Negative Cycle Canceling for Cost-Efficient Wind Farm Planning

Bachelor Thesis of

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Statement of Authorship

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Abstract

Planning cost-efficient offshore wind farms is a complex optimization problem. There is a variety of algorithms optimizing different cost factors of offshore wind farms. The Wind Farm Cabling Problem (WCP) deals with optimizing the inner array cable layout. The WCP optimizes this cable layout for a given set of turbines and substations. The inner array cables connect the turbines to the offshore substations and can use different cable types. The WCP problem is NP-hard and the optimal solution for large instances cannot be computed in reasonable time.

The algorithm proposed in [GUW+19] approaches this problem by improving an initial flow with Negative Cycle Cancelling. In this thesis, we extend this algorithm and propose a modifed WCP in which only a subset of turbines and substations is selected. This approach combines the optimization of the WCP with optimizing the offshore substation and turbine location. We also propose two strategies to find an initial flow and develop an escaping strategy to find flow changes that cannot be performed by cancelling a series of negative cycles. We evaluate the algorithm by comparing its solution to solutions we compute with a Mixed-integer Linear Program. We propose a cost model and generate benchmark sets based on this cost model to test our algorithm.

The results show that Negative Cycle Cancelling can be used for the modifed WCP, but the algorithm has issues with local optima. Our escaping strategy can help to escape some, but not all of those local optima. Furthermore, we could not decide which of the proposed initialisation strategies is to be preferred, but the results suggest that the algorithm performs best if the initial flow is close to the optimal solution.

Deutsche Zusammenfassung

Die Planung von kosteneffizienten Offshorewindparks ist ein komplexes Optimierungsproblem. Es gibt eine Vielzahl an Algorithmen für die Optimierung unterschiedlicher Kostenverursacher von Windparks. Das Wind Farm Cabling Problem beschäftigt sich mit der Optimierung der Verkabelung der Turbinen für gegebene Turbinen und Substations. Die von den Turbinen produzierte Energie wird von den Kabel zu den Offshore-Substations geleitet. Diese Problem ist NP-schwer und kann für große Instanzen nicht innerhalb sinnvoller Zeiten berechnet werden.

Der Algorithmus, der in [GUW+19] vorgestellt wird, benutzt Negative Cycle Cancelling, um eine initialen gültigen Fluss zu verbessern. In dieser Arbeit, erweitern wir diesen Algorithmus und schlagen ein modifiziertes WCP vor, bei dem nur eine Teilmenge der gegebenen Turbinen und Substations ausgewählt wird. Dadurch kombinieren wir die Optimierung des WCP mit der Optimierung der Positionen der Substations und Turbinen. Wir implementieren zwei Strategien um einen initialen Fluss zu finden und entwickeln eine Escaping-Strategy, um Flussänderungen zu finden, die die Lösung insgesamt verbessern, aber nicht durch Flussverschiebungen entlang mehreren negativen Kreisen erreicht werden können. Wir evaluieren den Algorithmus, indem wir seine Lösungen mit Lösungen vergleichen, die wir mit einem Mixed-integer Linear Program berechnet haben. Außerdem geben wir ein Kostenmodell an, mit dem wir Benchmarks erstellen. Dieses benutzen wir um unseren Algorithmus zu testen.

Die Ergebnisse der Evaluation zeigen, das Negative Cycle Cancelling für ein modifiziertes WCP verwendet werden kann. Allerdings hat der Algorithmus Probleme mit lokalen Optima. The Escaping-Strategy kann helfen, einige dieser lokalen Optima zu überwinden, sie findet aber nicht alle. Außerdem konnte nicht endgültig entschieden werden welche der Initialisierungsstrategien bessere Ergebnisse liefert. Die Ergebnisse zeigen jedoch, dass die Initialisierungsstrategie, deren initialer Fluss näher an der optimalen Lösung liegt, in den meisten Fällen bessere Lösungen liefert.

Contents

1. Introduction

Wind energy is one of the most important contributors towards renewable energy production. Over the last years the size and number of wind farms has increased and is still increasing. While the world wide capacity for wind energy was only 17.7 GW in the year 2000, it increased to 561 GW in 2018 with an average annual growth rate of 21% [SM19]. This growth rate emphasizes the importance and potential of the wind industry. Energy policies like the Renewable Energy Act (EEG) in Germany subsidise renewable energy and lead to more investments into wind energy. In 2019, more than 24% [Bur20] of all public electricity in Germany was produced by off- and onshore wind farms. Today, wind energy is cost-competitive to fossil resources like coal and gas and cheaper than most other renewable energy resources [SM19]. Nevertheless, with the increasing number of wind farms and improving technology, the generation of wind energy has become more competitive.

Especially in Europe, the potential for onshore wind farms is limited. Therefore, offshore wind was considered from the beginning. Constantly blowing winds on the sea and lower visual impact make offshore wind farms very suitable for the production of renewable energy. However, there are downsides to offshore wind farms: The environmental impact and higher investment costs cannot be neglected.

A lot of different factors contribute to the higher investment costs of planning an offshore wind farm, but optimizing the wind farm can increase its efficiency and reduce its costs. That is why planning efficient and cost-saving wind farms is an important challenge. The starting points for optimizing a wind farm diverse: The efficiency of turbines can be improved. The height and diameter of the turbines can be increased. Better electrical infrastructure can reduce energy losses. New maintenance strategies can lessen down time and repair costs. Optimized positions of turbines reduces wake effects between those turbines and increases the overall energy production.

5% [GU2] of the invested capital is used for acquisition and installation of sea cables. The electricity has to be collected from the turbines and brought to transformer stations called substations to transform the power for the transportation to the shore. Different cable types with different capacities and costs can be used for a connection between two turbines or between a turbine and a substation. Optimizing the cable layout for given turbine and substation positions is called Wind Farm Cabling Problem (WCP).

With the size of offshore wind farms increasing, algorithmic assistance for offshore wind farm planning has become indispensable. Therefore, solving the WCP is an algorithmic optimization problem of importance. However, this problem is difficult to solve: The WCP is NP-hard for general graphs, since it is already NP-hard on trees [Sta18]. The cables connecting turbines have different cost depending on their capacity. In addition, a huge number of cable connections are possible. Algorithms calculating the optimal solution have a long runtime and are unusable for interactive planning tools. The paper [GUW+19] approaches this issue with Negative Cycle Cancelling. The wind farm is modelled as a flow graph with levelized cost function. After a initial feasible flow is found, the solution is improved by adding flow along negative cycles in a residual graph. We want to determine if the approach can be extended to selecting only a subset of turbines and substations.

1.1 Related work

To design a cost-efficient wind farm, a cost model is needed to determine the cost of different wind farm instances. There are various cost models for offshore wind farms, which vary in their input parameters and cost factors they consider. The cost model in [SBE16] models the costs for the whole life cycle of offshore wind farms. It provides an overview for the cost distribution of the difference life cycle phases and cost factors, but does not go into detail how those costs can be estimated. A cost model with a lower quantity of input parameter is given in [GR17]. This paper reviews the cost for wind farm components from a variety of papers and focuses mainly on the costs, which occur until the wind farm is operational.

There are numerous algorithms to design cost-efficient wind farms. Hou [HZM+19] categorized them into two groups: Wind farm layout optimization and electrical system optimization. The wind farm layout optimization focuses on the location of the whole wind farm and the micro-siting of the turbines. The micro-siting of the turbines determines how turbines are placed inside the wind farm. Turbines extract energy from the wind, which causes a wake in the downstream of the turbine where the wind speed is reduced and can effect the efficiency of other turbines close by. This is called the wake effect. The algorithms that focus on the electrical system optimize wind farm regarding the cable connection layout, offshore substation locating, electrical component selection or voltage level selection.

There exist some algorithms that combine the optimization of the substation location, turbines positioning and cable layout. The paper [HHS+17] proposes an algorithm using adaptive particle swarm optimization to reduce wake effects and cable costs simultaneously, which suggests that harvesting the maximum amount of wind first and than optimizing the cable layout does not provide optimal solutions. However, most approaches optimize the turbine and substation location fist and then the cable layout.

To optimize the substation location before optimizing the cable layout, Li [LHS08] uses a genetic algorithm to minimize the distance of the turbines to the substations. The optimization of substation locations can be formulated as a Facility Location Problem. The position of facilities is chosen in order to minimize the costs for conveying from the customers to the facilities. In [PCJ+15] this is applied to the context of offshore substations.

The Wind Farm Cabling Problem (WCP) describes the algorithmic problem of finding a cable layout with minimum costs and can be placed in the cable connection layout category. The majority of WCP algorithms assumes the turbine positions and substation locations as given. In the work of Berzan et al. [BVMO11], two easier variations of this problem are considered: Optimizing the cable layout for only one given substation and optimizing the cable layout for multiple substations with only one cable type. In most papers regarding the WCP, the turbines and substations are fixed and multiple cable types are considered. Bauer and Lysgaard approach the WCP in [BL15] as a planar open vehicle routing problem, where they can ensure that no cables cross each other. They develop a heuristic method which was than compared to an integer programming formulation of this problem.

The exact solution of the WCP can be computed with a Mixed-integer Linear Program (MILP), but large wind farm instances cannot be solved by this strategy in reasonable time. One MILP model for the WCP is described [GWW20]. The same authors formulated the WCP as a flow problem and use Negative Cycle Cancelling to optimize an initial feasible flow in [GUW+19]. This algorithm has a shorter runtime than the MILP, and for most instance, the solution differs less than 2% from the MILP solution. This algorithm is the foundation of the algorithm proposed in this thesis.

1.2 Contribution

The WCP has been discussed in various papers using different approaches. In most of these papers, all of the given turbines and substations have to be used. In this thesis, we want to focus one the impact of omitting this precondition. While some other papers have additional conditions for the positions of turbines and substation as well as possible connections like focusing on a grid-based layout, we consider instances with no preconditions for the possible cable connections or position of the turbines and substations. More accurate, we focus on optimizing the cable layout of the electrical connections between turbines and the selection of the turbines and substations from a given set of possible turbines and substations. Therefore, we expand the WCP to not only select cables, but also a subset of substations and turbines to build. To do so, we expand the Negative Cycle Cancelling (NCC) algorithm proposed in [GUW+19] and adapt its model to fit the modifed WCP. We explore if optimizing the turbine and substation selection together with the cable layout, improves the overall cost and if this modifed WCP can be integrated in the existing NCC. To evaluate our algorithm, we select a cost model for wind farms and create benchmarks depicting different scenarios. For these instances, we calculated the optimal solution using Mixed-integer Linear Program (MILP) and compare it with different variants of our algorithm. Furthermore, using a heuristic method with NCC has proven promising for the WCP [GUW+19]. We examine in this thesis if this method can be adapted for the modifed WCP as a more broader definition of the WCP.

We optimize the cable layout simultaneous to the turbine and substation selection as opposed to approaches that use a iterative methods of optimizing the turbine and substation selection first and than optimizing the cables. In addition, our algorithm has a runtime of a few minutes in contrast to algorithms computing the correct solution with very long runtimes for large instances.

1.3 Outline of the thesis

The chapter 2 covers graph definitions later needed for defining the model and algorithm as well as a small introduction into offshore wind farms, in which the main components and phases of wind farms are explained. This information provides a better understanding of the terms used in later chapters and puts the decisions made in this thesis into a broader context. In chapter 3, we present our model for a wind farm. We model the modifed WCP as a flow graph with a cost function. After that, we use this model in the fourth chapter 4 which explains the actual algorithm. There, the details of the algorithm are explained and solutions for problems that might occur are presented. Then follows a chapter 5 about the evaluation of our proposed algorithm. It introduces the cost model and benchmark set we use for the evaluation as well as the model for the MILP we need to calculate an nearly optimal solution. After that, the conducted experiments are explained and their outcome is analysed. The last chapter 6 reflects on the outcome of the evaluation and provides an outlook on interesting aspects for further examination.

2. Preliminaries

This chapter provides insides into the basic tasks and components of wind farm planning. In the second section, we introduce graph definitions which we will need in the subsequent chapters.

2.1 Offshore wind farms

Offshore wind farms describe all wind farms in which the turbines are built off the coasts. They are more efficient than onshore wind farms. On the sea, no hills or trees obstruct the wind. Because of that, the wind conditions for offshore wind farms are generally better than at land. The full load hours of a turbine describe how many hours a turbine would need to run at full capacity to produce the same amount of energy it actual produced in a year. An offshore wind farm can have 3000-5000 full-load hours [Mac19, p. 69], while onshore wind farms only have 1000-2000 full-load hours [Mac19, p. 47]. Furthermore, offshore wind farms, especially when built far out at sea, have less impact on the landscape and cause no noise pollution for residents. This is an important factor considering a lot of onshore wind farms projects have to have a certain minimal distances to buildings and have to face initiatives of local residences. However, building offshore wind farms is more expensive than onshore wind farms: Onshore-produced wind energy can be consumed locally. In contrast, offshore energy has to be transported to the shore and then to more populated areas. The current electrical grid in Germany was not designed for large quantities of offshore wind. Furthermore, building an offshore wind farm itself is more expensive. Cables have to be buried into the sea bed and the foundation of the turbines have to hold up to the conditions at sea. All components have to be shipped from the nearest port and construction may be slowed down by downtime due to weather conditions. The turbines built offshore are exposed to higher wind speeds, storm and ocean weather, which leads to higher risks of damages and wear, so maintenance has to be performed more frequently and may be more difficult as well.

2.1.1 Components

The main components of an offshore wind farm are: turbines, turbine foundations, inner array cables, offshore substations, export cables and onshore substations.

A *turbine* is a generator with rotor blades, which converts the kinetic wind energy to electrical energy. A typical offshore wind turbine has a capacity between 3 MW and 8 MW

[RFB20, p. 17]. Though the capacity of offshore wind turbines has increased by 16% each year on average since 2014 [RFB20]. In that time frame, the average turbine diameter has increased from 60 m in the year 2000 to almost 150 m [Mac19]. The *installed capacity* (IC) of a wind farm is the capacity of all turbines combined.

The turbines have to be elevated above sea level. Although there are floating wind farms, most offshore wind turbines are anchored to the sea bed. The turbine foundation elevates the turbine to an average hub height of 60 m to 100 m, depending on the rotor diameter of the turbine [Mac19, p. 58]. Different types of foundations have to be used, depending on the water depths and sea bed: In deeper water, multiple anchor point are necessary while a steel cylinder buried deep into the sea bed is enough to support a turbine in more shallow water.

Inner array cables collect the energy produced by the turbines by connecting them to the offshore substation. They usually have a voltage of 33 kV, but in some resent projects, a higher voltage of 66 kV is considered [FdVFM12]. The inner array cables use alternate current. The sea cables are installed by trenching about 1 m [Uno09, p. 11] into the sea bed.

The *offshore substation* transforms the voltage to the higher voltage used by export cables. An offshore substation has to be elevated above sea level as well. Hence, they need a foundation as well. Depending on the size of the wind farm, it has one or multiple substations. The capacity of the offshore substation varies and depends on the number of substations and the installed capacity.

To reduce power losses, the *export cables* connecting the offshore substation to the shore use high-voltage altering current (HVAC) instead of the voltage of the inner array cables. If the wind farm is far away from the shore, export cables connect the the offshore substation to an AC/DC converter station which transform the alternate current to direct current in order to reduce the losses induced by the distance to the shore even further.

The export cables connect the offshore substation to the *onshore substation*. The onshore substation transforms the current according to the requirements of the local electrical grid.

2.1.2 Life cycle

In [SBE16], Shafiee describes the five phases of offshore wind farm projects: Pre-development and consenting (P&C), production and acquisition (P&A), installation and commissioning (I&C), operation and maintenance (O&M) and decommissioning and disposal (D&D).

The first phase (P&C) consists of administrative tasks like financing or negotiating with subcontractors. Permits have to be acquired and surveys have to be conducted to determine the feasibility. Furthermore, the actual wind farm has to be designed. The second phase (P&A) includes all activities related to the procurement of turbines, transmission system and monitoring system. The third phase, concerns the installation of the components as well as fees regarding the port and vessels and labour costs. Those three phases take place in the first five years of the wind farm's life cycle. Once those phases are completed, the wind farm is operational.

The fourth phase (O&M) starts when the wind farm is actual operational. Only in this phase, the wind farm is operational. The produced energy is fed in to the grid, which causes transmission fees and revenue. The maintenance activities can be categorized into two main components: Proactive maintenance and corrective maintenance. The first describes actions to reduce wear and prevent damages. The latter describes actions that are only carried out if a component fails, which is usually accompanied by downtime of that component.

Leases and insurance fees must be paid in this phase which lasts usually for about 20 years. All revenue generated in this phase has to finance the whole project.

The last phase (D&D) reverses the installation process. Some of the components can be recycled, others have to be removed. Depending on the decommissioning, process some components which may have not been removed after this phase, have to be monitored to identify potential future risks for the sea bed.

2.2 Graph definitions

We model a wind farm as a flow graph. A *directed graph* $G = (V, E)$ consists of a set vertices V and a set of edges E. E is a subset of the Cartesian product $V \times V$. We say an edge $e = (v, u)$ goes from start vertex v to end vertex u. For our purposes, a graph G has no loops, meaning there is no edge in E with the same vertex as start and end vertex. The *outgoing edges* of a vertex v are all edges $(v, u) \in E$. The *incoming edges* of a vertex v are accordingly all edges $(u, v) \in E$.

A *path* p in a graph G is a series of edges $p = \{e_1, .., e_k\} \in E$ where the start vertex of e_{i+1} equals the end vertex of e_i for all $i \in \{1, .., k\}$. A *cycle* C is a path p in G where the start vertex of the first edge equals the end vertex of the last edge.

For a graph G, we define a cost function $c : E \to \mathbb{R}$ which assigns a weight to each edge of the graph. The cost for a path is the sum of the costs of its edges. We write $c(p)$ for $\sum_{e \in p} c(e)$. A negative cycle is a cycle C in G with negative costs: $c(C) < 0$.

We define a capacity for all edges in G $cap : E \to \mathbb{R}_{\geq 0}$. A *flow* in G is a function which assigns a flow value to each edge $f : E \to \mathbb{R}$. If an edge has positive $f((v, u)) > 0$, we say that $f((v, u))$ units of flow go from v to u respective from u to v if an edge has negative $f((v, u)) < 0$. A *incoming flow* $f^-(v)$ from vertex v is defined as the sum of the flow of all incoming edges with a flow greater than zero plus the absolute value of the flow of all outgoing edges with negative flow. Respectively, the *outgoing flow* $f^+(v)$ for vertex v is defined as the absolute flow value of all incoming edges with a negative flow plus the sum of flows of all outgoing edges with a positive flow. A flow is feasible if the following conditions are met: The absolute value of the flow on each edge cannot exceed its capacity and the incoming flow of each vertex equals the outgoing flow:

$$|f(e)| \leq cap(e) \qquad \forall e \in E \tag{2.1}$$

$$f^-(v) = f^+(v) \qquad \forall v \in V \tag{2.2}$$

We *cancel* the flow along a cycle C about Δ by adding Δ units of flow to each edge of the cycle C. We can only cancel cycles if the remaining capacity of each edge in C is greater or equal to Δ. The flow conditions remain satisfied because for each vertex in C the cycle contains an incoming and outgoing edge.

3. Model

We model our modified WCP as a flow graph. We focus on the variable costs of a wind farm based on the number and positions of turbines, number and positions of substations as well as used cable types and length. Other variable costs of a wind farm, such as distance to shore, type of foundation or cost of connecting the wind farm to the grid, will be assumed as fixed. In the first paragraph, we describe what information an input instance has to provide. In the second paragraph, we define a flow graph representing a wind farm based on the information given by the input instances. Furthermore, we define additional flow conditions so that a feasible flow in our graph is a feasible solution of the modifed WCP. In the last paragraph, we propose two objectives to determine a cost-efficient solution of the modifed WCP.

Input

We use directed graphs G as input instances for our wind farm. The vertices consist of a set of possible turbines V_T and a set of possible substations V_S. If turbine v and turbine or substation u can be connected by a cable, G contains the edge (v, u) or (u, v). The direction can be chosen arbitrarily. In the following, we define cost functions for substations, turbines and cables, which have to be provided by a input instance:

The cost of inner array cables is defined by the cost of the chosen cable type per meter and the length of the cable. An input instance has to provide a function assigning costs to different cable types: Let $cost_{cableType}(k)$ with $k \in K$ be the cost function for the different cable types K. Furthermore, the length of the cable has to be given. The function $cable_{len}((v, u))$ represents the actual length of the cable represented by the edge (v, u). In most cases, this will be the Euclidean distance between the turbines connected by the cable plus a supplementary cable length needed to connect the cable to the turbines.

The input instance has to determine the capacity and costs of the turbines. In our model, all turbines $v \in V_T$ have the same capacity $cap_{turbine}(v) = 1$. The cost of a turbine with capacity $cap_{turbine}$ is given by $cost_{turbine}(v)$. The revenue is given by $rev_{turbine}(v)$.

The cost of a substation depends on its capacity and the distance to the next converter. The costs are modelled as $cost_{sub} : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ where $cost_{sub}(c, d)$ is the cost of a substation with capacity c and distance d to a common connection point which can be a converter or an onshore substation. Substations have a capacity $cap_{sub} : V_S \to \mathbb{N}$ representing the maximum number of turbines the substation can collect. In the following, we write $cost_{sub}(v)$ with $v \in V_S$ with a distance d for $cost_{sub}(cap_{sub}(v), d)$.

Flow graph

For a given wind farm instance G, we define a graph G' with a super substation s and a super turbine t. The super substation is connected to all substations. The t is connected to all turbines. The super substation and the super turbine are connected by an edge as well:

$$V(G') = V(G) \cup \{s, t\}$$

$$E(G') = E(G) \cup \{(t, u) : u \in V_T\} \cup \{(v, s), (s, v) : v \in V_S\} \cup \{(s, t)\}$$

We use this graph G' to model our WCP as a flow problem: A feasible flow has to determine which turbines and which substations will be built and which cables with which capacity will be needed.

Each original edge $e \in E(G)$ represents a cable. The absolute flow on these edges cannot exceed the capacity of the largest cable type:

$$|f(e)| \leq cap_{cable}(k_{max}) \qquad k_{max} = \max\{cap_{cable}(k) : k \in K\} \tag{3.1}$$

The edge from the super turbine t to a turbine v models whether or not the turbine v will be built. The flow on this edge can be either the capacity of the turbine, meaning the turbine produces electricity that has to be collected or the flow is zero. If the flow is zero, the turbine is not selected for the current solution. An unselected turbine cannot collect electricity from other turbines. Therefore, the incoming flow of this turbine has to be zero as well:

$$f((t, v)) \in \{0, cap_{turbine}(v)\} \qquad \forall v \in V_T \tag{3.2}$$

$$f^-(v) = 0 \qquad \forall v \in V_T : f((t, v)) = 0 \tag{3.3}$$

The amount of electricity collected by a substation can be modelled by the edge from the substation to the super substation s. The flow of this edge cannot exceed the capacity of the substation. Furthermore, no electricity can leave the substation though an inner array cable:

$$0 \leq f((v, s)) \leq cap_{sub}(v) \qquad \forall v \in V_S \tag{3.4}$$

$$f(v, u) \leq 0 \qquad \forall (v, u) \in E : v \in V_S \tag{3.5}$$

$$f(u, v) \geq 0 \qquad \forall (u, v) \in E : v \in V_S \tag{3.6}$$

A feasible flow for G' fulfils all of the conditions above and those defined in section 2.2. Thus, the incoming flow of each vertex equals the outgoing flow. All edges containing the super turbine except (s, t) are edges from the super turbine to turbines. Those edges can only have a flow greater or equal to zero. The sum of the flows of these edges are equal to the installed capacity cap_{total} of the wind farm. The incoming and outgoing flow of t are equal. Consequently, the flow of the edge from the super substation to the super turbine is the installed capacity of the wind farm:

$$cap_{total}(G'_f) = f((s, t)) \tag{3.7}$$

The total revenue of the wind farm for a given feasible flow equals the sum of the revenues of each selected turbine:

$$rev_{total}(G'_f) = \sum_{v \in V_T : f((t,v)) \neq 0} rev_{turbine}(v) \tag{3.8}$$

If a cable has zero flow, the costs of this cable are zero as well. If not, the costs are the length of the cable times the cost of the smallest cable type with a capacity greater or equal to the absolute flow:

$$cost_{cable}(e) = cable_{len}(e) \cdot cost_{cableType}(x) \quad x = \min\{k \in K : |f(e)| \leq cap_{cable}(k)\} \tag{3.9}$$

The cost of a wind farm represented by a feasible flow in G' can be calculated by the sum of the costs of all selected substations and turbines as well as the costs of all original edges:

$$cost_{total}(G'_f) = \sum_{e \in E} cost_{cable}(e) + \sum_{v \in V_T : f((t,v)) \neq 0} cost_{turbine}(v) + \sum_{u \in V_S : f((u,s)) \neq 0} cost_{sub}(u) \tag{3.10}$$

Objective

To plan a cost-efficient wind farm, we have to decide which factor we want to maximize or minimize. We consider two different approaches as an objective: In the first approach, we maximize the *absolute profit* of the wind farm. For the second approach, we maximize the profit per installed capacity (IC) i.e. the *rate of return*. From an economic perspective, both objectives have their merit. While relative profit maximizes the rate of return, the absolute approach maximizes the absolute profit even if the required capital can be significantly higher than other solutions with almost the same profit. For a given wind farm instance, every feasible flow is a solution of our modifed WCP. However, depending on whether we choose to optimize the absolute profit or the rate, we have different functions defining the quality of a solution.

If we select absolute profit as our objective, the best solution is a feasible flow with maximal revenue and minimal costs. This objective maximizes the profit of the wind farm. However, it does not relate the scale of the profit to the required capital. A solution with slightly more profit will be preferred to a wind farm with almost the same profit, but significantly smaller investment costs:

$$\text{maximize } rev_{total}(G'_f) - cost_{total}(G'_f) \qquad \textit{absolute profit} \tag{3.11}$$

We can also optimize the rate of return. The advantage of this objective is that it is independent from the revenue generated by a turbine but it can lead to solutions with a high profit per installed capacity, but very little installed capacity, which might not be desirable. To do so, we have to minimize the costs in relation to the installed capacity:

$$\text{minimize } \frac{cost_{total}(G'_f)}{cap_{total}(G'_f)} \qquad \textit{rate of return} \tag{3.12}$$

4. Algorithm

This chapter introduces a extended negative cycle cancelling algorithm Extended-NCC we propose which is base on the algorithm described in [GUW$^+$19]. That algorithm NCC optimizes the cable costs for a given set of turbines and substations. Our Extended-NCC optimizes the selection of substations and turbines as well as the cable costs. To do so, we have to adjust some parts of the algorithm to fit our model. After that, we propose strategies to improve the solutions of the algorithm because some flow changes that improve the solution cannot be detected by the NCC. Although this problem existed in the original version of the algorithm as well, its impact on the solution increases due to the different model.

4.1 NCC algorithm

The Extended-NCC is based on NCC algorithm proposed in [GUW$^+$19] and [GWW20], which optimizes the cable costs for a given set of turbines and substations. First, the algorithm's initialization strategy calculates an initial feasible flow. After that, in each iteration, a residual graph R is calculated for a specific change of flow Δ. If the residual graph has a negative cycle, the cycle is cancelled. The residual cost function ensures that cancelling a negative cycle in R improves the solution. If no negative cycle can be found, the delta strategy determines the next Δ. The proposed algorithm is faster than algorithms calculating the optimal solution, but it does not provide the optimal solution. Escaping strategies can be used to improve the solution. They are applied after finding no negative cycle for any Δ. If this improves the solution, the algorithm continues with the next Δ, otherwise the current solution is returned. The algorithm is described in algorithm 4.1. The paper [GUW$^+$19] proposed different strategies for initialisation and choosing the next Δ. We will use the strategies that generated the best results.

Initialisation strategy

The best initialisation strategy of the NCC uses Dijkstra's algorithm to calculate the shortest path from a turbine that is not yet connected to the nearest substation with free capacity. The search ignores edges where the cable capacity is reached. The length of an edge used in Dijkstra's algorithm is the length of the cable.

Algorithm 4.1: NEGATIVE CYCLE CANCELLING

Input: Graph G
Output: A feasible flow f in G

f = INITIALIZEFLOW(G), Δ = INITIALDELTA
while $\Delta \neq NULL$ **do**
    while $\Delta \neq NULL$ **do**
        (R, γ) = COMPUTERESIDUALGRAPH(G, f, Δ)
        $(f, found)$ = EXTRACTANDCANCELNEGATIVECYCLE(R, γ, f)
        Δ = NEXTDELTA(Δ, *found*)

    $(f, found)$ = ESCAPINGSTRATEGY(f)
    Δ = NEXTDELTA(Δ, *found*)
return f

Delta strategy

During the iterations of the NCC a delta strategy determines for which Δ the next residual graph is calculated and should generate all changes of flow that could improve the solution. The largest possible change of flow is changing the direction of a flow which has a value equal to the maximal cable capacity. Therefore, the delta strategy should generate all integer values from 1 to twice the maximum cable capacity. The best delta strategy proposed in [GUW$^+$19] starts with one and increments Δ until a negative cycle is cancelled. Then, it decrements Δ until it is 1 again. After that, the delta strategy starts again from the beginning. To improve the performance, all Δ during the incrementation can be skipped until the Δ is reached where the last negative cycle was found. If Δ is twice the maximal cable capacity and no negative cycle can be cancelled, Δ is set to null and the algorithm terminates.

Calculate residual graph and cancel negative cycles

The NCC computes a residual graph R for a given Δ. The graph R includes all edges of G and the reverse of those edges. The weight of an edge e is given by the residual cost function γ and represents the cost of increasing the flow along the corresponding edge in G by Δ or the cost of decreasing the flow if e is the reverse of an edge in G. Cancelling a cycle with negative costs in R improves cost of the flow in G. The residual cost function is also used to ensure that cancelling a negative cycle in R always results in an infeasible flow in G. The negative cycles are found with an implementation of the Bellman Ford algorithm.

Escaping strategy

The algorithm proposed in [GUW$^+$19] can get stuck in local minima where improving the solution cannot be performed by cancelling one negative cycle. Therefore, they modified their algorithm in [GWW20]. In this version, the algorithm does not stop if no negative cycles can be found, but performs an escaping strategy. Only if the escaping strategy cannot improve the solution as well, the algorithm stops. An escaping strategy can be any detection method of flow changes that result in a feasible flow with improved costs. In this thesis, we will not used any escaping strategy proposed in [GWW20], but create our own escaping strategy instead.

4.2 Initialisation

In our wind farm model, the total substation capacity does not have to match the total capacity of the turbines. To find an initial feasible flow, we propose two initialisation strategies. For the first initialisation strategy ANY, we follow the initialisation strategy in paragraph 10. We use Dijkstra's algorithm to calculate the shortest path to a substation with free capacity. We ignore edges if using this edge would exceed its capacity. Since a cable can be used in both direction, we have to consider the reversed edges as well. We increase the flow along this path by the turbine capacity and collect all turbines on the path to the substation as well. For all turbines we collect, we increase the flow from the super turbine to the turbine, the super substation to the super turbine, and the substation to the super substation by one as well to maintain a feasible flow. For the second initialisation strategy FILL, we follow the same process and connect the first turbine to the nearest substation with remaining capacity, but then, we connect all further turbines to the same substation until this substation has no free capacity left. After that, we repeat the same procedure until either all turbines are collected or no substation with free capacity is left. The flow created by both strategies meets the conditions for a feasible flow and is therefore a feasible solution for the WCP.

4.3 Residual costs

In the original version of the NCC, the residual costs were determined by the cost difference caused by the change of flow Δ. This is no longer applicable, since depending on which objective we use, we have to take either the revenue of the turbines or the installed capacity into account. We need a residual cost function $\gamma : E(R) \to \mathbb{R}$ where cancelling a negative cycle in R improves the solution of the WCP for the chosen objective.

A cycle in R that can change the installed capacity, has to contain edges from super turbine to turbines or edges from turbines to the super turbine. We can find residual cost functions for both objectives rate of return and absolute profit, which differ only for those edges between the super turbine and turbines. In the following, we define a cost function $\gamma : E(R) \to \mathbb{R}$ for all edges of the residual graph R:

The residual costs for edges from the super substation s to a substation v should ensure that the incoming flow of a substation will not exceed its capacity and that no flow will leave the super substation. Hence, we define γ as: $\gamma((s,v)) = \begin{cases} -cost_{sub}(v), & \text{for } f((v,s)) = \Delta \\ \infty, & \text{for } f((v,s)) < \Delta \\ 0, & \text{otherwise} \end{cases}$

$\gamma((v,s)) = \begin{cases} cost_{sub}(v), & \text{for } f((v,s)) = 0 \\ \infty, & \text{for } f((v,s)) + \Delta > cap_{sub}(v) \\ 0, & \text{otherwise} \end{cases}$ The residual costs for edges between the super substation and the super turbine are defined as:

$$\gamma((t,s)) = 0$$

$$\gamma((s,t)) = 0$$

Therefore, it is possible to find a negative cycle C containing either of these edges if a negative path increasing the capacity from t to s or a negative path reducing the capacity from s to t exists. If $(t,s) \in C$, the installed capacity increases by Δ. If $(s,t) \in C$, the installed capacity is reduced by Δ.

The residual costs for edges e between turbines or from a turbine to a substation are defined as:

$$\gamma(e) = (cost_{cable}(|f(e) + \Delta|) - cost_{cable}(|f(e)|)) \cdot cable_{len}(e)$$

If the flow of edge e exceeds the capacity of the largest cable type, the cable costs $cost_{cable}(|f(e) + \Delta|)$ are infinite. Consequently, the residual costs are infinite as well. This guarantees that the flow though a cable will never exceed its maximal capacity.

No flow can leave a substation. To ensure that, we define the residual costs for edges from a substation $s \in V_S$ to a turbine $v \in V_T$ as:

$$\gamma((s,v)) = \begin{cases} cost_{cable}(|f(e)+\Delta|) - cost_{cable}(|f(e)|) \cdot cable_{len}(e), & \text{for } f((v,s) \leq \Delta) \\ \infty, & \text{otherwise} \end{cases}$$

Absolute profit

To maximize the absolute profit, we consider the costs of a turbine minus the revenue of a turbine for the edges from the super turbine to a turbine. We define the residual costs for all edges form the super turbine t to a turbine v as:

$$\gamma((t,v)) = \begin{cases} cost_{turbine}(v) - rev_{turbine}(v), & \text{for } f((t,v)) = 0 \wedge \Delta = cap_{turbine}(v) \\ \infty, & \text{otherwise} \end{cases}$$

$$\gamma((v,t)) = \begin{cases} -(cost_{turbine}(v) - rev_{turbine}(v)) & \text{for } f((t,v)) = \Delta = cap_{turbine}(v) = f^{-}(v) \\ \infty, & \text{otherwise} \end{cases}$$

This cost function guarantees that the production of electricity is either zero or $cap_{turbine}(v)$ and that the flow can only be reduced if no other turbine uses v as connection point. The cost of a cycle containing either of these edges is the change of cost of all edges minus the change of revenue for all edges of the cycle containing the super turbine. Therefore, a negative cycle in R improves the absolute profit of the wind farm.

Rate of return

To minimize the costs per installed capacity, we require a residual cost function γ that represents the change of $\frac{cost_{total}}{cap_{total}}$ if the flow is changed by Δ. Cancelling a negative cycle C in R must reduce this ratio. Finding this cost function is difficult because the cost of a cycle is determined by the sum of the costs of its edges. A ratio cannot be calculated by a sum if the denominator is not known locally. In general, when calculating the residual costs of a edge, it is unknown how cancelling a cycle containing this very edge affects the installed capacity. However, it is not necessary to calculate the exact ratio before cancelling the cycle, since we are only interested in knowing if the cycle is negative. Increasing the flow along an edge from the super turbine to a turbine (t,v) by Δ increases the installed capacity by Δ. Equivalently, increasing the flow along (v,t) reduces the installed capacity by Δ. All cycles containing neither of these edges do not change the installed capacity. If the installed capacity changes by $cap'_{total} = cap_{total} + \alpha$ and $cap'_{total} \neq 0 \wedge cap_{total} \neq 0$, the change of the ratio for the entire wind farm can be calculated by:

$$\Delta\frac{cost_{total}}{cap_{total}} = \frac{cost_{total}}{cap_{total}} - \frac{cost'_{total}}{cap'_{total}} = \frac{cap'_{total} \cdot cost_{total} - cap_{total} \cdot cost'_{total}}{cap'_{total} \cdot cap_{total}}$$

$$= \frac{(cap_{total} + \alpha) \cdot cost_{total} - cap_{total} \cdot cost'_{total}}{cap_{total}(cap_{total} + \alpha)} = \frac{cap_{total} \cdot cost_{total\Delta} + \alpha \cdot cost_{total}}{cap_{total}(cap_{total} + \alpha)}$$

$$= \frac{1}{cap'_{total}} \cdot (cost_{total\Delta} + \frac{\alpha \cdot cost_{total}}{cap_{total}})$$

We know that the installed capacity can never be below zero therefore:

$$\Delta\frac{cost_{total}}{cap_{total}} < 0 \iff cost_{total\Delta} + \frac{\alpha \cdot cost_{total}}{cap_{total}} < 0$$

The first summand $cost_{total\Delta}$ is independent from the currently or future installed capacity. The capacity factor $\frac{\alpha \cdot cost_{total}}{cap_{total}}$ is only dependent on the previous ratio and the capacity change, but independent from the total cost change. As a consequence, we can define the residual cost of each edge by their actual cost change like proposed in the paragraphs above.

The costs of the capacity factor can be split between all edges in the cycle, which change the installed capacity, according to their contribution to the capacity change.

We define the residual costs for all edges from the super turbine t to a turbine v as:

$$\gamma((t,v)) = \begin{cases} cost_{turbine}(v) + \frac{cap_{turbine}(v) \cdot cost_{total}(G_f)}{cap_{total}(G_f)}, & \text{for } f((t,v)) = 0 \wedge \Delta = cap_{turbine}(v) \\ \infty, & \text{otherwise} \end{cases}$$

$$\gamma((v,t)) = \begin{cases} -(cost_{turbine}(v) + \frac{cap_{turbine}(v) \cdot cost_{total}(G_f)}{cap_{total}(G_f)}), & \text{for } f((t,v)) = \Delta = cap_{turbine}(v) = f^-(v) \\ \infty, & \text{otherwise} \end{cases}$$

This cost function guarantees that the electricity production is either zero or $cap_{turbine}(v)$ and that the flow can only be reduced if no other turbine uses v as connection point. The change of the installed capacity for a negative cycle C in R is given by:

$$\alpha = \sum_{(t,v) \in C \wedge v \in V_T} cap_{turbine}(v) - \sum_{(v,t) \in C \wedge v \in V_T} cap_{turbine}(v)$$

Thus, this definition of γ is valid to minimize the costs per installed capacity.

4.4 Escaping strategy: Free substation

The algorithm as currently proposed does not calculate the optimal solution. Some complex flow changes that improves the overall solution cannot be performed by cancelling a single cycle. This problem already existed in previous implementations of the algorithm [GWW20]. However, due to the modified model such complex flow changes might occur more frequently. The edges between a substation and the super substation change their costs only if all flow currently collected by this substations is redirected to other substations or those turbines are cancelled. Those flow changes cannot be performed by cancelling one cycle, since flow cannot leave the substation and in most case a substation has multiple edges with an incoming flow. As a consequence, the algorithm does not change the substation in most cases.

The number of substations should be as small as possible because substations are expensive compared to the cable costs. If a substation has an incoming flow greater than zero on more than one incoming edge, it is not possible to cancel all flow to this substation with only one cycle. However, cancelling multiple cycles can improve the overall costs of the wind farm. An example is illustrated in figure 4.4. The problem is that, except the last cycle which includes the negative costs of the substation, the residual costs of those cycles can be greater than zero. Therefore, the proposed NCC cannot find these combined negative cycles. To improve the substation induced costs, we propose two approaches: Selecting the substations in the beginning or using an escaping strategy to reduce the number of selected substations.

4.4.1 Selecting substations in the beginning

In the most common wind farm layouts, the costs of substations exceed the cost of cables. Hence, the costs of a wind farm are almost optimal if an optimum subset of substations is chosen in the beginning. If we use the initialisation strategy FILL section 4.2, we reduce the number of substations selected in the beginning. However, especially if the costs of substations differ, this approach cannot provide an optimal solution since the cost difference between two substations can be smaller than the cable costs.

4.4.2 Calculating shortest paths between substations

Another possible strategy is to explicitly search for a series of cycles $C_1, ..., C_k$ for which cancelling those cycles reduces the overall cost. We can search for such a combined cycle

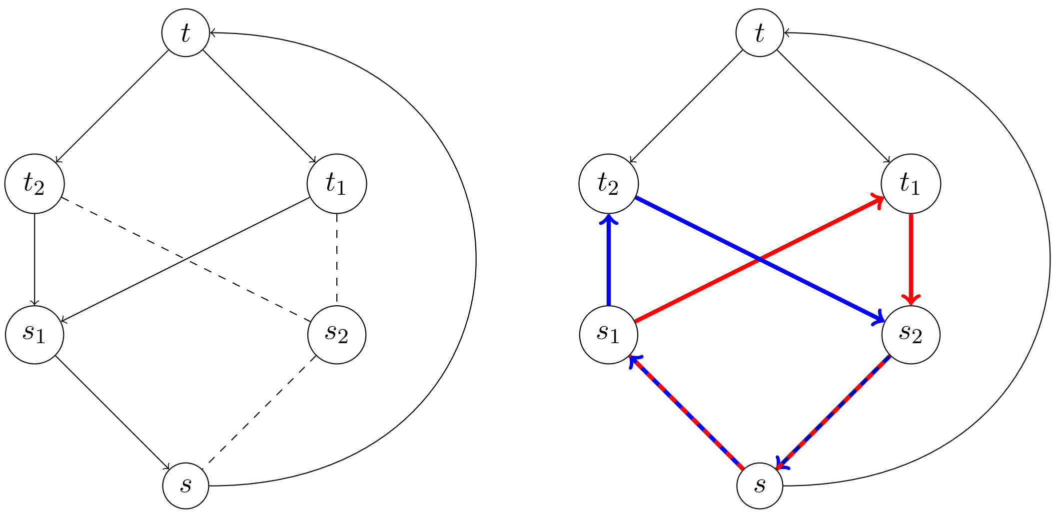


Figure 4.1: This graph shows a minimal example for a combined cycle. We assume in this case that substation s_2 is a lot cheaper than s_1. The left graph shows a flow graph. To free substation s_1 we need to move flow along the blue and read cycle shown in the right graph. In most cases the red and blue cycle do not have negative costs for $\Delta = 1$ since (s, s_1) has only negative costs if $\Delta = 2$. However, if we cancel the blue cycle first even if its costs are not negative and than cancel the red cycle, the total costs of cancelling those two cycles are negative if the cost difference of the substation exceeds the cable costs of this change of flow.

$C = C_1, ..., C_k$ for each substation. If C has negative costs, we cancel C, and if not, we continue with the next substation. Our current algorithm is not able to find those cycles, especially since $C_1, ..., C_k$ might not have the same Δ. We implement an escaping strategy which is called if no negative cycles can be found in the current graph for any Δ.

To find a combined cycle that frees a substation, we calculate, for each Δ, the shortest paths from each vertex to each substation and to the super turbine. We modify the implemented Bellman Ford Algorithm 4.2 to calculate those shortest paths. To ensure those paths only contain substations or the super turbine as an end point, we skip edges containing the super substation. A label for each vertex saves the two best distances to each substation and to the super turbine for each Δ from 1 to twice the maximum cable capacity as well as the parent pointer of the shortest path and the parent pointer of the second shortest path to that substation. In the beginning, the distances for each vertex are set to infinity. The distance from a substation to itself is set to zero. The distance from the super turbine to itself is zero for $\Delta = 1$. Since we need the paths to the substation, not from the substation, we use the incoming edges when relaxing a vertex.

After calculating all distances, we try to find a negative combined cycle for a substation s_i. If we cannot find a negative combined cycle, we try the next substation. If a combined cycle is found, we cancel it and continue with the normal NCC. Calculating those distances is expensive, but we calculate the distances only once and use them until we find a negative combined cycle.

To find a combined cycle for s_i, we follow four steps: Cancelling turbines if the flow to s_i is greater than the remaining capacity of the other substations, redirecting the flow from each incoming edge of s_i to other substations, adding as much turbines as possible to those other substations and cancelling turbines if their total profit is less than the cost of the substation they are connected to. We save all flow changes in a separate data structure and verify whether the combined cycle has negative costs before cancelling it.

Algorithm 4.2: Bellman Ford

Input: Graph $G = (V, E)$, maximum cable capacity Δ_{max}

Output: Distances distance(v, Δ), distanceTwo(v, Δ), parents parent(v, Δ), parentTwo(v, Δ) for all $v \in V_S$ and $1 \leq \Delta \leq \Delta_{max}$

```
for v ∈ V do
    Δ ← 1
    while Δ ≤ Δ_max do
        if v.isSubstation() and Δ ≤ v.freeCapacity() then
            v.distance(v, Δ) ← residualCosts((v, s), Δ)
        else if v.isSuperTurbine ∧Δ = 1 then
            v.distance(v, Δ) ← 0
        else
            v.distance(v, Δ) ← ∞
        v.parent(v, Δ) ←⊥
        v.parentTwo(v, Δ) ←⊥
        v.distance(v, Δ) ← ∞
        v.distanceTwo(v, Δ) ← ∞
        Δ ← Δ + 1
for n − 1 times do
    for (v, u) ∈ E do
        Δ ← 1
        while Δ ≤ Δ_max do
            for s ∈ V_S do
                if u.distance(s, Δ) + residualCosts((v, u), Δ) <
                 v.distance(s, Δ) then
                    if (v, u) ≠ v.parent(s, Δ) then
                        v.distanceTwo(s, Δ) ← v.distance(s, Δ)
                        v.parentTwo(s, Δ) ← v.parent(s, Δ)
                    v.distance(s, Δ) ← u.distance(s, Δ) +
                     residualCosts((v, u), Δ)
                    v.parent(s, Δ) ← (v, u)
            Δ ← Δ + 1
```

In the first step, we ensure that the flow to s_i can be redirected to other substations. If the flow to s_i is greater than the remaining capacity of the other substations, the surplus turbines have to be cancelled. If we have to cancel only one turbine, we add the cycle $C_1 = p_1 \cup \{(t,s),(s,s_i)\}$ to our combined cycle where p_1 is the shortest path from s_i to t. If we have to cancel more than one turbine, we remove the last edge from p_1 to get p_2. If the last vertex of p_2 has no incoming flow from other turbine, we add $C_2 = p_2 \cup \{(t,s),(s,s_i)\}$ to the combined cycle and proceed until the last vertex has an incoming flow greater than one. We add an edge with incoming flow that is not in p_i to this path p_i', until we reach t; This path is p_3. After that, we add $C_3 = p_3 \cup \{(t,s),(s,s_i)\}$ to the combined cycle. We continue until the flow to s_i is equal to the remaining capacity. For better performance, we stop if we want to cancel more profit from turbines than the cost of s_i. In that case, no negative combined cycle can be found. Before calling the escaping strategy, no negative cycles could be found by our algorithm. The following theorems 4.1 and 4.2 illustrate why p_i can be found:

Theorem 4.1. *If G' has a cycle $C_{G'} = \{u_1,..,u_n\}$ and $C_{G'}$ does not contain the super turbine, $f(u_i,u_{i+1}) \geq \Delta$ for all $i \in \{1,..,n-1\}$ and $f(u_n,u_1) \geq \Delta$ than the residual graph R from G' has a negative cycle C_R with $C_R = \{u_n,..,u_1\}$.*

Proof. We choose a maximal Δ, therefore, an edge e exists with $f(e) = \Delta$. The residual costs γ of $C_R = \{u_n,...,u_1\}$ for Δ are less or equal to zero, as reducing the flow of an edge (u_j,u_{j+1}) does not change or reduces the costs, because the cost function for substations and cables is monotonous increasing with the flow. Hence, $\gamma(u_{j-1},u_j) \leq 0$ for all edges in C_R. Furthermore, the current costs of e are greater than zero, but after reducing it by Δ edge e has zero costs. Thus, $\gamma(\bar{e}) < 0$. Consequently, $\gamma(C_R) = \gamma(C_R/\bar{e}) + \gamma(\bar{e}) \leq \gamma(\bar{e}) < 0$. □

Theorem 4.2. *If R for $\Delta = 1$ has no negative cycle without the super turbine, $f^-(t_1) \geq 1$ and p_1 is a path with finite costs from t to t_1 with $(t_1,t) \notin p_1$ then a cycle C exists with $(t_i,t) \in C$, finite costs and incoming flow $f^-(t_i) = 1$.*

Proof. Let $\Delta = 1$. If the incoming flow of t_1 is equal to one, all incoming flow is on the edge (t,t_1) and the residual costs are the negative profit of the turbine. Hence, $\gamma(C) < \infty$ with $C = p_1 \cup \{(t_1,t)\}$ and cancelling C will lead to a feasible flow. Turbines are only allowed to have an incoming flow from other turbines if they have an incoming flow from the super turbine as well. If the flow $f^-(t_1) > 1$, the edge from the super turbine to t_1 has flow $f(t,t_1) = 1$. As a consequence, there is another turbine t_2 with $f(t_2,t_1) > 0$. We choose $p_2 = p_1 \cup \{(t_1,t_2)\}$. The edge (t_2,t_1) has a flow > 0. Reducing the flow by $\Delta = 1$ will not rise the costs (the cable cost function increases monotonically with the flow). Thus, $\gamma(p_2) = \gamma(p_1) + \gamma(t_1,t_2) \leq \gamma(p_1) < \infty$. If $f^-(t_2) \geq 1$, we select an incoming edge (t_3,t_2) of t_2. As before, we can find a path p_3 with finite costs containing t_1,t_2,t_3. We continue until $f^-(t_i) = 1$. The residual costs for (t_i,t) are finite and the cycle $C_i = \{p_i,(t_i,t)\}$ has finite costs. Each turbine can only appear once in path p_i. Otherwise, we find a cycle $C_{jk} = \{(t_j,t_{j+1}),(t_{j+1},t_{j+2}),..,(t_k,t_j)\}$ with $f(C_{jk}) \geq 0$ (see theorem 4.1).The algorithm terminates because the number of turbines is finite. □

The second step is redirecting flow from s_i to other substations. For each edge (v,s_i) with positive flow, we try to find the shortest path p from v to a different substation s_j with $\Delta = f((v,s_i))$. We select s_j, by using the distances from the label of s_i, but then we add the cost of s_k if s_k has currently no incoming flow in G and is not used in any other cycle in our current combined cycle. Hence, substations will be preferred if they are already built or will be built anyway because they are used by a previously added cycle. After selecting the target substation s_j, we walk the path $p' = \{(s_i, v)\} \cup p$ to s_j using the parent pointers. At each step, we verify that the residual costs of the flow added to the current edge by p and by previously added cycles, are not infinity. This can happen because we do not recalculate the distanced after adding a cycle to the combined cycle. If an edge (w, u) has infinite residual costs, p' is split into p_1 and p_2 at w. The path p_1 is the same path as p', but Δ is reduced to the maximal change of flow where the residual costs of p_1 are not infinite. We add p_1 to our combined cycle and search for a second path p_2 from w to a substation. If p_2 wants to use the same edge as p_1, the second parent is used. We can split p_2 again if necessary. Otherwise, we add p_2 to our combined cycle. If no path is found, the escaping strategy failed for s_i. If successful, we continue with the next edge with an incoming flow at s_i.

After redirecting all flow from s_i, some additional substations might be build. If the cost of the combined cycle is greater than zero, it might be necessary to compensate the cost of those new substations by adding some turbines that are not yet selected. We walk the shortest path p from a currently not built turbine t_i to a substation with free capacity that is already built or is already used by a cycle in the current combined cycle. We find this path by walking the shortest path from t_i to a substation s_j and verifying that the residual costs are not infinite. If the costs are infinite, we try the second parent pointer. Before adding the cycle $C = p \cup \{(s_j, s), (s, t), (t, t_i)\}$, we verify that the costs of adding this cycle, after we would have added the current combined cycle are negative. As a consequence, only turbines improving the costs of the current combined cycle will be added.

If the costs of the combined cycle are still greater than zero, we try to cancel turbines collected by the last added substation s_j since it might have much free capacity left. To do so, we use the same algorithm like in step one, but with s_j as a source substation.

5. Evaluation

To evaluate our algorithm, we compare its results to almost optimal solutions for a variety of instances. To do so, we have to decide on a cost model and create benchmark sets. We calculate the almost optimal solution with MIXED-INTEGER LINEAR PROGRAM solver. Therefore, we have to formulate both objectives as a MILP. Based on the results of the MILP, we decide whether we maximize the absolute profit or the rate of return. Then, we compare the results of the Extended-NCC for different combinations of initialization and escaping strategies to the MILP solution and analyse the results based on running time and quality.

5.1 Cost model

The costs of offshore wind farms are determined by the acquisition and installation costs as well as the yearly cash flow. The acquisition and installation costs can be assigned to the following components: turbines, foundations, cable, substations and high voltage connection. The yearly cash flow consists of operation and maintenance costs, price per MW h and subsidies, deprecation, and annual taxes.

There is a great variety of different models to calculate the costs of a wind farm. They differ in the number of input parameters and life cycle phases they cover. We choose a model that matches most the required input parameters of our model. Therefore, the costs have to be assignable to either turbines, substations or cables. An optimal cost model would have only the following input parameters: number and capacity of turbines, number and capacity of substations, and length of inner array cable for a variety of cable types.

The cost model proposed in [GR17] focuses on the acquisition and installation phases of offshore wind farms and provides cost functions for cables, turbines and substations. However, it does not provide any information about the costs during the other phases of the wind farm. In addition, the cost model only considers the costs for building the wind farm and does not include revenues generated in any life cycle phase.

We choose this cost model because its input factors match the input factors of our algorithm. One disadvantage of this model is that it does not provide a cost model for the operation and maintenance phase of a wind farm, but other cost models that provide more detailed information about this life cycle phase cannot allocate those costs directly to our input factors. Therefore, we decided to only consider the costs of the acquisition and installation phases. The profit of a wind farm differs from country to country and is usually not

I_{max} in A	Cost in €/m	#Turbines
380	128	≤ 5
680	321	≤ 9
780	481	≤ 10
900	506	≤ 11

Table 5.1: This table shows the costs and maximal capacity of the four proposed cable types.

included in cost models, but it can be calculated with the feed-in remuneration of the country the wind farm is built in. Since the revenue of a wind farm must refinance the wind farm throughout all life cycle phases, we take only a percentage of the total wind farm profits into account proportional to the costs induced by the acquisition and installation phases compared to the total life cycle costs.

Turbine cost

To calculate the cost of a turbine, we need to calculate the costs for the foundation and the turbine itself. The costs of acquisition, shipping, assembling and electrical installation of a turbine can be modelled with $cost_{WT} = 1\,374\,000\,€ \cdot Cap^{0.87}$ [GR17, p.13] where Cap represents the capacity in MW. The cost of the foundation consists of transport, installation and scour protection and depends on the type of foundation. It can be calculated with $cost_{foundation} = 363\,000\,€ \cdot Cap^{1.06}$ [GR17, p.14]. We assume that all turbines have a capacity of 3.6 MW. Therefore, the total costs of a turbine v are $cost_{turbine}(v) = 5\,598\,841\,€$.

Substation cost

The costs of a substation depend on the costs of the substation itself and its foundation. The costs of the substation, its foundation and the installation can be modelled with $cost_{substation} = 539\,000\,€ \cdot Cap^{0.678}$ [GR17, p.15], where Cap represents the capacity in MW.

Inner array cable cost

The inner array cable cost consists of costs for acquisition and costs for installation and depends on the length and capacity of the cable. We assume installation cost of 331 €/m [GR17, p.16]. The costs for acquisition depends on the capacity of the cable. We select four cable types with a maximal current of 380 A, 430 A, 780 A and 900 A with prices of 128 €/m, 192 €/m, 481 €/m and 506 €/m [GR17, p.15] respectively. We follow a paper by Dutta and Overbye [DO11] to calculate the amount of turbines that can be collected by one cable. In this paper, a turbine has a capacity of 3.6 MW, a lagging power factor of 0.8 and the inner array cables have a voltage of 34.5 kV. The maximum current of these cables is $\frac{3.6 \cdot 10^6}{34.5 \cdot 10^3 \cdot \sqrt{3} \cdot 0.8} = 75.31\,\text{A}$. The calculated capacity of the different cable types is displayed in table 5.1. To connect a cable to a turbine, a supplementary cable extension of 40 m [GR17, p.14] must be added for each turbine.

Export cable costs

Export cables connect a substation to an onshore substation or an offshore connection point. The cost of export cables depends on the cable length and capacity. Since not all substation have the same distance to a common connection point, the cost of export cables should be considered when optimizing the wind farm layout. We use a 132 kV cable with a capacity of 138 MW and price of 518 €/m [GR17, p.16]. Therefore, this cable is suitable for substations that can collect up to 38 turbines with a capacity of 3.6 MW. The installation costs vary depending on water depth and distance to shore. We assume installation costs of 331 €/m [GR17, p.16]. If the wind farm contains a substation that can collect more than 38 turbines, a second cable type with a capacity of 250 MW and price of 843 €/m [GR17, p.16] is used. This cable type can collect up to 69 turbines.

Revenue of offshore wind farms

The price per MW h differs depending on the installed capacity, year and country. We use current prices from Germany. In the first 12 years, the price is 0.154 €/(kW h) and after that, it is 0.039 €/(kW h)[EEG17]. Most wind farms in Germany have between 2500 and 4500 full-load hours [IWE]. With a turbine capacity of 3.6 MW and assumed 3500 full load hours per year as well as an operating period of 20 years [SBE16], the revenue for one turbine is about 27 216 000 €. We only calculate 58% of the revenue because we do not consider operation and maintenance or decomposition and 62% of the life cycle costs are used for acquisition and installation [SBE16], but about 4% of the life cycle costs are induced by the acquisition and installation onshore substations which we do not consider in our model.

Summary

We assign the following values and functions to the functions defined in the model based on the previously described cost model in section 5.1. The model requires natural numbers as turbine capacity. We set the turbine capacity to one to represent a capacity of 3.6 MW. Accordingly, the substation capacity in the model has to be multiplied by 3.6 to get the actual capacity needed for the cost function.

$$cap_{turbine}(v) = 1 \tag{5.1}$$
$$cable_{len}(e) = \text{len}(e) + 2 \cdot 40\,\text{m} \tag{5.2}$$
$$cost_{cableType}(k_1) = 128\,€/\text{m} + 331\,€/\text{m} \tag{5.3}$$
$$cost_{cableType}(k_2) = 192\,€/\text{m} + 331\,€/\text{m} \tag{5.4}$$
$$cost_{cableType}(k_3) = 481\,€/\text{m} + 331\,€/\text{m} \tag{5.5}$$
$$cost_{cableType}(k_4) = 506\,€/\text{m} + 331\,€/\text{m} \tag{5.6}$$
$$cap_{cable}(k_1) = 5 \tag{5.7}$$
$$cap_{cable}(k_2) = 9 \tag{5.8}$$
$$cap_{cable}(k_3) = 10 \tag{5.9}$$
$$cap_{cable}(k_4) = 11 \tag{5.10}$$
$$cost_{export}(u) = \begin{cases} export_{len}(u) \cdot (518\,€/\text{m} + 331\,€/\text{m}) & \text{if } cap_{sub}(u) \leq 38 \\ export_{len}(u) \cdot (843\,€/\text{m} + 331\,€/\text{m}) & \text{if } cap_{sub}(u) \leq 69 \end{cases} \tag{5.11}$$
$$cost_{sub}(u) = 539\,000\,€ \cdot (cap_{sub}(v) \cdot 3.6)^{0.678} + cost_{export}(u) \tag{5.12}$$
$$cost_{turbine}(v) = 5\,598\,841\,€ \tag{5.13}$$
$$rev_{turbine}(v) = 0.58 \cdot 27\,216\,000\,€ \tag{5.14}$$

There are still parameters left to assign after these functions have been assigned: the layout of the input graph, capacities of the substations, and the length of the export cables. Those parameters vary from instance to instance.

5.2 Benchmark set generation

We generate benchmarks based on the benchmarks proposed in [LRWW17]. They were used to evaluate NCC algorithm described in [GUW+19]. The benchmarks consist of five sets $N_1, ..., N_5$. They vary in number of turbines and number of substations. All substations of one instance have the same capacity and the capacity tightness (the ratio between the total turbine capacity and total substation capacity) is between 0.87 and 1. To evaluate our algorithm, we have to add the costs for turbines and substations to the benchmarks as well as modify the substation capacities.

For our purpose, it is important that the scale of the wind farm is realistic since we use absolute costs and revenues of turbines. Previous algorithms that were evaluated with these benchmark sets considered only cable costs. Therefore, the scale of the wind farm was irrelevant. To evaluate the correct scaling factor, we compare the length of the shortest edge between two turbines for each instance. For benchmark sets N_1, N_2, N_3, N_4 and N_5 the minimal edge length is about 70. The minimal distance between two turbines is dependent on the turbine size and should be between 5.9 and 7.5 rotor diameters [LLC18]. In our case, that leads to a minimal distance between 708 m and 900 m because we use turbines with a capacity of 3.6 MW and a rotor diameter of 120 m [SIE]. Hence, we scale all benchmarks by a factor of 10.

To determine the costs of vertices, we use the proposed cost model in section 5.1. We generate five variants from one existing benchmark instance to test the effect of selecting only a subset of turbines and substations. For each instance, the cost of every turbine is equal and is calculated based on the cost function for turbines with a capacity of 3.6 MW. Accordingly, the costs of the substation are calculated. We describe the details of the different variants in the following paragraph. An overview of the used substation capacities and export cable lengths is given in table 5.2. If the calculated substation capacity is not a natural number, we round it up to next natural number, which might effect the number of substations and turbines that will be selected.

For the first variant V_1, the substation capacities sum up to half of the total capacity of the turbines. With this variant, we want to analyse how the algorithm behaves if not all turbines should be built.

Total substation capacity of the second variant equals the total turbine capacity to simulate that not all turbines can be built.

The third variant V_3 has twice the substation capacity from V_2. Thus, not all substations need to be build.

The fourth variant has the same capacities as the third, but the cost of connecting the substations to a common connection point are considered. As a consequence, not all substations with equal capacity have the equal costs. We use the distance to the coordinate origin as cable length to the common connection point.

For the last variant V_5, we create instances for which the number of turbines and capacity of substations are chosen as following: First, we connect as many turbines a possible to substations, so those substations operate at full capacity. After that, the number of turbines remaining has to be one, if not, we remove turbines until one remains. This way, we can evaluate for what instances the profit of one turbines and saved costs from shorter cables can refinance an additional substation. Therefore, V_5 has to satisfy the following condition:

$$|V_T| \cdot cap_{turbine} = \beta \cdot cap_{sub} + 1 \text{ with } \beta \in \mathbb{N}_{>0}$$

All substations of an instance of V_5 have the same capacity cap_{sub}, but we choose cap_{sub} at random. The lower bound for the capacity has to be 19 because there are instances with only 20 turbines. The upper bound for the capacity is the minimum of 69 and the number of turbines the instance has minus one. The upper bound of 69 is given by the maximal number of turbines the export cable can collect. Those bounds ensure that we only have

V_i	cap_{sub}	$export_{len}$
V_1	$\lceil \frac{0.5 \cdot \sum cap_{turbine}}{\lvert V_S \rvert} \rceil$	0
V_2	$\lceil \frac{cap_{turbine}}{\lvert V_S \rvert} \rceil$	0
V_3	$\lceil \frac{2 \cdot \sum cap_{turbine}}{\lvert V_S \rvert} \rceil$	0
V_4	$\lceil \frac{2 \cdot \sum cap_{turbine}}{\lvert V_S \rvert} \rceil$	$dist(s_i, (0,0))$
V_5	$cap_{sub} \in [19, min(69, \lvert V_T \rvert - 1)]$	0

Table 5.2: This table shows how we choose the substation capacities for the different variants. Furthermore, we see that we consider the cost of export cables only in variant V_4. Therefore, all other variant have a export cable length of zero. For variant V_4, we use the distance to a common connection point as export cable length.

to remove turbines to full fill the proposed condition. After we choose a value for cap_{sub}, we remove turbines until the condition is satisfied. Variant V_5 can only be calculated for N_2-N_5 since the number of substations is one for all instances of V_1.

5.3 MILP

To evaluate the quality of the solutions calculated by the Extended-NCC, we need to compare it to the optimal solution. We can calculate the optimal solution as a Mixed-integer Linear Program and use a MILP solver. We introduce two MILPs because we evaluate two different objectives. The runtime of these MILPs increase with the size of the instance. We limit the run time of the MILP solver to one day and use the best solution to this point for our evaluation. The MILP solver provides the current upper and lower bound of the optimal solution. These values can be used to determine if the MILP solution is good enough for further evaluations after one day.

MILP **maximize absolute profit**

We minimize the sum of costs from the turbines, substations and cables minus the revenue of the wind farm. Variable y in equation 5.16 represents whether a substation or turbine will be built. Variable x in equation 5.17 defines whether a cable type will be used for an edge. The flow of an edge has to be a integer and is defined in equation 5.18. The constraint 5.19 ensures that if a turbine will be built, the net flow (incoming minus outgoing flow) is the capacity of the turbine, otherwise, the net flow has to be zero. A substation can have a net flow between its capacity and zero if it will be built, otherwise, the net flow has to be zero (see equation 5.20). Flow cannot leave a substation (see constraints 5.21 and 5.22). Only one cable type can be used per edge (see 5.23). The flow of an edge cannot exceed the capacity of the selected cable type, which is ensured in equation 5.24. An edge cannot be use if either of its vertices will not be built (see 5.25 and 5.26).

$$min \sum_{e \in E} \sum_{k \in K} c_k \cdot x(e,k) \cdot cable_{len}(e) + \sum_{v \in V_T} y(v) \cdot (cost_{turbine}(v) - rev_{turbine}(v)) + \sum_{v \in V_S} y(v) \cdot cost_{sub}(v) \quad (5.15)$$

$$y(v) \in \{0,1\} \qquad \forall v \in V_T \cup V_S \tag{5.16}$$
$$x(e,k) \in \{0,1\} \qquad \forall e \in E, k \in K \tag{5.17}$$
$$f(e) \in \mathbb{Z} \qquad e \in E \tag{5.18}$$
$$f_{\text{net}}(v) = (-1) \cdot cap_{turbine}(v) \cdot y(v) \qquad \forall v \in V_T \tag{5.19}$$
$$0 \leq f_{\text{net}}(v) \leq cap_{sub}(v) \cdot y(v) \qquad \forall v \in V_S \tag{5.20}$$
$$f(v,u) \leq 0 \qquad \forall (v,u) \in E : v \in V_S \tag{5.21}$$
$$f(u,v) \geq 0 \qquad \forall (u,v) \in E : v \in V_S \tag{5.22}$$
$$\sum_{k \in K} x(e,k) \leq 1 \qquad \forall e \in E \tag{5.23}$$
$$|f(e)| \leq \sum_{k \in K} x(e,k) \cdot cap_k \qquad \forall e \in E \tag{5.24}$$
$$|f(v,u)| \leq y(u) \cdot max\{cap_k : k \in K\} \qquad \forall (v,u) \in E, k \in K \tag{5.25}$$
$$|f(v,u)| \leq y(v) \cdot max\{cap_k : k \in K\} \qquad \forall (v,u) \in E, k \in K \tag{5.26}$$

MILP minimize costs per installed capacity

To minimize the costs per installed capacity, we define an additional variable u and change the objective to minimizing u (see equation 5.27). The constrain 5.28 ensures that u is greater or equal than the ratio $\frac{cost_{total}}{cap_{total}}$. Therefore, minimizing u minimizes the ratio. The installed capacity cannot be zero. Hence, at least one turbine has to be build (see constraint 5.29). All other constraints 5.16 - 5.26 remain unchanged.

$$\text{minimize } u \tag{5.27}$$

$$\sum_{e \in E} \sum_{k \in K} c_k \cdot x(e,k) \cdot cable_{len}(e) + \sum_{v \in V_T} y(v) \cdot cost_{turbine}(v) + \sum_{v \in V_S} y(v) \cdot cost_{sub}(v) \leq u \cdot \sum_{v \in V_T} y(v) \cdot cap_{turbine}(v) \tag{5.28}$$

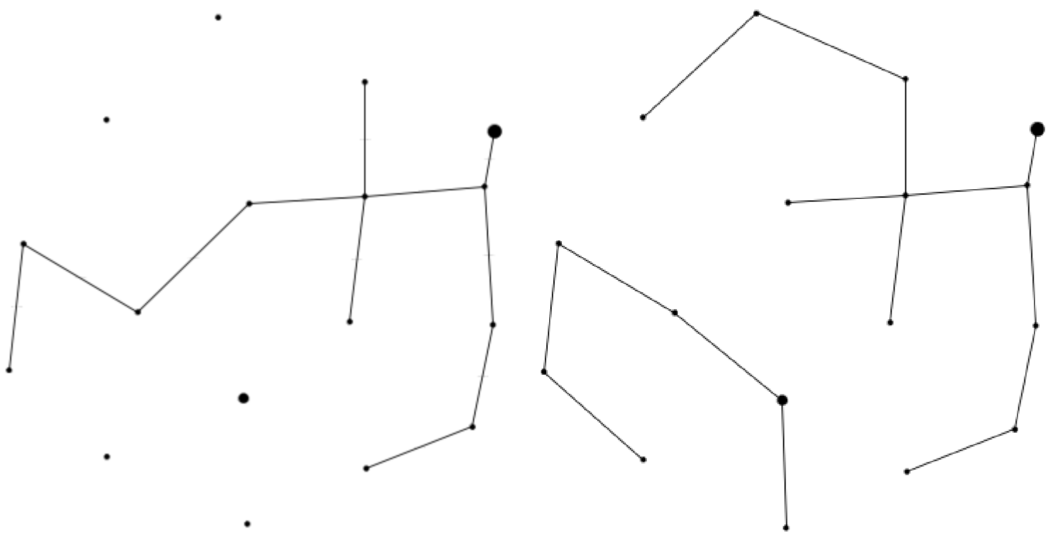
$$\sum_{v \in V_T} y(v) \geq 1 \tag{5.29}$$

5.4 MILP results

To get a better understanding for the solutions of our modifed WCP, we evaluate the solutions computed with the MILP. First, we decided which objective is more promising for our problem, so that we can execute all further experiments with this objective. After that, we select 80 random instances from each benchmark set and generate 20 instances for each variant. We use different instances for V_1, V_2, V_3 and V_5. The variant V_4 uses the same instances as V_3. For each instance, we calculate the MILP solution with a running time of a day. We use those results to evaluate the Extended-NCC, but first, we analyse them based on the runtime, solution quality, and number of turbines and substations selected as well as costs.

5.4.1 Select the best objective

We can optimize the modifed WCP based on the absolute profit or the rate of return. From an economic perspective, both objectives have advantages and disadvantages. If maximizing the rate of return leads to solutions with at high rate of return but very little installed capacity, this objective is not suitable. On the other hand, if we maximize the absolute profit, we need to avoid solutions with a high absolute profit but a small rate of return.



(a) Rate of return (b) Absolute profit

Figure 5.1: This figure shows the flow graph calculated by the MILP with objective maximize rate of return (see 5.1(a)) and with the objective maximize the absolute profit (see 5.1(b)). If we maximize the rate of return, only one substation is chosen. The more suitable solution is calculated if the objective is to maximize the absolute profit.

We computed the MILP solution of both objectives for a few instances and compared the flow graph for both objectives for each instance.

For all tested instances, the objective rate of return leads to solutions where only one substation is selected. The objective absolute profit leads to solutions where sometimes every and sometimes only a subset of substations and turbines is selected. The direct comparison for one instance is displayed in figure 5.1.

Based on these results, it is clear that the better objective is to maximize the absolute profit. A cost-minimal cable layout mostly consists of disjoint trees where each tree has only one substation. To maximize the rate of return, the tree with the best ratio should be chosen. Adding any other tree to the solution increases the ratio and thus, worsens the rate of return. In reality, this result does not match the desired outcome. As a consequence, the rate of return cannot be used for our problem. However, if the number of substations in the solution is given, the objective could still be interesting to determine if the maximal possible number of turbines should be built. In addition, solving a flow problem with a global ratio as costs function could be interesting for other applications. Based on the flow graphs of those tested instances, maximizing the absolute profit seems to lead to the desired outcome, but we still need to determine if this objective leads to a small rate of return for some instances. All further MILP solutions will be calculated with the objective absolute profit.

5.4.2 Runtime and quality

The optimal solution for large wind farm instances cannot be calculated in reasonable time. Therefore, we use the best solution the MILP found after one a day. We need to make sure that the MILP solutions after one day are close enough to the optimal solution to justify using them for our evaluation.

To determine the quality of the MILP solutions, we examine for how many instances the optimal solution could be found within the time limit. Furthermore, the MILP solver provides the current upper and lower bound of the optimal solution. The gap of the upper to lower bound in relation to the currently best solution gives us an understanding how far the optimal solution differs from the current solution. In addition, we evaluate this gap in relation to the cable costs of the current solution to determine how exactly the cable layout of the current solution represents the cable layout of the optimal solution.

From all 480 MILP instance, only 44 instances finished within a day. All of them belong to either test set one or two which contain the smallest instances. This subset of instances with an optimal solution contains instances from all variants. The exact distribution is displayed in figure 5.4.2 and shows that the MILP could not calculate the optimal solution for almost all instances. Figure 5.3(a) displays the distribution of the gap between the upper bound and lower bound of the optimal solution provided by the MILP solver in relation to the best provided solution. The solution of most instances is within 2% of the optimal solution. Only a few instances from variant V_4 and V_5 differ up to 4%. Figure 5.3(b) shows the gap in relation to the cable cost of the current best solution. The gap between upper and lower bound of the optimal solution can account for up to 60% of the cable costs.

In comparison to the scale MILP solution, this gap is small and emphasises that we can use the solutions to evaluate the Extended-NCC. However, it is important to note that, while the absolute profit might be close to the optimal solution, other aspects of the solution might not even be near the optimal solution. The cable costs are only a fraction of the total wind farm costs. The gap between upper and lower bound is small compared to the total profit but not compared to the cable costs. Therefore, cable costs and other cost factors that make up only a fraction of the absolute profit have to be used with caution in the evaluation.

5.4.3 Turbine and substation selection

To get a better understanding of the effects of selecting only a subset of turbines and substations, we analyse the MILP solution based on how many substations and turbines will be built. We use those results to determine if selecting a subset of turbines and substations should be optimized simultaneously with the cable layout.

For each instances, we examine how many possible turbines and substations this instance has and what percentage of them is selected in the MILP solution. Furthermore, we divide the number of built turbines by the substation capacity and round the solution up to get the minimal the number of substations needed to collect all turbines. We subtract this value from the number of substations built to get the number of additional substations. Since the total substation capacity varies from variant to variant, we evaluate all variants separately.

Figure 5.4(a) shows the percentage of turbines built and figure 5.4(b) the percentage of substations built for all instances of the five different variants. For the first variant, 100% of all substations and between 50-60% of all turbines are selected. For variants V_2 and V_3, all instances select 100% of the turbines. Variants V_4 and V_5 have instances where at least one turbine is cancelled, but the majority of instances selects all available turbines. All except a few instances of V_2 select all substations as well. For variant V_3 and V_4 the distribution is very similar but not equal. Between 50% and 100% of all substation are built. The distribution of the percentage of substations built shows no specific pattern for variant V_5. Figure 5.4.3 shows how many additional substations are built per instance and variant. In V_3-V_5, a few instances build additional substations. The other variants select no additional substations.

In variant V_1, the total substation capacity is about half the total turbine capacity. The obvious solution would be to select all substations and the maximal possible number of

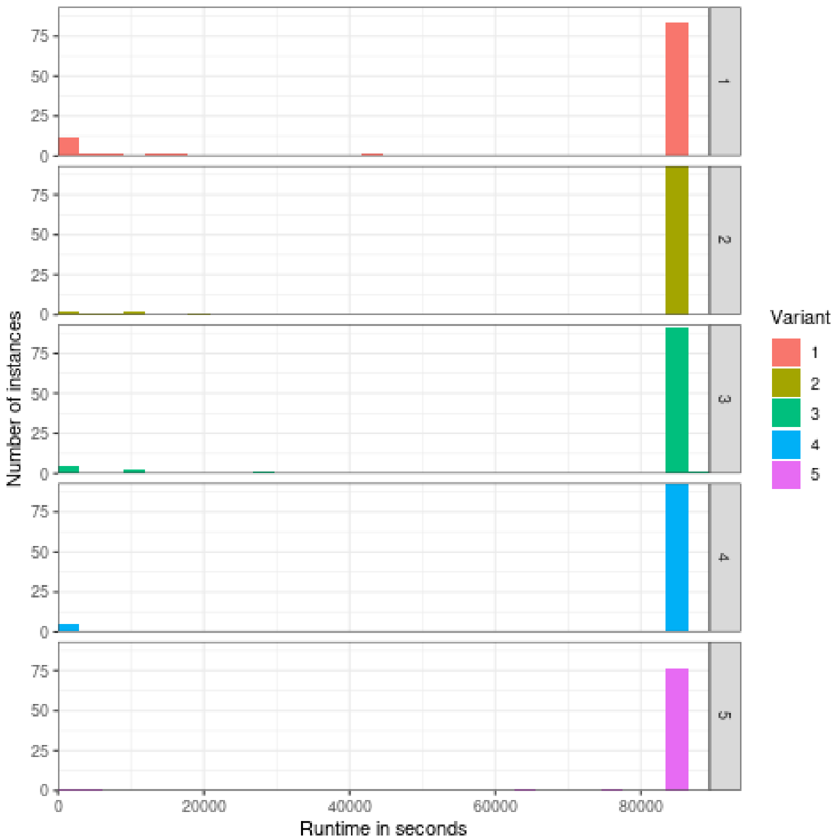
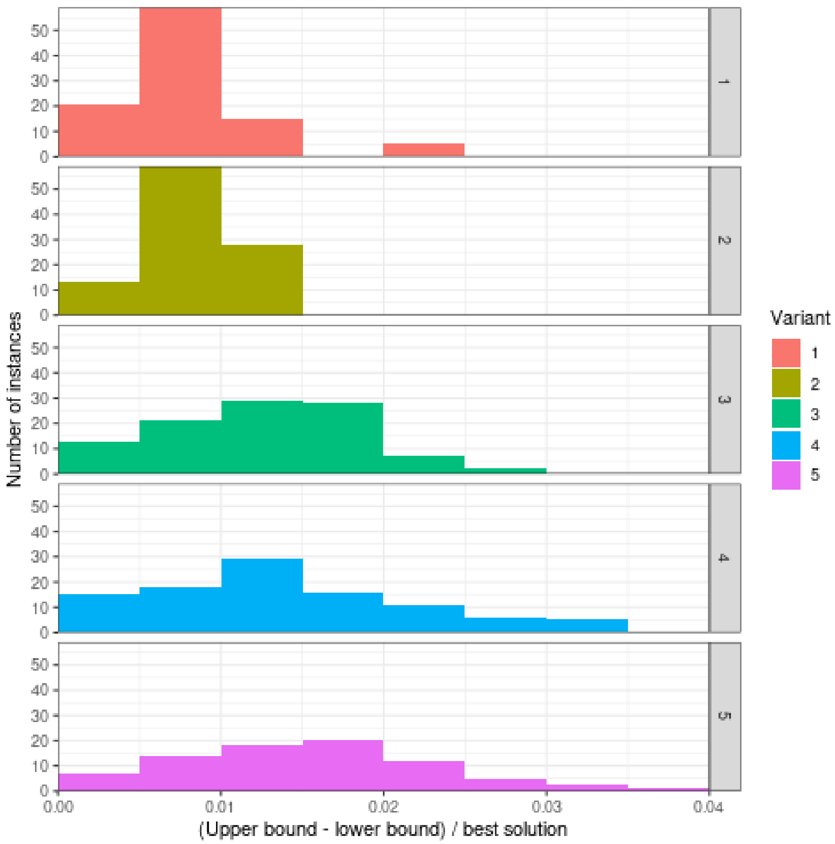
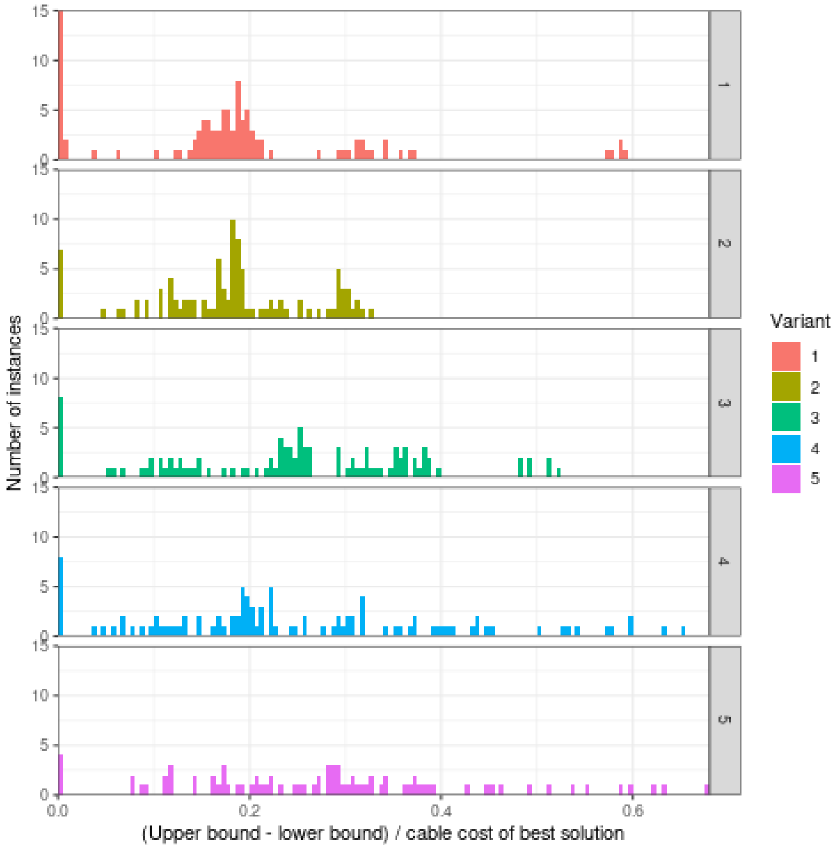


Figure 5.2: This figure displays the distribution of the runtime of the MILP with a time limit of one day for the five different variants. The runtime is given in ms.

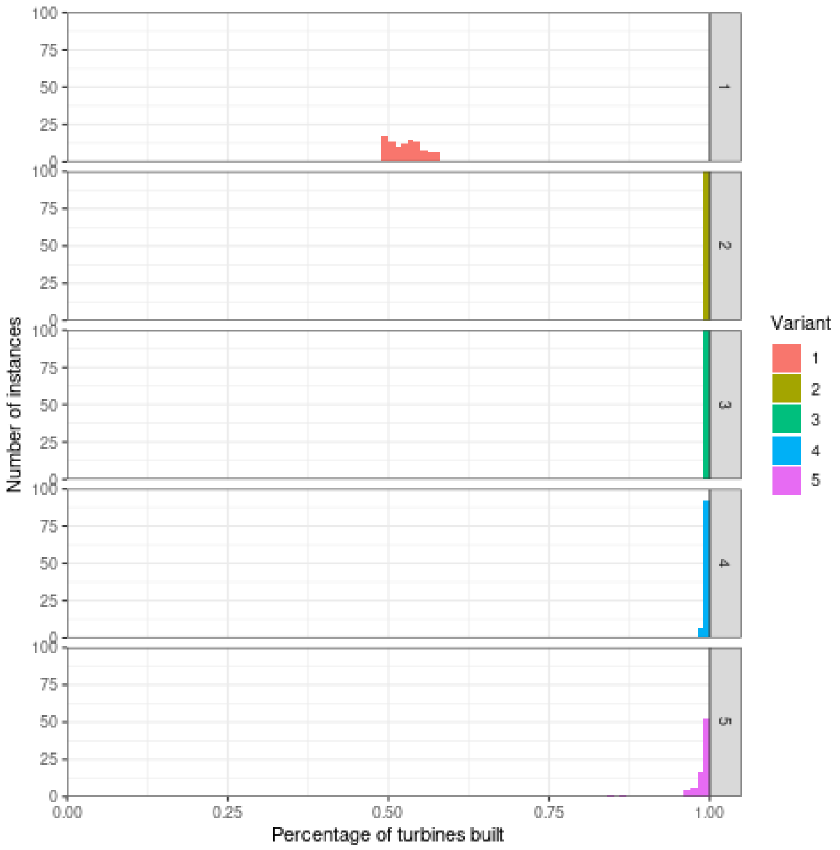


(a) Gap in relation to the absolute profit of the best MILP solution

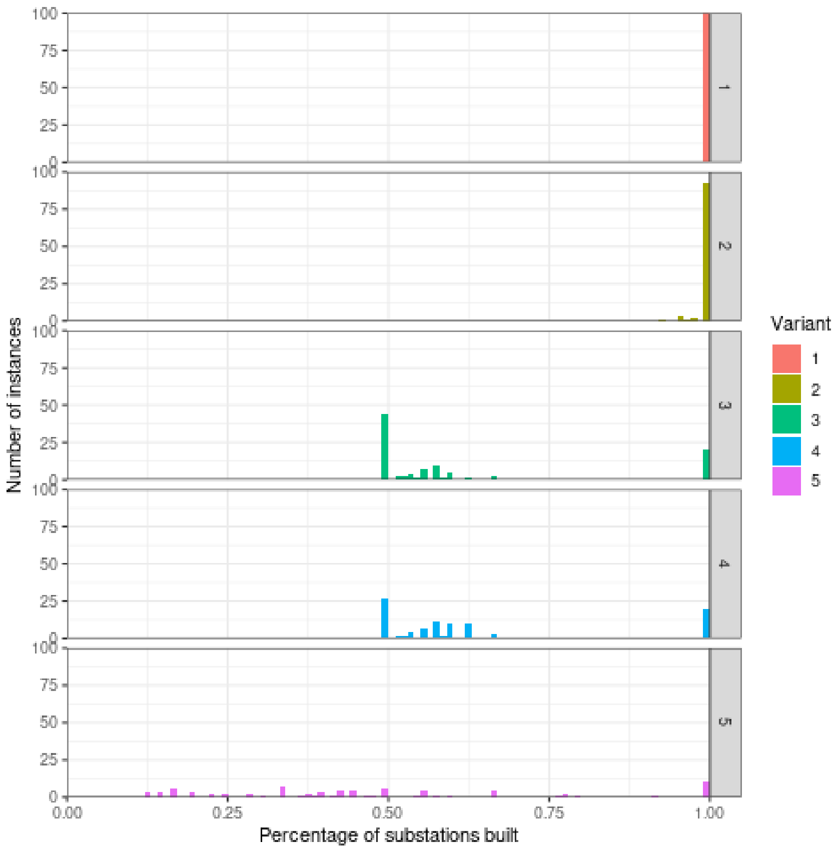


(b) Gap in relation to the cable cost of the best MILP solution

Figure 5.3: Figure 5.3(a) shows the gap between upper and lower bound of the optimal solution in proportion to the absolute profit of the MILP solution after one day. The second figure 5.3(a) shows this gap in proportion to the cable costs of the MILP solution.



(a) Turbines



(b) Substations

Figure 5.4: This figure shows what percentage of the possible turbines (see fig. 5.4(a)) and what percentage of possible substations (see fig. 5.4(b)) is selected in the MILP solution. We display at each variant separately.

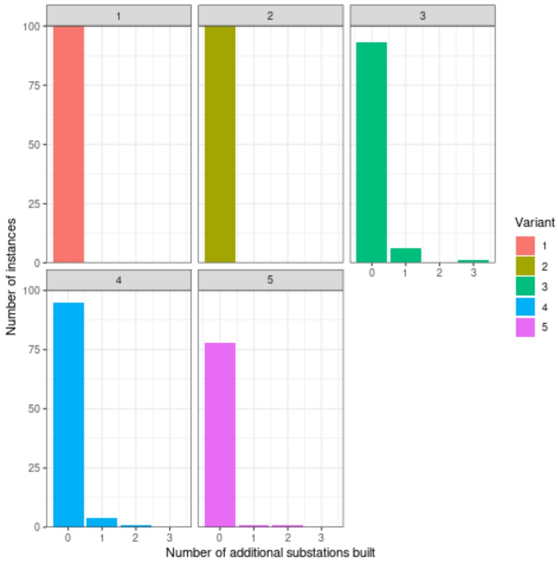


Figure 5.5: This shows the number of substations built more than necessary to collect all selected turbines.

turbines. Those instances with a percentage higher than 50% can be explained by the calculation process of the substation capacity. If the calculated value is not an integer value, the capacity was rounded up to the next integer. The summed up capacity of all built turbines equals the built substation capacity for all instances of V_1, which supports this hypothesis.

The total substation capacity of instances in V_2 matches the total turbine capacity or is slightly greater. Therefore, the expected results would be that all turbines and as much substations as needed to collect all turbines will be selected. There are a few instances where not all substations are selected. This, again, is caused by rounding up the substation capacity when generating the benchmark sets. Depending on the number of turbines and substations, rounding the capacity of the substation to the next integer can result in instances in V_2 where the capacity of all expect one substation is enough to collect all turbines. This explanation can be supported by the fact that no additional substation was built for any instance of V_2.

The third variant has twice as much the substation capacity than turbine capacity. The expected result would be that all turbines and only as many substations as necessary will be built. The instances where slightly more substations than 50% are selected could be explained by rounding the capacity. If the number of turbines is not divisible by the number of substations, at least one of the selected substation has free capacity left. However, figure 5.4.3 shows that some additional substations are selected for V_3. There are also 20 instances where all substations are built. These are caused by the instances generated from test set one. In this test set, each instance has only one substation. Therefore, all substations have to be built if any turbine is collected.

The fourth variant uses the same instances as the third only the costs of substations are modified to represent the cost of connecting the substation to a connection point. Therefore, all substations have the same capacity, but not all substations have the same costs. If those costs were irrelevant to determine how many turbines and substations should be built, the results would be the same as for variant V_3. However, the experiments showed that this is not the case. For all instances, at least half the substations were built. Although, the selected substation have enough capacity to collect all turbines, there are instances where not all turbines were built. These results suggest that depending on which substations we use, there can be turbines for which the cable costs exceed the profit of that turbine and the substation costs induced by this turbine. For this variant, less additional substations are selected than in V_3. This can be explained with the higher substation costs of this instance.

With the fifth variant, we want to evaluate if there are instances where not all substations and not all turbines will be built. Since the substation capacity is chosen at random, the percentage of substations built cannot provide any information. The distribution of the percentage of turbines built shows that there are instances where not all turbines will be built. Whether those instances correspond to instances where not all substations will be built requires further analysing. For almost all instances of this variant where exactly one turbine was not built, at least one substation was not built as well. Otherwise, we would have more instances with additional substations.

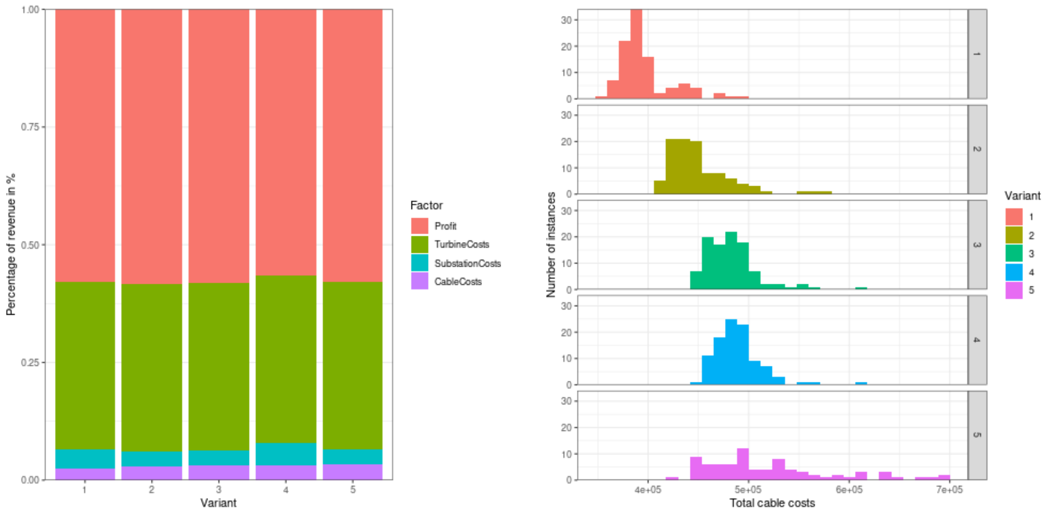
5.4.4 Costs

To get a better understanding of the solution provided by the MILP, we analyse its costs. The largest cost factor are the turbines. However, each turbine generates a revenue. Especially interesting is the question whether higher substation costs can be compensated by lower cable costs.

We calculate the percentage of profit, turbine costs, substation costs and cable costs for each instance. After that, we calculate average values for each variant. Furthermore, we take a closer look at the cable costs by calculating the cable costs per installed capacity for each instance. For variant V_5, we separate all instances into two groups. The first group contains all instances where all turbines are selected. The second group contains all instances where at least one turbine was cancelled.

Figure 5.6(a) shows the average division of the revenue of a wind farm instance used for profit, turbine costs, substation cost and cable costs for variants V_1-V_5. It shows that this division is very similar for V_1-V_3 and V_5. The variant V_4 uses a higher percentage for substations on average. Figure 5.6(b) shows the cable costs per installed capacity. The cable costs for variant V_1 are the smallest and the cable costs for V_5 vary the most within the instance. The cable costs for instances of V_5 where no turbine was cancelled are slightly smaller than for those instances where at least one turbine was cancelled and the substation costs for instances where no turbine was cancelled, are higher than for those instances where at least one turbine was not selected.

Variant V_4 has higher substation costs since the cost for export cables are considered for those instances. The variant V_5 is designed in a way that if the last turbine is built the substations still have remaining capacity. The revenue of one turbine is not enough to compensate the costs for that substation if the substation can collect more than 22 turbines. The question is whether the cable costs that will be saved by this additional substation can compensate the difference. This differs from instance to instance. Figure 5.7(a) suggest that if the last turbine and the last substation are built, the substation costs are higher and the cable costs lower. This would support the hypothesis that the last turbine is built if the saved cable cost can compensate additional substation costs left after subtracting the turbine profit of that last turbine. However, those results have to be treated with caution since the MILP solution is not optimal. The other possible factor to decide whether that last turbine will be built, is the cost of an additional substation which depends on the substation capacity. If the cable cost were irrelevant for determining whether the turbine will be built, there would be a threshold capacity. From this threshold on the substation would cost much more than the profit of one turbine that the the profit of the additional turbine could not compensate the substation. All instances with a substation capacity larger than that threshold would not built the additional turbine. However in figure 5.7(b),



(a) Percentage of revenue used by the cost factors (b) Cable costs per installed capacity

Figure 5.6: This figure provides an overview of the costs of the MILP solutions. For each variant, the left figure 5.6(a) shows what percentage of the revenue is used for profit, substation costs, turbine costs, and cable costs. For each instance and variant, the right figure 5.6(b) shows the ratio cable costs per installed capacity.

we see that there is no substation capacity threshold that determines if the last turbine will be built. This suggests that it is useful to optimize the substation and turbine selection at the same time as optimizing the cable layout at least for instances that are similar to those in V_5. On the other hand, instances that are created in a way to intentionally force a lot of free substation capacity if no turbine is cancelled might not be applicable in reality. A more practical solution for those instances would be to reduce the substation capacity.

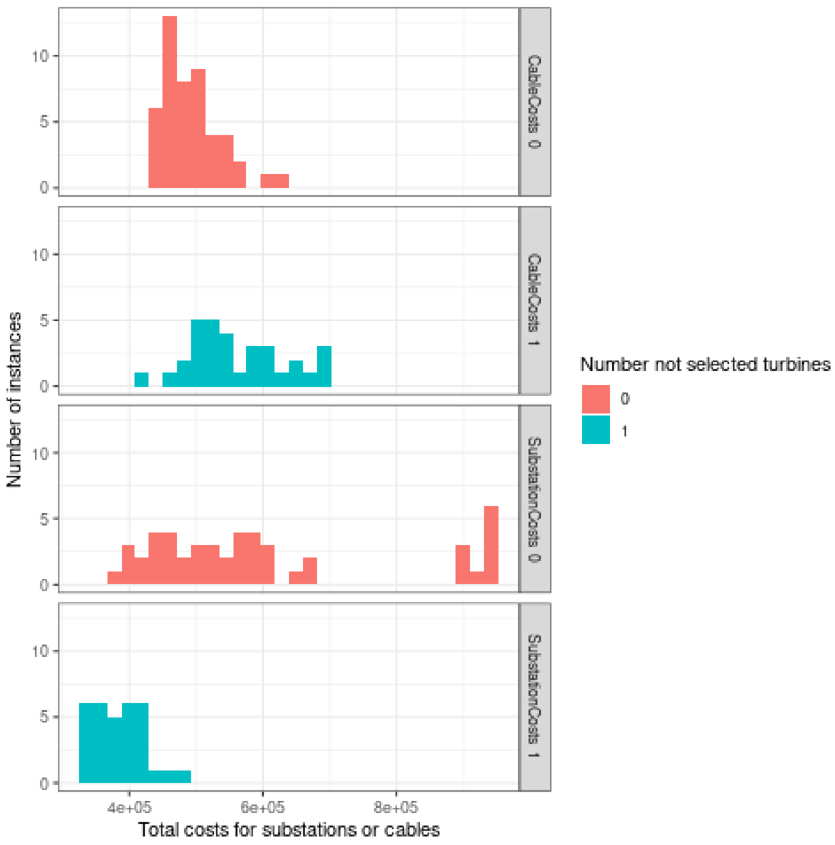
5.5 Extended-NCC

To evaluate our Extended-NCC, we compare its solutions to the MILP solutions for the same instances. Since we can run the Extended-NCC with different configurations, we run the Extended-NCC three times for each instance: Initialisation strategy FILL with escaping strategy FREESUB (FILL-FREESUB), initialisation strategy ANY with escaping strategy FREESUB (ANY-FREESUB) and initialisation strategy FILL without any escaping strategy (FILL-NOESC). We analyse the results based on runtime, number of turbines selected and number of substation selected, profit, and costs. We compare those factors to the MILP solution and among the different Extended-NCC configurations.

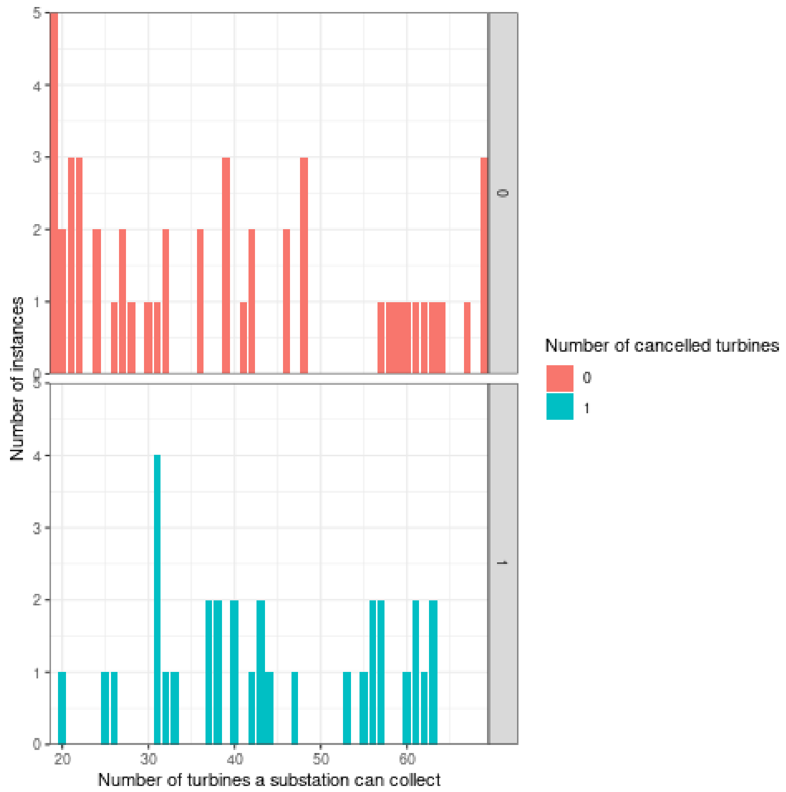
5.5.1 Runtime

The main advantage of the Extended-NCC is that it is faster than calculating the optimal solution. In this section, we examine which configuration is faster for which instances.

To analyse the runtime of the different configurations, we first compare the total runtime, the runtime of the initialisation strategy and the runtime after the initial flow is found for the three configurations. To get a better understanding which initialisation strategy has the better runtime, we compare the ratio of the runtime of the configuration FILL-FREESUB to the runtime of the configuration ANY-FREESUB for each instance and analysed them separately for each variant. This shows us which percentage of the runtime of the configuration ANY-FREESUB the configuration FILL-FREESUB needs. Again, we compare

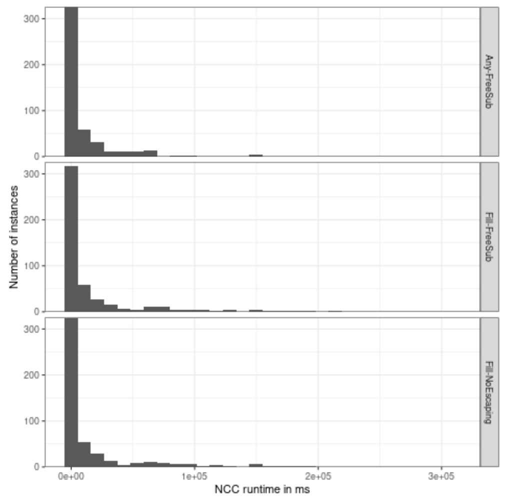


(a) Substation and cable costs for V_5

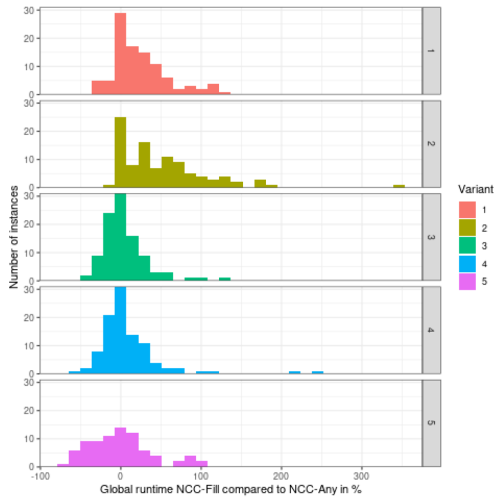


(b) Substation capacity for V_5

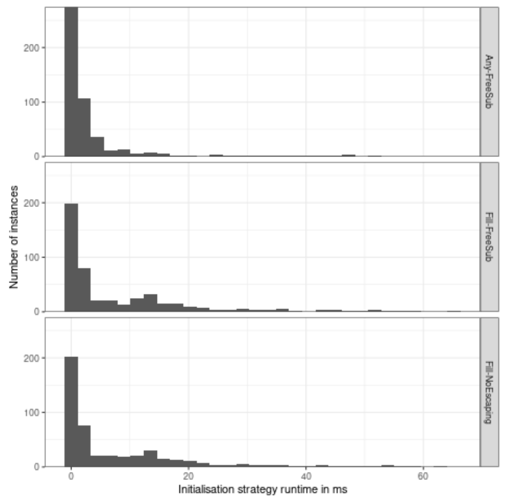
Figure 5.7: In this figure, we separated the instances of V_5 into two group based on whether at least one turbine is not selected. Figure 5.7(a) shows the distribution of the substation and cable cost for those two groups. The second figure 5.7(b) shows how the substation capacity is spread among those two groups.



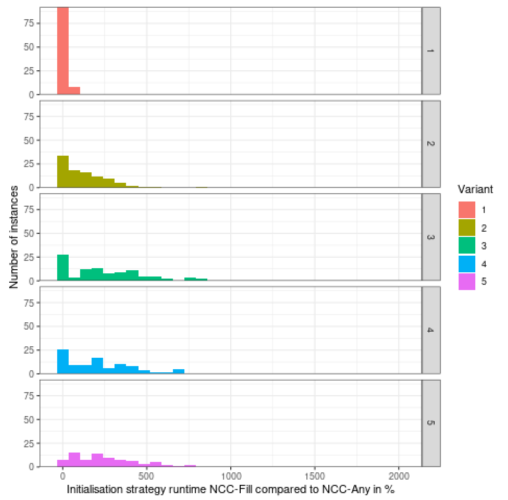
(a) Total runtime



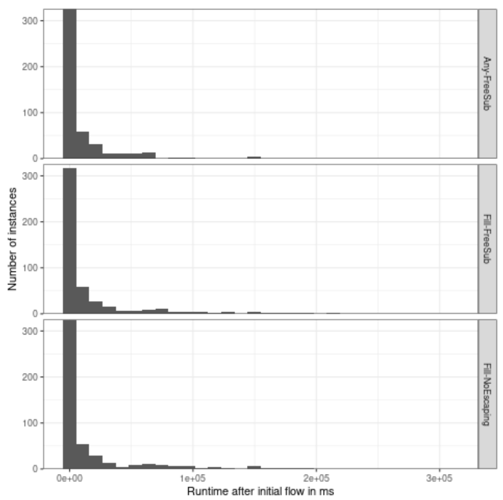
(b) Total runtime comparision



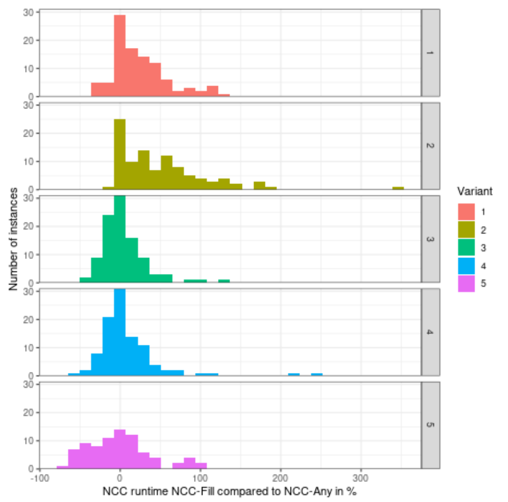
(c) Runtime initialisation strategy



(d) Runtime initialisation strategy comparision



(e) Runtime after initial flow



(f) Runtime after initial flow comparision

Figure 5.8: For the three configurations of the Extended-NCC, the three left figures show the total runtime, the runtime of the initialisation strategy and the runtime after the initial flow is found. The figures on the right side compare those runtimes for the configuration FILL-FREESUB and ANY-FREESUB. Those figures show the runtime of the configuration FILL-FREESUB divided by the runtime of corresponding instance of the configuration ANY-FREESUB.

this aspect for the total runtime, the initialisation strategy runtime, and the runtime after an initial flow is found.

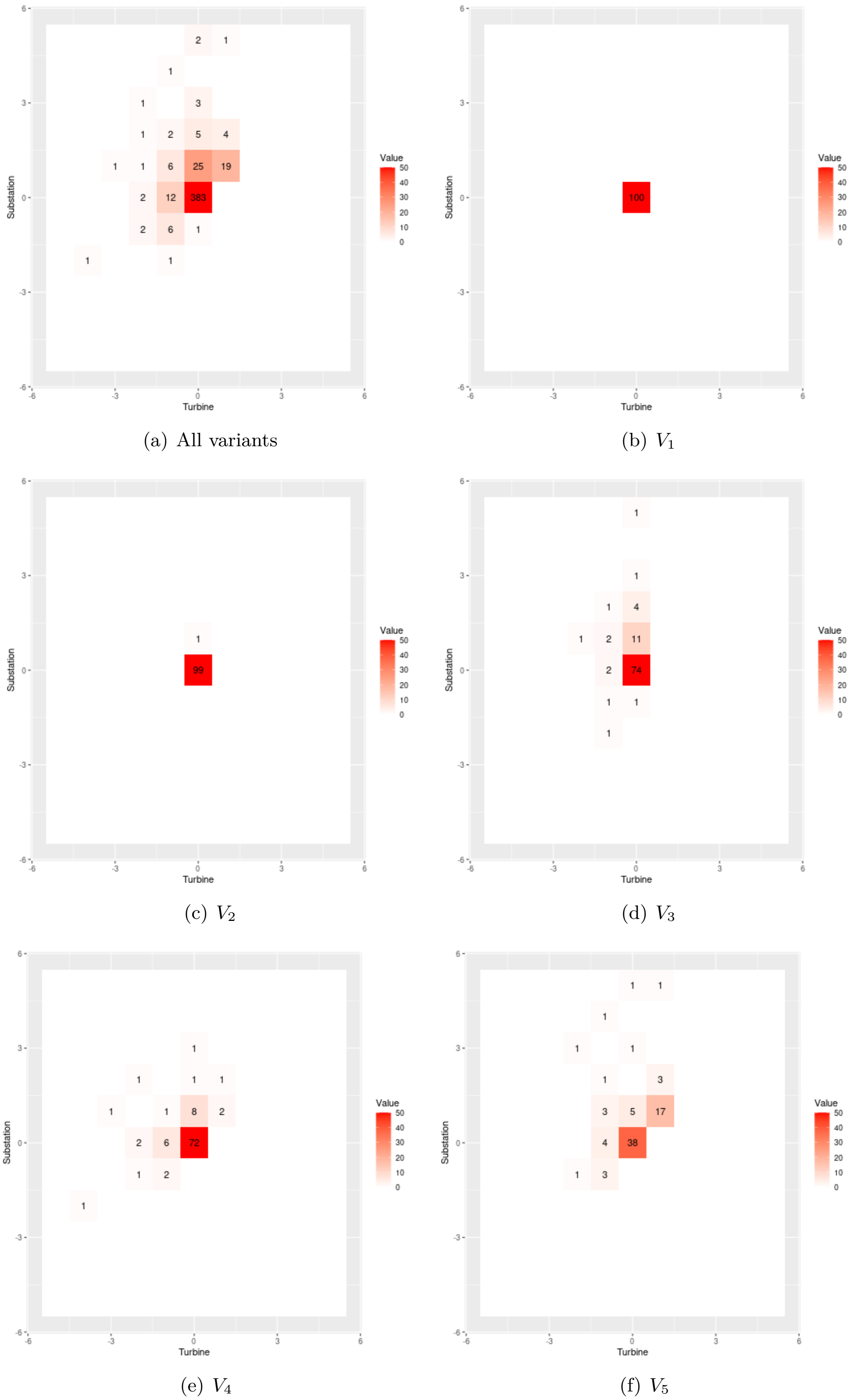
All configurations of the Extended-NCC are faster than computing the MILP solution. Most instances finished within one minute. On the first look, the total runtime for each configuration is very similar for the three configurations (see 5.8(a)). Most instances finish within a minute and some instance need two to three minutes. The runtime after the initial flow is found can be seen in 5.8(e). It shows almost the same results as the total runtime. The runtime for the initialisation strategy is much shorter. Figure 5.8(c) shows that all initialisation strategies finish within milliseconds. Since the initialisation strategy FILL is used in two configurations, those runtimes for the initialisation strategy are almost identical. The initialisation strategy ANY is slightly faster. If we compare the runtimes of the configuration FILL-FREESUB directly with the configuration ANY-FREESUB, we see in 5.8(d) that the initialisation strategy ANY is faster or at least equally fast as than the strategy FILL. The initialisation strategy FILL takes for some instance five times as long as the initialisation strategy ANY. The total runtime compared in figure 5.8(b) and the runtime after the initial flow, which is displayed in figure 5.8(f), are almost identical. For variant V_1 and V_2, the initialisation strategy ANY is faster for almost all instances. For variants V_3-V_5, initialisation strategy FILL has sightly better results but there are also a lot of instances where initialisation strategy ANY results in a better runtime.

The runtime of the initialisation strategy is only a fraction of the total runtime. Therefore, the total runtime and the runtime after the initial flow was found are almost identical. The runtime of most instances is less than a minute which is a significant improvement towards the MILP runtime. It is also notable that the runtime of the original NCC was on average shorter, but in similar magnitude [GUW$^+$19] but we have to consider that while we based our benchmark on the same benchmarks that were used for the evaluation of the original NCC, we modify them. Therefore, the runtime can only be compared with caution between those algorithms. However, since we add additional edges for the super turbine to the residual graph and implement an escaping strategy it makes sense that our Extended-NCC has a longer runtime. The initialisation strategy FILL takes longer than ANY for all instances because in the initialisation strategy ANY `Dijkstra`'s algorithm terminates at the first feasible substation but the initialization strategy FILL needs to find the path to a specific substation. However, this is irrelevant for the total runtime since both initialisation strategies run only for a fraction of the total runtime. More interesting is how the initial flow effects the runtime of the rest of the algorithm. The Extended-NCC algorithm with the configuration ANY-FREESUB performs better for variant V_1 and V_2. The strategy ANY connects the turbines to the nearest substation. Therefore, for almost all instances in V_1 and V_2, the initial flow uses all substations. As a consequence, the initial flow resembles the solution based on the selected substations and turbines. Furthermore, less negative cycles have to be cancelled to achieve the same solution. For the other variants V_3-V_5, the better runtime varies from instance to instance.

5.5.2 Turbine and substation selection

The largest cost factor are the number turbines and substations. Therefore, a good indicator for the quality of the solution is whether the number of selected substation and turbines resembles the MILP solution. However, there could be solutions with almost the same profit but less turbines and less substations or more turbines and more substations.

We compute how many turbines and how many substations the Extended-NCC selected more than the MILP solution for each instance and configuration. To see if selecting less turbines results in less substations, we analyse them together by creating a heat map, which shows how many instances have the exact same number of substations and turbines selected more than the MILP solution.



(a) All variants

(b) V_1

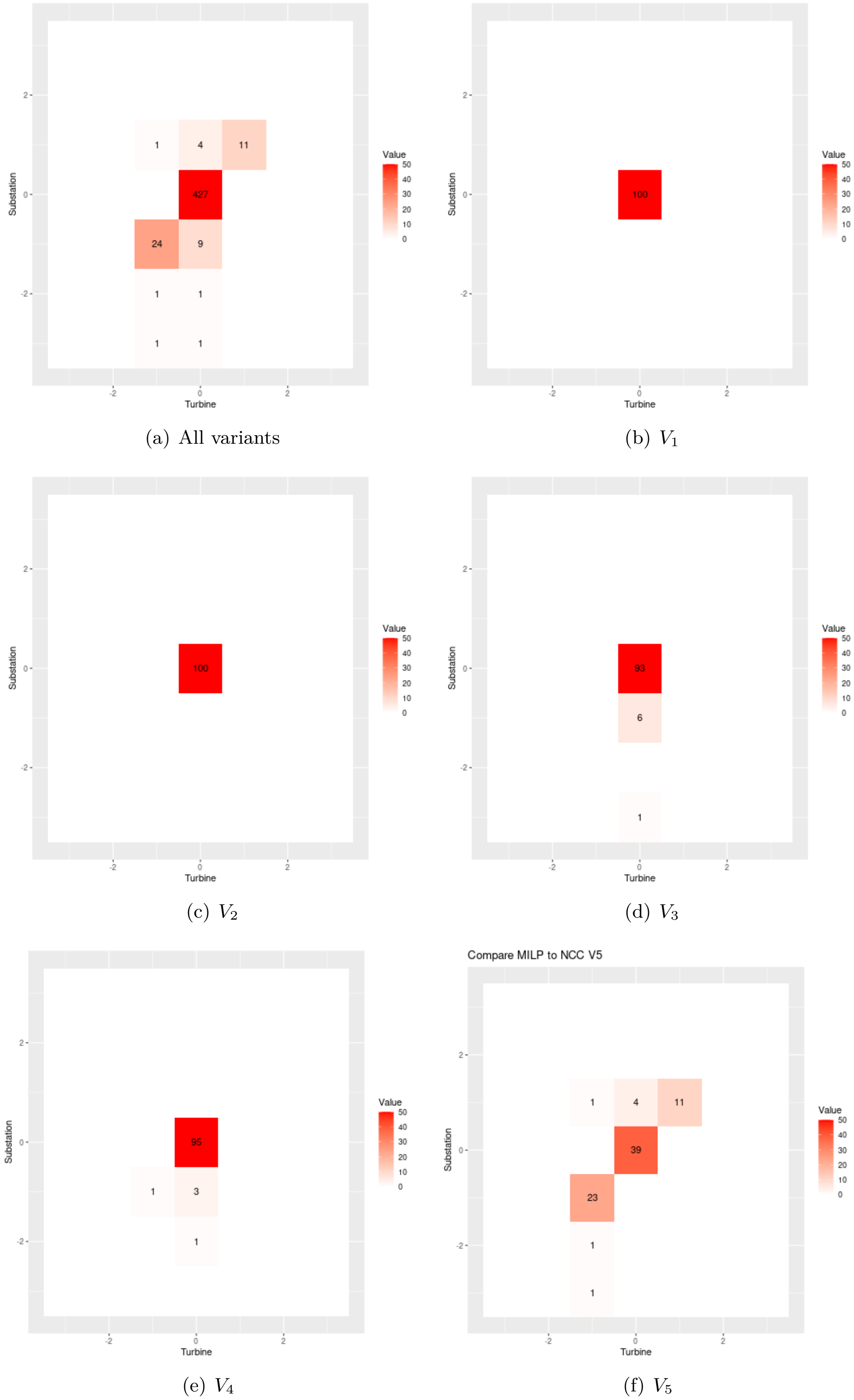
(c) V_2

(d) V_3

(e) V_4

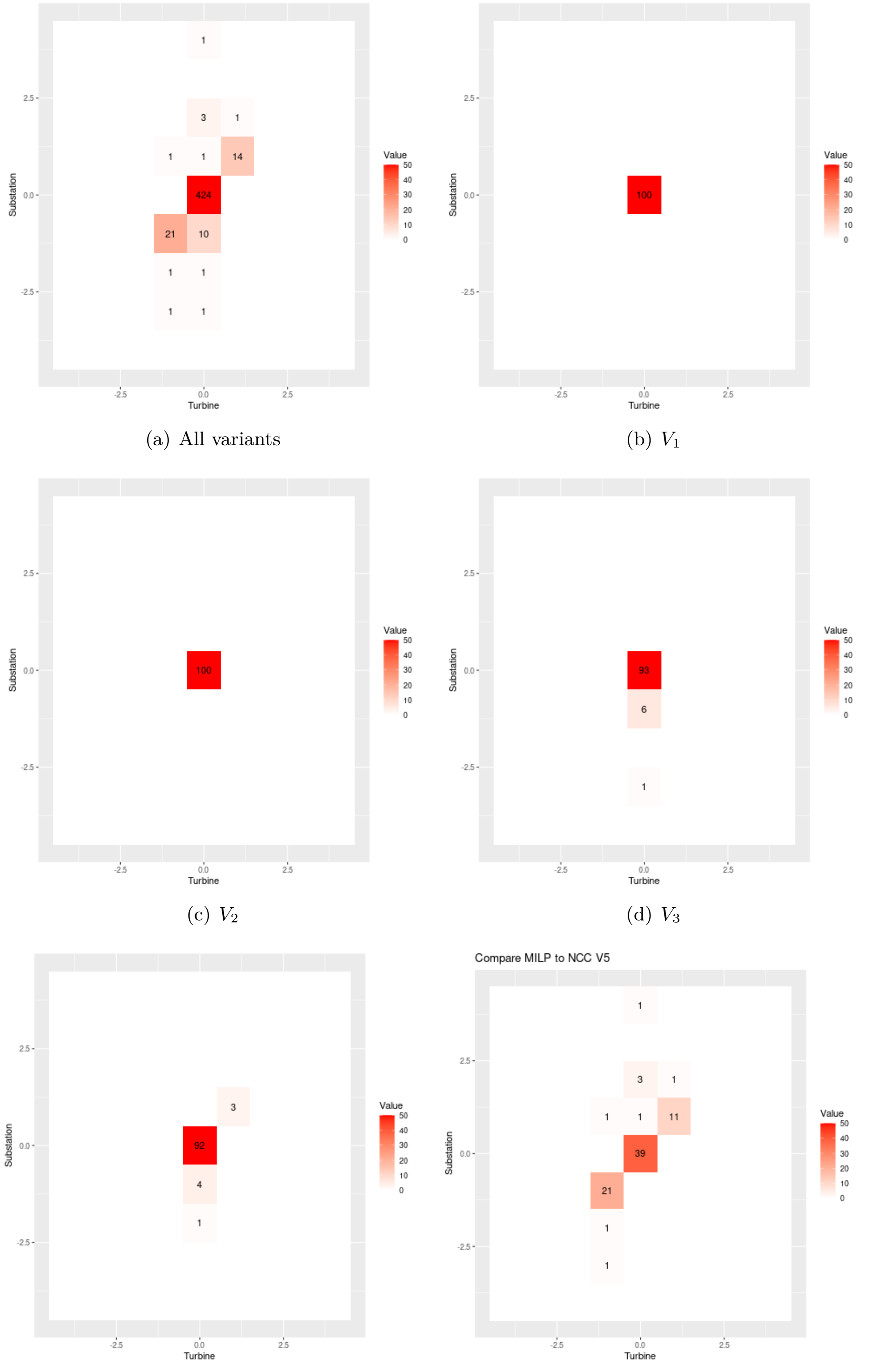
(f) V_5

Figure 5.9: These graphs show the distribution of how many turbines and substation the Extended-NCC configuration ANY-FREESUB selects more than the MILP solution of the same instances.



(a) All variants
(b) V_1
(c) V_2
(d) V_3
(e) V_4
(f) V_5

Figure 5.10: These graphs show the distribution of how many turbines and substation the Extended-NCC configuration FILL-FREESUB selects more than the MILP solution of the same instances.



(a) All variants (b) V_1

(c) V_2 (d) V_3

(e) V_4 (f) V_5

Figure 5.11: These graphs show the distribution of how many turbines and substation the Extended-NCC configuration FILL-NOESC selects more than the MILP solution of the same instances.

Figure 5.5.2 shows how many turbines or substations the configuration ANY-FREESUB built more than the MILP solution of the same instances. This configuration selects on average more substations than the MILP solution. For the variant V_1 and V_2, the same amount of substations and turbine as in the MILP solution is selected for all instances except one. The largest differences occur in variant V_5. Figure 5.5.2 shows the same diagrams for the results of configuration FILL-FREESUB. It shows that this initialisation strategy has better results for selecting the same amount of turbines and substations as the MILP solution. On average, this configuration selects less substations than the MILP solution and builds only one turbine too much or too little. If we compare the results of the configuration FILL-FREESUB to the results of the same initialisation strategy without any escaping strategy, we see in figure 5.5.2 that the results are very similar but the results from the configuration with our escaping strategy are slightly better.

The configuration ANY-FREESUB selects all substation for the initial flow. The previously described results suggest that there are substation whose flow cannot be shifted to other substations even if they have enough capacity. Therefore, on average, more substations than in the MILP solution are built. For the configuration FILL-FREESUB, the number of selected turbines and substations is already close to the MILP solution after an initial flow is found. The last substation added to the initial flow is the only substation with free capacity. Especially, if only one turbine is collected by this substation like it is the case for instances of V_5, cancelling this substation can be performed with one regular cycle. This explains why on average this configuration builds less substations than the MILP solution. If we compare the configuration FILL with and without the escaping strategies, we can see the effect the escaping strategy has on the solution. The variants V_1-V_3 show no effect but both variants V_4 and V_5 have instances that were improved by the escaping strategy. Those results suggest, that the escaping strategy can improve the solution, but its effect has limitations. In general, the solution is closer to the MILP solution if the initial flow is closer to the expected solution.

5.5.3 Cost

While the number of selected turbines and substations is a good indicator whether the solution is close to the MILP solution, the crucial factor is whether the profit of the wind farm is close to the profit of the MILP solution.

We compare about how many percent the profit of the Extended-NCC solutions is less or more than the corresponding MILP solution. We analyse this separately for all configurations and variants. Furthermore, we compare the profit of the configuration FILL-FREESUB directly to the profit of configuration ANY-FREESUB and examine for which instances in which variant which configuration generates more profit or whether the profit is equal for both configurations. We do not consider the results of the configuration FILL-NOESC since the profit of this configuration cannot be better than the results of the same initialisation strategy with escaping strategy because the escaping strategy is first applied when no negative cycles can be found.

Figure 5.12(a) shows about how many percent the profit of the Extended-NCC solution with configuration FILL-FREESUB differs from the MILP solution. Most instances have less than 2% less profit than the MILP solution. For variants V_1-V_3, almost all instance differ less than 1% from the MILP solution. There are instances from all variants that generate more profit than the MILP solution. Figure 5.12(b) shows the same diagram for the configuration ANY-FREESUB. On the first look, both configurations provide similar results. However, the configuration ANY-FREESUB has slightly more instances for which the profit is more than 2% worse than the MILP solution but seems to perform better for variant V_1 and V_2. Figure 5.5.3 shows about how many percent the Extended-NCC solution with configuration

FILL-NOESC differs from the MILP solution. The results without escaping strategy show a similar distribution as the results of FILL-FREESUB but there are some instance that are more than 7% worse than the MILP solution whereas the worst solutions for configuration FILL-FREESUB are only 5% worse and the worst solutions for configuration ANY-FREESUB are only 6% worse than the MILP solution.

Those results show that the Extended-NCC does provide solutions similar to the MILP solution in regards to the generated profit. The variants V_1 and V_2 are closer to the MILP solution if calculated with the configuration ANY-FREESUB. Equivalently to the explanation for the shorter runtime of this configuration and those variants, this can be explained by the closeness of the initial flow to the MILP solution. Especially, if we compare both initialisation strategies instance for instance, we can see that the configuration ANY-FREESUB is the better choice for V_1 and V_2. However, there are also instances for which the configuration FILL-FREESUB achieves better solutions for both variants. For the other variants V_3-V_5 the preferable configuration is not as clear but especially for V_4 and V_5 the configuration FILL-FREESUB has the slightly better outcome. If we compare the configuration FILL-FREESUB with the configuration without an escaping strategy, we see that particularly the solutions that differ most from the MILP can be improved by the escaping strategy. This emphasizes that the escaping strategy FREESUB has the ability to improve the solution and should be applied especially since we could not observe a strong influence of this escaping strategy on the total runtime of the Extended-NCC. However, those results show as well that the Extended-NCC in its current implementation can get stuck in local minima. This problem seems to have the most impact for instances where we do not know in advanced how many substations and turbines we should select for the optimal solution. Unfortunately, those are the most interesting instances for this algorithm. All in all, the cost analysis shows that the best results are achieved if the initial flow is close to the optimal solution and if the escaping strategy FREESUB is applied. For instances for which the optimal number of substations and turbines is unknown in advance, the escaping strategy does help to improve the solution but could be further improved in the future to achieve better results for those difficult instances.

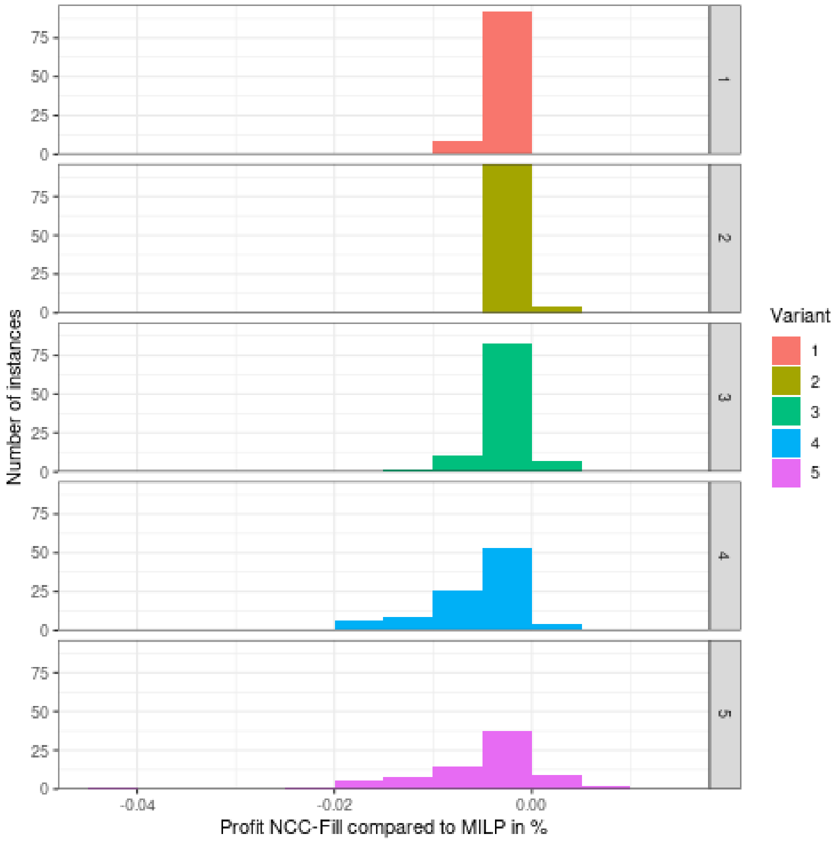
5.5.4 Escaping strategy FREESUB

We already saw in the previous sections that the escaping strategy improves the solution. Now, we evaluate how often the escaping strategy is successful, to determine if the escaping strategy should be used.

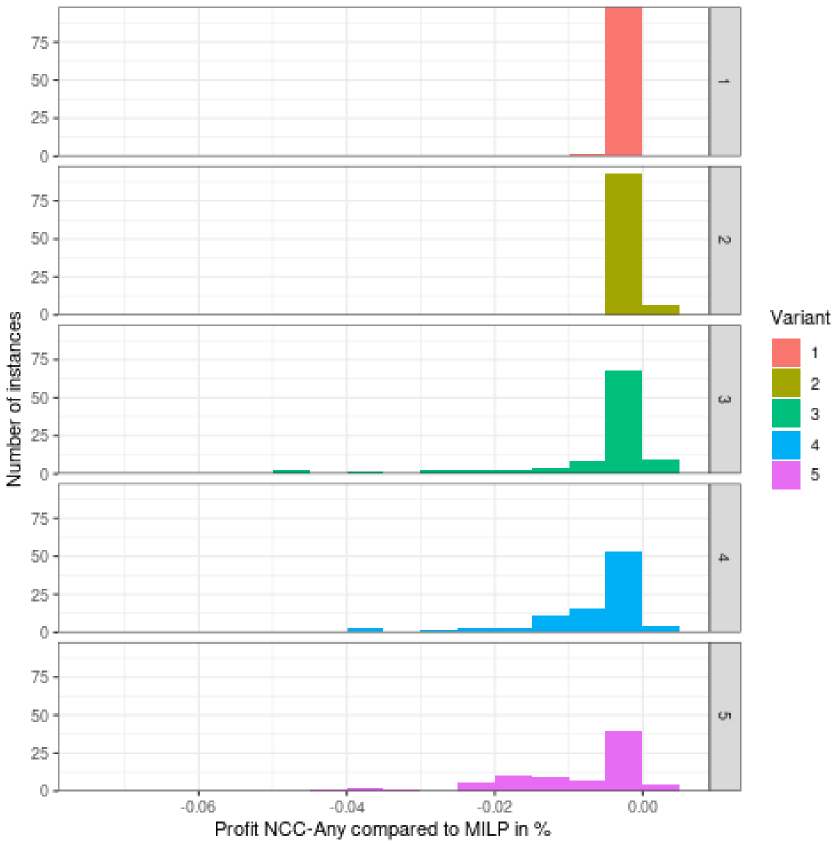
We log how many negative combined cycles the Extended-NCC finds for each instance. Therefore, how often the escaping strategy is used to improve the solution. We analyse the number of combined cycles based on the configuration and variant we use.

Figure 5.15(a) shows how many combined cycles are cancelled by the Extended-NCC for the configuration FILL-FREESUB. It shows that the escaping strategy does not improve the solution for V_1-V_3. For the variant V_4 and V_5, a few instances found one or two combined cycles. Figure 5.15(b) shows the same diagram for the configuration ANY-FREESUB. In general, the escaping strategy is successful more often for this configuration. For V_3-V_5, less than a quarter of all instances find no combined cycles and there are even instances with more than ten negative combined cycles. In V_2 only a few instances and in V_1 no instance found any combined cycles.

For instances from V_1 and for most instances of V_2, no combined cycle can be found since the initial solution uses all substations in both initialisation strategies. Freeing any of those substations is only possible if all turbines collected by this substation are cancelled. This however does not improve the solution in most cases since the profit of those turbines exceeds the substation and cable costs in most cases where the substations operate almost



(a) FILL-FREESUB



(b) ANY-FREESUB

Figure 5.12: The difference of profit from the Extended-NCC solution to the MILP solution in relation to the profit of the MILP solution is shown in figure 5.12(a) for the configuration FILL-FREESUB and in figure 5.12(b) for the configuration ANY-FREESUB.

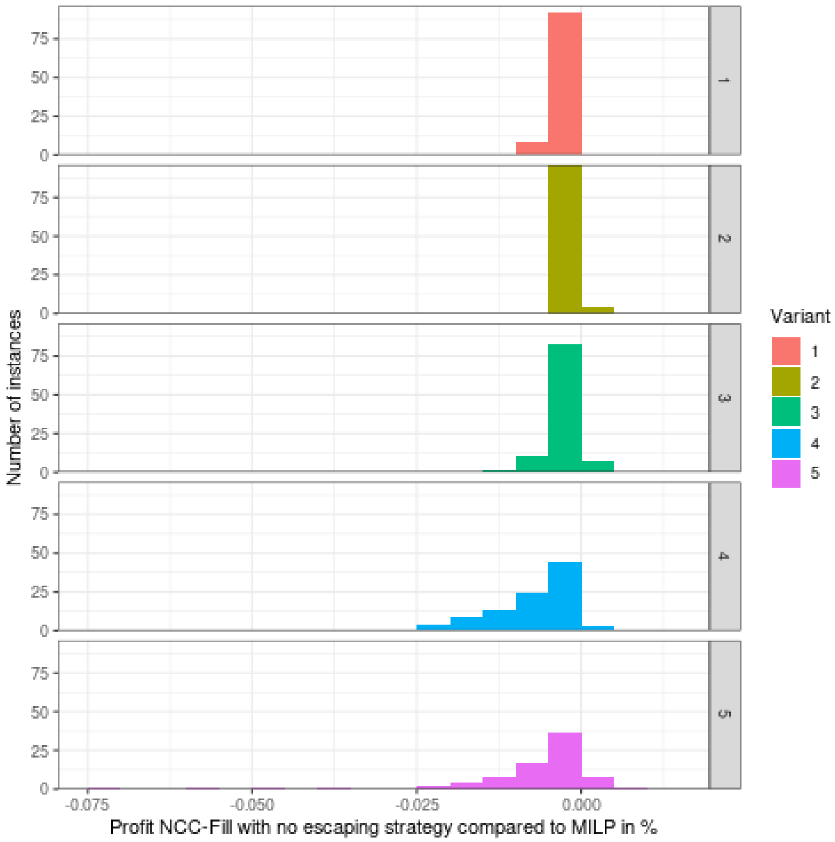


Figure 5.13: This figure shows the distribution of the difference of profit from the Extended-NCC solution to the MILP solution in relation to the profit of the MILP solution for the configuration FILL-NOESC

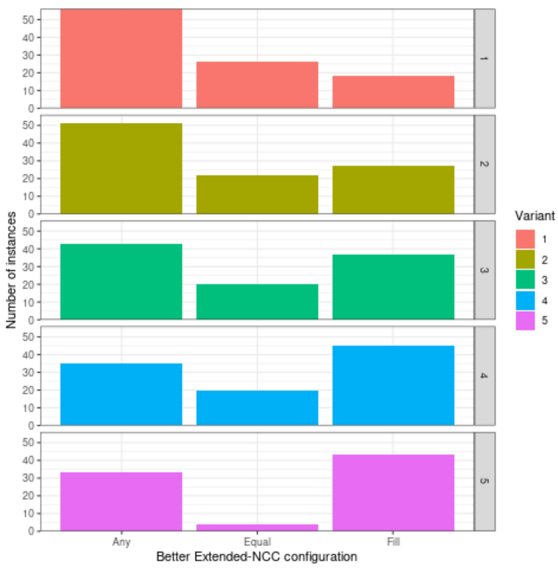
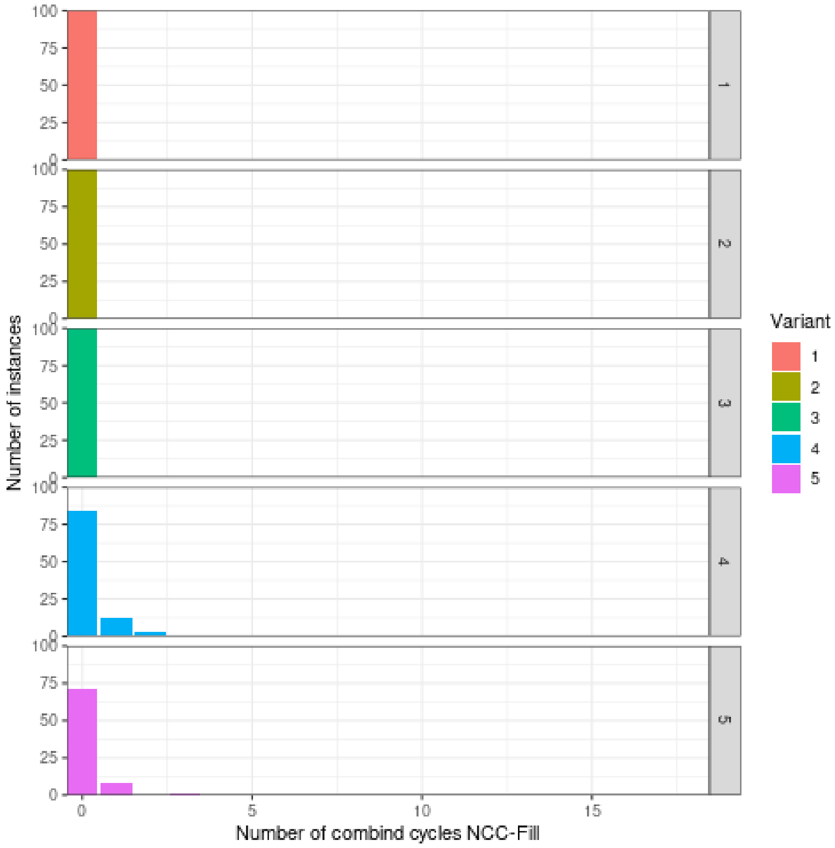
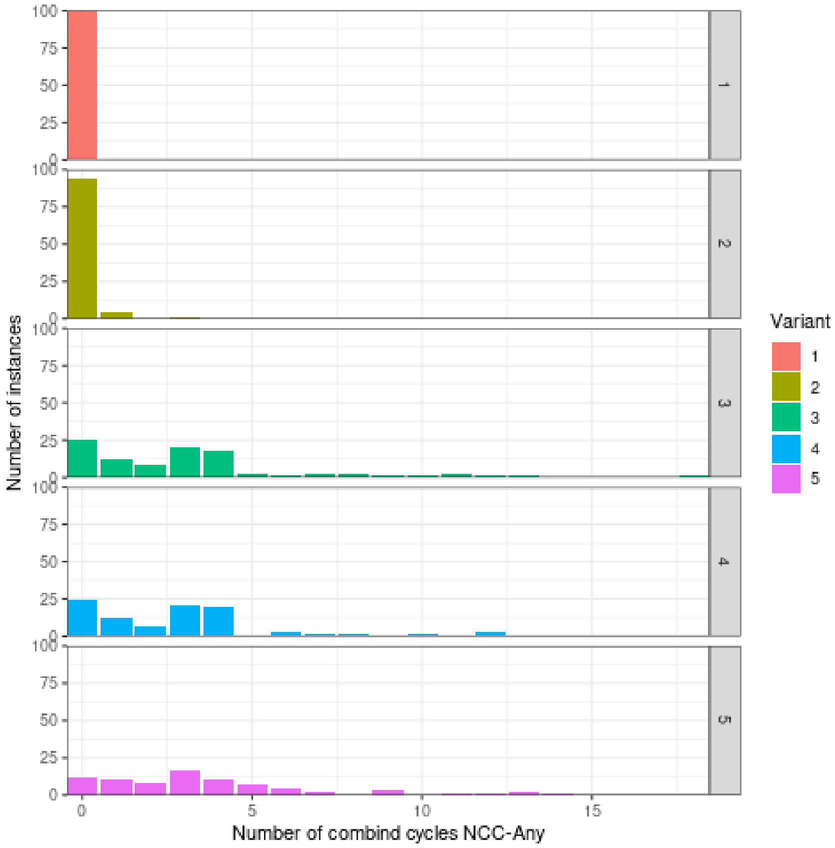


Figure 5.14: This figure displays for how many instances the configuration ANY-FREESUB or FILL-FREESUB provides solutions with a better absolute profit and for how many instance the profit of both configurations is equal.



(a) FILL-FREESUB



(b) ANY-FREESUB

Figure 5.15: This figure show the distribution of how many combined cycles per instance were found by the escaping strategy FREESUB for the Extended-NCC configurations FILL-FREESUB 5.15(a) and ANY-FREESUB 5.15(a).

at full capacity. For V_3-V_4, not all substations should be chosen. The initialisation strategy FILL already selects a minimal subset. Therefore, V_3 has no combined cycles. In V_4, the instances are the same as in V_3 but the substations do not all have the same costs. Therefore, switching a substation could improve the solution. This can be performed by cancelling a combined cycle. The initial flow found by initialisation strategy ANY uses the nearest substation for each turbine. Therefore, the initial flow will use almost all substation. For V_3-V_5, multiple combined cycles have to be cancelled to reduce the number of substations. Those results show that the escaping strategy can improve the solution if not all substations are selected in the optimal solution and if a suboptimal subset of substations is used for the initial flow. Together with the results of the previous section this emphasizes that the Extended-NCC needs an escaping strategy to move flow between substation. The current implementation of the FREESUB can perform such flow changes but it can get stuck in local optima. Combined cycles do not only occur for substation but can also occur in the original NCC. If we can select a subset of promising edges that might be in such a negative cycle, we could apply the escaping strategy FREESUB with some modification to the the NCC.

6. Conclusion

We propose an algorithm for a modified Wind Farm Cabling Problem where we optimize not only the cable layout but also select a subset of turbines and substations that should be built. This Extended-NCC algorithm is based on the NCC algorithm proposed in [GUW$^+$19] which models the WCP as a flow problem and improves an initial feasible flow by cancelling negative cycles in residual graph. We adjusted its model and residual cost function in a way that turbines and substations can be represented by flow on edges in the flow graph and a negative cycle in the a residual graph improves the flow. Not all flow changes that can improve the solution can be performed by a series of negative cycles. Especially, moving flow from one substation to an other substations can often not be realized by cancelling negative cycles. We implement an escaping strategy to find flow changes that free a substation. These flow changes can be performed by a combined cycle, a series of cycles that have aggregated negative costs.

To evaluate this algorithm, we generated benchmarks for five different variants based on benchmarks which were proposed in [LRWW17]. To do so, we propose a cost model determining the cost and revenue of substations, turbines and cables, which are mainly based on the costs given in [GR17]. We examined two objectives: Optimizing the absolute profit or the rate of return. If installed capacity is not restricted in any way optimizing the rate of return leads to solution where only one substation is connected. Therefore, we decide to maximize the absolute profit. Furthermore, we analysed the MILP solutions which show that in most cases all turbines will be built. Only if the total substation capacity limits the maximal number of turbines or if the number of turbines cannot be divided by the number of substations, not all substation are selected. If the number of turbines cannot be divided by the substation capacity, the substations have a lot of free capacity. As a consequence, it seems more interesting to optimize only substations in combination with the cables instead of optimizing the turbine and substation selection simultaneously with the cable layout.

We compare the solutions of the Extended-NCC to the solutions of a Mixed-integer Linear Program after one day. We tested three configurations of our algorithm: Initialisation strategy ANY with escaping strategy FREESUB, initialisation strategy FILL with and without the escaping strategy FREESUB. The most solutions of all configuration are with in 2% of the MILP solution. Both initialisation strategies outperformed each other for some instances. There is no clear winner, which initialisation strategy is better. However, the results suggest that the initialisation strategy closest to the optimal solution performs better. Therefore, the better initialisation strategy is ANY for instances where all substation are selected and FILL for instance where only a subset of substations is chosen. There are instance for both initialisation strategies that are improved by the escaping strategy. However, the number of instances that where improved by the escaping strategy after using the initialisation strategy FILL is very limited. Nevertheless, those result emphasize that there are flow changes that improve the solution substantially, which cannot be found by Negative Cycle Cancelling.

6.1 Future work

The cost model we use for our evaluation does not consider the operation and maintenance phase of the wind farm. The Extended-NCC could be evaluated with a more detailed cost models and different benchmarks instance to evaluate the Extended-NCC on a greater variation of instances.

The evaluation shows that there are a few instances for which the profit varies more from the MILP result than usual. It could be useful to examine those instances more closely to get an understanding of what flow changes that could improve the solution, cannot be found by the current implementation of the Extended-NCC.

The escaping strategy FREESUB shows that escaping strategies are needed to improve the solution. In the future, the algorithm could used additional escaping strategies to find more flow changes that improve the solution. The paper [GWW20] already proposed multiple escaping strategies for the NCC. Those and new escaping strategies could be incorporated into the Extended-NCC.

In addition, the evaluation showed that an initial flow closer to the optimal solution improves the outcome of the algorithm. Further initialisation strategies are imaginable to improve the initial flow. For example an initialisation strategy could decide based on each instance which one of the two initialisation strategies ANY and FILL is more promising.

Furthermore, the results of the evaluation suggest that for most instances all turbines are built. Based on this, it might be interesting to investigate how the algorithm performs if the turbines are given but the a subset of substations is selected. The algorithm could not only select which substation will be built but also what capacity the substation should have. For each substation, the algorithm can choose a capacity from a set of given capacity with corresponding costs. This could be easily integrated in the current implementation of the algorithm since the substations are already represented by an edge in the flow graph and the algorithm already deals with levelized costs functions for the edges that represent the inner array cables. This variation of the Extended-NCC could determine if a small number of substations with a huge capacity or a larger number of small substations is more cost-efficient.

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