Algorithmic Graph Theory

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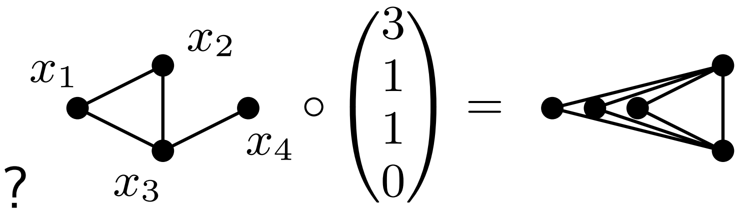
Update from the faculty

- exams may be taken in English or German

Graph G on vertices $x_1, \dots, x_n$, tuple $h = (h_1, \dots, h_n)$ of nonnegative integers
Recall $H = G \circ h$: $V(H) = \{x_i^1, \dots, x_i^{h_i} \mid i \in [n]\}$, $E(H) = \{x_i^a x_j^b \mid x_i x_j \in E(G), a \in [h_i], b \in [h_j]\}$

New definition: $H = G \ominus h$ with
$V(H) = \{x_i^1, \dots, x_i^{h_i} \mid i \in [n]\}$, $E(H) = E(G) \cup \{x_i^a x_j^1 \mid x_i x_j \in E(G), a \in [h_i], h_j > 0\}$



(1) What is the difference between $\circ$ and $\ominus$?

(2) Can $\circ$, resp. $\ominus$, be realized by elementary operations?
I.e., is there a sequence $h^1, h^2, \dots$ of tuples, each with all entries 1 except for one 0 or 2, such that $G * h = G * h^1 * h^2 * \dots$ for each $* \in \{\circ, \ominus\}$? If so, does the order matter?

(3) Find a largest cycle, a largest induced cycle, ω, χ, α, and κ of $K_n \ominus (2, \dots, 2)$.

(4) Prove or disprove: If G is perfect, then ...
- $\omega(G \ominus h) = \chi(G \ominus h)$ (follow a proof of the lecture)
- $G \ominus h$ is perfect

Let $G_0 = K_2$ and $H_i = G_{i-1} \ominus (2, \dots, 2)$, $G_i = H_i + u + \{uv \mid v \in V(H_i) - V(G_{i-1})\}$, where u is a new vertex, $i \geq 1$

(5) Prove that $G_1, G_2, \dots$ are not perfect.

(6) Prove that $G_0, G_1, \dots$ are "far from perfect". For this, find $\omega(G_i)$ and $\chi(G_i)$, $i \geq 0$.