Algorithmic Graph Theory
Summer Term 2025
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Exercise Sheet 2

Discussion: 21 May 2025

Exercise 1

We say a graph class $\mathcal{G}$ is *hereditary* if for every graph $G \in \mathcal{G}$ and every induced subgraph $H \subseteq_{\text{ind}} G$ we have that $H \in \mathcal{G}$. We say a graph class $\mathcal{G}$ is *monotone* if for every graph $G \in \mathcal{G}$ and every subgraph $H \subseteq G$ we have that $H \in \mathcal{G}$.

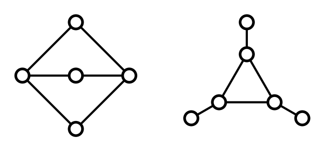
(a) Prove or disprove the following statements:
   (i) The class of all perfect graphs is hereditary.
   (ii) The class of all perfect graphs is monotone.

(b) Give a smallest hereditary graph class that contains every complete graph.

(c) Give a smallest monotone graph class that contains every complete graph.

Exercise 2

Which of the graphs depicted on the right are circle graphs, circular-arc graphs and interval graphs, respectively?



Exercise 3

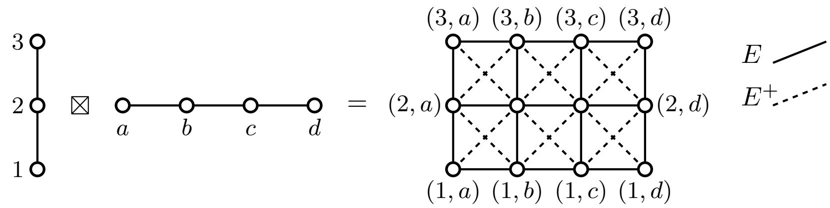
Show that for every graph $G = (V, E)$ there exists a family $\mathcal{F}$ of subsets of E such that G is the intersection graph of $\mathcal{F}$.

Exercise 4

The *strong product* $G_1 \boxtimes G_2$ of two graphs $G_1 = (V_1, E_1)$ und $G_2 = (V_2, E_2)$ is defined as $G_1 \boxtimes G_2 = (V_1 \times V_2, E \cup E^+)$ with

- $E = \{\{(v_1, v_2), (v_1', v_2')\} \mid$ either $v_1 = v_1'$ and $\{v_2, v_2'\} \in E_2$ or $v_2 = v_2'$ and $\{v_1, v_1'\} \in E_1\}$
- $E^+ = \{\{(v_1, v_2), (v_1', v_2')\} \mid \{v_1, v_1'\} \in E_1$ and $\{v_2, v_2'\} \in E_2\}$

For $P_3 \boxtimes P_4$ this looks like this:



Prove the following statements for all graphs G_1 and G_2:

(a) $\chi(G_1 \boxtimes G_2) \geq \max\{\chi(G_1), \chi(G_2)\}$

(b) $\omega(G_1 \boxtimes G_2) = \omega(G_1) \cdot \omega(G_2)$

(c) $\alpha(G_1 \boxtimes G_2) \geq \alpha(G_1) \cdot \alpha(G_2)$

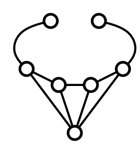
(d) $k(G_1 \boxtimes G_2) \leq k(G_1) \cdot k(G_2)$

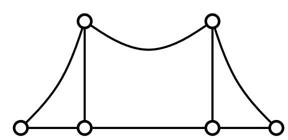
Bonus: When do the statements hold with equality, when do they not? Give an example for each of these cases.

Exercise 5

Is the "bull head" an interval graph?

Is the complement of the "suspension bridge" an interval graph?





Exercise 6

Six group members were in the office the day the coffee beans were stolen. Every group member arrived at a certain time, stayed for some time and then left. If two group members were present at the same time, at least one of them saw the other.

During questioning they say the following:

- *Torsten* claims to have seen *Laura* and *Kolja*,
- *Laura* claims to have seen *Torsten* and *Marco*,
- *Miriam* claims to have seen *Samuel* and *Marco*,
- *Samuel* claims to have seen *Torsten* and *Marco*,
- *Kolja* claims to have seen *Laura* and *Miriam*, and
- *Marco* claims to have seen *Miriam* and *Kolja*.

To deviously misdirect the suspicion to one of their colleagues, the thief claimed to have seen people that they did not see. Who is the thief?