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Computational Geometry
Summer term 2018

Exercises 6

Discussion: Friday, July 20th, 2018

Lecture 12 – Range Queries II (04.07.2018)

Exercise 1 – Interval-Trees. In the lecture we have considered the following problem for querying segments with arbitrary orientation.

SegmentQuery
Given: n disjoint segments and a axis-aligned rectangle $R = [x, x'] \times [y, y']$
Find: All segments that intersect R.

We have solved this problem using segment trees. Can we also use interval trees? Which problems may arise?

Exercise 2 – Segment-Trees – Construction. In the lecture we have seen the following theorem:

Theorem 1. *Let S be a set of horizontal (axis-parallel) line segments in the plane. All k line segments that intersect a vertical query segment (an axis-parallel rectangle R) can be found in $O(\log^2(n) + k)$ time. The data structure requires $O(n \log n)$ space and $O(n \log n)$ construction time.*

We now show that the data structure can be constructed in $O(n \log n)$ time. Let $s, s' \in S$ be two line segments. The segment s lies *below* s' ($s \prec s'$), if there are points $p \in s$ and $p' \in s'$ with $x(p) = x(p')$ and $y(p) < y(p')$, where $x(p)$ denotes the x-coordinate of p and $y(p)$ denotes y-coordinate of p.

1. Show that the relation $\prec$ defines a acyclic relation on S.
2. Describe an algorithm, that computes such an order on S in $O(n \log n)$ time.
   Hint: Use a sweep-line-procedure.
3. Show that the data structure of Theorem 1 can be constructed in $O(n \log n)$ time.

Exercise 3 – Counting Intervals. Let I be a set consisting of n intervals. Describe a data structure, by means of which the number of intervals containing a given point $p \in \mathbb{R}$ can be determined. To that end consider the following variants.

1. The data structure is based on interval trees.
2. The data structure is based on segment trees.
3. The data structure uses neither interval trees nor segment trees.

Exercise 4 – Intersection of Rectangles. Let $\mathcal{R}$ denote a set of n axis-aligned rectangles in the plane. For a point $p \in \mathbb{R}^2$ the number of rectangles containing p is denoted by $w_{\mathcal{R}}(p)$. Describe an algorithm that computes $\max_{p\in\mathbb{R}^2} w_{\mathcal{R}}(p)$ in $O(n \log n)$ time.

Hint: Use segment trees and a sweep-line-procedure.

Lecture 14 – Duality (13.07.2018)

Exercise 5 – Duality I. In the lecture we have seen that the dual of a line segment is a double wedge, with a wedge left and right to the point that is dual to the line containing the line segment.

1. What is the dual of a triangle with vertices p, q and r?
2. What is the dual of a circle through the points p, q and r?

Exercise 6 – Duality II. Let L be a set of n lines in the plane. We want to find an axis-aligned rectangle $\mathcal{B}(L)$ that contains all vertices of the arrangement $\mathcal{A}(L)$. Describe an algorithm that computes $\mathcal{B}(L)$ in $O(n \log n)$ time.

Exercise 7 – Duality III. Let R be a set of n red points in the plane, and let B be a set of n blue points in the plane. We call a line ℓ a *separator* of R and B, if all blue points lie on one side and all red points on the other side of ℓ.

1. Describe an algorithm that decides in $O(n \log n)$ time whether a separator of R and B exists.
2. Describe an randomized algorithm, that decides in $O(n)$ expected time whether a separator of R and B exists.

Exercise 8 – Duality IV. Let S be a set of n points in the plane. Describe an $O(n^2)$ algorithm that computes the line on which most points of S lie.