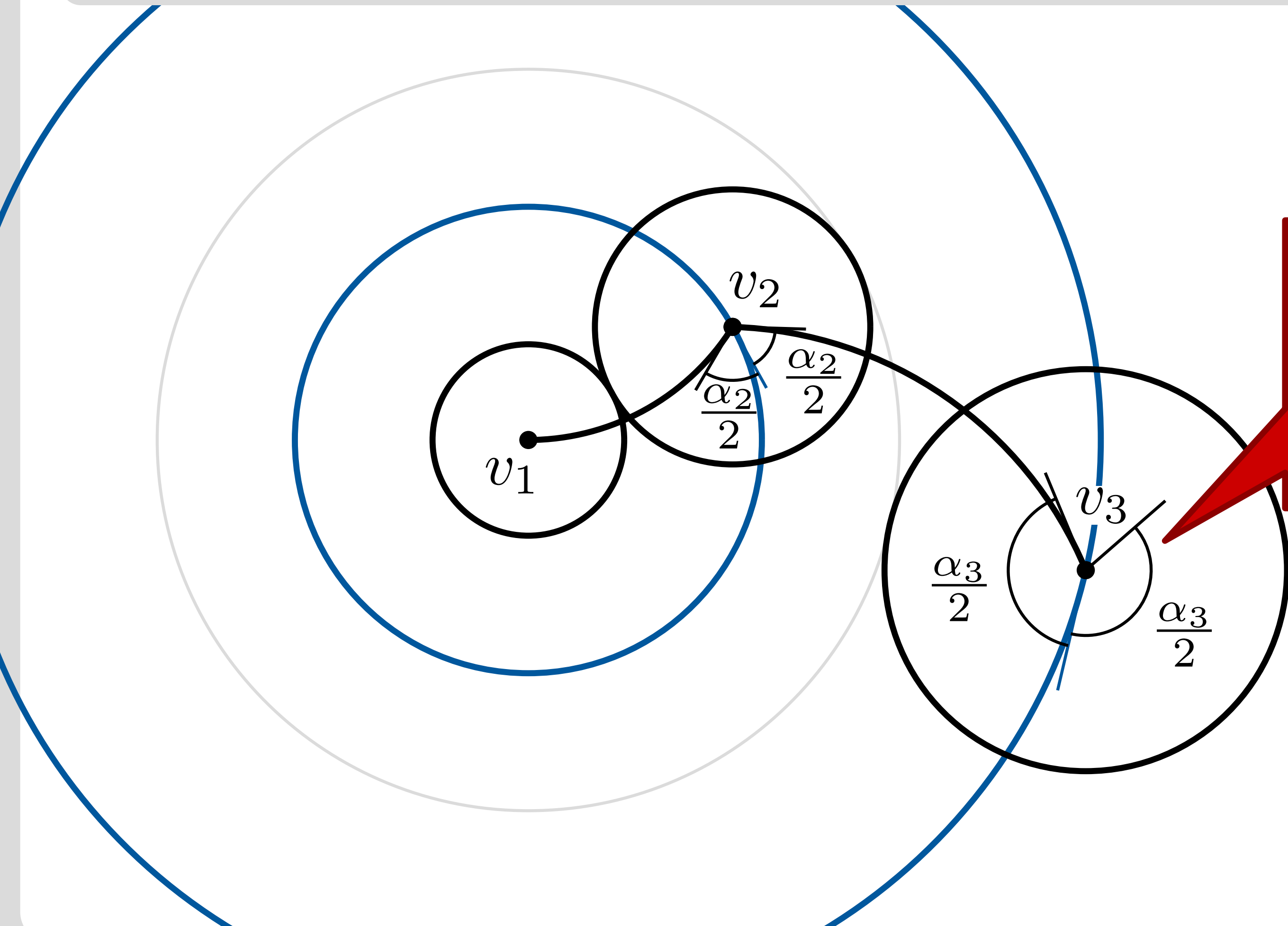

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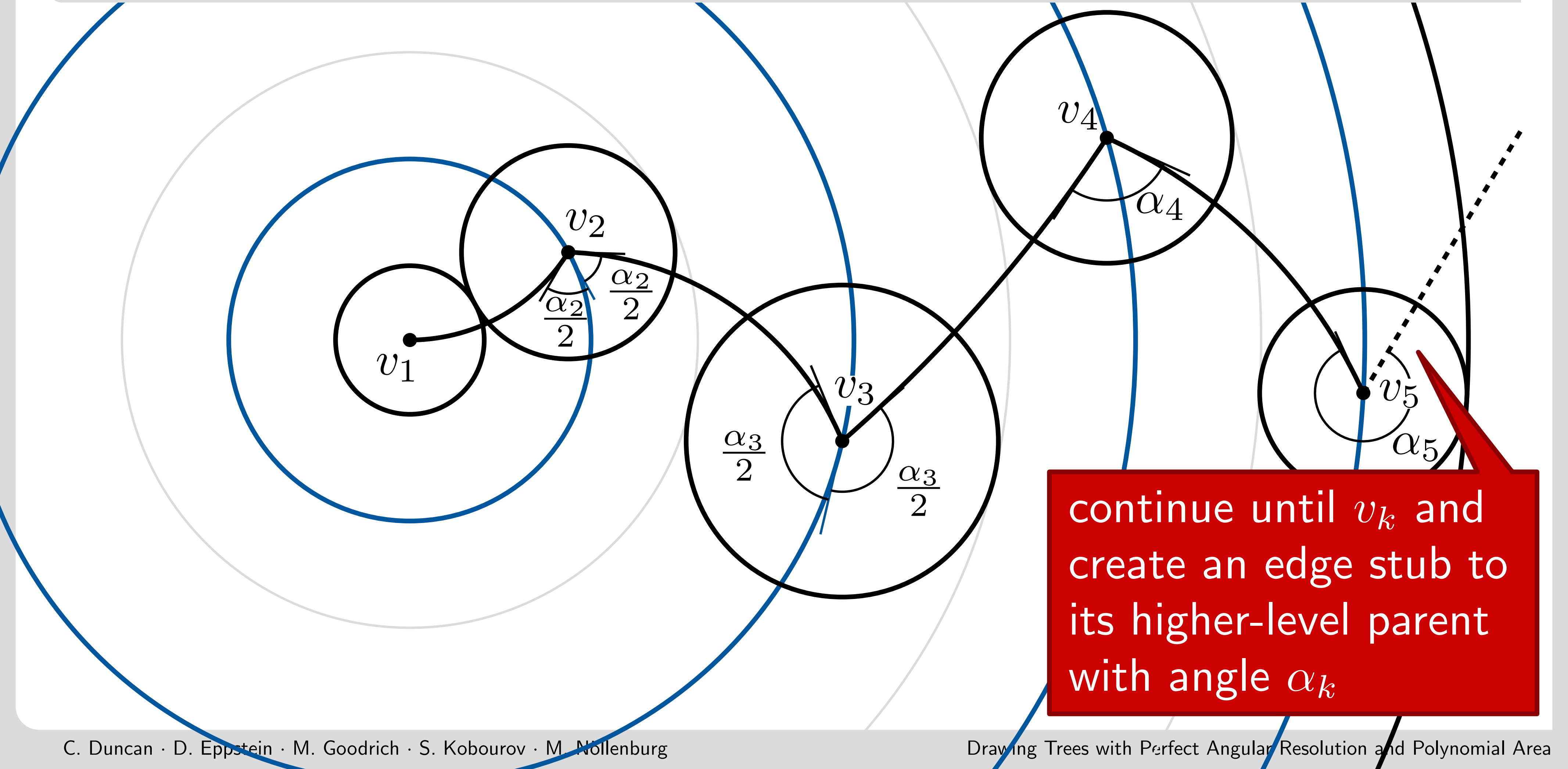

continue until v_k and create an edge stub to its higher-level parent with angle α_k

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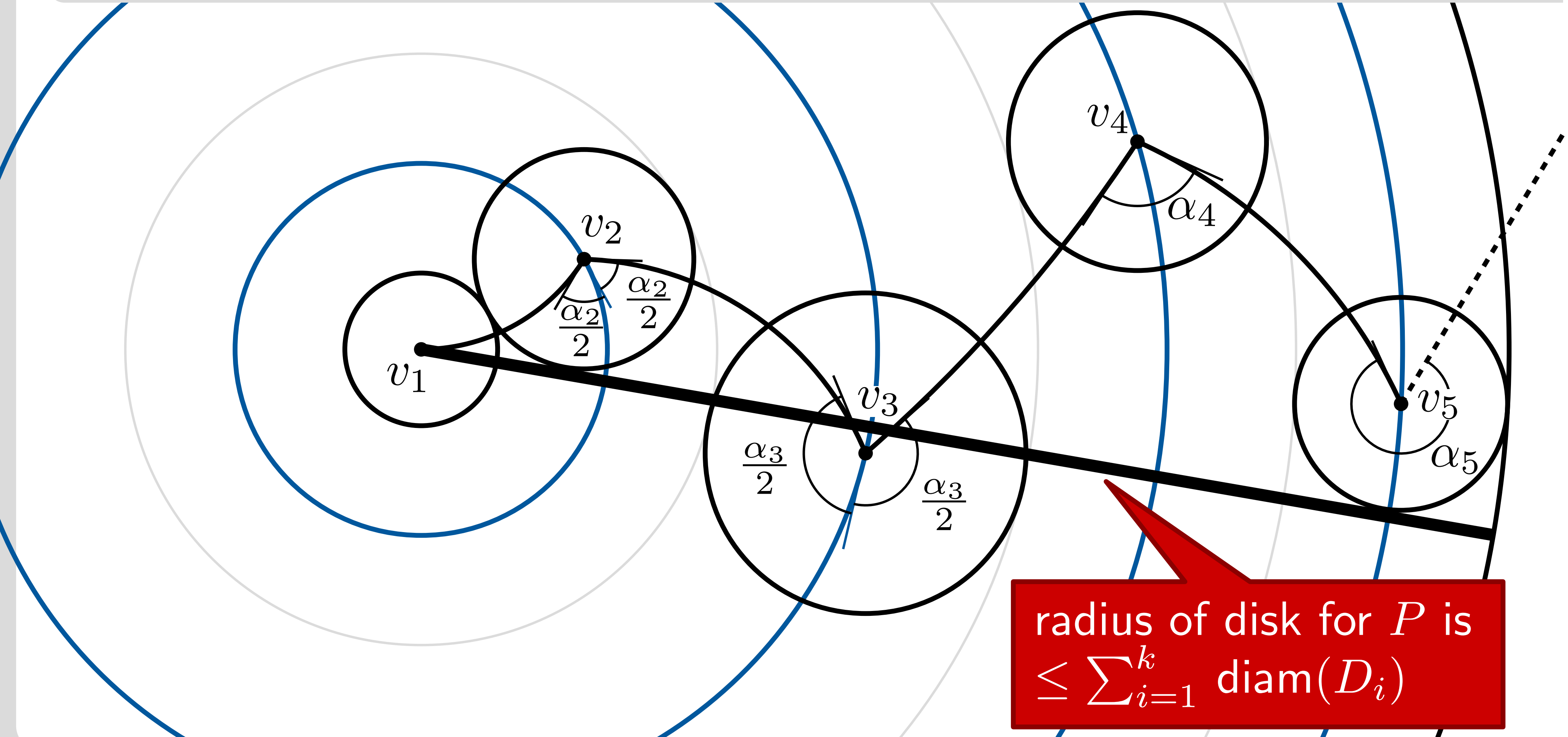


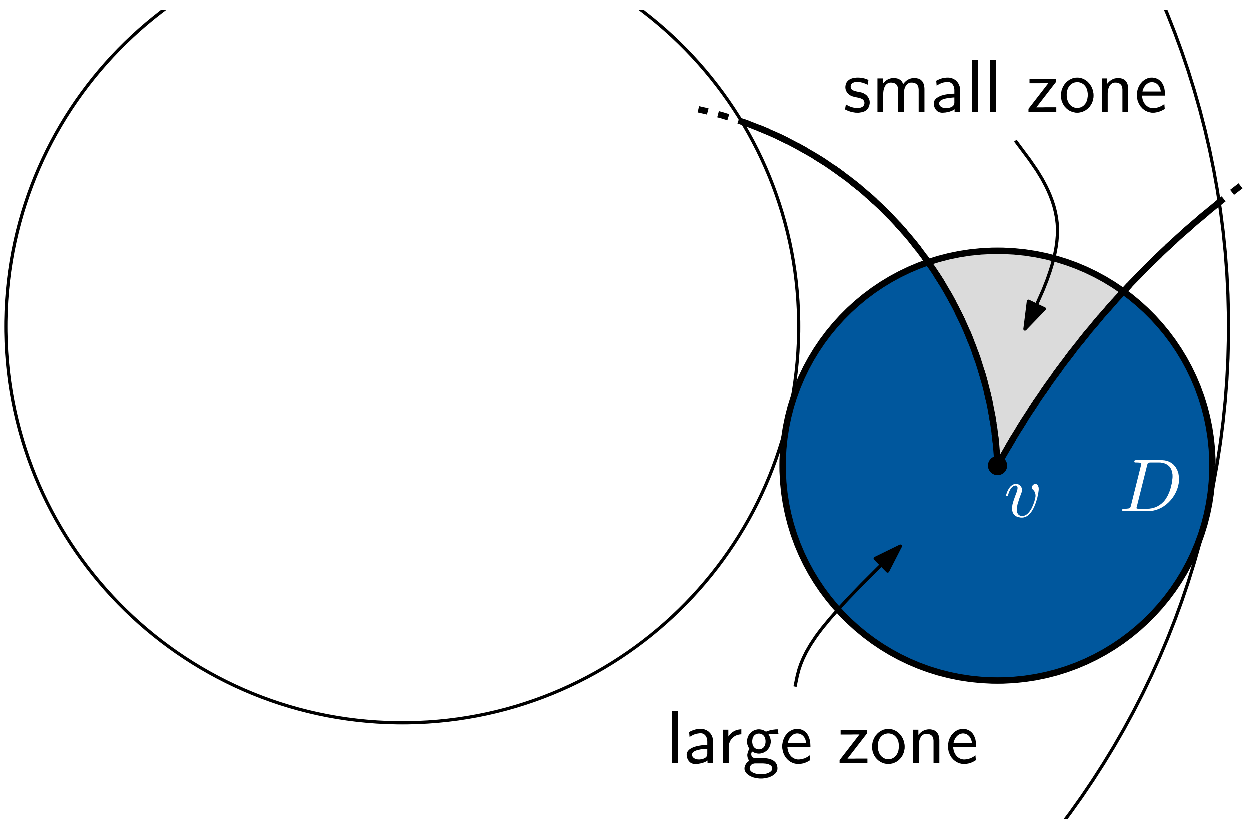
radius of disk for P is $\leq \sum_{i=1}^{k} \text{diam}(D_i)$

Step 2: Drawing light children

Distinguish two cases

Given a node v of a heavy path draw its light children in the given order within disk D. The two heavy edges incident to v divide D into

- **small zone** with opening angle $\leq \pi$
- **large zone** with opening angle $> \pi$

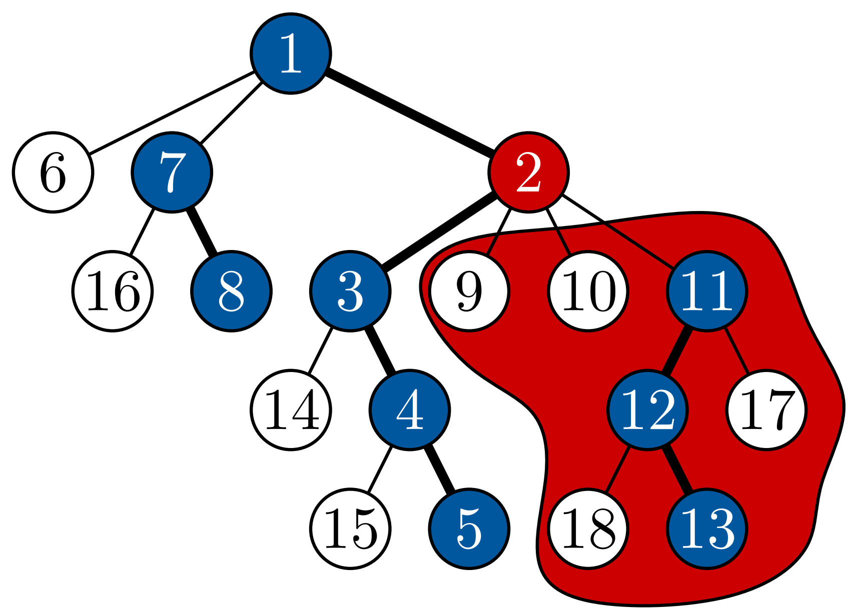


Choose the right disk radii

Definition

For heavy path node v at level j of HPD $H(T)$ define disk radius

$$r_v = 4^{h-j}(1 + \sum_{u \in L(v)} |T_u|).$$



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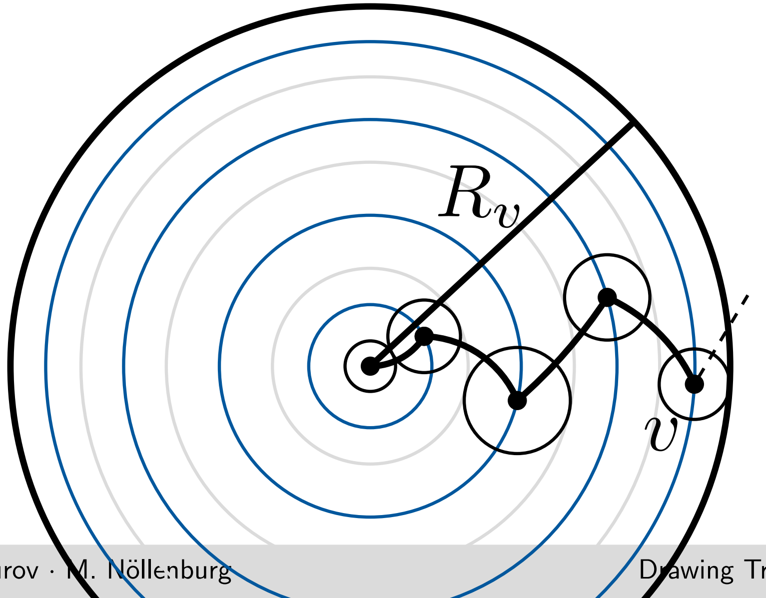

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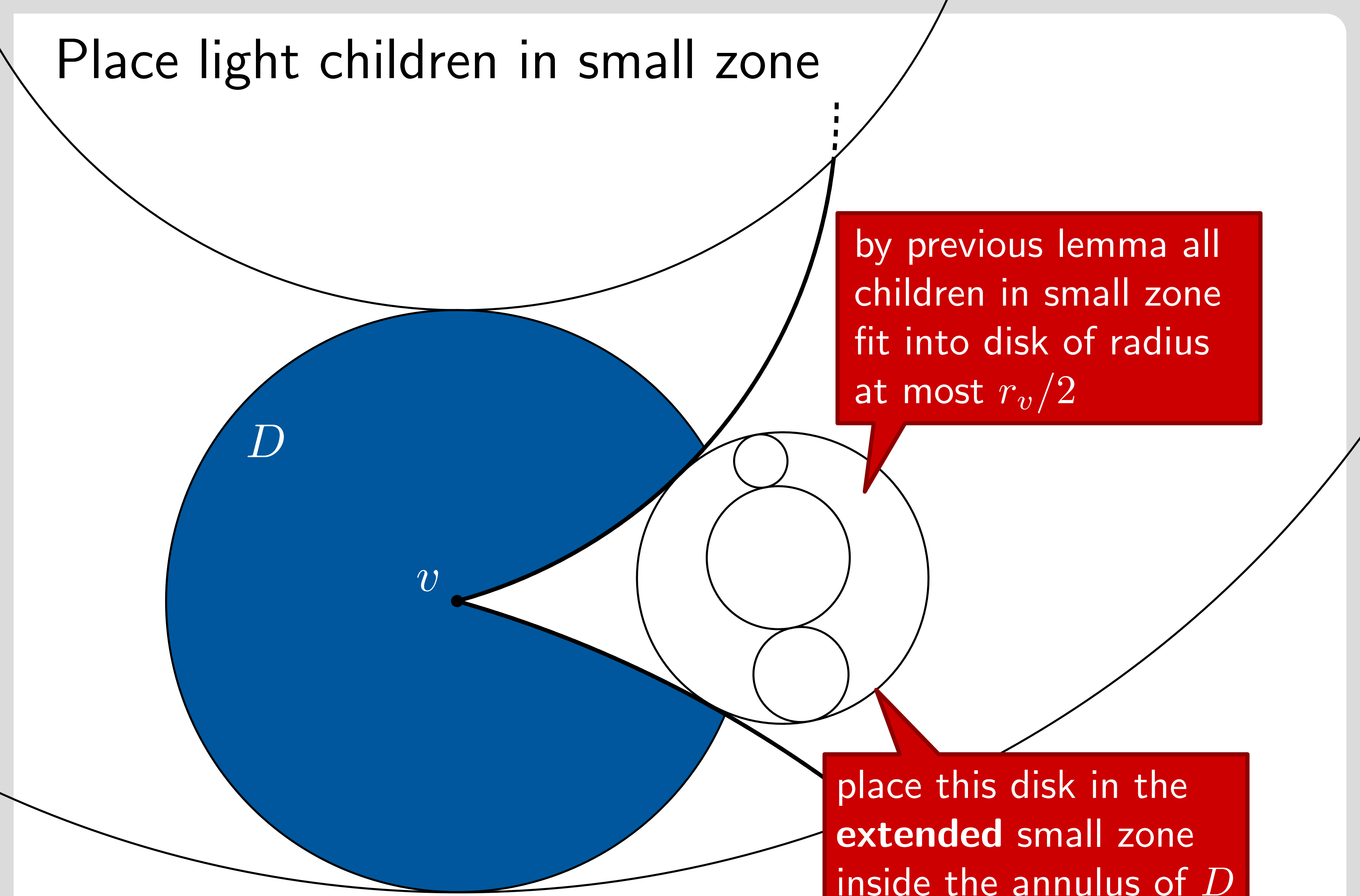
Lemma

Every light child of a heavy path node v at level j is the root of a heavy path at level $j+1$. Hence

$$\sum_{u \in L(v)} R_u = \sum_{u \in L(v)} (2 \cdot 4^{h-j-1}|T_u|) = \frac{1}{2} \cdot 4^{h-j} \sum_{u \in L(v)} |T_u| \leq \frac{r_v}{2}.$$

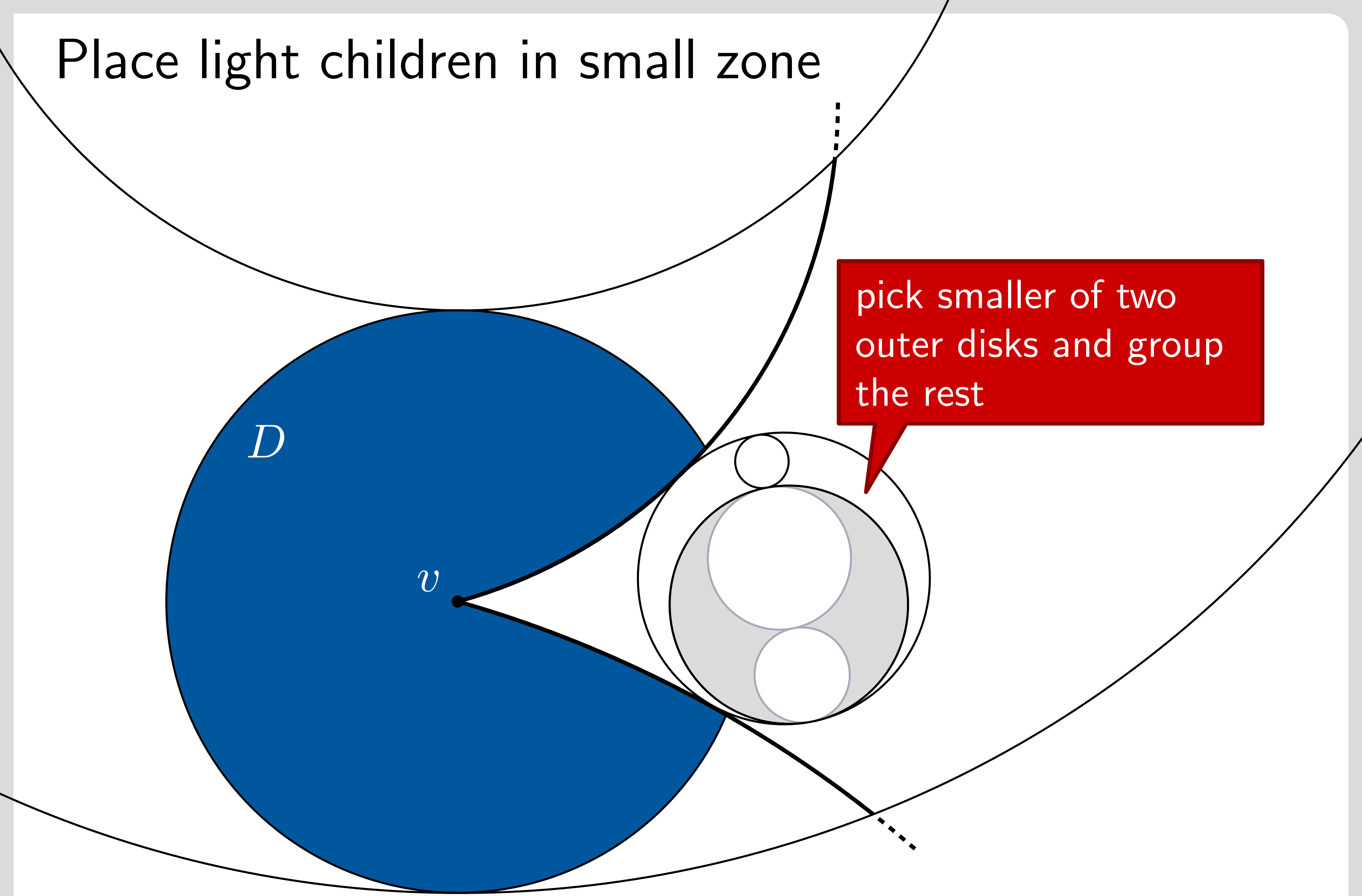
Place light children in small zone



by previous lemma all children in small zone fit into disk of radius at most $r_v/2$

place this disk in the **extended** small zone inside the annulus of D

Place light children in small zone



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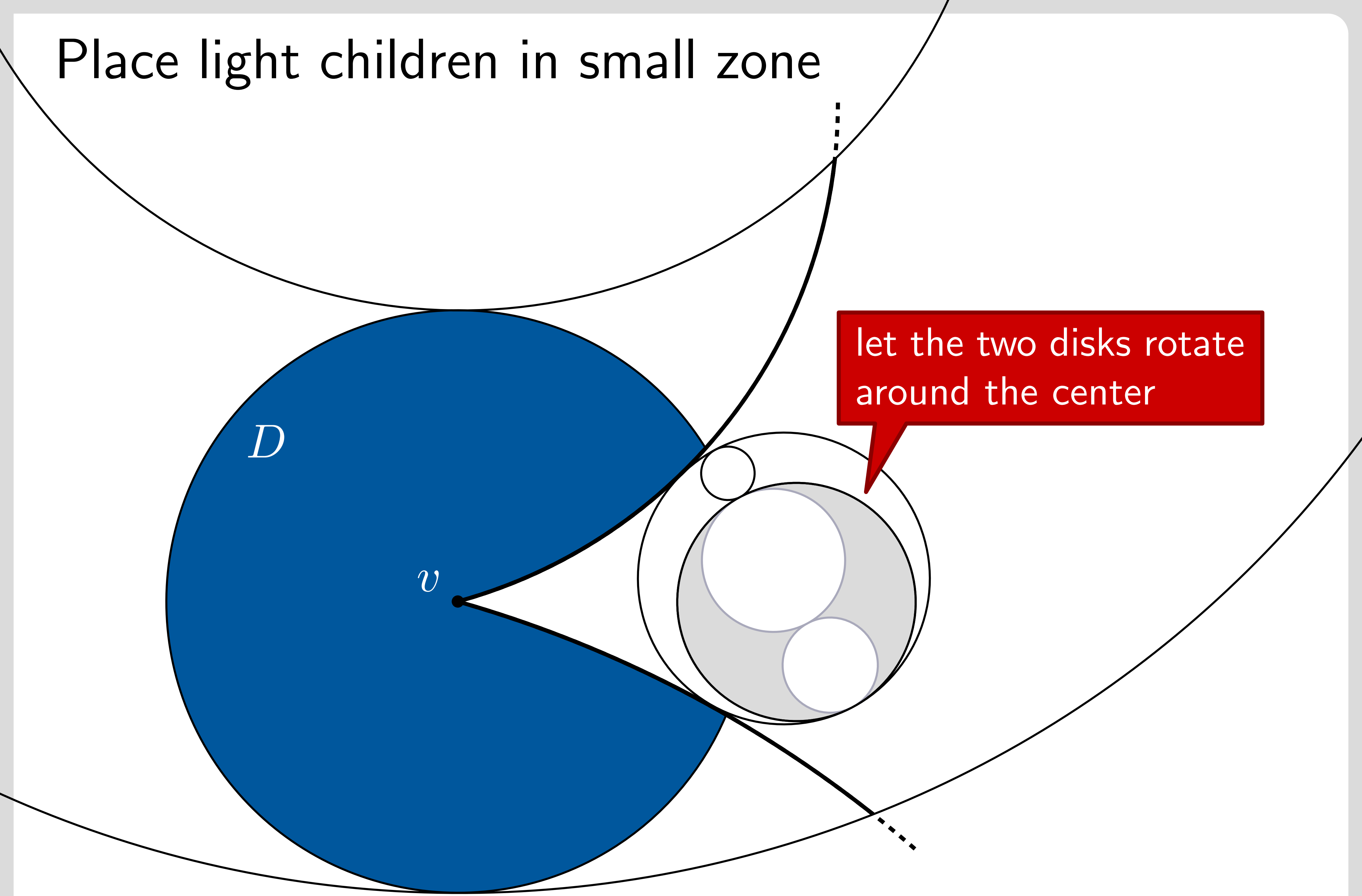

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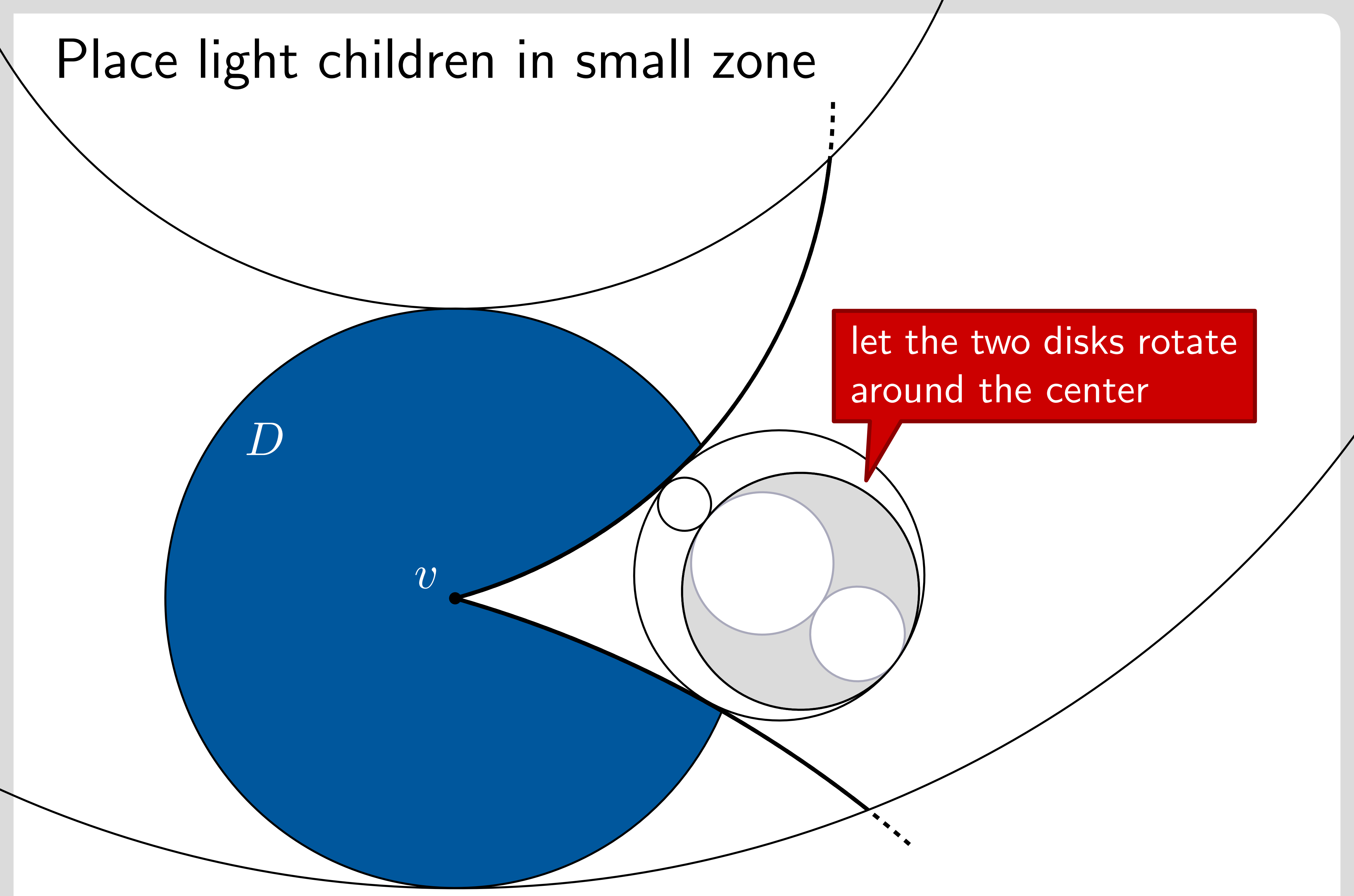

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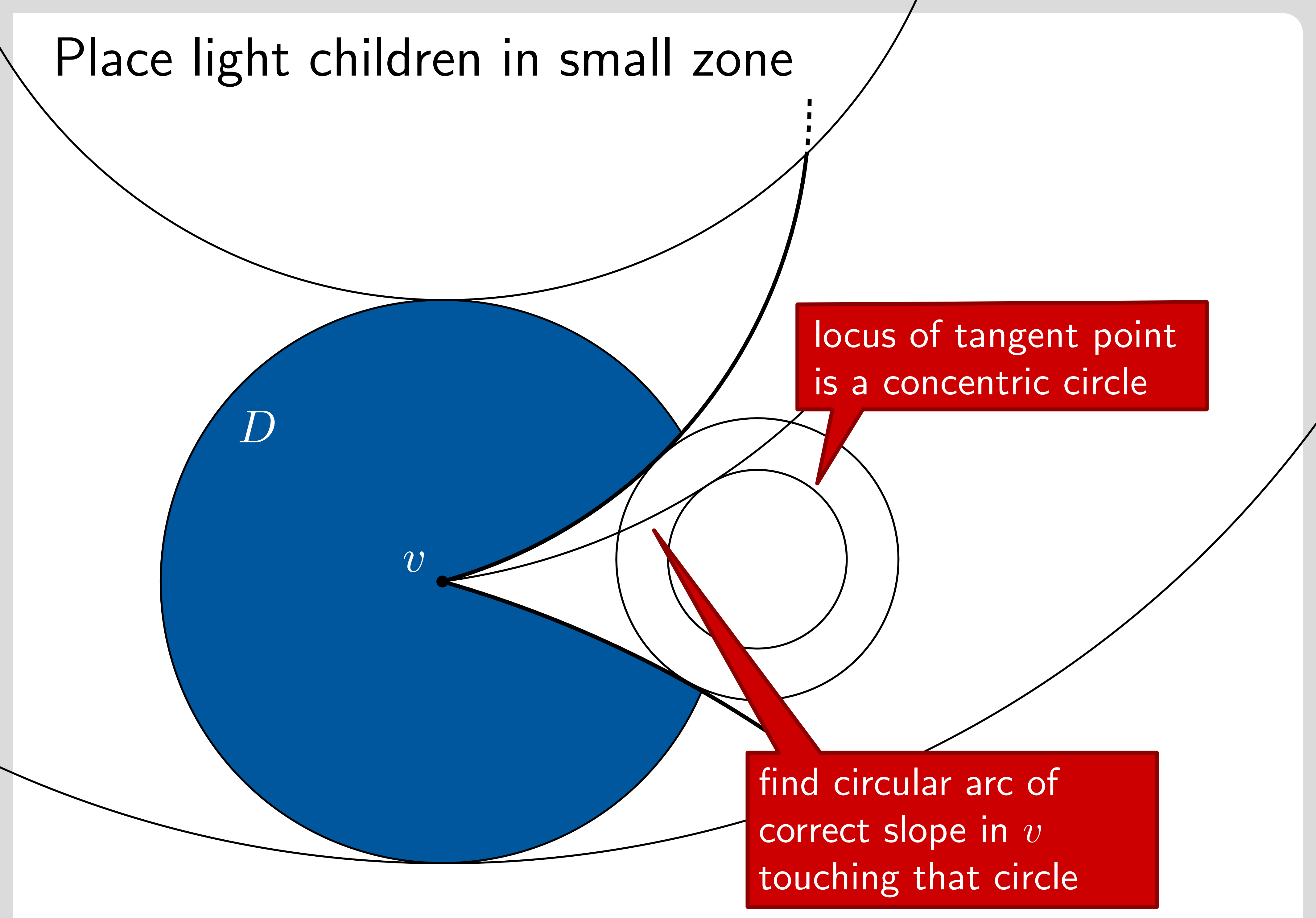

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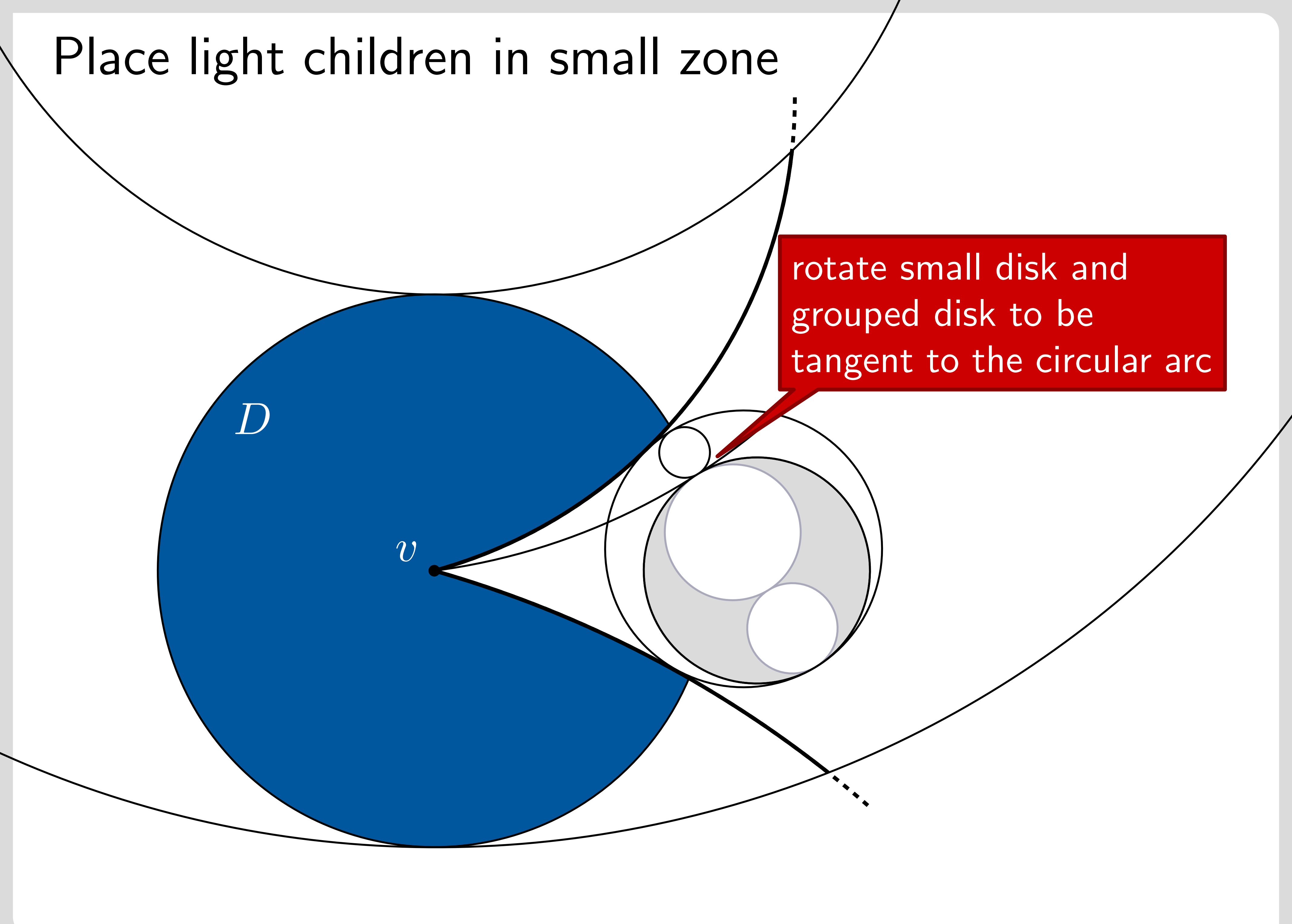

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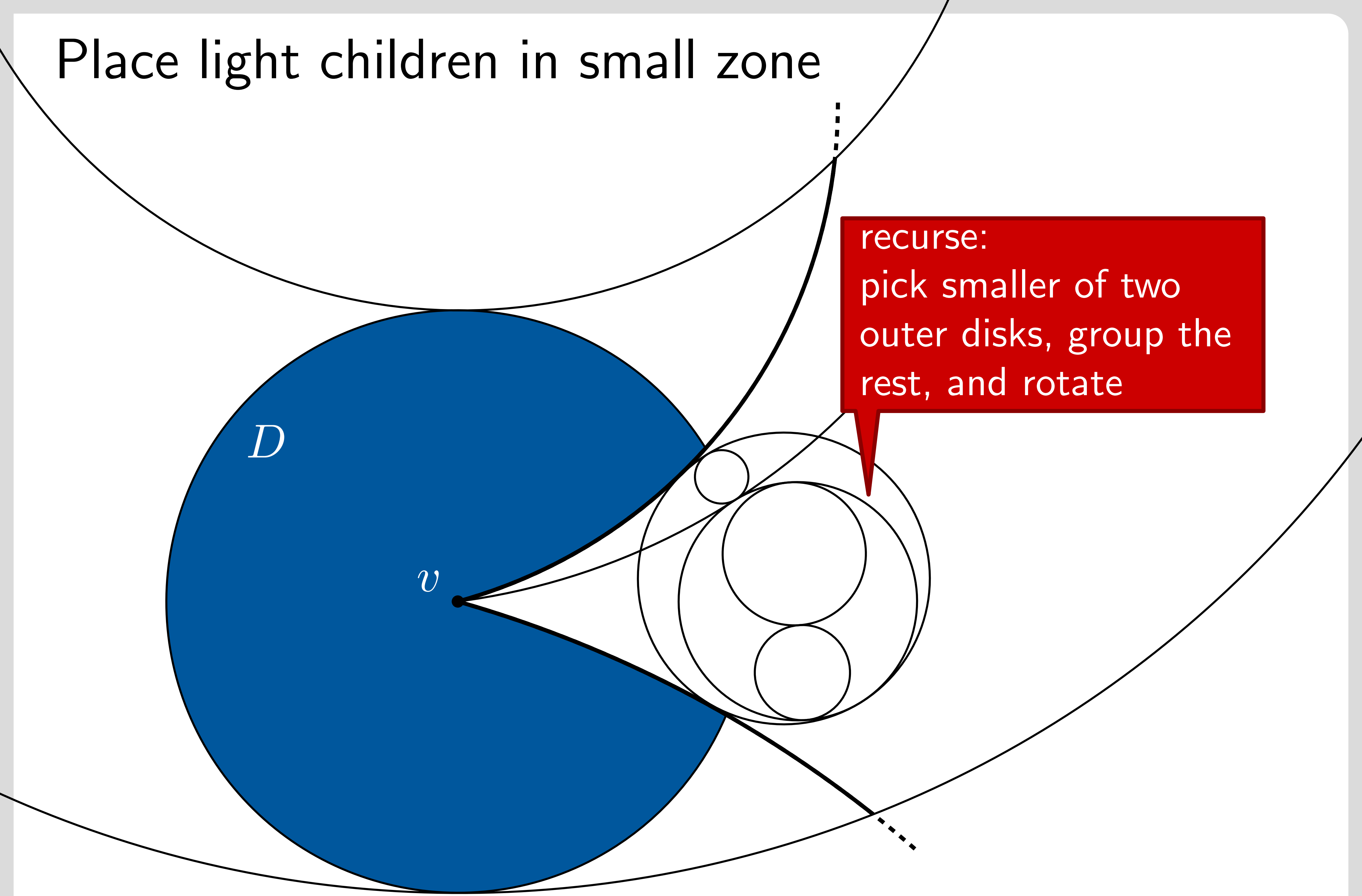

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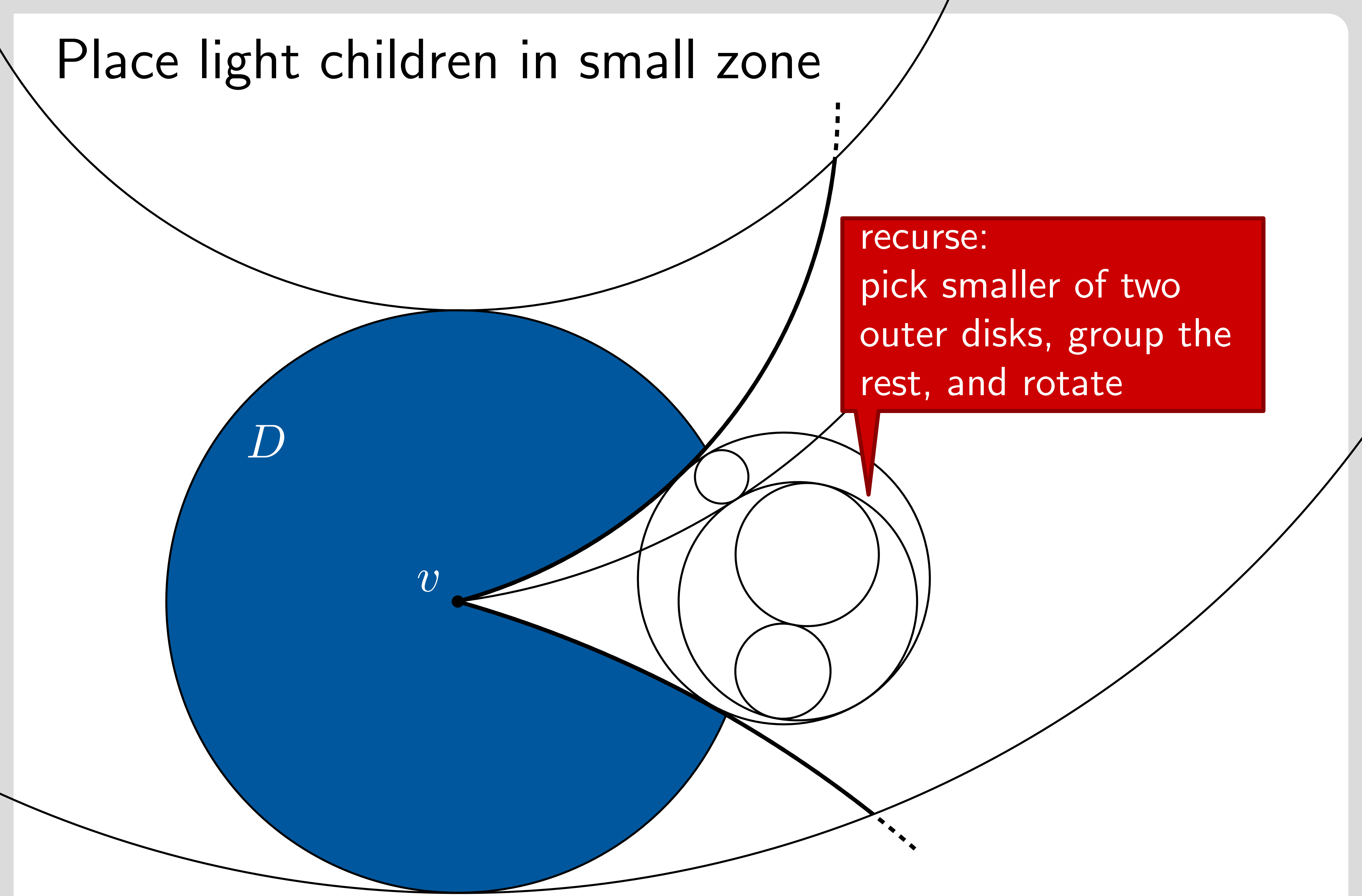

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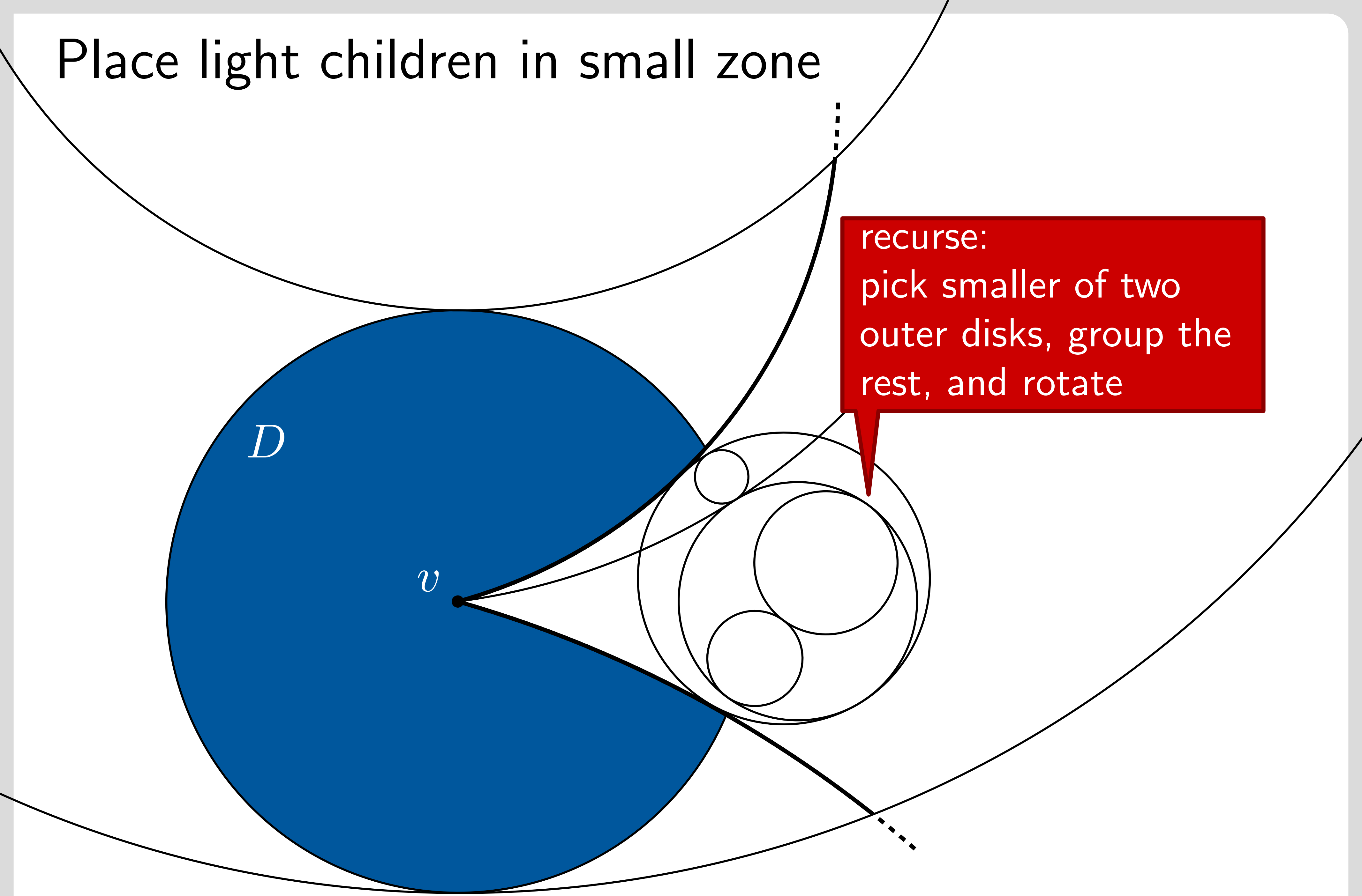
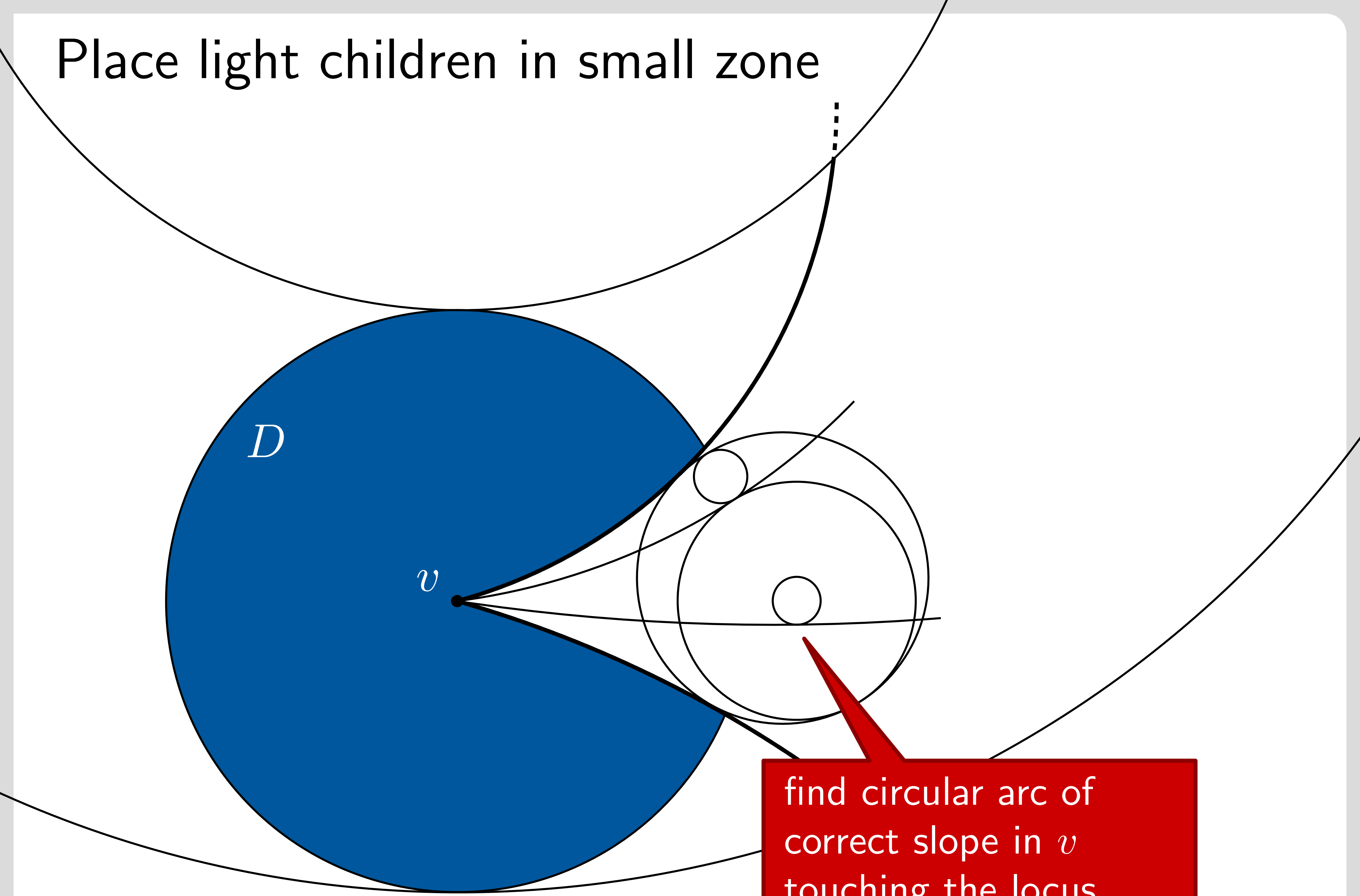


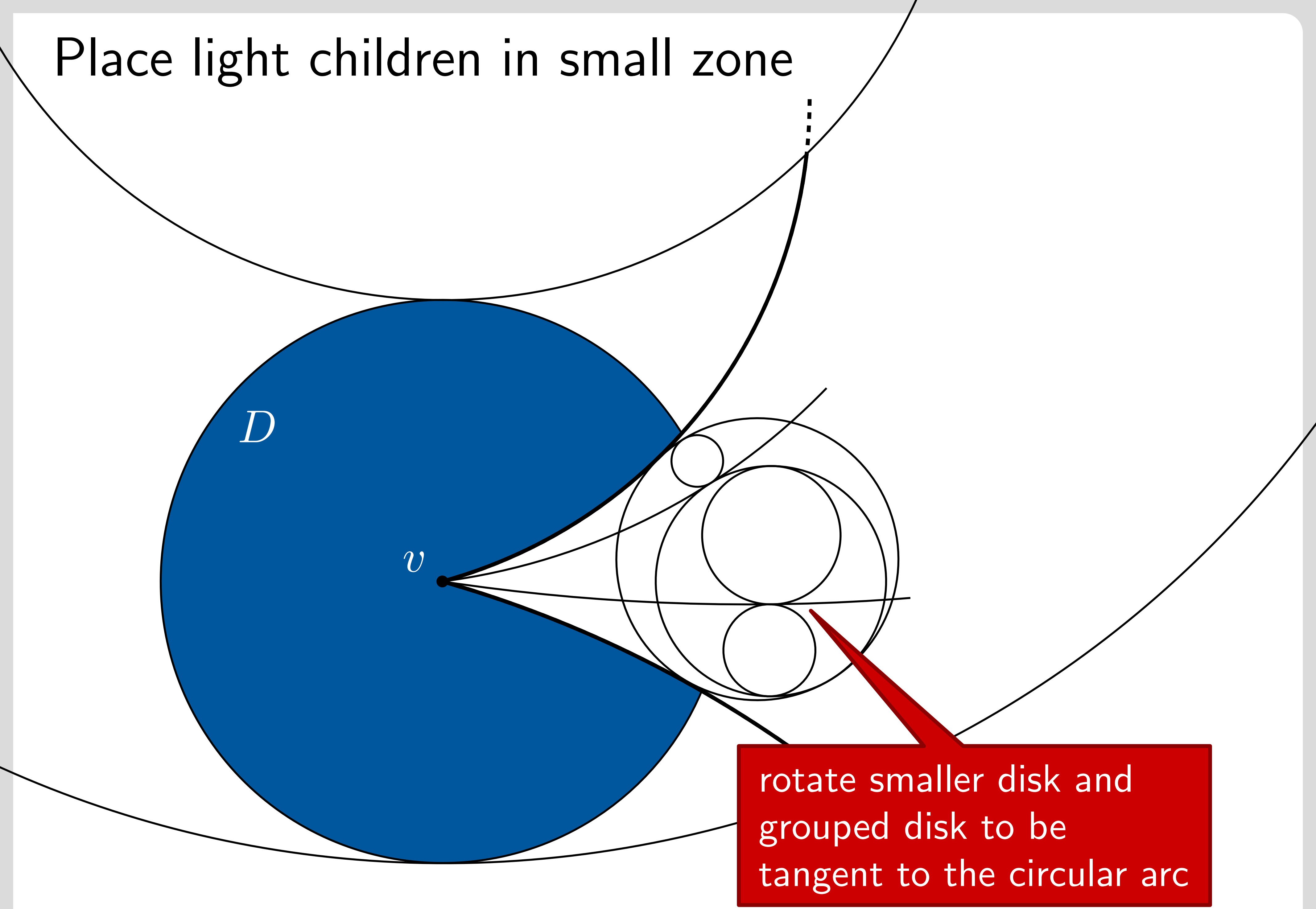
recurse:
pick smaller of two outer disks, group the rest, and rotate

Place light children in small zone



find circular arc of correct slope in v touching the locus circle of tangent point

Place light children in small zone



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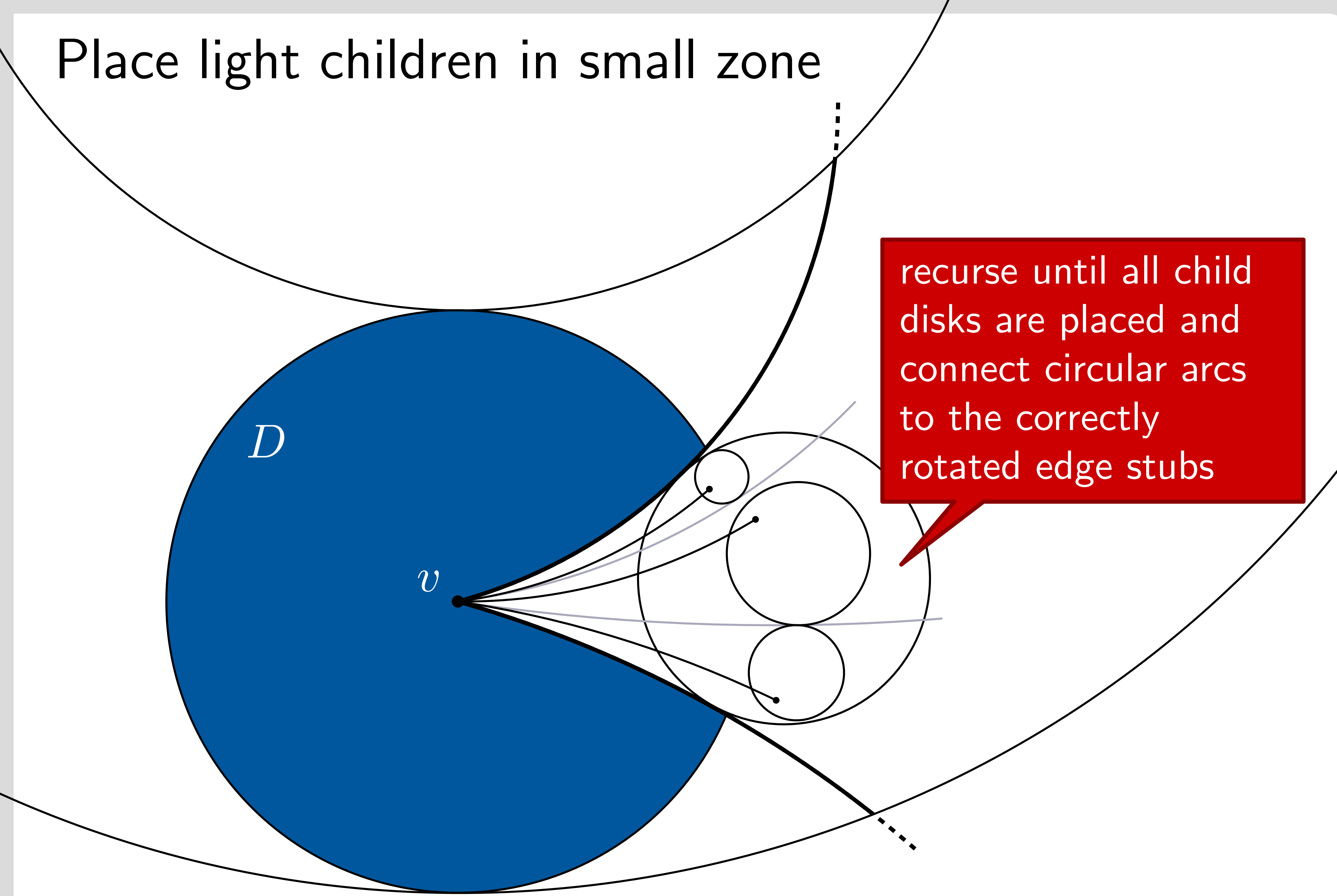
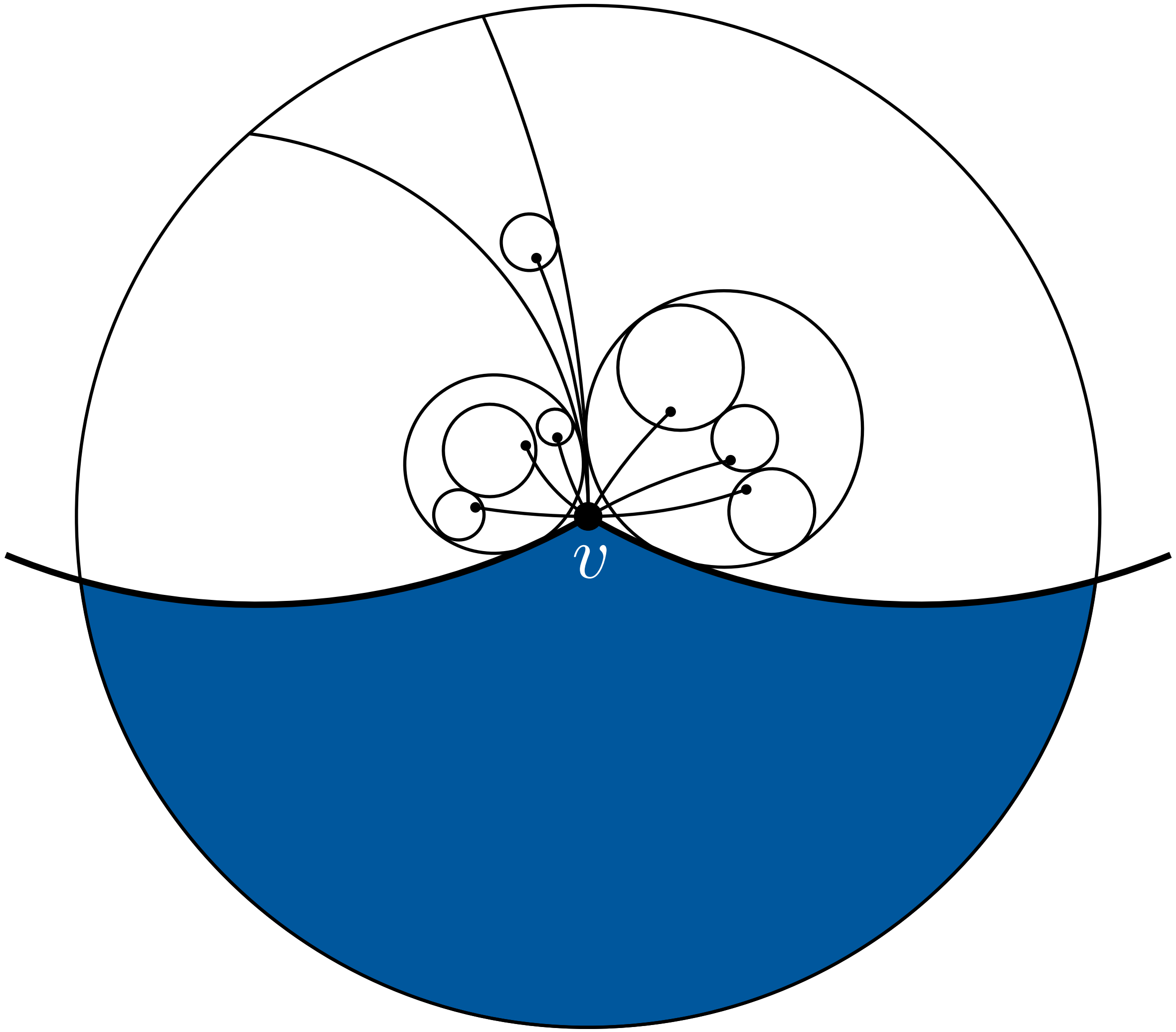


recurse until all child disks are placed and connect circular arcs to the correctly rotated edge stubs

Place light children in large zone



placement similar to the small zone:
split large zone into two small parts and apply same method

Main result

Theorem

Given an ordered tree T with n nodes our algorithm finds a drawing of T with crossing-free Lombardi edges and perfect angular resolution within a disk of radius $2n^3$.

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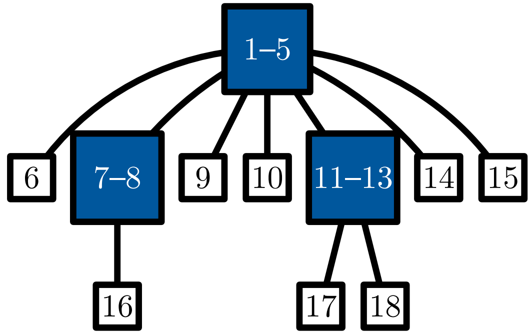
Proof:

- inductive construction places every heavy path P at level j with root node v within a disk of radius $R = 2 \cdot 4^{h-j}|T_v|$
- heavy path at level 0 corresponds to full tree T
- height of the HPD is $h \leq \log_2 n$
- radius of disk containing T is $R = 2 \cdot 4^{\log_2 n} n = 2 \cdot n^3$



$\square$

Summary

Any tree has a drawing with

- crossing-free edges
- perfect angular resolution
- polynomial area

	unordered trees	ordered trees
straight edges	✓	✗
Lombardi edges	✓	✓

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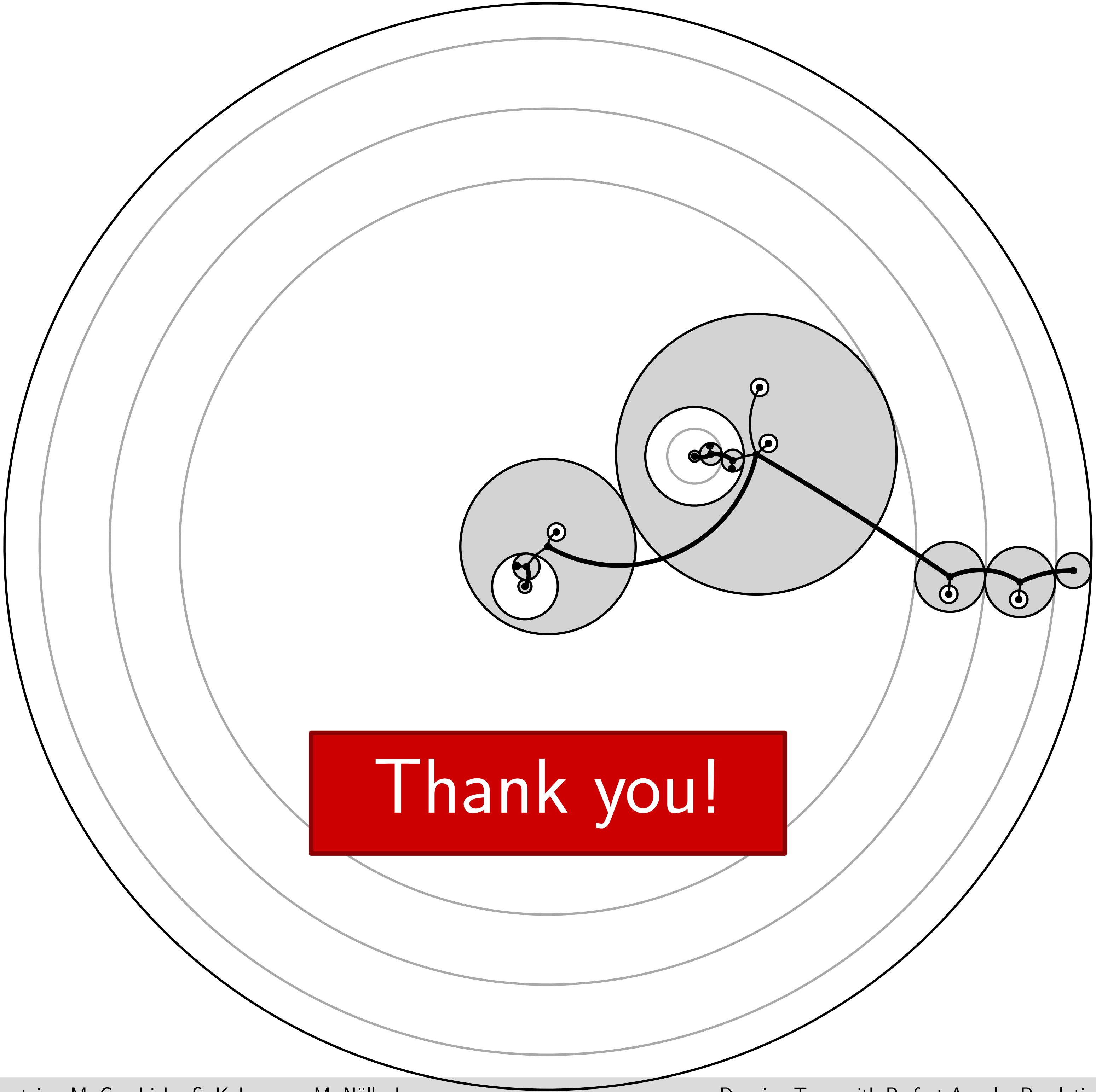
Open questions

- area bound unlikely to be tight
- find simpler algorithms for special classes of trees (e.g. bounded degree)



Thank you!