Algorithmic Graph Theory
Summer Term 2025
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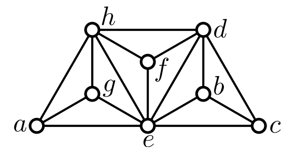
Exercise Sheet 4

Discussion: 02 July 2025

Exercise 1

Let $\sigma = [a, b, c, d, e, f, g, h]$ be a vertex ordering of the graph G on the right. Run the linear time algorithm from the lecture to check if σ is a perfect elimination scheme of G.



Exercise 2

Let G be a graph. A vertex v of G is called *universal* if v is adjacent to all other vertices of G. The graph G is called *triangulated* if G is planar and every face of G is incident to exactly three vertices.

Prove the following statements:

1. There are infinitly many triangulated graphs that are not chordal.
2. There are infinitly many triangulated graphs that are chordal.
3. Every triangulated graph that has a universal vertex is chordal.

Exercise 3

Let σ be a perfect elimination scheme and let K_v be the clique consisting of v and its subsequent neighbors with regard to σ. Prove that K_v is a maximal clique if and only if there is no predecessor u of v such that $K_v \subseteq K_u$.

Exercise 4

Show that a minimum vertex cover can be computed efficiently on chordal graphs.

Exercise 5

Let G be a connected graph. Prove that G is a tree if and only if every family of paths in G fulfills the Helly property.

Exercise 6

Let G be a graph. The *line graph* $L(G)$ has the set $E(G)$ as vertices. Two vertices $e_1, e_2 \in V(L(G))$ are adjacent if $e_1 \cap e_2 \neq \emptyset$, i. e. if the respective edges in G share a vertex.

~~Prove that if G is chordal then $L(G)$ is chordal.~~ Prove that if $L(G)$ is chordal, then G is chordal. Show that the reverse does not hold.

Exercise 7

This is a bonus exercise that won't be discussed in the upcoming exercise class. Instead we will provide solutions in form of a reference to a paper. This exercise is most likely more difficult than the other exercises.

Prove that a graph has treewidth at least three if and only if it contains K_4 as a topological minor.