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Computational Geometry
Summer term 2018

Exercises 3 – Well-Separated Pair Composition

Discussion: Friday, June 1st, 2018

Well-Separated Pair Decomposition (16.05.2018 & 25.05.2018)

Exercise 1 – Foundations. Let $s > 0$ and let $x := 2/s + 1$. Further, let $S := \{x^i \mid 0 \leq i \leq n-1, i \in \mathbb{N}\}$ and let $\{A_j, B_j\}$ $(1 \leq j \leq m)$ be an arbitrary s-WSPD for S. Show that

$$\sum_{j=1}^{m}(|A_j| + |B_j|) = \binom{n}{2} + m$$

Hint: For each j at least one of both sets A_j and B_j is a singleton.

Exercise 2 – Neighbor I. Let P be a set of n points in $\mathbb{R}^d$. Let $p \in P$ and let $q \in P$ be the next neighbor of p in P, i.e., $|pq| = \min\{|pr| : r \in P, r \neq p\}$. Consider an arbitrary s-WSPD for P with $s > 2$.

1. Let $\{A, B\}$ be a pair in this decomposition and assume that p lies in A and q lies in B. Show that A only contains p.
2. Show that the size of an arbitrary s-WSPD with $s > 2$ is at least $n/2$.

Exercise 3 – Neighbor II. Let P be a set of n points in $\mathbb{R}^d$. Further, let $p, q \in P$ be a pair of points with minimal distance to each other, i.e., $|pq| = \min\{|ab| : a \in P, b \in P\}$. Consider an arbitrary s-WSPD $\mathcal{W}$ for P with $s > 2$. Show that $\mathcal{W}$ contains the pair $\{\{p\}, \{q\}\}$.

Exercise 4 – Closest Pair of Points. Consider a point set P, let x and y be the closest pair of points and let p_u and p_v be the representatives from their associated well-separated pair. Show that it is $\|xy\| = \|p_u p_v\|$.