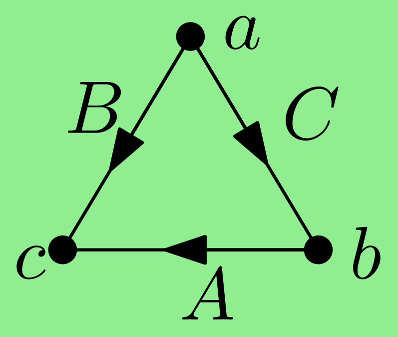
Lemma 4.3 (Triangle Lemma)
$A, B, C \in \mathcal{I}(G), A \neq B, C^{-1}$
(i) $b'c' \in A \Rightarrow ab' \in C, ac' \in B$
(ii) $b'c' \in A, a'b' \in C \Rightarrow a'c' \in B$



G-decomposition $[B_1, \ldots, B_k]$
- $E = \hat{B}_1 + \cdots + \hat{B}_k$
- $B_i \in \mathcal{I}(\hat{B}_i + \cdots + \hat{B}_k), i \in [k]$

rainbow triangle $\{a, b, c\}$
- $\hat{bc} \in \hat{A}, \hat{ac} \in \hat{B}, \hat{ab} \in \hat{C}$
- $\hat{A}, \hat{B}, \hat{C} \in \hat{\mathcal{I}}(G)$ pw. distinct

Thm 4.1 $A \in \mathcal{I}(G)$, F transitive
$\Rightarrow F \cap \hat{A} = A$ or $F \cap \hat{A} = A^{-1}$

Thm 4.7
(i) G comparabilty graph
$\Leftrightarrow$ *(ii)* $A \cap A^{-1} = \emptyset, A \in \mathcal{I}(G)$
$\Leftrightarrow$ *(iii)* $\forall$ G-decomposition $[B_1, \ldots, B_k]$
it holds $B_i \cap B_i^{-1} = \emptyset, i \in [k]$

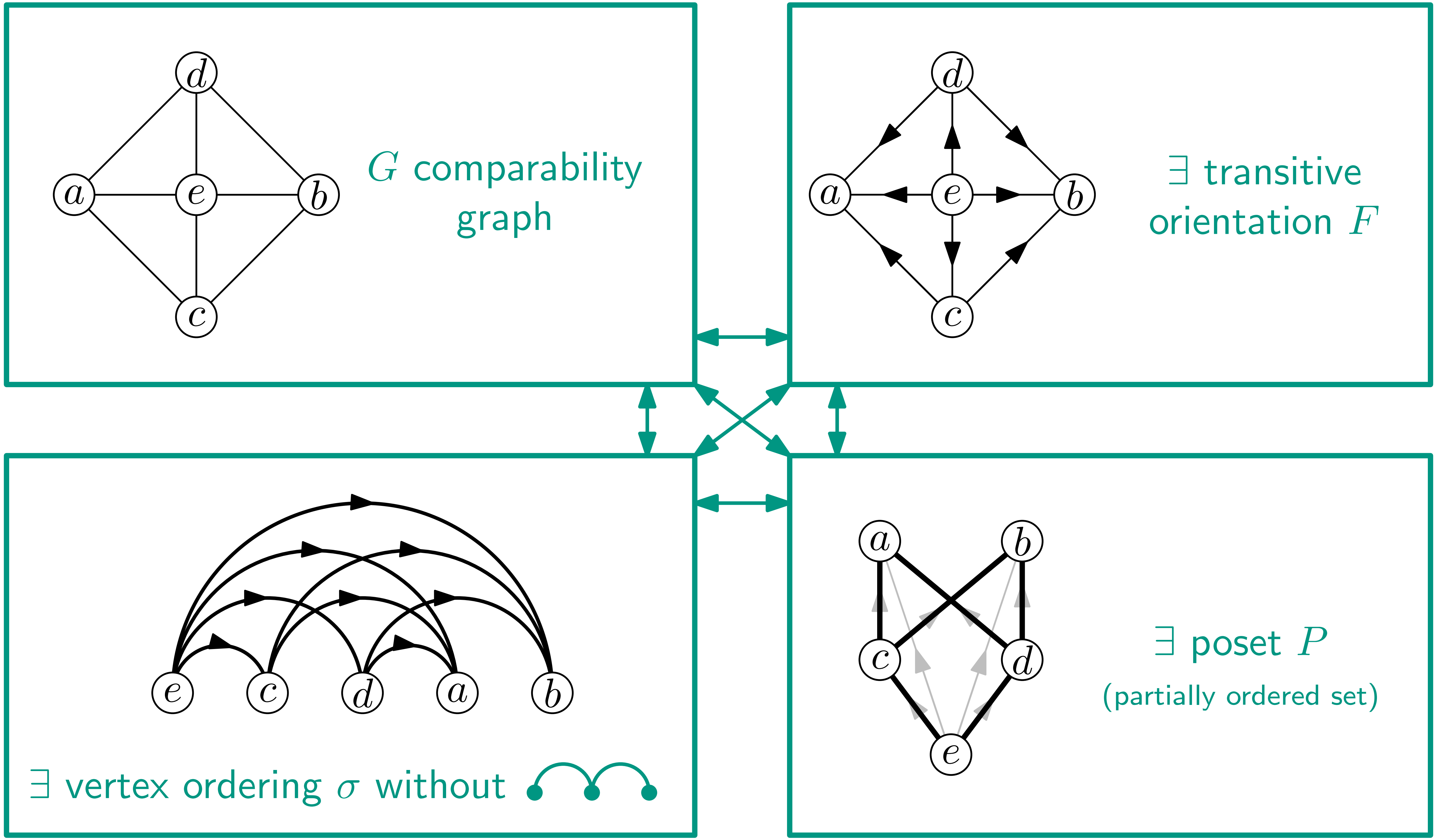
Thm 4.4 $A \in \mathcal{I}(G)$
$\Rightarrow A = A^{-1}$ or $A \cap A^{-1} = \emptyset$
and A, A^{-1} transitive

Thm 4.6 $A \in \mathcal{I}(G), D \in \mathcal{I}(E - \hat{A})$
(i) $D \in \mathcal{I}(G)$ and $A \in \mathcal{I}(E - \hat{D})$
or *(ii)* $D = B + C$, $\hat{A}, \hat{B}, \hat{C}$ in rainbow triangle

Input : undirected graph $G = (V, E)$.
Output : transitive orientation T, if it exists.

$T \leftarrow \emptyset$;
$i \leftarrow 1$; $E_i \leftarrow E$;
while $E_i \neq \emptyset$ **do**
    choose $x_i y_i \in E_i$ arbitrarily;
    determine implication class B_i of E_i containing $x_i y_i$;
    if $B_i \cap B_i^{-1} \neq \emptyset$, **then**
        return *"G is no comparability graph"*;
    end if
    add B_i to T;
    $E_{i+1} \leftarrow E_i - \hat{B}_i$;
    $i \leftarrow i + 1$;
end while
return T;

Algorithm 7 : Recognition of comparabilty graphs


G comparability graph
∃ transitive orientation F
∃ vertex ordering σ without
∃ poset P
(partially ordered set)

Input : comparability graph $G = (V, E)$.
Output : vertex coloring h and clique C.

compute transitive orientation F of G;
compute tological ordering σ of (V, F);
for $i \leftarrow 1$ **to** n **do**
    $v \leftarrow \sigma(i)$;
    $h(v) \leftarrow 1 + \max\{h(w) \mid wv \in F\}$;
    $\chi \leftarrow \max\{\chi, h(v)\}$;
    $w \leftarrow \mathrm{argmax}\{\chi, h(v)\}$;
end for
for $i \leftarrow \chi$ **to** 1 **do**
    $C \leftarrow \{w\}$;
    $w \leftarrow \mathrm{argmax}\{h(v) \mid vw \in F\}$;
end for
return h and C;

Algorithm 8 : Compute $\chi(G)$ and $\omega(G)$