Algorithmic Graph Theory
Summer Term 2025
Laura Merker, Samuel Schneider, Torsten Ueckerdt

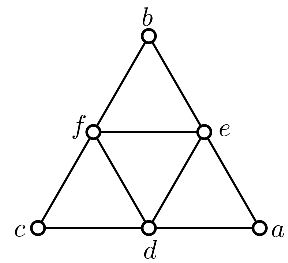
Exercise Sheet 5

Discussion: 16 July 2025

Exercise 1

Give an efficient algorithm that takes a chordal graph G and computes a set of subtrees of a tree such that G is the intersection graph of these subtrees. Assume that you are already given a perfect elimination scheme of G.

Apply your algorithm to the graph on the right. Use that $\sigma = [a, b, c, d, e, f]$ is a perfect elimination scheme.



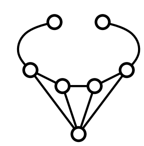
Exercise 2

Prove the following equivalences.

$$ab\ \Gamma\ a'b' \Leftrightarrow ba\ \Gamma\ b'a'$$
$$ab\ \Gamma^*\ a'b' \Leftrightarrow ba\ \Gamma^*\ b'a'$$

Exercise 3

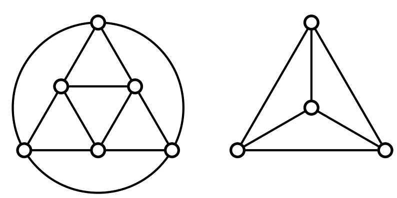
Prove that the "bull head" is not transitively orientationable. To do this prove that there is an implication class A such that $A = A^{-1}$.



Exercise 4

Apply the algorithm for computing a transitive orientation to the graphs on the right.

For each step additionally list all implication classes and observe how they change when the edges of a color class are removed.



Exercise 5

Show that the algorithm for computing a transitive orientation can be implemented to run in $\mathcal{O}(\Delta \cdot |E|)$ time and $\mathcal{O}(|V| + |E|)$ space with Δ denoting the maximum degree of a vertex.

Hint: Show that the computation of an implication class B_i can be implemented to run in $\mathcal{O}(|V| + |E|)$ time.

Exercise 6

Let $G = (V, E)$ be a graph. Prove the following statements.

1. A vertex order σ is a perfect elimination scheme of G if and only if $a <_\sigma b <_\sigma c$ with $ab, ac \in E$ implies that $bc \in E$.
2. G is a comparability graph if and only if there is a vertex oder σ such that $a <_\sigma b <_\sigma c$ with $ab, bc \in E$ implies that $ac \in E$.

Exercise 7

This is a bonus exercise that won't be discussed in the upcoming exercise class. Instead we will provide solutions in form of a reference. This exercise is most likely more difficult than the other exercises.

Let G be a transitively oriented comparability graph with only one sink and one source. Prove that

If there is a topological ordering σ of its vertex set $V(G)$ and a planar straight-line drawing of the transitive reduction of G such that $y(v) < y(w)$ if and only if $v <_\sigma w$,

then there are two linear orders φ, φ' of $V(G)$ such that $(v, w) \in E(G)$ if and only if $v <_\varphi w$ and $v <_{\varphi'} w$.

Prove that the reverse does not hold.