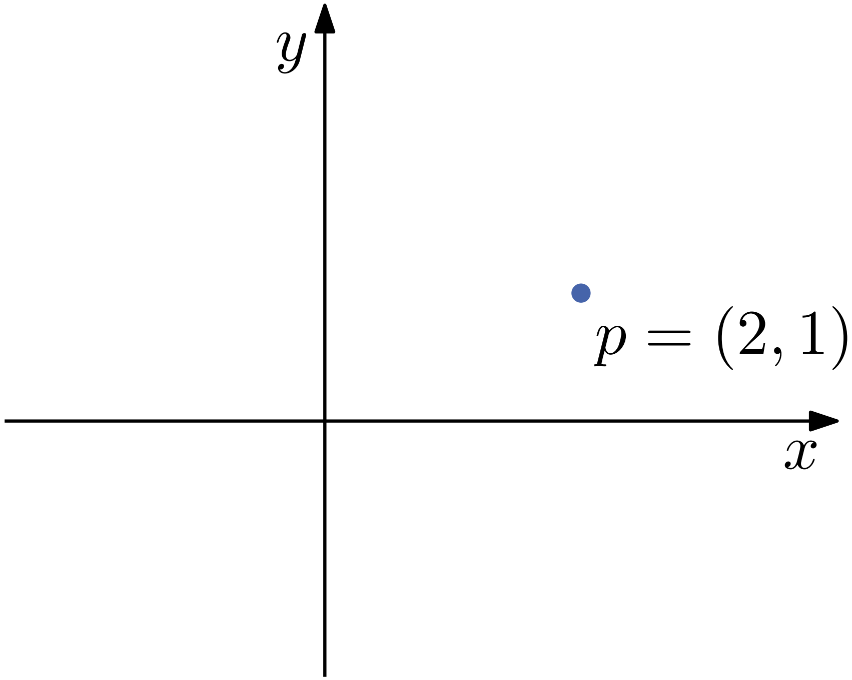
Computational Geometry – Exercise
Duality

LEHRSTUHL FÜR ALGORITHMIK I · INSTITUT FÜR THEORETISCHE INFORMATIK · FAKULTÄT FÜR INFORMATIK

Guido Brückner
20.07.2018

Duality Transforms

We have seen duality for planar graphs and duality of Voronoi diagrams and Delaunay triangulations. Here we will see a duality of points and lines in $\mathbb{R}^2$.



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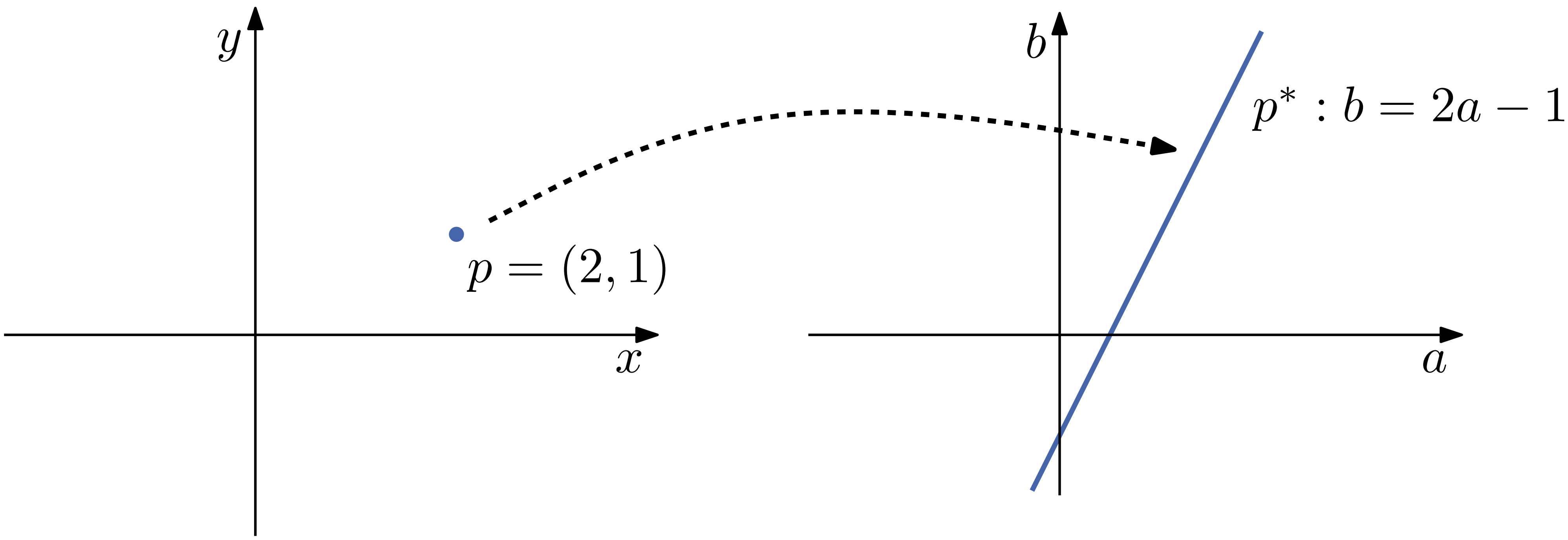

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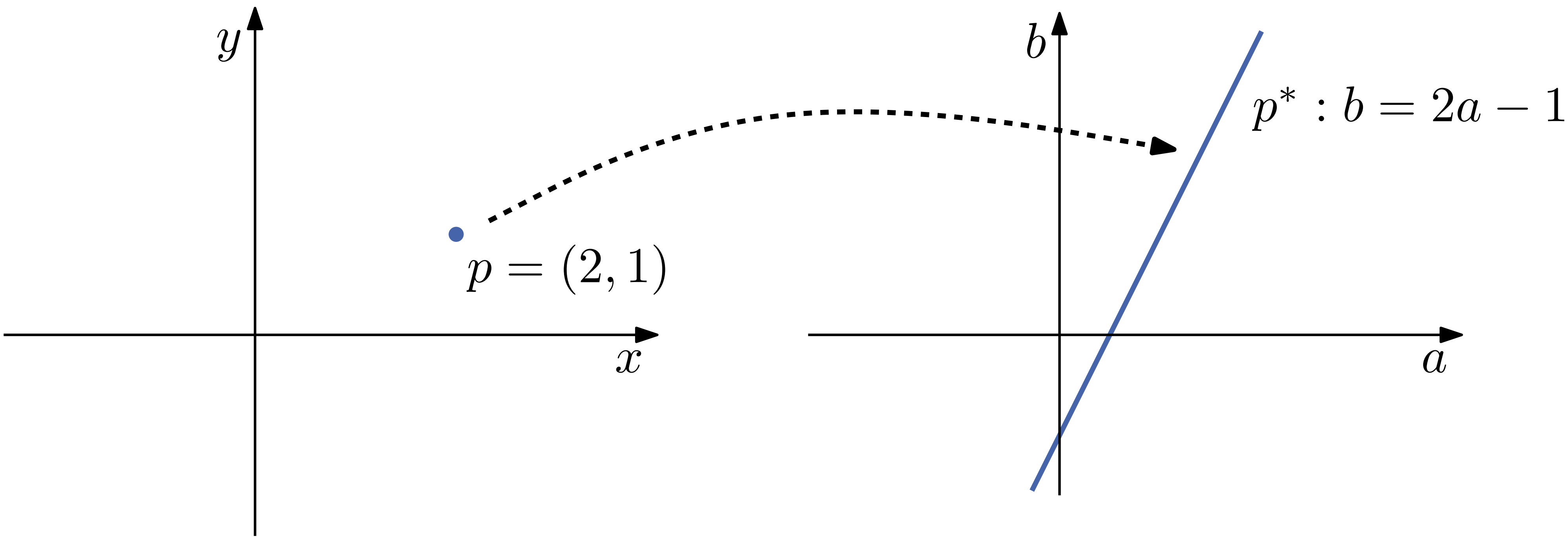

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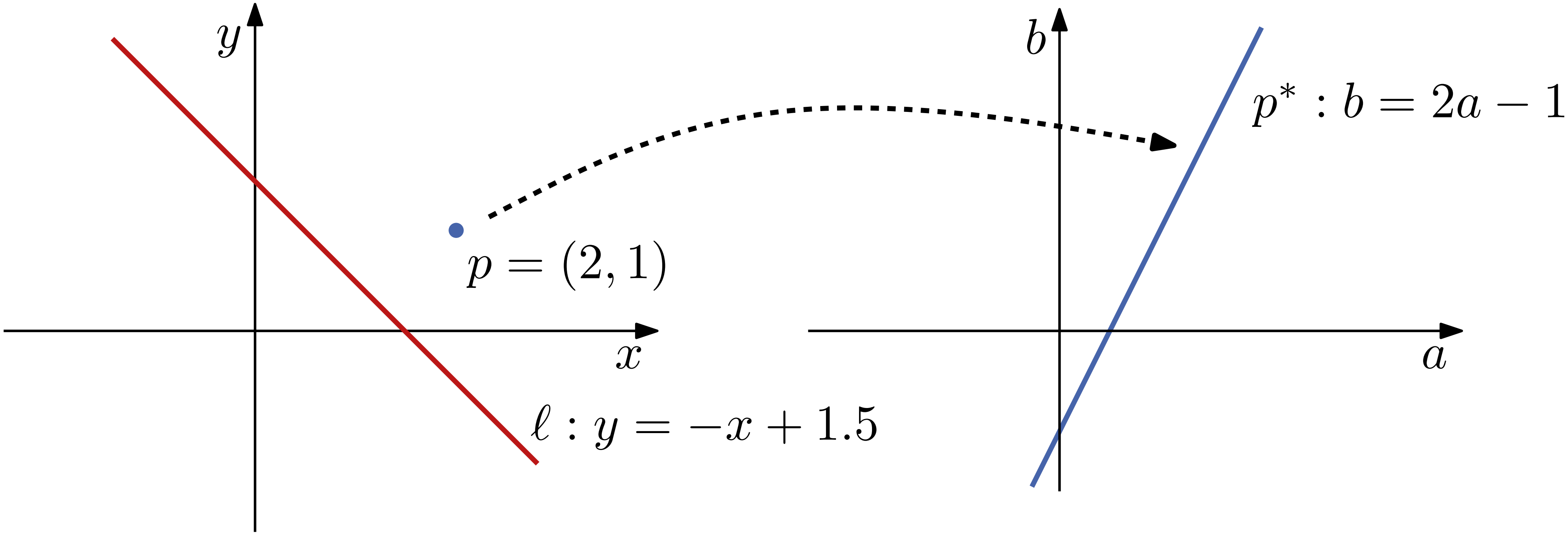

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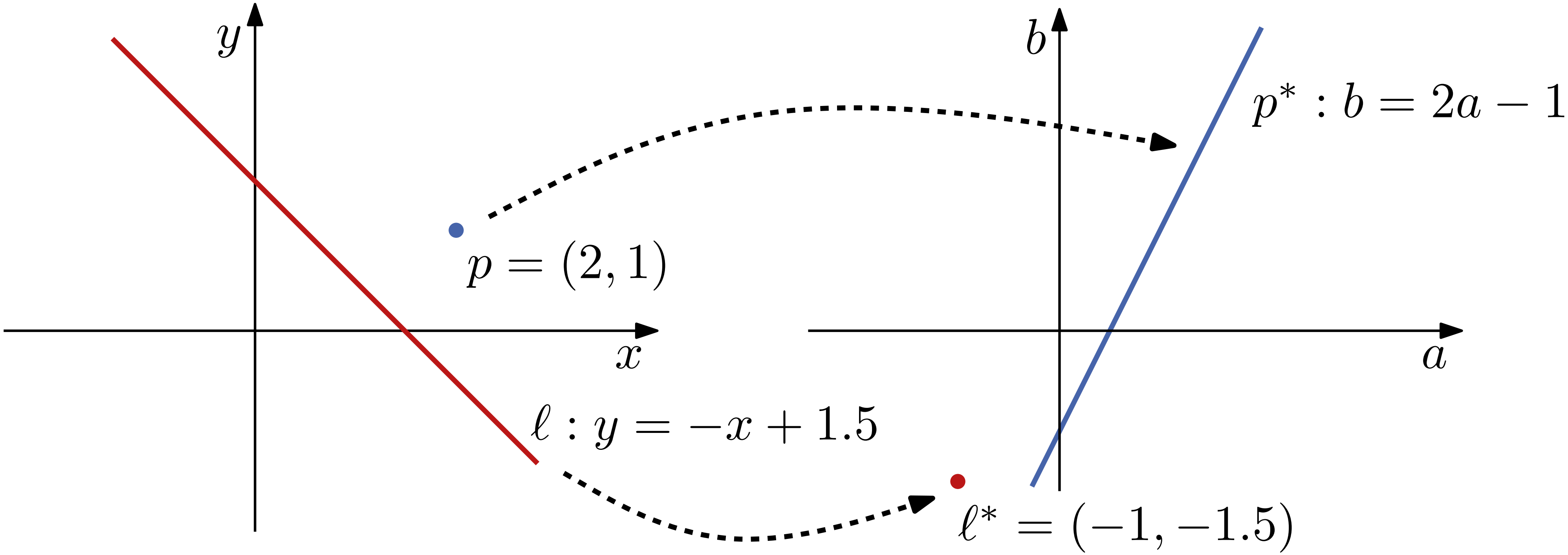

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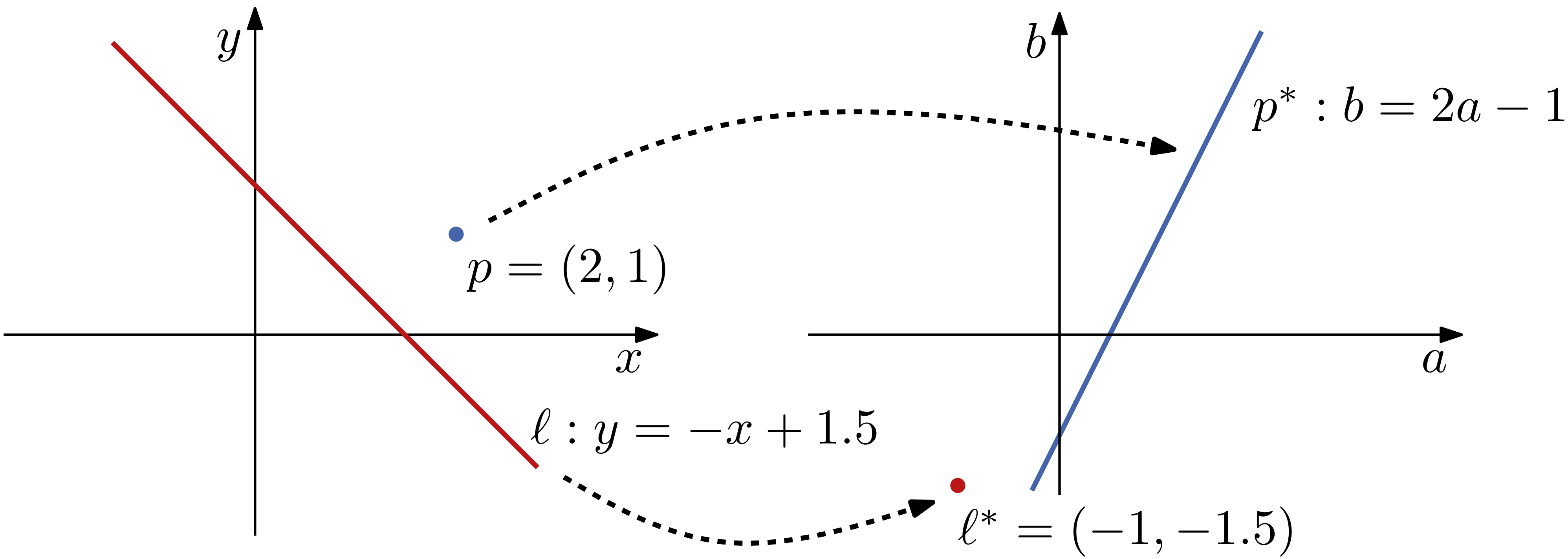

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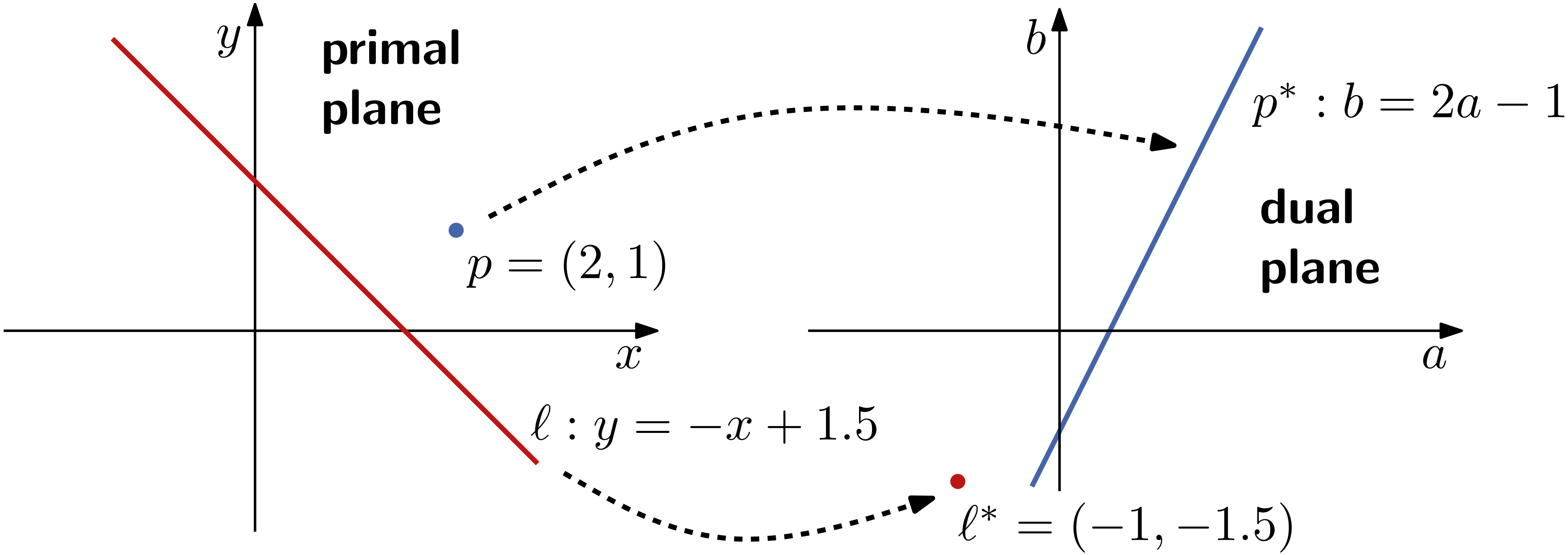

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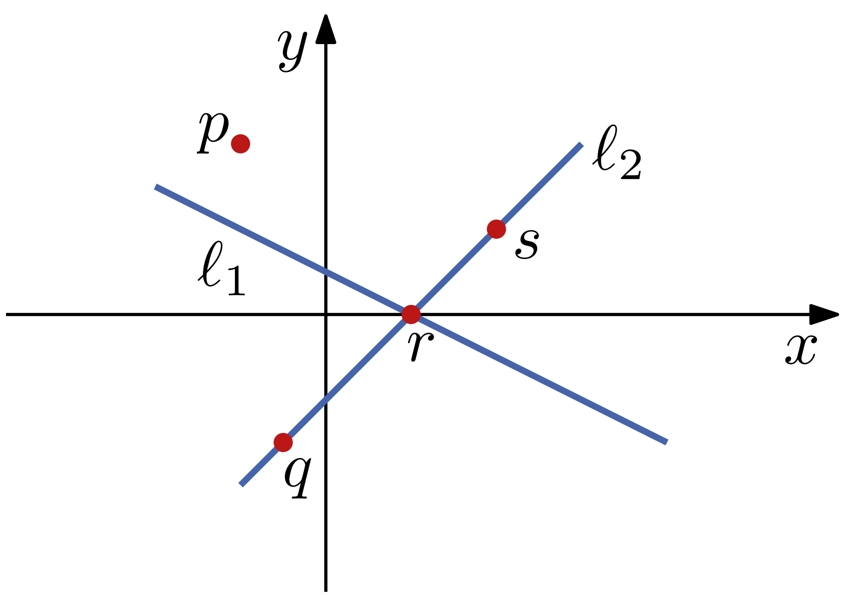

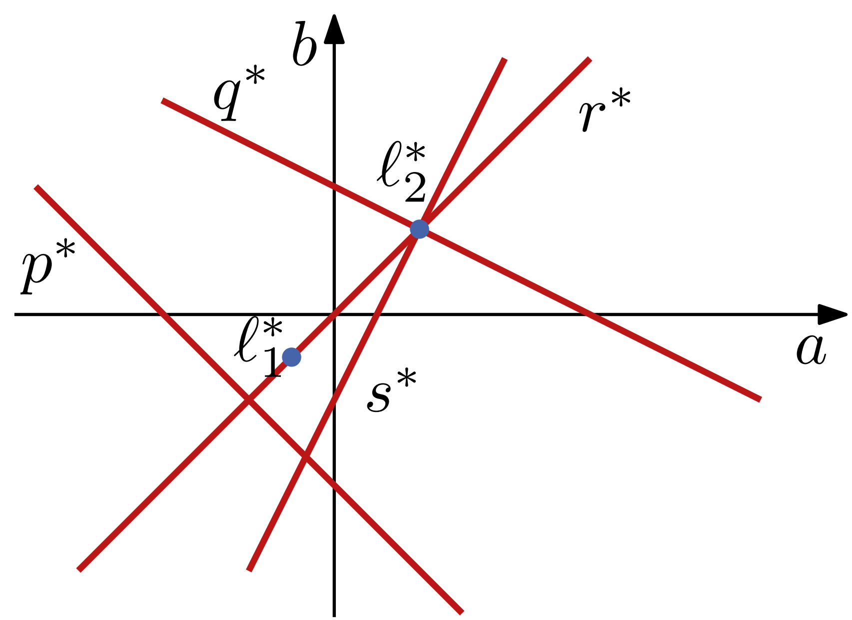

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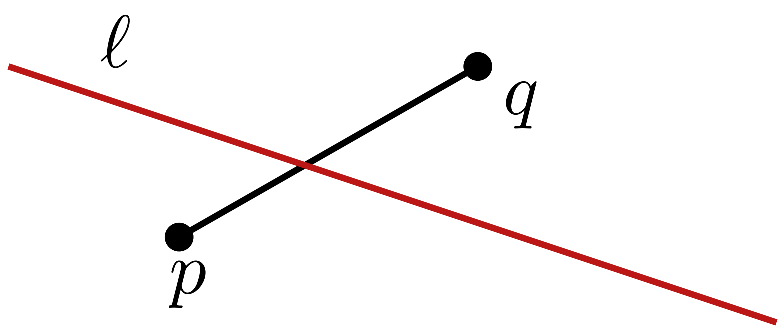

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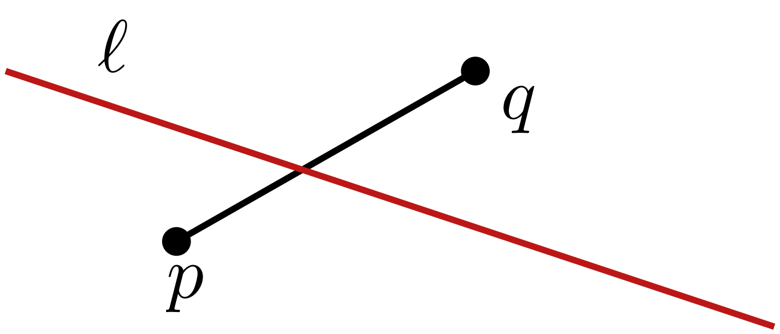
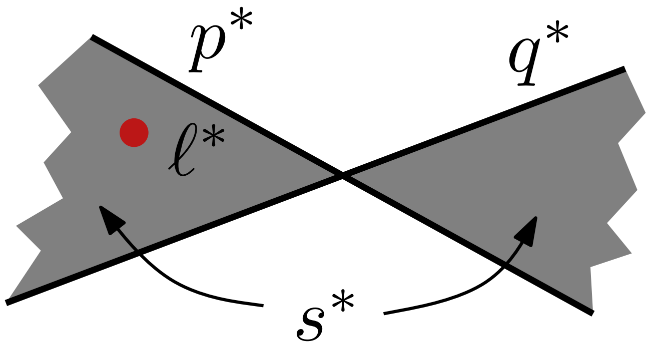
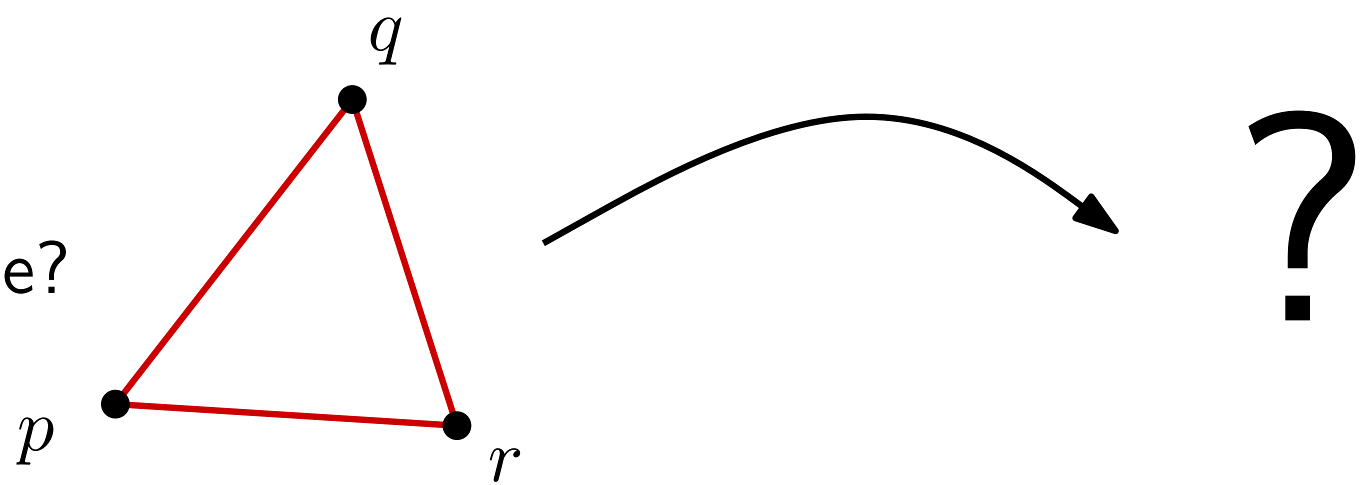




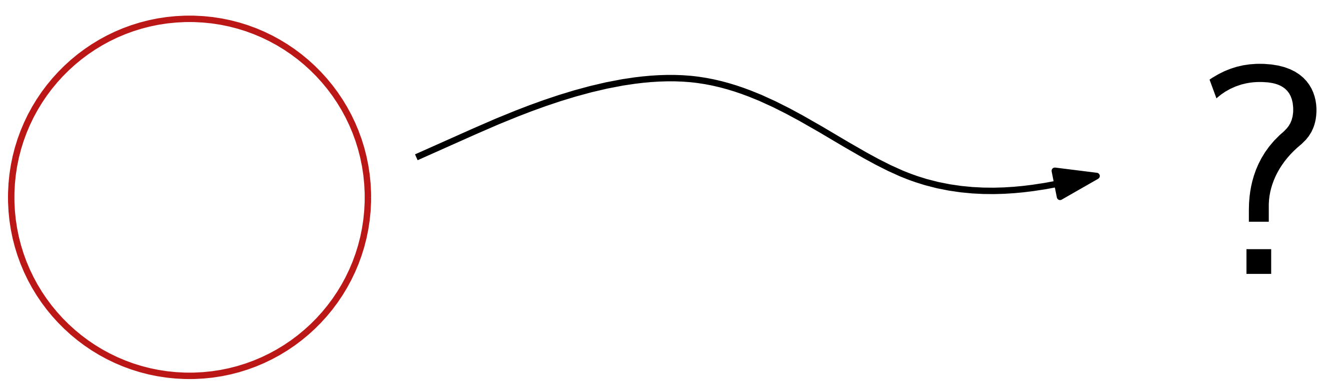
Exercise 5

Problem:
What is the dual of ...

- ... a triangle?



- ... dual of a circle?



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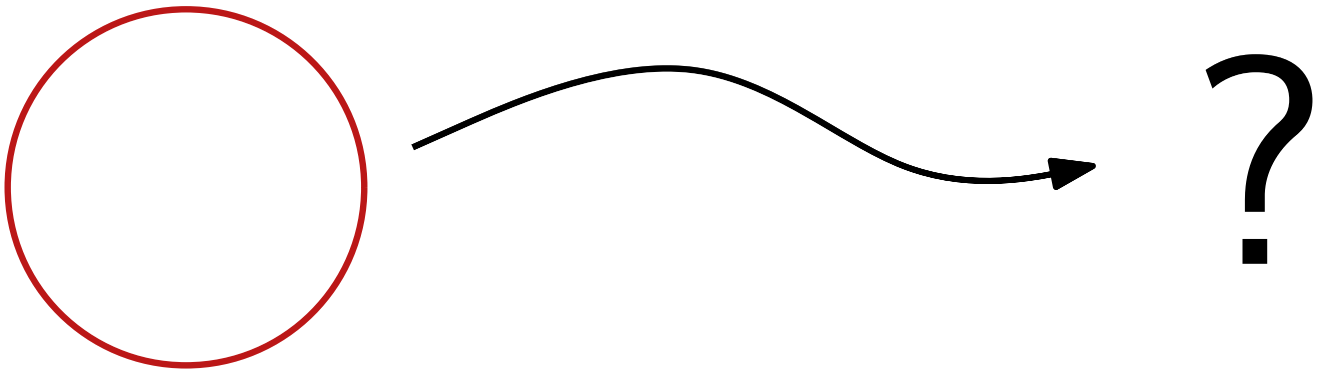

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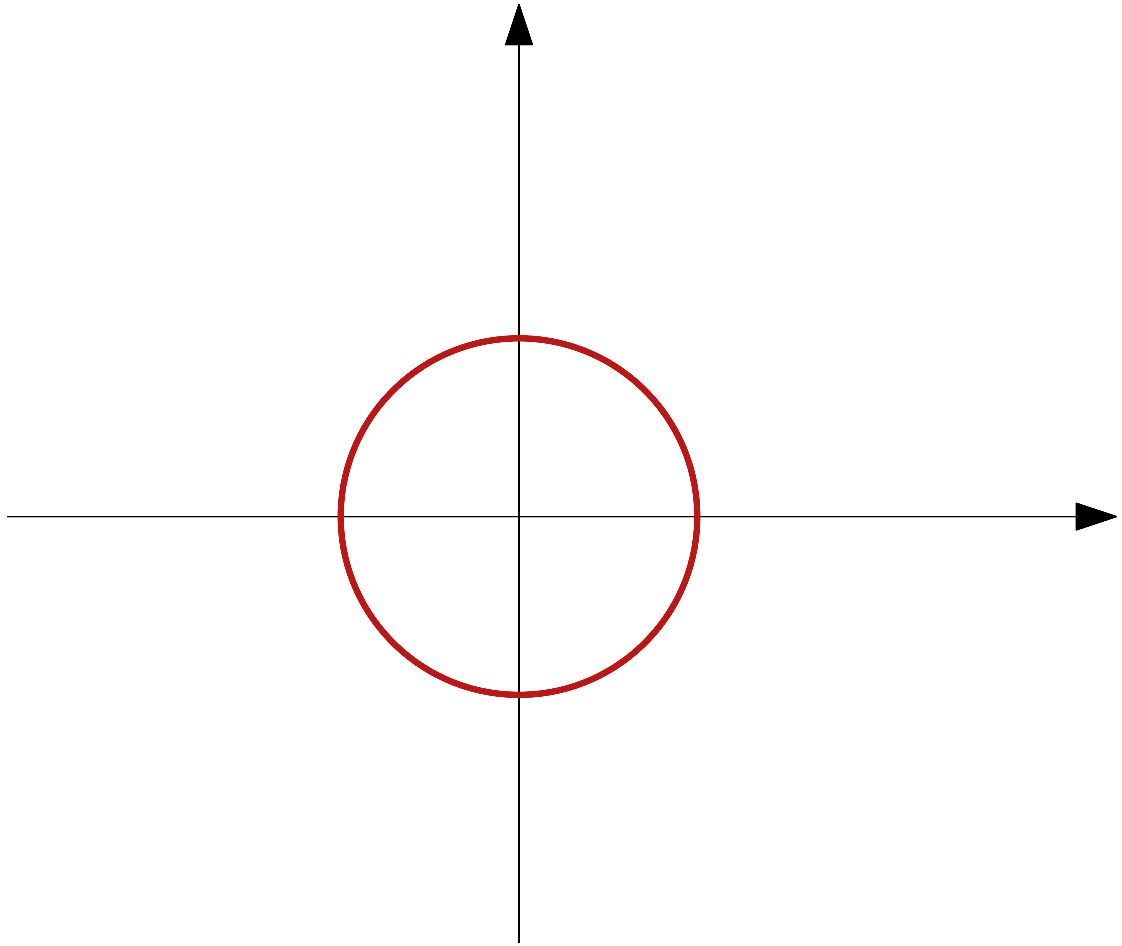

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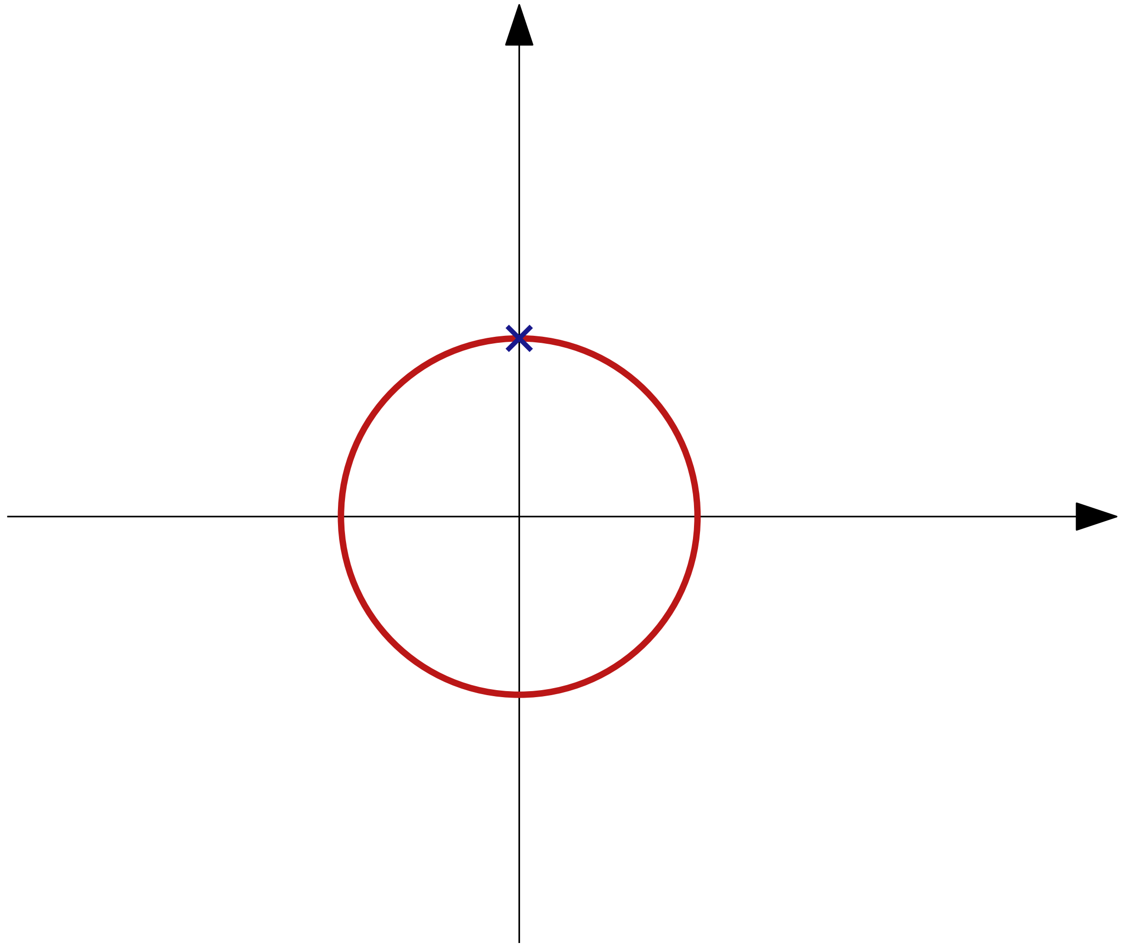

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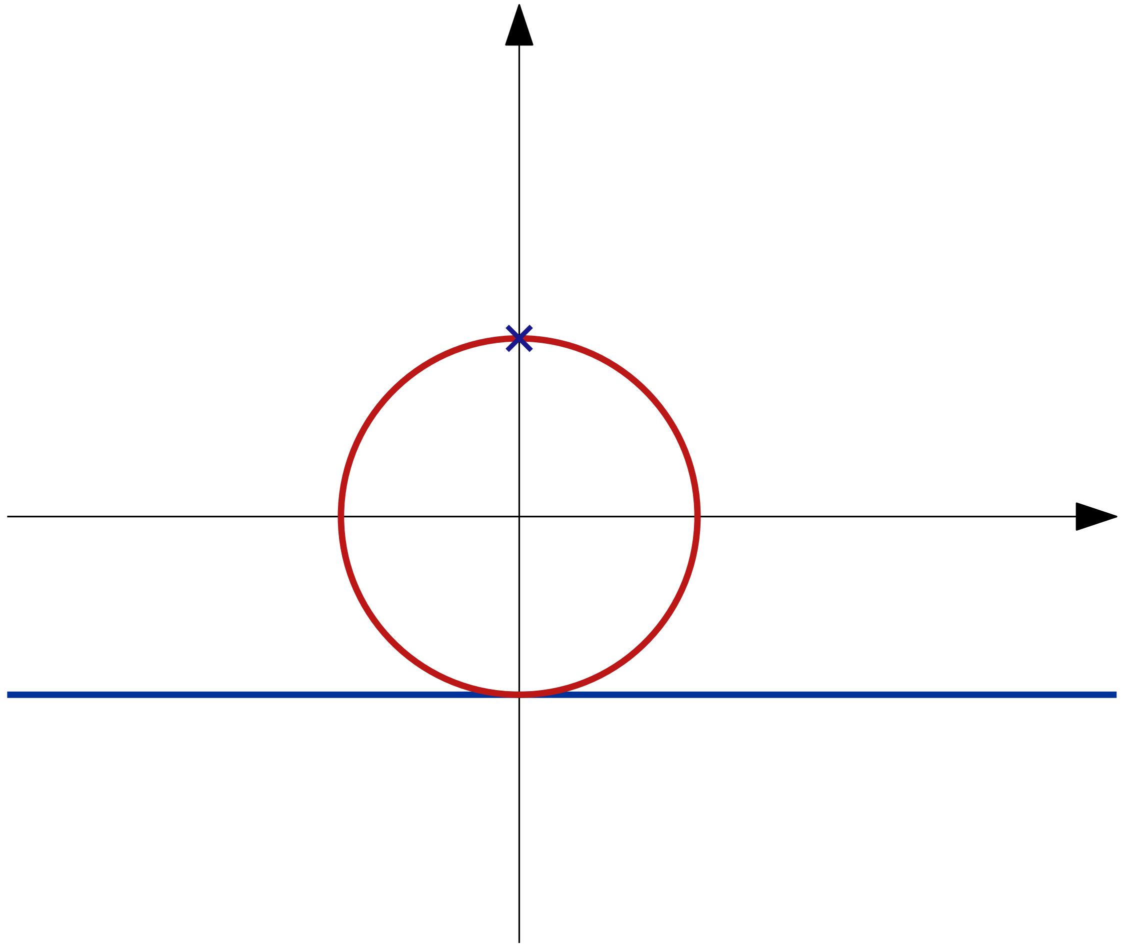

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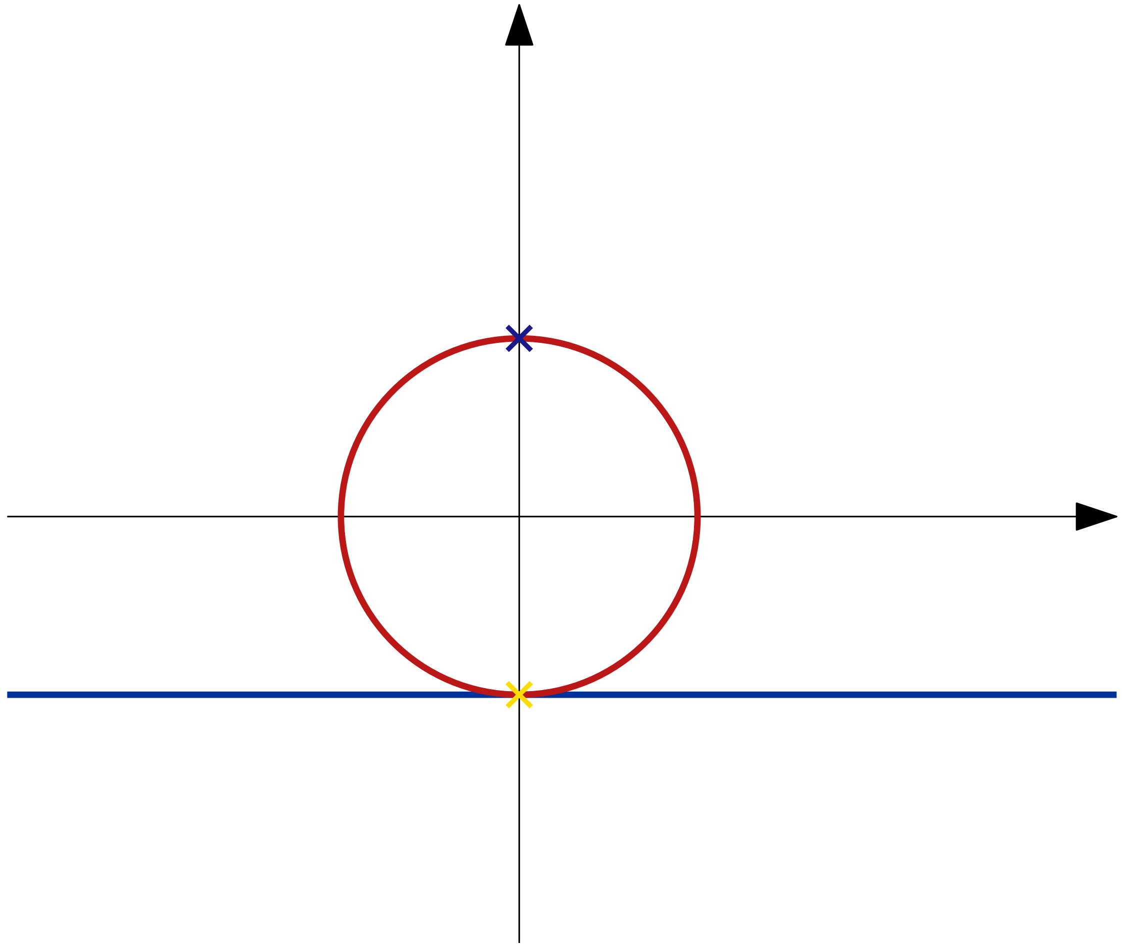

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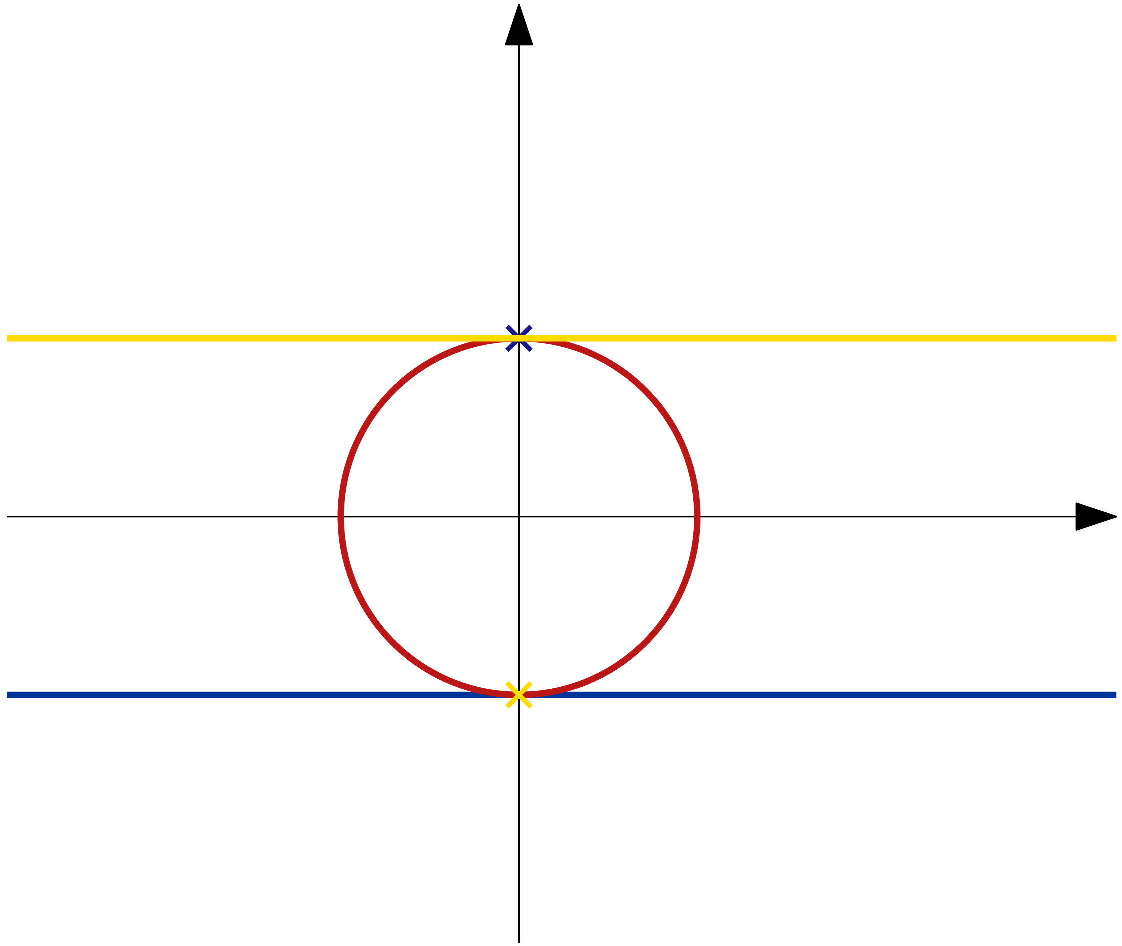

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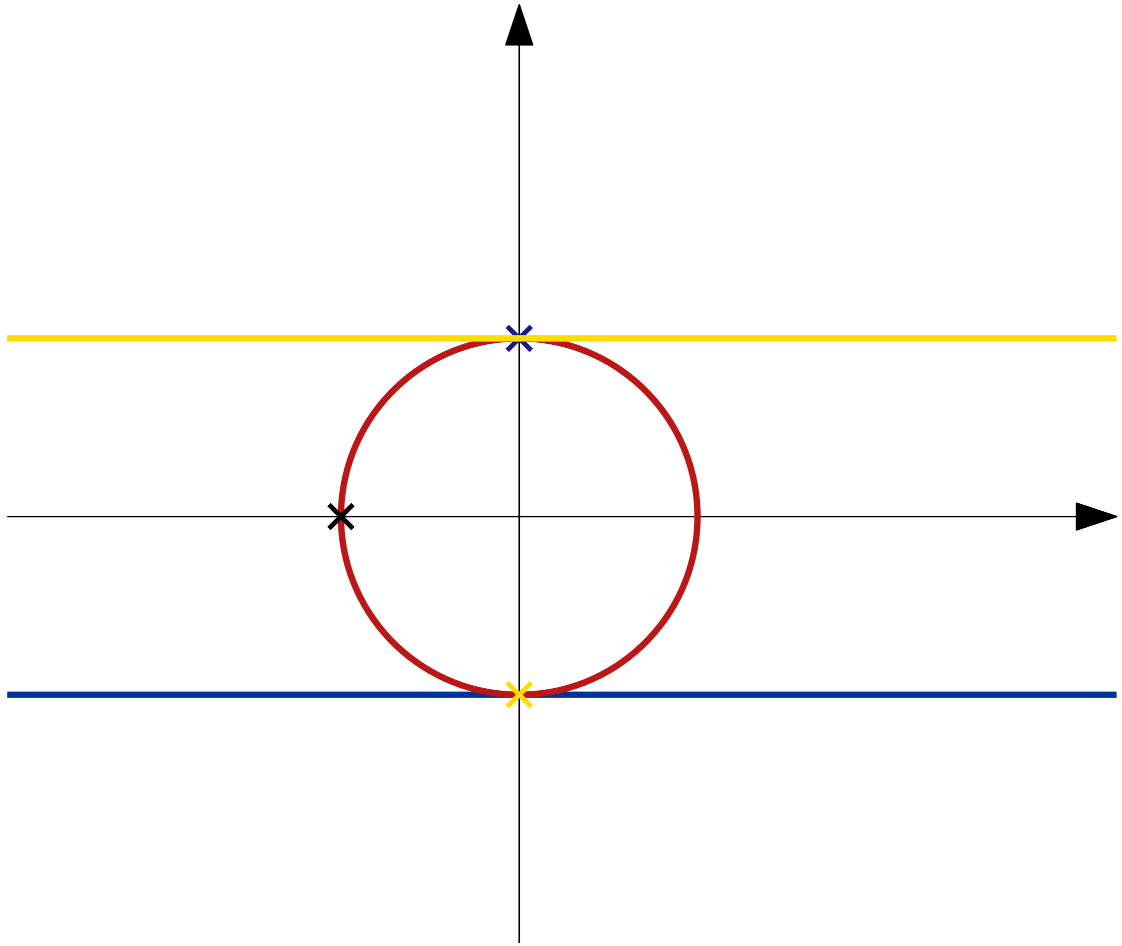

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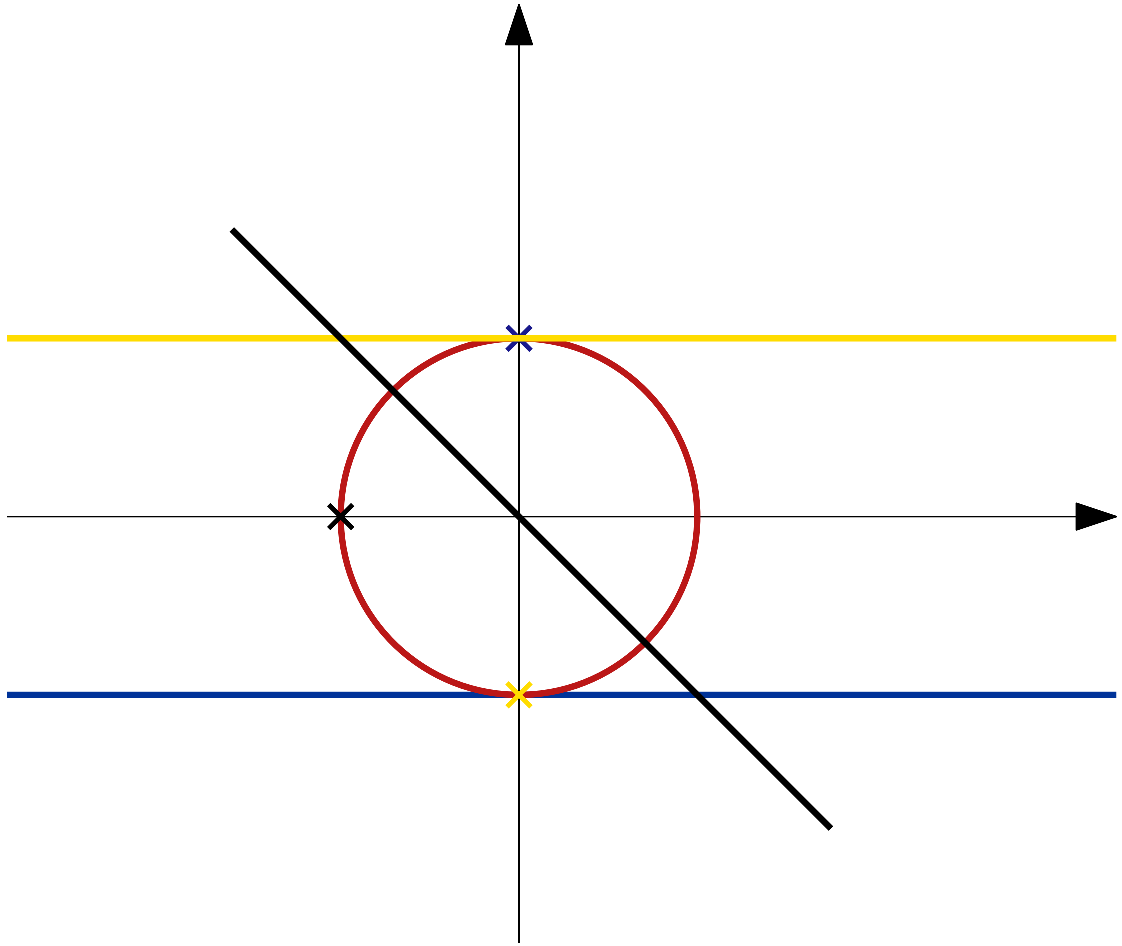

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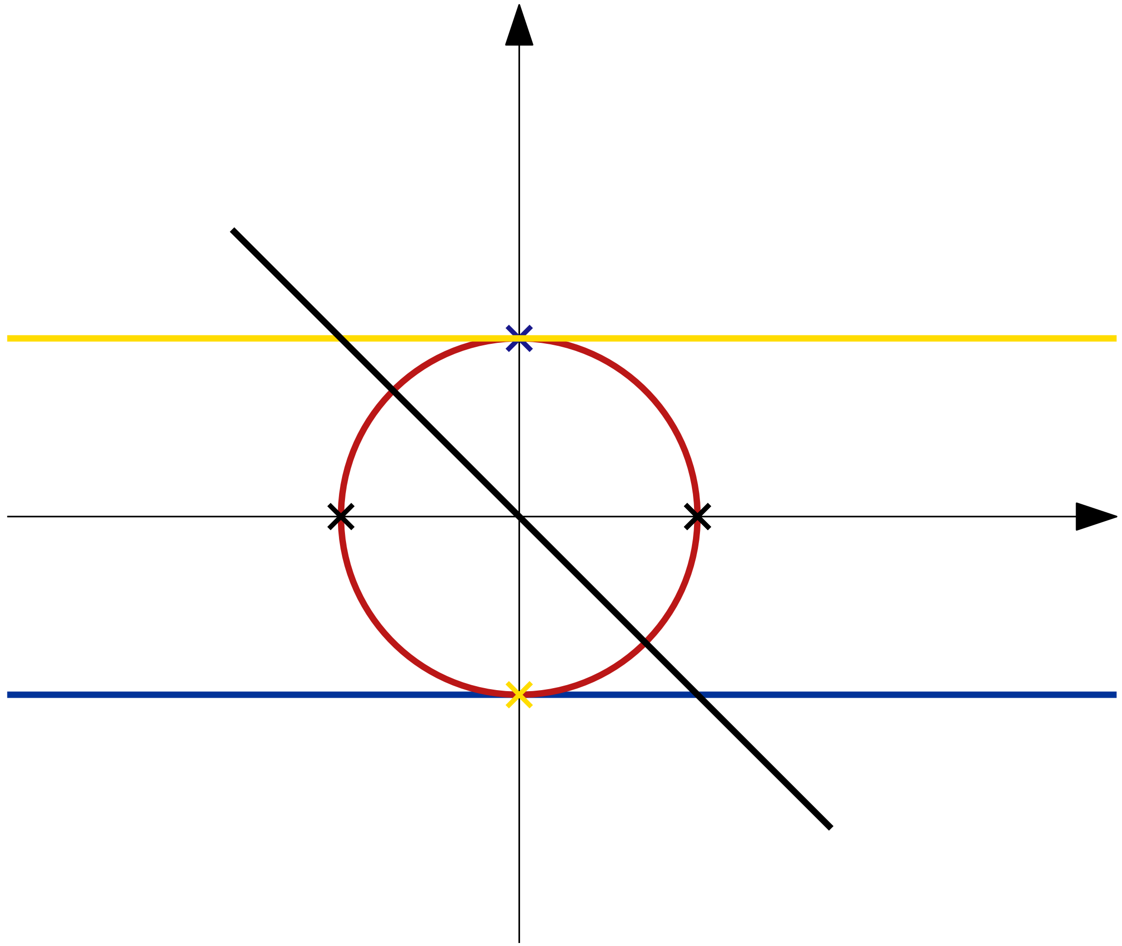

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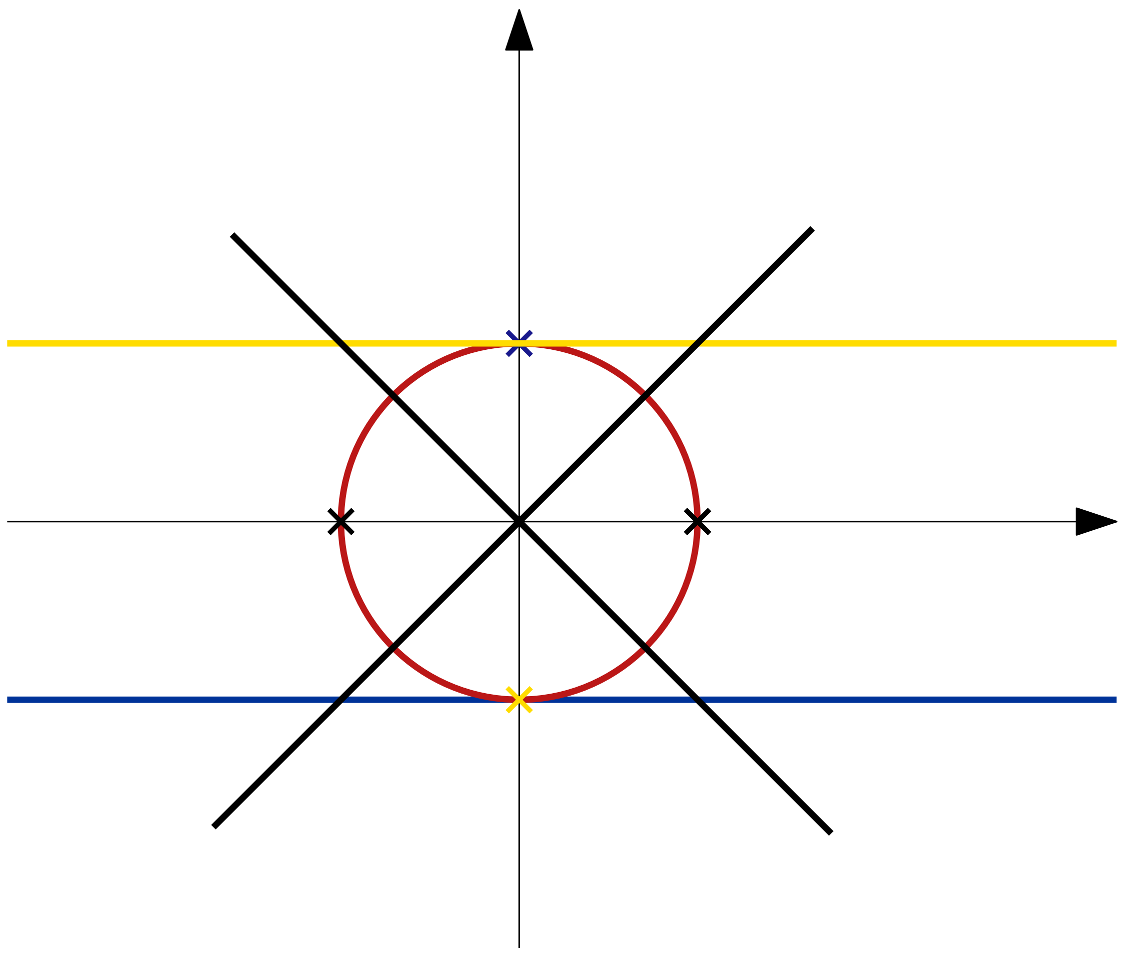

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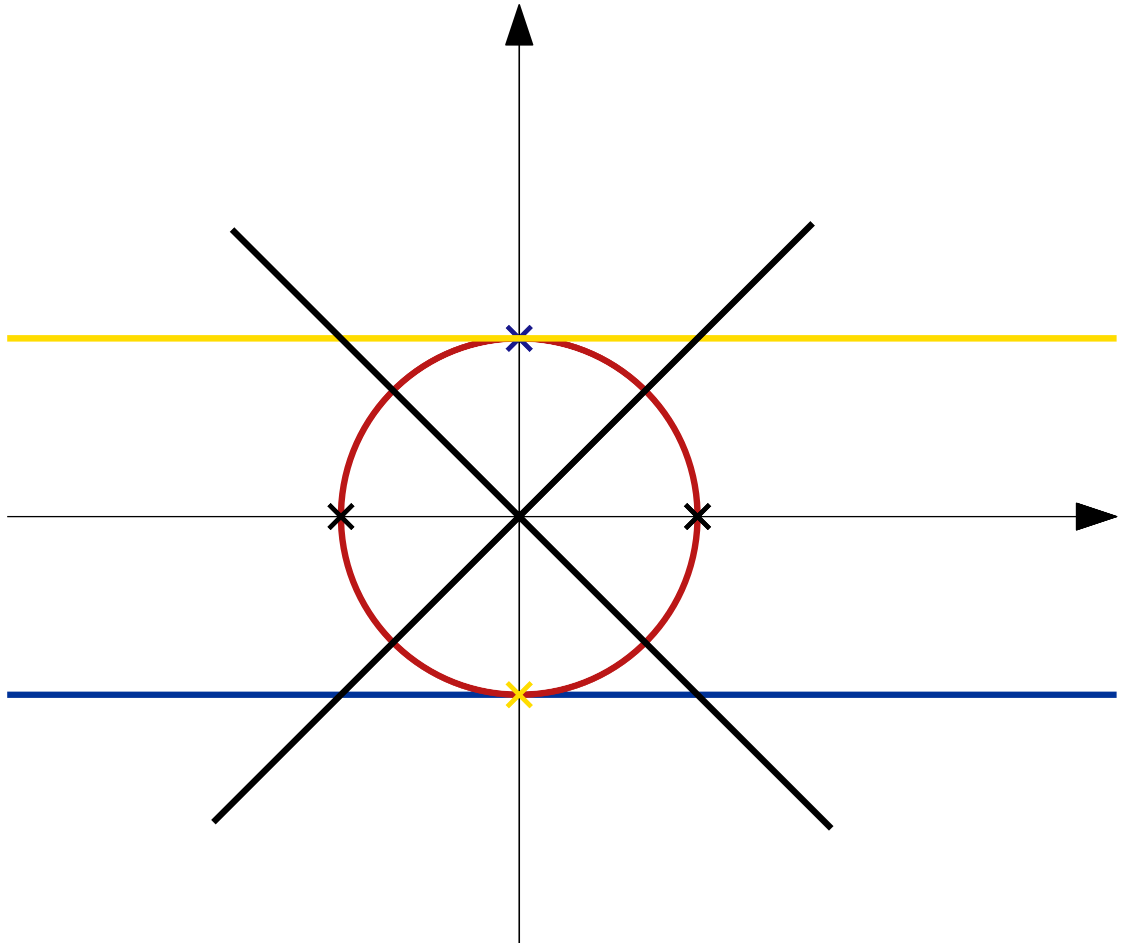

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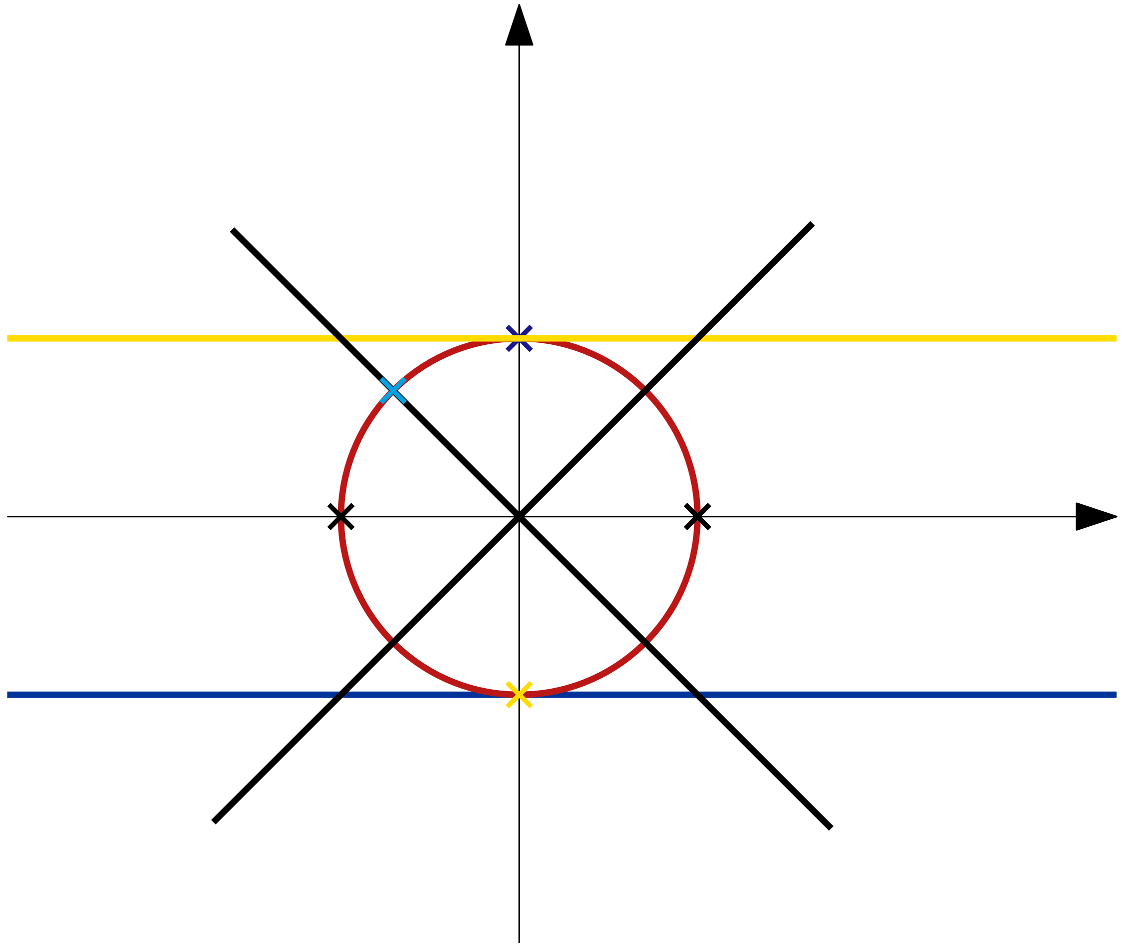

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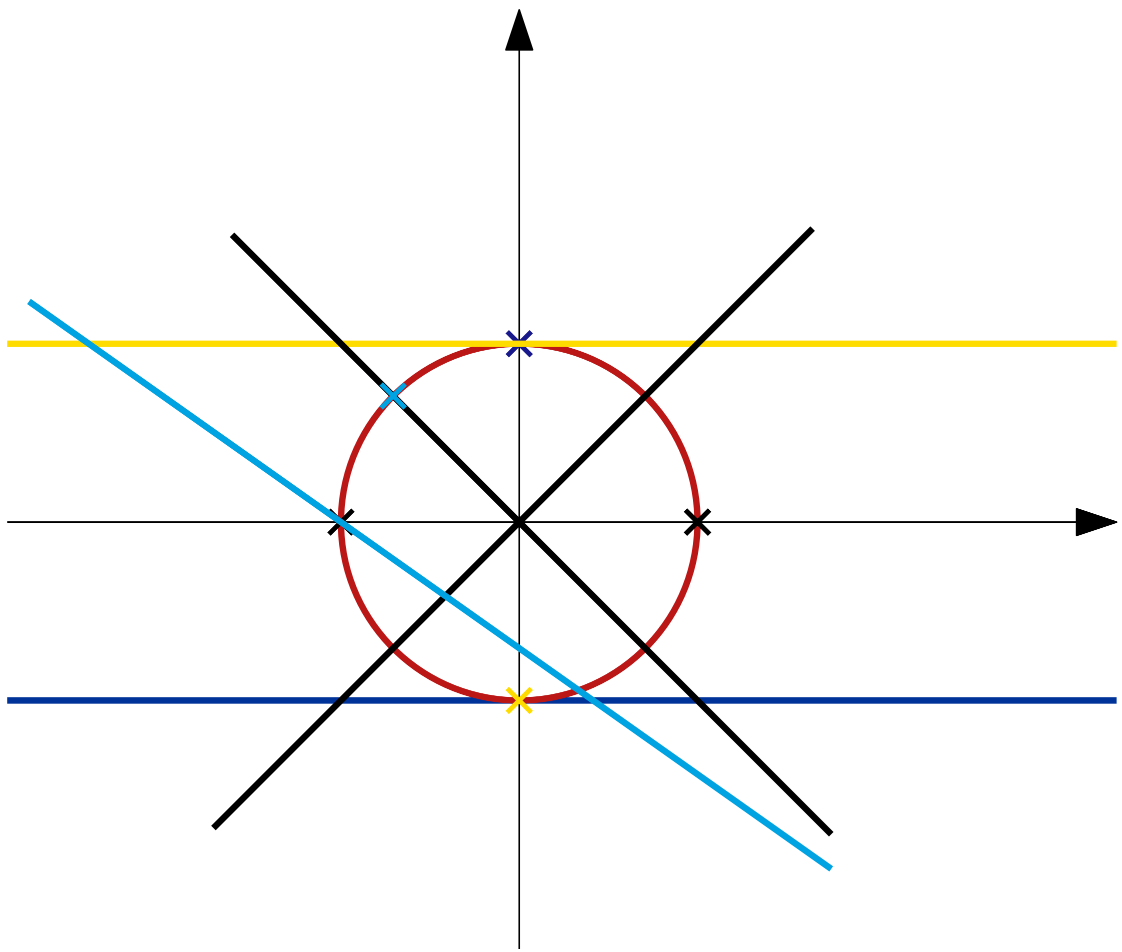

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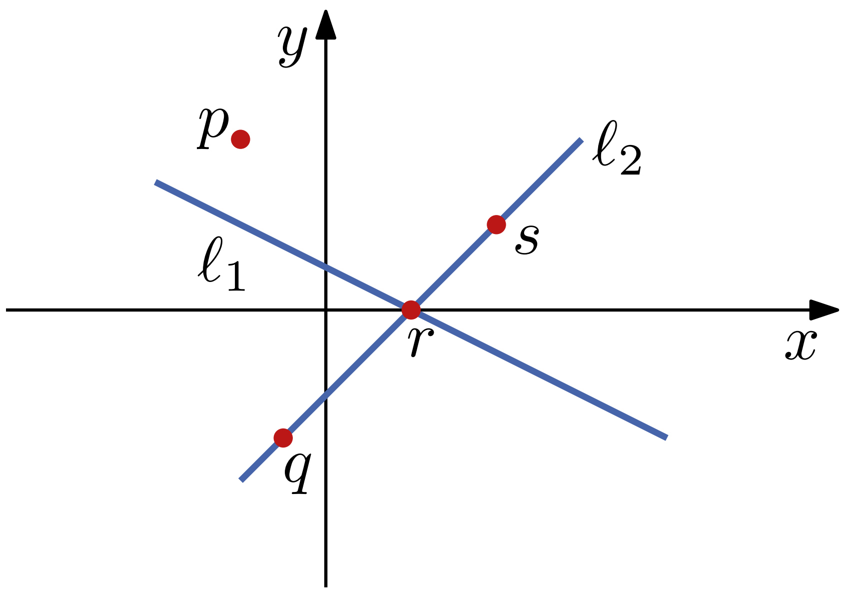

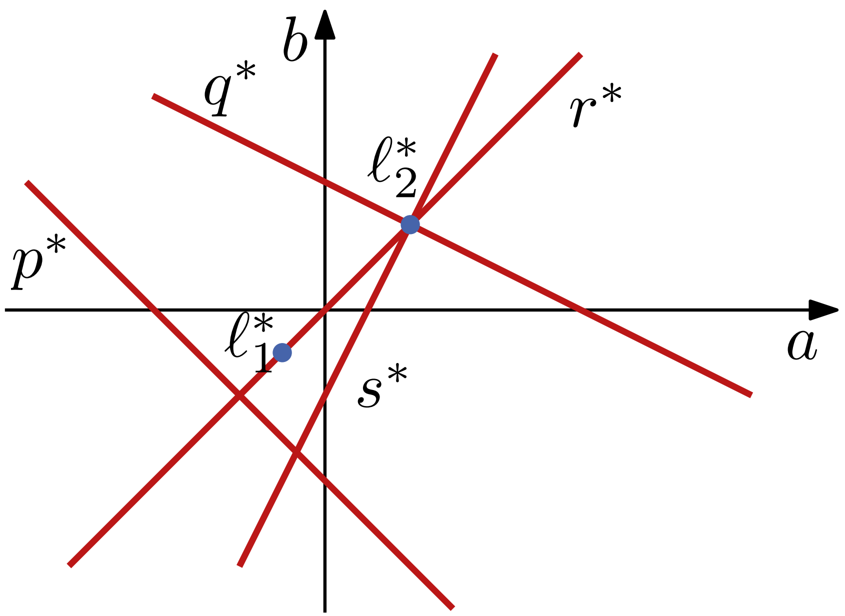

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Given: Set L consisting of n lines.

Find: Axis-aligned rectangle that containes all vertices of the arrangement $\mathcal{A}(L)$.

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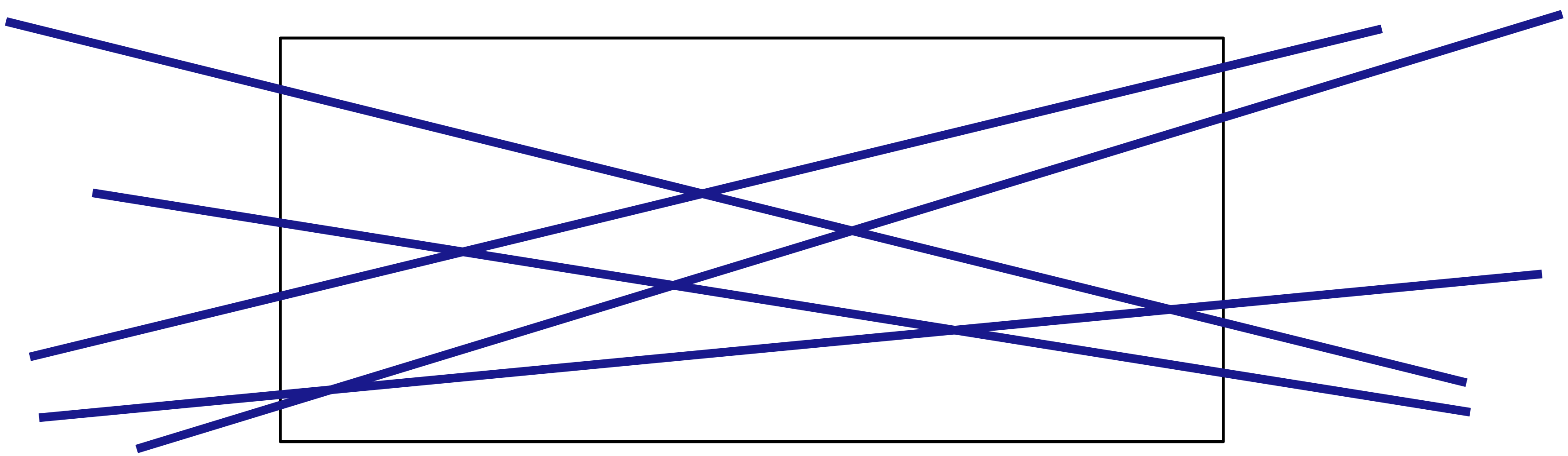
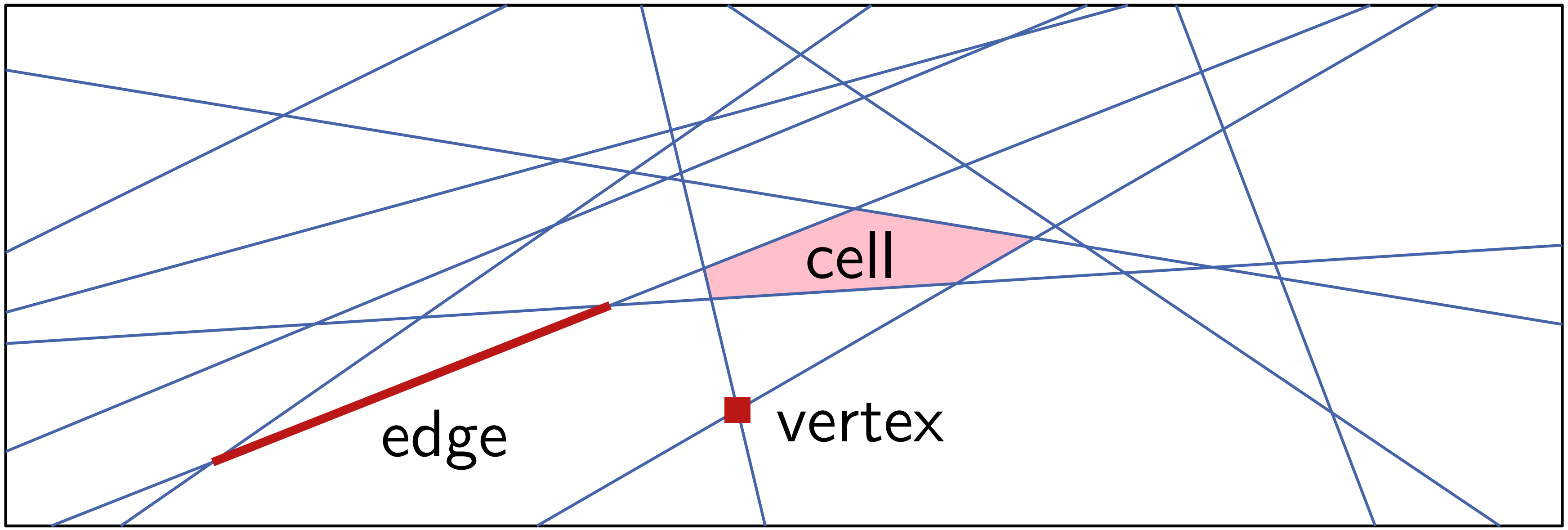


Line Arrangements



Def: A set L of lines defines a subdivision $\mathcal{A}(L)$ of the plane (the **line arrangement**) composed of vertices, edges, and cells (poss. unbounded).
$\mathcal{A}(L)$ is called **simple** if no three lines share a point and no two lines are parallel.

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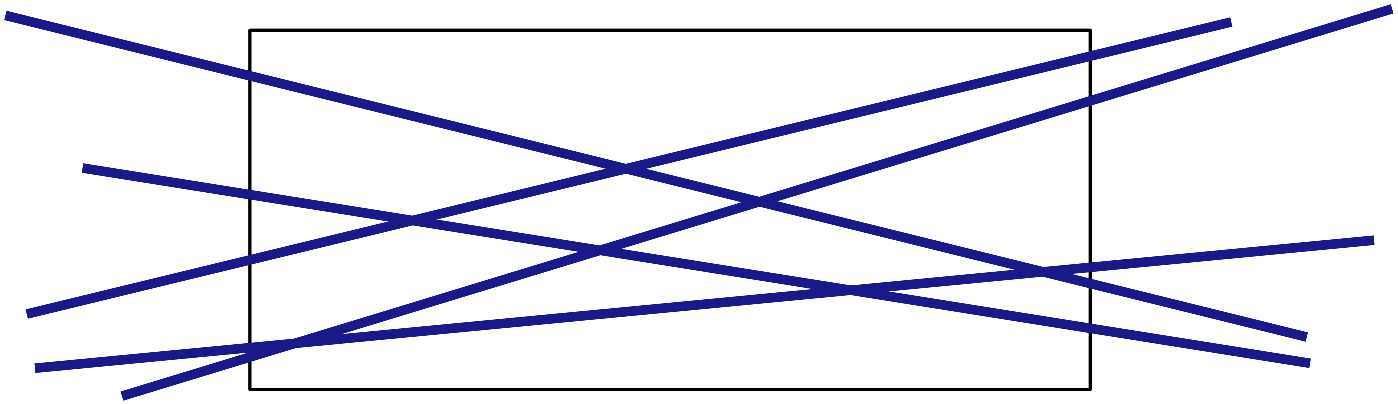
Gesucht: Axis-aligned rectangle that containes all vertices of the arrangement $\mathcal{A}(L)$.



Determine left side of rectangle (similar other side):

Sort all lines with respect to their slopes (in increasing order).

Determine the intersections of lines that are adjacent in that order.

Left side of the rectangle must lie to the left of the leftmost intersection point.

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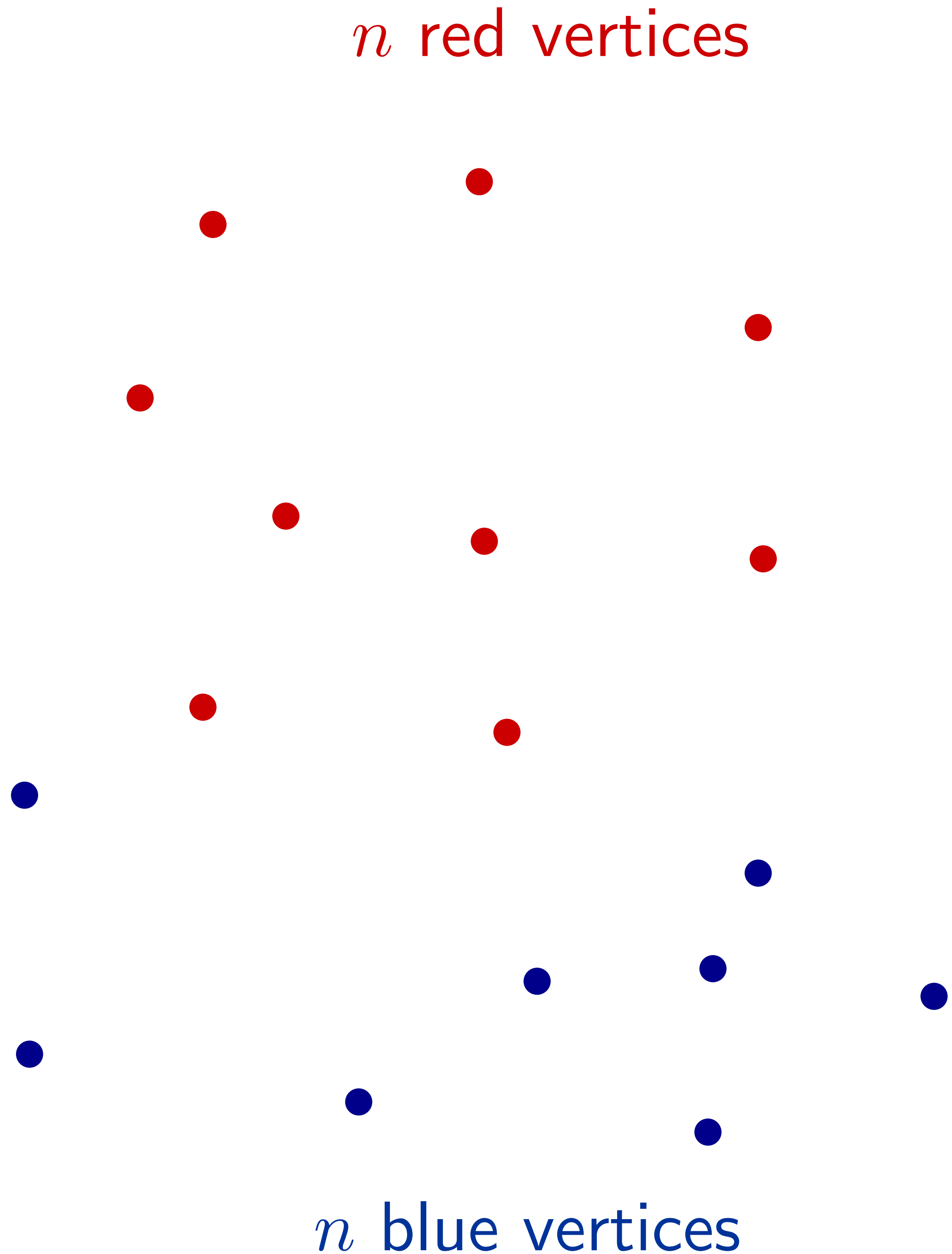

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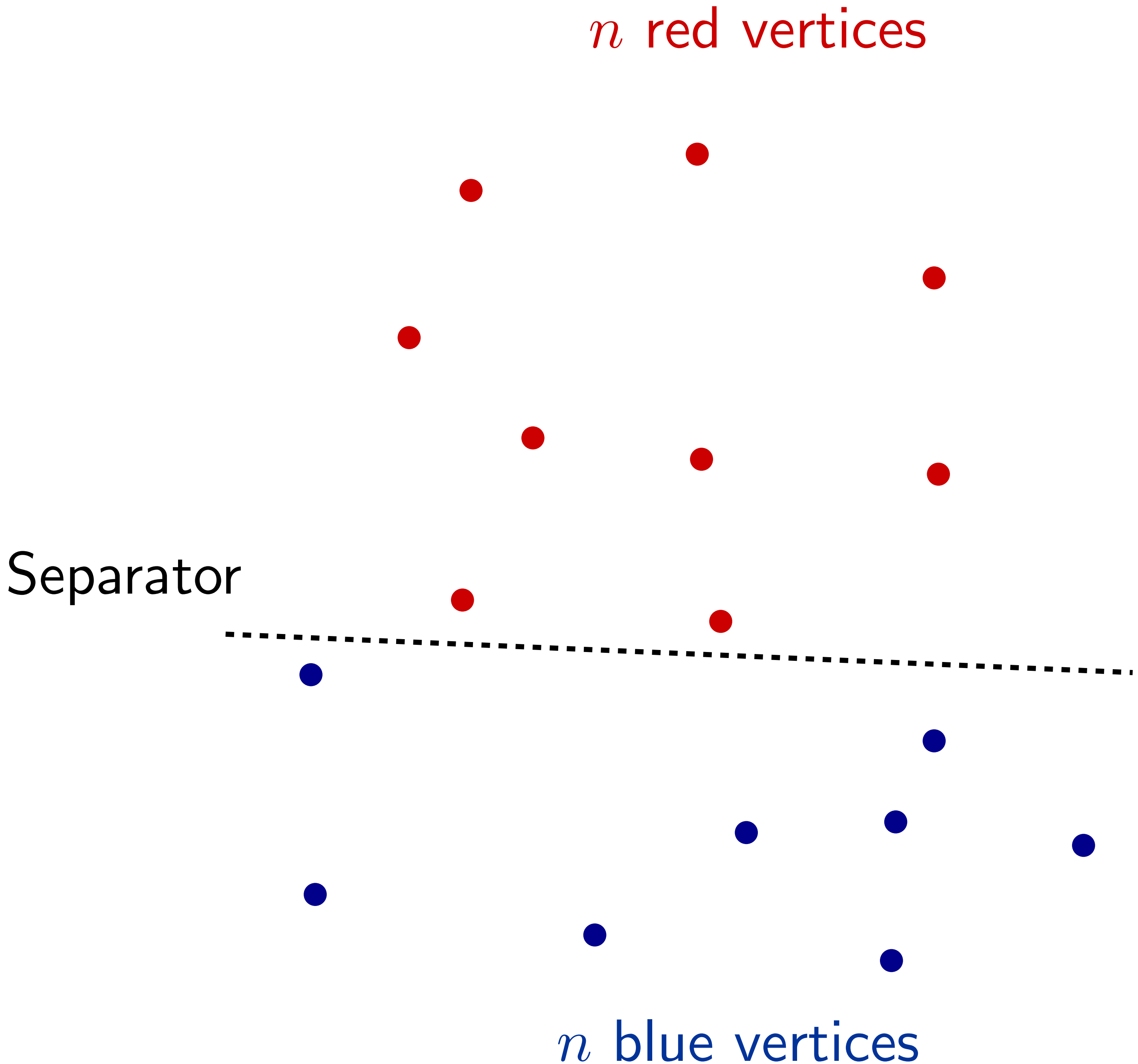

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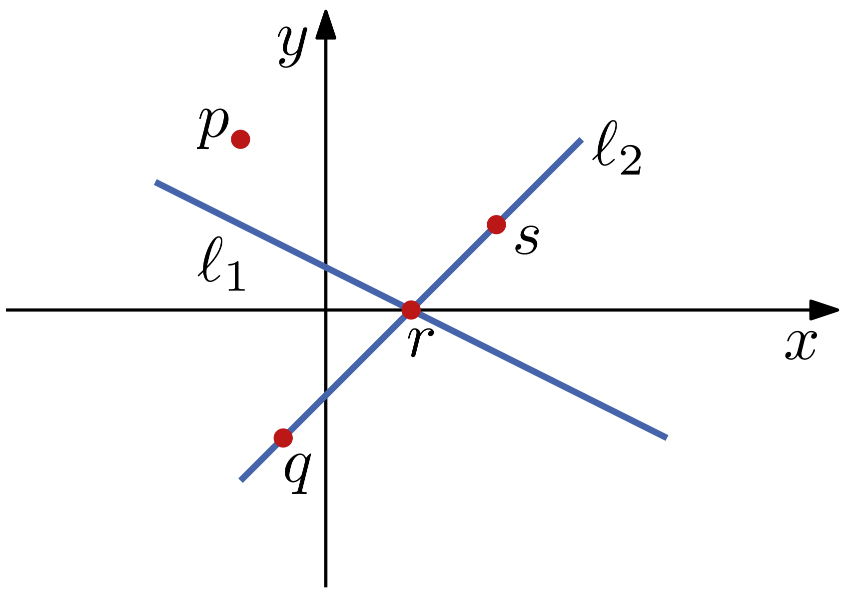

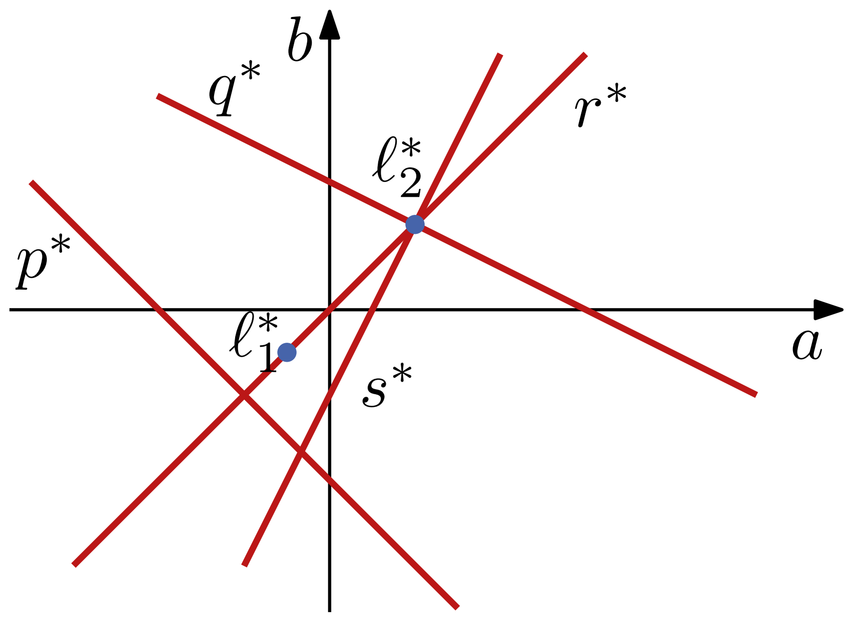

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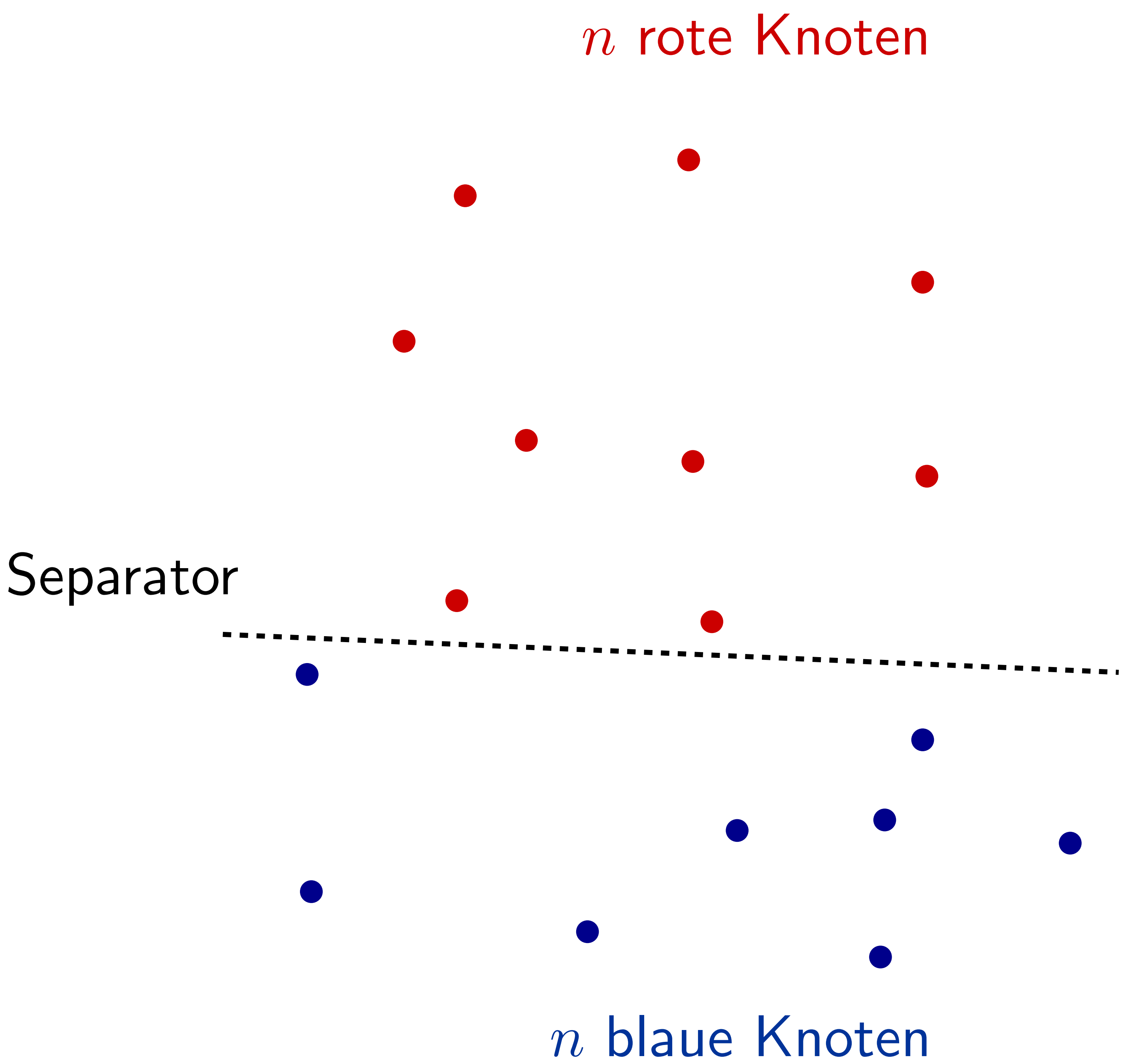

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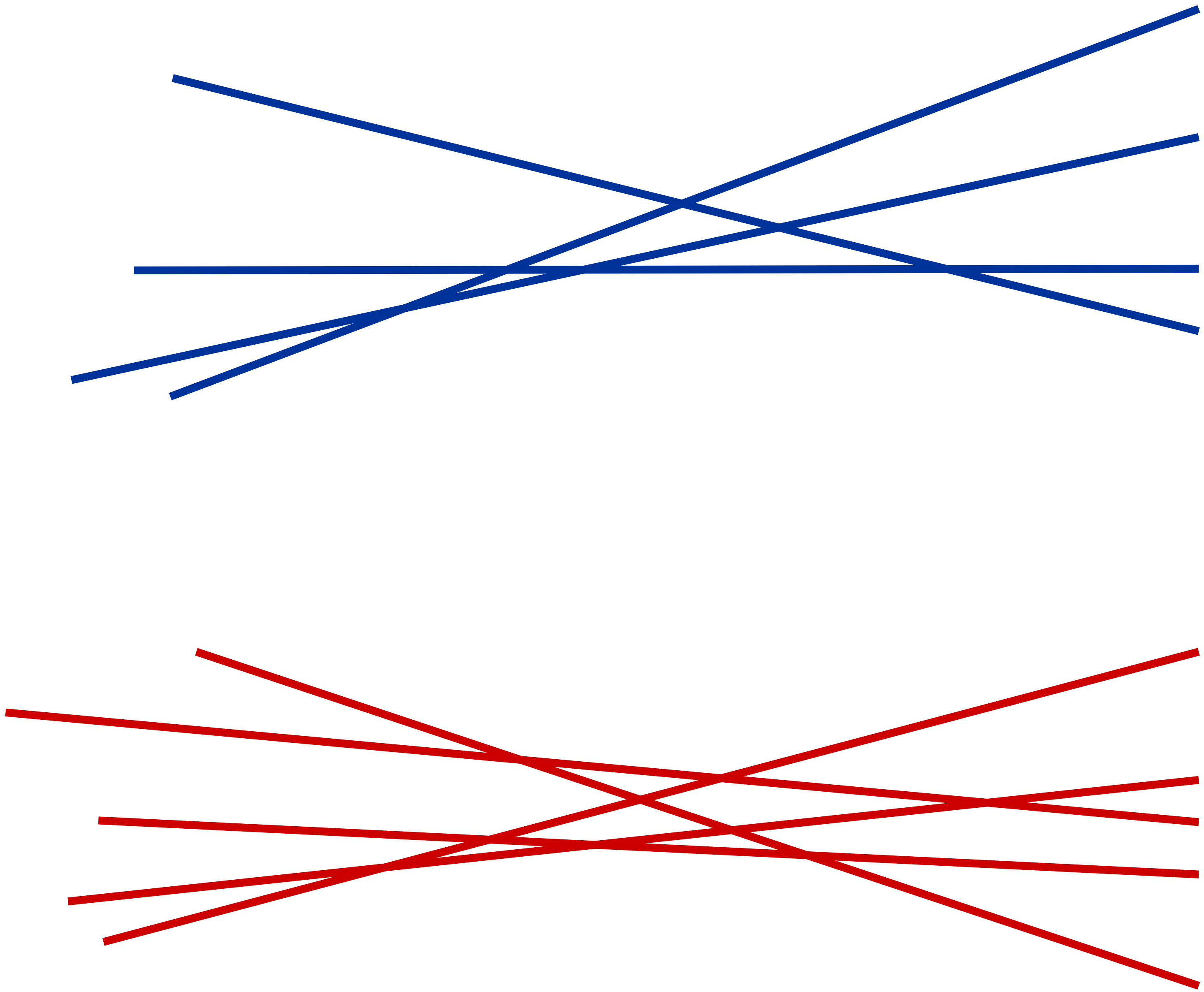

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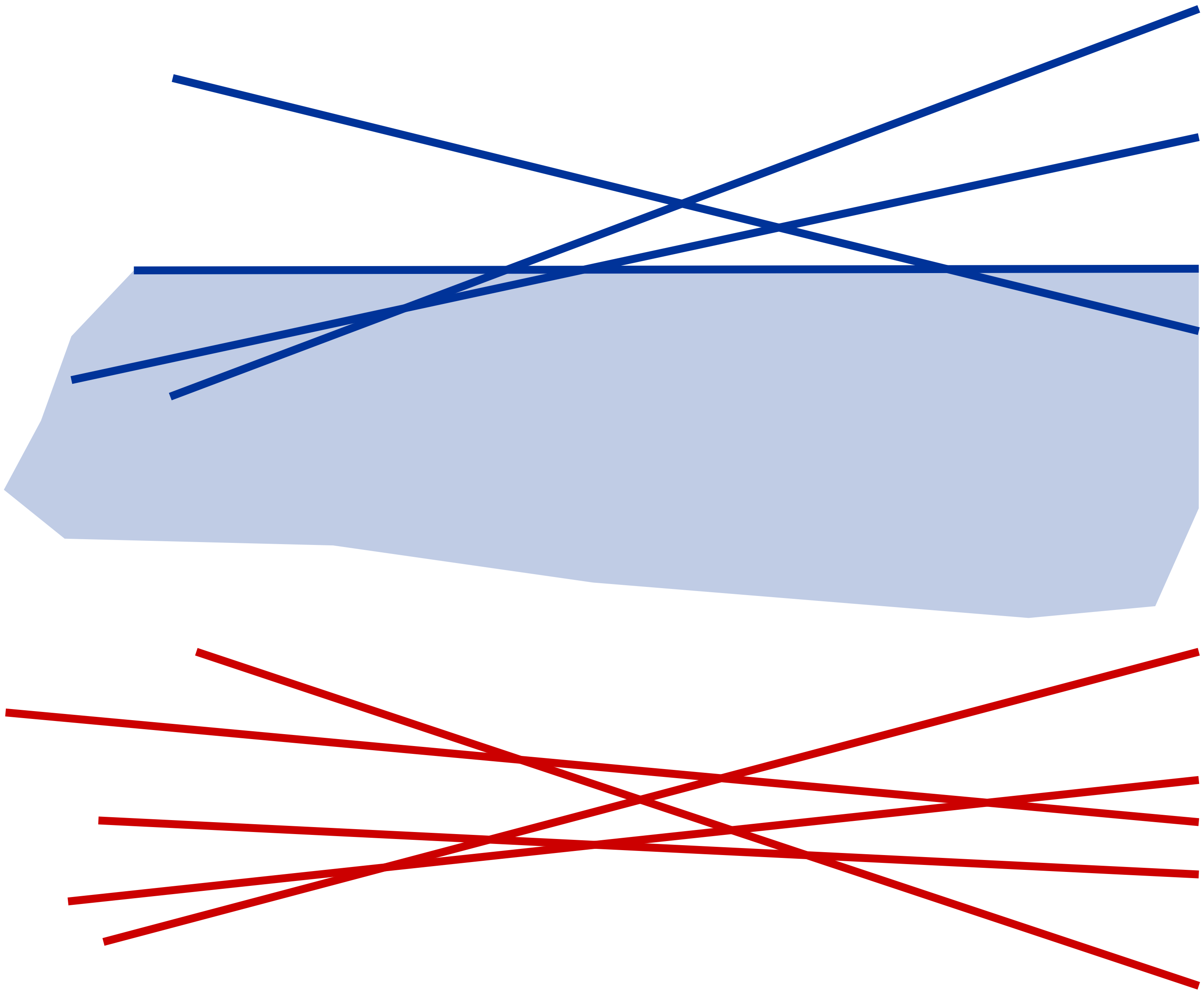

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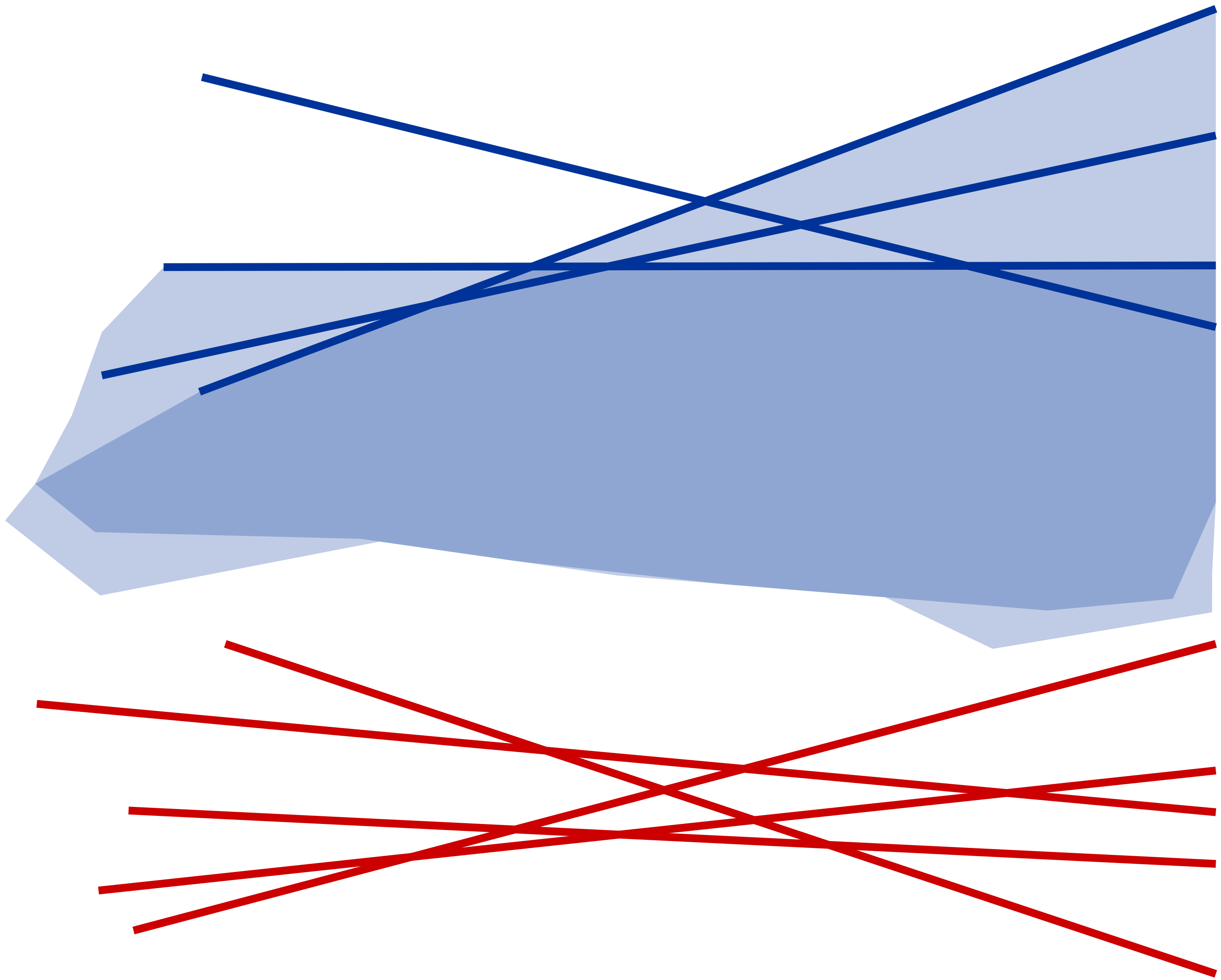

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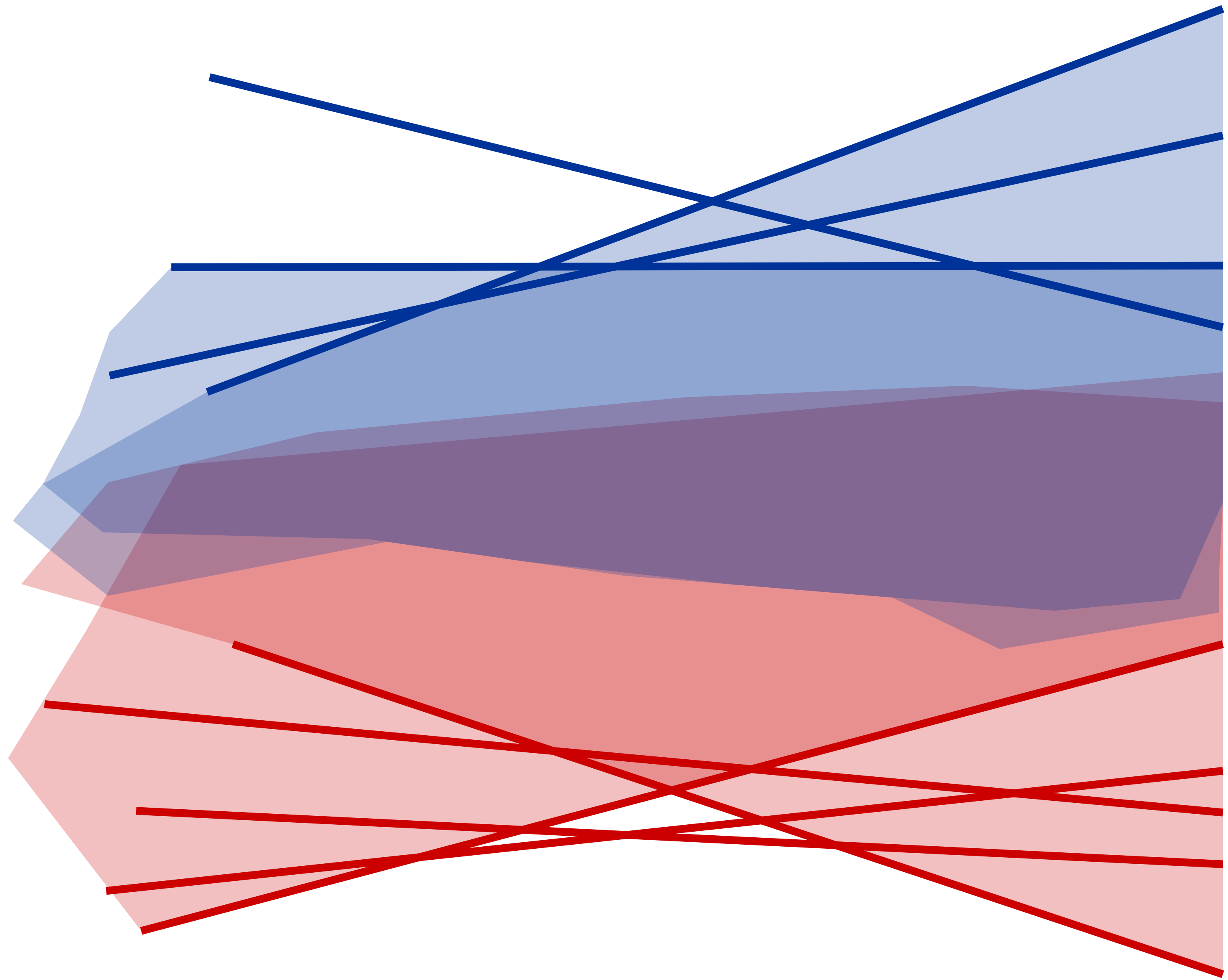

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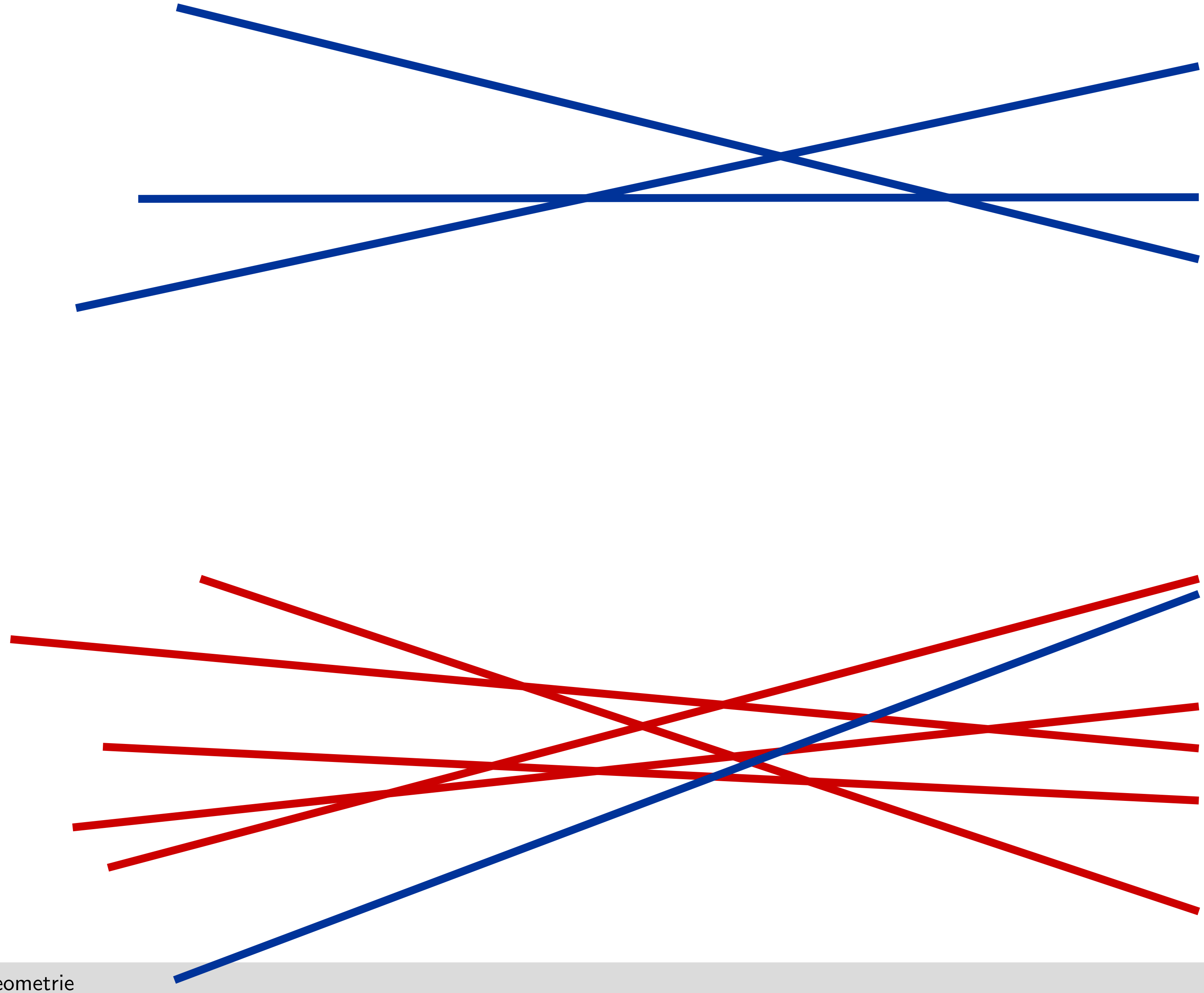

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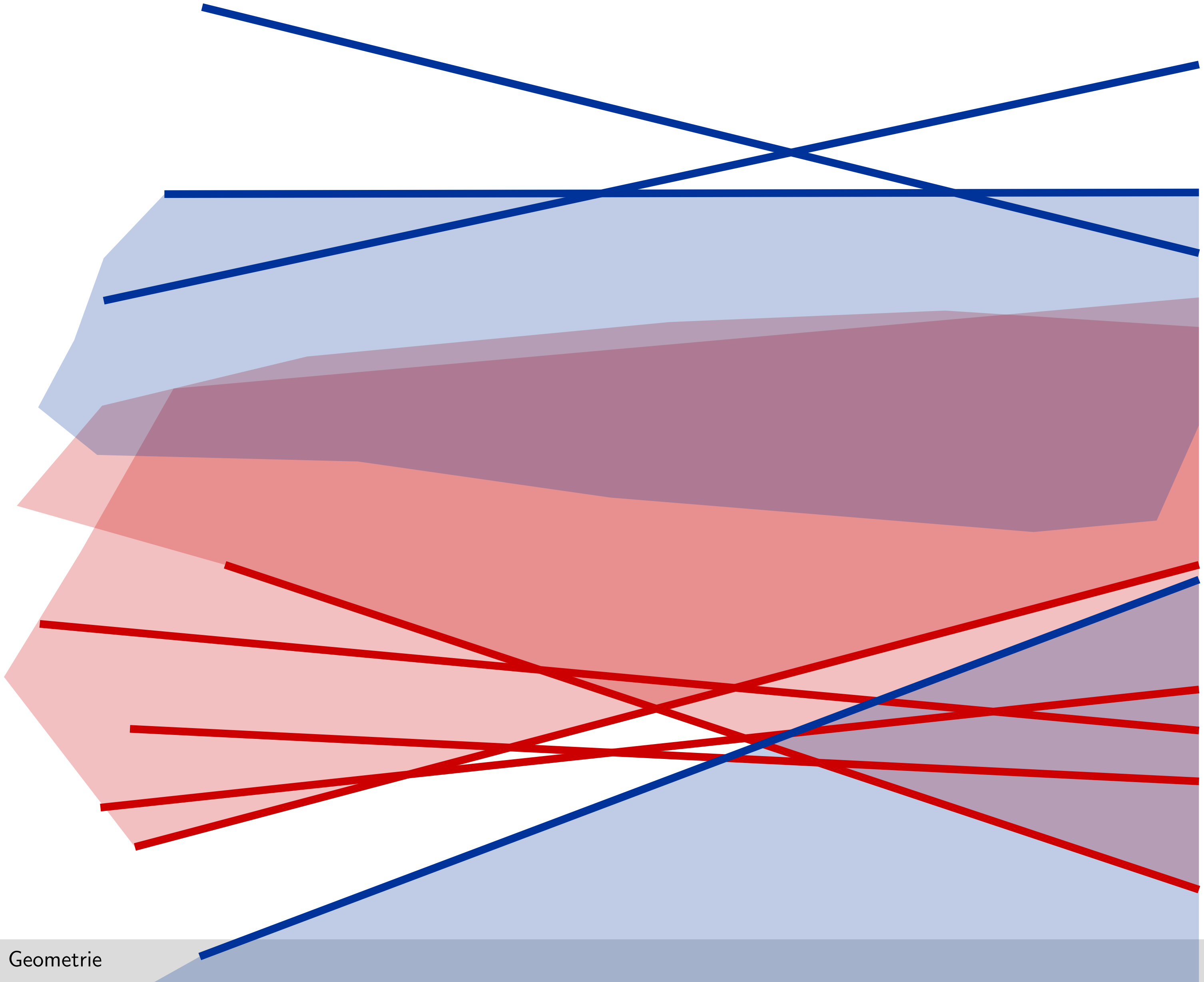

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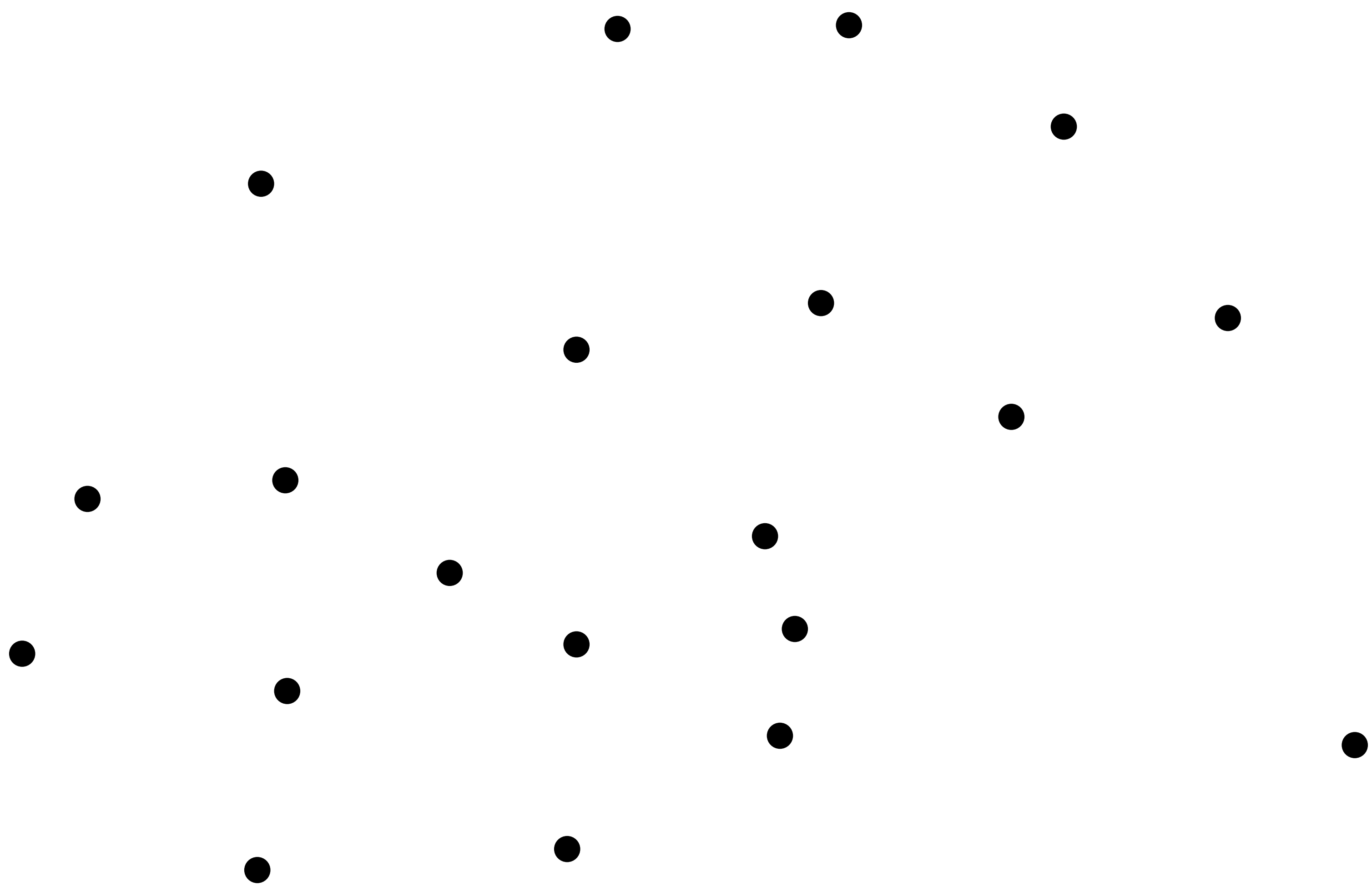
Find line that goes through the most points in S, [in $O(n^2)$].



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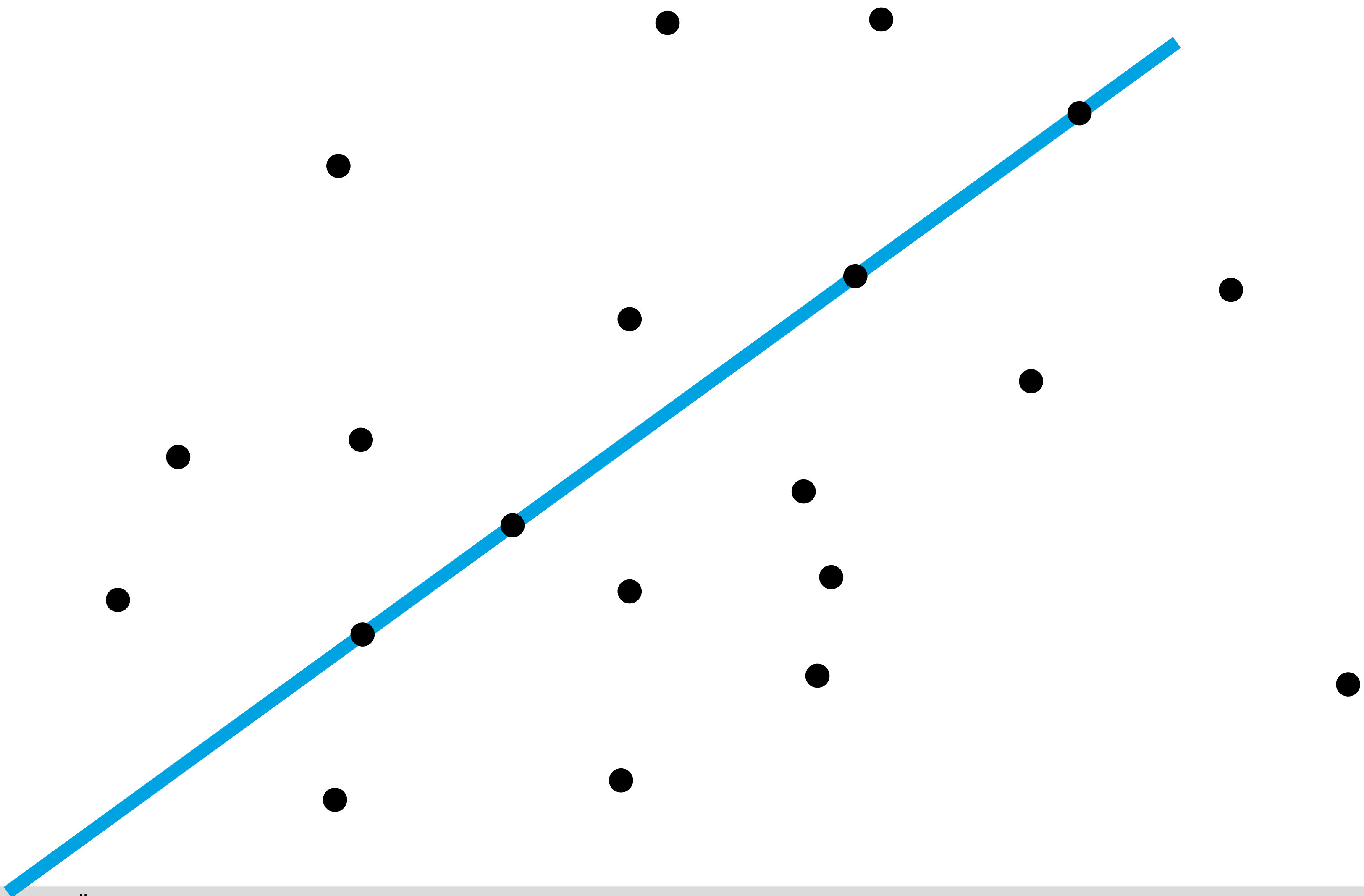

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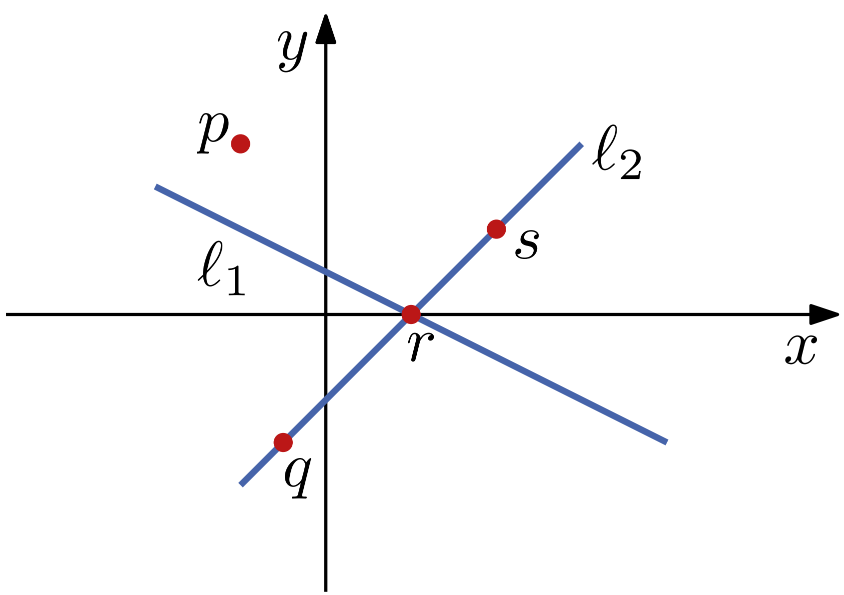

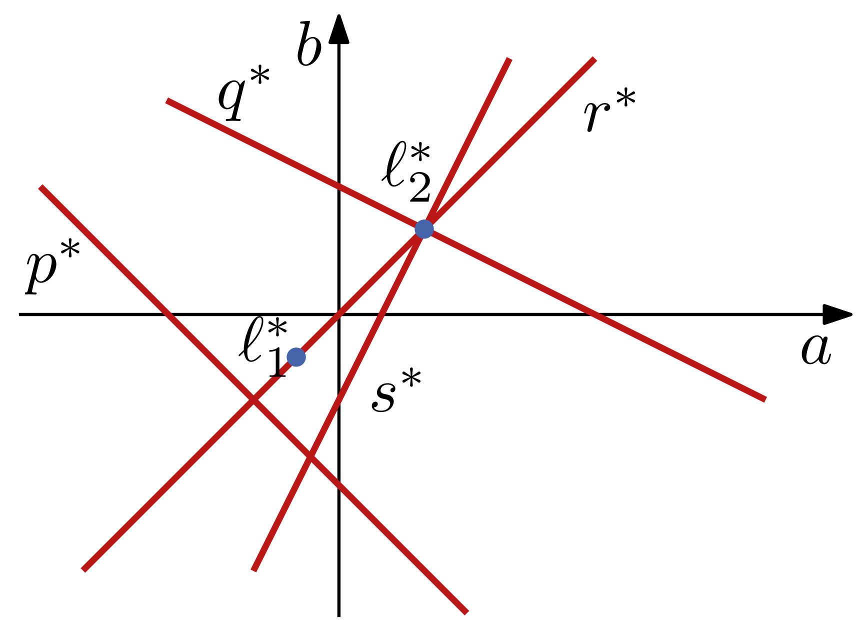

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Karlsruhe Institute of Technology

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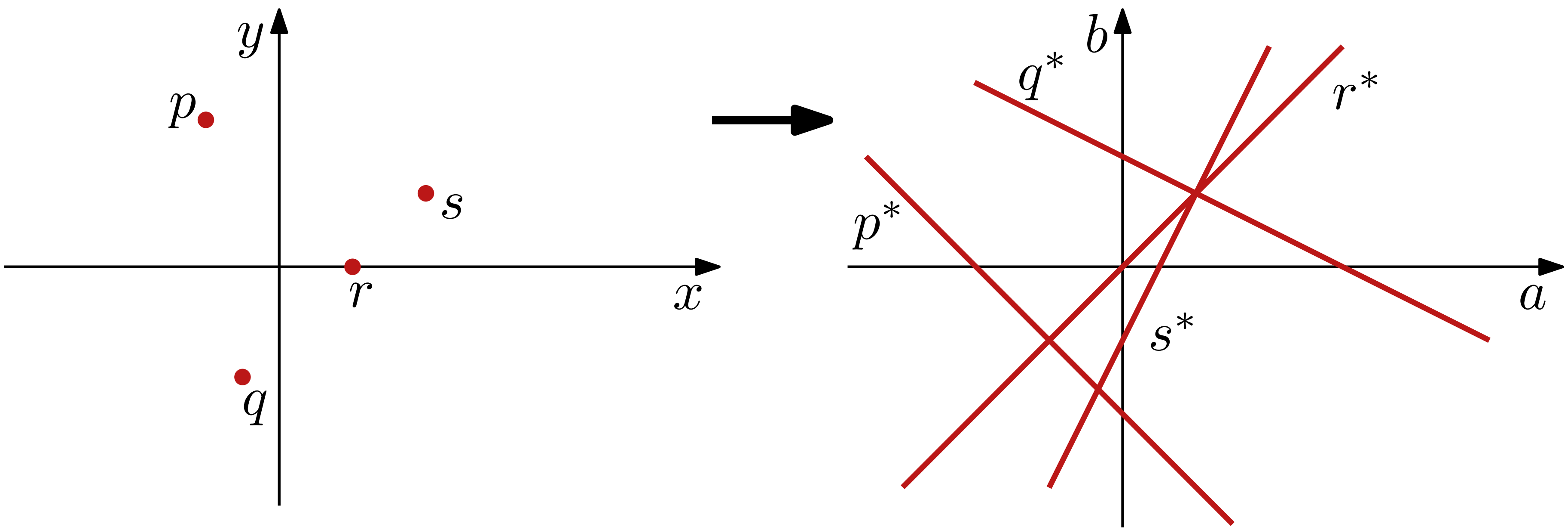

1. Transform all points into lines.
2. Compute arrangement.
3. Determine vertex with highest degree.

Reason: Co-linear points in the primal space are lines in the dual space that intersect in point.