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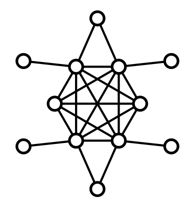
Exercise Sheet 3

Discussion: 04 June 2025

Exercise 1

List all simplicial vertices in the graph on the right.



Exercise 2

Let S be a minimal vertex separator in a chordal graph $G = (V, E)$. Prove that every component of G_{V-S} contains a vertex c such that $S \subseteq N(c)$.

Hint: Compare with a similar statement from the lecture.

Exercise 3

Let G be an interval graph. Give two different proofs for the chordality of G by proving the following statements:

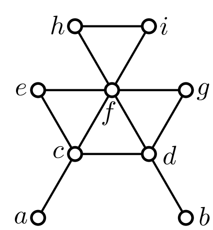
1. G has a perfect elimination scheme.
2. Every minimal vertex separator of G is a clique.

Exercise 4

Run a lexicographic BFS on the graph on the right.

Give the current status of the queue data structure (including all necessary pointers) for every step of the algorithm.

Bonus: Run a lexicographic DFS on the graph on the right. For that change the update step in the LexBFS algorithm to label$(w) \leftarrow (n-i)$.label(w). What do you observe?



Exercise 5

Give a graph with a perfect elimination scheme σ such that σ cannot be computed using a lexicographic BFS.

Exercise 6

The class of k-trees is recursively defined as follows. The complete graph K_k is a k-tree. Let T be a k-tree with a clique $C = \{x_1, \ldots, x_k\}$ of size k. Then the graph obtained by adding a vertex to T that is adjacent to the vertices in C is a k-tree as well.

Prove that k-trees are chordal. Give a chordal graph that is not a k-tree.