

Cartograms: Drawing Weighted Graphs as Maps Gastvortrag Vorlesung Algorithmen II

Dr. Martin Nöllenburg Institut für Theoretische Informatik YIG Algorithmen zur Geovisualisierung

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Dr. Martin Nöllenburg · Gastvortrag Algorithmen II



How to best visualize statistics about spatial data? **Example:** population in the USA



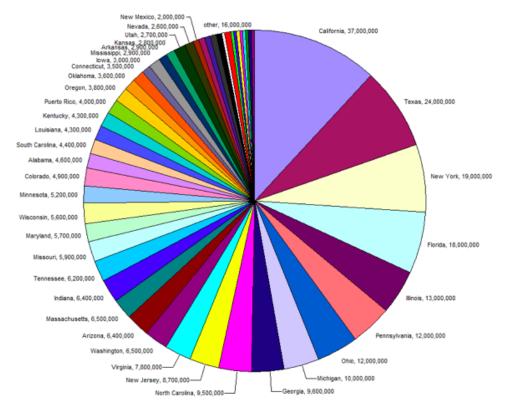
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		Kentucky	4,369,356	Oklahoma	3,791,508
State	2011	Louisiana	4,574,836	Oregon	3,871,859
	рор.	Maine	1,328,188	Pennsylvania	12,742,886
<u>Alabama</u>	4,802,740	Maryland	5,828,289	Rhode Island	1,051,302
<u>Alaska</u>	722,718	Massachusetts	6,587,536	South	4,679,230
<u>Arizona</u>	6,482,505	Michigan	9,876,187	Carolina	
Arkansas	2,937,979	Minnesota	5,344,861	South Dakota	824,082
<u>California</u>	37,691,912	Mississippi	2,978,512	Tennessee	6,403,353
<u>Colorado</u>	5,116,796	Missouri	6,010,688	Texas	25,674,681
Connecticut	3,580,709	Montana	998,199	<u>Utah</u>	2,817,222
Delaware	907,135	Nebraska	1,842,641	Vermont	626,431
DC	617,996	Nevada	2,723,322	Virginia	8,096,604
Florida	19,057,542	New	1,318,194	Washington	6,830,038
Georgia	9,815,210	Hampshire		West Virginia	1,855,364
Hawaii	1,374,810	New Jersey	8,821,155	Wisconsin	5,711,767
<u>ldaho</u>	1,584,985	New Mexico	2,082,224	Wyoming	568,158
Illinois	12,869,257	New York	19,465,197		
Indiana	6,516,922	North Carolina	9,656,401		
lowa	3,062,309	North Dakota	683,932		
Kansas	2,871,238	<u>Ohio</u>	11,544,951		



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a pie chart?



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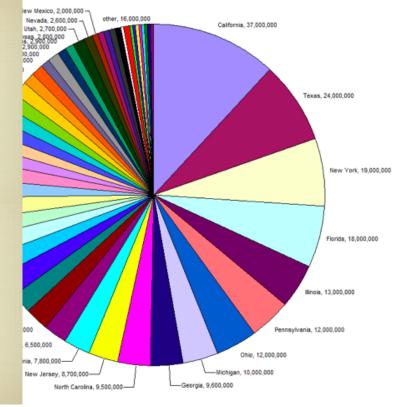
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Census 2010 Population By State

Alabama	**********	
Alaska	***	
Arizona	*****************	
Arkansas	******	
California		*****
Colorado	***********	
Connecticut	******	
Delaware	***	
D.C.		
Florida	***************************************	
Georgia	*****************************	
Hawaii	*****	
Idaho	*****	
Illinois	***************************************	
Indiana	***************	
lowa	******	
Kansas	*****	
Kentucky	*********	
Louisiana	**********	
Maine	*****	
Maryland	***************	
Massachusetts	******************	
Michigan	*********************************	
Minnesota	**********	
Mississippi	********	
Missouri	******	
Montana	****	
Nebraska	******	
Nevada	******	
New Hampshire		
New Jersey	************************	
New Mexico	++++++	
New York		
North Carolina	+++++++++++++++++++++++++++++++++++++++	
North Dakota	**	
Ohio		
Oklahoma	*****	-
Oregon		
Pennsylvania		-
Puerto Rico	*****	
Rhode Island	+++++	-
South Carolina		
South Dakota	***	-
Tennessee		-
Texas		
Utah		m
Vermont	**	-
Virginia		-
Washington		-
West Virginia		-
Wisconsin		
	#	-
	TT Source: US Census 2010 Data	
wyoming		
wyoming		
Wyoming	Made by CU	віт
• = 250,000) People Made by CU www.cubitplanning	

a bar chart?



a pie chart?



How to best visualize statistics about spatial data?

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Nevada #####	

New Mexico #####	

North Dakota ##	

Oregon titte	*****

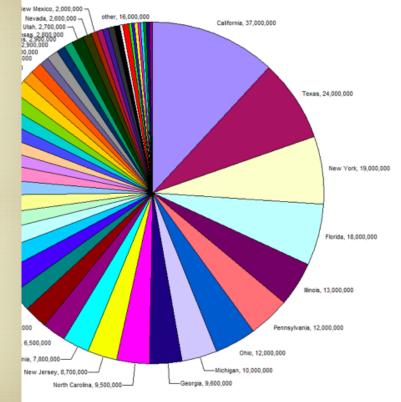
Rhode Island ###	

South Dakota ##	

Utah IIIII	
Vermont ##	

West Virginia #####	

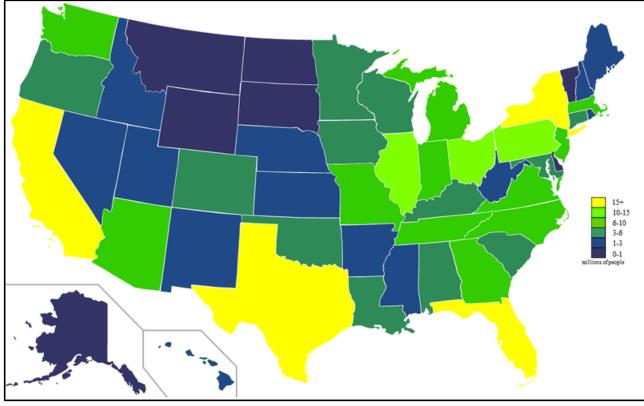
Wyoming #	
	Source: US Census 2010 Data
1 = 250,000 Peo	ple Made by CUB www.cubitplanning.cu
2	bar chart?



a pie chart?

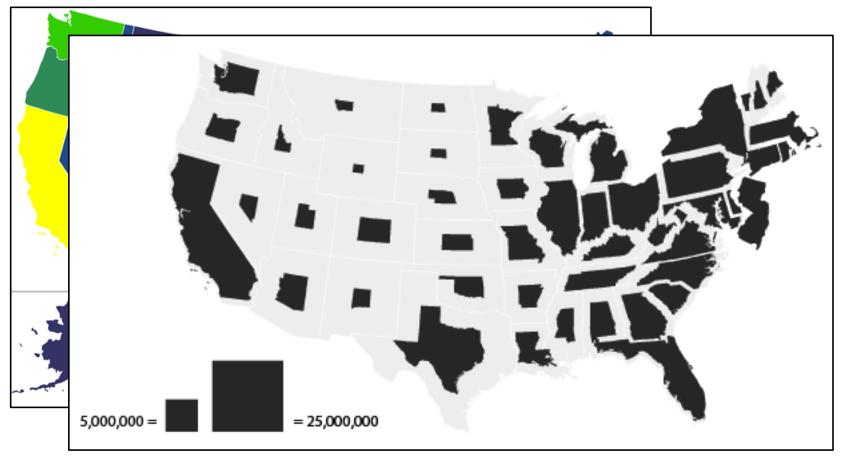
Problem: standard methods don't show the spatial patterns!





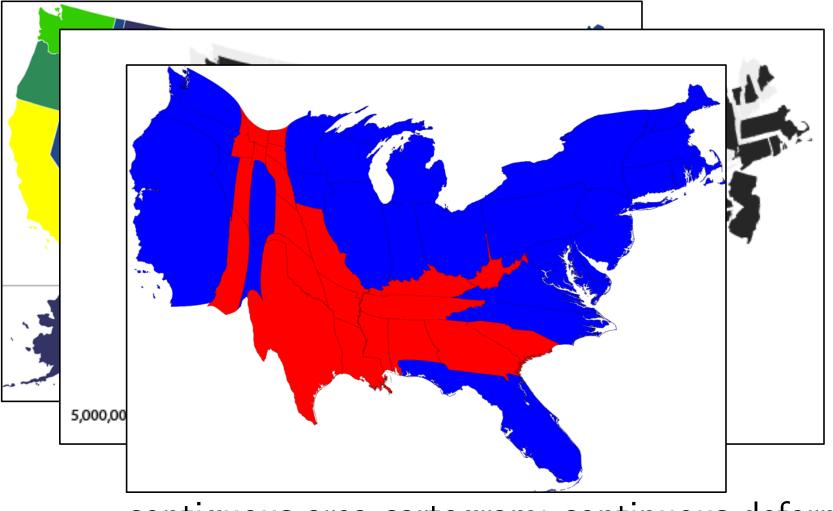
choropleth map: use colors to show stats





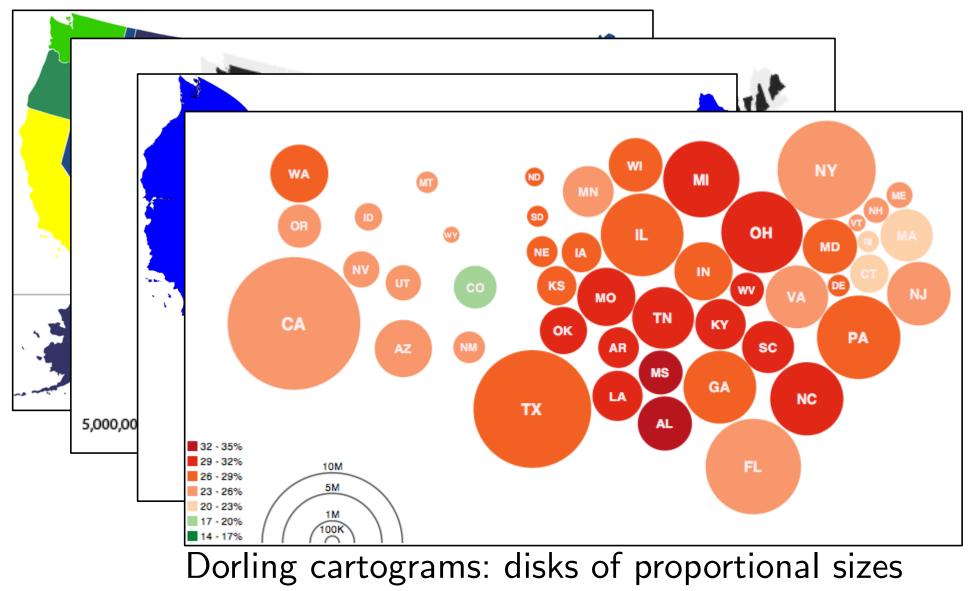
non-contiguous area cartogram: area proportional to population



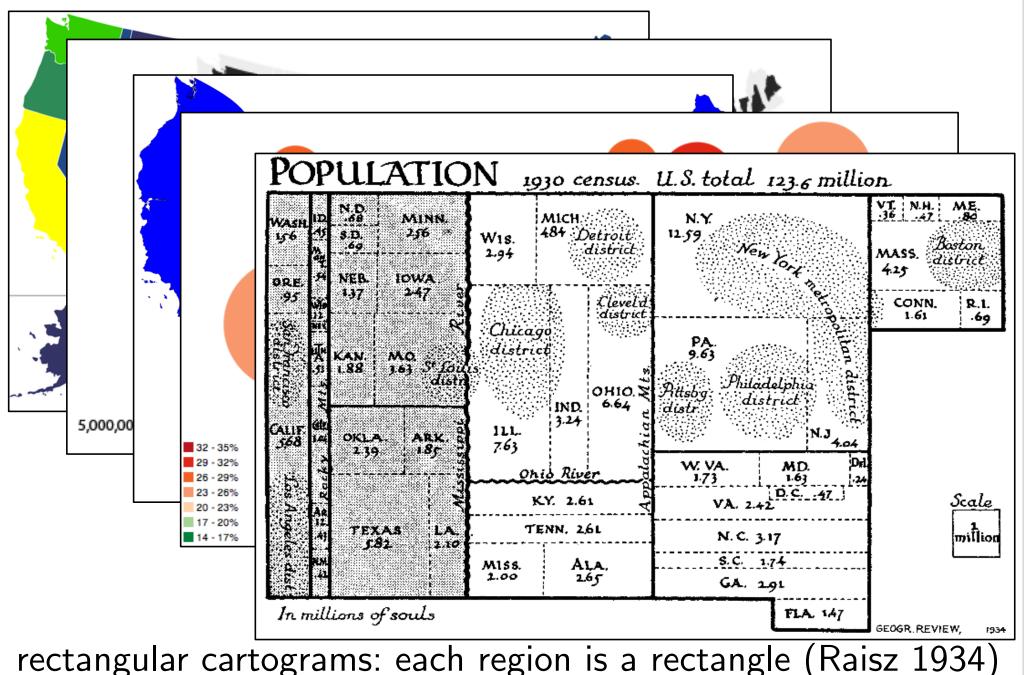


contiguous area cartogram: continuous deformation (Gastner, Newman 2004)







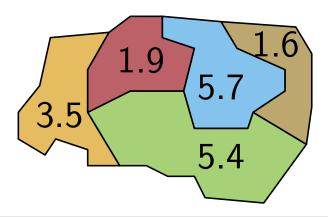


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Cartograms: Drawing Weighted Graphs as Maps



Input: political map M (subdivision of a rectangle), and a positive value w_i for each region R_i

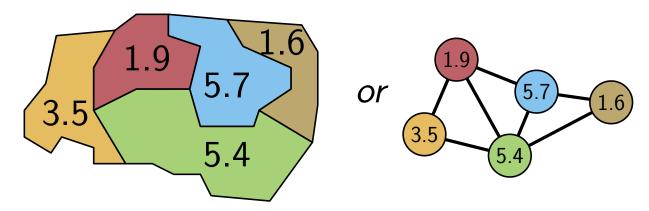


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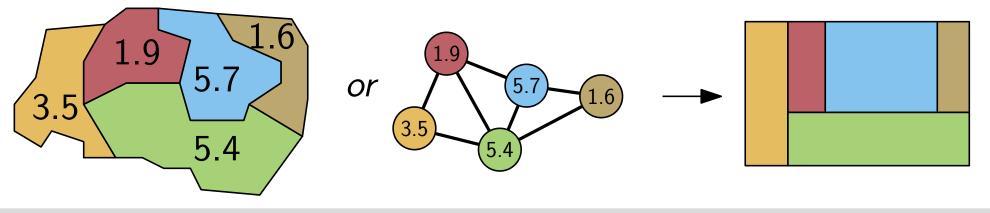


- **Input:** political map M (subdivision of a rectangle), and a positive value w_i for each region R_i
- or: vertex-weighted inner-triangulated plane graph G dual to M, where each vertex v_i is a region R_i , edges connect adjacent regions, and vertex weights are w_i



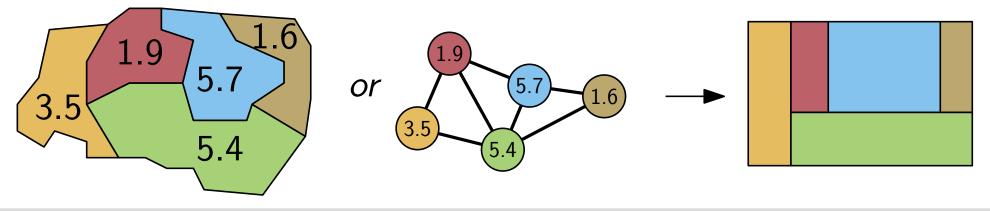


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- or: area-proportional contact representation of G, where each vertex v_i is represented as a geometric shape s_i of area w_i and two shapes s_i, s_j touch iff $v_i v_j \in E$



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Cartograms: Drawing Weighted Graphs as Maps

What is known for unweighted graphs?

 every planar graph has a disk contact representation [Koebe 1936] What is known for unweighted graphs?

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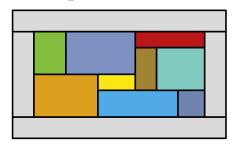
 every planar graph has a hole-free contact representation of convex hexagons [Gansner et al. 2010]



What is known for unweighted graphs?

 every planar graph has a disk contact representation [Koebe 1936]

- every planar graph has a hole-free contact representation of convex hexagons [Gansner et al. 2010]
- every planar graph satisfying that
 1. every inner face is a triangle, the outer face a quadrangle
 2. there are no separating triangles
 has a rectangular dual [Koźmiński, Kinnen 1985]



What is known for vertex-weighted graphs?



 rectangular cartograms with low error [van Kreveld, Speckmann 2005]

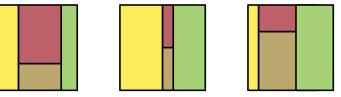


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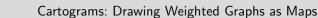
characterization of area-universal rectangular cartograms
 [Eppstein et al. 2012]



What is known for vertex-weighted graphs?

rectangular cartograms with low error [van Kreveld, Speckmann 2005]

- characterization of area-universal rectangular cartograms [Eppstein et al. 2012]
- every inner-triangulated planar vertex-weighted graph has a rectilinear cartograms with octagons [Alam et al. 2012] octagons are actually necessary for some graphs [Yeap, Sarrafzadeh 1993]









Rectilinear cartograms with 10-sided polygons

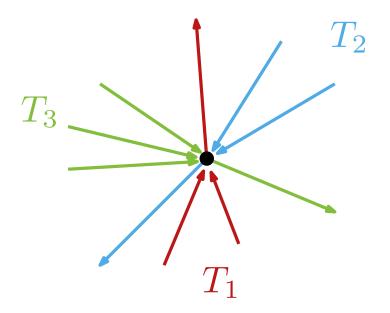
Md. J. Alam, T. Biedl, S. Felsner, A. Gerasch, M. Kaufmann, S. G. Kobourov. *Linear-time algorithms for proprtional contact representations*. Proc. ISAAC 2011.

thanks to Md. Jawaherul Alam for letting me use some of his slides

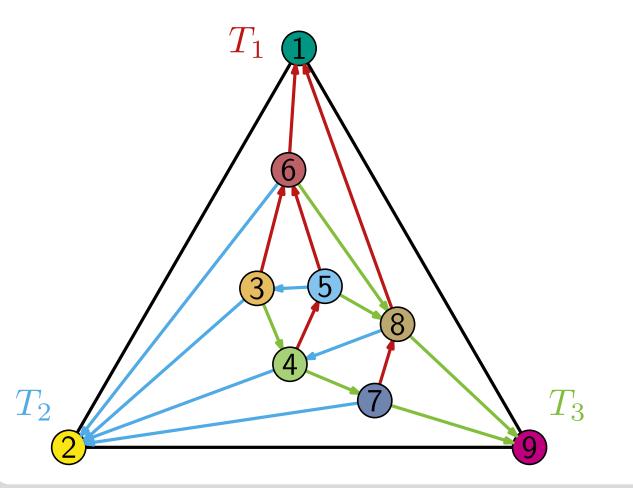


Let G be a fully triangulated planar graph. A Schnyder realizer partitions the internal edges into three sets T_1 , T_2 , T_3 of directed edges so that

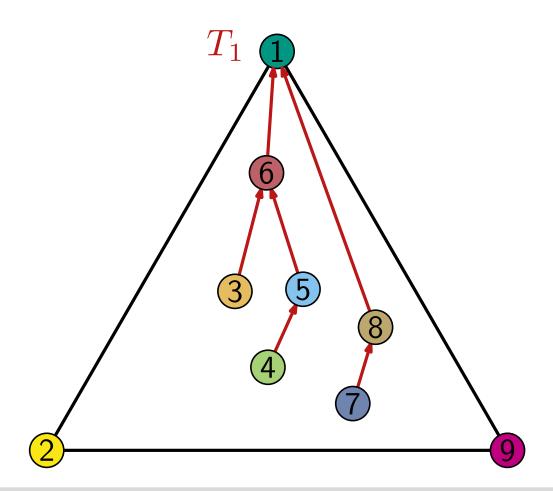
- every internal vertex v has exactly one edge in each T_i^{out}
- $\hfill \ensuremath{\bullet}$ the ccw ordering of edges around any v is
 - T_1^{in} , T_3^{out} , T_2^{in} , T_1^{out} , T_3^{in} , T_2^{out}



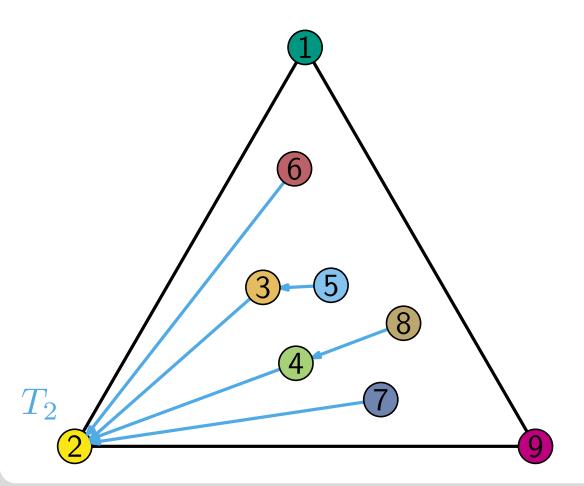




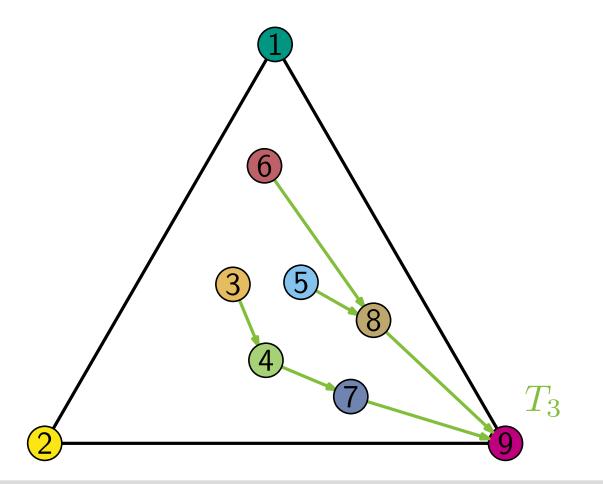




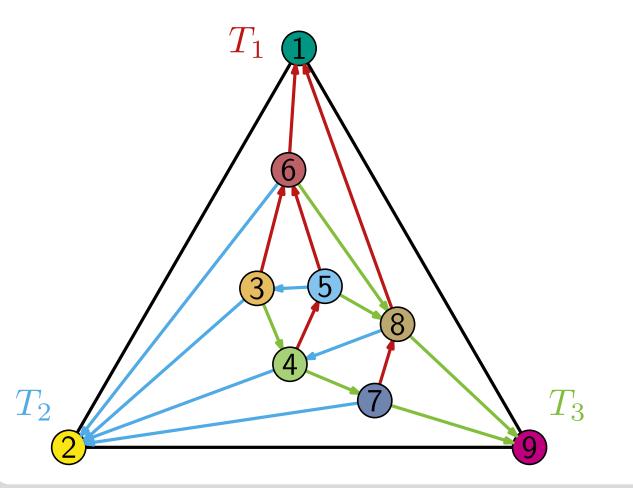




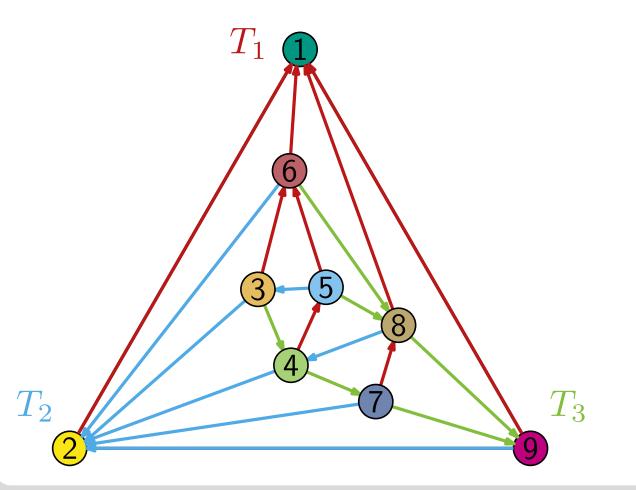






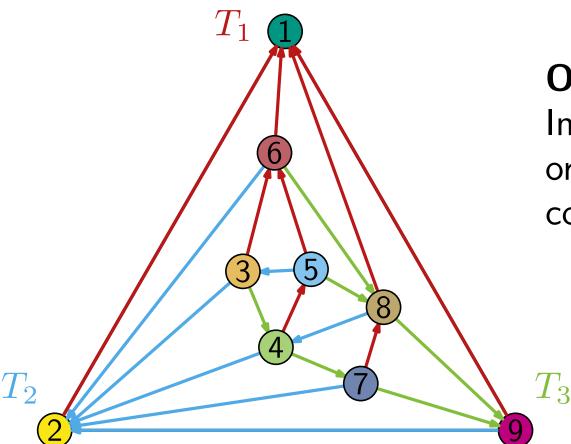








Theorem: Each set T_i (i = 1, 2, 3) is a spanning tree of the inner vertices and one outer vertex. Every triangulated graph has a Schnyder realizer and it can be computed in O(n) time.

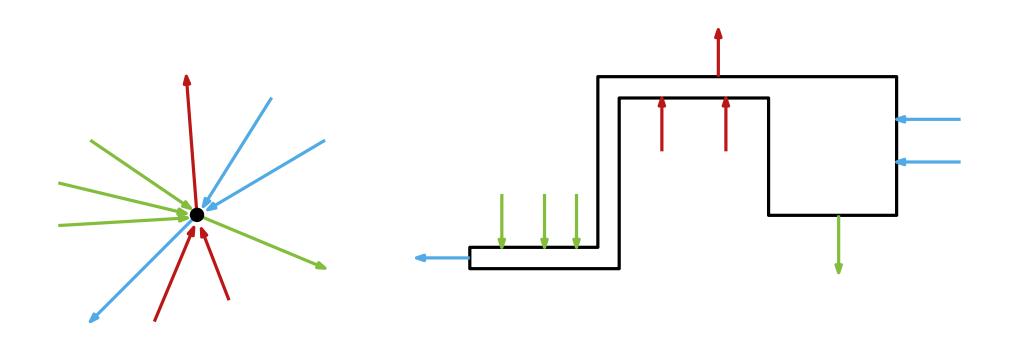


Observation:

In the left-to-right DFS order of T_1 , parents in T_2 come before their children.

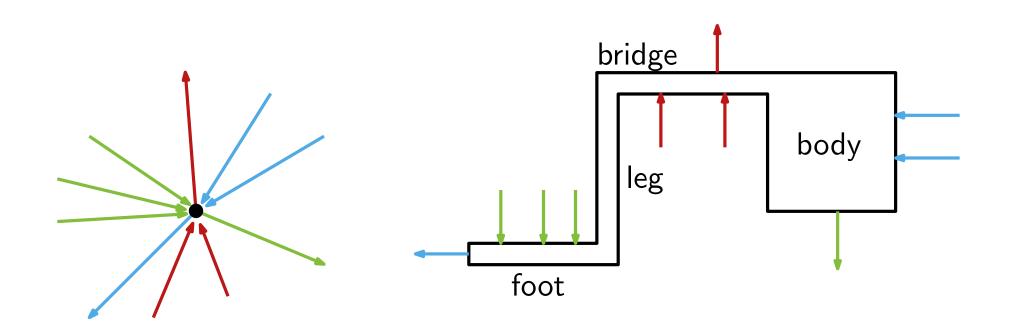


Every vertex is represented by a rectilinear 10-gon.



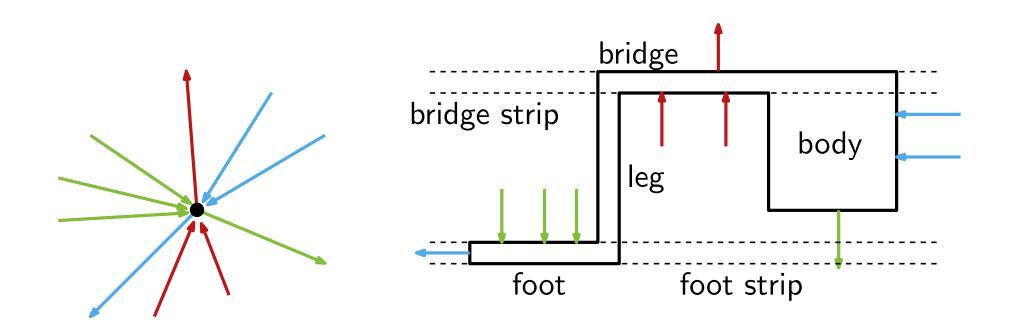


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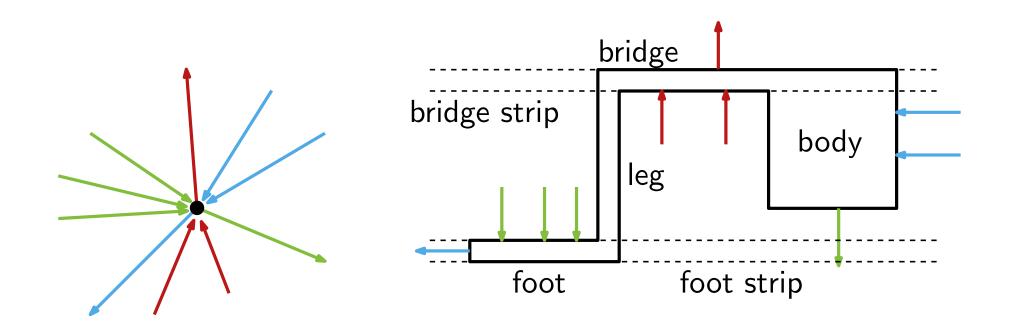


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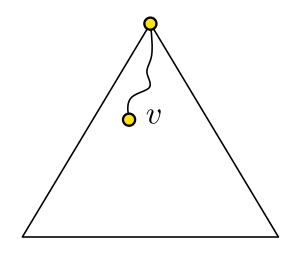


Foot, leg, and bridge use small area; the body carries almost all weight.

Layout Algorithm



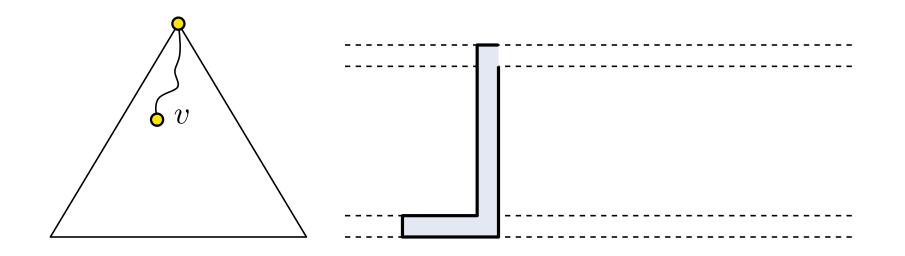
Compute polygons in left-to-right DFS order of T_1 .



Layout Algorithm



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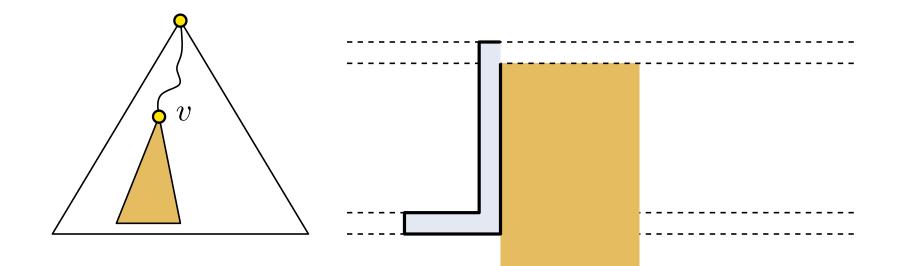


fix foot, leg, and bridge strip

Layout Algorithm



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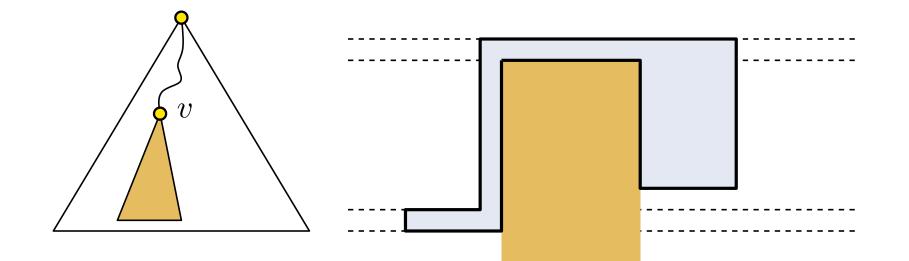


- fix foot, leg, and bridge strip
- compute child polygons

Layout Algorithm



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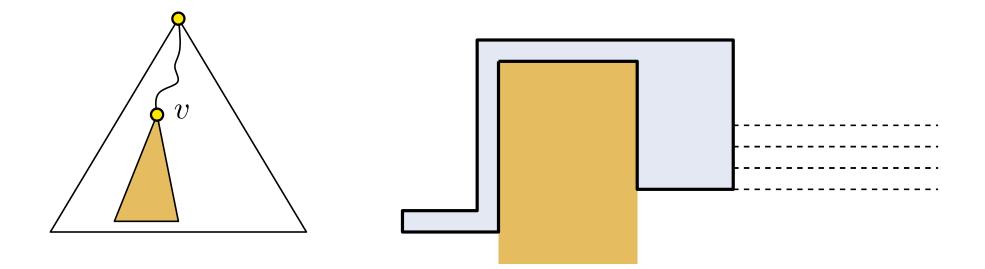


- fix foot, leg, and bridge strip
- compute child polygons
- fix bridge and body

Layout Algorithm

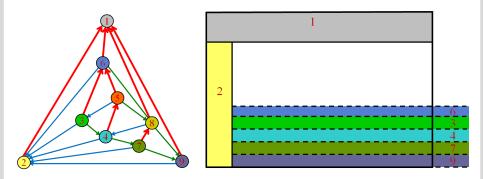


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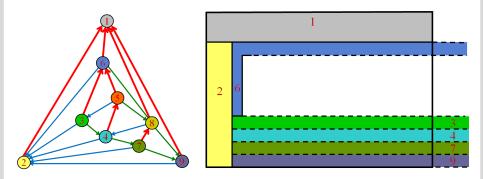


- fix foot, leg, and bridge strip
- compute child polygons
- fix bridge and body
- fix foot strips for children in T_2

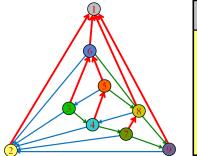


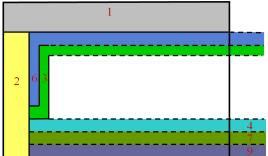




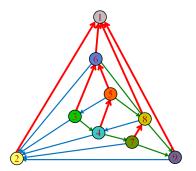


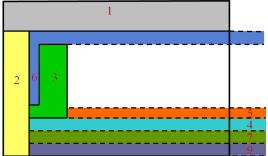




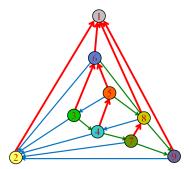


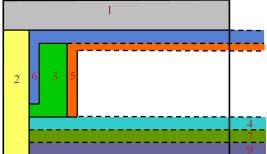




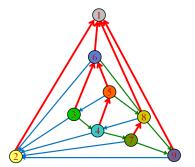


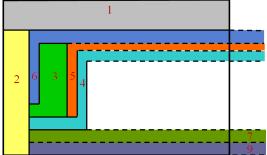




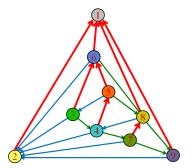


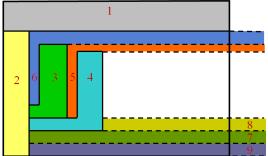




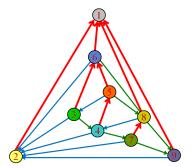


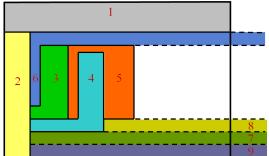




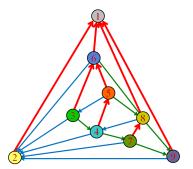


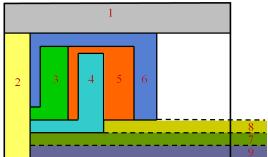




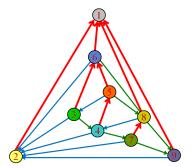


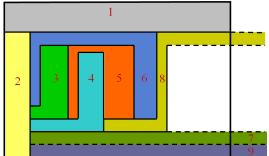




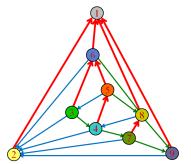


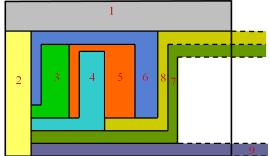




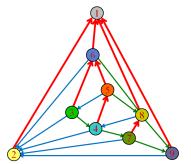


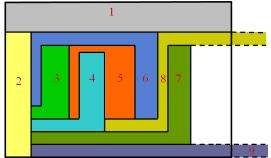




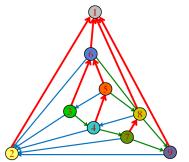


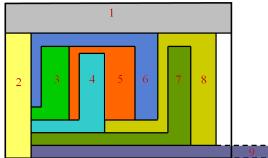




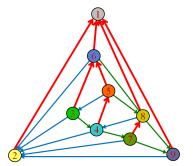


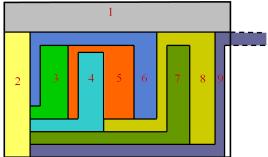




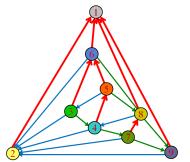


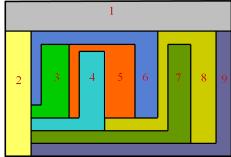








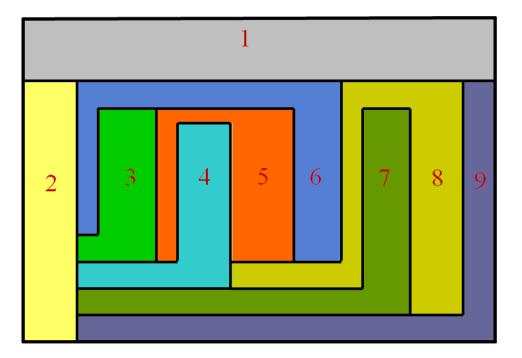




Summary



- every vertex-weighted planar triangulated graph can be drawn as a rectilinear contact representation with 10-gons
 running time of the algorithm is O(n)
- running time of the algorithm is O(n)
- actually 8-sided polygons are always sufficient, but a constructive algorithm is still missing



Lust auf mehr?



Regelmäßige Master-Vorlesungen:

- Algorithmische Kartografie (erstmals SS 2013)
- Algorithmische Geometrie (im SS, nicht 2013)
- Algorithmen zur Visualisierung von Graphen (im WS)

Themengebiete für praktische und theoretische Abschlussarbeiten:

- Geovisualisierung
- Graphenzeichnen
- Algorithmische Geometrie
- . . .

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