Algorithmic Graph Theory
Summer Term 2025
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Exercise Sheet 6

Discussion: 30 July 2025

Exercise 1

Let G be a comparability graph and $[B_1, \dots, B_k]$ a G-decomposition. A tupel $(e_1, \dots, e_k)$ of edges is called a *decomposition scheme* of G if there is a G-decomposition $[B_1, \dots, B_k]$ such that $e_i \in B_i$ for all $i \in [k]$.

Let G be a comparability graph and $(e_1, \dots, e_k)$ a decomposition scheme of G. Prove that every permutation of $(e_1, \dots, e_k)$ is a decomposition scheme as well.

Exercise 2

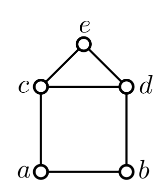
Let G be a graph. Prove that every G-decomposition has the same length. As an intermediate step prove that for every pair of decomposition schemes $(e_1, \dots, e_k)$ and $(f_1, \dots, f_l)$ of G there is a $j \in [l]$ such that $(f_1, \dots, f_{j-1}, e_1, f_{j+1}, \dots, f_l)$ is a decomposition scheme of G.

Exercise 3

A graph can be chordal, co-chordal, a comparability graph or a co-comparability graph. Prove that these properties are independent by giving an example for each of the 16 possible combinations.

Exercise 4

Let H be the graph depicted on the right. Prove that H is a permutation graph by finding a matching representation of H. Furthermore, prove that H and its complement $\overline{H}$ are comparability graphs.

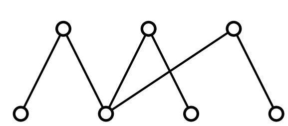


Is there a matching representation of H such that the vertices in the top row are ordered a, b, c, d, e?

Exercise 5

Which trees are permutation graphs?

Hint: Consider *spiders* (depicted on the right).



Exercise 6

This is a bonus problem that is most likely harder than the other problems. We might discuss it only briefly in the problem session.

For a fixed linear order $\prec$ of a graph, we say that two edges vw, xy with $v \prec w$ and $x \prec y$ *nest* if $v \prec x \prec y \prec w$ or $x \prec v \prec w \prec y$. A *rainbow* (w.r.t. a linear vertex order) is a set of edges that are pairwise nesting. The *rainbow number* of a graph is the smallest k such that there is a linear vertex order whose largest rainbow has size at most k. The *queue number* of a graph is the smallest k such that there is a linear vertex order and a partition of the edges into at most k sets such that no two edges in the same part nest.

Prove that the rainbow number and the queue number are equal for every graph.